$\pi^0 \to \gamma\gamma$ and chiral anomaly on the lattice

[arXiv:1206.1375, PRL 109 (2012) 182001]

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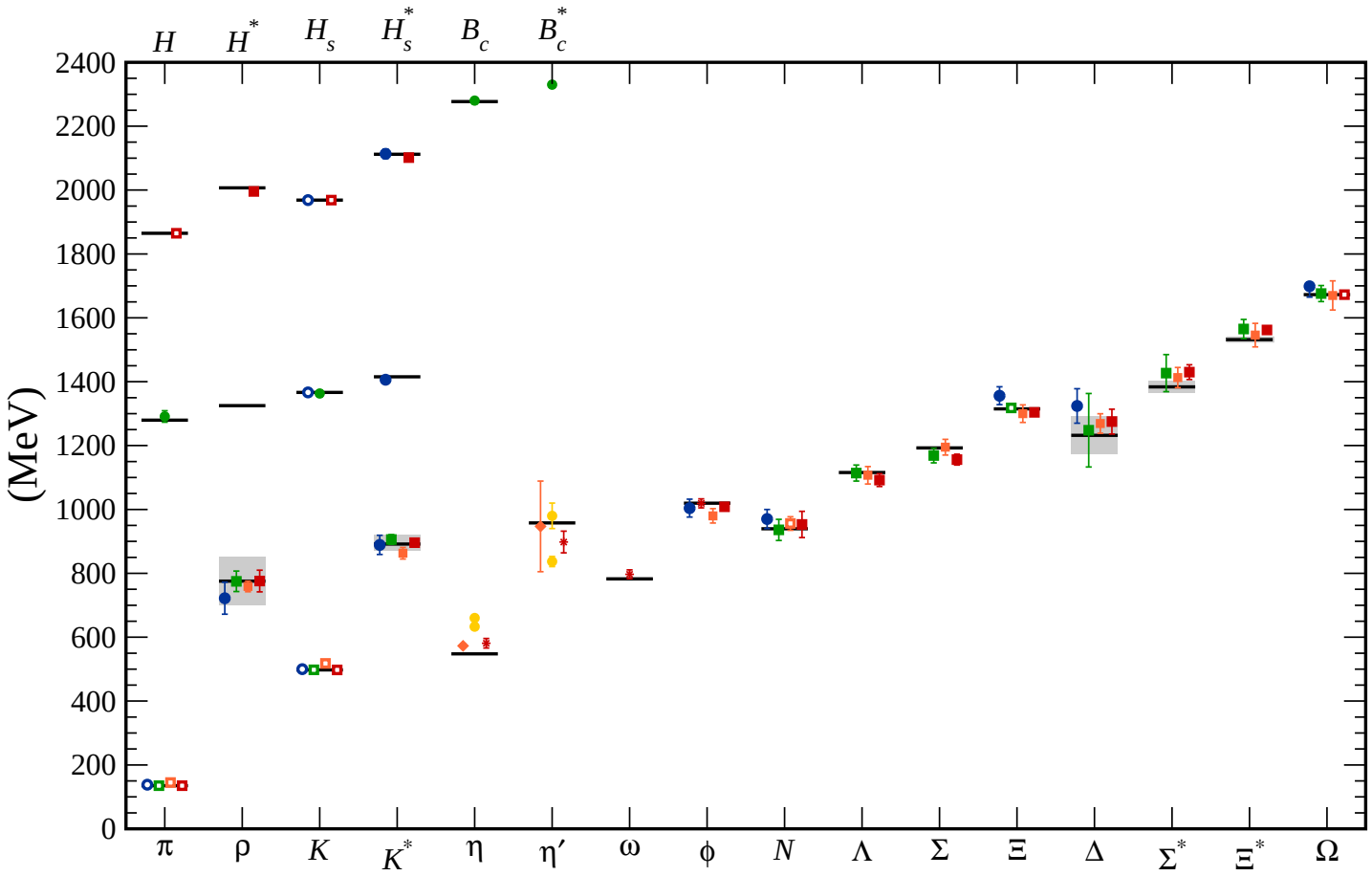
Hadron mass spectrum from lattice QCD, plot by Kronfeld



- hadron spectrum from different group, e.g. diamond η & η' from RBC-UKQCD [Q. Liu et. al., RRL 105 (2010) 241601]
- input: $\alpha_s^{\rm lat}$, $m_{u,d}+m_s$, output: hadron spectrum v.s. experiment

"Gold-plated" quantities and beyond

- lattice QCD works well in, e.g.
  - 2-point correlation function, hadron spectrum
  - some 3-pt function with single hadron or vacuum as initial (final) state
- many interesting physics beyond "gold-plated" quantities
  - $\Delta I = 3/2$ & $1/2$ $K \to \pi\pi$ decay [RBC-UKQCD, Wilson Award 2012] involving multi-hardon state
  - Long distance contribution to ΔM_K [J. Yu et. al. arXiv:1212.5931, RBC-UKQCD]:

$$\int d^4x \int d^4y \langle \bar{K}^0 | \mathrm{T}\{H_W(x) H_W(y)\} | K^0 \rangle$$

    involving a 4-point correlation function (also multi-hadron state)
  - $\pi^0 \to \gamma\gamma$, involving the non-QCD state

$$\pi^0 \to \gamma\gamma$$

$\pi^0 \to \gamma\gamma$

- π^0: lightest hadron, cannot decay into other hadrons
- main decay channel $\pi^0 \to \gamma\gamma$ with a branch rate of 98.8%

- $\pi^0 \to \gamma\gamma$ decay width is related to transition amplitude

$$\langle \gamma(p_1, \lambda_1)\gamma(p_2, \lambda_2)|\pi^0(q)\rangle$$

- γ couples to the hadronic vector current J_μ, integrating out γ-field

$$\int d^4x e^{ip_1 x}\langle\Omega|T\{J_\mu(x)J_\nu(0)\}|\pi^0(q)\rangle = \epsilon_{\mu\nu\alpha\beta}p_1^\alpha p_2^\beta F_{\pi^0\gamma\gamma}(m_\pi^2, p_1^2, p_2^2)$$

  - $\mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, p_1^2, p_2^2)$: $\pi^0 \to \gamma^*\gamma^*$ transition form factor
  - $\epsilon_{\mu\nu\alpha\beta}p_1^\alpha p_2^\beta$: induced by the negative parity of the π^0

Sutherland & Veltman Paradox

- decay width is given by $\Gamma_{\pi^0\gamma\gamma} = (\pi/4)\alpha_e^2 m_\pi^3 \mathcal{F}^2_{\pi^0\gamma\gamma}(m_\pi^2, 0, 0)$
- $\mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, 0, 0)$ can be determined from $\langle 0|J_\mu(p_1)J_\nu(p_2)|\pi^0(q)\rangle$
- using LSZ reduction formula

$$\langle 0|J_\mu(p_1)J_\nu(p_2)|\pi^0(q)\rangle = \langle 0|J_\mu(p_1)J_\nu(p_2)\phi_\pi(q)\rangle(m_\pi^2 - q^2)$$

- using PCAC relation $\partial_\lambda J^5_\lambda = m_\pi^2 F_\pi \phi_\pi$

$$\langle 0|J_\mu(p_1)J_\nu(p_2)\phi_\pi(q)|0\rangle m_\pi^2 F_\pi = q_\lambda \langle 0|J_\mu(p_1)J_\nu(p_2)J^5_\lambda(q)|0\rangle$$

- using V-WI and A-WI, Sutherland & Veltman prove in the chiral limit

$$q_\lambda \langle 0|J_\mu(p_1)J_\nu(p_2)J^5_\lambda(q)|0\rangle|_{p_1^2=p_2^2=0} = q^2 A(q^2)\epsilon_{\mu\nu\alpha\beta}p_{1\alpha}p_{2\beta}$$

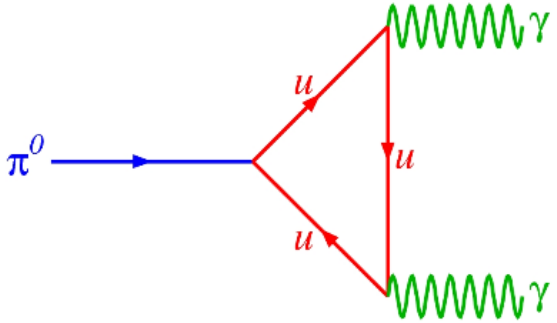
- we finally have $\mathcal{F}_{\pi^0\gamma\gamma}(q^2, 0, 0) = (m_\pi^2 - q^2)/(m_\pi^2 F_\pi)q^2 A(q^2)$

$$q \to 0, \quad \mathcal{F}^2_{\pi^0\gamma\gamma}(0, 0, 0) = 0$$

- pion should *not* decay, but experimentally it decays!

Chiral anomaly

- Paradox is solved by existence of chiral anomaly (ABJ anaomaly)
  - classical level, PCAC relation is valid
  - considering quatumn fluctuation, PCAC relation has to be modified



$$\partial_\lambda J^5_\lambda = m_\pi^2 F_\pi \phi_\pi + \frac{e^2}{8\pi}\epsilon_{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$$

  - ABJ anomaly valid to all perturbative orders [Adler, Bardeen, 1969]
  - Anomaly leads to

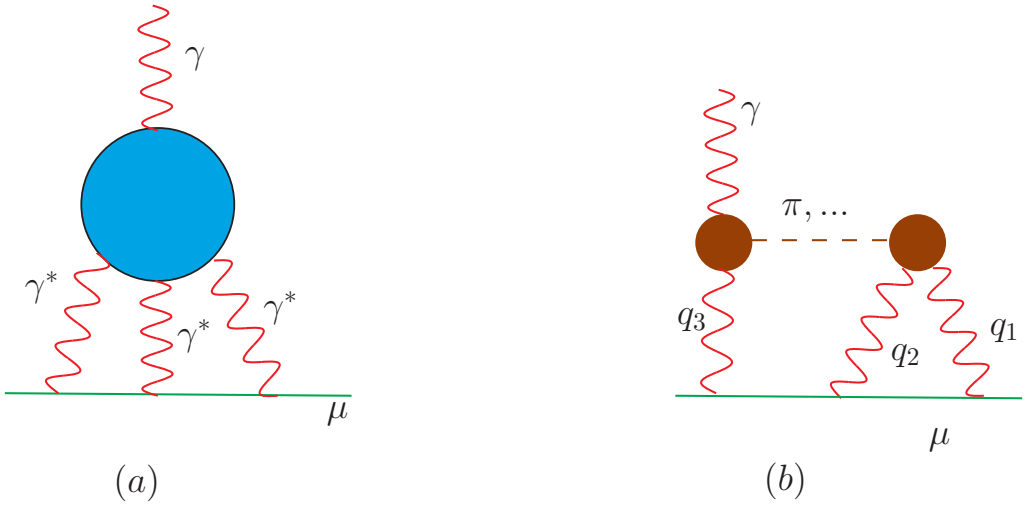
$$\mathcal{F}_{\pi^0\gamma\gamma}(0,0,0) = \frac{N_c}{12\pi^2 F_\pi} \quad \Rightarrow \quad \Gamma_{\pi^0\gamma\gamma} = 7.76\ \mathrm{eV}$$

