# Transport phenomena in NJL-type models

## Shear Viscosity

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# Motivation

## From experiments to theory: ratio $\eta/s$

elliptic flow $v_2$ $\oplus$ hydrodynamic simulations $\oplus$ CGC/Glauber

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measurement/extraction of a **constant** ratio $\eta/s$

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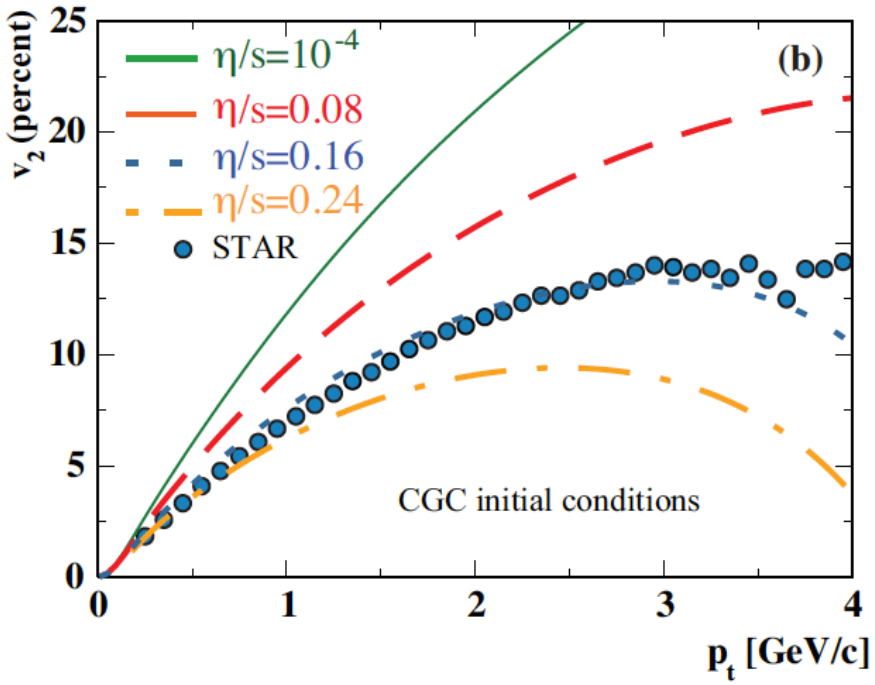
The QGP is known to be

- almost-perfect fluid,
- strongly interacting.

[Facilities at RHIC]

[Luzum, Romatschke, PRC (2008) **78** 034915]

[Snellings, arXiv:1102.3010]



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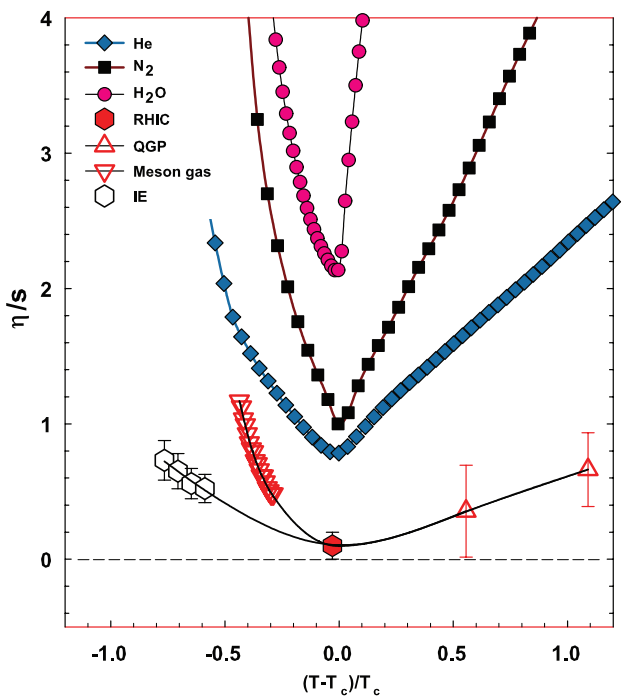

