Dirac spectrum and chiral and $U_A(1)$ symmetries at finite temperature with the highly improved staggered quarks

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Plan of this talk

- Introduction
  - Symmetries in QCD
  - Order of the phase transition in N_f=2 and $U_A(1)$ symmetry
  - Dirac spectrum and the symmetries
- Results
- Conclusions

Symmetries in QCD

- QCD Lagrangian with N_f–flavor massless quarks is invariant under:

$$SU_L(N_f) \times SU_R(N_f) \times U_V(1) \times U_A(1)$$

  – $U_V(1)$: corresponds to the baryon number conservation
  – $SU_L(N_f) \times SU_R(N_f)$: spontaneously **broken** in the vacuum
  – $U_A(1)$: explicitly **broken** due to the quantum corrections (chiral anomaly)

$\mathbf{SU_L(N_f) \times SU_R(N_f)}$ symmetry

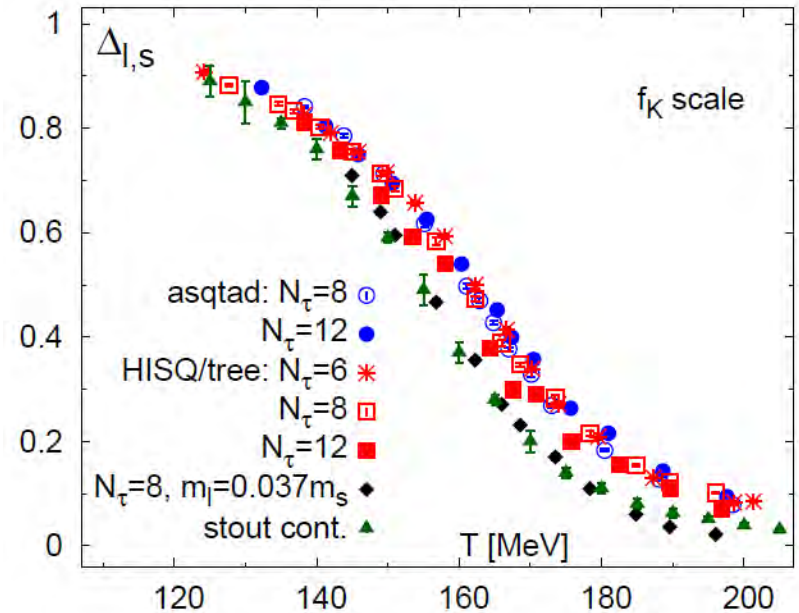
- $SU_L(N_f) \times SU_R(N_f)$ symmetry is expected to be **restored** at finite temperature.
- Chiral condensate $\Sigma \equiv -\langle \bar{\psi}\psi \rangle$: order parameter

– **chiral broken phase**

$\rightarrow \Sigma \neq 0$

– **chiral symmetric phase**

$\rightarrow \Sigma = 0$



A. Bazavov *et al*. [HotQCD Collaboration] (2011)

$\mathrm{U_A(1)}$ symmetry

- $U_A(1)$ symmetry is **broken** at any finite temperature due to the anomaly:

S. Adler, J. S. Bell and R. Jackiw (1969)

$$\partial_\mu J^{\mu 5}(x) = -\frac{g^2}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}(x)F_{\rho\sigma}(x)$$