- PrimEx@Cornell (1974) measured $\Gamma_{\pi^0\gamma\gamma} = 8.02(42)$ eV
  - stringent test for existence of anomaly
  - one of the evidences for $N_c = 3$

Current motivation

- PrimEx@JLab: $\Gamma_{\pi^0\gamma\gamma} = 7.82(22)$ eV [PrimEx, PRL106, 2011]
  - Precision: 2.8% $\rightarrow$ 1.4% (projected goal)
  - Benchmark test of chiral anomaly in QCD

- Muon g-2 provides a test for the existence of NP beyond SM
  - HLbL is one of the dominant theoretical error for muon g-2



  - Difficult: HLbL involves $\langle J_\mu J_\nu J_\rho J_\sigma \rangle$
  - Better to start with $\pi^0(\eta,\ \eta') \rightarrow \gamma^*\gamma^*$
  - pion-exchange contribution: $\mathcal{F}_{\pi^0\gamma\gamma}(p^2, q_3^2, 0)\frac{K(q_1,q_2,q_3)}{p^2-m_\pi^2}\mathcal{F}_{\pi^0\gamma\gamma}(p^2, q_1^2, q_2^2)$

Lattice QCD and chiral anomaly

Anomaly on the lattice

- Conventional fermion formulation, e.g. Wilson, break chiral symmetry
  - No conserved current: $\partial_\mu A_\mu - 2mP \neq 0$
  - Non-zero term yeilds anomaly in continuum limit [Karsten, Smit, 1981]
- Ginsparg-Wilson fermions $\Rightarrow$ modified chiral symmetry on the lattice

$$\delta\bar{\psi} = i\alpha\bar{\psi}(1 - aD/2)\gamma_5, \quad \delta\psi = i\alpha\gamma_5(1 - aD/2)\psi$$

  - measurement produces a Jacobian [Adams 98, Fujikawa 00]

$$\mathcal{D}\bar{\psi}'\mathcal{D}\psi' = \mathcal{J}\mathcal{D}\bar{\psi}\mathcal{D}\psi, \quad \mathcal{J} = \exp[-i\mathrm{Tr}\ \alpha\gamma_5 D] \neq 1$$

  - if gauge config. sufficiently smooth $\Rightarrow$ chiral anomaly

$$\frac{1}{2}\mathrm{Tr}\ \gamma_5 D = \frac{1}{32\pi^2}\epsilon_{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

Lattice setup

- we are using overlap fermion to test the chiral anomaly
  - at $a \sim 0.1$ fm, gauge field is far from smooth $\Rightarrow$ chiral anomaly may not be guaranteed
- calculation of $\pi^0 \to \gamma\gamma$ is nontrivial
  - γ is not an asympototic state of QCD
  - conventional method to extract the eigenstate fails
  - 1^{--} interpolating operator yields vector meson rather than γ
- new method is needed

Analytic continuation method

Non-QCD final state

$\pi^0 \to \gamma\gamma$ has non-QCD final state

$$\langle \gamma(p_1, \lambda_1)\gamma(p_2, \lambda_2)|\pi(q)\rangle$$

- intergrating out the photon fields, leaving

$$\langle \gamma(p_1, \lambda_1)\gamma(p_2, \lambda_2)|\pi(q)\rangle \quad \Rightarrow \quad \int d^4x\, e^{ip_1x}\langle \Omega|T\{J_\mu(x)J_\nu(0)\}|\pi^0(q)\rangle$$

- a simpler case: hadronic vacuum polarization (HVP)

$$\int d^4x\, e^{ipx}\langle \Omega|T\{J_\mu(x)J_\nu(0)\}|\Omega\rangle$$

Analytical continuation in the HVP

- HVP function in Euclidean space-time

$$\Pi_{\mu\nu}(p) = \int d^4x\, e^{ipx} \langle \Omega | T\{J^E_\mu(x) J^E_\nu(0)\} | \Omega \rangle$$

  - On lattice, discrete FFT assign the discrete momentum $p_i = n_i(2\pi/L)$
  - $p^2 = -\sum_i p_i^2 < 0$, always space-like
- what we use (proposed by Ji & Jung, 2001)

$$\int dt\, e^{\omega t} \int d^3\vec{x}\, e^{i\vec{p}\vec{x}} \langle \Omega | T\{J^E_\mu(x) J^E_\nu(0)\} | \Omega \rangle$$

  - $p_0 \to -i\omega$: analytical continuation
  - ω means photon energy, input by hand, thus can be tunned continously
  - $p^2 = \omega^2 - \vec{p}^2$, both space-like and time-like
  - $\langle \Omega | T\{J^E_\mu(t) J^E_\nu(0)\} | \Omega \rangle$ exponentially decrease as $e^{-E_V t}$, $\omega < E_V$

Demonstration: Minkowski space-time

- in Minkowski space-time, in the framework of QCD+QED

$$\langle\gamma(k,\lambda)|J_\nu^M(0)|\Omega\rangle$$

- Using the LSZ reduction formula

$$\langle\gamma(k,\lambda)|J_\nu^M(0)|\Omega\rangle = i \lim_{k'\to k} \varepsilon_\mu^*(k,\lambda)\, k'^2 \int d^4x\, e^{-ik'x} \langle\Omega|\mathrm{T}\{A_\mu^M(x) J_\nu^M(0)\}|\Omega\rangle$$

  where $A_\mu^M(x)$ is a photon field defined in Minkowski space-time

- pQED: $\exp(iS_{\rm int}) = 1 + iS_{\rm int} + \cdots$, $S_{\rm int} = e\int d^4x\, A_\mu^M(x) J_\mu^M(x)$
- At $O(e)$ we have

$$\langle\gamma(k,\lambda)|J_\nu^M(0)|\Omega\rangle = i\, e\, \varepsilon_\mu^*(k,\lambda) \int d^4y\, e^{-iky} \langle\Omega|\mathrm{T}\{J_\mu^M(y) J_\nu^M(0)\}|\Omega\rangle$$

Demonstration: Euclidean space-time

- in Euclidean space-time, look at

$$\begin{aligned} M_{\mu\nu}(t_0,\vec{k}) &= \int d^3\vec{x}\, e^{-i\vec{k}\vec{x}} \langle\Omega|A^E_\mu(\vec{x},t_0)J^E_\nu(0)|\Omega\rangle \\ &\to \sum_\lambda \langle\Omega|A^E_\mu(\vec{k})|\gamma(k,\lambda)\rangle e^{-\omega t_0}\langle\gamma(k,\lambda)|J^E_\nu(0)|\Omega\rangle \end{aligned}$$

  - $\omega < E_V$: neglect hadronic states, suppressed by $e^{-(E_V-\omega)t_0}$
  - neglect three-photon states, suppressed by powers of QED coupling
- Using the pQED

$$M_{\mu\nu}(t_0,\vec{k}) = e\int dt \int d^3\vec{y}\, e^{-i\vec{k}\vec{y}} D_{\mu\rho}(\vec{k},t_0-t)\langle\Omega|J^E_\rho(\vec{y},t)J^E_\nu(0)|\Omega\rangle$$

- propagator $\int_{-\infty}^{\infty}\frac{dk_0}{2\pi}D_{\mu\nu}(k)e^{-ik_0t} = \int_{-\infty}^{\infty}\frac{dk_0}{2\pi}\frac{\delta_{\mu\nu}}{k_0^2+\vec{k}^2}e^{-ik_0t} = \frac{e^{-\omega|t|}}{2\omega}\delta_{\mu\nu}$
- we can write

$$M_{\mu\nu}(t_0,\vec{k}) \sim e\int dt\, e^{-\omega|t_0-t|}\int d^3\vec{y}\, e^{-i\vec{k}\vec{y}}\langle\Omega|J^E_\mu(\vec{y},t)J^E_\nu(0)|\Omega\rangle$$

Demonstration of the analytical continuation

- we get in Euclidean space-time

$$\langle\gamma(k,\lambda)|J^E_\nu(0)|\Omega\rangle = e\,\varepsilon^*_\mu(\lambda)\int dt\; e^{\omega t}\int d^3\vec{y}\; e^{-i\vec{k}\vec{y}}\langle\Omega|J^E_\mu(\vec{y},t)J^E_\nu(0)|\Omega\rangle$$

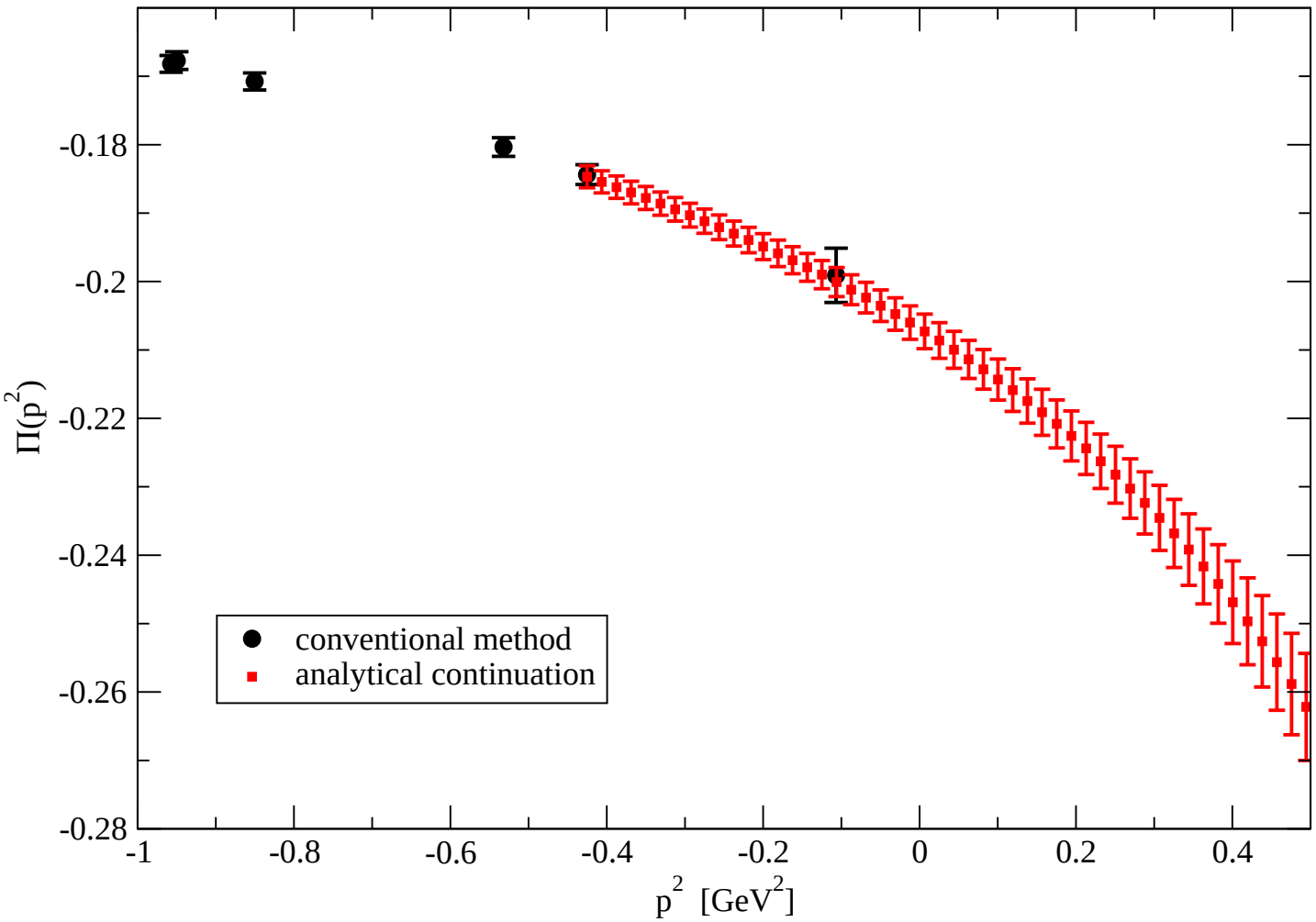
- remember that in Minkowski space-time

$$\langle\gamma(k,\lambda)|J^M_\nu(0)|\Omega\rangle = i\;e\,\varepsilon^*_\mu(k,\lambda)\int d^4y\; e^{-iky}\langle\Omega|\mathrm{T}\{J^M_\mu(y)J^M_\nu(0)\}|\Omega\rangle$$