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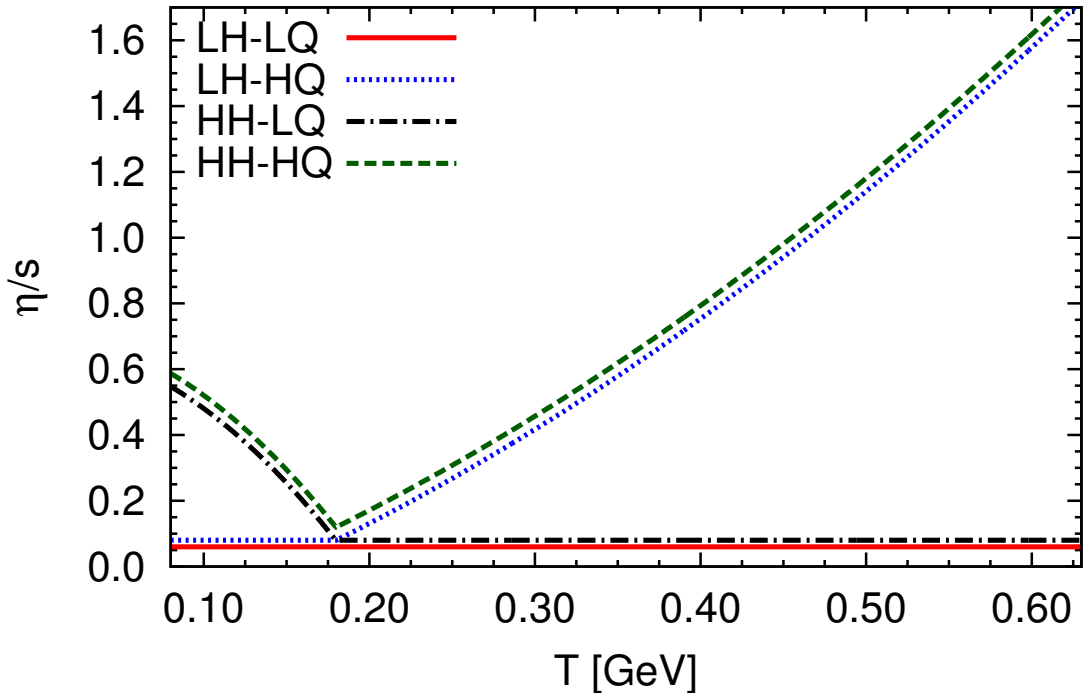
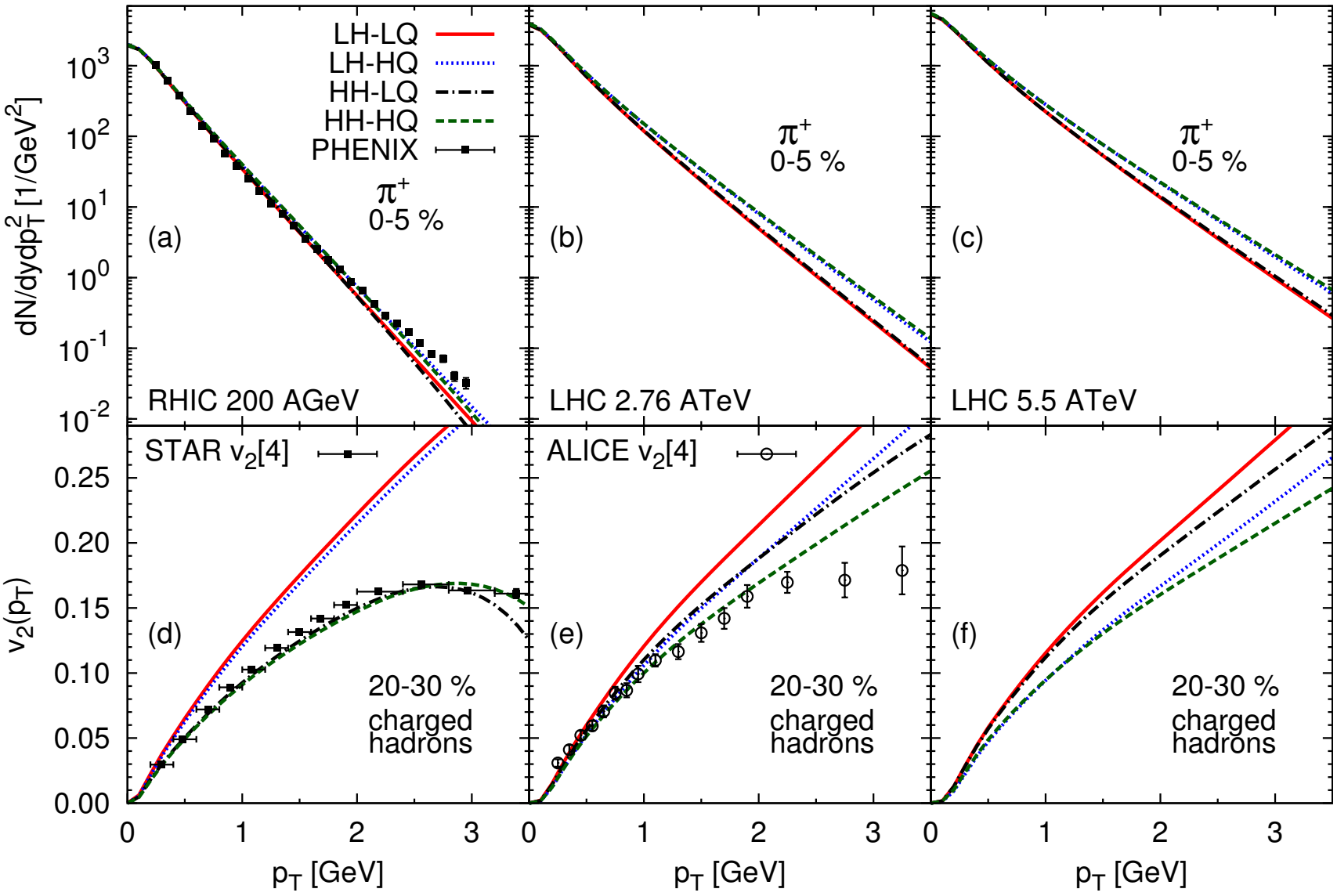
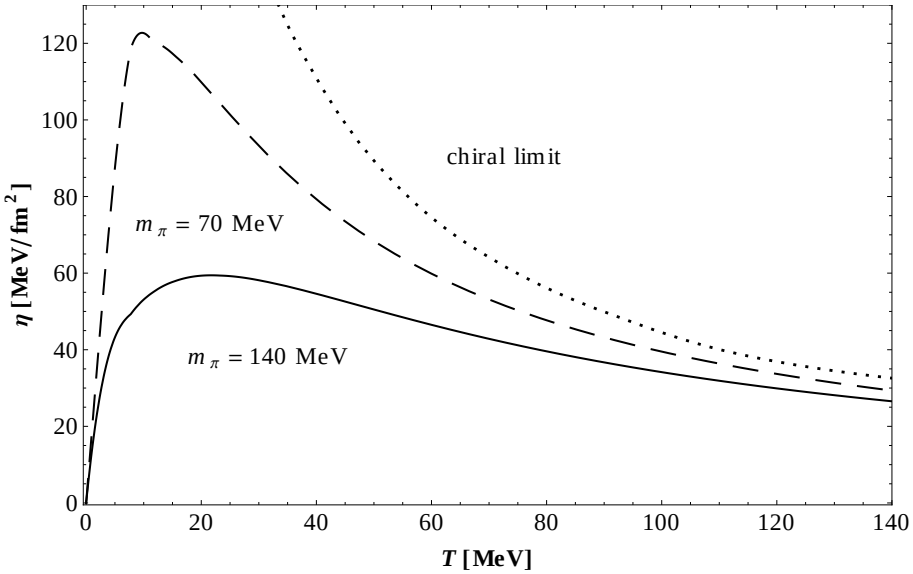


# Motivation II

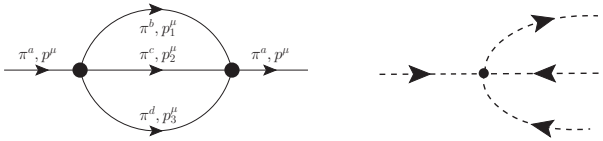



[Niemi, Denicol, Molnár, Rischke, PRL (2011) **106** 212302]

# Shear viscosity of pions in low-temperature $\chi$PT – Results




- $2^{\text{nd}}$ order chiral Lagrangian in $\sigma$ gauge
- includes $2 \to 2$ pion scattering

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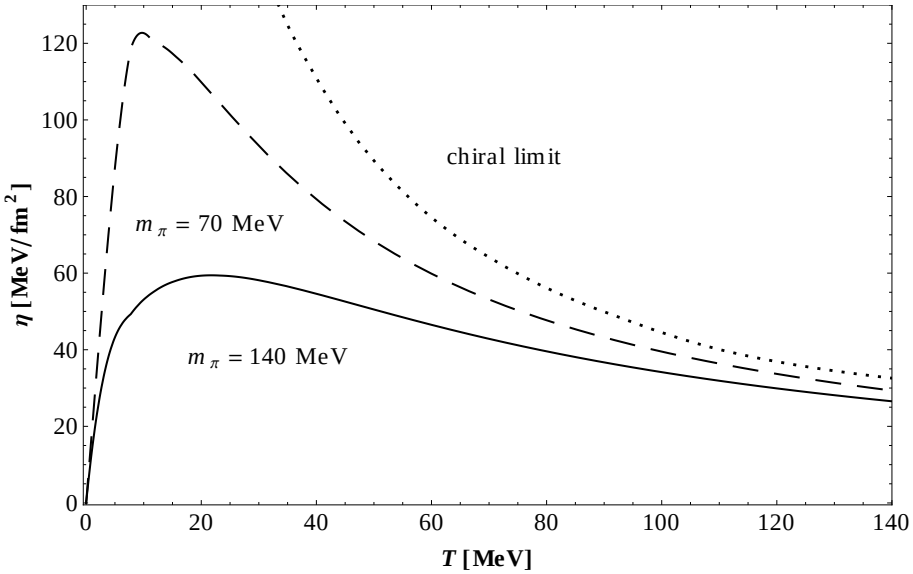


