Non-Abelian Vortex in Dense QCD

--- orientational moduli dynamics---

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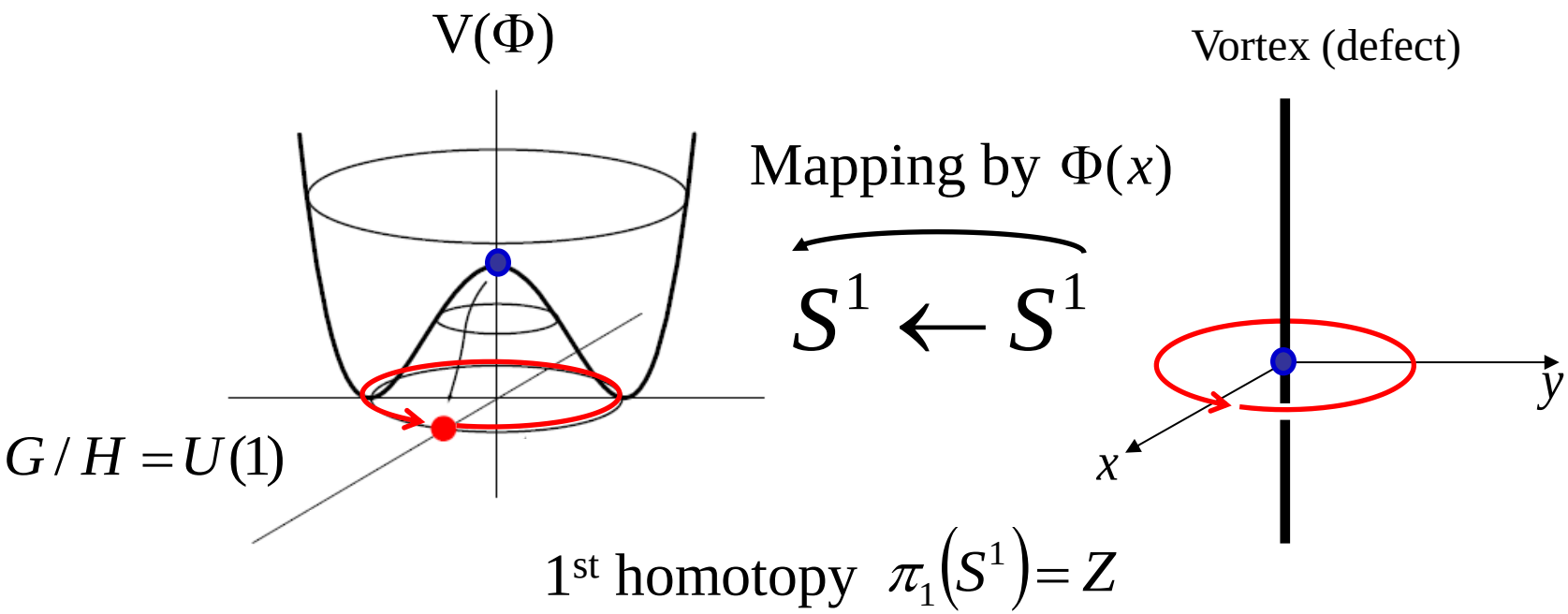
1. Review on Abelian U(1) vortices

U(1) vortices :

- topological soliton w/SSB
- particle like (2+1d), string like (3+1d)
- quantized flux if gauged

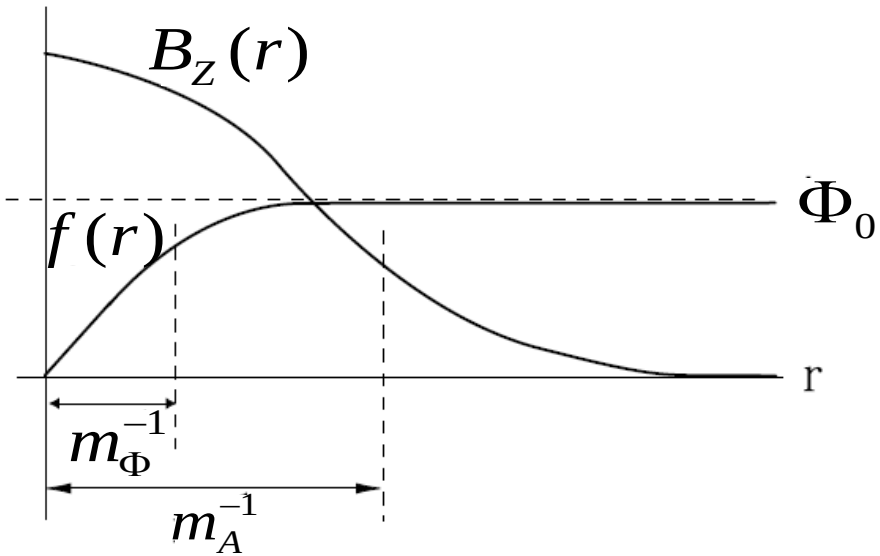
$$L = |D\Phi|^2 + m^2|\Phi|^2 - \lambda|\Phi|^4 - \tfrac{1}{4}FF$$

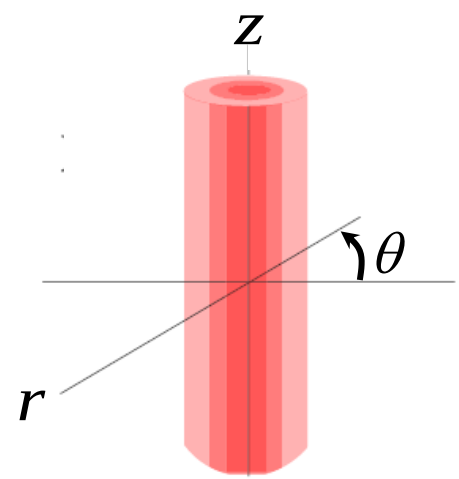


e.g., Gauged *U(1)* Vortex by A.A.Abrikosov ('57) and Nielsen-Olesen ('73)

$$\Phi(r,\theta) = e^{i\theta} f(r)$$

$$A_i(r) = \frac{1-h(r)}{e} e^{-i\theta}\partial_i e^{i\theta} \;\Rightarrow\; A_\theta(r) = \frac{h(r)}{er}$$





Penetration depth : $m_A^{-1} = (e\Phi_0)^{-1}$

Coherence length : $m_\Phi^{-1} = (\sqrt{\lambda}\Phi_0)^{-1}$

- quantized flux: $n = \frac{e}{2\pi}\oint A_i \mathrm{d}x^i = \frac{e}{2\pi} B_z \in \pi_1[U(1)] = Z$

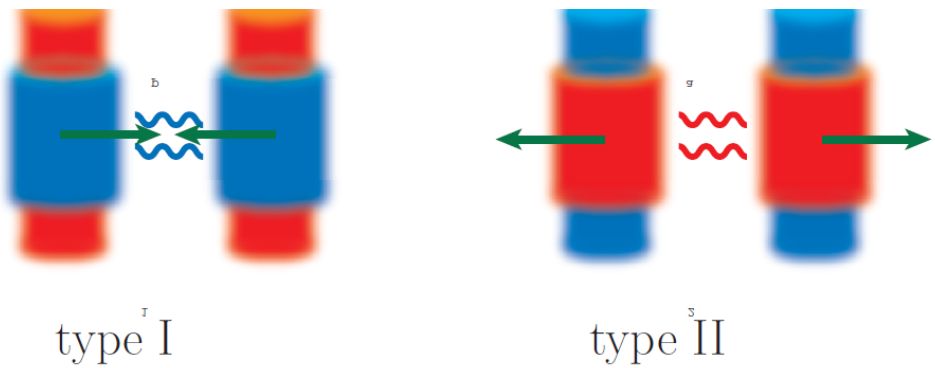
Long Range Interactions:

1) Local (gauged) Vortex Strings (3+1 dim)

$$\text{Type I}: \quad m_\Phi^{-1} > m_A^{-1} \Rightarrow \text{attractive force} \Rightarrow \text{unstable}$$

$$\text{Type II}: \quad m_\Phi^{-1} < m_A^{-1} \Rightarrow \text{repulsive force} \Rightarrow \text{stable} \Rightarrow \text{Lattice}$$

$$\text{Critical (BPS)}: \quad m_\Phi^{-1} = m_A^{-1} \Rightarrow \text{no force} \Rightarrow \text{moduli dynamics}$$



2) Global Vortex Strings (3+1 dim)

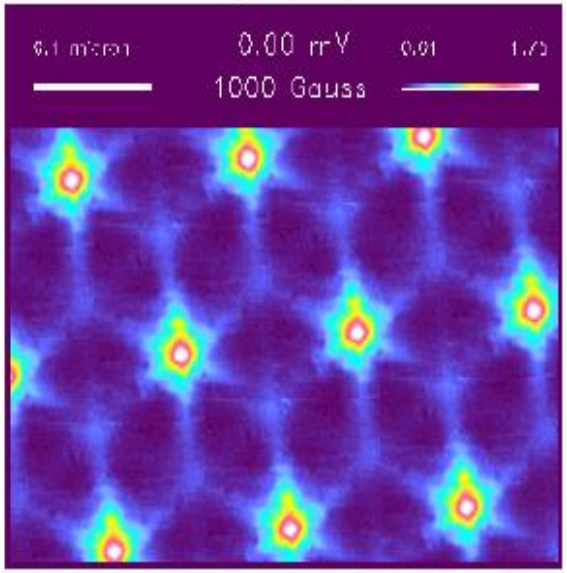
Logarismic divergent energy: $E_{\text{int}}(r) = 4\pi \log[\Lambda / r]$

Repulsive force: $F(r) = -\dfrac{\partial E_{\text{int}}}{\partial r} = \dfrac{4\pi}{r}$

Vortex strings appear in variety of physics:

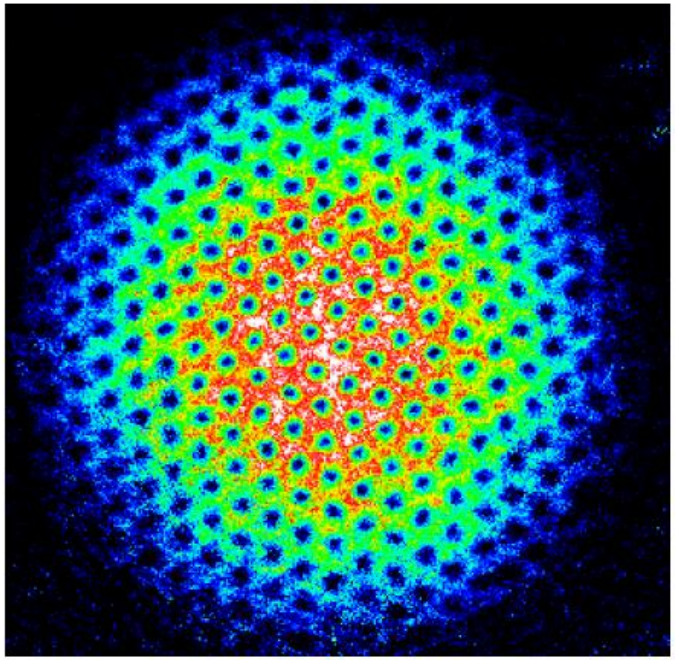
1. Condensed matter: Superconductors (Abrikosov), Superfluid (Mermin-Ho)
2. High energy: Dual for confinement (Nambu), Monopole ends (Nielsen)
3. Cosmology : Cosmic strings
4. QCD: Color superconductor, Chiral condensation

Type-II superconductors



Bell Labs

Dilute gas BEC



JILA
MIT

Superfluid and Superconducting vortices

- Superfluid vortex

  created by rotated with angular velocity > critical one.

- Superconducting vortex

  created by external magnetic field

- Non-Abelian Vortex in Dense QCD

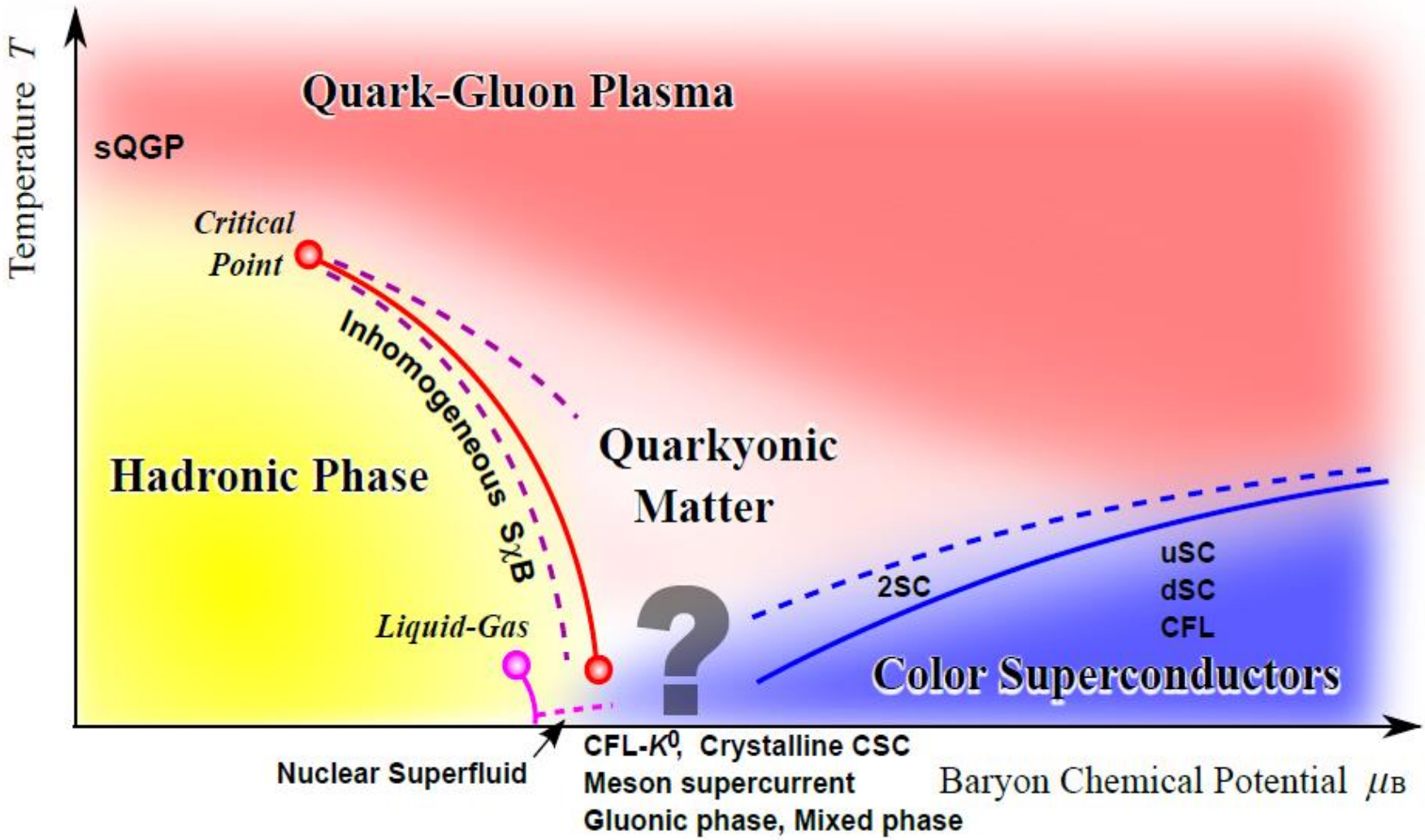
  has both of superfluid and (color) superconducting natures:

  ⇒ semi-superfluid vortex.