- this method is originally proposed by Ji & Jung [PRL 86 (2001) 208, PRD 64 (2001) 034506]

- first used in dynamical Lattice QCD calc. by Dudek & Edwards, charmonium radiative decay [PRL 97 (2006) 172001]

Results for HVP function



- conventional method: $p^2 = -\sum_i p_i^2$, $p_i = n_i(2\pi/L)$
- analytical continuation: $p^2 = \omega^2 - \vec{p}^2$, $\vec{p}$ discrete but ω continous

Analytical continuation: $\pi^0 \to \gamma\gamma$

- $\pi^0 \to \gamma\gamma$

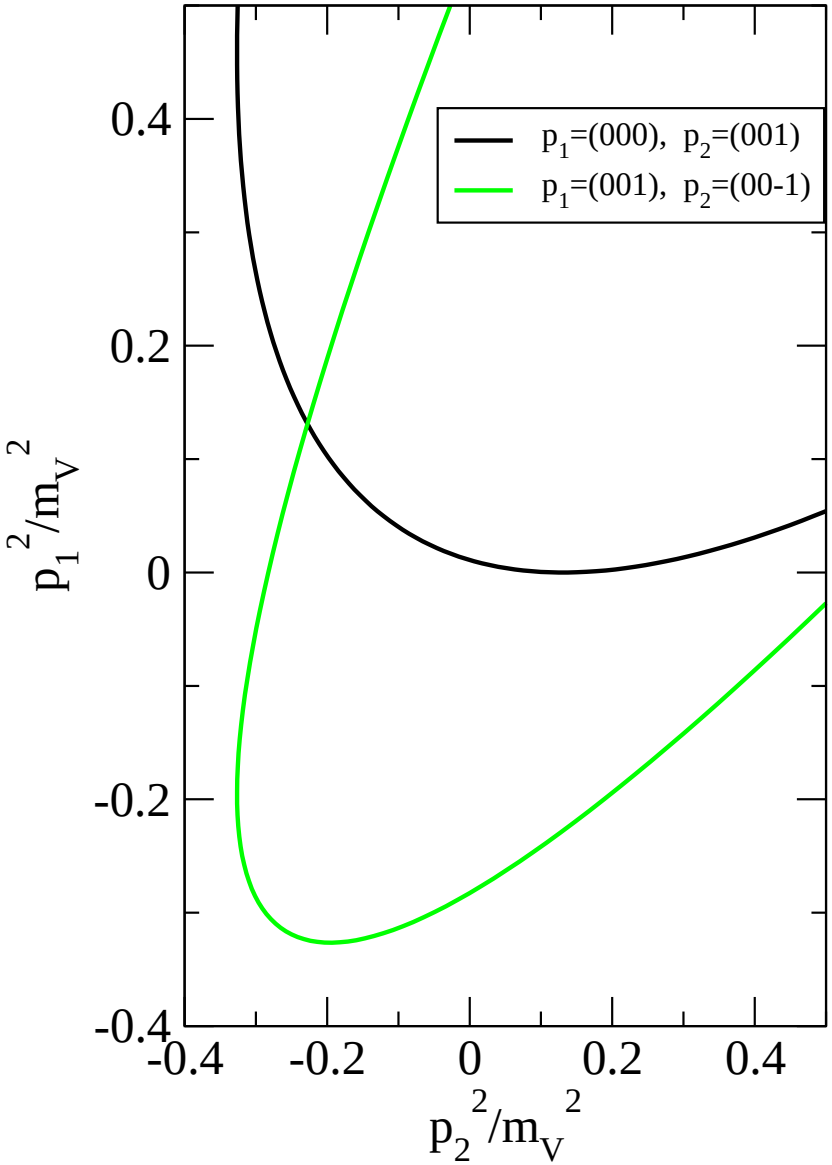
$$\langle\gamma(p_1,\lambda_1)\gamma(p_2,\lambda_2)|\pi(q)\rangle \quad \Rightarrow \quad \int d^4x\, e^{ip_1x}\langle\Omega|T\{J_\mu(x)J_\nu(0)\}|\pi^0(q)\rangle$$

- apply analytical continuation method

$$\mathcal{M}_{\mu\nu} = \int dt\, e^{\omega t} \int d^3\vec{x}\, e^{-i\vec{p}_1\cdot\vec{x}}\langle\Omega|T\{J_\mu(x)J_\nu(0)\}|\pi^0(q)\rangle$$

- pion is on-shell: $q = (E_\pi, \vec{q}) \Rightarrow q^2 = m_\pi^2$
- photon 4-momentum: $p_1 = (\omega, \vec{p}_1)$ and $p_2 = (E_\pi - \omega, \vec{q} - \vec{p}_1)$
- requirement: $p_{1,2}^2 < m_\rho^2$ or $E_{\pi\pi}^2$ (hadron production threshold)
- transition form factor $\mathcal{M}_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta}p_1^\alpha p_2^\beta F_{\pi^0\gamma\gamma}(m_\pi^2, p_1^2, p_2^2)$

Momentum domain



- perform integration:
  $\int dt\ e^{\omega t} \langle \Omega | T\{J_\mu(\vec{p}_1, t) J_\nu(\vec{p}_2, 0)\} | \pi^0 \rangle$
  $\Rightarrow \mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, p_1^2, p_2^2)$
  - form factor relies on p_1^2 & p_2^2
- $\vec{p}_1$, $\vec{p}_2$, $\vec{q}$ for two γ and π^0 fixed
  $\langle \Omega | T\{J_\mu(\vec{p}_1, t) J_\nu(\vec{p}_2, 0)\} | \pi^0(q) \rangle$
- tunning ω
  $p_1^2 = \omega^2 - \vec{p}_1^2$
  $p_2^2 = (E_\pi - \omega)^2 - \vec{p}_2^2$
- diff. spatial momentum setup,
  diff. contour of (p_1^2, p_2^2)

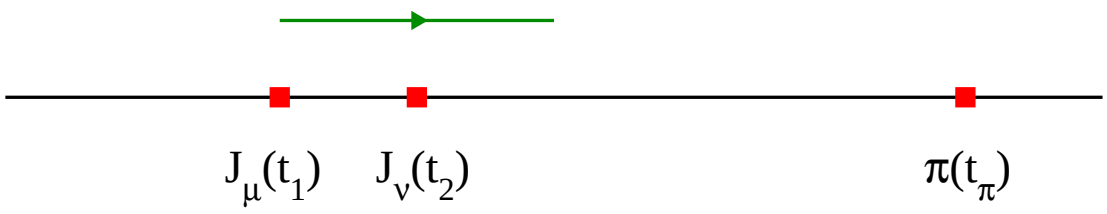
Amplitude $A_\pi(\tau)$

- observable:

$$\mathcal{M}_{\mu\nu} = \int dt\ e^{\omega t} \int d^3\vec{x} e^{-i\vec{p}_1\cdot\vec{x}} \langle\Omega| T\{J_\mu(x) J_\nu(0)\} |\pi^0(q)\rangle$$

- correlation function:

$$C_{\mu\nu} = \langle\Omega| J_\mu(\vec{p}_1, t_1) J_\nu(\vec{p}_2, t_2) \pi^0(-\vec{q}, t_\pi) |\Omega\rangle$$



- set $\tau = t_1 - t_2$ and $t = \min\{t_1, t_2\}$

$$A_\pi(\tau) \equiv \lim_{t - t_\pi \to \infty} C_{\mu\nu}(t_1, t_2, t_\pi) / e^{-E_\pi (t - t_\pi)}$$

- matrix element $\mathcal{M}_{\mu\nu}$ is given by

$$\mathcal{M}_{\mu\nu} = \frac{2E_\pi}{\phi_\pi} \left(\int_0^\infty d\tau\ e^{\omega\tau} A_\pi(\tau) + \int_{-\infty}^0 d\tau\ e^{(\omega - E_{\pi,\vec{q}})\tau} A_\pi(\tau) \right)$$

All-to-all propagators

- use stochastic source $\eta(y)$, propagator $\phi(x) = D^{-1}(x,y)\eta(y)$

$$\eta(y)\eta^*(y') = \delta_{y,y'} + \text{noise} \quad \Rightarrow \quad \phi(x)\eta^*(y) = D^{-1}(x,y) + \text{noise}$$

- propagagtor can also be constructed from the eigenvectors

$$D\ u^{(k)}(x) = \lambda^{(k)} u^{(k)}(x) \quad \Rightarrow \quad D^{-1}(x,y) = \sum_k \frac{1}{\lambda^{(k)}} u^{(k)}(x) u^{(k)\dagger}(y)$$

low-lying eigenvectors dominate long-distance behavior of correlator [T. DeGrand et. al., PRD 64 (2001) 034512]

- all-to-all propagator

$$D^{-1}(x,y) = \sum_{k=1}^{N_{ev}} \frac{1}{\lambda^{(k)}} u^{(k)}(x) u^{(k)\dagger}(y) + \sum_{d=1}^{N_d} \left[D^{-1}_{\text{high}}\ \eta^{(d)} \right](x) \eta^{(d)\dagger}(y)$$