$\sim \rho_{\mathrm{top}}(x)$: topological charge density

- Above the critical temperature T_c ,

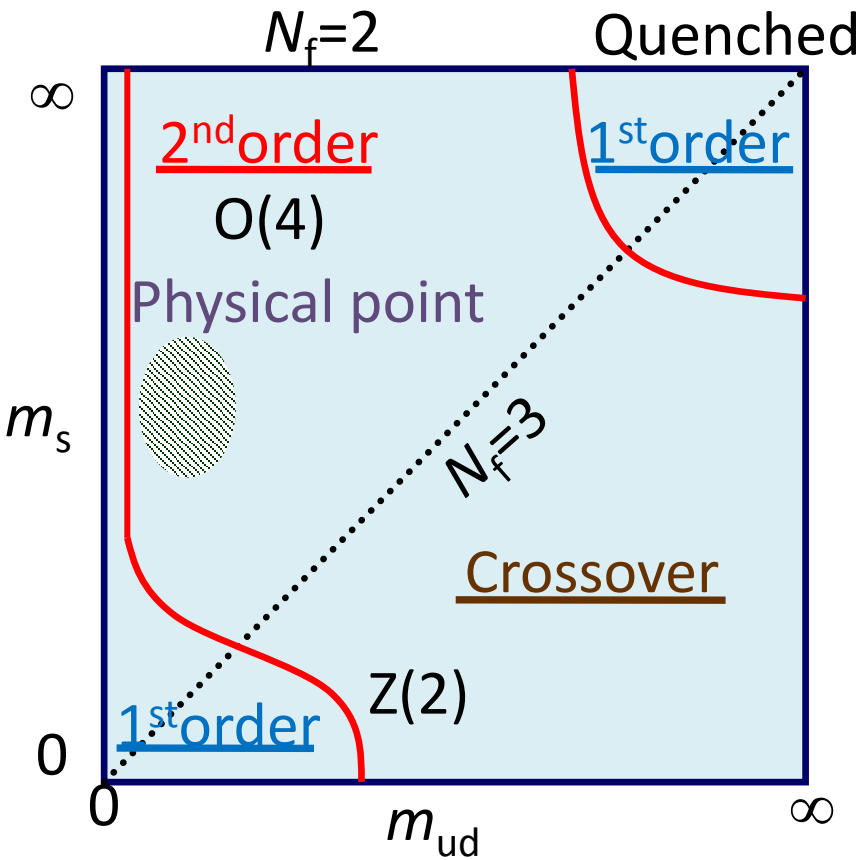
$\rightarrow n(T) = n(0)\exp(-AT^2)$: instanton density

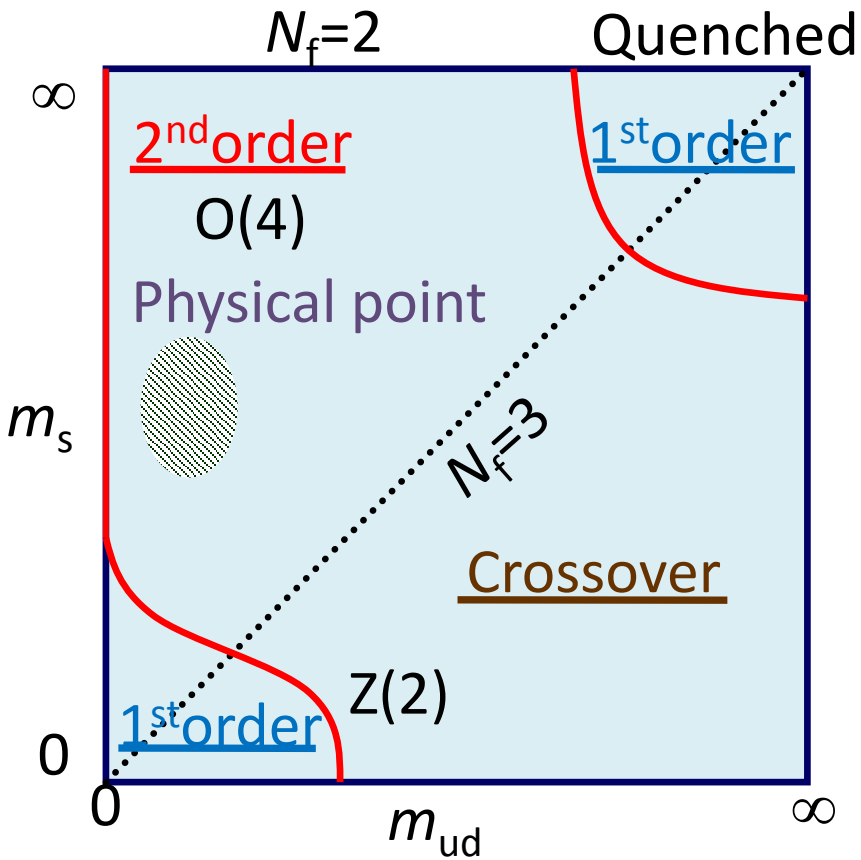
D. J. Gross, R. D. Pisarski and L. G. Yaffe (1981)

$\rightarrow U_A(1)$ symmetry is **effectively restored** as $T \rightarrow \infty$

