PQCD factorization for heavy quarkonium production and fragmentation functions

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Based on works done with: Y.-Q Ma and J. Qiu

Outline

- ✧ Introduction
- ✧ PQCD factorization for heavy quarkonia production
- ✧ Apply NRQCD to calculate fragmentation functions
- ✧ Summary and outlook

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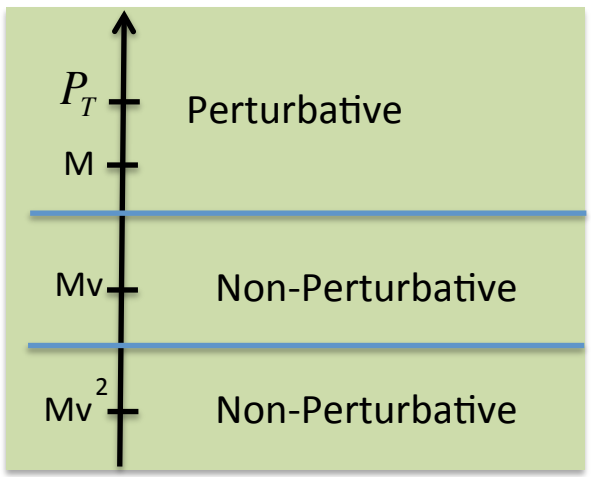
Heavy Quarkonium

- Effectively, a non-relativistic QCD bound state

  Charmonium: $v^2 \approx 0.3$ Bottomonium: $v^2 \approx 0.1$

- Multiple Scales



Detect full energy range of QCD

Hard—Production of $Q\bar{Q}$

Soft—Relative Momentum

Λ_{QCD}

Ultrasoft—Binding Energy

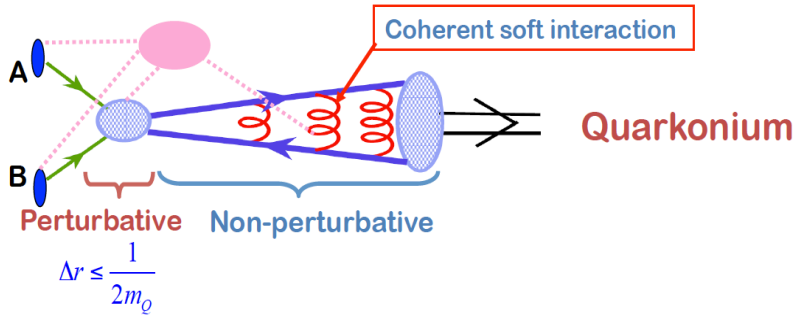
- Spectroscopic notation

$c\bar{c}$: $\eta_C(^1S_0)$, $J/\psi(^3S_1)$, $\chi_{CJ}(^3P_J)$ … 2S, 2P…

$b\bar{b}$: …$\Upsilon(^3S_1)$, $\chi_{bJ}(^3P_J)$ … 2S, 2P…

Historical Models: CSM

- **Conjectured** factorization form



$$\sigma_{AB\to J/\psi} = \sum_{\text{states}} \int d\Gamma_{Q\bar{Q}} \left[\frac{d\hat{\sigma}(Q^2)}{d\Gamma_{Q\bar{Q}}}\right] F_{\text{states}(Q\bar{Q})\to J/\psi}(p_Q, p_{\bar{Q}}, P_{J/\psi})$$

production of heavy quark pair

different models have different assumptions

- Color Singlet Model (CSM): 1975-

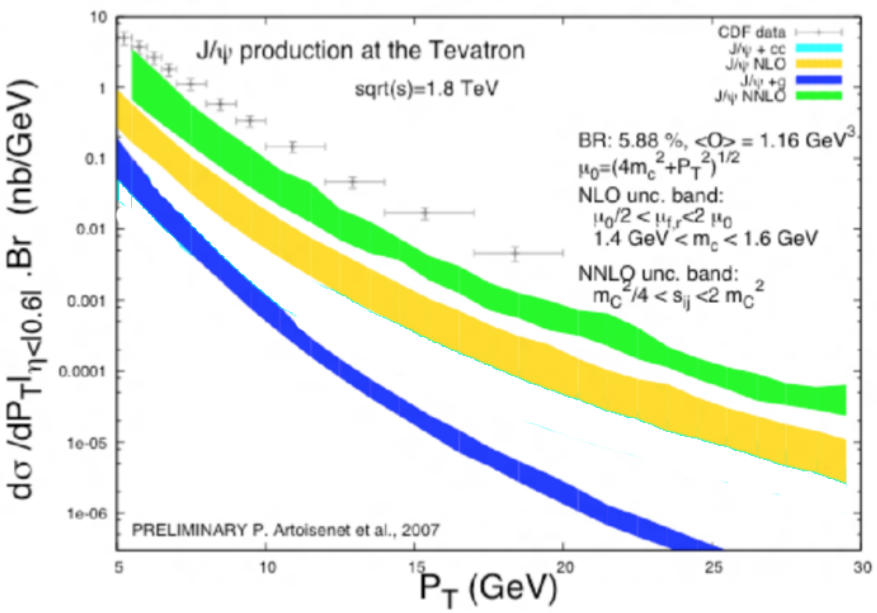
Assumes $Q\bar{Q}$ evolves into quarkonium as isolated particles, no interaction with environment.

$Q\bar{Q}$ must have same quantum number (color singlet, J^{PC}) as quarkonium.

Einhorn and Ellis (1975), Chang (1980), Berger and Jone (1981)

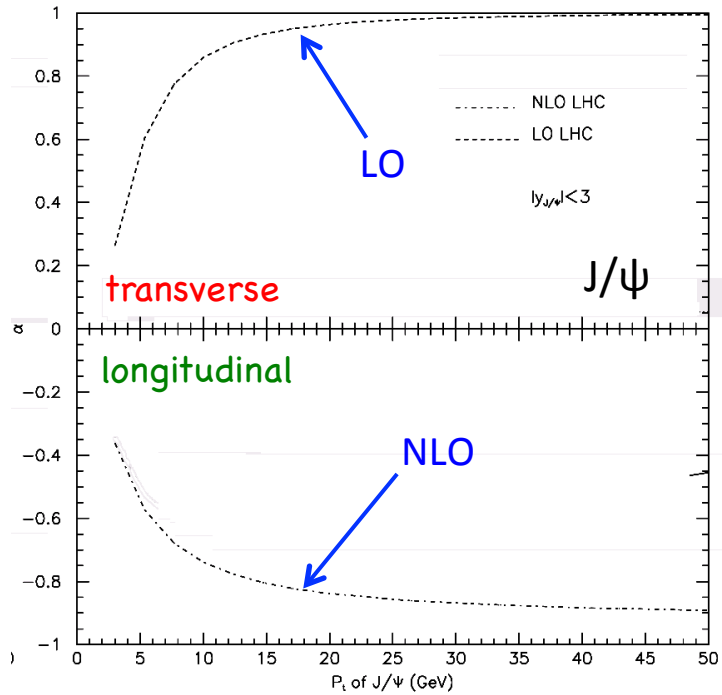
Historical Models: CSM

- Color Singlet Model (cont'd)
  - ✓ No free parameter
  - ✧ Large corrections from higher orders.
  - ✧ P-wave is IR-divergent—CSM is incomplete!



NLO is more than 10 times larger than LO!

F. Maltoni QWG 2007



NLO flips the polarization of LO!

B. Gong et, al. Phys. Rev. Lett, 100 (2008) 232001

Historical Models: CEM

- Color Evaporation Model (CEM): 1977-

Fritsch (1977), Halzen (1977), …

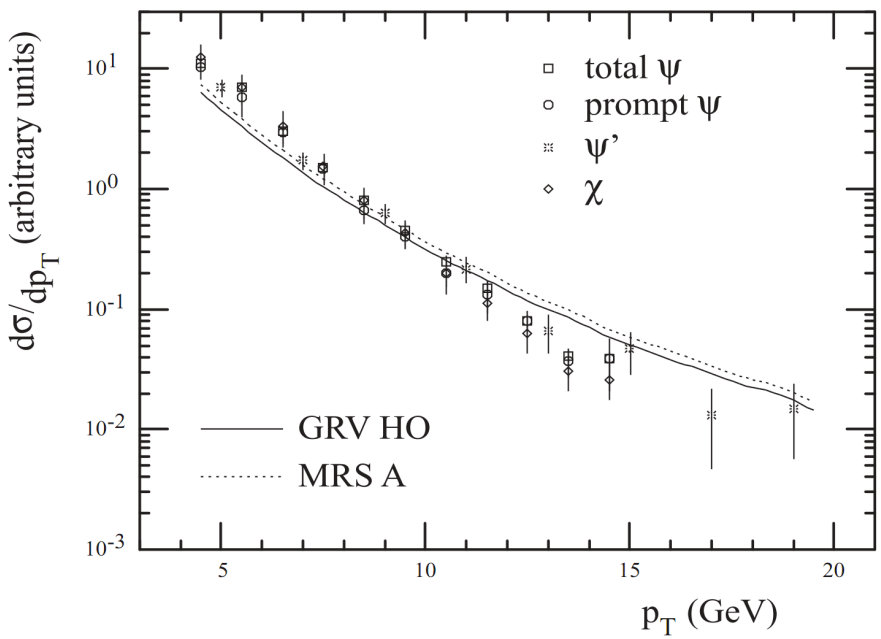
Invariant mass > open charm/bottom threshold → open charm/bottom

Otherwise → a fixed possibility to go to each state.