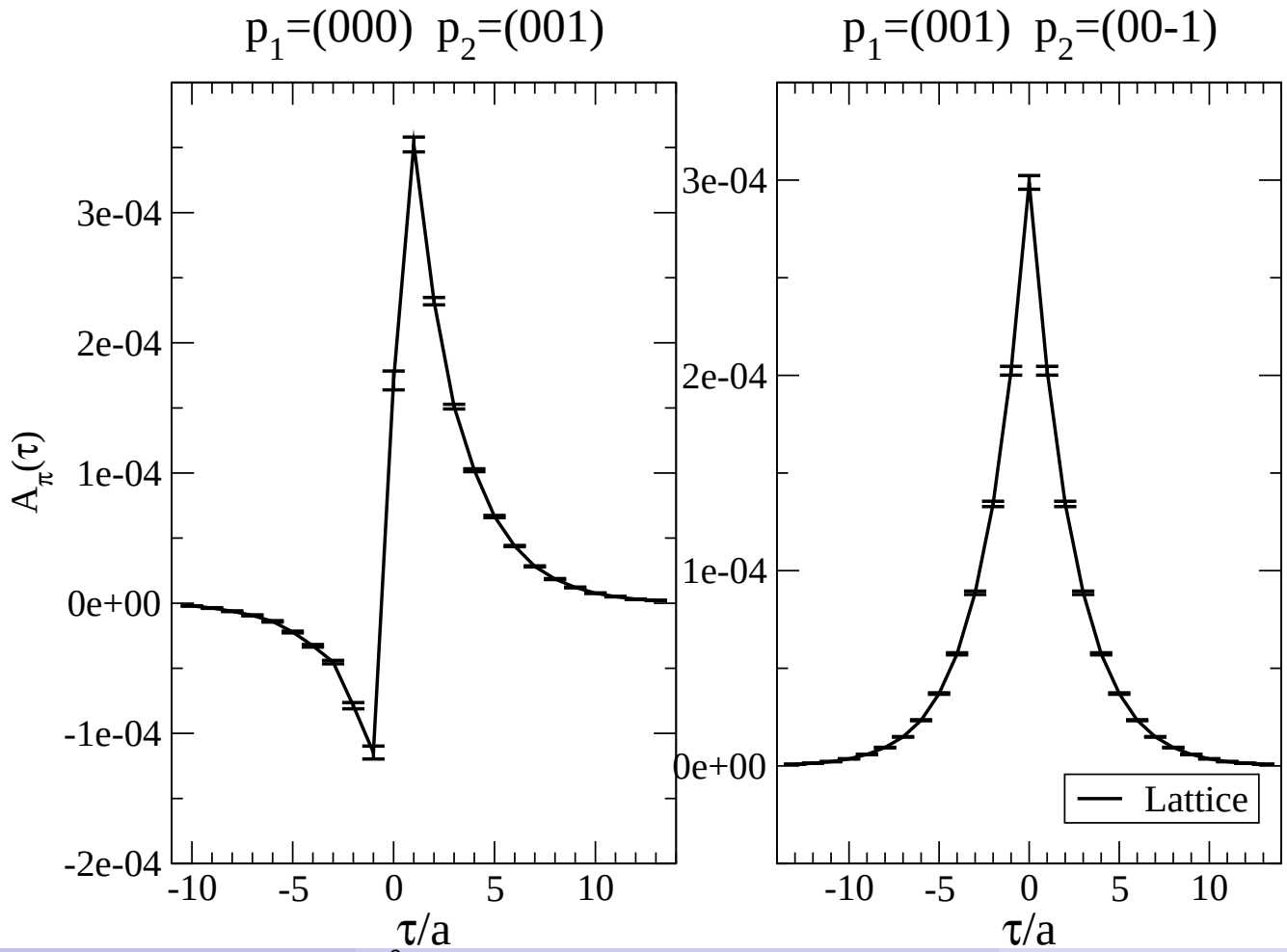
- $24^3 \times 48$ lattice: $N_{ev} = 120 \times 2, \quad N_d = (T/2) * N_s * N_c = 288$
- calculate $\langle J_\mu(t_1) J_\nu(t_2) \pi^0(t_\pi) \rangle$ at any t_1, t_2, t_π with a good precision
- all mode averaging method [T. Izubuchi et. al. arXiv:1208.4349]

Numerical results

Amplitude $A_\pi(\tau)$: result

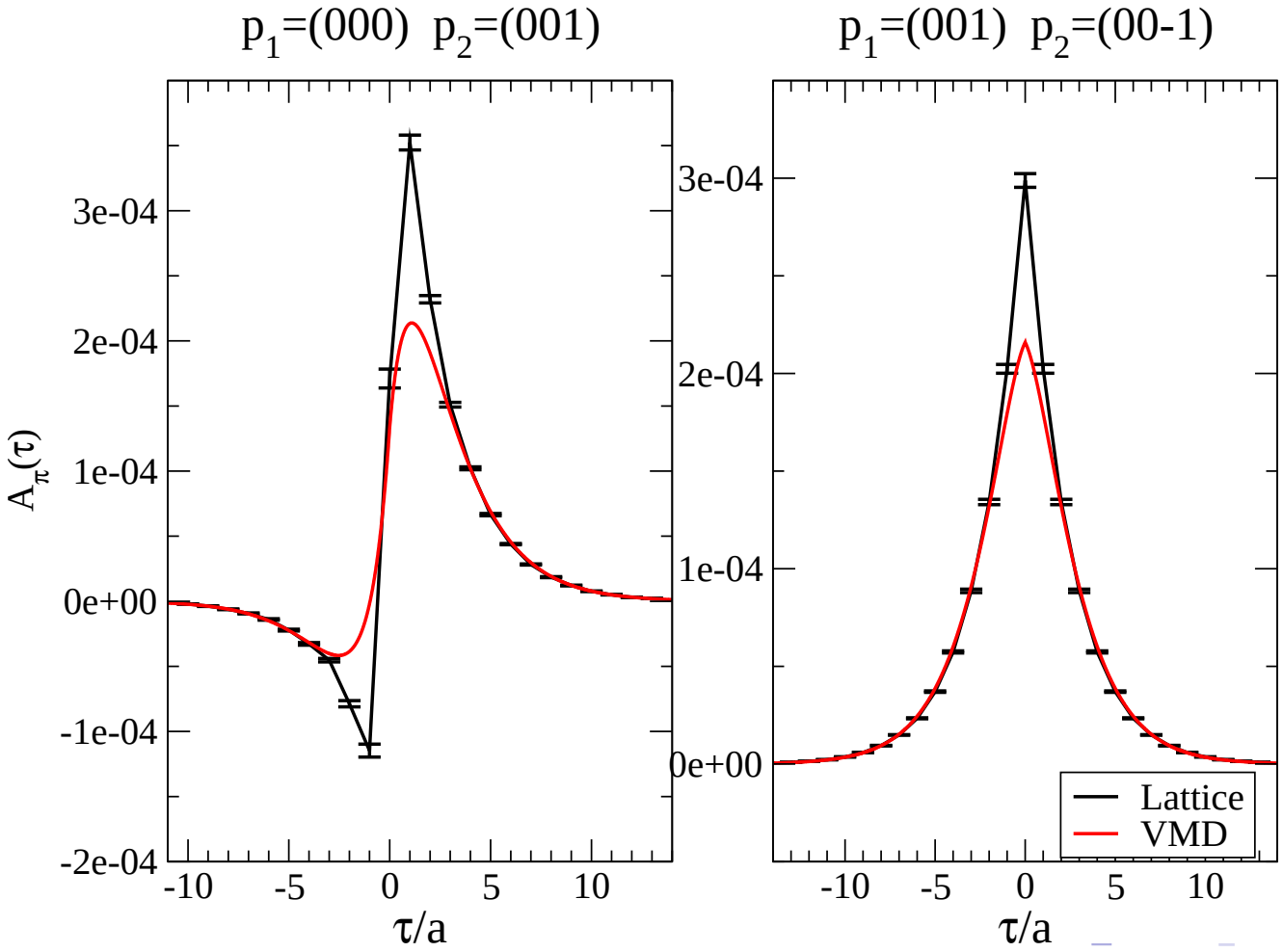
- extract $|\pi^0(q)\rangle$: set $\tau = t_1 - t_2$ and $t = \min\{t_1, t_2\}$

$$A_\pi(\tau) \equiv \lim_{t-t_\pi\to\infty} \langle\Omega|J_\mu(\vec{p}_1, t_1)J_\nu(\vec{p}_2, t_2)\pi^0(-\vec{q}, t_\pi)|\Omega\rangle / e^{-E_\pi(t-t_\pi)}$$



Time dependence of $A_\pi(\tau)$

- VMD model: $\mathcal{F}^{\rm VMD}_{\pi^0\gamma\gamma}(m_\pi^2, p_1^2, p_2^2) = c_V G_V(p_1^2) G_V(p_2^2)$
- $G_V(p^2) = M_V^2/(M_V^2 - p^2)$ is the (lightest) vector meson propagator



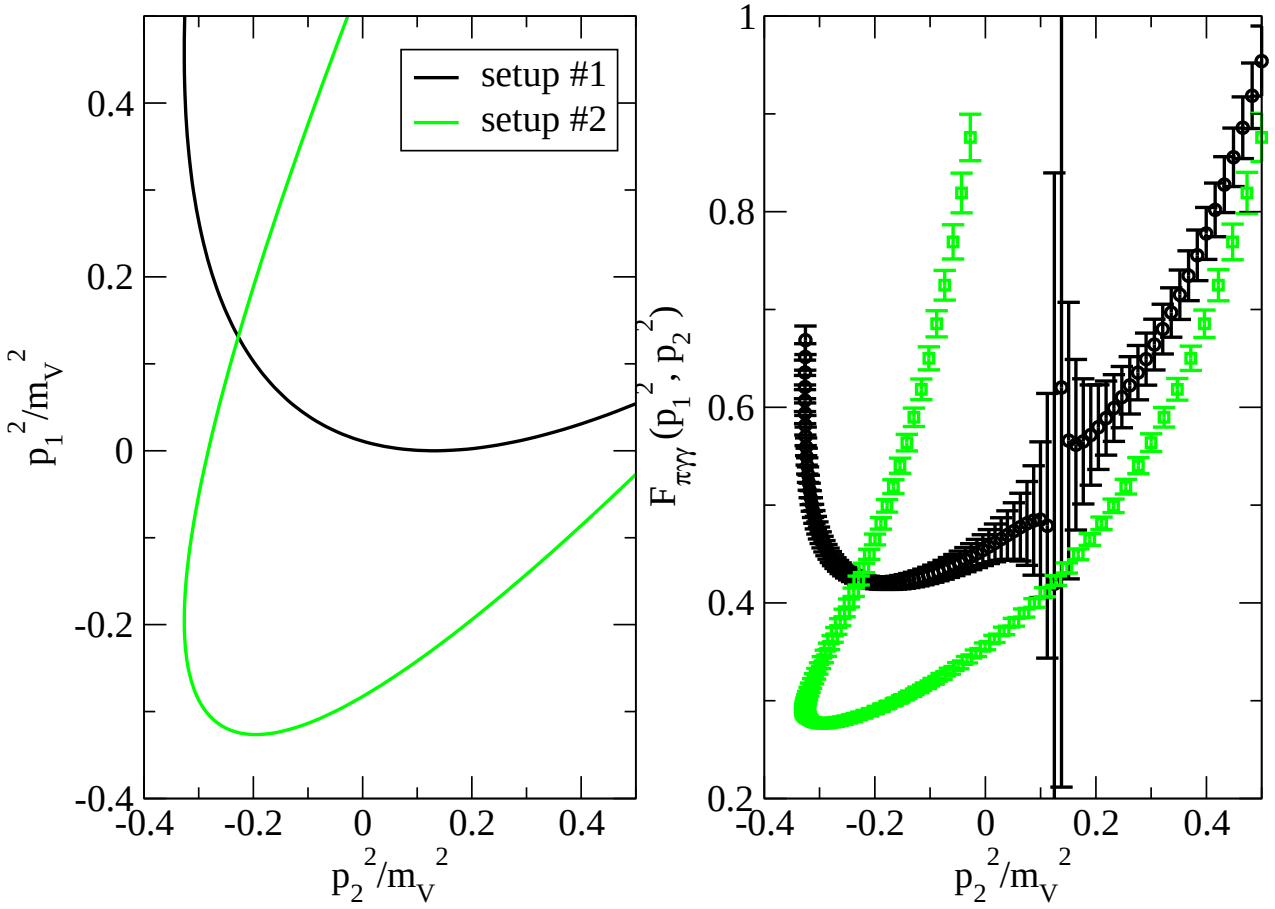
Fit ansatz

- lowest vector meson effects should be accounted for first
- corrected by including excited-state effects
- includes contributions from excited states as a polynomial of $p_{1,2}^2$

