Numerical Simulation of Anomalous Hydrodynamics

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June 27th, 2013 RIKEN Lunch seminar

Outline

I. Motivation
- ◇ Electromagnetic Field in Heavy Ion Collisions
- ◇ Chiral Magnetic/Separation Effect
- ◇ Chiral Magnetic Wave

II. Anomalous Hydrodynamics
- ◇ Relativistic Hydrodynamics in external field
- ◇ Anomalous Hydrodynamics

III. Results of Numerical simulation
- ◇ Chiral Magnetic Wave in uniform/expanding plasma
- ◇ Charge dependent elliptic flow

IV. Summary

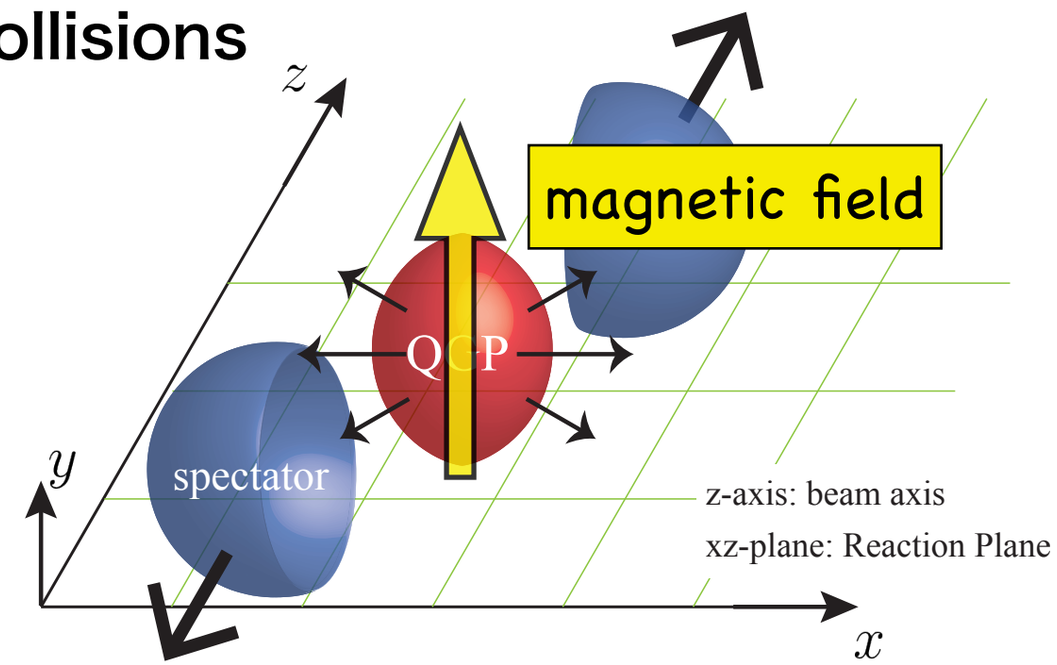
Motivation

Heavy Ion Collisions

■ Quark Gluon Plasma in Heavy Ion Collisions

LHC(CERN) : Pb+Pb $\sqrt{s_{NN}} = 2.76$ TeV

RHIC(BNL) : Au+Au $\sqrt{s_{NN}} = 200$ GeV



◇ Observation of Elliptic flow v_2

→ QGP is like a perfect fluid, $\frac{\eta}{s} \sim \frac{1}{4\pi}$

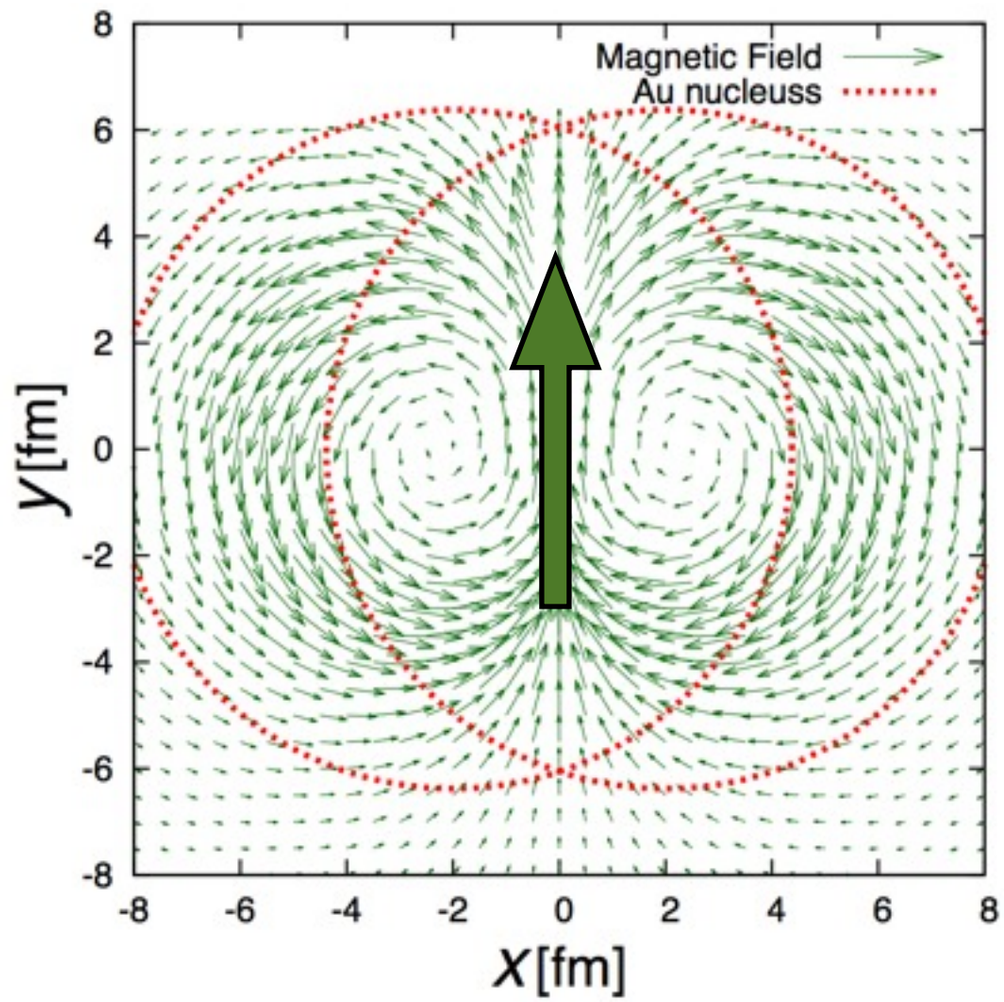
■ Intense magnetic field in peripheral collisions

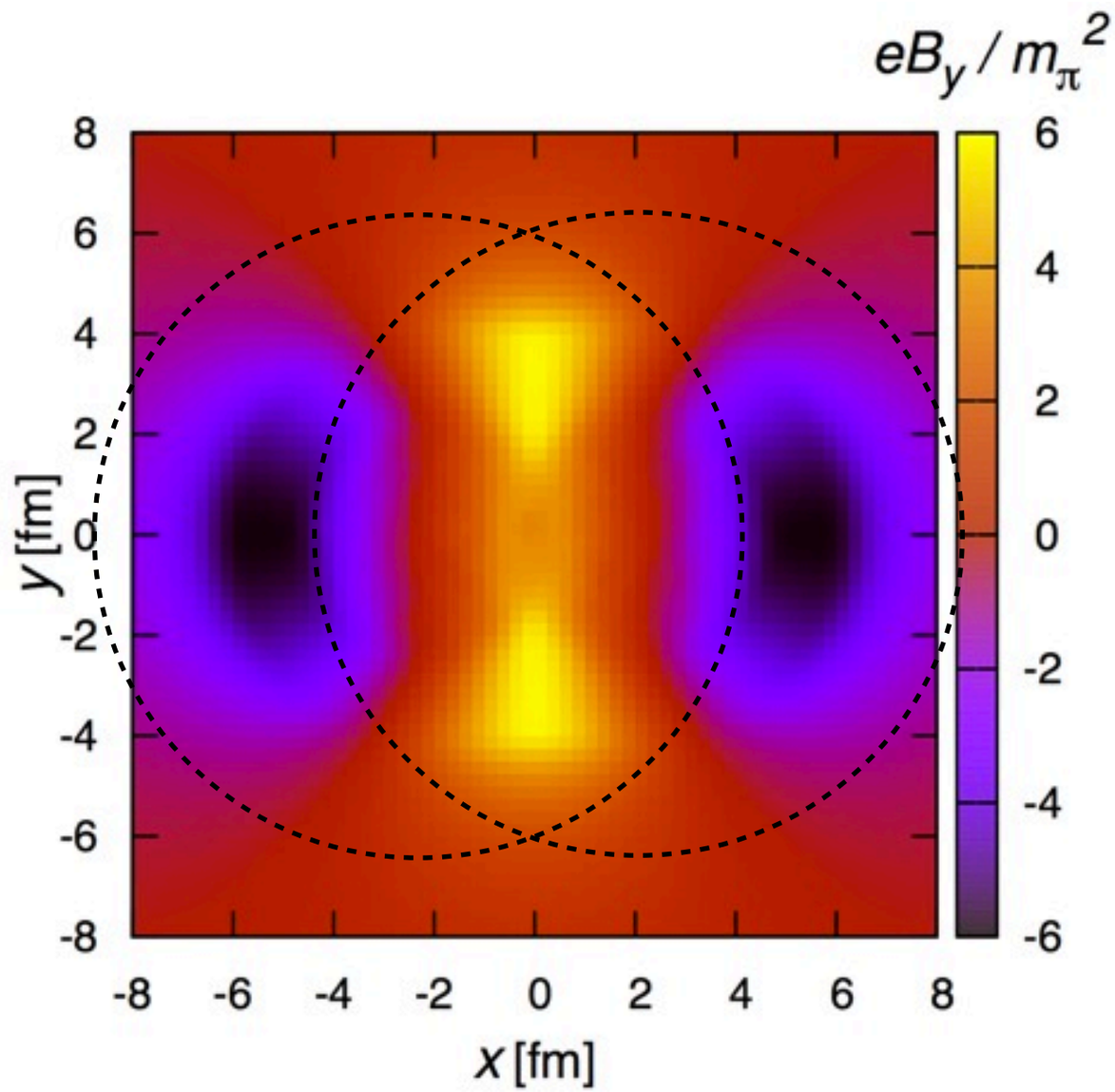
$$e|\vec{B}| \sim m_\pi^2 \sim 10^{14}\ \mathrm{T}$$

cf.) Compact Star : $10^8 - 10^{11}$ T

Intense Magnetic field in HIC

- **Event-averaged magnetic field with impact parameter** $b = 4\ \mathrm{fm}$

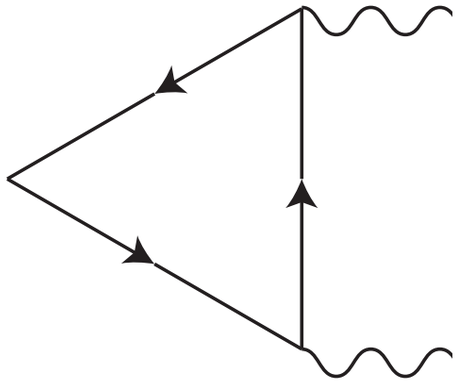




QED Chiral Anomaly

■**Axial current non-conservation** (Adler, Bell, Jackiw (1969))

$$\partial_\mu j_5^\mu = -\frac{N_c e^2}{16\pi^2} F^{\mu\nu} \tilde{F}_{\mu\nu}$$



Chiral Magnetic/Separation Effect

■ Chiral Magnetic Effect(CME)

Fukushima et al. (2008) Vilenkin(1980)

Magnetic field+chirality imbalance

$$\vec{j} = \frac{N_c e}{2\pi^2}\mu_5 \vec{B}$$

chiral chemical potential:

$$\mu_5 \equiv \mu_R - \mu_L$$

$$n_5 \equiv n_R - n_L \neq 0$$

vector current || magnetic field

■ Chiral Separation Effect(CSE)

Son et al. (2004)

Magnetic field+charge density

$$\vec{j}_5 = \frac{N_c e}{2\pi^2}\mu \vec{B}$$

charge chemical potential:

$$\mu \equiv \mu_R + \mu_L$$

$$n \equiv n_R + n_L \neq 0$$

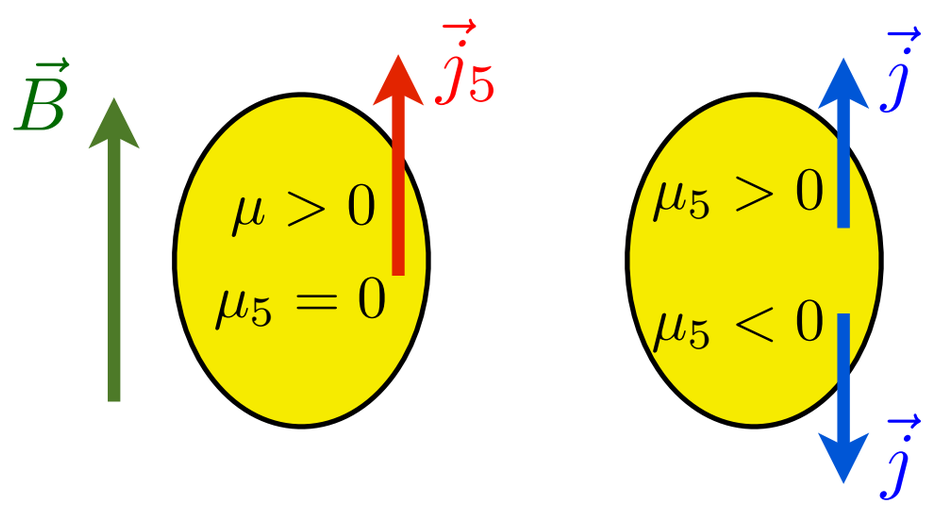
axial current || magnetic field

Chiral Magnetic Wave

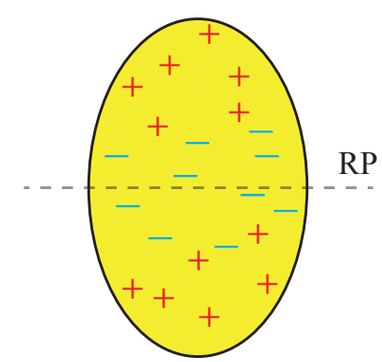
Chiral Magnetic+Separation Effect → **Chiral Magnetic Wave**

$$\vec{j}_5 = \frac{N_c e}{2\pi^2}\mu\vec{B} \qquad \vec{j} = \frac{N_c e}{2\pi^2}\mu_5\vec{B}$$