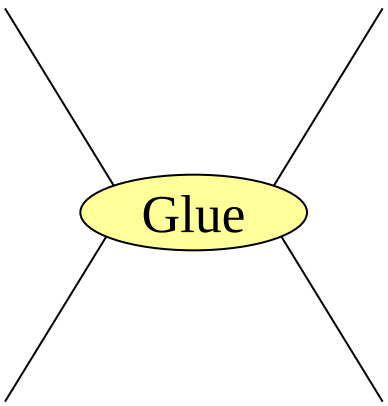
  Balachandran-Digal-Matsuura (PRD73,074009)

2. Color-Flavor Locking (CFL) in dense QCD

QCD phase structure K. Fukushima & T. Hatsuda, 10



Quark Cooper pairing in CFL phase



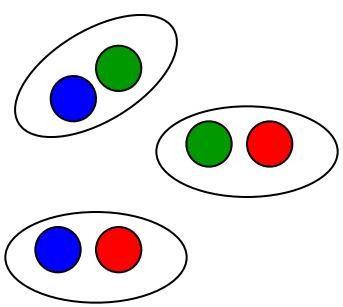
: quark-quark interaction at $N_F = N_C = 3$
attractive in color **anti-symmetric** channels

$\Rightarrow$ quark cooper pairing

$$\left\langle \psi_i^{\alpha} C\gamma_5 \psi_j^{\beta} \right\rangle = \varepsilon^{\alpha\beta\gamma} \varepsilon^{ijk} \Phi_{\gamma k}$$

Color: $3 \times 3 = \bar{3} + 6$

Flavor: $3 \times 3 = \bar{3} + 6$



$$m_u = m_d = m_s = 0$$

Color-Flavor locking (CFL) : $\Phi_{\gamma k} = \Delta \delta_{\gamma k}$

Alford, Rajagopal, Wilczek '99

Spontaneous symmetry breaking in CFL

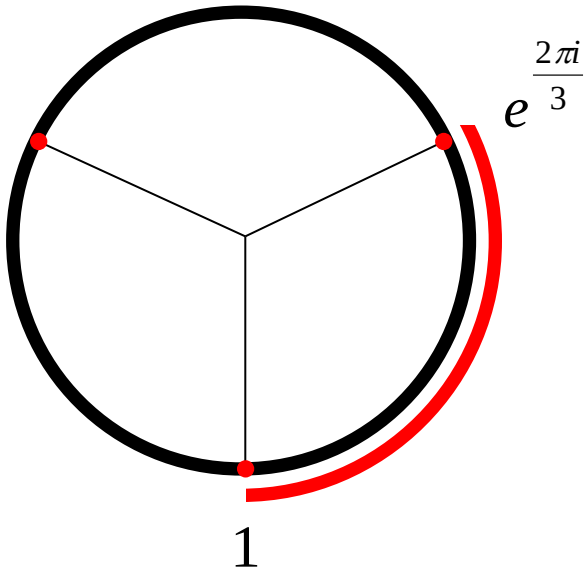
$U(1)_B$

$$e^{i\alpha} U_c \Phi U_f^{-1} \rightarrow \Delta e^{i\alpha} U_c \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} U_f^{-1}$$

$$G = \frac{SU(3)_C \times SU(3)_F \times U(1)_B}{Z(3)_{C+B} \times Z(3)_{F+B}} \Rightarrow H = \frac{SU(3)_{C+F} \times Z(3)_{C-F+B}}{Z(3)_{C+B} \times Z(3)_{F+B}}$$

$$G/H = \frac{SU(3)_{C-F} \times U(1)_B}{Z(3)_{C-F+B}} = U(3)$$

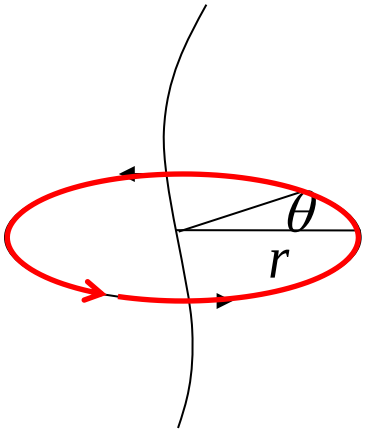
1st homotopy in CFL phase: $\pi_1\left[\dfrac{SU(3)_{C-F}\times U(1)_B}{Z(3)_{C-F+B}}\right]=Z$



Fundamental mapping:

$$\Phi(r,\theta)\propto \exp i\left[\frac{1}{3}-\sqrt{\frac{2}{3}}\lambda_8\right]\theta$$

$$=\begin{pmatrix} e^{i\theta} & & \\ & 1 & \\ & & 1 \end{pmatrix}$$



$$0\le\theta\le 2\pi$$

$U(1)_B$ circle and $SU(3)$ rotation

$$e^{-\frac{4\pi i}{3}\sqrt{\frac{2}{3}}\lambda_8}e^{\frac{2\pi i}{3}}=\begin{pmatrix} e^{\frac{4\pi i}{3}} & 0 & 0 \\ 0 & e^{-\frac{2\pi i}{3}} & 0 \\ 0 & 0 & e^{-\frac{2\pi i}{3}} \end{pmatrix}\begin{pmatrix} e^{\frac{2\pi i}{3}} & 0 & 0 \\ 0 & e^{\frac{2\pi i}{3}} & 0 \\ 0 & 0 & e^{\frac{2\pi i}{3}} \end{pmatrix}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3 type of mapping for single-valued Φ :

$$\Phi(r,\theta) \propto \exp i\left[\frac{1}{3} - i\sqrt{\frac{2}{3}}\left(Q_3\lambda_3 + Q_8\lambda_8\right)\right]\theta$$

$$\Rightarrow \begin{pmatrix} e^{i\theta} & & \\ & 1 & \\ & & 1 \end{pmatrix} \text{ or } \begin{pmatrix} 1 & & \\ & e^{i\theta} & \\ & & 1 \end{pmatrix} \text{ or } \begin{pmatrix} 1 & & \\ & 1 & \\ & & e^{i\theta} \end{pmatrix}$$

SU(3) part: three color fluxes

$$(Q_3, Q_8) = \underset{\text{Red}}{(0,1)}, \quad \underset{\text{Green}}{\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)}, \quad \underset{\text{Blue}}{\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)}$$

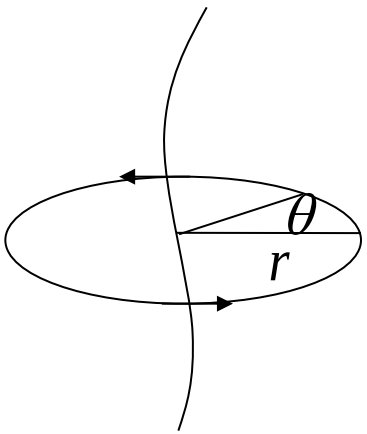
$$\pi_1\left[SU(3)/Z(3)\right] = Z(3)$$

3. Non-Abelian Vortex (NAV) in CFL phase

NAV solution of GL Lagrangian

Pisarski '00, Iida-Baym '01, Giannakis-Ren '02

$$L_{GL} = \mathrm{tr}\left|D\Phi\right|^2 - m^2\mathrm{tr}\left|\Phi\right|^2 - \tfrac{1}{4}F_aF_a - \lambda_1\left(\mathrm{tr}\left|\Phi\right|^2\right)^2 - \lambda_2\mathrm{tr}\left|\Phi\right|^4$$