$$\begin{aligned}\mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, p_1^2, p_2^2) &= c_V G_V(p_1^2) G_V(p_2^2) \\ &+ \sum_m c_m \left((p_2^2)^m G_V(p_1^2) + (p_1^2)^m G_V(p_2^2)\right) \\ &+ \sum_{m,n} c_{m,n} (p_1^2)^m (p_2^2)^n\end{aligned}$$

Form factor

- along contour of (p_1^2, p_2^2) (left), $\mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, p_1^2, p_2^2)$ (right) determined

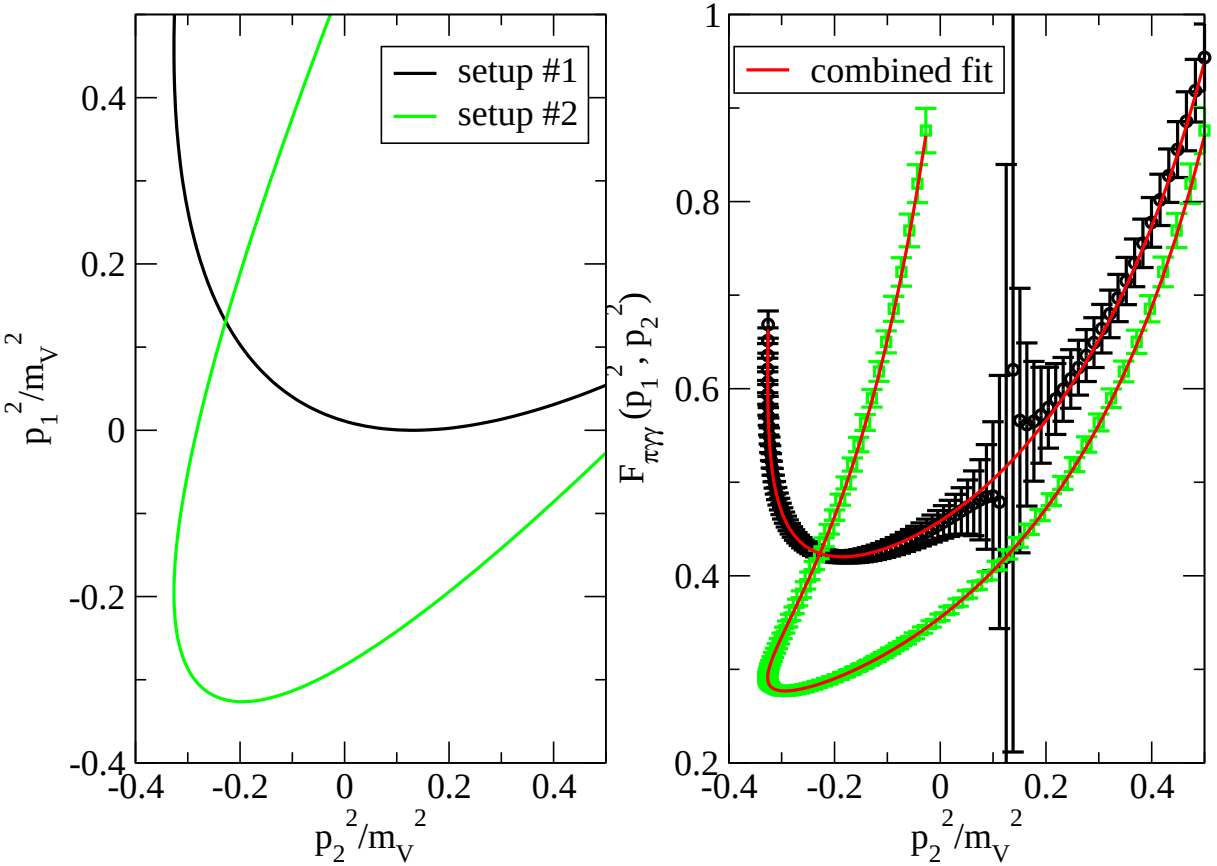


- $\mathcal{F}_{\pi^0\gamma\gamma} = \mathcal{M}_{\mu\nu}(\omega)/\epsilon_{\mu\nu\alpha\beta}p_1^\alpha p_2^\beta$, for $\omega = \omega_0$, $\epsilon_{\mu\nu\alpha\beta}p_1^\alpha p_2^\beta = 0$

Combined fit

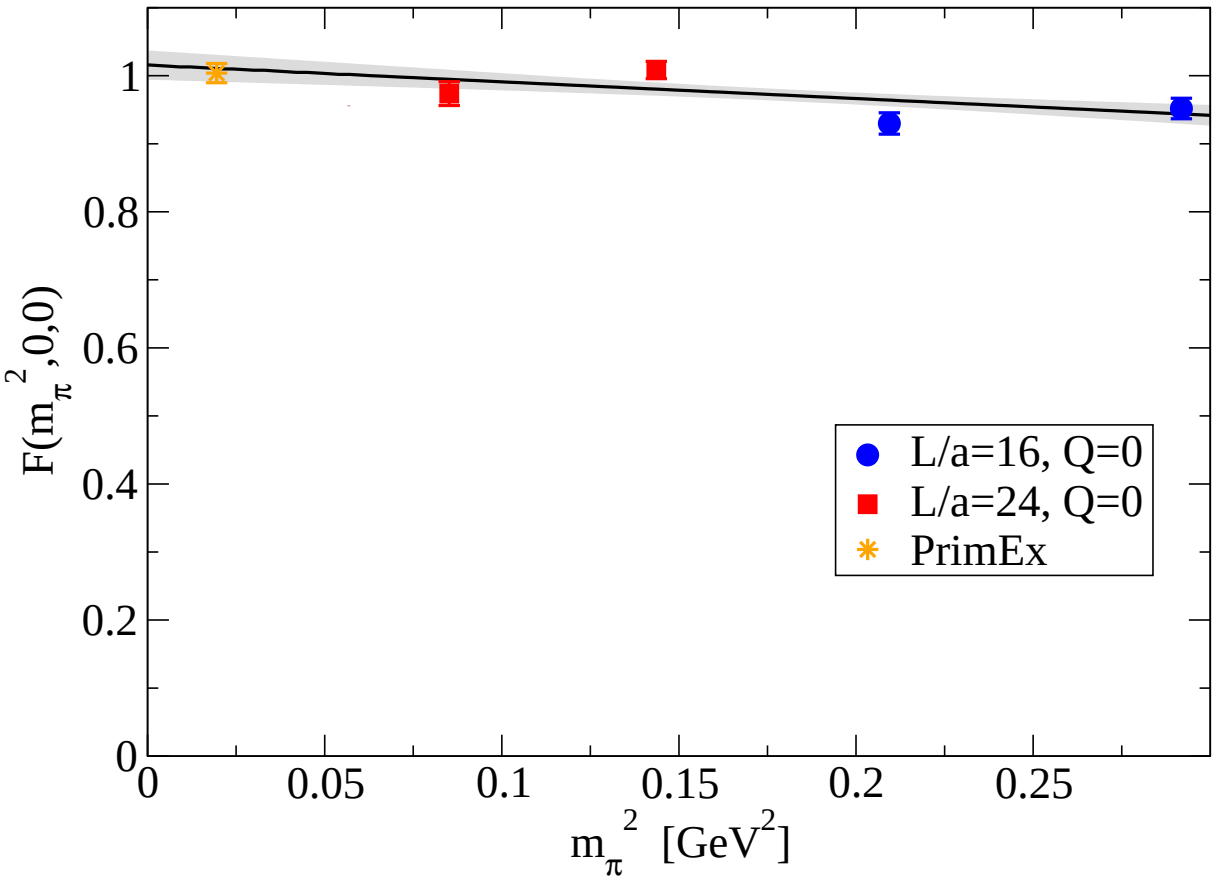
- fit ansatz $\mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, p_1^2, p_2^2) = c_V G_V(p_1^2) G_V(p_2^2) +$

$$\sum_m c_m \left((p_2^2)^m G_V(p_1^2) + (p_1^2)^m G_V(p_2^2) \right) + \sum_{m,n} c_{m,n} (p_1^2)^m (p_2^2)^n$$



- determine c_V, c_0, $c_{0,0}$, $c_{0,1} = c_{1,0}$, non-zero value $\Rightarrow$ diff. from VMD

On-shell photon limit



- $F(m_\pi^2, 0, 0) \equiv \mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, 0, 0)/\mathcal{F}^{\rm ABJ}_{\pi^0\gamma\gamma}$
- known from ChPT: up to NLO, $F(m_\pi^2, 0, 0)$ involves no chiral log [Donoghue et. al. 1985, Bijnens et. al. 1988]
- data with $m_\pi L \geq 4$: consistent with ABJ and PrimEx

Results

- use the data with $m_\pi L < 4$ to check finite-size effect
- use ensembles with $Q = 0$ and $Q = 1$ to check fixing-topology effect
- include disconnected diagrams
- after examining possible systematic effects

$$\begin{aligned} F(0,0,0) &= 1.009(22)(29) \\ F(m^2_{\pi,\mathrm{phy}},0,0) &= 1.005(20)(30) \\ \Gamma_{\pi^0\gamma\gamma} &= 7.83(31)(49)\ \mathrm{eV} \end{aligned}$$

- ABJ anomaly and PrimEx measurement

$$\begin{aligned} F(0,0,0) &= 1 \\ F(m^2_{\pi,\mathrm{phy}},0,0) &= 1.004(14) \\ \Gamma_{\pi^0\gamma\gamma} &= 7.82(22)\ \mathrm{eV} \end{aligned}$$