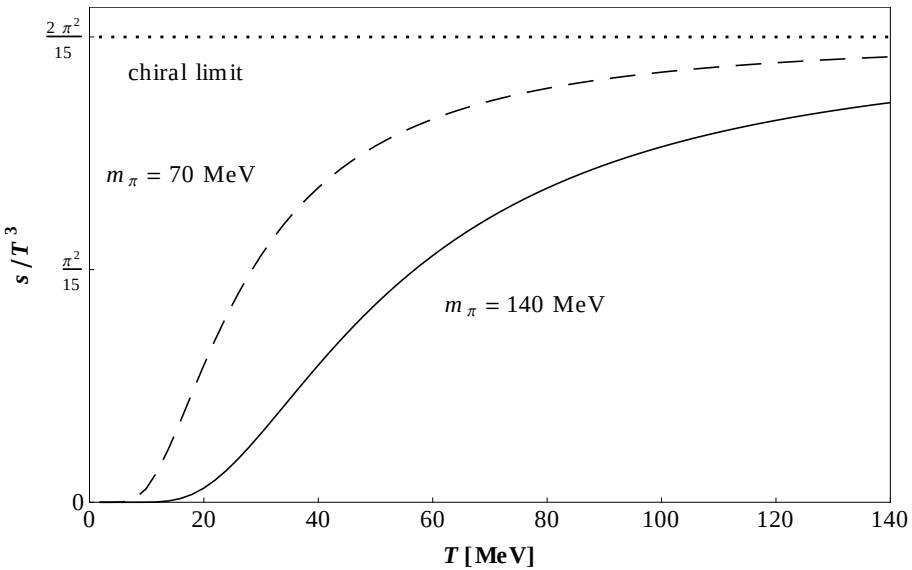


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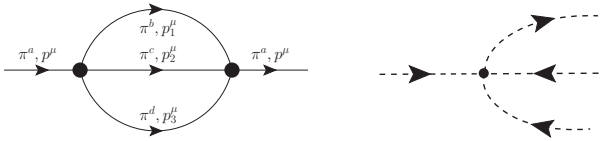

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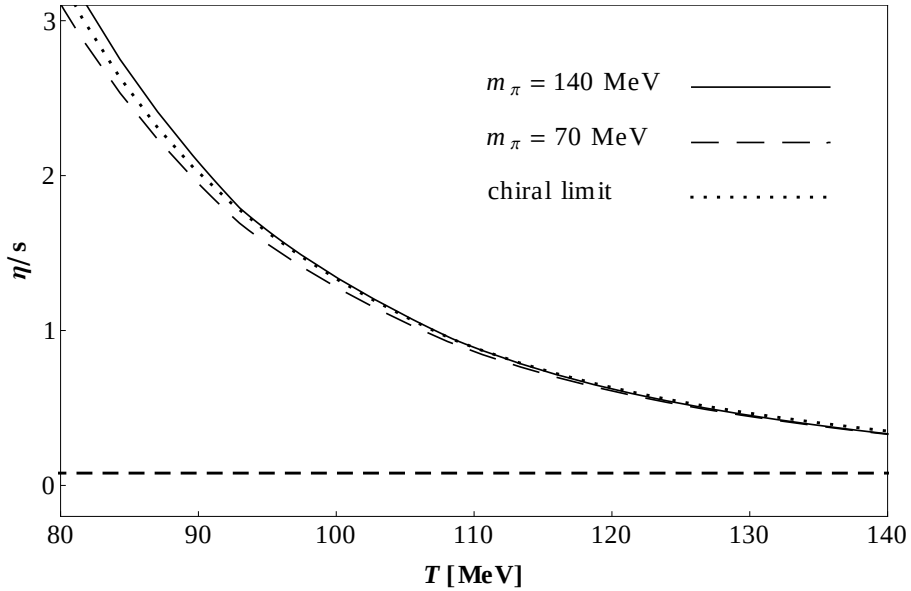


$$\eta/s(T \lesssim 140 \text{ MeV}) > 1/4\pi$$

# NJL model in short

$$\mathcal{L} = \bar{\psi}(\mathrm{i}\partial\!\!\!/ - m)\psi + \frac{G}{2}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}\mathrm{i}\gamma_5\vec{\tau}\psi)^2\right]$$

- imitate the (approximate) symmetries of QCD: $\mathrm{SU}(3)_\mathsf{c} \times \mathrm{SU}(2)_\mathsf{L} \times \mathrm{SU}(2)_\mathsf{R} \times \mathrm{U}(1)_\mathsf{V}$

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$\mathcal{O}(1)$ :

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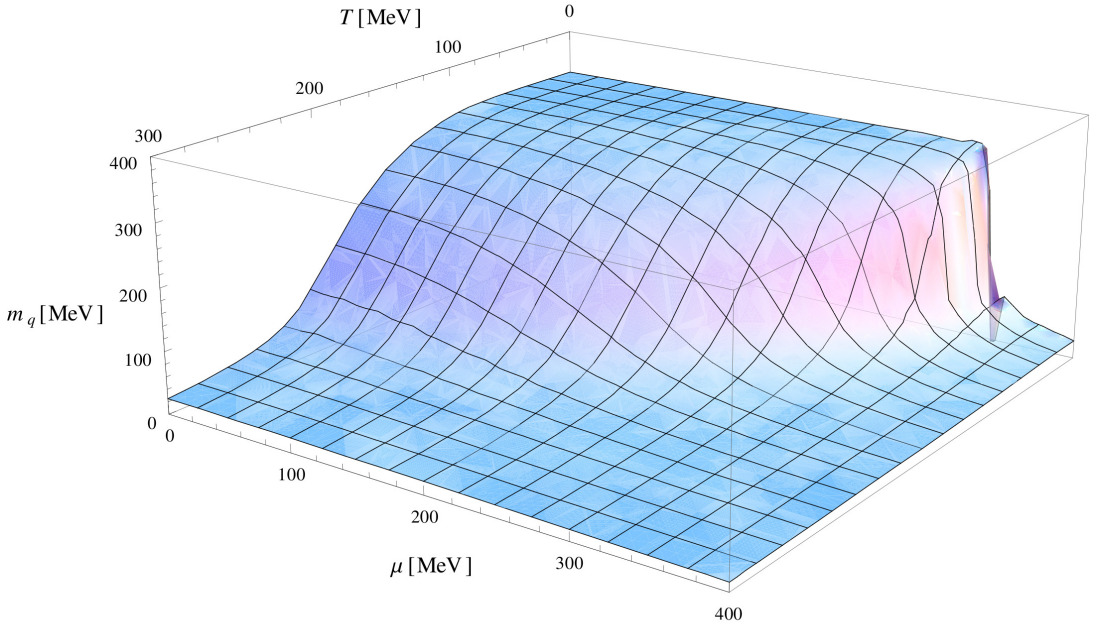
$\mathcal{O}(1)$ :

## Input parameters

$\Lambda = 651$ MeV
$m_0 = 5.5$ MeV
$G = 10.08$ GeV$^{-2}$

## Results at $(T, \mu) = (0, 0)$

$m = 326$ MeV
$\langle\bar{\psi}\psi\rangle = (251\ \mathrm{MeV})^3$



# Formulas to calculate the shear viscosity

## Transport coefficients? - Kubo formula!

$$\eta(\boldsymbol{x}, t) = -\frac{1}{10} \int \mathrm{d}^3\boldsymbol{x}' \int_{-\infty}^{t} \mathrm{d}t' \int_{-\infty}^{t'} \mathrm{d}\tilde{t} \, \langle \pi^{\mu\nu}(\boldsymbol{x}, t), \pi_{\mu\nu}(\boldsymbol{x}', \tilde{t}) \rangle_{\mathrm{ret}}$$