Vortex ansatz

$$\begin{cases} \Phi(r,\theta) = \exp\left[i\dfrac{\theta}{3} - i\sqrt{\dfrac{2}{3}}\lambda_8\theta\right]\Phi_0\mathrm{diag}(f,g,g) = \Phi_0\begin{pmatrix} e^{i\theta}f(r) & & \\ & g(r) & \\ & & g(r)\end{pmatrix}, \\ A^8_\theta(r) = \dfrac{\xi}{g_s r}h(r), \end{cases}$$

Flux:

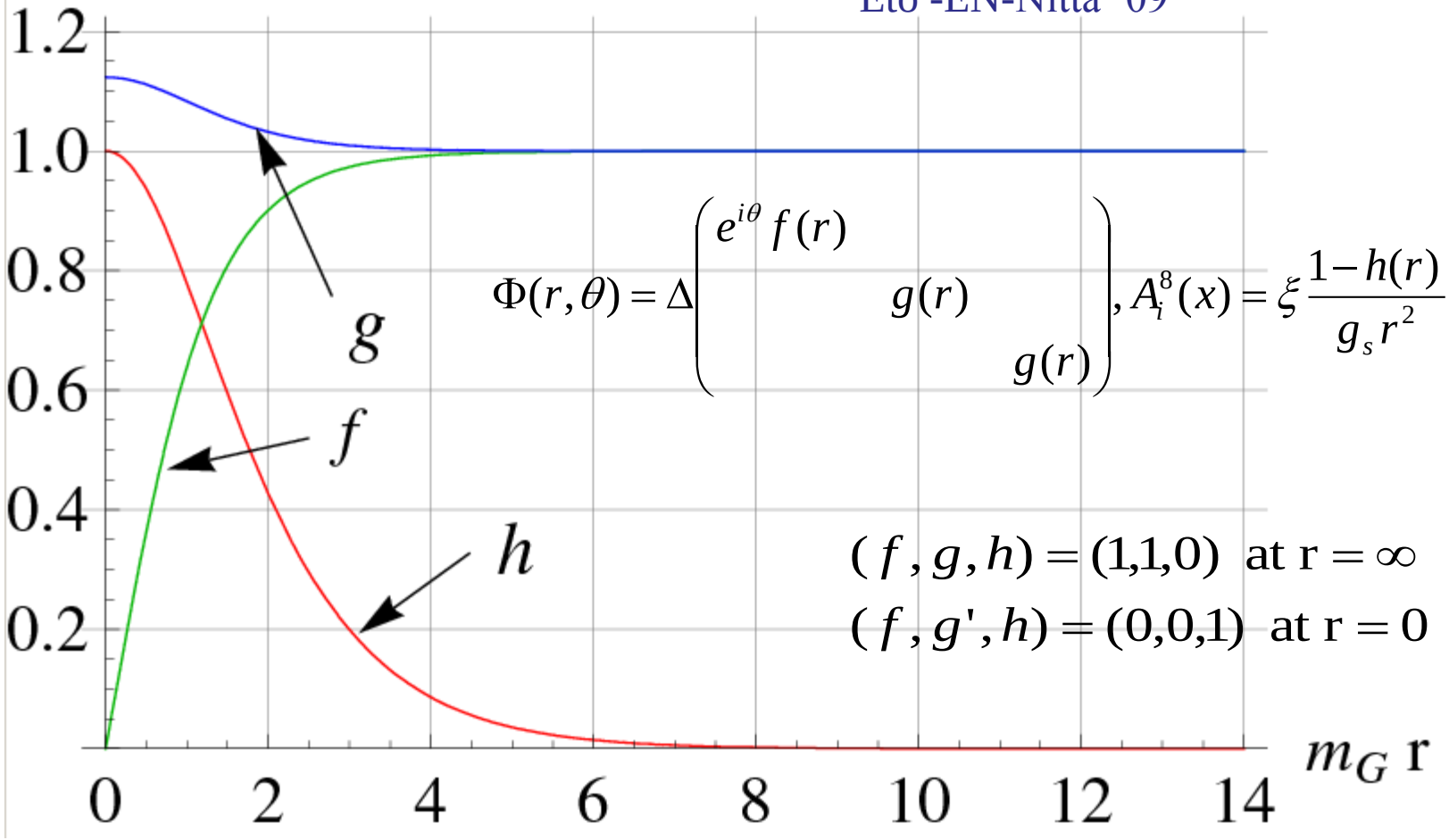
$$F^8_{12} = \frac{1}{r}\frac{d}{dr}\left[rA^8_\theta\right] = \frac{\sqrt{6}}{3g_s}\frac{h'(r)}{r}$$

B.C.

$$\begin{cases} V(\Phi)|_{r\to\infty} = V(\Phi_0), \\ F_\alpha F_\alpha|_{r\to\infty} = 0, \\ \mathrm{tr}\left|D\Phi\right|^2|_{r\to\infty} = \left[\left(\xi + \sqrt{\tfrac{2}{3}}\right)^2 + \tfrac{1}{3}\right]/r^2, \\ \text{minimized at } \xi = -\sqrt{\tfrac{2}{3}} \end{cases}$$