QCD phase transition in $\mathrm{N_f} = 2$

- Order of the phase transition: R. D. Pisarski and F. Wilczek (1984)
  a) $U_A(1)$ breaking effect is negligible at T_c → **1st order**
  b) Otherwise → **2nd order**, $O(4)$ universality class





Symmetry relations

$$SU_L(N_f) \times SU_R(N_f)$$

$$\pi : \bar{\psi} i\gamma_5 \frac{\tau}{2} \psi \quad \longleftrightarrow \quad \sigma : \bar{\psi}\psi$$

$$U_A(1) \qquad\qquad U_A(1)$$

$$SU_L(N_f) \times SU_R(N_f)$$

$$\delta : \bar{\psi} \frac{\tau}{2} \psi \quad \longleftrightarrow \quad \eta : \bar{\psi} i\gamma_5 \psi$$

$$\chi_\pi \equiv \frac{T}{V} \langle \mathrm{Tr}(M^{-1}\gamma_5 M^{-1}\gamma_5) \rangle$$

$$\chi_\delta = \chi_{\mathrm{con}} \equiv \frac{T}{V} \langle \mathrm{Tr}(M^{-1} M^{-1}) \rangle$$

$$\chi_\sigma = \chi_{\mathrm{con}} + \chi_{\mathrm{disc}}$$

$$\chi_{\mathrm{disc}} \equiv \frac{T}{V} [\langle (\mathrm{Tr} M^{-1})^2 \rangle - \langle \mathrm{Tr} M^{-1} \rangle^2]$$

$SU_L(N_f) \times SU_R(N_f)$ restoration

$\Rightarrow \chi_\pi = \chi_\sigma \rightarrow \chi_\pi - \chi_\delta = \chi_{\mathrm{disc}}$

$U_A(1)$ restoration

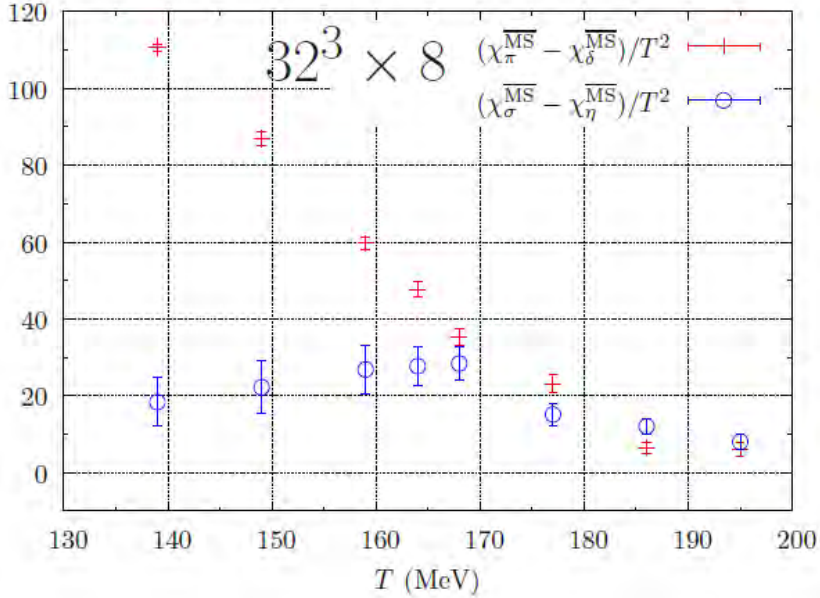
$\Rightarrow \chi_\pi = \chi_\delta \rightarrow \chi_\pi - \chi_\delta = 0$

Is $\mathrm{U_A(1)}$ symmetry restored at $\mathrm{T = T_c}$?