[Kubo, JPSJ (1957) **12**(6) 570]

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## NJL model ⇒ shear viscosity of quarks (first order in skeleton expansion)

$$\eta = \frac{64 N_{\mathrm{c}} N_{\mathrm{f}}}{15\pi T} \int_{-\infty}^{\infty} \frac{\mathrm{d}\epsilon}{2\pi} \int_{0}^{\infty} \frac{\mathrm{d}p}{2\pi} \frac{p^6 m^2 \Gamma^2(p)\, n_{\mathrm{F}}(\epsilon)[1-n_{\mathrm{F}}(\epsilon)]}{\left[\left(\epsilon^2 - p^2 - m^2 + \Gamma^2(p)\right)^2 + 4m^2\Gamma^2(p)\right]^2} \qquad (*)$$

[Iwasaki, Ohnishi, Fukutome, JPG (2008) 35 035003]

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## $\chi$PT $\Rightarrow$ shear viscosity of pions (first order in skeleton expansion)

$$\eta = \frac{1}{5\pi T} \int_{0}^{\infty} \frac{\mathrm{d}p}{2\pi} \frac{p^6\, n_{\mathrm{B}}(E_p)[1+n_{\mathrm{B}}(E_p)]}{E_p^2\, \Gamma(p)} + \mathcal{O}(\Gamma(p))$$

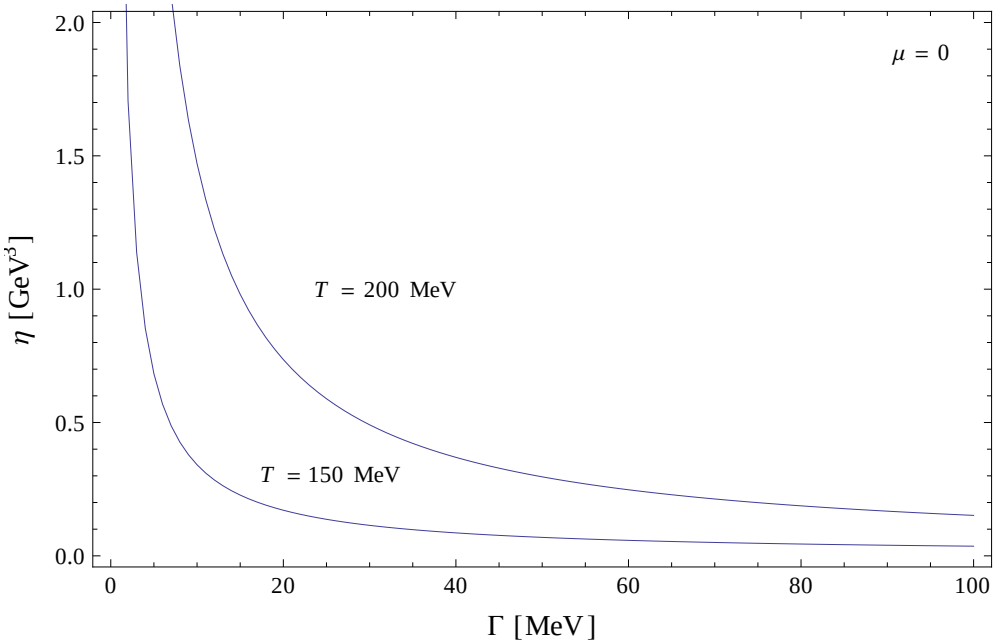
# Study with constant $\Gamma$

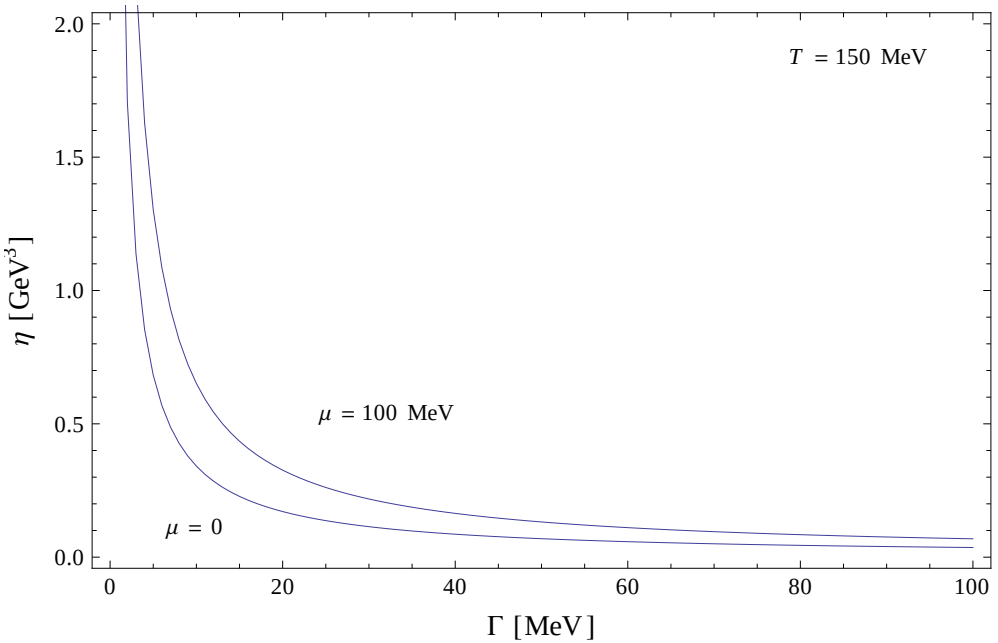
For a constant spectral width $\Gamma$, the $p$ integration of $(*)$ is **analytical**!

- $\eta$ diverges for $\Gamma \to 0$
- dependence on $T$ is stronger than on $\mu$

Compare to scales in $\chi$PT:

$$\eta \approx 50\,\mathrm{MeV/fm}^2 \approx 10^{-3}\,\mathrm{GeV}^3$$



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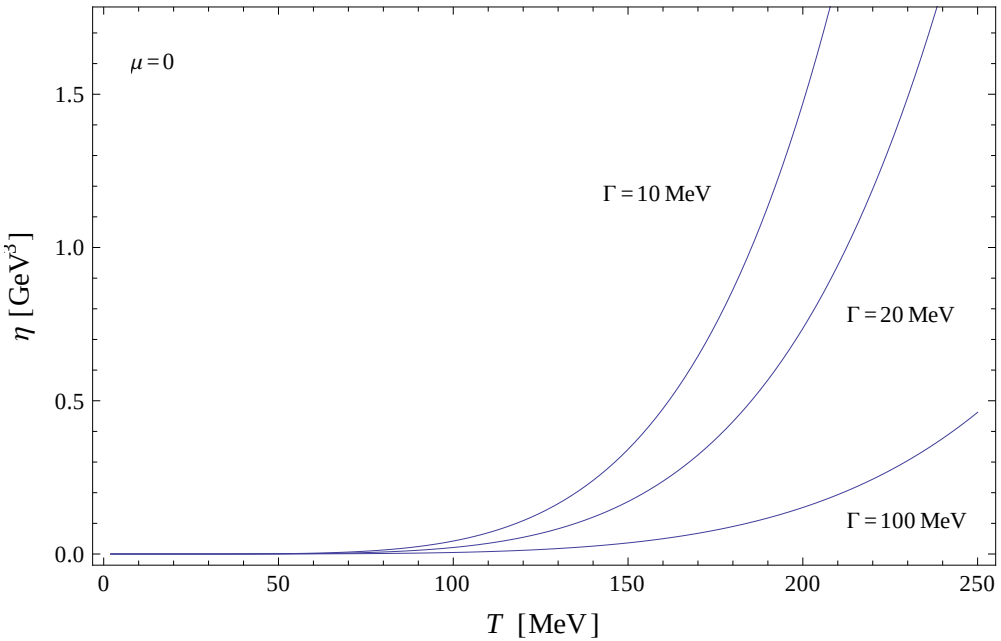
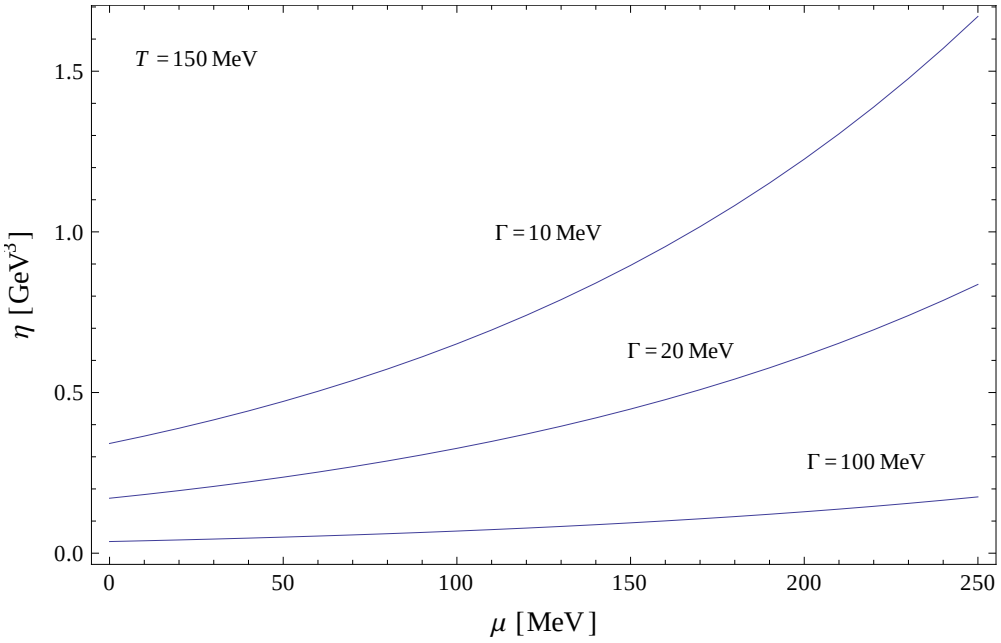
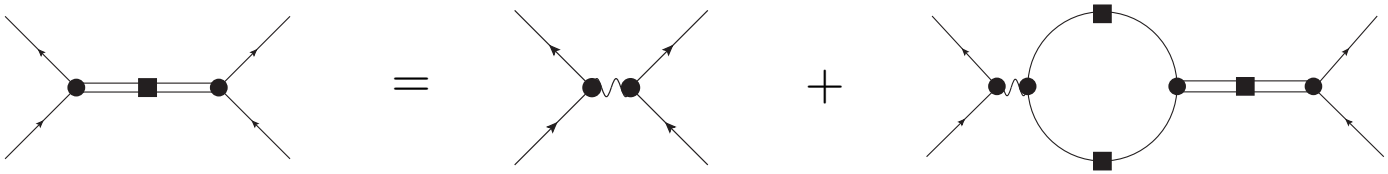


# Pion masses in the NJL model – NLO in $1/N_\mathrm{c}$ expansion

**Bethe-Salpeter equation for mesonic mode**

$\mathcal{O}(\frac{1}{N_\mathrm{c}})$ :



Calculate the quark-antiquark loop $\Pi$ for the **pion propagator** $T = \frac{G}{1-G\Pi}$.

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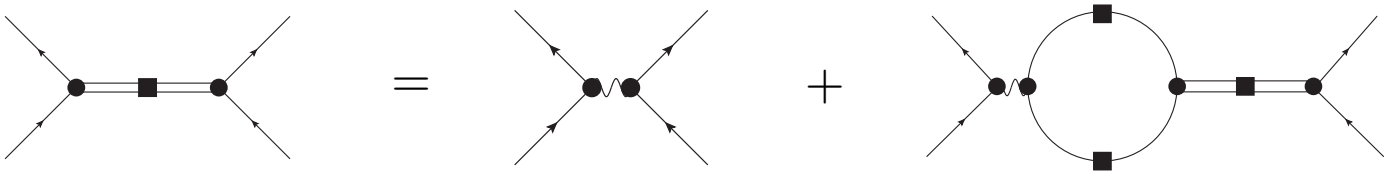

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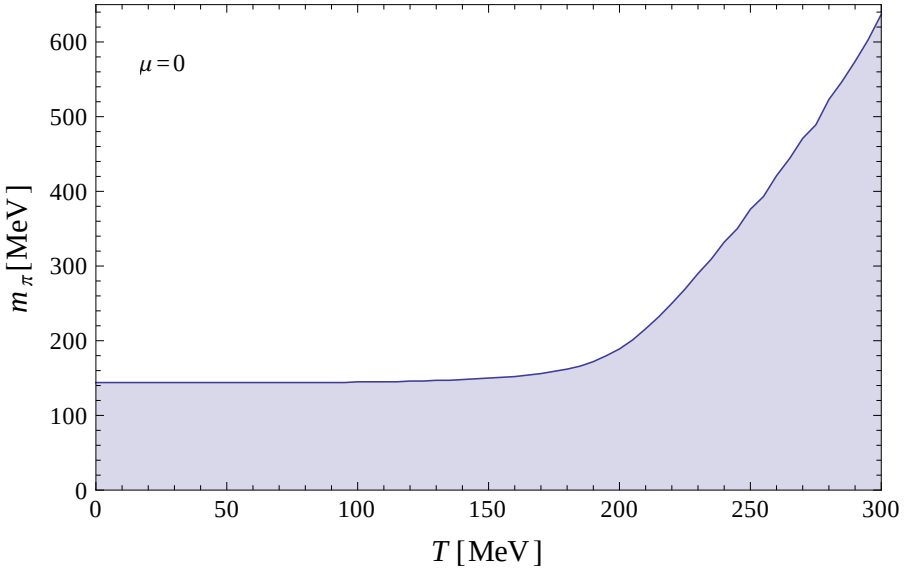