Predictive power: one parameter per quarkonium state.

✧ Predicts less suppression;

✧ Predicts $\sigma(A)/\sigma(B)$ is process-independent, which conflicts with data.



CEM shows less P_T suppression.

J. F. Amundson et al, Phys. Lett. B 390 (1997) 323

NRQCD: framework

- NRQCD Lagrangian

$$\mathcal{L}_{\text{NRQCD}} = \mathcal{L}_{\text{light}} + \mathcal{L}_{\text{heavy}} + \delta\mathcal{L}_{\text{bilinear}} + \ldots + \delta\mathcal{L}_{\text{4-fermion}}$$

Caswell, Lapage (1986)

$$\mathcal{L}_{\text{heavy}} = \psi^{+}\left(iD_t + \frac{D^2}{2M}\right)\psi + \chi^{+}\left(iD_t - \frac{D^2}{2M}\right)\chi$$

$$\delta\mathcal{L}_{\text{bilinear}} = \frac{c_1}{8M^3}\left[\psi^{+}(D^2)^2\psi - \chi^{+}(D^2)^2\chi\right]$$

Relativistic correction at $O(v^2)$

$$+\frac{c_2}{8M^2}\left[\psi^{+}(D\cdot gE - gE\cdot D)\psi + \chi^{+}(D\cdot gE - gE\cdot D)\chi\right]$$

$$+\frac{c_3}{8M^2}\left[\psi^{+}(iD\times gE - gE\times iD)\cdot\sigma\psi + \chi^{+}(iD\times gE - gE\times iD)\cdot\sigma\chi\right]$$

$$+\frac{c_4}{2M}\left[\psi^{+}(gB\cdot\sigma)\psi - \chi^{+}(gB\cdot\sigma)\chi\right]$$

$\delta\mathcal{L}_{\text{4-fermion}}$ non-renormalizable terms

- NRQCD factorization for quarkonium production

Bodwin, Braaten, Lepage (1995)

$$\sigma(H) = \sum_n \frac{F_n(\Lambda)}{M^{d_n-4}}\langle 0|O_n^H|0\rangle$$

$$O_n^H = \chi^{+}\kappa_n\psi\sum_X |H+X\rangle\langle H+X|\,\psi^{+}\kappa'_n\chi$$

$Q\bar{Q}$ and H can have different quantum number.
later transition effect is included.

Factorization is not proved!

NRQCD: Predictive Power

- Double Expansion of α_s and v
  - Short distance coefficient: calculated perturbatively
  - Non-perturbative matrix elements:

    color singlet ME: related to wave functions as CSM
    color octet ME: extracted from experimental data

  e.g. J/ψ production to order O(v^4), three parameters: $\langle {}^1S_0^{(8)} \rangle$, $\langle {}^3S_1^{(8)} \rangle$ and $\langle {}^3P_0^{(8)} \rangle$

- CSM and CEM are special cases of NRQCD Bodwin, Braaten, Lee, PRD (2005)

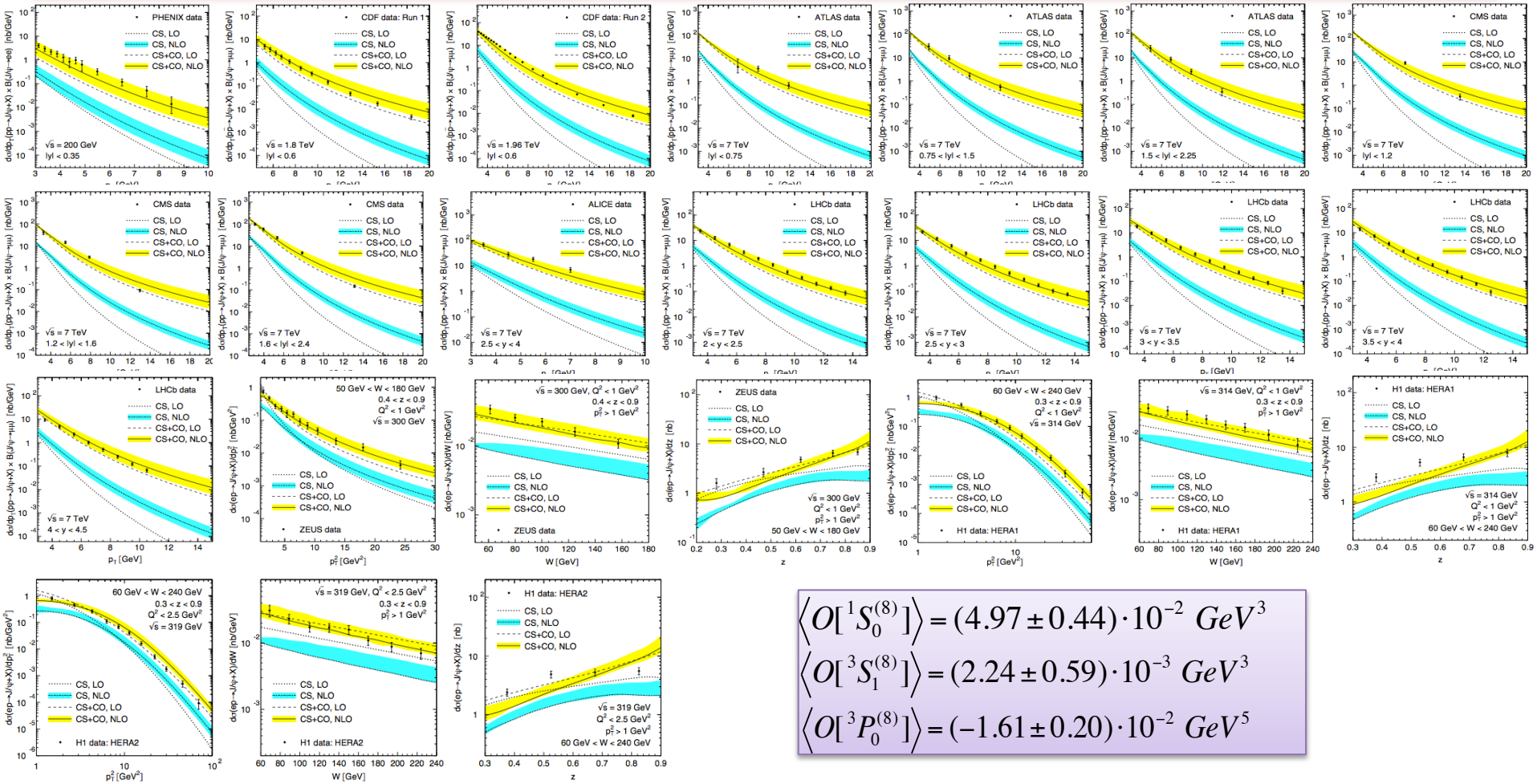
- P-wave divergence is cancelled by color octet contribution

  In leading order of v, p-wave $Q\bar{Q}$ production has two contributions:

  (1) Hard part produces p-wave $Q\bar{Q}$ directly.
  (2) Hard part produces s-wave $Q\bar{Q}$, which later transits to p-state by emitting a gluon.

- ✧ Factorization form is not proved.
- ✧ As in CSM, NRQCD calculation suffers from higher order corrections

NRQCD: Universality of LDME

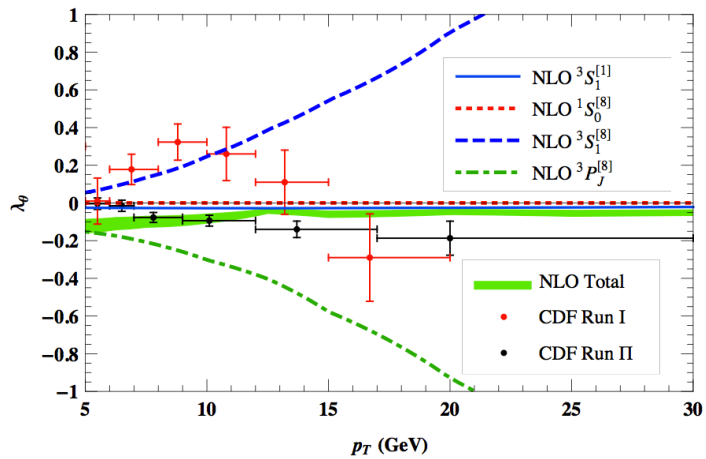


$$\langle O[^1S_0^{(8)}]\rangle = (4.97 \pm 0.44)\cdot 10^{-2}\ GeV^3$$
$$\langle O[^3S_1^{(8)}]\rangle = (2.24 \pm 0.59)\cdot 10^{-3}\ GeV^3$$
$$\langle O[^3P_0^{(8)}]\rangle = (-1.61 \pm 0.20)\cdot 10^{-2}\ GeV^5$$