◇ In the case of finite chemical potential (low energy collisions)



Quadruple deformation



(Burnier et al, (2012))

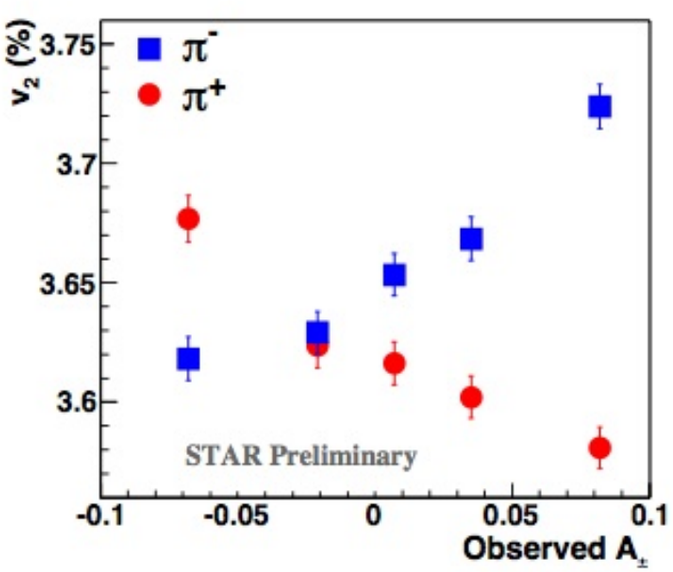
Charge dependent elliptic flow

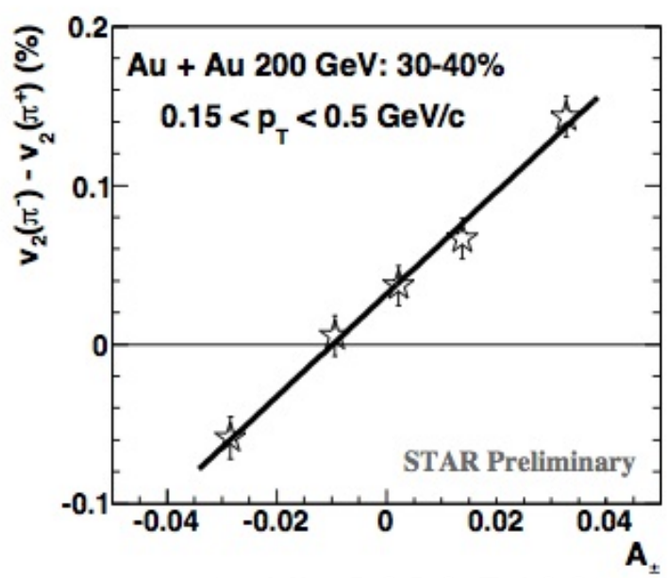
■**Observable：**

$$\frac{d\left(N_+ + N_-\right)}{d\phi} = \left(\overline{N}_+ + \overline{N}_-\right)\left[1 + 2v_2\cos(2\phi)\right],$$

$$\frac{d\left(N_+ - N_-\right)}{d\phi} = \left(\overline{N}_+ - \overline{N}_-\right)\left[1 + 2r_e\cos(2\phi)\right]$$

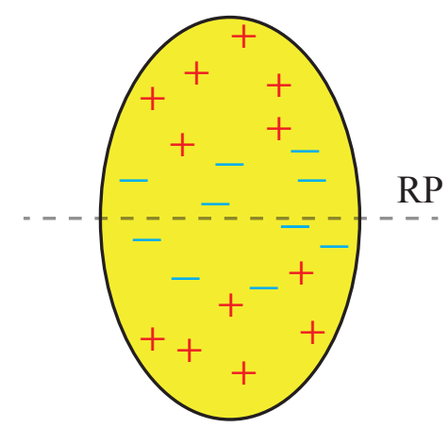
Charge dependent v_2： $v_2^{\pm} = v_2 \mp r_e A$ $\qquad A \equiv \frac{\overline{N}_+ - \overline{N}_-}{\overline{N}_+ + \overline{N}_-}$





(arXiv:1210.5498)

Quadrupole deformation



$$r_e \sim 10^{-2} > 0$$

Effect of the CME/CSE？

(Burnier et al, (2012))

Origins of $\Delta v_2^{\pm}$

charge dependent v_2

Chiral Magnetic Wave

Lorentz force

Conducting current

Local Charge Conservation

full-3D anomalous hydrodynamic simulation

Anomalous Hydrodynamics

Hydrodynamics in External Fields

■ Relativistic hydrodynamic equation

– Energy-momentum conservation of the fluid

$$\partial_\mu T^{\mu\nu} = 0$$

$T^{\mu\nu}$: energy-momentum tensor of the QGP fluid

■ Relativistic hydrodynamic equation in external fields

$$\partial_\mu T^{\mu\nu} = \underline{F^{\nu\lambda} j_\lambda} \qquad \partial_\mu j^\mu = 0$$

Energy-momentum source due to **Lorentz Force**

$F^{\mu\nu}$: field strength tensor

j^μ : U(1)$_V$ current

Anomalous Hydrodynamics

(Son, Surowka(2009))

◇ Conservation law :

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda \qquad \partial_\mu j^\mu = 0 \qquad \partial_\mu j_5^\mu = -CE^\mu B_\mu$$

◇ Constitutive equation :

$$j^\mu = nu^\mu + \underbrace{\kappa_B B^\mu}_{\text{CME}} + \underbrace{\kappa_\omega \omega^\mu}_{\text{CVE}}$$

Chiral Magnetic/Vortical Effect(CME/CVE)

$$j_5^\mu = n_5 u^\mu + \underbrace{\xi_B B^\mu}_{\text{CSE}} + \underbrace{\xi_\omega \omega^\mu}_{\text{CVE}}$$

Chiral Separation Effect ... (CSE)

◇ Coefficients :

$$\kappa_B = C\mu_5 \left(1 - \frac{\mu n}{e+p}\right) \qquad \kappa_\omega = 2C\mu\mu_5 \left(1 - \frac{\mu n}{e+p}\right)$$

$$\xi_B = C\mu \left(1 - \frac{\mu_5 n_5}{e+p}\right) \qquad \xi_\omega = C\mu^2 \left(1 - \frac{2\mu_5 n_5}{e+p}\right)$$

Numerical Simulations

Simulation Setup (1)

◇ **Hydrodynamic equation**

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda \qquad \partial_\mu j^\mu = 0 \qquad \partial_\mu j_5^\mu = -C E^\mu B_\mu \qquad C = \frac{1}{2\pi^2}$$

◇ **CME and CSE at Leading order**

$$j^\mu = n u^\mu + C\mu_5 B^\mu \qquad j_5^\mu = n_5 u^\mu + C\mu B^\mu$$

◇ **EoS: 1-component Fermion ideal gas**

$$p = \frac{1}{3}e = \frac{7\pi^2}{180}T^4 + \frac{1}{6}\left(\mu^2 + \mu_5^2\right)T^2 + \frac{1}{12\pi^2}\left(\mu^4 + 6\mu^2\mu_5^2 + \mu_5^4\right)$$

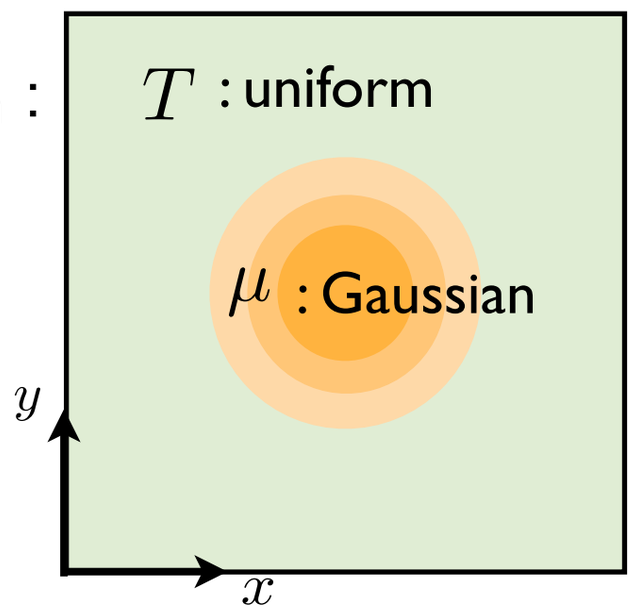
$$n = \frac{1}{3\pi^2}\mu^3 + \frac{1}{3}\mu\left(T^2 + \frac{3}{\pi^2}\mu_5^2\right), \; n_5 = \frac{1}{3\pi^2}\mu_5^3 + \frac{1}{3}\mu_5\left(T^2 + \frac{3}{\pi^2}\mu^2\right)$$

→ **Solve these equations numerically without linearization**

Simulation Setup (2)

1. Test case : Charge in uniform temperature

Initial Condition :



without anomaly

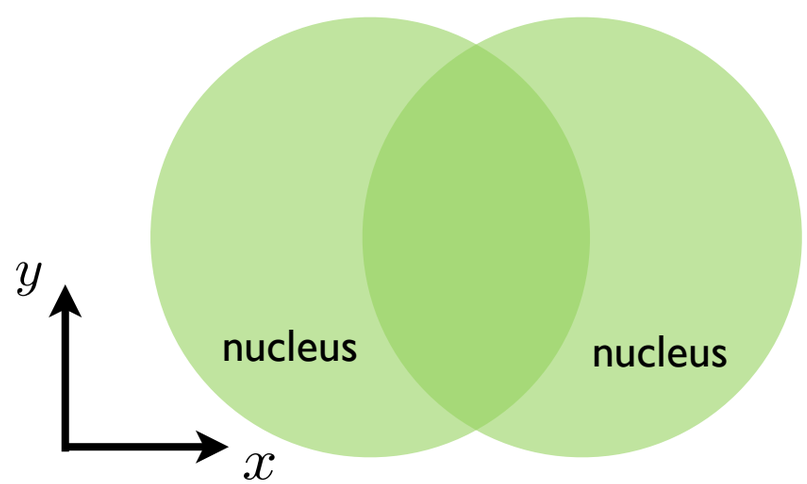
with anomaly

Lorentz force only

Effects of Anomaly

CME/CSE

2. More realistic case : Expanding geometry



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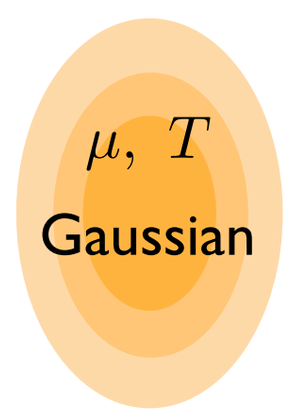
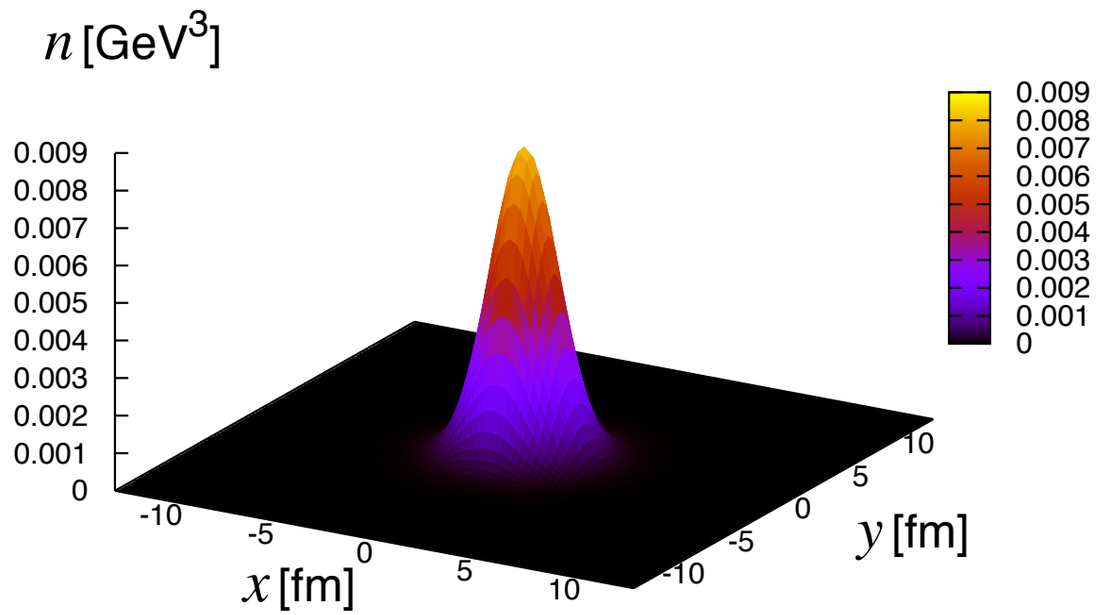


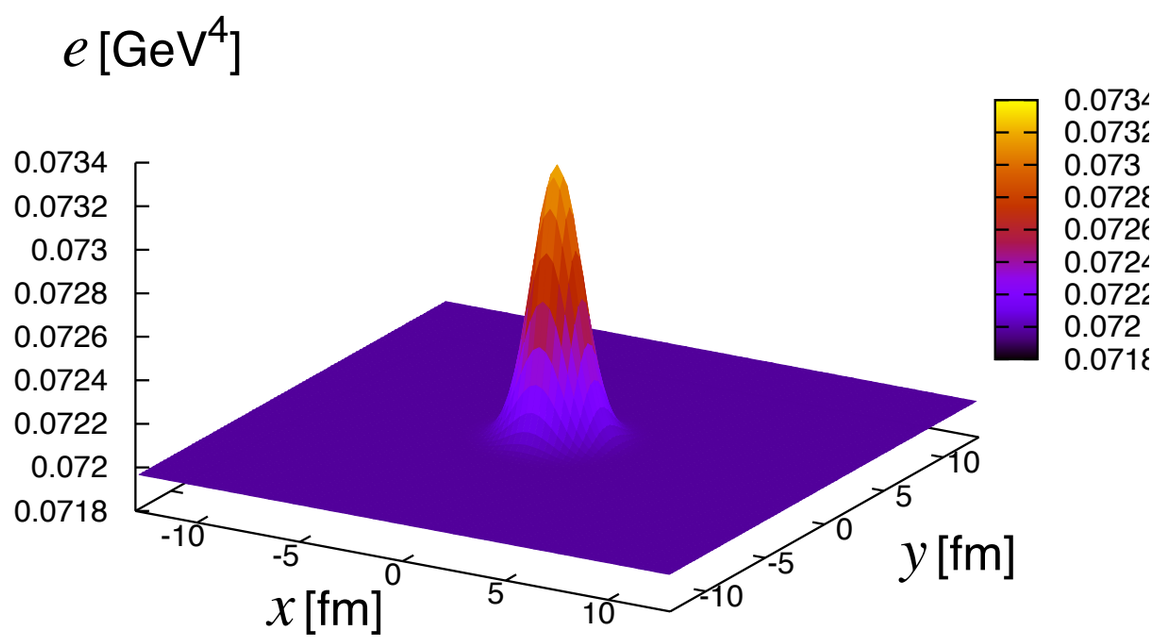
- CMW in expanding plasma
- Source term by Anomaly
- Lorentz force