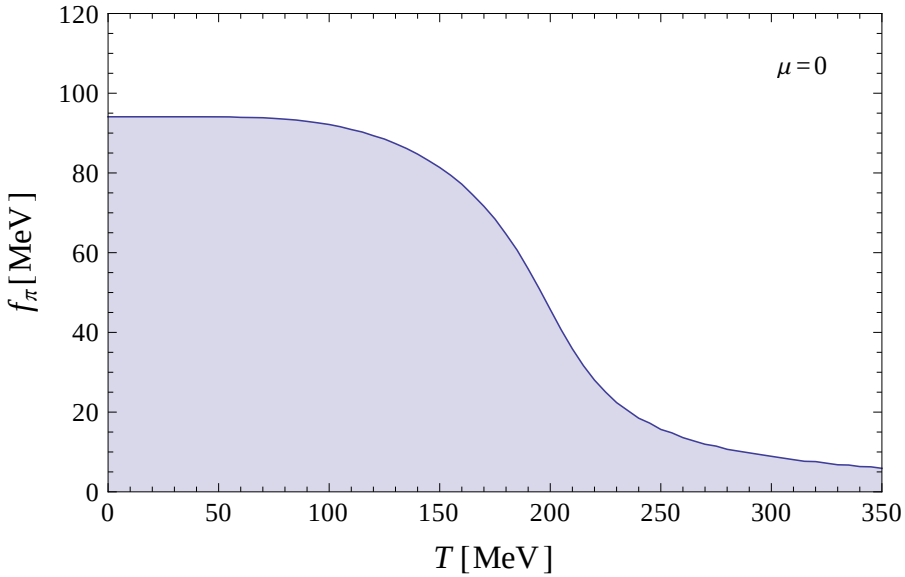


**Results at $(T, \mu) = (0, 0)$**

$$m_\pi = 144 \text{ MeV} \qquad f_\pi = 94.1 \text{ MeV}$$

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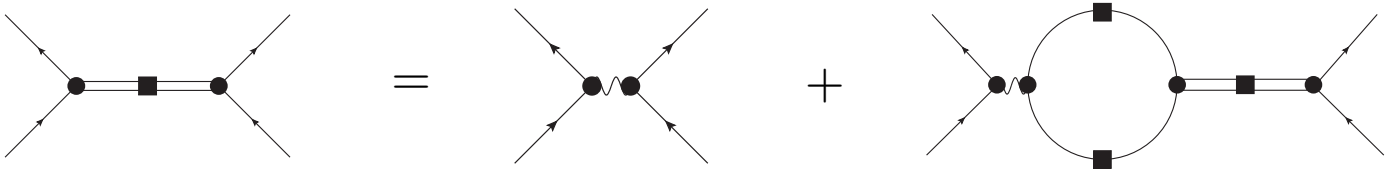

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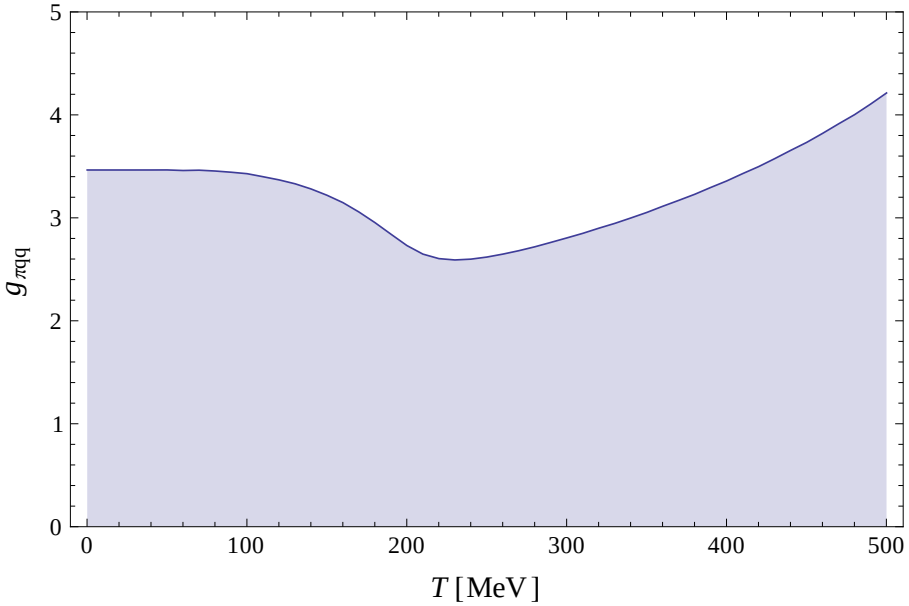
## coupling between pions and quarks

Goldberger-Treiman relation

$$g_{\pi qq} = \frac{m}{f_\pi}$$

"$g_{\pi qq}$ is constant for small $T$"

[Eletsky, Kogan, PRD (1994) **49** R3083]

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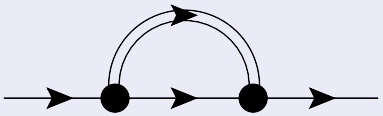
# Consistent gap equation at NLO in $1/N_\mathsf{c}$ expansion

## Quarks propagating through the heat bath



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with the Fermi distributions $n_\mathsf{F}^\pm(E) = \left( \mathsf{e}^{\beta(E \mp \mu)} + 1 \right)^{-1}$ and antiquark-energy boundaries

$$E_\mathsf{max,min}(p) = \frac{1}{2m^2} \left[ \left( m_\pi^2 - 2m^2 \right) \sqrt{m^2 + p^2} \pm m_\pi p \sqrt{m_\pi^2 - 4m^2} \right].$$

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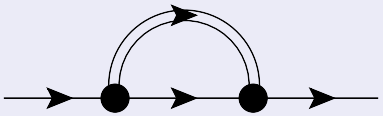

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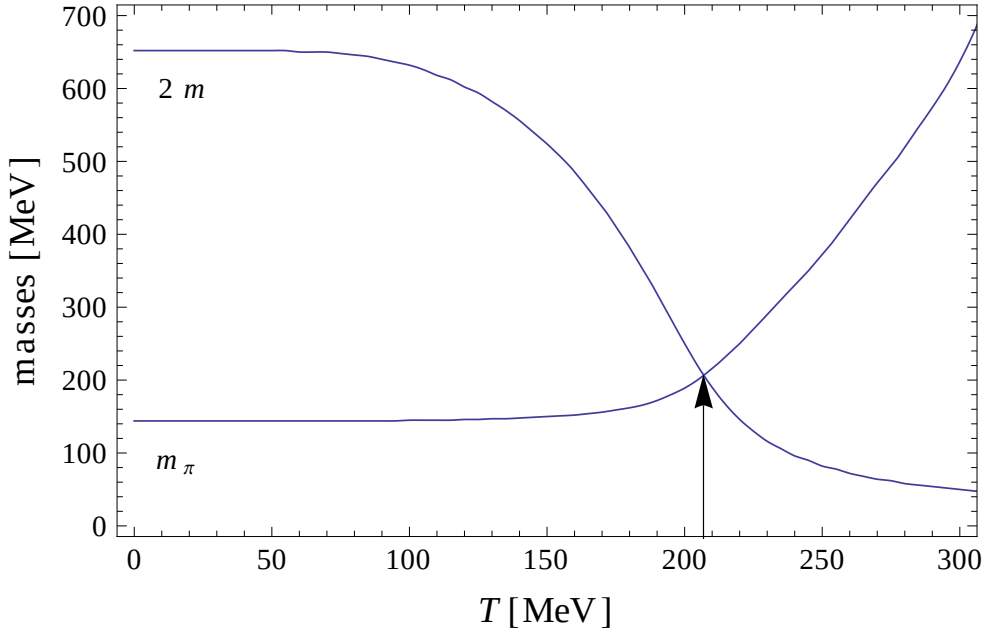
**Kinematics of the one-loop diagram**

For $\Gamma \neq 0$ one needs

$m_\pi > 2m \quad \Leftrightarrow \quad T \gtrsim 210$ MeV



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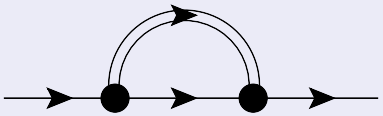