194 data from 10 experiments $\chi^2 / d.o.f. = 857/194 = 4.42$

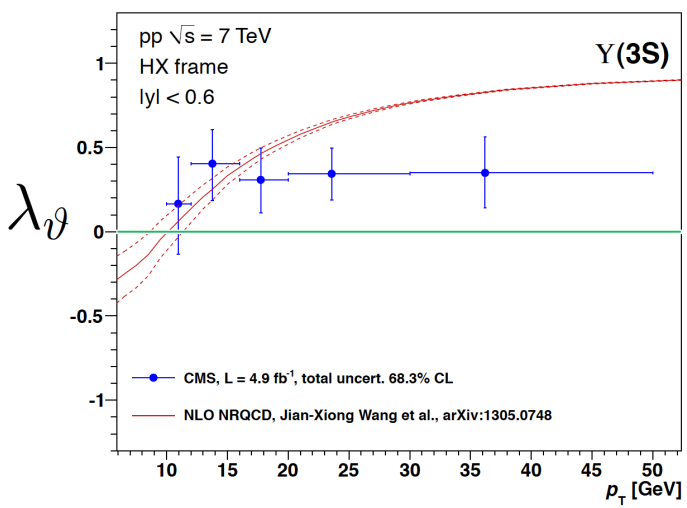
Butenschoen and Kniehl, PRD (2011)

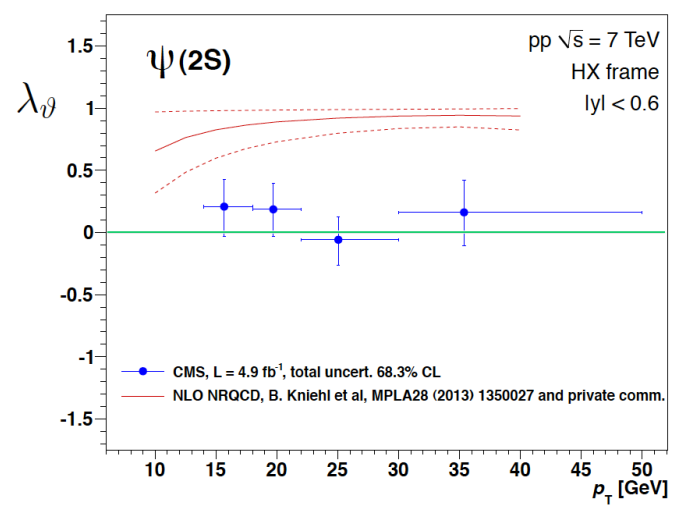
NRQCD: Compare With Data Polarization



Explains J/ψ polarization

K.T. Chao et al, PRL (2012)



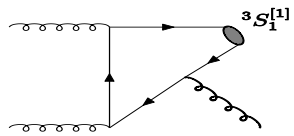


Fails to explain the polarization of ψ(2S) and Υ(3S)

Lourenco, LHCP 2013

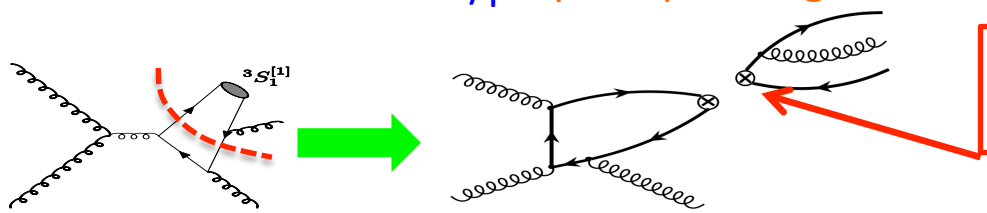
Explain the large corrections

- Other inconsistencies may relate to large high-order correction
- Origin of large high-order correction
  - LO in α_S but NNLP in $1/p_T$



  - NLO in α_S but NLP in $1/p_T$ quark pair fragmentation

Kang, Qiu and Sterman, 1109.1520

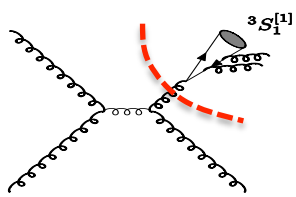


Relativistic Projector to all "spin states"

$$\frac{d\hat{\sigma}^{\rm NLO}}{dp_T^2} \rightarrow \frac{\alpha_S^3(p_T)}{p_T^6} \otimes \alpha_S(\mu)\, \log(\mu^2/\mu_0^2)\, m_Q^2 \qquad \mu_0 \gtrsim 2m_Q$$

  - NNLO in α_S but NNLP in $1/p_T$ gluon fragmentation

Braaten, Yuan, 9303205



$$\frac{\alpha_S^2(p_T)}{p_T^4} \otimes \alpha_S^{\,3+m}(\mu)\, \log^{\rm m}(\mu^2/\mu_0^2)\, m_Q^2$$

At large P_T, M_Q/P_T gives much stronger suppression than α_S

Need to expand by M_Q/P_T first, and resum $\mathrm{Log}[P_T^2/M_Q^2]$

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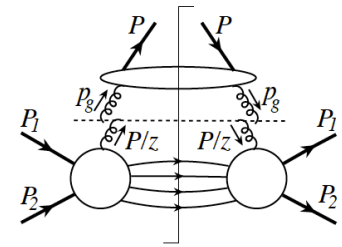
pQCD Factorization Approach: Formalism

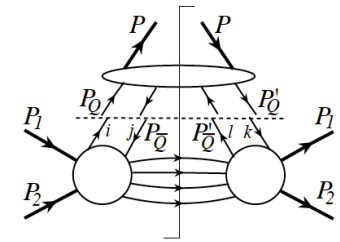
Nayak, Qiu and Sterman, PRD (2005)
Kang, Qiu and Sterman, PRL (2011)

- Expand first in $\frac{1}{p_T^2}$, then α_s
- Factorization formalism up to NLP **Strictly Proved!**

$$d\sigma_{A+B\to H+X}(p_T) = \sum_f d\hat{\sigma}_{A+B\to f+X}(p_f = p/z) \otimes D_{H/f}(z, m_Q)$$
$$+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta)/2z, p(1\pm\zeta')/2z) \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q)$$
$$+ \mathcal{O}(m_Q^4/p_T^4)$$



$\frac{1}{p_T^4}$



$\frac{1}{p_T^6}$

κ = V, A, T for spin, and 1, 8 for color

- Projection operators

Leading power: top $\widetilde{\mathcal{P}}_{\mu\nu}(p) = \frac{1}{2}\left[-g_{\mu\nu} + \frac{p_\mu n_\nu + n_\mu p_\nu}{p\cdot n} - \frac{p^2}{(p\cdot n)^2} n_\mu n_\nu\right]$

bottom $\mathcal{P}_{\mu\nu}(p) = -g_{\mu\nu} + \bar{n}_\mu n_\nu + n_\mu \bar{n}_\nu \equiv d_{\mu\nu}$

Next leading power:

Hard part is insensitive to final quarkonium states

top
$$\widetilde{\mathcal{P}}_v^L(p) = \frac{1}{4p\cdot n}\gamma\cdot n$$
$$\widetilde{\mathcal{P}}_a^L(p) = \frac{1}{4p\cdot n}\gamma\cdot n\gamma^5$$
$$\widetilde{\mathcal{P}}_t^L(p) = \frac{1}{4p\cdot n}\gamma\cdot n\gamma_\perp^\alpha$$

bottom
$$\mathcal{P}_v^L(\hat{p}_Q,\hat{p}_{\bar{Q}}) = \gamma\cdot\hat{p} = \gamma\cdot(\hat{p}_Q + \hat{p}_{\bar{Q}})$$
$$\mathcal{P}_a^L(\hat{p}_Q,\hat{p}_{\bar{Q}}) = \gamma_5\gamma\cdot\hat{p} = \gamma_5\gamma\cdot(\hat{p}_Q + \hat{p}_{\bar{Q}})$$
$$\mathcal{P}_t^L(\hat{p}_Q,\hat{p}_{\bar{Q}}) = \gamma\cdot\hat{p}\gamma_\perp^\alpha = \gamma\cdot(\hat{p}_Q + \hat{p}_{\bar{Q}})\gamma_\perp^\alpha$$

pQCD Factorization Approach: Predictive Power

- Short-distance hard part can be calculated in pQCD

  Power series in α_s , without large logarithms

- Evolution equations

  Independence on the factorization scale $\frac{d}{d\ln(\mu)}\sigma_{A+B\to HX}(P_T) = 0$

$$\frac{d}{d\ln\mu^2}D_{H/f}(z,m_Q,\mu) = \sum_j \frac{\alpha_s}{2\pi}\gamma_{f\to j}(z)\otimes D_{H/j}(z,m_Q,\mu)$$
$$+\frac{1}{\mu^2}\sum_{[Q\bar{Q}(\kappa)]}\frac{\alpha_s^2}{(2\pi)^2}\Gamma_{f\to[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta')\otimes\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q,\mu)$$

If only keep LP DGLAP evolution

$$\frac{d}{d\ln\mu^2}\mathcal{D}_{H/[Q\bar{Q}(c)]}(z,\zeta,\zeta',m_Q,\mu) = \sum_{[Q\bar{Q}(\kappa)]}\frac{\alpha_s}{2\pi}K_{[Q\bar{Q}(c)]\to[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta')$$
$$\otimes\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q,\mu)$$

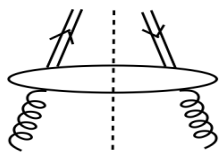
- Evolution kernels can be calculated in pQCD

  Power series in α_s , scheme in choosing factorization scale μ

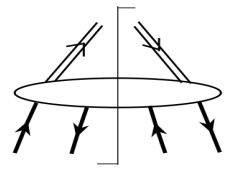
Predictive Power: depend on input FFs

- To give prediction, it is crucial to known the fragmentation functions. Based on evolution equations, only need to know FFs at initial scale.