Without anomaly

1. Charge in uniform temperature

■ Initial Condition





Parameters

- Temperature $T_0 = 0.5\ \mathrm{GeV}$
- Chemical potential

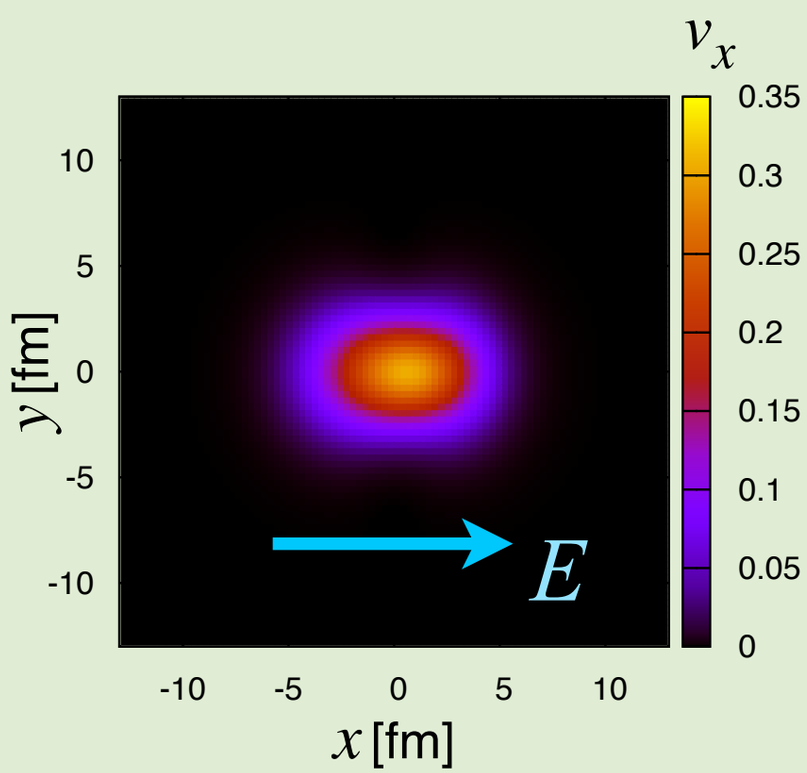
$$\mu(\vec{r}) = \mu_0 \exp\left(-\frac{\vec{r}^{\,2}}{2\sigma^2}\right)$$

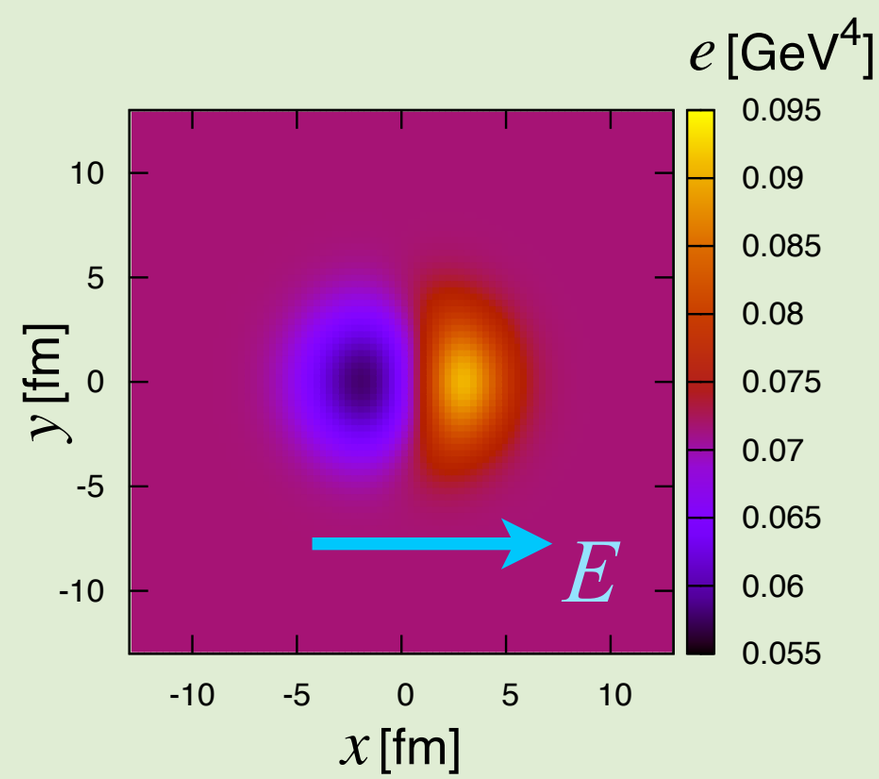
$$\mu_0 = 0.1\ \mathrm{GeV} \qquad \sigma = 1.5\ \mathrm{fm}$$

Without CME/CSE 1-1. Electric field

■ **In case of stationary Electric field** $eE_x = 0.1\ \mathrm{GeV}^2$

◇ x-component of velocity, energy distribution($t = 6.0\ \mathrm{fm}\ \ z = 0\ \mathrm{fm}$)

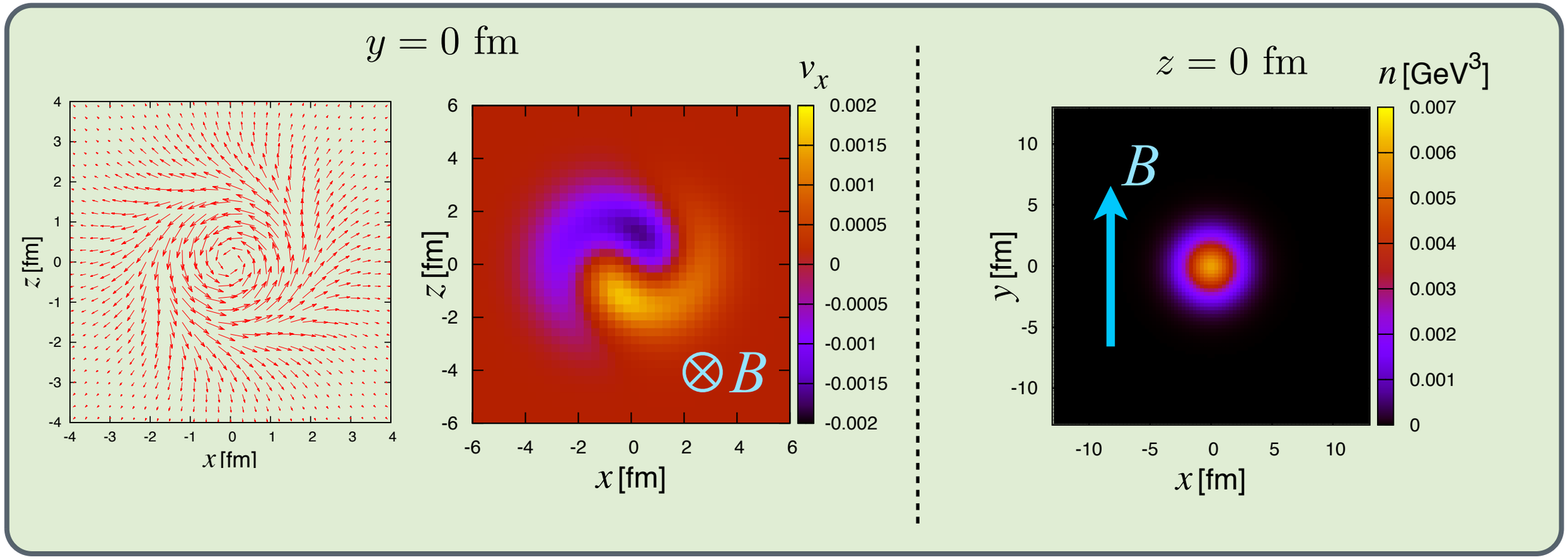




Without CME/CSE 1-2. Magnetic field

■ **In case of stationary magnetic field** $eB_y = 1.0\ \mathrm{GeV}^2$

◇ Velocity field, Charge distribution($t = 3.0\ \mathrm{fm}$)



– Rotational velocity field in the plane ⊥ magnetic field

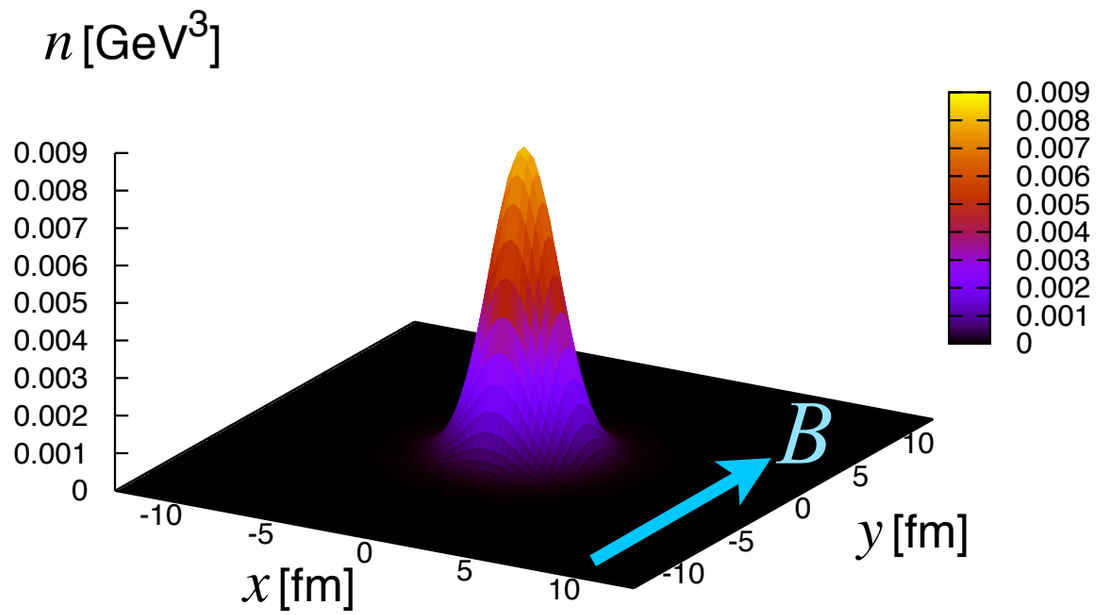
  – Magnitude of velocity: ~0.002c

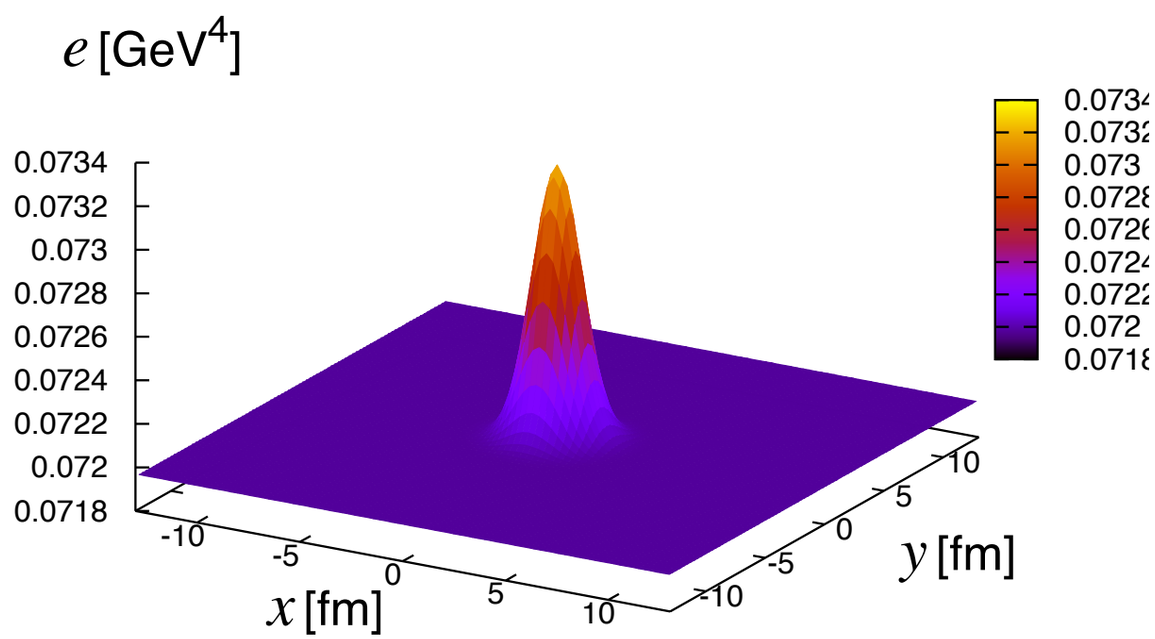
– There is no Charge transportation along magnetic field

With anomaly

1. Charge in uniform temperature

■ Initial Condition





Parameters (IC & EM field)