Discussion

- our results
  - agree with ABJ and PrimEx, but with larger err $\Rightarrow$ better precision
  - $m_u = m_d$, isospin breaking effects are not yet studied
  - pion mass dependence mild
- pheno analysis, review by Bernstein & Holstein [arXiv:1112.4809]
  - ChPT, sum rule $\cdots$, predicts a 4-5% enhancement in $\Gamma_{\pi^0\gamma\gamma}$ vs. ABJ
  - precisions of these predications are at 1% level
  - mainly due to the isospin breaking, where π^0 mix with η & η'
- future task
  - improve statistics, understanding isospin breaking effect

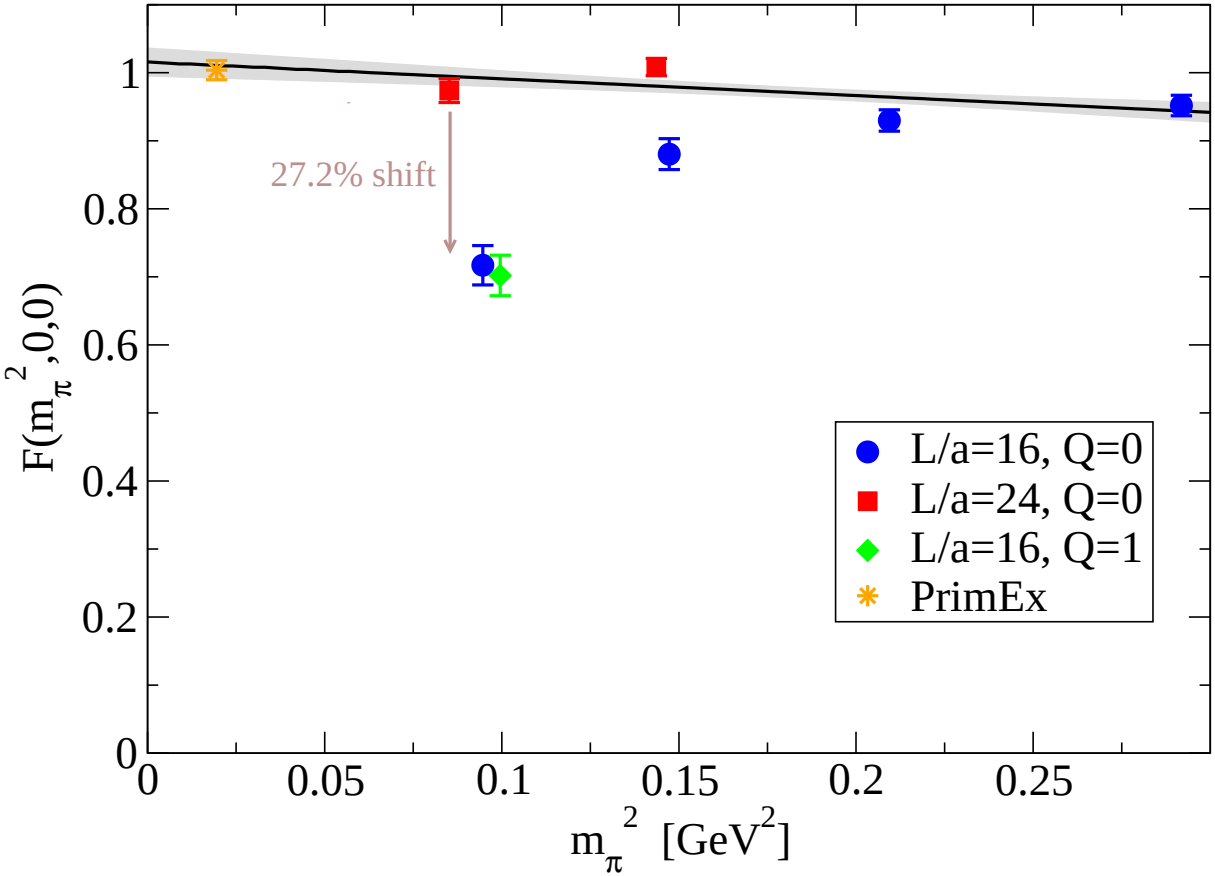
Conclusions

- $\pi^0 \to \gamma\gamma$ calculation is done
  - by analytic continuation
  - using all-to-all propagators
- chiral lattice fermion works well here
  - ABJ anomaly confirmed in the chiral limit
  - although expensive, theoretically clean formulation is helpful
- Non-QCD state can be treated on the lattice, extend to new project
  - $\eta, \eta' \to \gamma\gamma$ or other hadron to $\gamma\gamma$
  - charged pion radiative decay: $\pi^+ \to l^+\nu_l\gamma$, one γ plus one W^+
  - muon $g-2$, light-by-light, dominated by $\gamma\gamma^* \to \pi^0 \to \gamma^*\gamma^*$
  - photon-Nucleon Compton scattering: $\gamma^* + N \to \gamma^* + N$

Backup slides

Analysis of systematic effects

On-shell photon limit



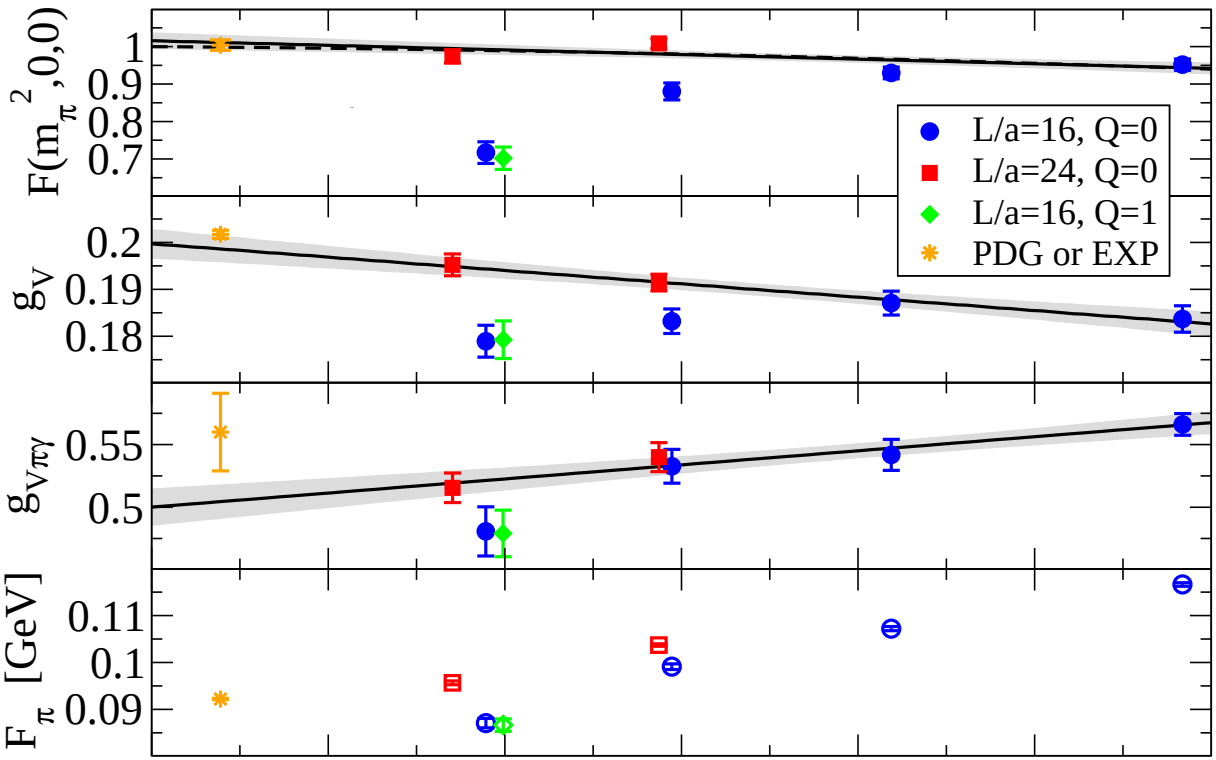
- $F(m_\pi^2, 0, 0) \equiv \mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, 0, 0)/\mathcal{F}^{\rm ABJ}_{\pi^0\gamma\gamma}$
- data with $m_\pi L \geq 4$: consistent with ABJ and PrimEx
- $L = 16$: smallest two quark mass, $m_\pi L < 4$, big FS effects
- FS effects checked at topological sector $Q = 0$ and 1

Finite-size effects

- expand the correlator into three hadronic matrix elements:

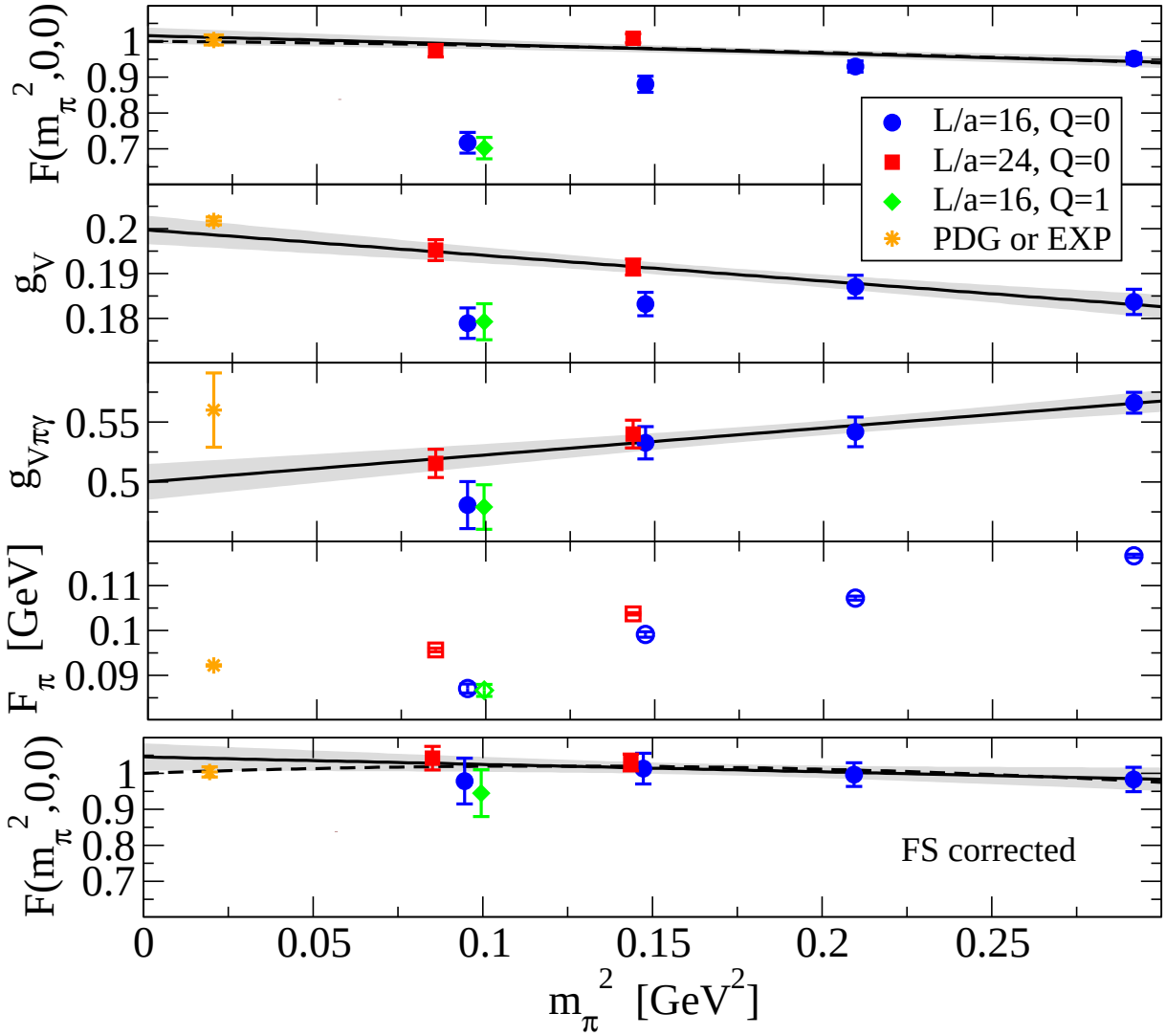
$$\langle J_\mu J_\nu \pi^0 \rangle \to \langle \Omega | J_\mu | V \rangle \langle V | J_\nu | \pi^0 \rangle \langle \pi^0 | \pi^0 | \Omega \rangle$$