- Input fragmentation functions



$D_{H/f}(z, m_Q, \mu_0)$



$\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z, \zeta, \zeta', m_Q, \mu_0)$

  - Different quarkonium states require different input distributions!
  - In principle, should be extracted from experimental data.

- Too many unknown fragmentation functions: impossible in practice

e.g. for $J/\psi(^3S_1)$ **8 functions to fit**

Leading power: 2 functions

Next-to Leading power: 6 functions

Quarkonium FFs are perturbative

- To produce a quarkonium, the virtuality of the initial partons should be at least at the scale of $\mu_0 > 2m_Q >> \Lambda_{QCD}$

Fragmentation functions to heavy quarkonium is potentially perturbatively calculable!

- Compare with fragmentation functions like $D_{\pi/f}$: no large scales, and nonperturbative.

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Motivation

- If NRQCD factorization is correct, it can be used to calculate the FFs at initial scale

  For initial fragmentation function at scale m_Q, no large logrithmic term, α_S expansion is reliable.

  For $J/\psi(^3S_1)$, **THREE** unknown matrix elements up to order v^4: comparing with 8 unknown functions if direct fit FFs.

- No proof for NRQCD factorization yet.

  Although not proved, NRQCD is proven to be valid to NLO.
  For some particular case, NNLO can also proven. Nayak, Qiu and Sterman, 2006

- If NRQCD factorization is not correct to all order, calculation of FFs using NRQCD can be thought of as a reasonable model: determine the functional form of FFs

Apply NRQCD To FF's

- Definition of FF's

$$D_{g->H}(z,\mu)=\frac{(-g_{\mu\nu})z^{D-4}}{16\pi(D-2)p^{+}}\int_{-\infty}^{+\infty}dy^{-}e^{-i(p^{+}/z)y^{-}}\langle 0|G_{c}^{+\mu}(0)\mathcal{E}^{+}(0^{-})_{cb}|H(p^{+})X\rangle\langle H(p^{+})X|\mathcal{E}(y^{-})_{ba}G_{a}^{+\nu}(y^{-})|0\rangle$$

$\mathcal{E}(x^{-})_{ba}=\mathrm{P}\exp[ig\int_{y^{-}}^{\infty}dz^{-}A^{+}(0^{+},z^{-},0_{\perp})]_{ba}$ is Wilson line.

$$\mathcal{D}_{H/[Q\bar{Q}(a8)]}(z,\zeta,\zeta')=\sum_{X}\int\frac{p^{+}dy^{-}}{2\pi}\,\mathrm{e}^{-i(p^{+}/z)y^{-}}\times\int\frac{p^{+}dy_{1}^{-}p^{+}dy_{2}^{-}}{(2\pi)^{2}}\mathrm{e}^{i(p^{+}/2z)(1-\zeta)y_{1}^{-}}\,\mathrm{e}^{-i(p^{+}/2z)(1-\zeta')y_{2}^{-}}$$
$$\times\frac{4}{(N^{2}-1)}\langle 0|\,\psi_{i}(0)\frac{\gamma^{+}\gamma_{5}}{4p^{+}}\,(t^{a})_{ij}\,\overline{\psi}_{j}(y_{2}^{-})|H(p^{+})X\rangle$$
$$\times\langle H(P^{+})X|\psi_{l}(y^{-}+y_{1}^{-})\frac{\gamma^{+}\gamma_{5}}{4p^{+}}\,(t^{a})_{lk}\,\overline{\psi}_{k}(y^{-})\,|0\rangle$$

- Apply NRQCD to FF's

Perturbative

$$D_{g\to H}(z,\mu_{0},m_{Q})\to\sum_{[Q\bar{Q}(c)]}\hat{d}_{g\to[Q\bar{Q}(c)]}(z,\mu_{0},m_{Q})\langle O_{[Q\bar{Q}(c)]\to H}\rangle$$

$$D_{[Q\bar{Q}(\kappa)]\to H}(z,\mu_{0},m_{Q})\to\sum_{[Q\bar{Q}(c)]}\hat{d}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(c)]}(z,\mu_{0},m_{Q})\langle O_{[Q\bar{Q}(c)]\to H}\rangle$$

Non-perturbative

Calculate Coefficients

- Perturbative coefficients are insensitive to final state.
  For LP FF, match final quarkonium states to $Q\bar{Q}[H]$

$$D_{g\to[Q\bar{Q}(\kappa)]}(z,\mu_0,m_Q)\Big|_{Pert.QCD} = \sum_{[Q\bar{Q}(c)]} \hat{d}_{g\to[Q\bar{Q}(c)]}(z,\mu_0,m_Q)\left\langle O_{[Q\bar{Q}(c)]\to[Q\bar{Q}(\kappa)]}\right\rangle\Big|_{Pert.NRQCD}$$

Expand by α_S

$$D^{(0)}_{g\to[Q\bar{Q}(\kappa)]}(z,\mu_0,m_Q) = \sum_{[Q\bar{Q}(c)]} \hat{d}^{(0)}_{g\to[Q\bar{Q}(c)]}(z,\mu_0,m_Q)\left\langle O_{[Q\bar{Q}(c)]\to[Q\bar{Q}(\kappa)]}\right\rangle^{(0)} = \hat{d}^{(0)}_{g\to[Q\bar{Q}(\kappa)]}(z,\mu_0,m_Q)$$

$$D^{(1)}_{g\to[Q\bar{Q}(\kappa)]}(z,\mu_0,m_Q) = \sum_{[Q\bar{Q}(c)]} \hat{d}^{(1)}_{g\to[Q\bar{Q}(c)]}(z,\mu_0,m_Q)\left\langle O_{[Q\bar{Q}(c)]\to[Q\bar{Q}(\kappa)]}\right\rangle^{(0)}$$
$$+ \sum_{[Q\bar{Q}(c)]} \hat{d}^{(0)}_{g\to[Q\bar{Q}(c)]}(z,\mu_0,m_Q)\left\langle O_{[Q\bar{Q}(c)]\to[Q\bar{Q}(\kappa)]}\right\rangle^{(1)}$$

Solve the above equations

$$\hat{d}^{(0)}_{g\to[Q\bar{Q}(\kappa)]}(z,\mu_0,m_Q) = D^{(0)}_{g\to[Q\bar{Q}(\kappa)]}(z,\mu_0,m_Q)\Big|_{Pert.QCD}$$

$$\hat{d}^{(1)}_{g\to[Q\bar{Q}(\kappa)]}(z,\mu_0,m_Q) = D^{(1)}_{g\to[Q\bar{Q}(\kappa)]}(z,\mu_0,m_Q)\Big|_{Pert.QCD}$$
$$- \sum_{[Q\bar{Q}(c)]} \hat{d}^{(0)}_{g\to[Q\bar{Q}(c)]}(z,\mu_0,m_Q)\left\langle O_{[Q\bar{Q}(c)]\to[Q\bar{Q}(\kappa)]}\right\rangle^{(1)}\Big|_{Pert.NRQCD}$$

Divergences cancel, $\hat{d}^{(1)}_{g\to[Q\bar{Q}(\kappa)]}(z,\mu_0,m_Q)$ is always finite.

- Perturbative coefficients for NLP is calculated in the same way.

Leading Power FF's: LO

- Leading order in α_S

  Gluon: color octet, $J^P = 1^-$

  Only survived channel: $g \to {}^3S_1^{[8]}$

$$D^{(0)}_{g\to Q\bar{Q}(^3S_1^8)}(z,\mu_0,m_Q) = \mathrm{Tr}[(-ig_S\mu^\varepsilon\gamma^\lambda T^c)(-\frac{1}{\sqrt{2m_Q}}\frac{\sqrt{2}T^a}{\sqrt{N_C^2-1}})\,\gamma^\alpha(\frac{\gamma\cdot p}{2}+m)]$$

$$\times\,\mathrm{Tr}[(ig_S\mu^\varepsilon\gamma^\tau T^d)(-\frac{1}{\sqrt{2m_Q}}\frac{\sqrt{2}T^b}{\sqrt{N_C^2-1}})(\frac{\gamma\cdot p}{2}+m)\,\gamma^\beta]$$

$$\times\frac{i}{p^2}(-g^{\lambda\mu})\times\frac{-i}{p^2}(-g^{\tau\nu})\times\frac{1}{D-1}(-g^{\alpha\beta}+\frac{p^\alpha p^\beta}{4m_Q^2})\,\delta_{ab}$$

$$\times\,\delta(z-\frac{p\cdot n}{p_C\cdot n})\frac{1}{D-2}(-g^{\mu\nu}+\frac{p_C^\mu n^\nu+p_C^\nu n^\mu}{p_C\cdot n}-\frac{p_C^2}{(p_C\cdot n)^2}n^\mu n^\nu)\,\delta_{cd}$$