- Temperature $T_0 = 0.5\ \mathrm{GeV}$
- Chemical potential

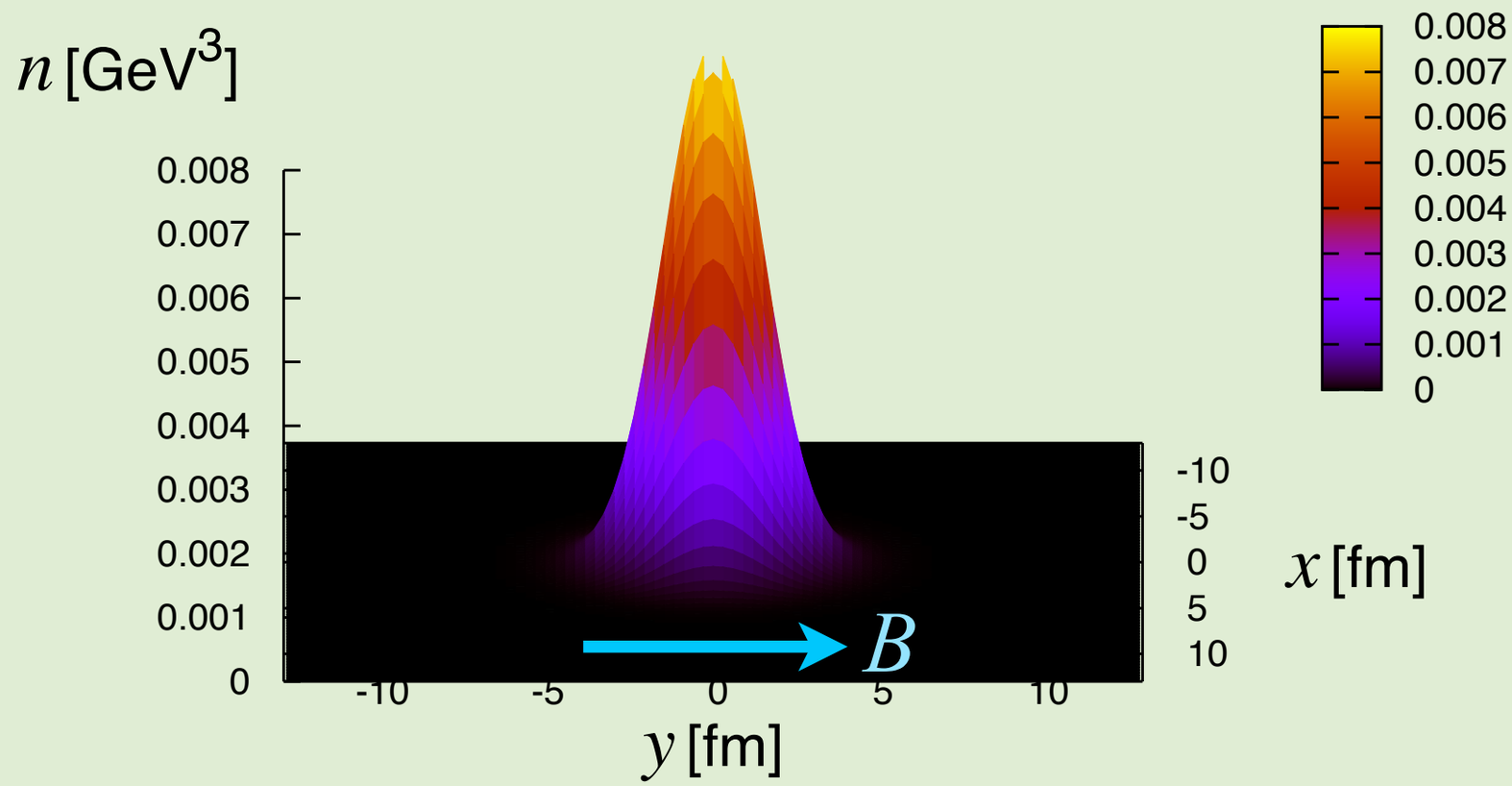
$$\mu(\vec{r}) = \mu_0 \exp\left(-\frac{\vec{r}^{\,2}}{2\sigma^2}\right)$$

$$\mu_0 = 0.1\ \mathrm{GeV} \qquad \sigma = 1.5\ \mathrm{fm}$$

- EM field

$$eB_y = 1.0\ \mathrm{GeV}^2$$

■ Time evolution of the charge distribution ($t = 0 - 9.0$ fm)



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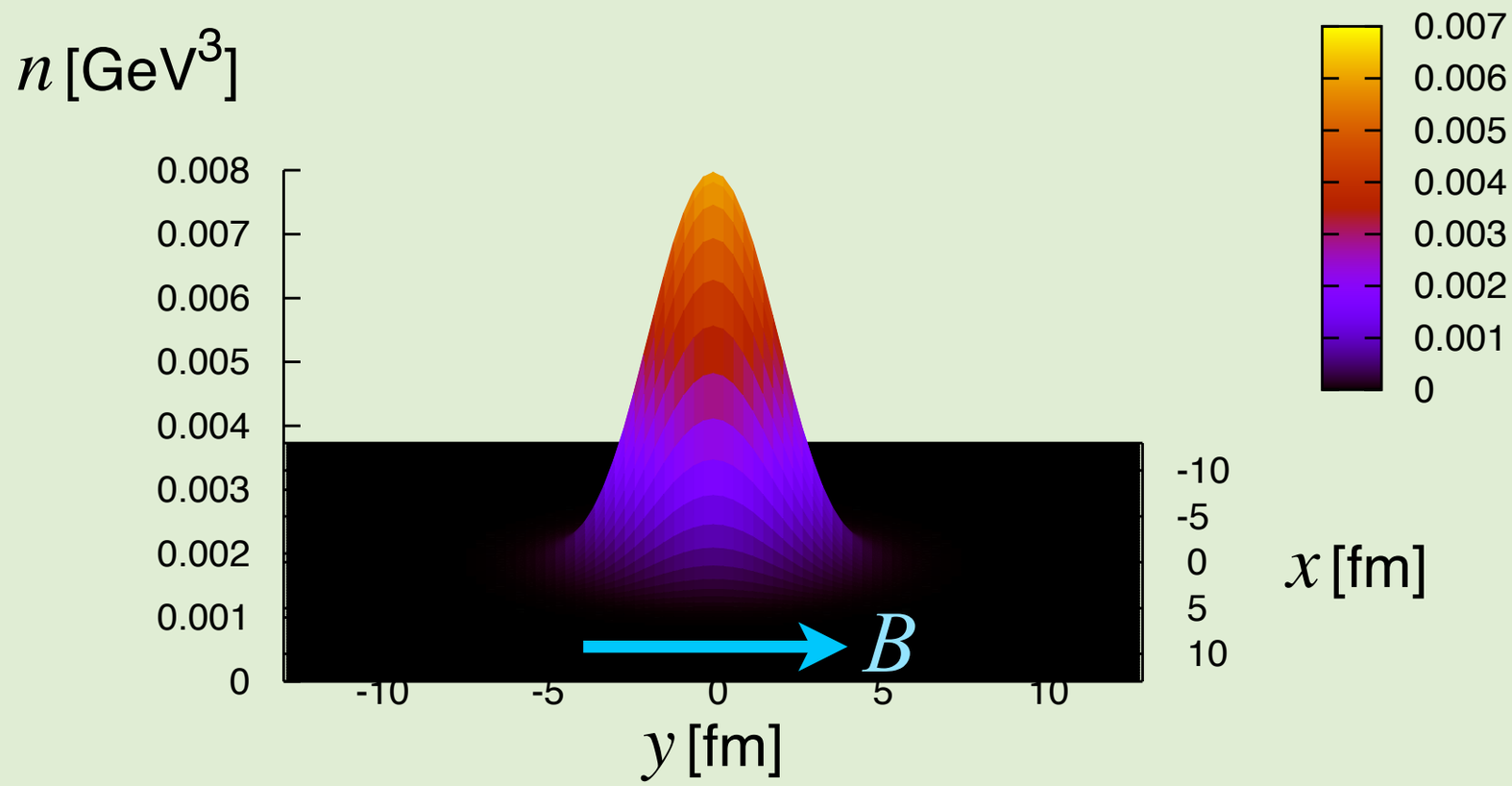

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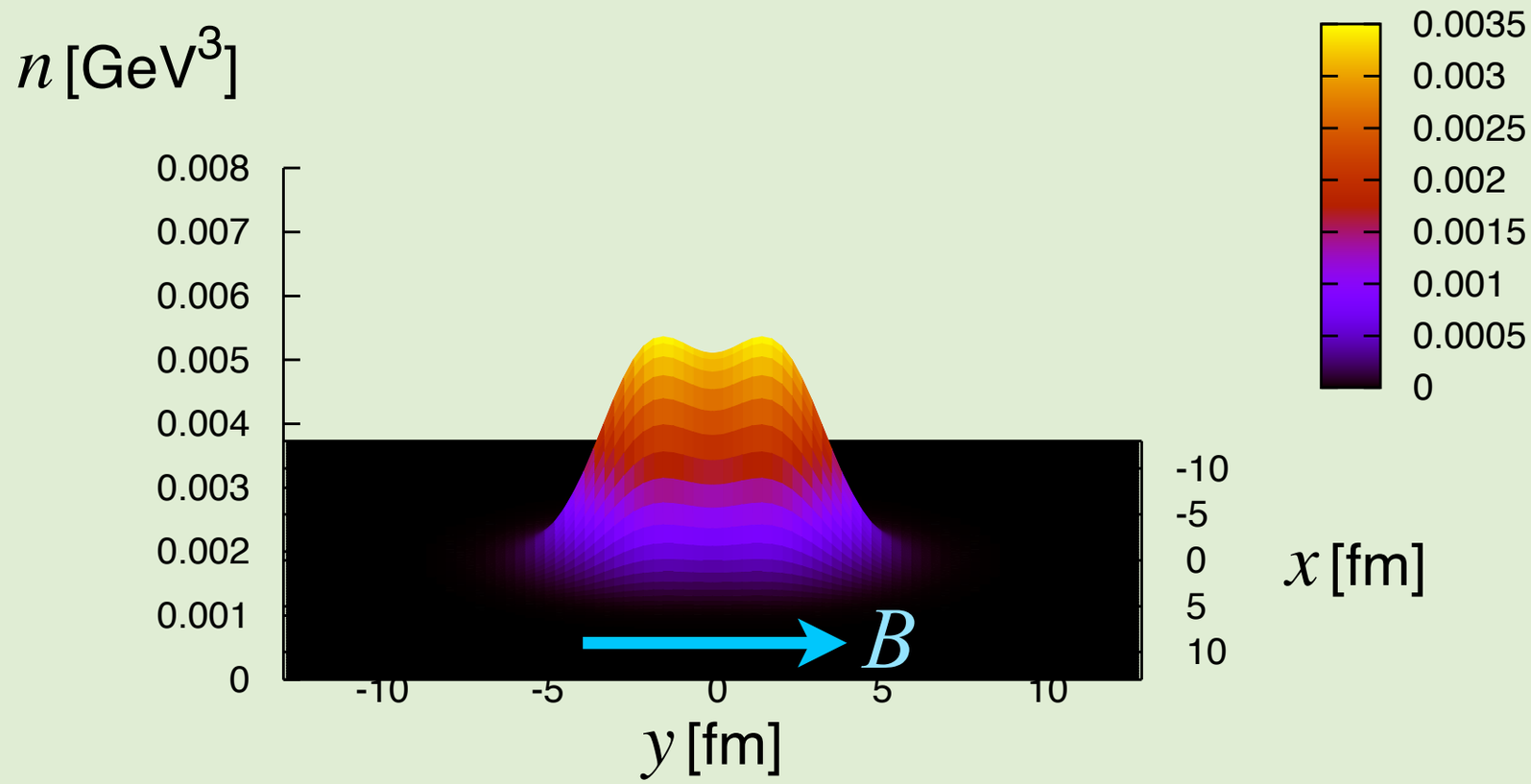
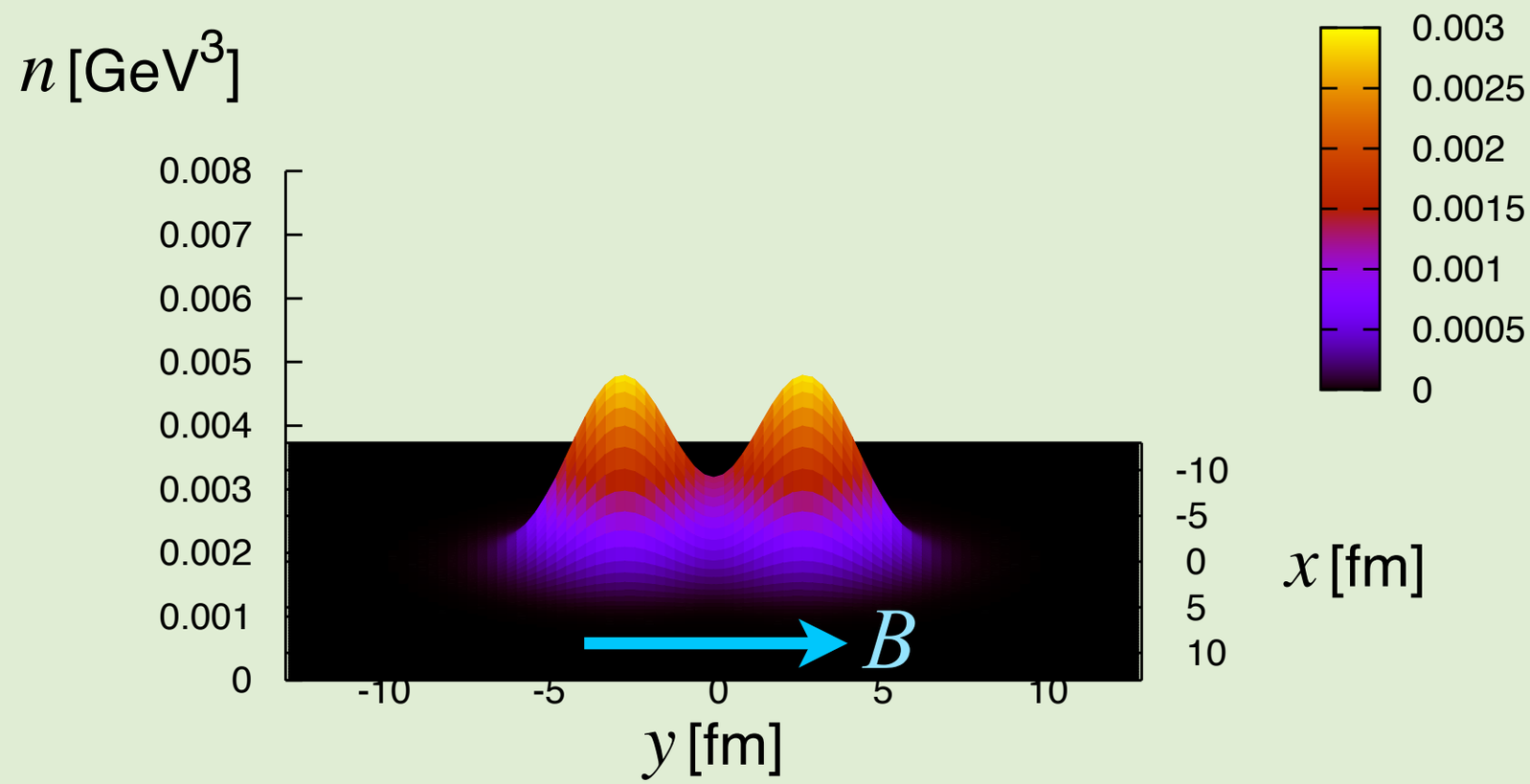


■ Time evolution of the charge distribution ($t = 0 - 9.0$ fm)