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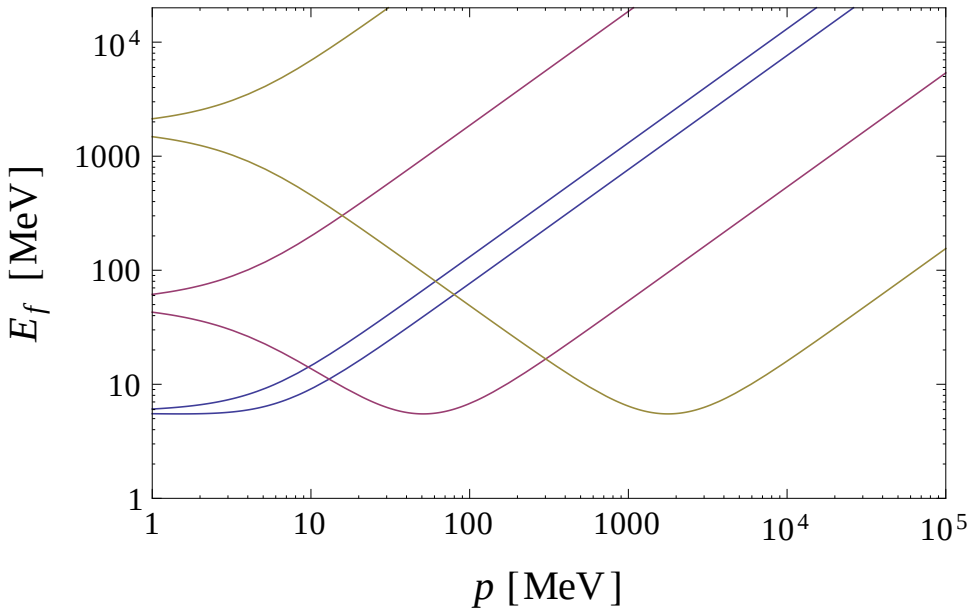
**Kinematics of the one-loop diagram**

$\rightarrow$ **different mass ratios**
(parametrized plot only)

blue: $m_\pi/m = 2.1$
red: $m_\pi/m = 10$
yellow: $m_\pi/m = 25$



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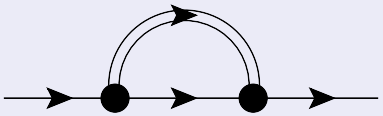

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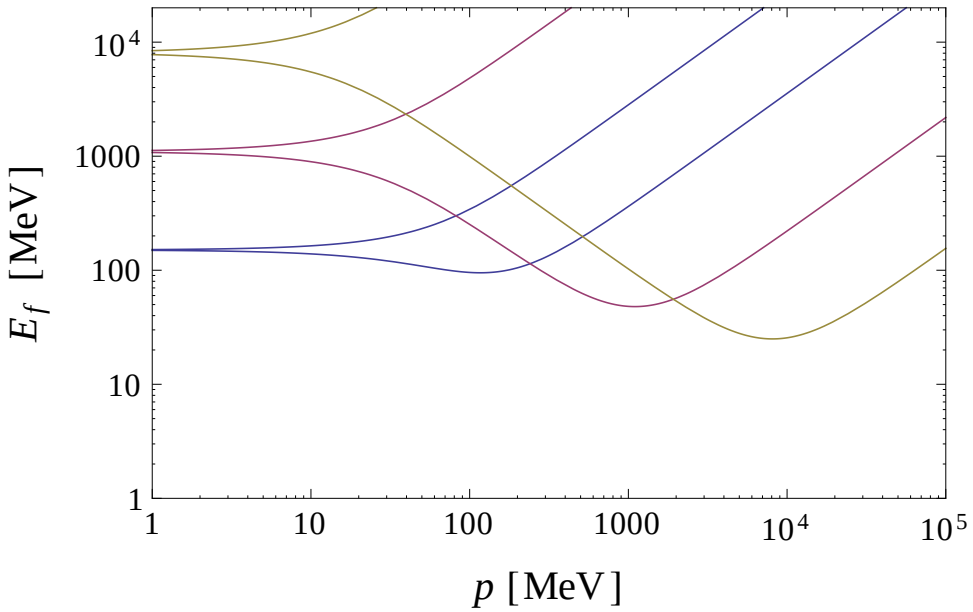
**Kinematics of the one-loop diagram**

$\rightarrow$ **different temperatures**
(plot includes all NJL input)

blue: $T = 210$ MeV
red: $T = 240$ MeV
yellow: $T = 300$ MeV



# Results: physical spectral width

## Quarks AND antiquarks propagating through the heat bath

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$\Gamma_{\mathsf{q}}$, $\Gamma_{\bar{\mathsf{q}}}$ **not invariant** under $\mu \mapsto -\mu$