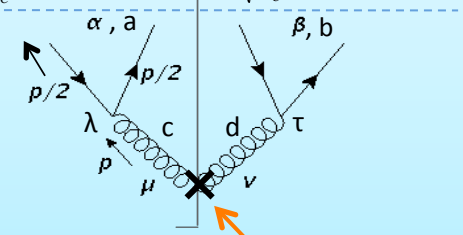
$$=\frac{\pi\alpha_S\mu^{2\varepsilon}}{(D-1)(N_C^2-1)m_Q^3}\delta(1-z)$$

$$\hat{d}^{(0)}_{g\to Q\bar{Q}(^3S_1^8)}(z,\mu_0,m_Q) = \frac{\pi\alpha_S\mu^{2\varepsilon}}{(D-1)(N_C^2-1)m_Q^3}\delta(1-z)$$

- Projection operators

$$\frac{1}{D-1}(-g^{\alpha\beta}+\frac{p^\alpha p^\beta}{4m_Q^2})$$

$$-\frac{\sqrt{2}T^a}{\sqrt{N_C^2-1}}m\gamma^\alpha(\frac{\gamma\cdot p}{2}+m) \qquad -\frac{\sqrt{2}T^b}{\sqrt{N_C^2-1}}m(\frac{\gamma\cdot p}{2}+m)\gamma^\beta$$



$$\frac{1}{D-2}(-g^{\mu\nu}+\frac{p_C{}^\mu n^\nu+p_C{}^\nu n^\mu}{p_C\cdot n}-\frac{p_C{}^2}{(p_C\cdot n)^2}n^\mu n^\nu)\,\delta_{cd}$$

projection operators to ${}^3S_1^{[8]}$ state

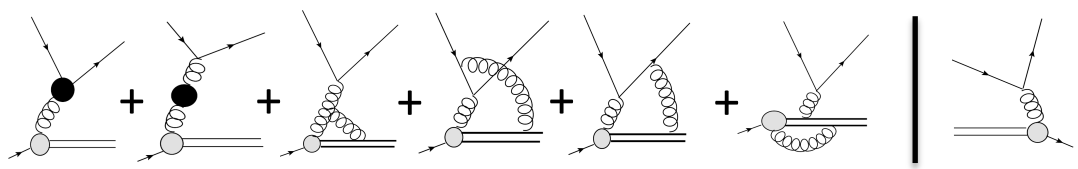
Leading Power FF's: NLO S-wave

- Next-to Leading order in α_S

$$\hat{d}^{(1)}_{g\to Q\bar{Q}(^3S_1^8)}(z,\mu_0,m_Q) = D^{(1)}_{g\to Q\bar{Q}(^3S_1^8)}(z,\mu_0,m_Q) - \hat{d}^{(0)}_{g\to Q\bar{Q}(^3S_1^8)}(z,\mu_0,m_Q)\left\langle O_{Q\bar{Q}(^3S_1^8)\to Q\bar{Q}(^3S_1^8)}\right\rangle^{(1)}$$

higher order in v

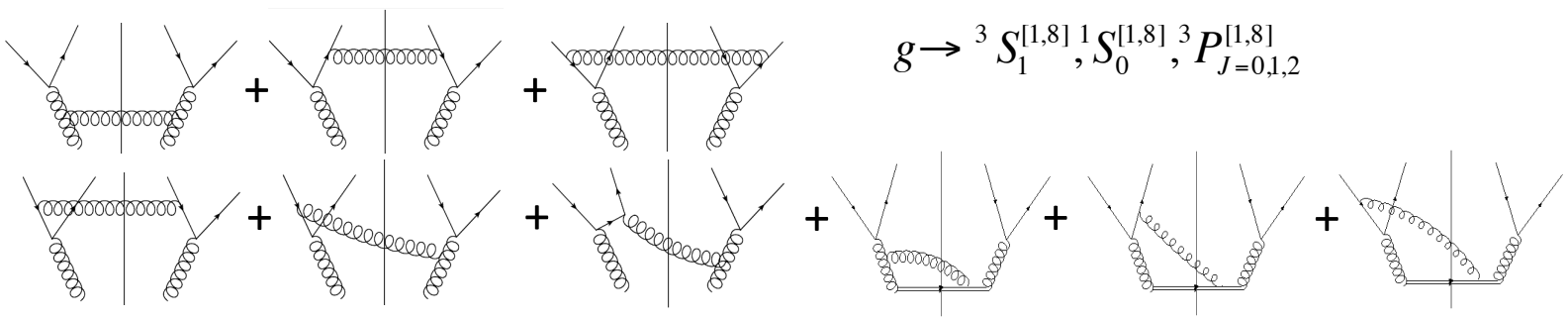
- Virtual diagrams (10+10 c.c.) $g\to {}^3S_1^{[8]}$



+c.c.

- Real diagrams (9+6 c.c.)

$g\to {}^3S_1^{[1,8]}, {}^1S_0^{[1,8]}, {}^3P_{J=0,1,2}^{[1,8]}$



+c.c.

Lightcone gauge: No Wilson line, but more complex gluon propagator.

We tried each of these two gauges and cross checked the results.

Leading Power FF's: NLO S-wave Divergencies

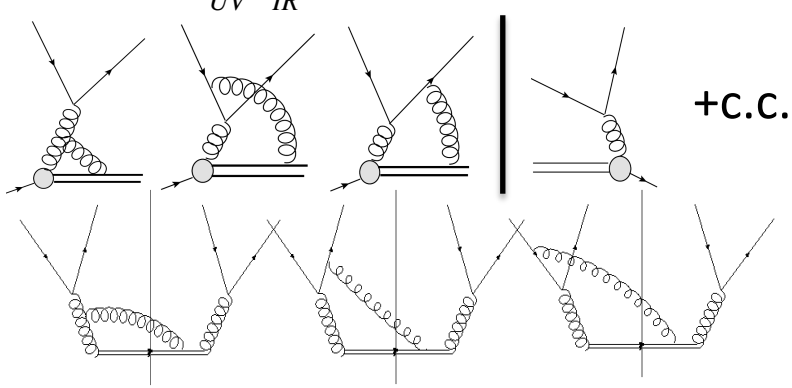
- UV divergence —loop integration
- IR divergence

Soft gluon: use plus function to separate the divergence for real diagram.

$$\frac{1}{(1-z)^{1+2\varepsilon}} = -\frac{1}{2\varepsilon_{IR}}\delta(1-z) + \frac{1}{(1-z)_+} - 2\varepsilon\left(\frac{\ln(1-z)}{1-z}\right)_+$$

Rapidity: $\int d^4k \frac{1}{k\cdot n + i\varepsilon} \ldots$ with k // n $\qquad n^\mu = \frac{1}{\sqrt{2}}(1,0,0,-1)^\mu$

Diagrams with $\frac{1}{\varepsilon_{UV}\varepsilon_{IR}}$:



+c.c.

Leading Power FF's: NLO Results

$$d_{real} = \frac{\alpha_S^2 \mu^{2\varepsilon}}{m_Q^3}\left(\frac{\pi\mu^2}{m_Q^2}\right)^{\varepsilon}\Gamma(1+\varepsilon)\times\left\{-\frac{3(1-\varepsilon)}{2\varepsilon_{UV}\varepsilon_{IR}}\delta(1-z)+\frac{3(1-\varepsilon)}{\varepsilon_{UV}}\left[\frac{z}{(1-z)_+}+\frac{1}{z}-z^2+z-1\right]\right.$$

$$\left.-\frac{6}{z}\left(\frac{\ln(1-z)}{1-z}\right)_+ +6(z^2-z+2)\ln(1-z)+3\left(\frac{1}{z}-1\right)\right.$$

$$d_{virtual} = \frac{\alpha_S^2 \mu^{2\varepsilon}}{m_Q^3}\left(\frac{\pi\mu^2}{m_Q^2}\right)^{\varepsilon}\delta(1-z)\times\left\{\frac{3(1-\varepsilon)}{2\varepsilon_{UV}\varepsilon_{IR}}\Gamma(1+\varepsilon)+\beta_0\frac{\Gamma(1+\varepsilon)}{\varepsilon_{UV}}+\frac{177-10n_f}{18}-\frac{\pi^2}{2}+8\ln 2+6\ln^2 2\right\}$$

$$d_{ren} = \frac{\alpha_S^2 \mu^{2\varepsilon}}{m_Q^3}\left(4\pi e^{-\gamma}\right)^{\varepsilon}\frac{(-3)}{\varepsilon_{UV}}\times\left\{\frac{z}{(1-z)_+}+\frac{1-z}{z}+z(1-z)+\frac{\beta_0}{3}\delta(1-z)\right\}$$

$$d_{all} = \frac{\alpha_S^2 \mu^{2\varepsilon}}{m_Q^3}\left\{-\frac{6}{z}\left(\frac{\ln(1-z)}{1-z}\right)_+ +6(z^2-z+2)\ln(1-z)+3\left(\frac{1}{z}-1\right)+\frac{1}{2}\left[\ln\left(\frac{\mu^2}{4m_Q^2}\right)-1\right]P_{gg}(z)\right.$$

$$\left.+\delta(1-z)\left[\frac{\beta_0}{2}\left(\ln\left(\frac{\mu^2}{4m_Q^2}\right)+\frac{7}{3}\right)+\frac{2}{3}-\frac{\pi^2}{2}+8\ln 2+6\ln^2 2\right]\right\}$$

$$\beta_0 = \frac{33-2n_f}{6}$$

$$P_{gg}(z) = 6\left[\frac{z}{(1-z)_+}+\frac{1-z}{z}+z(1-z)+\frac{\beta_0}{6}\delta(1-z)\right]$$

$\frac{1}{\varepsilon_{UV}\varepsilon_{IR}}$ cancel between real and virtual diagrams. Final result is finite.