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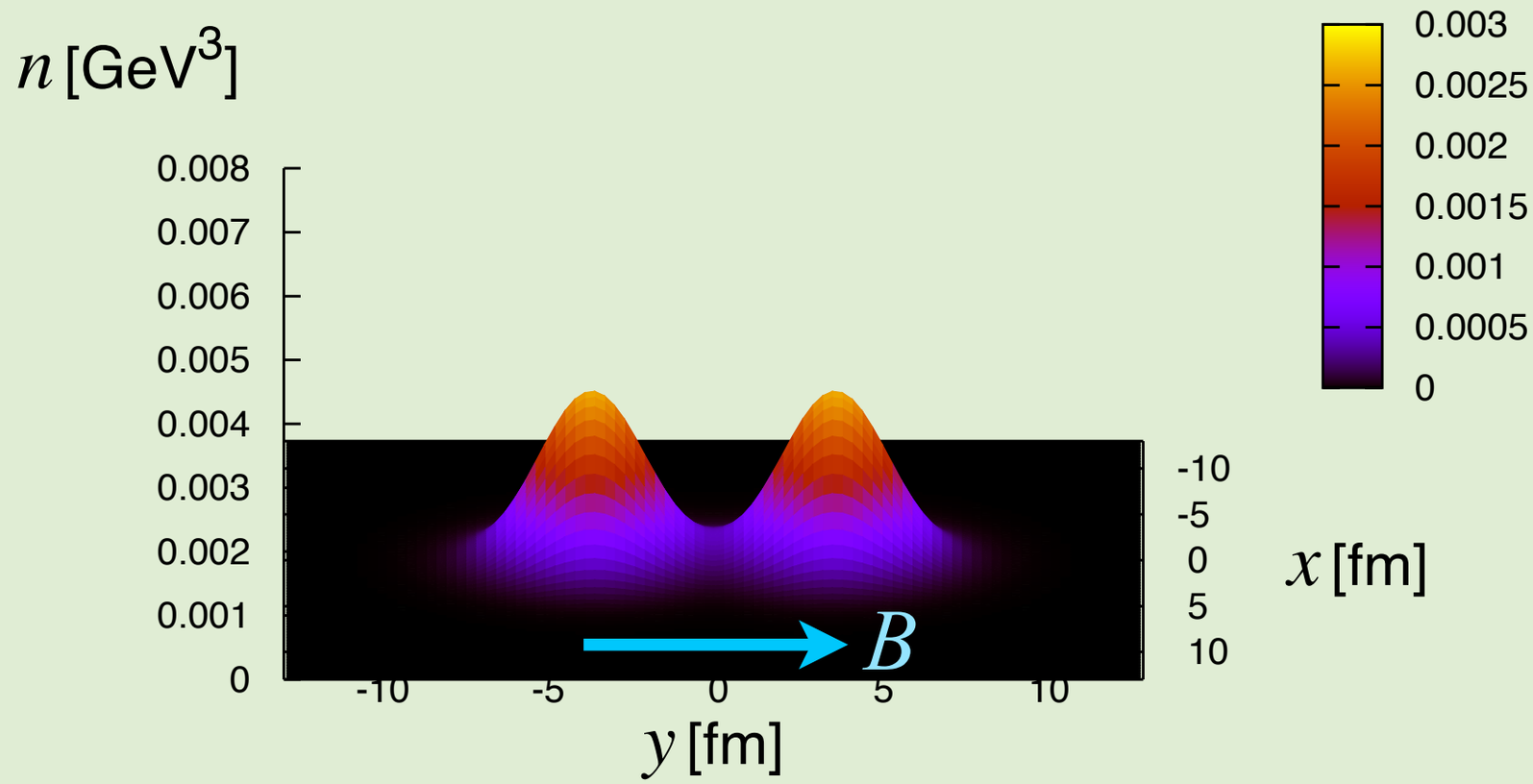

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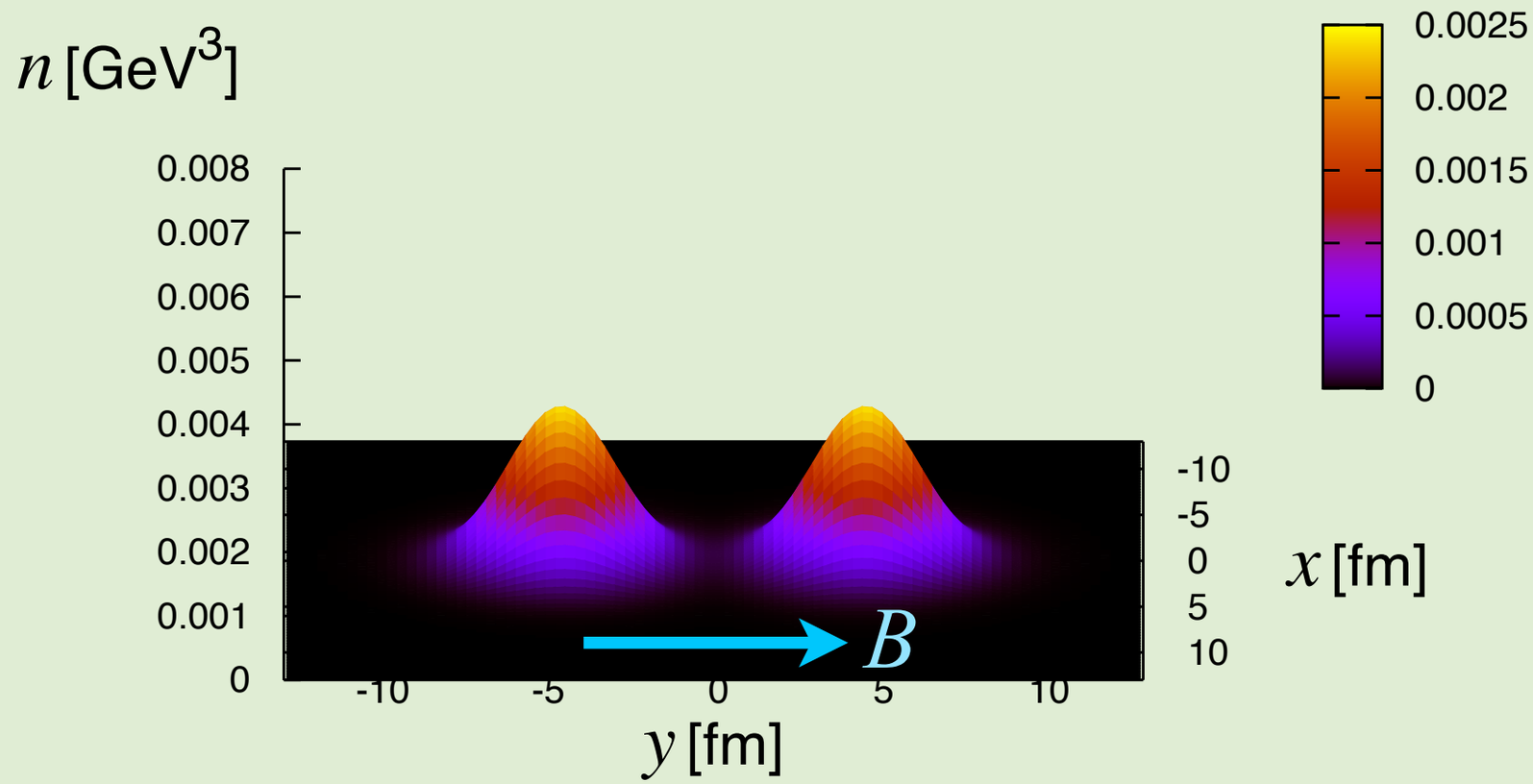
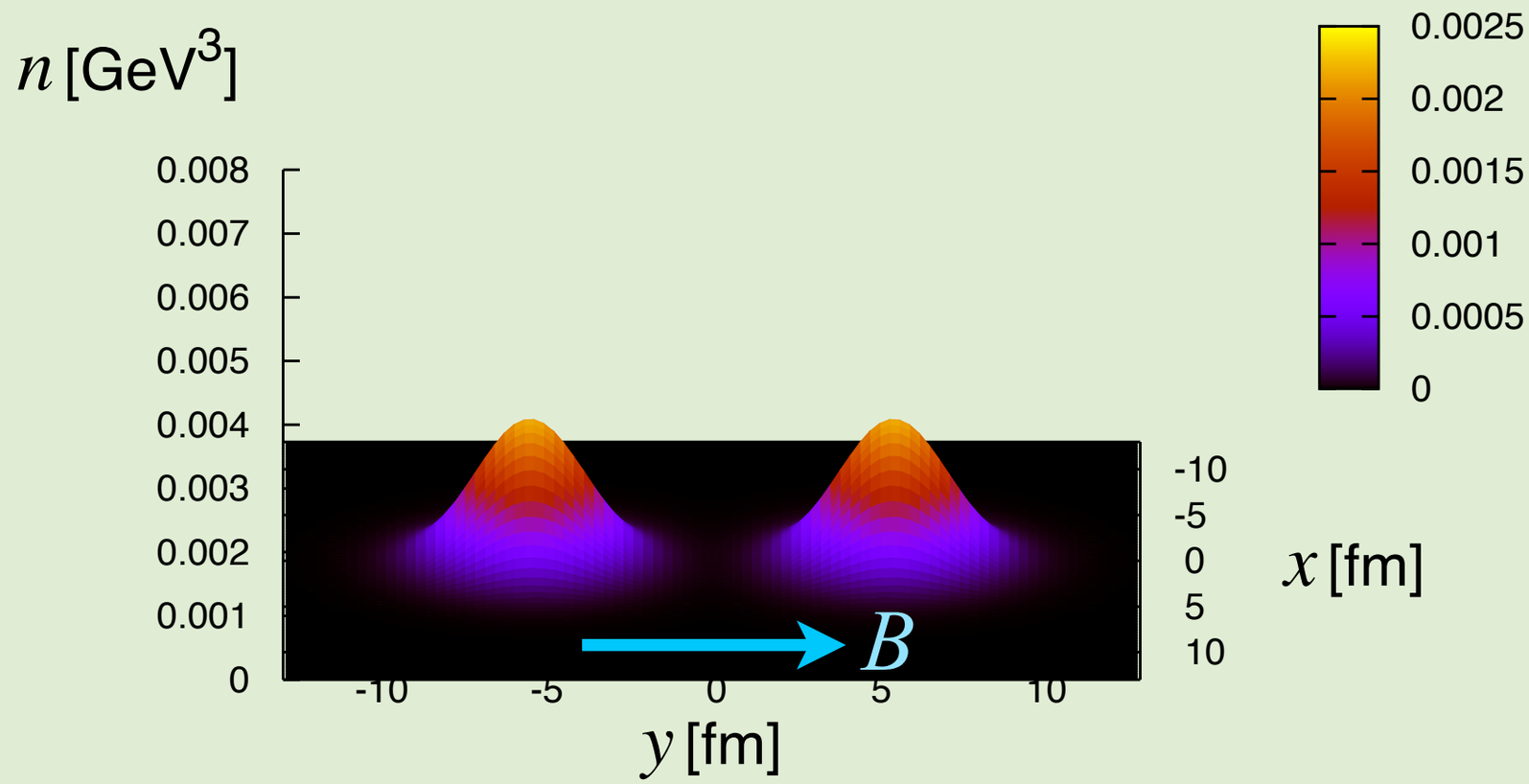
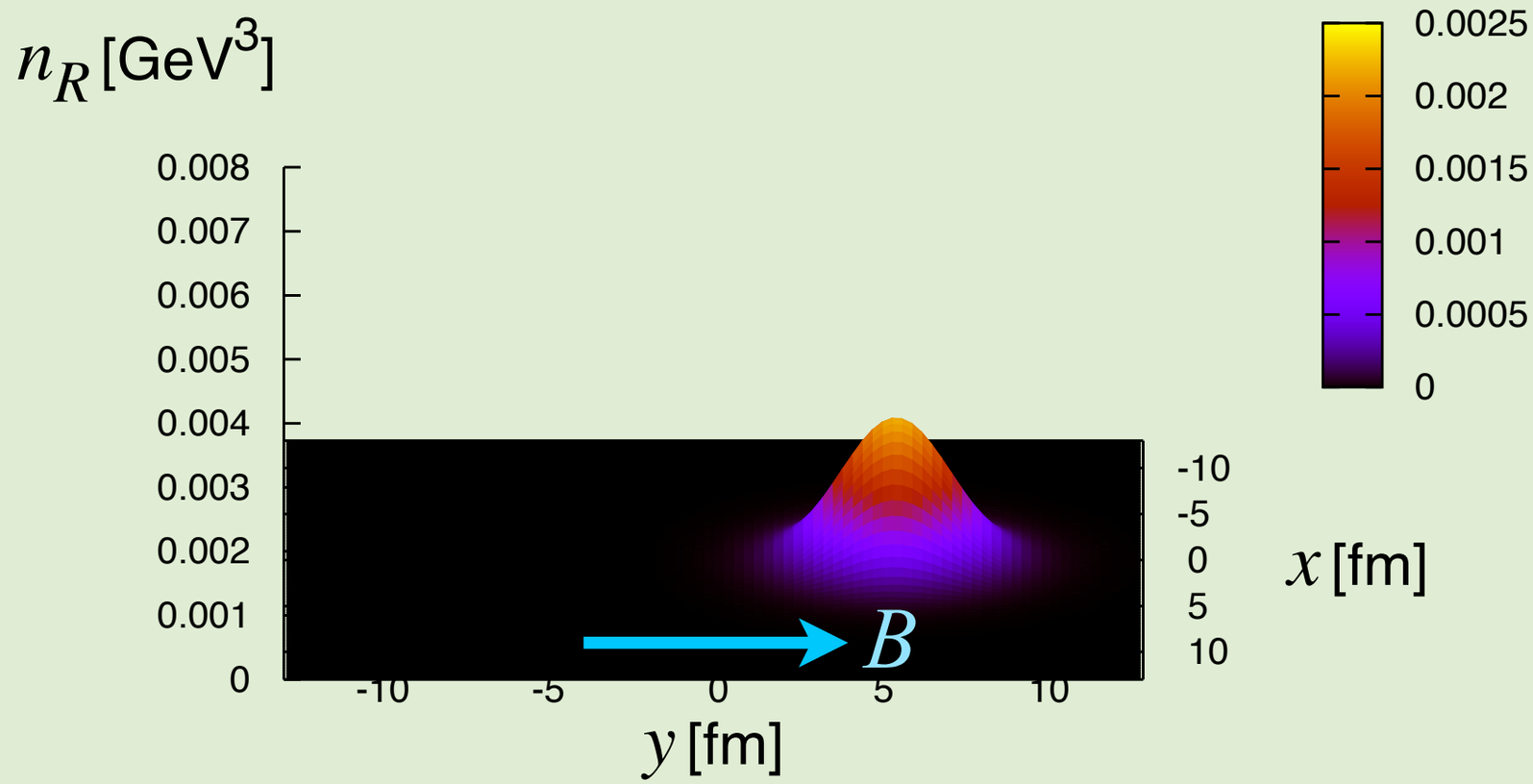