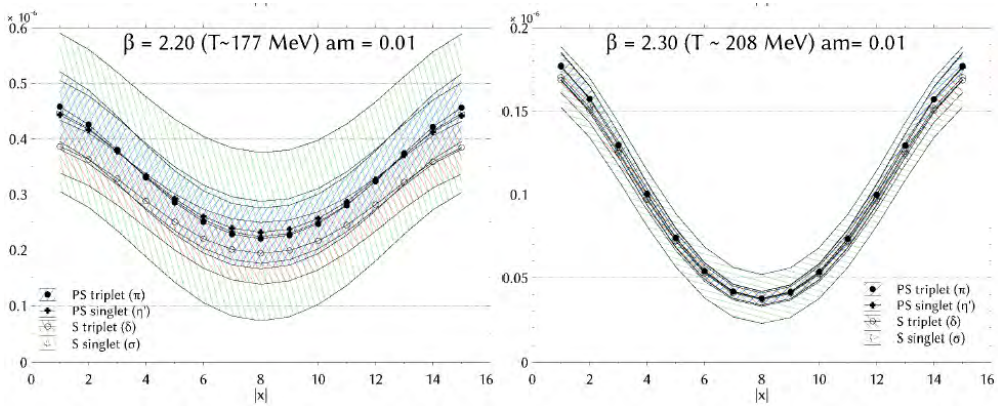
- T. D. Cohen (1996) → **Yes**
  - Above T_c, $\chi_\sigma - \chi_\delta = \chi_{\rm disc} \sim O(m_q^2) \to 0 \ \ (m_q \to 0)$
- S. H. Lee and T. Hatsuda (1996) → **No**
  - $\chi_\sigma - \chi_\delta = \chi_{\rm disc} \sim O(m_q^2) + \Delta$
  - $\Delta \sim O(m_q^{N_f - 2})$: topological non-trivial contribution
- S. Aoki, H. Fukaya and Y. Taniguchi (2012) → **Yes**
  - Assumption: VEV of m-independent observable is an analytic function of m^2
  - Constraint: $\lim_{m\to 0}\langle\rho(\lambda)\rangle_m = \lim_{m\to 0}\langle\rho_3\rangle_m \frac{|\lambda|^3}{3!} + O(\lambda^4)$

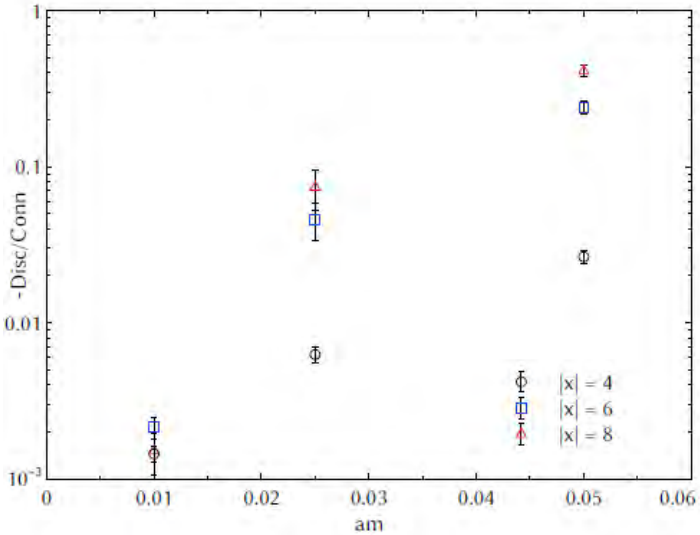
Recent lattice results

M. I. Buchoff *et al*. [LLNL/RBC Collaboration] (2013)



G. Cossu *et al*. (2013)





Results with the domain wall fermion (left):

$U_A(1)$-violating susceptibility differences differ from zero even at about T=200 MeV

→ **$U_A(1)$ symmetry is still broken.**

Results with the overlap fermion (right):

Degeneracy of the correlators is found.

Contribution from the disconnected diagram vanishes. → **restoration of $U_A(1)$ symmetry.**

Dirac spectrum and chiral symmetry

- Relation between Σ and eigenvalues of the Dirac operator

$$\Sigma = \int_0^\infty d\lambda \frac{2m\rho(\lambda, m)}{m^2 + \lambda^2}$$

$$i\not{D}\phi = \lambda\phi\,, \quad \rho(\lambda, m) \equiv \frac{1}{V}\langle \sum_k \delta(\lambda - \lambda_k(m))\rangle$$: eigenvalue density

- Banks-Casher relation T. Banks and A. Casher (1980)

$$V \to \infty, m \to 0 \qquad \Sigma = \pi\rho(0)$$

$T < T_c$ $\quad \Sigma \neq 0 \quad \Rightarrow \quad \rho(0) \neq 0$

$T > T_c$ $\quad \Sigma = 0 \quad \Rightarrow \quad \rho(0) = 0$

If the transition is continuous, $\rho(0) \to 0$ as $T \to T_c$

Dirac spectrum and $\mathrm{U_A(1)}$ symmetry

- A probe of the $U_A(1)$ breaking

$$\chi_\pi - \chi_\delta = \int_0^\infty \frac{4m^2\rho(\lambda, m)}{(\lambda^2 + m^2)^2} d\lambda$$