Full numerical solution

Eto -Nitta ’09
Eto -EN-Nitta ’09

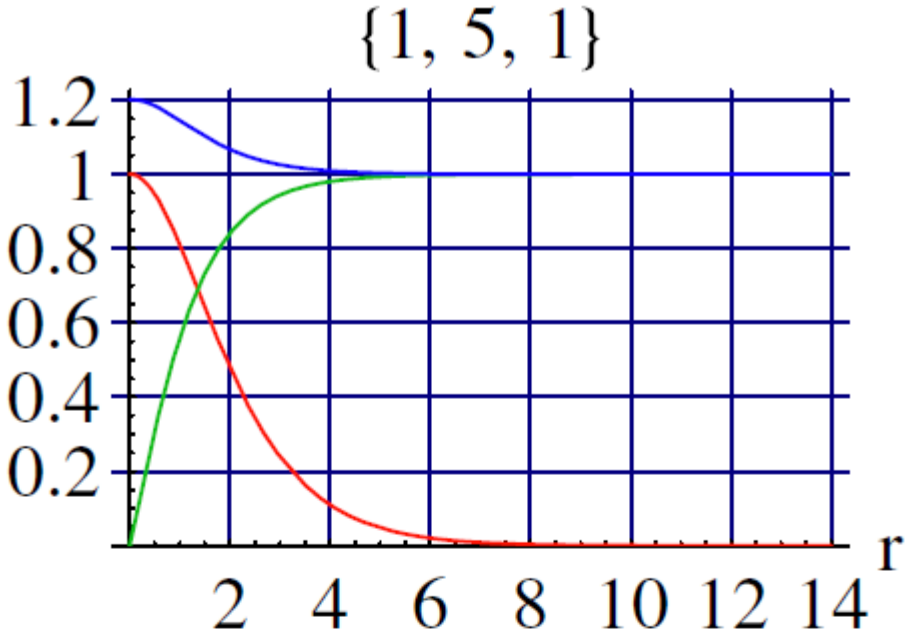


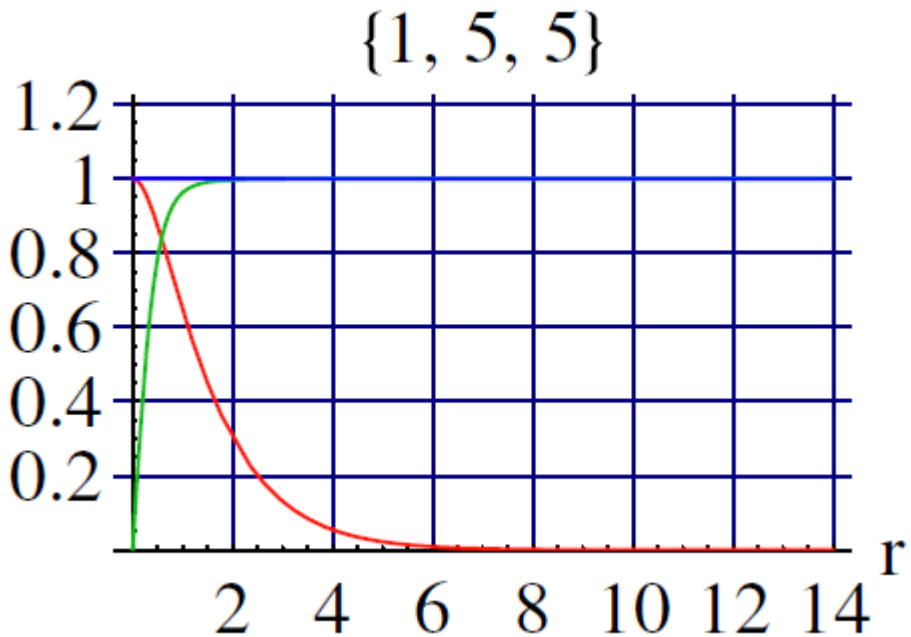
$$\Phi(r,\theta) = \Delta \begin{pmatrix} e^{i\theta} f(r) & & \\ & g(r) & \\ & & g(r) \end{pmatrix}, \quad A_i^8(x) = \xi \frac{1-h(r)}{g_s r^2} \varepsilon^{ij} x_j,$$

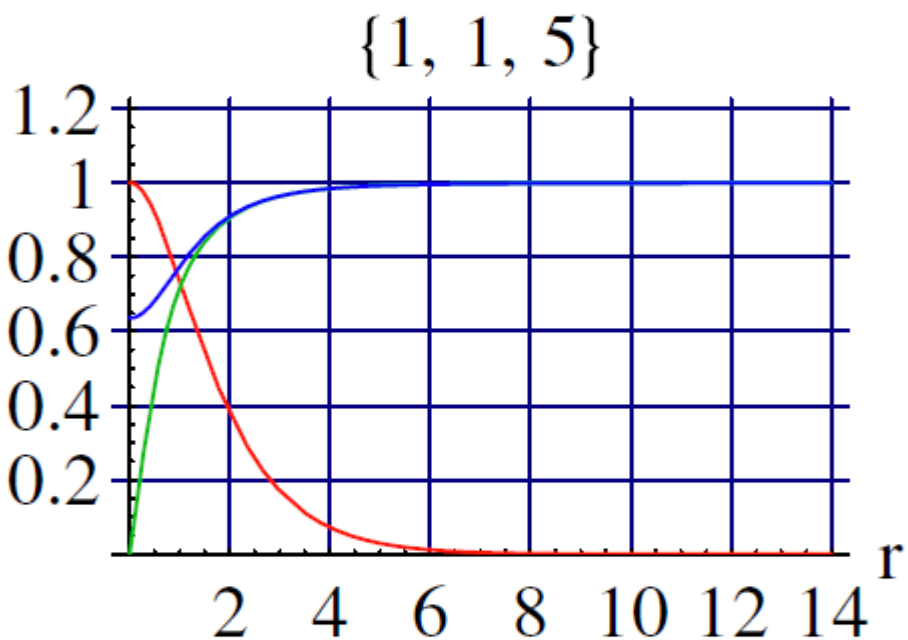
$$(f, g, h) = (1,1,0) \text{ at } r = \infty$$

$$(f, g', h) = (0,0,1) \text{ at } r = 0$$

$$m_G^2 = 2g^2 v^2, \quad m_1^2 = -2m^2, \quad m_8^2 = 4\lambda_2 v^2$$


{1, 5, 1}
1.2
1
0.8
0.6
0.4
0.2
2 4 6 8 10 12 14
r


{1, 5, 5}
1.2
1
0.8
0.6
0.4
0.2
2 4 6 8 10 12 14
r


{1, 1, 5}
1.2
1
0.8
0.6
0.4
0.2
2 4 6 8 10 12 14
r

4. Orientational moduli (NG modes)

Orientational moduli (NG modes)

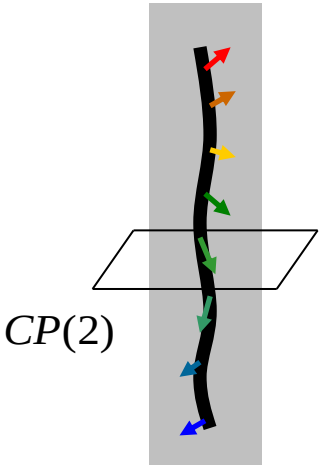
Nakano-Matsuura-Nitta '08, Eto-Nakano-Nitta '09

$$\Phi(r,\theta) = \begin{pmatrix} e^{i\theta} f(r) & & \\ & g(r) & \\ & & g(r) \end{pmatrix}$$

Existence of NA string further breaks H :

$$H = SU(3)_{C+f}$$

$$\to H' = SU(2)_{C+f} \times U(1)_{C+f}$$



Moduli space (orientation) along NA string:

$$H / H' = \frac{SU(N)_{C+F}}{SU(N-1)_{C+F} \times U(1)_{C+F}} \cong CP(N-1)\,|_{N=3}$$

Definition of Non-AbelianVortex (?)

$$\pi_1[G/H] \neq 0$$

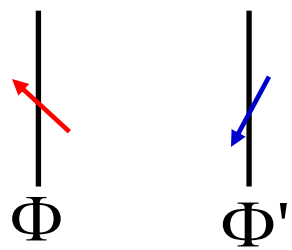
$$\text{w/ Non-Abelian } H.$$

e.g., Supersymmeric theories, superfluid Helium3, …

N=2 Supersymmetric Hanany-Tong ’03, Auzzi etal ‘03
Non Higgs phase: Alford, Coleman, Wilczek ’89
Quiver gauge: Lechtenfeld, Popov, Szabo ‘03

Where is the orientational moduli ?
⇒ normalizable or not?

Consider two NAVs with relatively different orientations



$$\Phi' = U\Phi U^{-1}, \quad U \in H = SU(3)_{C+F}$$

For relative orientation, 2 by 2 matrix is enough,

$$\Phi' = U\Phi U^{-1} = U\begin{pmatrix} \begin{pmatrix} e^{i\theta} f & 0 \\ 0 & g \end{pmatrix} u^{-1} & 0 \\ 0 & g \end{pmatrix}$$

where $u = \begin{pmatrix} a^* & -b \\ b^* & a \end{pmatrix} \in \mathrm{SU}(2)_{\mathrm{C+F}}$ with $|a|^2 + |b|^2 = 1$

Then, make an another color transformation:

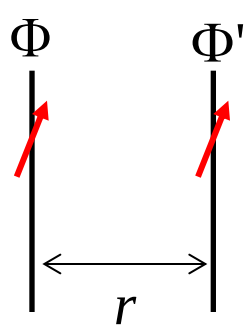
$$\underline{u_c u^{-1}} u \begin{pmatrix} e^{i\theta} f & 0 \\ 0 & g \end{pmatrix} u^{-1} = u_c \begin{pmatrix} e^{i\theta} f & 0 \\ 0 & g \end{pmatrix} u^{-1}$$