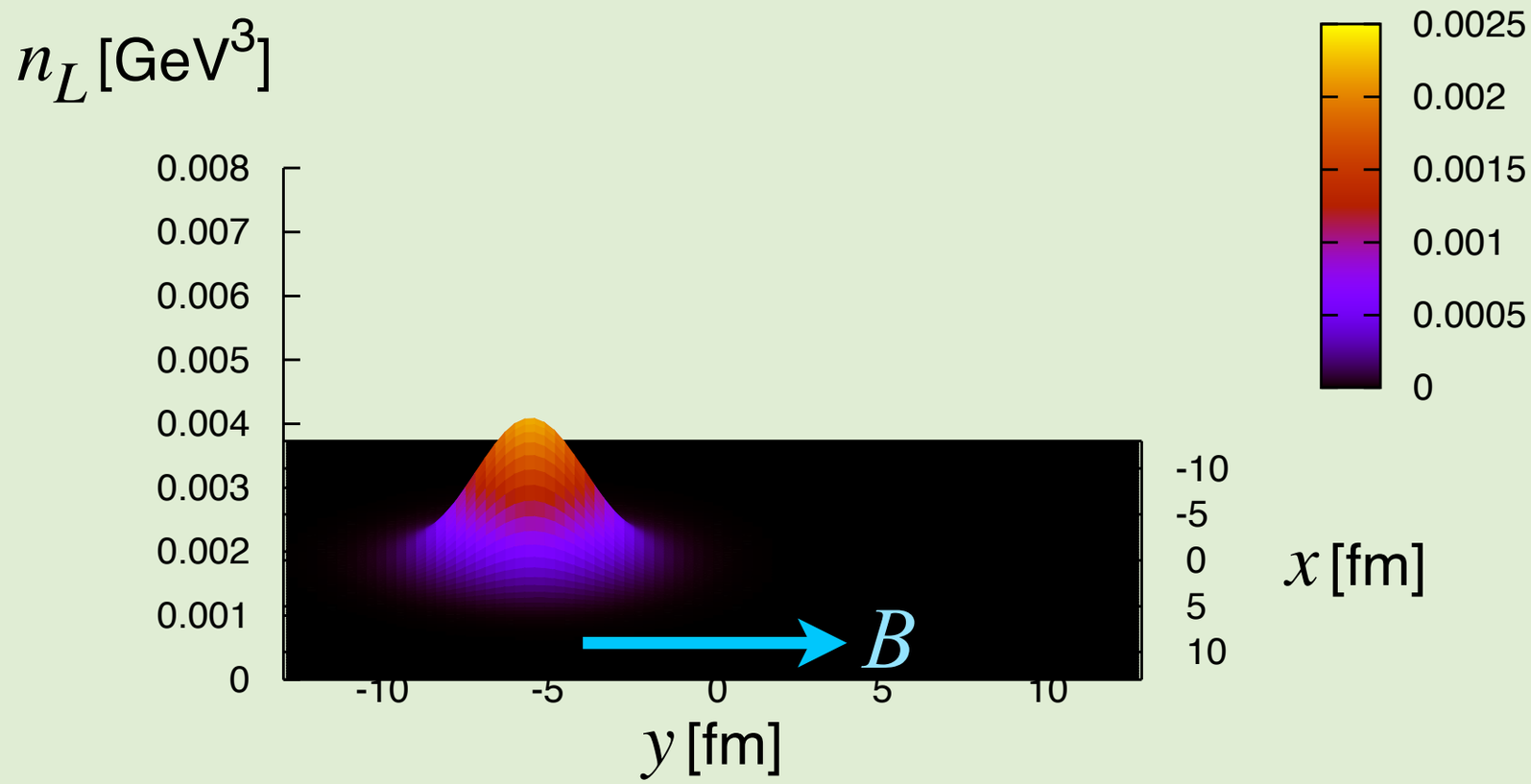
■ Time evolution of the charge distribution ($t = 0 - 9.0$ fm)



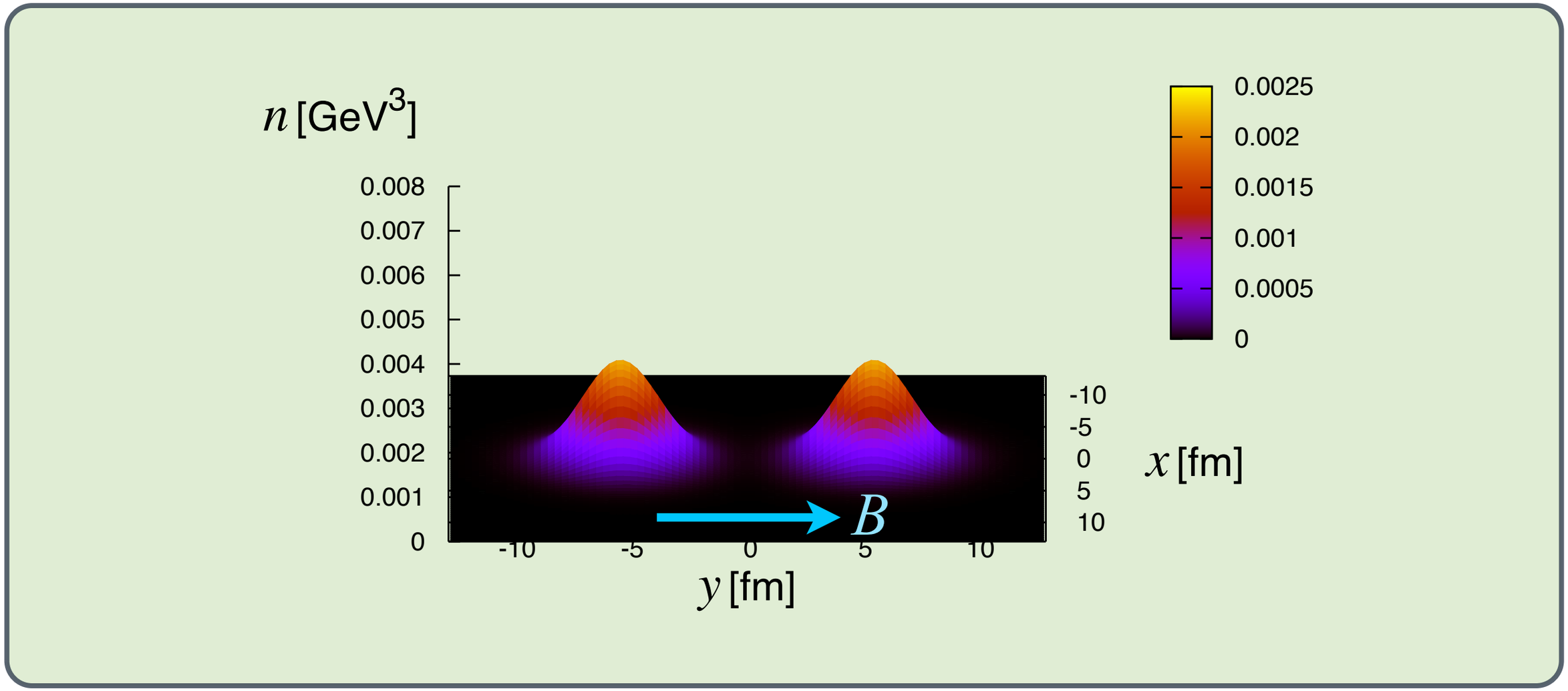
■ Time evolution of the charge distribution ($t = 0 - 9.0$ fm)



■ Time evolution of the charge distribution ($t = 0 - 9.0$ fm)



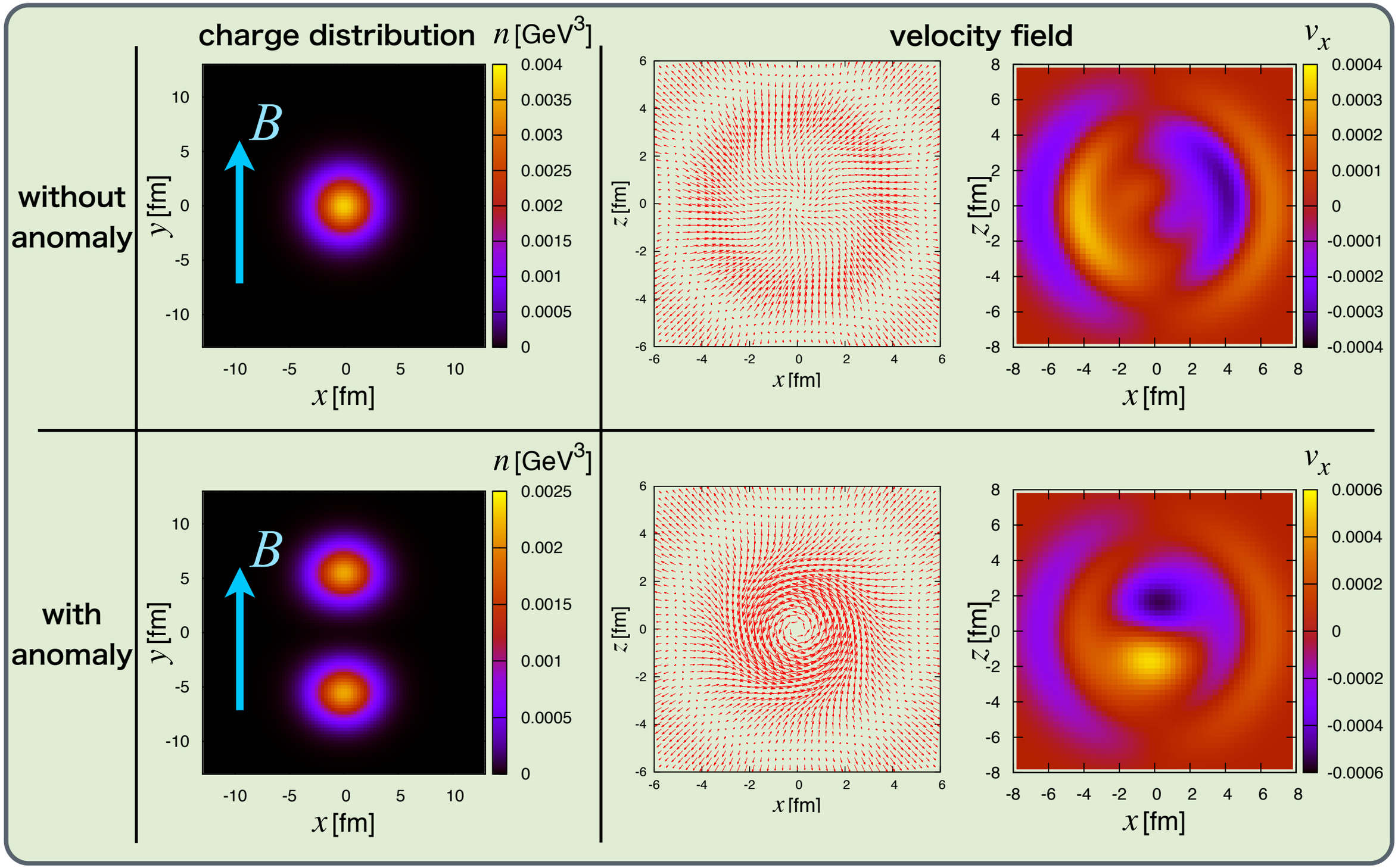
■ Time evolution of the charge distribution ($t = 0 - 9.0$ fm)



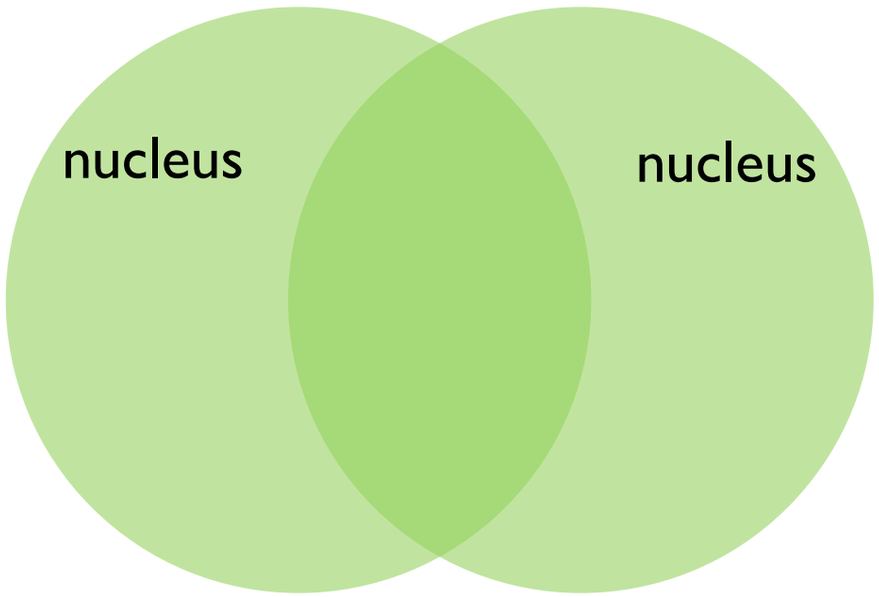
– Charge propagation along magnetic field = Chiral Magnetic Wave

– Estimated $v_\chi \simeq 0.58$ is consistent with a linearized calculation $v_\chi \simeq 0.61$

■Comparison with cases with/without anomaly ($t = 9.0$ fm)