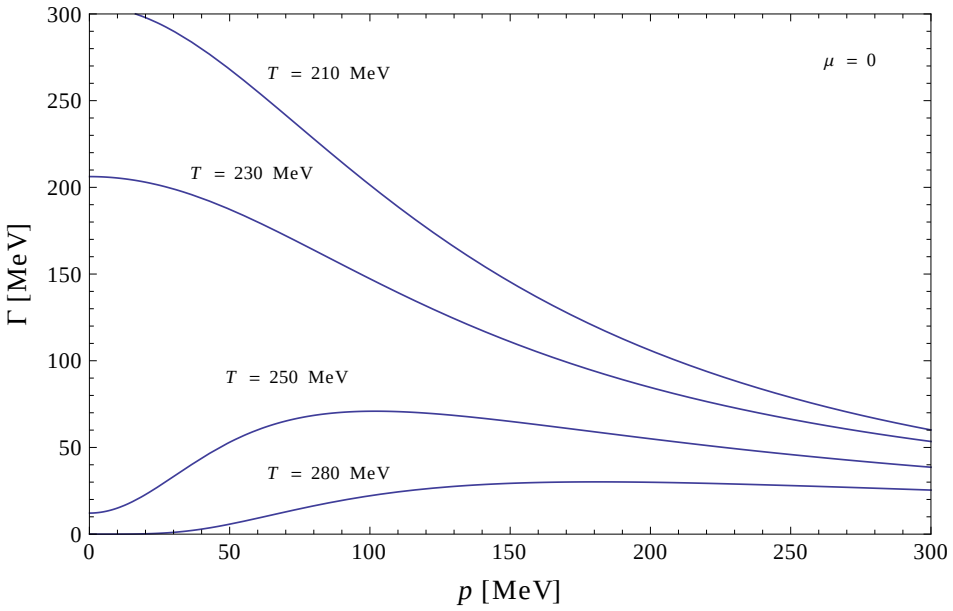
# Results: physical spectral width

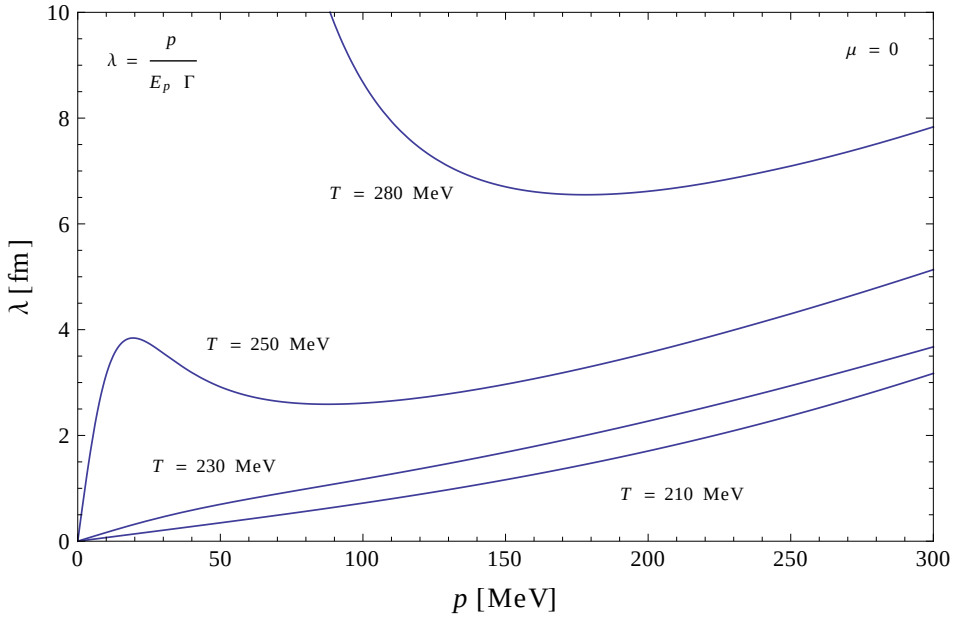
## Quarks AND antiquarks propagating through the heat bath

$\Rightarrow$ **combine** the spectral width for quarks and antiquarks:

$$\Gamma = \frac{1}{2}\left(\Gamma_{\mathsf{q}} + \Gamma_{\bar{\mathsf{q}}}\right) = \frac{m g_{\pi\mathsf{qq}}^2 N_{\mathsf{c}} N_{\mathsf{f}}}{8\pi p} \int_{E_{\mathsf{min}}}^{E_{\mathsf{max}}} \mathrm{d}E_f \left[ n_{\mathsf{B}}(E_b) + \frac{n_{\mathsf{F}}^-(E_f) + n_{\mathsf{F}}^+(E_f)}{2} \right]$$

$\Gamma$ **invariant** under $\mu \mapsto -\mu$



# Results: shear viscosity in the NJL model at one-loop level

## Reminder: Kubo-type formula for shear viscosity of quarks

$$\eta = \frac{64 N_{\mathsf{c}} N_{\mathsf{f}}}{15\pi T} \int_{-\infty}^{\infty} \frac{\mathsf{d}\epsilon}{2\pi} \int_{0}^{\infty} \frac{\mathsf{d}p}{2\pi} \frac{p^6 m^2 \Gamma^2(p)\, n_{\mathsf{F}}(\epsilon)[1 - n_{\mathsf{F}}(\epsilon)]}{\left[\left(\epsilon^2 - p^2 - m^2 + \Gamma^2(p)\right)^2 + 4m^2\Gamma^2(p)\right]^2} \qquad (*)$$

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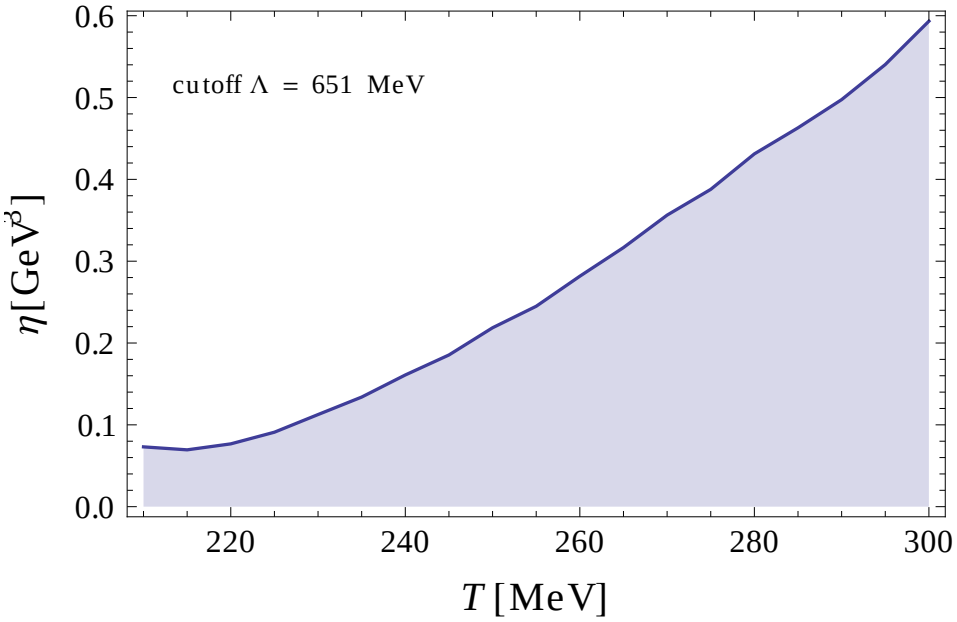


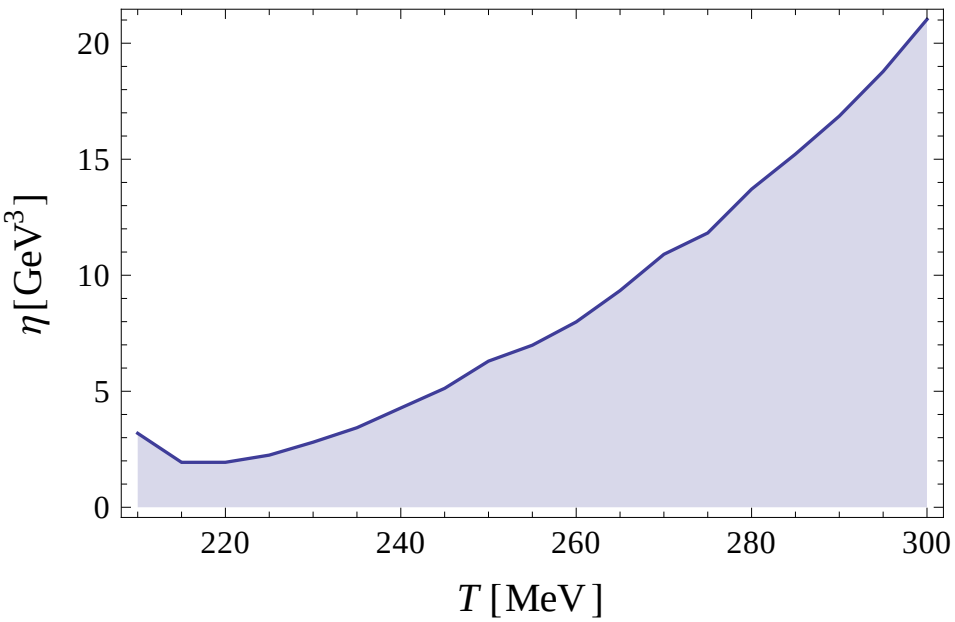

# Summary & Outlook

**Most important points:**

- **gap equation** (for quarks) and **Bethe-Salpeter equation** (for mesons) follow from a $1/N_c$ expansion in the NJL model
- Kubo-type formula with **skeleton expansion** lead to the shear viscosity $\eta[\Gamma]$
- $\eta(T)$ in the NJL model is **rising function** with strong **cut-off dependence**

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**Thank you for your attention!!**