If taking the following form,

$$u_c = \begin{pmatrix} a^* e^{i\theta} f & -b e^{i\theta F(r)} \\ b^* e^{i\theta F(r)} & a \end{pmatrix}$$

at large distance, with b.c. $F(0) = 0, F(\infty) = 1$

$$u_c \begin{pmatrix} e^{i\theta} f & 0 \\ 0 & g \end{pmatrix} u_f^{-1} \to \begin{pmatrix} e^{i\theta} & 0 \\ 0 & 1 \end{pmatrix} \quad \text{at } r >> m_G^{-1}, m_\Phi^{-1}$$

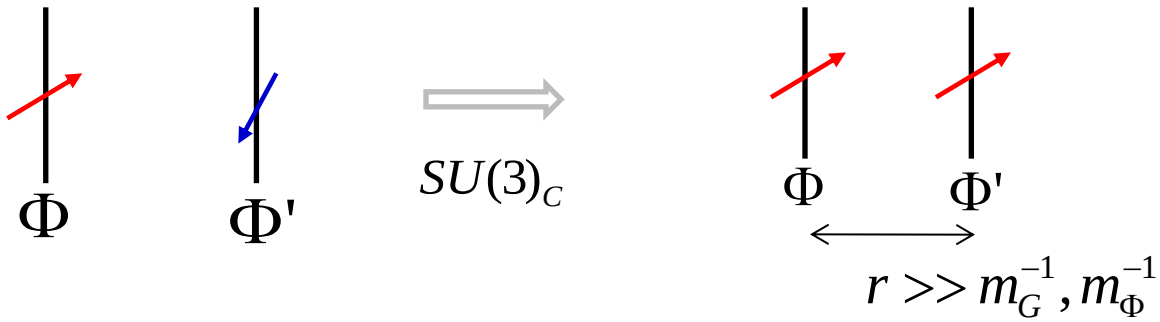


implying
the orientation moduli is normalizable,
i.e., localized along the vortex core

5. Long-range force between 2 NAVs

Eakano-Matsuura-Nitta ‘08

A regular color gauge transformation can make the relative orientation aligned at large distance.



$$\left[\begin{array}{l} \text{Abrikosov ansatz}: \ \Phi_{tot} = \Phi.\Phi', \quad A^\mu_{tot} = A^\mu + A'^\mu \\ \quad + \\ \text{GL potential}: \qquad E_{\text{int}}(r) = E(\Phi_{tot}) - E(\Phi) - E(\Phi') \end{array}\right.$$

Force between vortices : $f(r) = \dfrac{\mathrm{d}E_{\text{int}}(r)}{\mathrm{d}r} \cong \pm \dfrac{1}{3}\dfrac{2\pi}{r}$

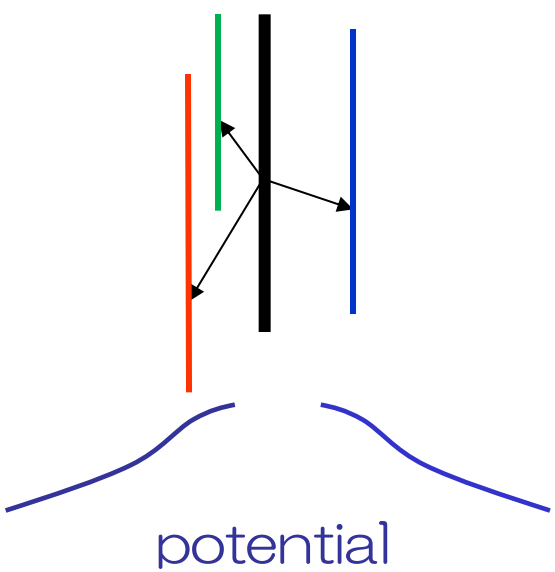
This is repulsive force !
At large distance NAV looks like Global *U(1)* Vortex each other,
But force is reduced by 1/3.

If a $U(1)_B$ vortex is created in CFL phase by rotation, it would decay into 3 NAVs.

$$U(1)_B \,\mathrm{Vortex} = \mathrm{diag}(e^{i\theta}, e^{i\theta}, e^{i\theta}),$$

Decay!

$$3\,\mathrm{NAVortex} = \mathrm{diag}(e^{i\theta},1,1),$$
$$\mathrm{diag}(1,e^{i\theta},1),$$
$$\mathrm{diag}(1,1,e^{i\theta}),$$



⇒Stability of NAV in CFL phase.

3 Color Fluxes might merge to vanish at confinement boundary.
⇒ U(1)B Vortex (Iida-Baym'02, Fobes-Zhitnitsky'02)

How about force at intermediate range?

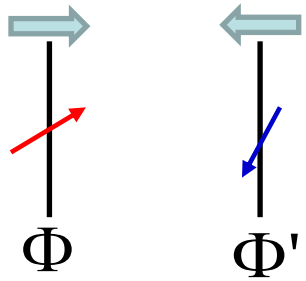
Dual transformation : clear to see interaction between
vorticity tensor and bulk particle (gluons and U(1) phonon)

Hirono et al '11

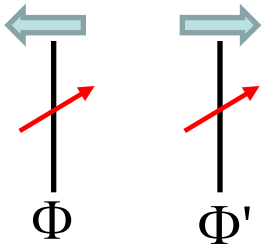
$$L_{coupling} = g_a \int \mathrm{d}\sigma^a_{\nu\mu} B^a_{\nu\mu}$$

$$\Downarrow$$

$$\mathrm{E} \sim \int d\vec{X}_1 d\vec{X}_2 \phi_1^\dagger T^a \phi_1 \ \phi_2^\dagger T^a \phi_2 \ \frac{\exp(-M_G|\vec{X}_1 - \vec{X}_2|)}{\sqrt{|\vec{X}_1 - \vec{X}_2|}}$$



Most attractive
for the different orientation

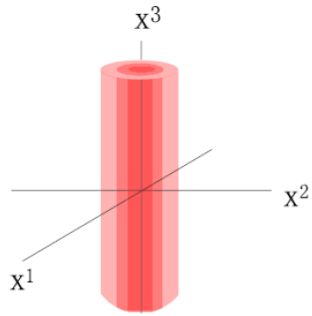


Most repulsive
for the same orientation

6. EFT

Eto-Nakano-Nitta '09

1+1 dim World Sheet Effective Field Theory of *CP(2)* orientational moduli, *a la* Chiral perturbation theory



$$A_{x,y}, \Phi : \text{Vortex solution}$$

$$U = \text{collective modes around it :}$$

$$\Phi(U) = U\Phi U^{-1}$$
$$A_{x,y}(U) = UA_{x,y}U^{-1}$$
$$U \in H = SU(3)_{C+F}$$

Identify CP(2) modes

$$\Phi \propto \#I_3 + \#\lambda_8 \quad \Rightarrow \quad \boxed{U\lambda_8 U^{-1} \equiv \phi\phi^\dagger - \frac{1}{3}I_3}$$

$$\phi^\dagger\phi = 1, \quad e^{i\alpha}\phi \sim \phi$$

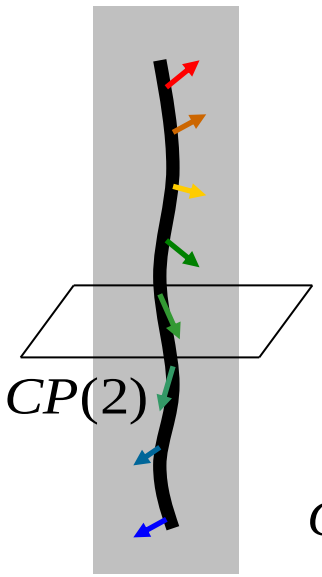
CP(2) homogeneous coordinate $\phi = (\phi_{ds}, \phi_{su}, \phi_{ud})^T \in CP(2)$

Then promote it to be field, $\phi \to \phi(t, z)$

$\phi \to \phi(t,z)$ induces gauge fluctuations of *0* and *z* components:

$$A_{0,z} = \frac{i\rho_{0,z}(r)}{g}\left[\phi\phi^{+}, \partial_{0,z}\left(\phi\phi^{+}\right)\right]$$