2. Expansion & Anomaly Source

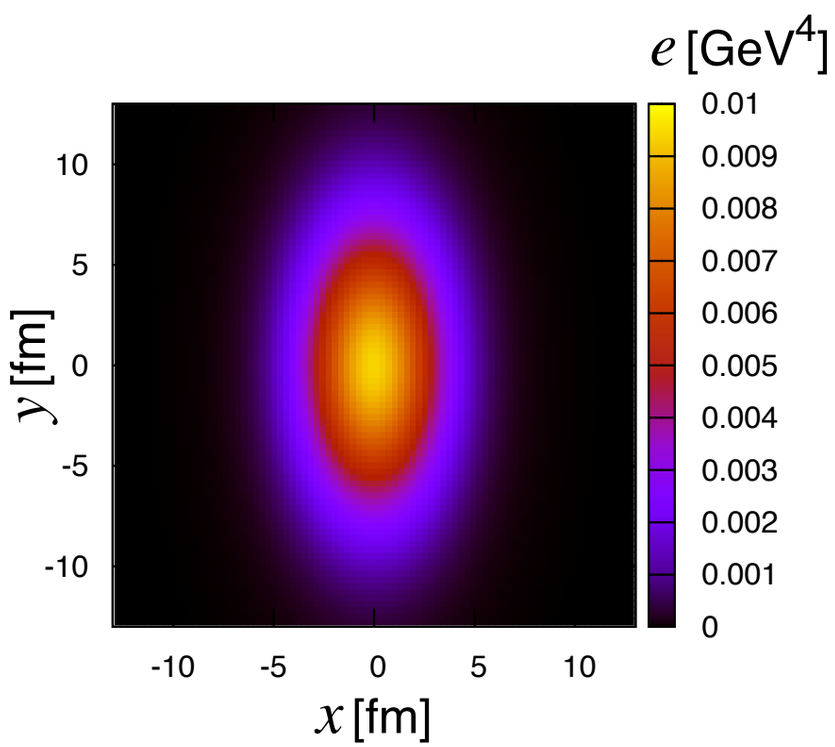


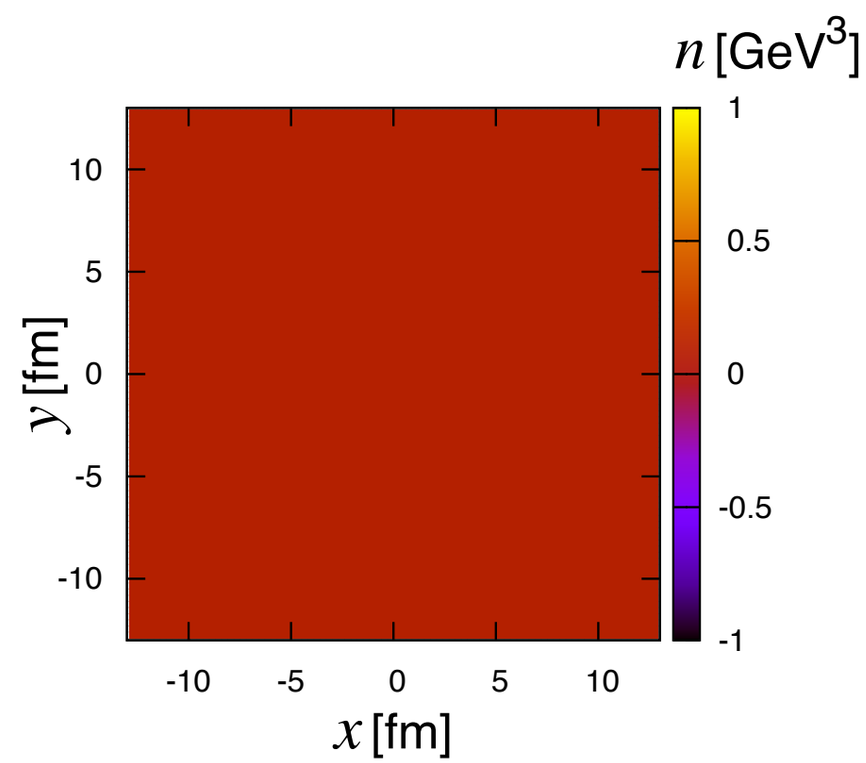
Parameters

$$T(\vec{r}) = T_0 \exp\left(-\frac{r'^2}{2\sigma_T^2}\right) \qquad T_0 = 0.3\ \text{GeV} \quad \sigma_T = 5\ \text{fm}$$

$$\mu(\vec{r}) = \mu_0 \exp\left(-\frac{r'^2}{2\sigma_\mu^2}\right) \qquad \mu_0 = 0\ \text{GeV} \quad \sigma_\mu = 2\ \text{fm}$$

$$r' = \sqrt{x^2 + (y/2)^2 + z^2}$$





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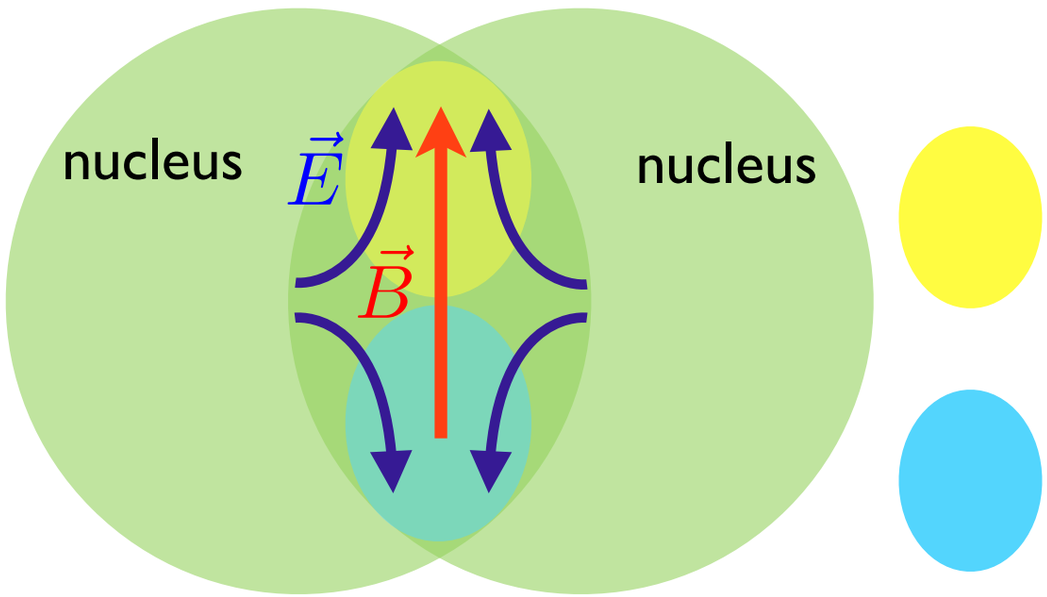
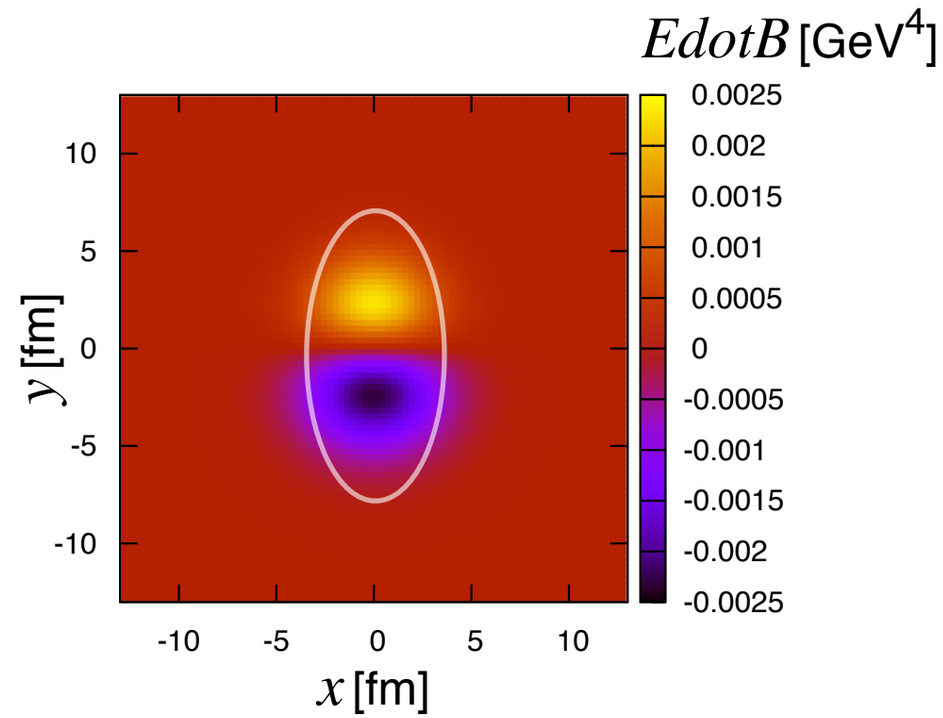


$\vec{E}\cdot\vec{B} > 0$

$\vec{E}\cdot\vec{B} < 0$



Parameters

$$eB_y(\vec{r}) = eB_0 \exp\left(-\frac{r''^2}{2\sigma_B^2}\right) \times \exp\left(-\frac{t}{\tau_B}\right)$$

$$eB_0 = 0.08\ \mathrm{GeV}^2$$

$$\sigma_B = 4\ \mathrm{fm} \qquad \tau_B = 3\ \mathrm{fm}$$

$$eE_y(\vec{r}) = y \times eE_0 \exp\left(-\frac{r''^2}{2\sigma_E^2}\right) \times \exp\left(-\frac{t}{\tau_E}\right)$$

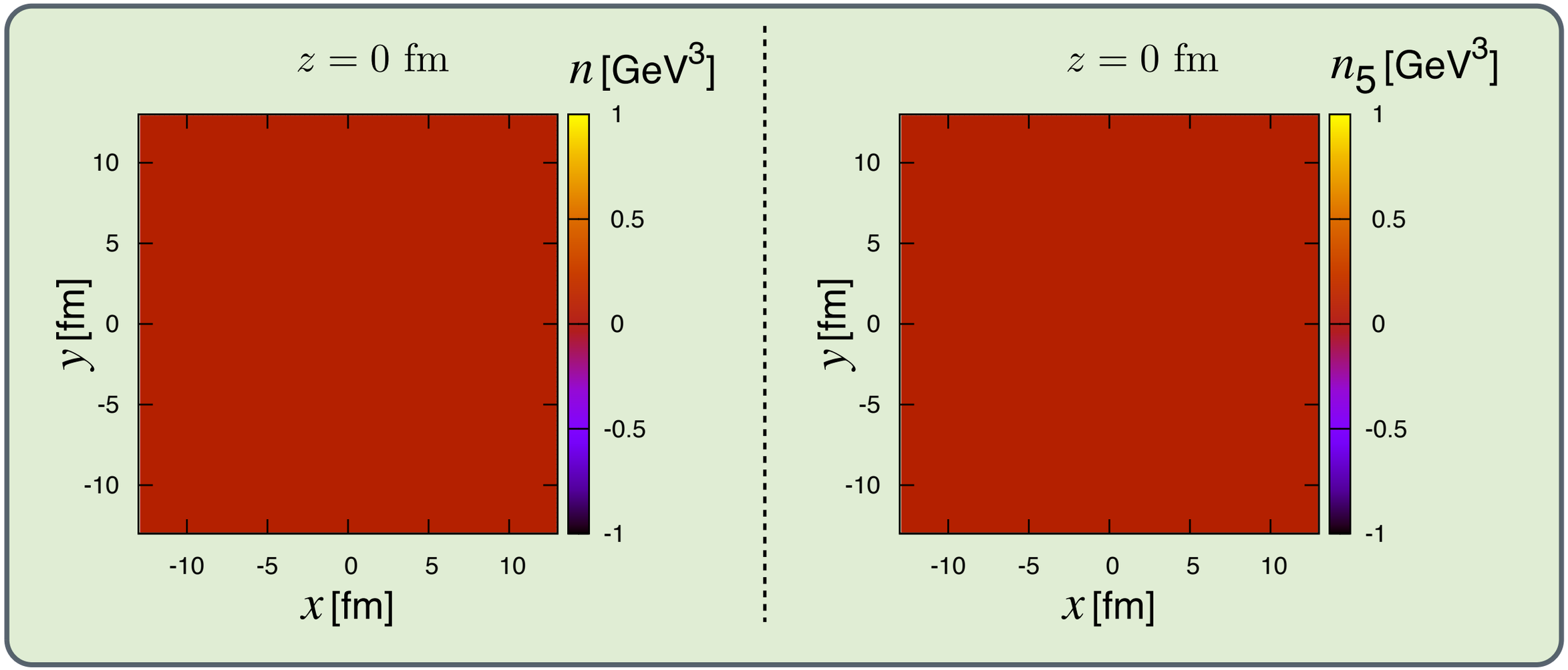
$$eE_0 = 0.02\ \mathrm{GeV}^2$$

$$\sigma_E = 3\ \mathrm{fm} \qquad \tau_E = 1\ \mathrm{fm}$$

$$r'' = \sqrt{x^2 + y^2 + (2z)^2}$$

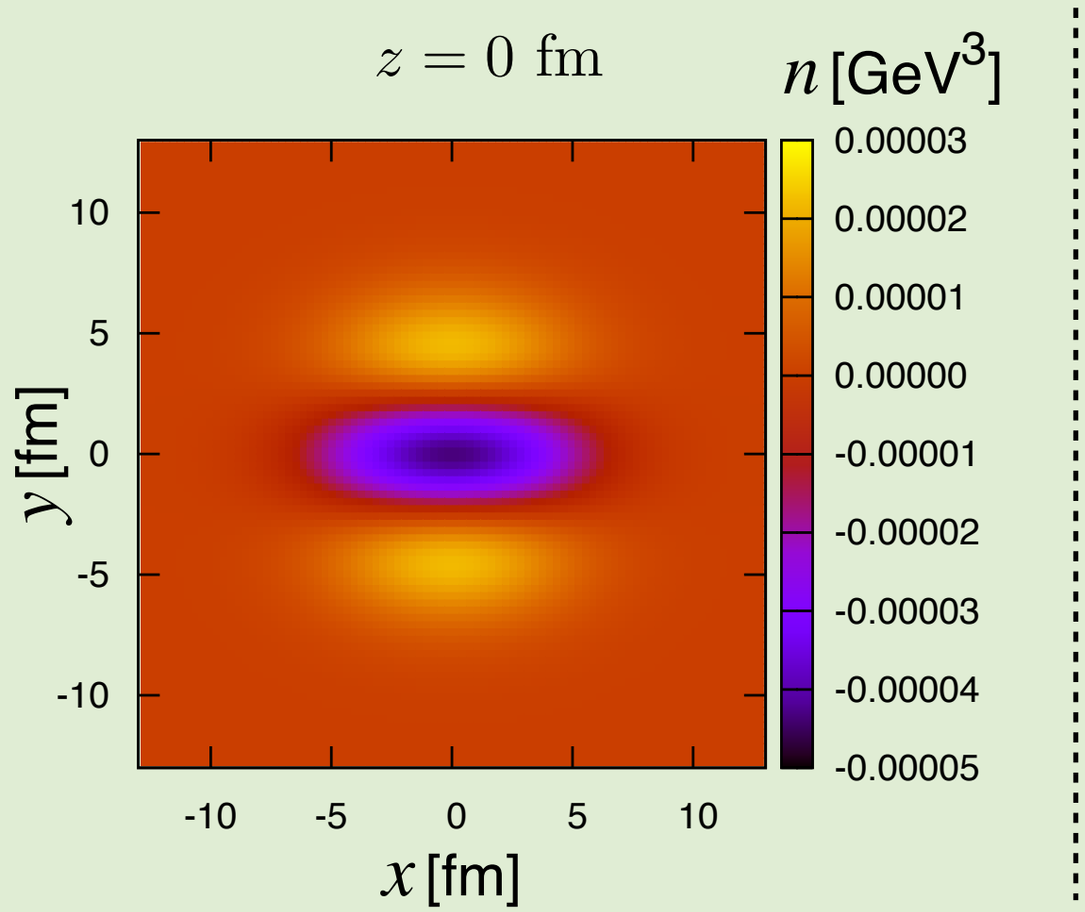
■ Deformation of the charge distribution