g_V $g_{V\pi\gamma}$ F_π



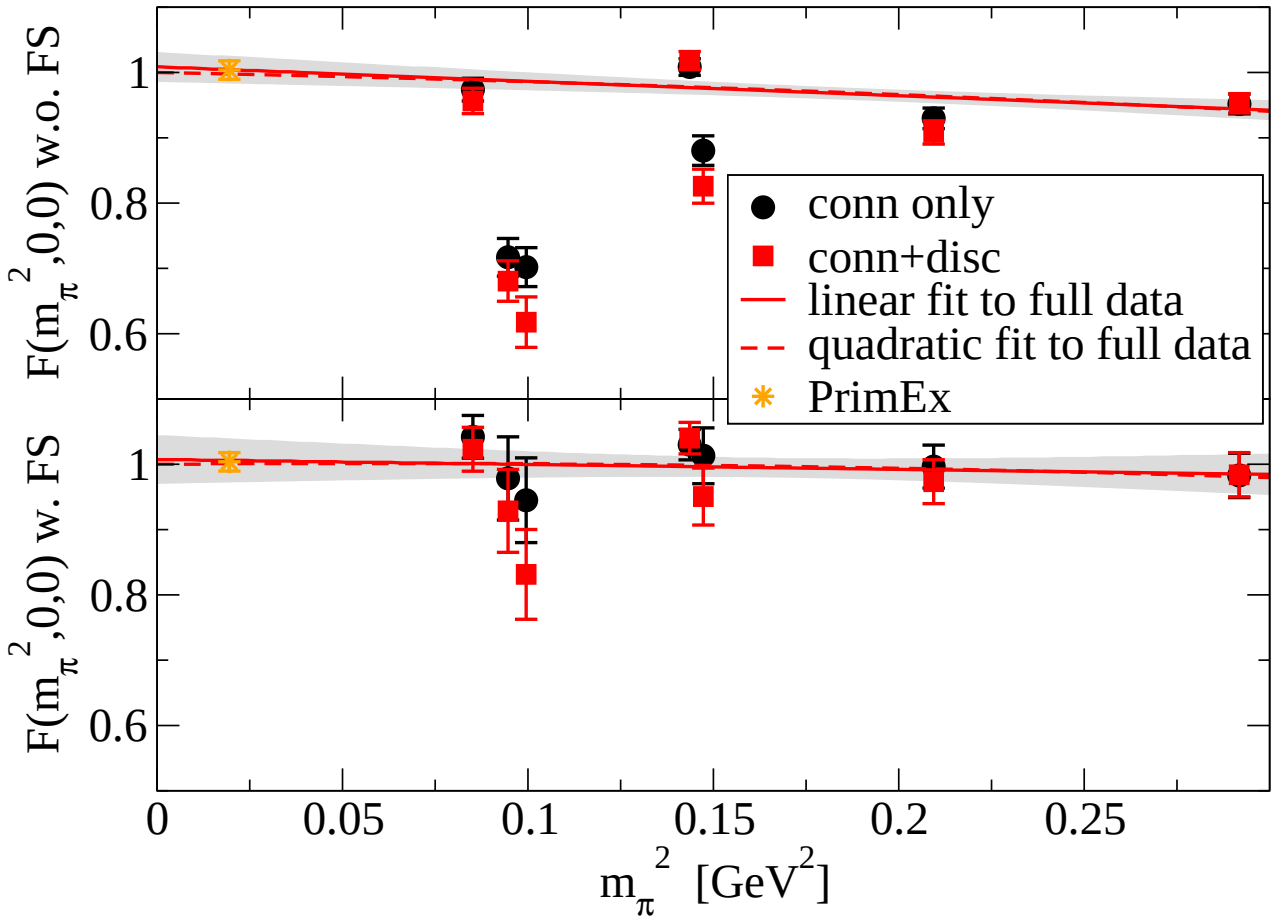
- FS effects in g_V, $g_{V\pi\gamma}$, F_π accumulate and add up to a large effect

Finite-size corrections



- FS corrections $R_{\mathcal{O}} \equiv \mathcal{O}(\infty)/\mathcal{O}(L)$, assume $R_{F(m_\pi^2,0,0)} = R_{g_\rho} R_{g_{\rho\pi\gamma}} R_{F_\pi}$

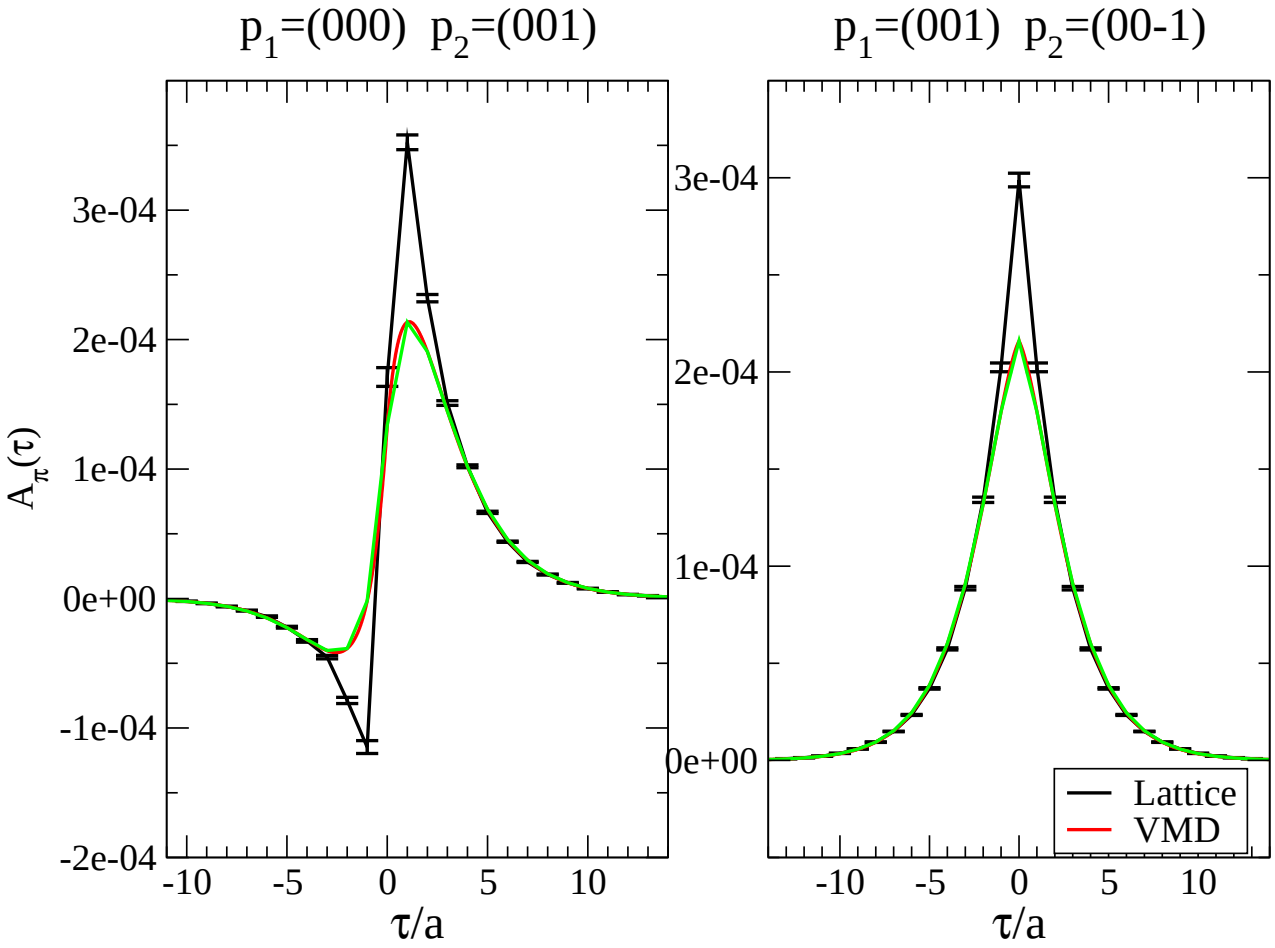
Disconnected-diagram effects



- all-to-all propagator: control error of disc. contribution
- although not significant, conn+disc systematically shift down
- precision level (3% for form factor): disc. diagram should be included

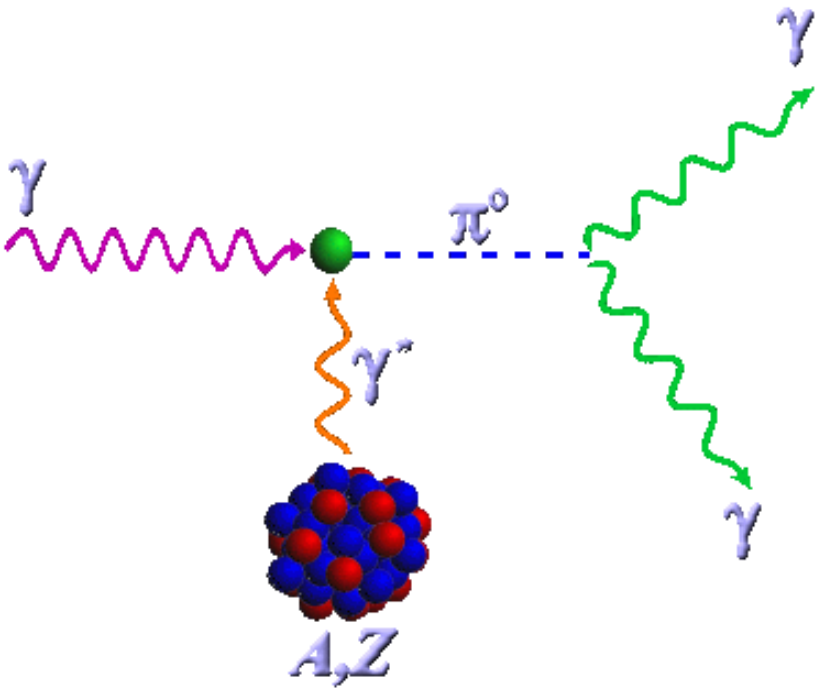
Lattice artifacts

- discrete data v.s. continuum case?