- Expected behavior of $\rho(\lambda \to 0, m \to 0)$ above T_c
  - $\rho(\lambda, m)$ has **a gap**

    → $\Sigma = 0,\ \chi_\pi - \chi_\delta = 0$

    → $U_A(1)$ symmetry is **restored**
  - $\rho(\lambda, m) \sim m^2\delta(\lambda)$ or $\sim \lambda$ or $\sim m$

    → $\Sigma = 0, \chi_\pi - \chi_\delta$: finite

    → $U_A(1)$ symmetry is **broken**

This study

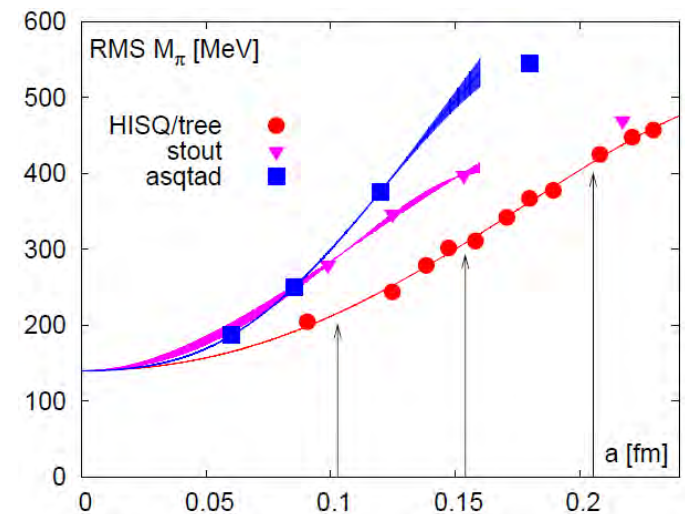
- Investigating the chiral phase transition and $U_A(1)$ symmetry at finite temperature
  - in terms of the Dirac spectrum
  - using lattice QCD simulations
- Trying to find
  - whether there is a gap in the Dirac spectrum $\rho(\lambda, m)$
  - what kind of behavior $\rho(\lambda, m)$ has around $\lambda = 0$ above T_c in the chiral limit.

Various lattice fermion formalism

Fermion	Chiral symmetry	Computational cost
Staggered	Broken, only remnant U(1)	Cheap
Wilson	Broken	-
Domain wall	Broken but controlled by Ls	Expensive
Overlap	Exact	Very expensive

- Our choice: Highly Improved Staggered Quark (HISQ)
  - can reduce the taste symmetry violation and improve the cutoff effect better than the other staggered actions