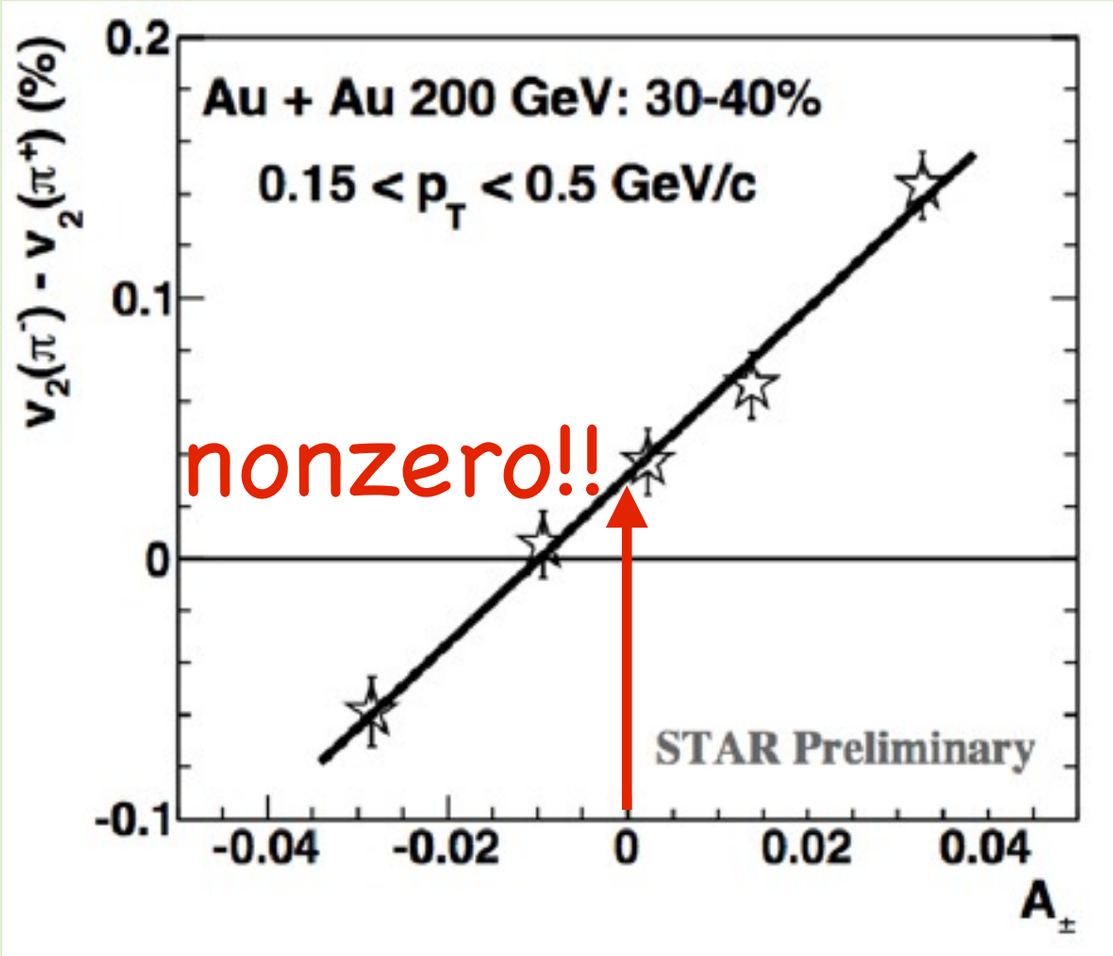
◇ Vector/Axial charge distribution($t = 0.0$ fm)



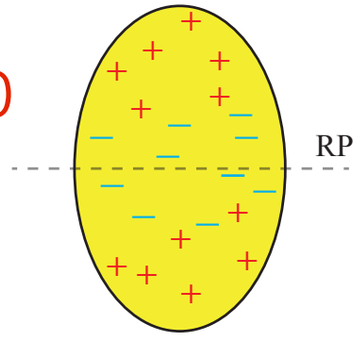
■ Deformation of the charge distribution

◇ Vector/Axial charge distribution($t = 6.0$ fm)





– Quadrupole deformation of charge distribution even if $A_\pm$=0



→ Origin of $v_2(\pi^-) > v_2(\pi^+)$ in high energy collision?

Charge dependent v_2

Numerical simulation for v_2

Initial condition: $T(\vec{r})$ $\mu(\vec{r})$ $\mu_5(\vec{r})$ ... **at t = 0 fm**

$\mu_0 = 0,\ 0.05, ...\mathrm{GeV}$

Anomalous hydro simulation

Freeze out time: $T(\vec{r})$ $\mu(\vec{r})$ $\mu_5(\vec{r})$ ... **at t = 6 fm**

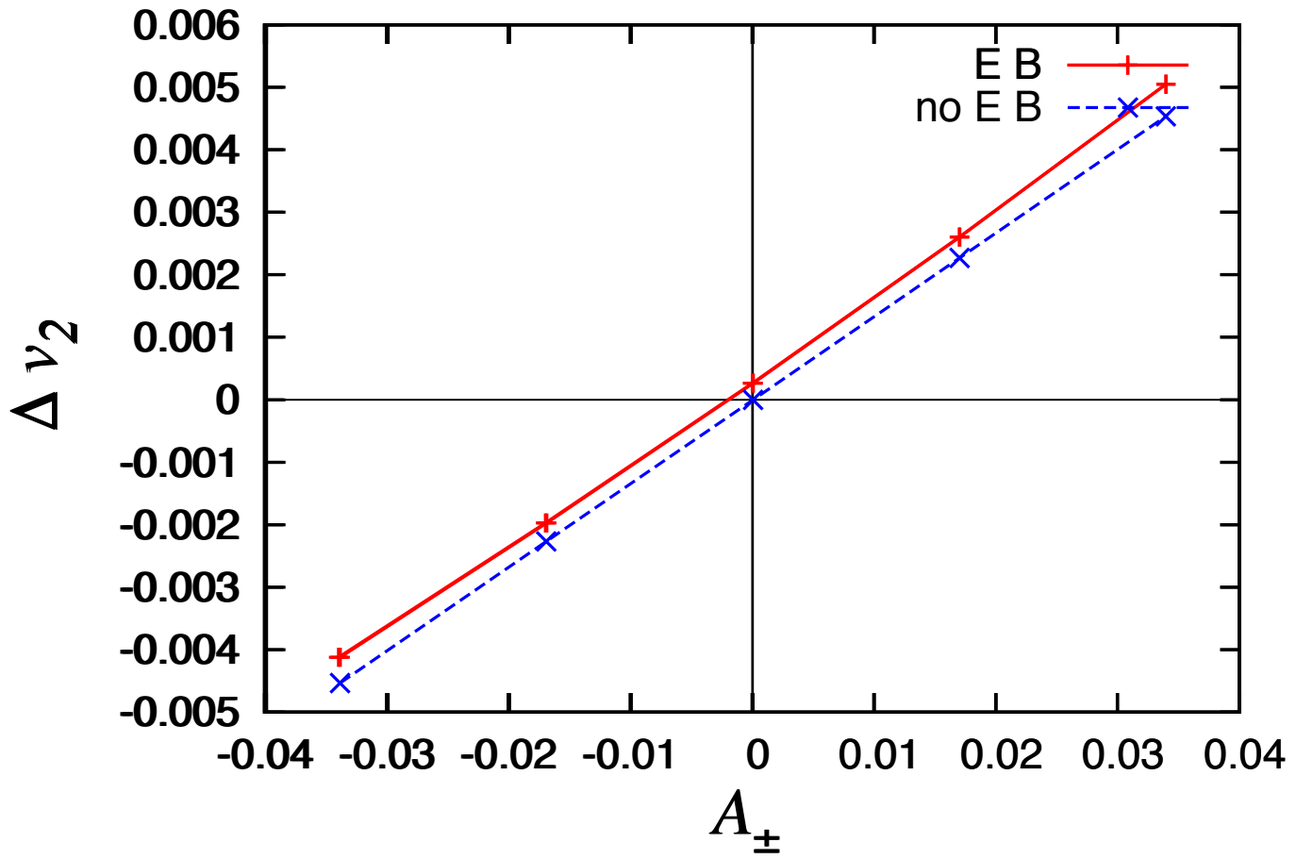
Cooper-Frye formula

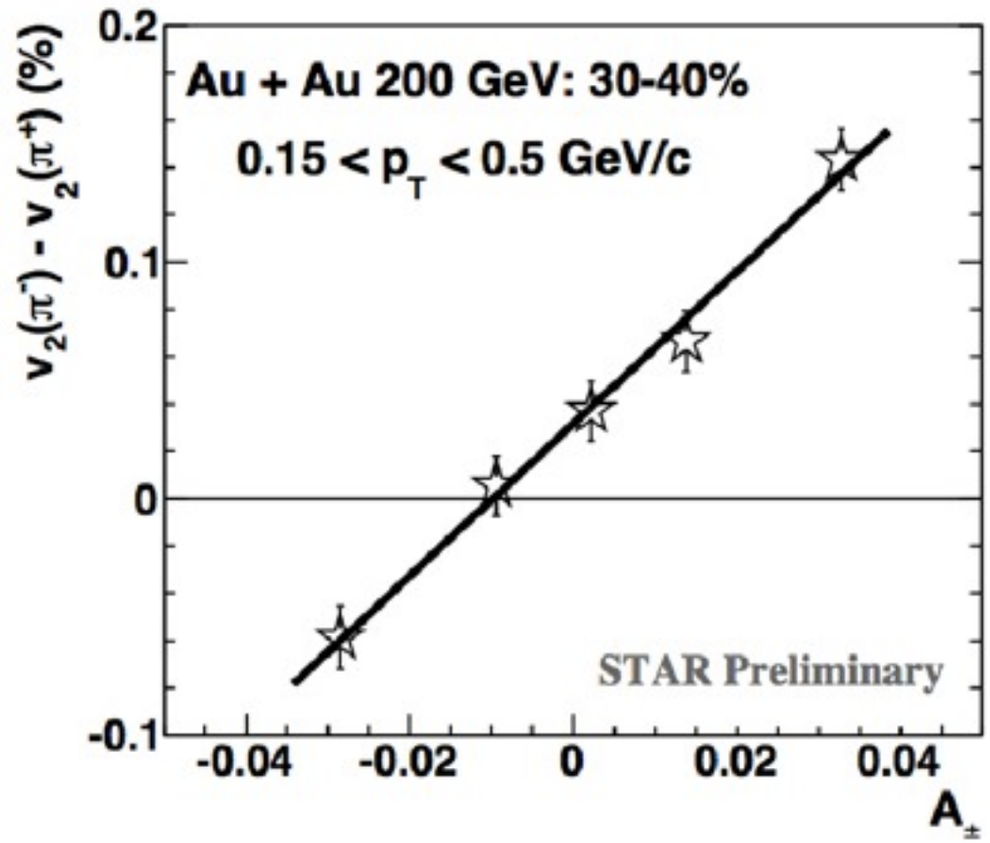
Azimuthal particle distribution: $\frac{dN_\pm}{d\phi}$ $A = \frac{\overline{N}_+ - \overline{N}_-}{\overline{N}_+ + \overline{N}_-}$

$A = \bigcirc,\ \triangle, ...$

Charge dependent v_2

Results of $v_2^{\pm}$





- Finite difference in v_2 even if A=0
- Initial charge fluctuation itself has a effect on v_2

→ **Need more realistic study by anomalous hydro**

Summary

Summary

■ Anomalous Hydrodynamics

– Lorentz force from external fields

– Chiral Magnetic/Separation Effect, Chiral Magnetic Wave

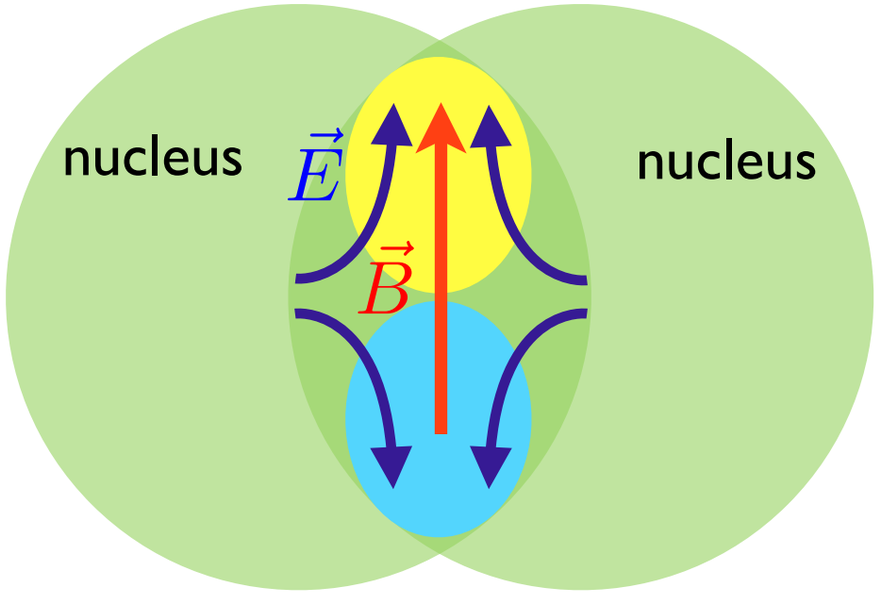
■ Numerical Simulation

– Effect of Lorentz force

– Chiral Magnetic Wave in an expanding matter

■ Suggestion for Heavy-Ion-Collisions

– Charge dependent elliptic flow $v_2^- > v_2^+$

Outlook

- Realistic setup for Heavy Ion Collisions
- Dissipative effects (conductivity, diffusion)
- Including Chiral Vortical Effect
- Dynamical Electromagnetic field