Putting all into GL Lagrangian and integrate x, y :



$$L_{eff}^{CP(2)}[\phi] = \int dxdy\, L_{GL}[A_{\mu}^{a}(\phi), \Phi(\phi)]$$

$$= \boxed{\sum_{\alpha=0,z} C_{\alpha}\left[\partial_{\alpha}\phi^{\dagger}\partial^{\alpha}\phi + \left(\phi^{\dagger}\partial_{\alpha}\phi\right)\left(\phi^{\dagger}\partial^{\alpha}\phi\right)\right]}$$

1+1 dim CP(2) Non-linear sigma model

$$C_{0,z} = \frac{1}{2}\int\left\{m_G^{\,2}\left(\left(1-\rho_{0,z}\right)\left(f-g\right)^2 + \frac{\rho_{0,z}^{\;2}}{2}\left(f^2+g^2\right)\right) + \frac{\left(1-\rho_{0,z}\right)^2 h^2}{r^2} + \rho_{0,z}'^{\;2}\right\} r dr$$

Numerically solve EOM with vortex background

$\to C_{0,z} \sim O(1)$ FINITE $\Rightarrow$ normalizable mode

■ Taking quantum effect and θ-vacua into account,

D'adda-Luscher-DiVecchia '79, Witten '80, '98

$$L_{CP(N-1)} = \sum_{\alpha=t,z} \mathrm{C}_\alpha \left(\partial_\alpha \phi^+ \partial_\alpha \phi + \phi^+ \partial_\alpha \phi \phi^+ \partial_\alpha \phi\right) - \sigma\left(\phi^+ \phi - 1\right)$$

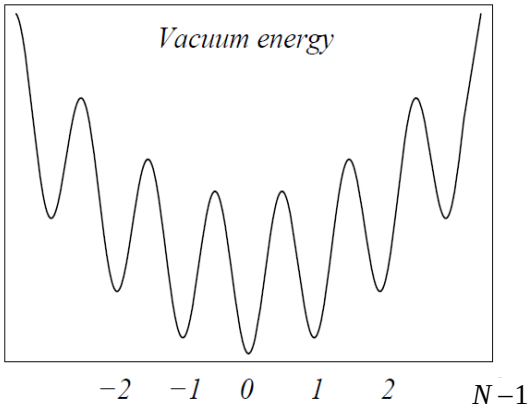
$\rightarrow$ Leading order at Large N_C (at the end $N_C = 3$)

$$L^{Quant}_{CP(N-1)} = \left(\partial_\alpha - iA_\alpha\right)\phi^+ \left(\partial^\alpha + iA^\alpha\right)\phi - M^2 \phi^+ \phi - \frac{N_C}{48\pi M^2} F^2_{\alpha\beta}$$

$$A_\alpha = \phi^+ \partial_\alpha \phi, \;\; M \sim \Delta e^{-c(\mu/\Delta)^2}$$

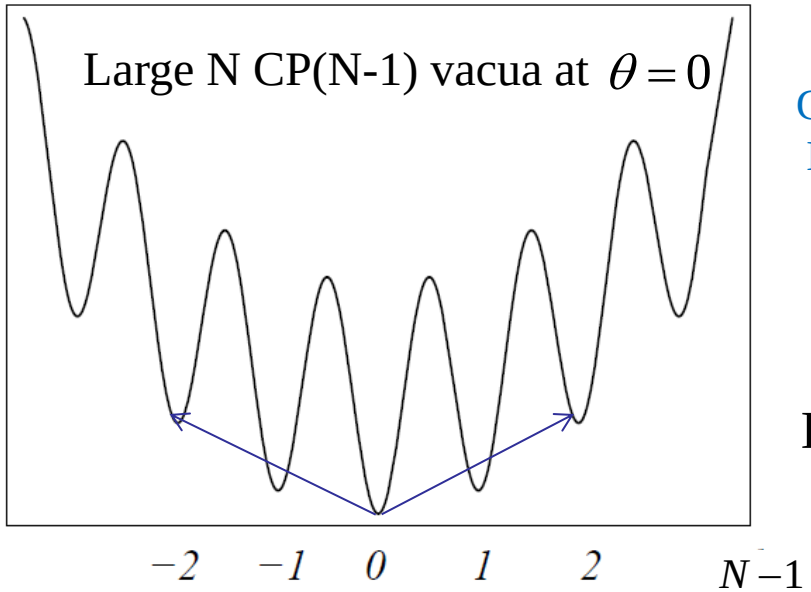
■ Energy with periodicity and reflection of θ

$$E(\theta)|_{\theta=0} = E_0 + c\frac{k^2}{N_C}, \; k = 0,1,2,\cdots N_C - 1$$



$\Rightarrow$ Kink-antiKink excitation in *CP(2) in world sheet EFT*

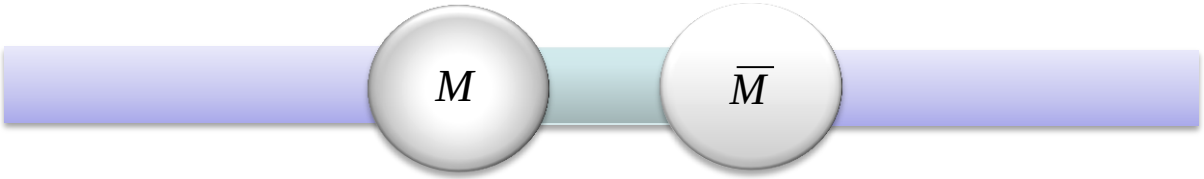
= looks Monopole - antiMonopole bound in bulk :



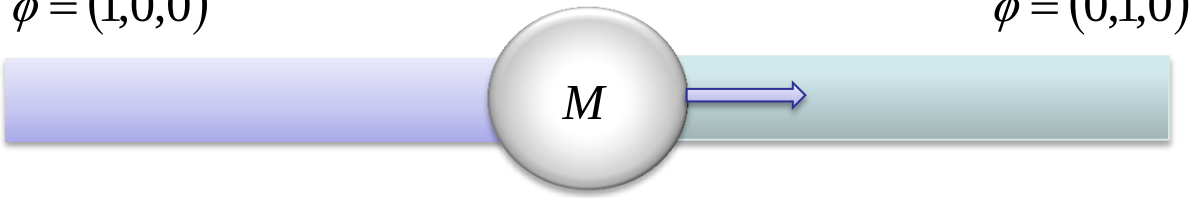
Gorsky-Shifman-Yung '11
Eto-Nitta-Yamamoto '11

Kink-antiKink

Confined monopoles (in Higgs phase)



$\phi = (1,0,0)$ $\phi = (0,1,0)$



$(Q_3, Q_8) = (1,0)$ $SU(3)$ fundamental

7. Summary

- Non-Abelian vortex in CFL phase
- Orientational moduli and its EFT
- 2-body forces (important for many-body physics):

1) Long range $\gg m_\Phi^{-1}, m_A^{-1}$
massless $U(1)_B$ phonon exchange
$\Rightarrow$ Universal Repulsion and stability

2) Intermediate range $> m_\Phi^{-1}$, Hirono et al. '11
massive gluon exchange
$\Rightarrow$ Attraction for different orientations
Repulsion for same orientations

3) Short range $< m_\Phi^{-1}$
- massive scalar exchange ?
- role of monopole excitations?
- genuine 3-body force?

Other topics

- Realistic situations

Quark masses &Beta equilibrium effects on zero modes

$\Rightarrow$ specific orientation (CP(2) quantum effect swept out?)