E. Follana *et al*. (2007)



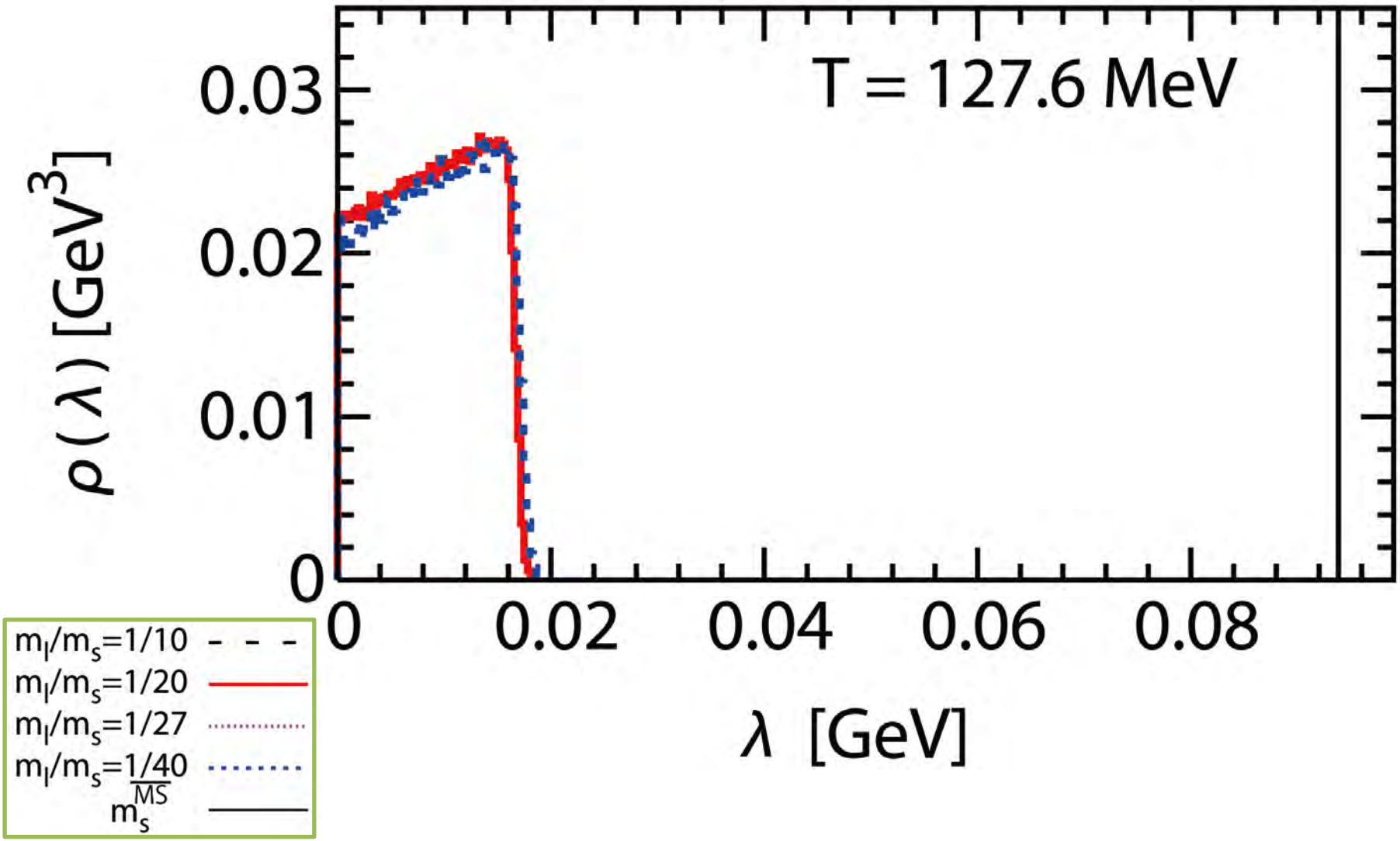
A. Bazavov *et al*. [HotQCD Collaboration] (2011)

Simulation setup

- Action
  - Tree level improved gauge
  - (2+1)-flavor HISQ
- Lattice size
  - 32^3x8, 48^3x8
- Quark masses
  - physical strange quark mass
  - a couple of light quark masses around physical point
- Temperature
  - about 128～330 MeV
- 100 low-lying eigenvalues

β	a [fm]	T [MeV]	$m_s a$	# configurations			
				$m_l/m_s = 1/10$	1/20	1/27	1/40
$N_s^3 \times N_\tau = 32^3 \times 8$							
6.195	0.1847	127.7	0.0880	-	702	-	360
6.245	0.1759	134.6	0.0830	-	792	-	822
6.260	0.1733	136.8	0.0810	-	768	-	834
6.285	0.1691	140.4	0.0790	-	714	-	882
6.315	0.1642	144.9	0.0760	-	684	-	936
6.341	0.1601	148.9	0.0740	-	612	-	402
6.354	0.1581	151.0	0.0728	-	534	1158	600
6.390	0.1527	156.8	0.0694	-	600	1458	600
6.423	0.1478	162.2	0.0670	312	696	1362	540
6.445	0.1447	165.9	0.0652	-	600	1146	522
6.460	0.1426	168.5	0.0640	-	582	-	-
6.488	0.1388	173.4	0.0620	-	678	-	-
6.550	0.1307	184.8	0.0582	-	750	-	-
6.664	0.1171	207.7	0.0514	-	594	-	-
6.800	0.1029	237.0	0.0448	-	594	-	-
6.950	0.0894	273.8	0.0386	-	594	-	-
7.150	0.0744	330.1	0.0320	-	594	-	-
$N_s^3 \times N_\tau = 48^3 \times 8$							
6.423	0.1478	166.8	0.0670	-	144	-	-
6.445	0.1447	170.5	0.0652	-	156	-	-
7.150	0.0744	331.6	0.0320	-	306	-	-