Leading Power FF's: P-wave Divergence

- For final state $Q\bar{Q}(^3P_1^{[8]})$, at leading order of v

$$\hat{d}^{(1)}_{g\to Q\bar{Q}(^3P_1^8)}(z,\mu_0,m_Q) = D^{(1)}_{g\to Q\bar{Q}(^3P_1^8)}(z,\mu_0,m_Q) - \hat{d}^{(0)}_{g\to Q\bar{Q}(^3S_1^8)}(z,\mu_0,m_Q)\left\langle O_{Q\bar{Q}(^3S_1^8)\to Q\bar{Q}(^3P_1^8)}(0)\right\rangle^{(1)}$$

CSM don't have this term
and therefore IR divergent.

- Results

$$D^{(1)}_{g\to Q\bar{Q}(^3P_1^8)}(z,\mu_0,m_Q) = \frac{N_C^2-4}{2N_C(N_C^2-1)}\frac{4\alpha_S^2\mu^{2\varepsilon}}{27m_Q^5}\left(\frac{\pi\mu^2}{m_Q^2}\right)^{\varepsilon}\Gamma(1+\varepsilon)\left\{\left(\boxed{-\frac{3}{2\varepsilon_{IR}}}-\frac{5}{8}\right)\delta(1-z)-\frac{3z(1+4z)}{4}+\frac{3z}{(1-z)_+}\right\}$$

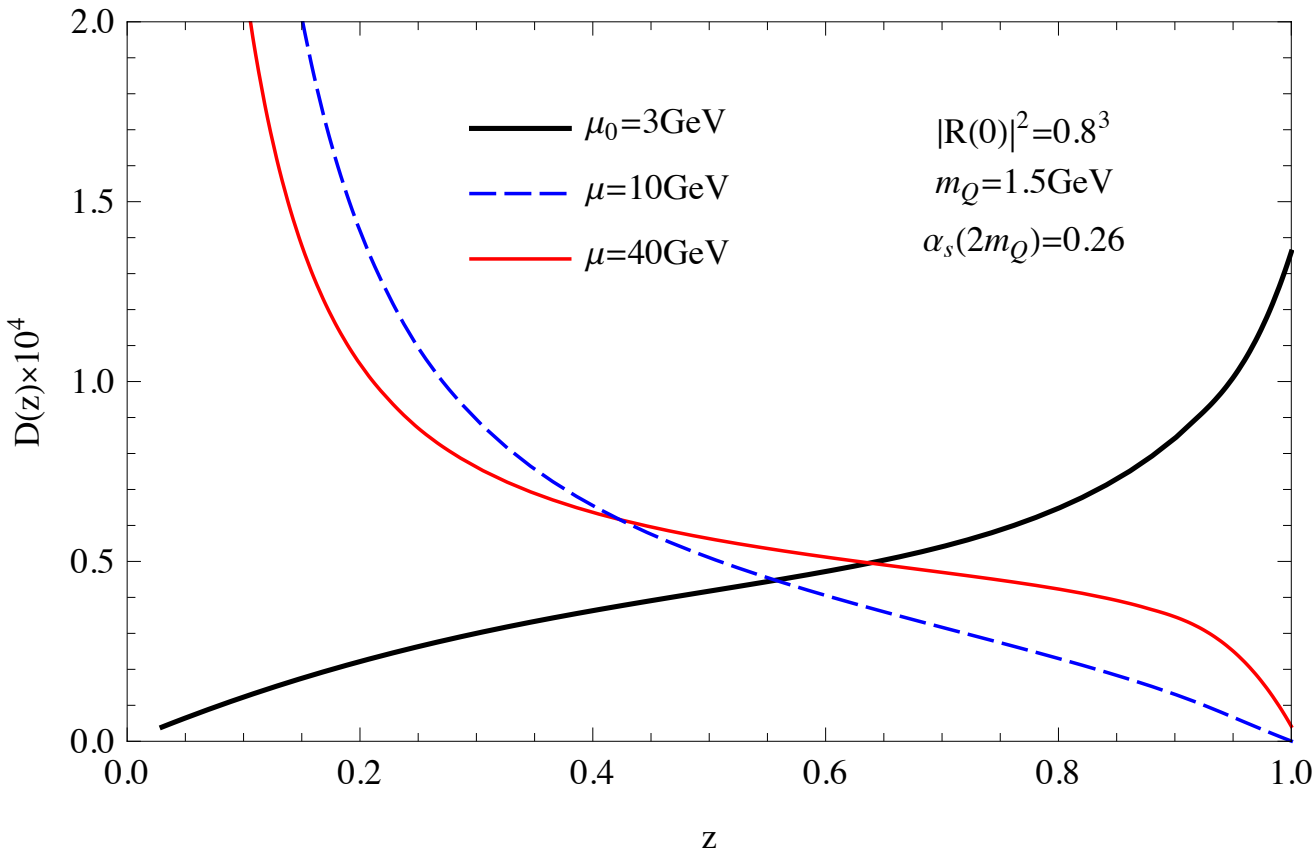
$$\left\langle O_{Q\bar{Q}(^3S_1^8)\to Q\bar{Q}(^3P_1^8)}(0)\right\rangle^{(1)} = -\frac{N_C^2-4}{4N_C}\left(\boxed{\frac{1}{\varepsilon_{IR}}}+\ln(4\pi)-\gamma_E\right)\left(\frac{\mu}{\mu_\Lambda}\right)^{2\varepsilon}\frac{4\alpha_S}{3\pi m_Q^2}$$

$$d^{(1)}_{g\to Q\bar{Q}(^3P_1^8)}(z,\mu_0,m_Q) = \frac{N_C^2-4}{2N_C(N_C^2-1)}\frac{4\alpha_S^2\mu^{2\varepsilon}}{27m_Q^5}\left\{\left(-\frac{3}{2}\ln\left(\frac{\mu_\Lambda^2}{4m_Q^2}\right)+\frac{3}{8}\right)\delta(1-z)-\frac{3z(1+4z)}{4}+\frac{3z}{(1-z)_+}\right\}$$

Finite.

Leading Power FF's: evolution effect

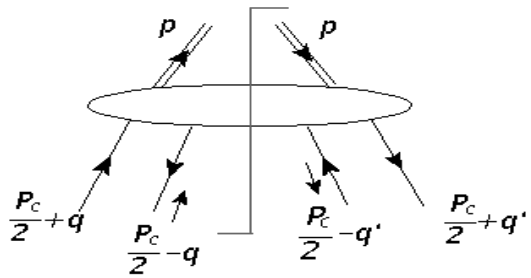
$$\hat{d}^{(1)}_{g\to {}^1S_0^{[8]}}(z,\mu_0,m_Q) \to \frac{N_C}{2(N_C^2-1)}\frac{\alpha_S^2}{2m_Q^3}\left\{z(3-2z)+2(1-z)\ln(1-z)\right\}$$



Next-to Leading Power: Definition

- Double parton FF

Relative momentum does not need to be same on both sides.



$$\mathcal{D}_{H/[Q\bar{Q}(a8)]}(z,\zeta,\zeta')$$

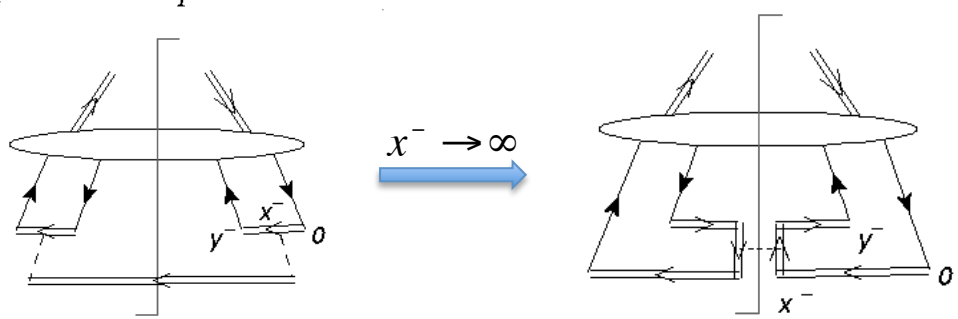
$$\zeta = 2q^+ / p_C^+$$

$$\zeta' = 2q'^+ / p_C^+$$

$$z = p^+ / p_C^+$$

- Wilson line

$$\frac{d}{dx^-}\langle 0|\psi_{i'}(0)\frac{\gamma^+\gamma_5}{4p^+}U^{(f)}_{i'i}(0,x^-)(t^a)_{ij}U^{(f)}_{jj'}(x^-,y^-)\,U^{(A)}_{aa'}(x^-,\infty)\,\overline{\psi}_{j'}(y_1^-)|H(p^+)X\rangle = 0$$