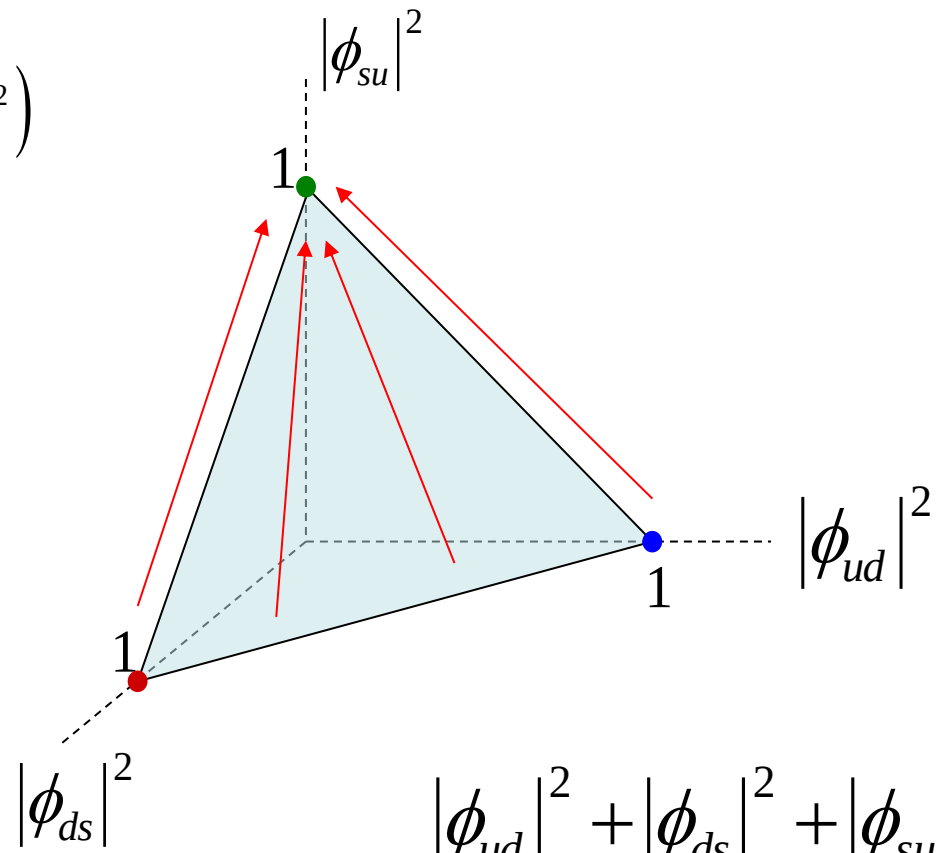
Eto et al '10, Gorsky et al '11

e.g., $V_{CP} \sim \varepsilon\left(|\phi_3|^2 - 1\right)$,

or $V_{CP} \sim \varepsilon\left(|\phi_3|^2 - |\phi_2|^2\right)$

Iida et al '04,

$$SU(3)_{C+L+R} \to U(1) \times U(1)$$

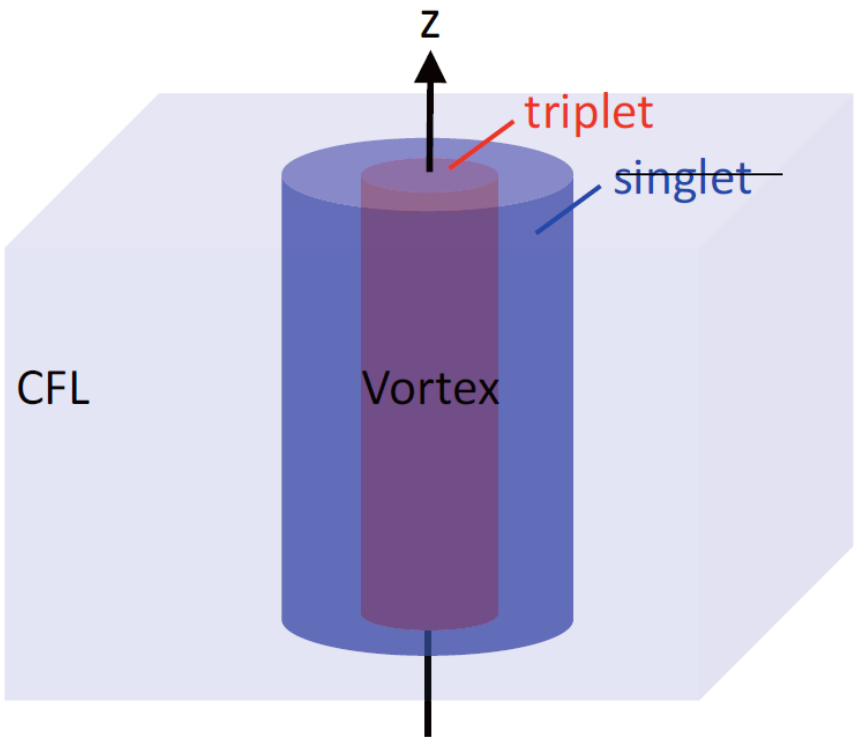


$$|\phi_{ud}|^2 + |\phi_{ds}|^2 + |\phi_{su}|^2 = 1$$

■ Fermion zero modes inside vortex core Yasui et al '10

Fujiwara et al '12

⇒statistical feature of vortex strings



Fermion wave functions :

Multiplets of $H' = SU(2)_{C+F} \times U(1)_{C+F}$

Global Non-Abelian vortex strings in Chiral transition:

$$G = SU(N)_L \times SU(N)_R \times U(1)_A$$

$$\Rightarrow H = SU(N)_{L+R} \times Z_N$$

$$G/H = \frac{SU(N)\times U(1)}{Z_N} = U(N)$$

- **Order parameter always respects U(1)_B sym.**
- **In absence of chiral anomaly.**

$U(1)_A$ vortex string (η' string)

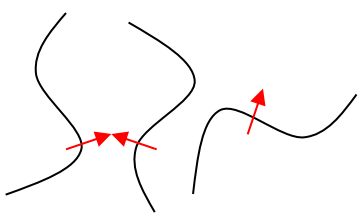
Interaction dependent on orientation (not normalizable):

$$f(\rho) = (1+\cos[\alpha])\frac{\pi}{N\rho}$$

$$\mathrm{diag}(e^{i\theta},1,....,1), \quad \alpha = 0$$

$$\mathrm{diag}(1,e^{i\theta},1,....,1), \quad \alpha = \pi$$

$$\mathrm{diag}(e^{i\theta},1,....1), \quad \alpha = 2\pi N$$



Relax only via NG boson radiation (might be observed in RHIC).

General expression of 3 fundamental vortices :

where $F \equiv f + 2g, \quad G \equiv f - g$

$$\Phi(r,\theta) = \exp\left[i\frac{\theta}{3} - i\sqrt{\frac{2}{3}}\left(Q_3\lambda_3 + Q_8\lambda_8\right)\theta \right]\left[\frac{F(r)}{3} - \sqrt{\frac{2}{3}}\left(Q_3\lambda_3 + Q_8\lambda_8\right)G(r) \right]$$

$$\Rightarrow \begin{pmatrix} e^{i\theta} f(r) & & \\ & g(r) & \\ & & g(r) \end{pmatrix} \text{ or } \begin{pmatrix} g(r) & & \\ & e^{i\theta} f(r) & \\ & & g(r) \end{pmatrix} \text{ or } \begin{pmatrix} g(r) & & \\ & g(r) & \\ & & e^{i\theta} f(r) \end{pmatrix}$$

SU(3) part: three color fluxes

$(Q_3, Q_8) = (0,1)$ Red, $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ Blue, $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ Green

$$\pi_1\left[SU(3)/Z(3)\right] = Z(3)$$