- disc. effects in VMD model: less than 5×10^{-4}, neglegiable

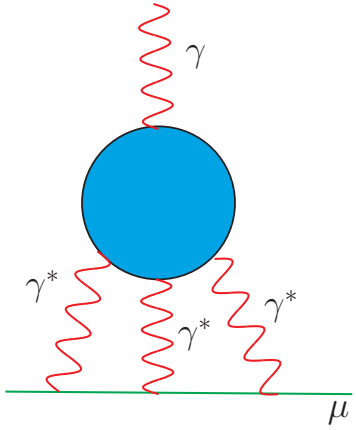
Primakoff effect



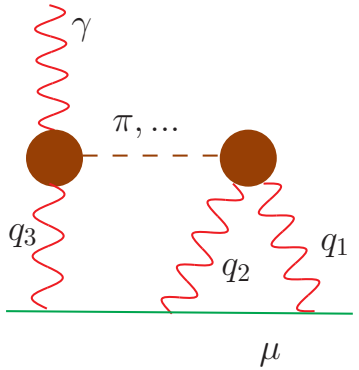
- high-energy photon interact with an atomic nucleus
- at small angles this reaction is dominated by $\gamma + \gamma^* \to \pi^0$
- γ^* is due to the Coulomb field of the nucleus

Motivated by muon $g_\mu - 2$

- Exp. determination of $g_\mu - 2$ to 0.54 ppm [E821@BNL, PRD73, 2006]
- S.M. prediction of $g_\mu - 2$ to 0.51 ppm [Jegerlehner, EPJC71, 2011]
- Discrepancy: $3.3\sigma \Rightarrow$ New Physics ??
- HLbL is predicted to be dominant error in the next round



(a)



(b)

- Difficult: HLbL involves $\langle J_\mu J_\nu J_\rho J_\sigma \rangle$
- Better to start with $\pi^0(\eta,\ \eta') \to \gamma^*\gamma^*$

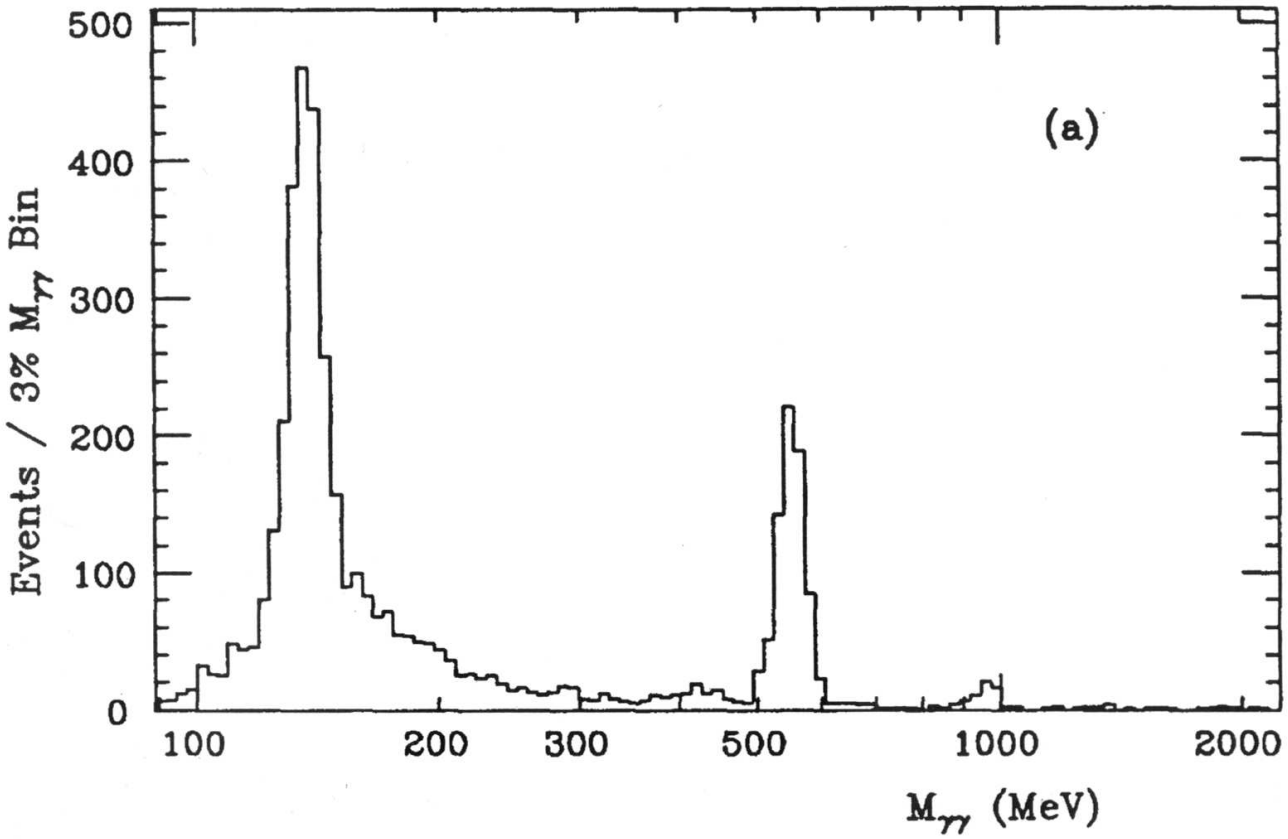
π^0 contribution

Contribution	BPP	HKS	KN	MV	BP	PdRV	N/JN
π^0, η, η'	85±13	82.7±6.4	83±12	114±10	–	114±13	99±16
π, K loops	−19±13	−4.5±8.1	–	–	–	−19±19	−19±13
π, K loops + other subleading in N_c	–	–	–	0±10	–	–	–
axial vectors	2.5±1.0	1.7±1.7	–	22± 5	–	15±10	22± 5
scalars	−6.8±2.0	–	–	–	–	−7± 7	−7± 2
quark loops	21± 3	9.7±11.1	–	–	–	2.3	21± 3
total	83±32	89.6±15.4	80±40	136±25	110±40	105±26	116±39

- summary table [Jegerlehner, Nyffeler, Phys.Rept.477:1-110,2009]
  - $\pi^0(\eta,\ \eta') \to \gamma^*\gamma^*$ are consistent to total contributions
  - Among three PS mesons, π^0 takes about $\tilde{7}0\%$
  - calulation on the $\pi^0 \to \gamma^*\gamma^*$ can be duplicated to the η, η' sector

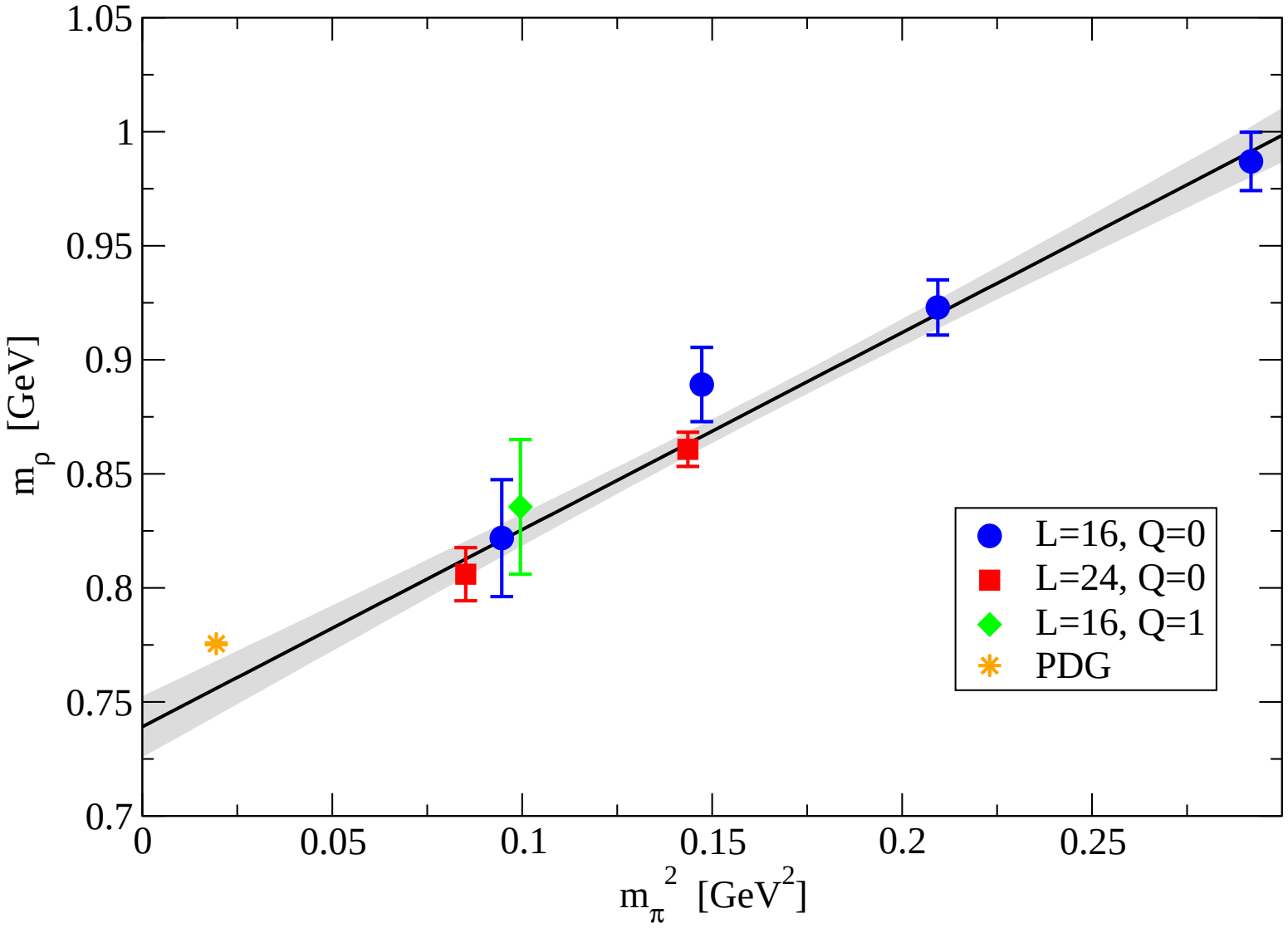
Non-perturbative nature

- Invariant mass spectrum for $\gamma\gamma$ [Crystal Ball Dector @SLAC, 1988]



- Three spikes present three bound states: π^0, η, η'
- Bound states $\rightarrow$ confinement $\rightarrow$ Lattice QCD

Rho mass


1.05
1
0.95
0.9
0.85
0.8
0.75
0.7
0
0.05
0.1
0.15
0.2
0.25
m_ρ [GeV]
m_π^2 [GeV^2]
L=16, Q=0
L=24, Q=0
L=16, Q=1
PDG