- No momentum flows to infinity along Wilson line;
- Easier to handle in program

Next-to Leading Power

- Matching in NRQCD factorization

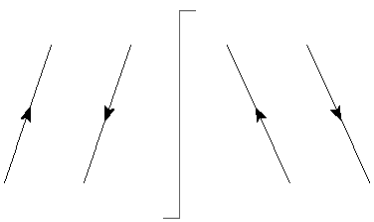
$$D_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(H)]}(z,\mu_0,m_Q)\Big|_{Pert.\ QCD} \to \sum_{[Q\bar{Q}(c)]} \hat{d}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(c)]}(z,\mu_0,m_Q)\left\langle O_{[Q\bar{Q}(c)]\to[Q\bar{Q}(H)]}(0)\right\rangle\Big|_{Pert.\ NRQCD}$$

Initial states: $Q\bar{Q}[\kappa] = V^{[1,8]}, A^{[1,8]}, T^{[1,8]}$

Final states: $Q\bar{Q}[H] = {}^1S_0^{[1,8]}, {}^3S_1^{[1,8]}, {}^1P_1^{[1,8]}, {}^3P_{J=0,1,2}^{[1,8]}$

72 channels!

- Leading Order in α_S



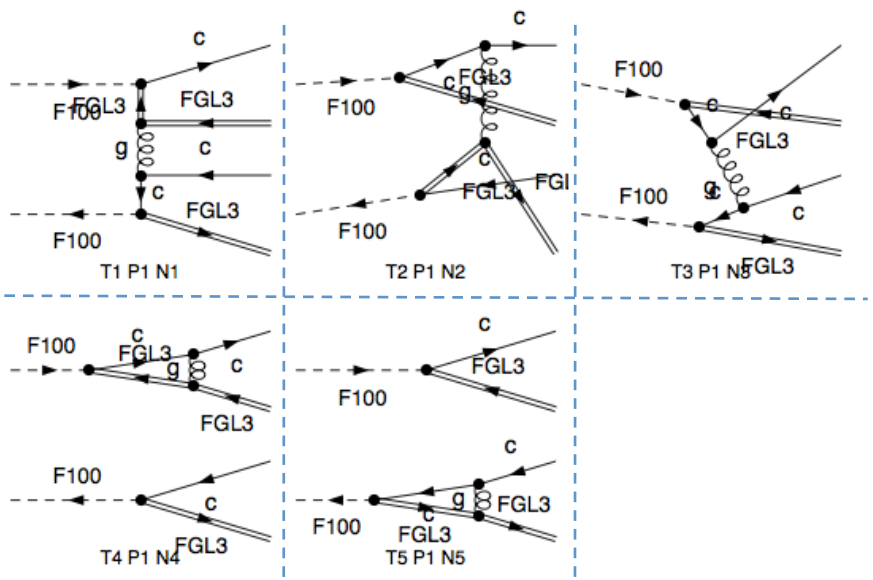
20 Open Channels:

$V^{[1]} \to {}^3S_1^{[1]}$	$V^{[8]} \to {}^3S_1^{[8]}$	$T^{[1]} \to {}^3S_1^{[1]}$	$T^{[8]} \to {}^3S_1^{[8]}$
$V^{[1]} \to {}^3P_{J=0,2}^{[1]}$	$V^{[8]} \to {}^3P_{J=0,2}^{[8]}$	$T^{[1]} \to {}^1P_1^{[1]}$	$T^{[8]} \to {}^1P_1^{[8]}$
$A^{[1]} \to {}^1P_1^{[1]}$	$A^{[8]} \to {}^1P_1^{[8]}$	$T^{[1]} \to {}^3P_{J=1,2}^{[1]}$	$T^{[8]} \to {}^3P_{J=1,2}^{[8]}$
$A^{[1]} \to {}^3P_1^{[1]}$	$A^{[8]} \to {}^3P_1^{[8]}$		
$A^{[1]} \to {}^1S_0^{[1]}$	$A^{[8]} \to {}^1S_0^{[8]}$		

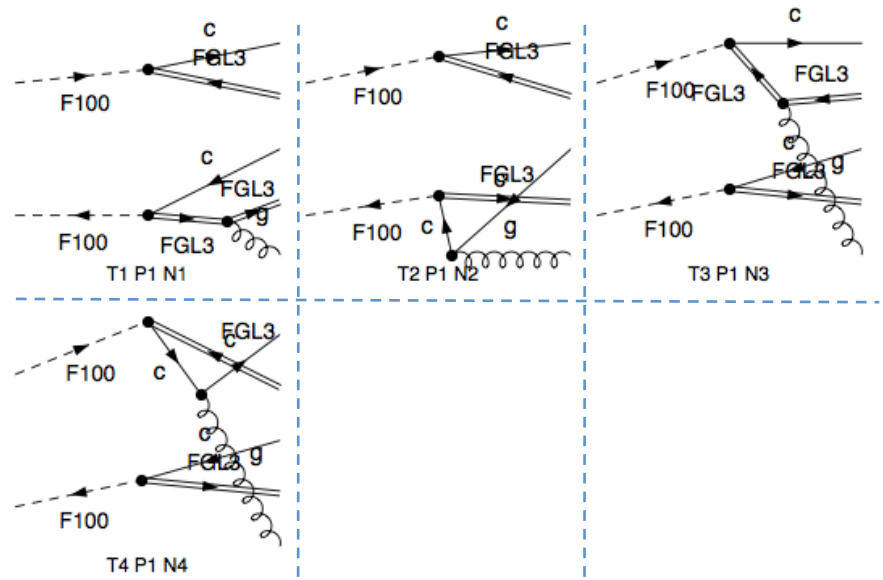
Bad news: NLO virtual diagrams not zero for 20 channels.

Next-to Leading Power: Diagrams

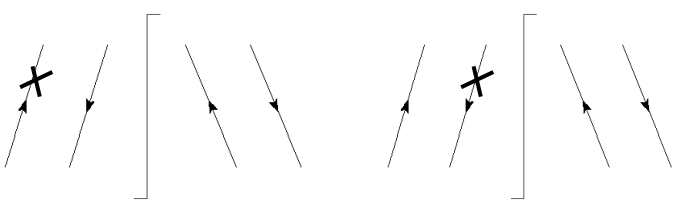
- Virtual amplitudes



- Real amplitudes



- Counter-term Diagrams



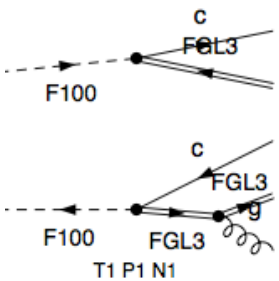
+ c.c.

	# of open channels	# of amplitudes	# of cut diagrams
Virtual	20	100	100+100c.c.
Real	50	200	> 400

Impossible to calculate by hand!

Mathematica Packages

- FeynArts –diagrams and Feynman amplitudes generator



Original package: no Wilson line, connected diagrams only.

Improvement:

(1) Include new particle family "gauge link" and its Feynman rule;
(2) Include the nice double line for Wilson line;
(3) Allow user-defined "seeds" for generating any type of diagrams.

- FeynCalc –Feynman amplitude calculator

Original package: (1) Use different notations as in FeynArts;
(2) Cannot deal with $\int d^4k \frac{1}{k\cdot n + i\varepsilon} \cdots$.

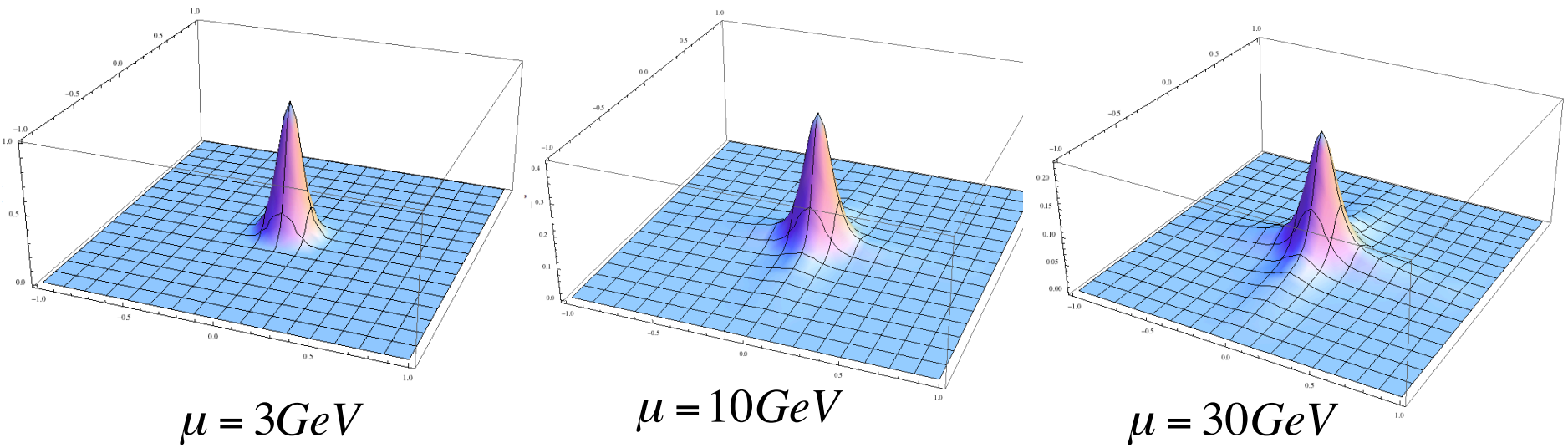
Improvement: (1) Include the translator to link FeynArts and FeynCalc;
(2) Isolate unknown integrations and wait for user's command;
(3) Many new functions to facilitate calculation.

Human brain + computer's speed

Next-to Leading Power: evolution effect

- Evolution of double parton fragmentation

Starting profile: $\delta(\zeta)\delta(\zeta') \rightarrow \mathrm{e}^{-100\zeta^2} e^{-100\zeta'^{\,2}}$



When in higher energy scale,
Peak gets lower and wider while evolving to larger μ.

Summary

- At large $p_T >> m_Q$, calculations based on early models are not perturbatively stable. LO in α_s-expansion does not coincide with LP term in $1/p_T$-expansion.
- PQCD factorization is a strictly proved factorization approach. It expands $1/p_T$ before α_s expansion. When pT is large enough, perturbative expansion converges much faster than any other model.
- With the evolution equations, the predictive power of pQCD factorization for heavy quarkonium production relies completely on the initial distributions
- Different from fragmentation to pion, fragmentation to heavy quarkonium is perturbatively calculable because of the large mass scale. NRQCD is applied to calculate the fragmentation functions at scale $\mu \sim 2m_Q$.
- With our results, several unknown fragmentation functions are reduced into a few parameters, which is much easier to be extracted from data.

Thank you!

pQCD Factorization: Evolution Kernels

$$
\begin{aligned}
P_{v1\to v1} = P_{a1\to a1} =& (3-S_0)C_F\Delta_0 + C_F\Delta_v,\\
P_{t1\to t1} =& (3-S_0)C_F\Delta_0 + C_F\tilde{\Delta}_v,\\
P_{v8\to v8} = P_{a8\to a8} =& (3-S_0)C_F\Delta_0 - \frac{1}{2N_c}\Delta_v + \frac{1}{2N_c}\frac{z}{2(1-z)_+}S_+\Delta_+^{[8]},\\
P_{t8\to t8} =& (3-S_0)C_F\Delta_0 - \frac{1}{2N_c}\tilde{\Delta}_v + \frac{1}{2N_c}\frac{z}{2(1-z)_+}\left(S_+\Delta_+^{[8]} + S_-\Delta_-^{[8]}\right)
\end{aligned}
$$

$$
\begin{aligned}
P_{v8\to v1} = P_{a8\to a1} =& \frac{z}{2(1-z)}S_+\Delta_-^{[1]},\\
P_{t8\to t1} =& \frac{z}{2(1-z)}\left(S_+\Delta_-^{[1]} + S_-\Delta_+^{[1]}\right),\\
P_{v8\to a1} = P_{a8\to v1} =& \frac{z}{2(1-z)}S_-\Delta_-^{[1]},\\
P_{v8\to a8} = P_{a8\to v8} =& \frac{1}{2N_c}\frac{z}{2(1-z)}S_-\Delta_+^{[8]},\\
P_{X1\to Y8} =& \frac{N_c^2-1}{4N_c^2}P_{X8\to Y1}, \quad \text{for} \quad X,Y = v,a,t.
\end{aligned}
$$

Kang, Y.-Q. Ma, Qiu, Sterman, 1306.xxxx

$$
\begin{aligned}
S_0 =& \ln(u\bar{u}v\bar{v}), \quad S_\pm = \left(\frac{u}{u'} \pm \frac{\bar{u}}{\bar{u}'}\right)\left(\frac{v}{v'} \pm \frac{\bar{v}}{\bar{v}'}\right),\\
\Delta_0 =& \delta(1-z)\delta(u-u')\delta(v-v')\\
\Delta_\pm^{[1]} =& \left[\delta(u-zu') \pm \delta(\bar{u}-z\bar{u}')\right]\left[\delta(v-zv') \pm \delta(\bar{v}-z\bar{v}')\right],\\
\Delta_\pm^{[8]} =& \left\{(N_c^2-2)\left[\delta(u-zu')\delta(v-zv') + \delta(\bar{u}-z\bar{u}')\delta(\bar{v}-z\bar{v}')\right]\right.\\
& \left.\pm 2\left[\delta(u-zu')\delta(\bar{v}-z\bar{v}') + \delta(\bar{u}-z\bar{u}')\delta(v-zv')\right]\right\},\\
\Delta_v =& \delta(1-z)\delta(u-u')\left[\left\langle\frac{1}{v-v'}\right\rangle_{+1} - \left\langle\frac{\bar{v}'}{v}\right\rangle_{+0}\right] + u\leftrightarrow v,\\
\tilde{\Delta}_v =& \delta(1-z)\delta(u-u')\left[\left\langle\frac{1}{v-v'}\right\rangle_{+1} - \left\langle\frac{1}{v}\right\rangle_{+0}\right] + u\leftrightarrow v,
\end{aligned}
$$

Light quark case:

$$
\begin{aligned}
\gamma^{(2)}_{[Q\bar{Q}(v8)]/q} &= \alpha_s^2\frac{N_c^2-1}{N_c}\frac{8(1-z)}{z^2}\\
\gamma^{(2)}_{[Q\bar{Q}(v1)]/q} &= \gamma^{(2)}_{[Q\bar{Q}(a8)]/q} = \gamma^{(2)}_{[Q\bar{Q}(a1)]/q} = \gamma^{(2)}_{[Q\bar{Q}(t8)]/q} = \gamma^{(2)}_{[Q\bar{Q}(t1)]/q} = 0
\end{aligned}
$$

Heavy quark case:

$$
\begin{aligned}
\gamma^{(2)}_{[Q\bar{Q}(v8)]/Q} &= \alpha_s^2\frac{N_c^2-1}{2N_c^3}\frac{1}{\bar{u}\bar{v}}\frac{1-z}{z^2}\frac{4N_c\bar{u}(1-zu)+z(1+z\bar{u})}{1-zu}\\
&\quad\times\frac{4N_c\bar{v}(1-zv)+z(1+z\bar{v})}{1-zv}\\
\gamma^{(2)}_{[Q\bar{Q}(v1)]/Q} &= \alpha_s^2\left(\frac{N_c^2-1}{N_c}\right)^2\frac{1-z}{\bar{u}\bar{v}}\frac{(1+z\bar{u})(1+z\bar{v})}{(1-zu)(1-zv)}\\
\gamma^{(2)}_{[Q\bar{Q}(a8)]/Q} &= \alpha_s^2\frac{N_c^2-1}{2N_c^3}\frac{1-z}{\bar{u}\bar{v}}\frac{(1+z\bar{u})(1+z\bar{v})}{(1-zu)(1-zv)}\\
\gamma^{(2)}_{[Q\bar{Q}(a1)]/Q} &= \alpha_s^2\left(\frac{N_c^2-1}{N_c}\right)^2\frac{1-z}{\bar{u}\bar{v}}\frac{(1+z\bar{u})(1+z\bar{v})}{(1-zu)(1-zv)}\\
\gamma^{(2)}_{[Q\bar{Q}(t8)]/Q} &= \gamma^{(2)}_{[Q\bar{Q}(t1)]/Q} = 0
\end{aligned}
$$

Gluon case:

$$
\begin{aligned}
\gamma^{(2)}_{[Q\bar{Q}(v8)]/g} &= \alpha_s^2\frac{1}{4u\bar{u}v\bar{v}}\left\{\frac{N_c}{z^2}\left[4(1-z)^2 - 4(1-2u\bar{u}-2v\bar{v})(1-z)^2(z+2)\right.\right.\\
&\quad\left.+(u-\bar{u})^2(v-\bar{v})^2(2z^4+2z^3-3z^2-4z+4)\right]\\
&\quad\left.+\frac{N_c^2-4}{N_c}(u-\bar{u})(v-\bar{v})[z^2+(1-z)^2]\right\}\\
\gamma^{(2)}_{[Q\bar{Q}(v1)]/Q} &= \alpha_s^2\frac{(u-\bar{u})(v-\bar{v})}{u\bar{u}v\bar{v}}[z^2+(1-z)^2]\\
\gamma^{(2)}_{[Q\bar{Q}(a8)]/Q} &= \alpha_s^2\frac{1}{u\bar{u}v\bar{v}}\left[\frac{N_c}{2}(\bar{u}\bar{v}+uv) - \frac{1}{N_c}\right][z^2+(1-z)^2]\\
\gamma^{(2)}_{[Q\bar{Q}(a1)]/Q} &= \alpha_s^2\frac{1}{u\bar{u}v\bar{v}}[z^2+(1-z)^2]\\
\gamma^{(2)}_{[[Q\bar{Q}(t8)]/Q} &= \gamma^{(2)}_{[[Q\bar{Q}(t1)]Q} = 0
\end{aligned}
$$