Results: eigenvalue distribution (low *T*)



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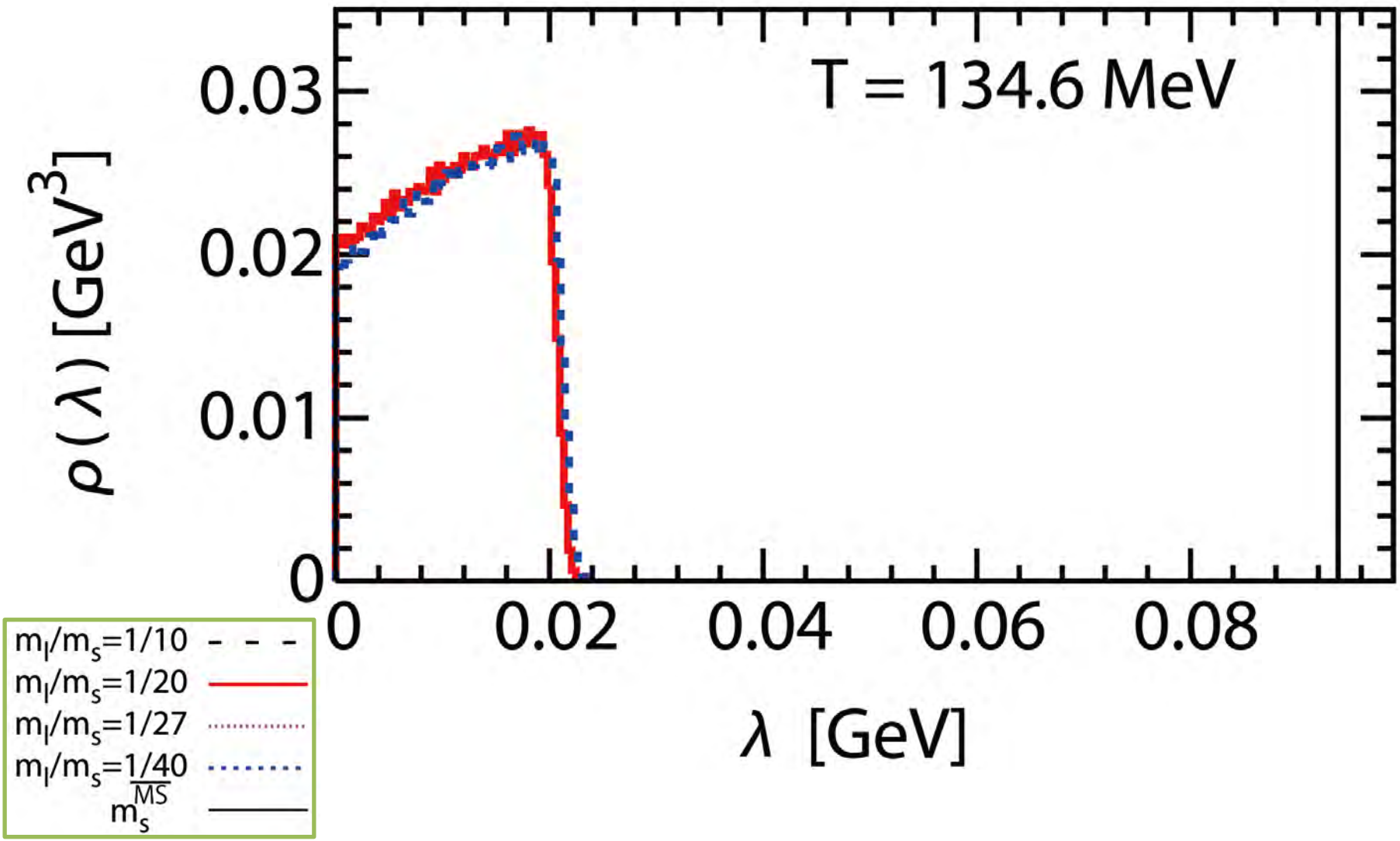

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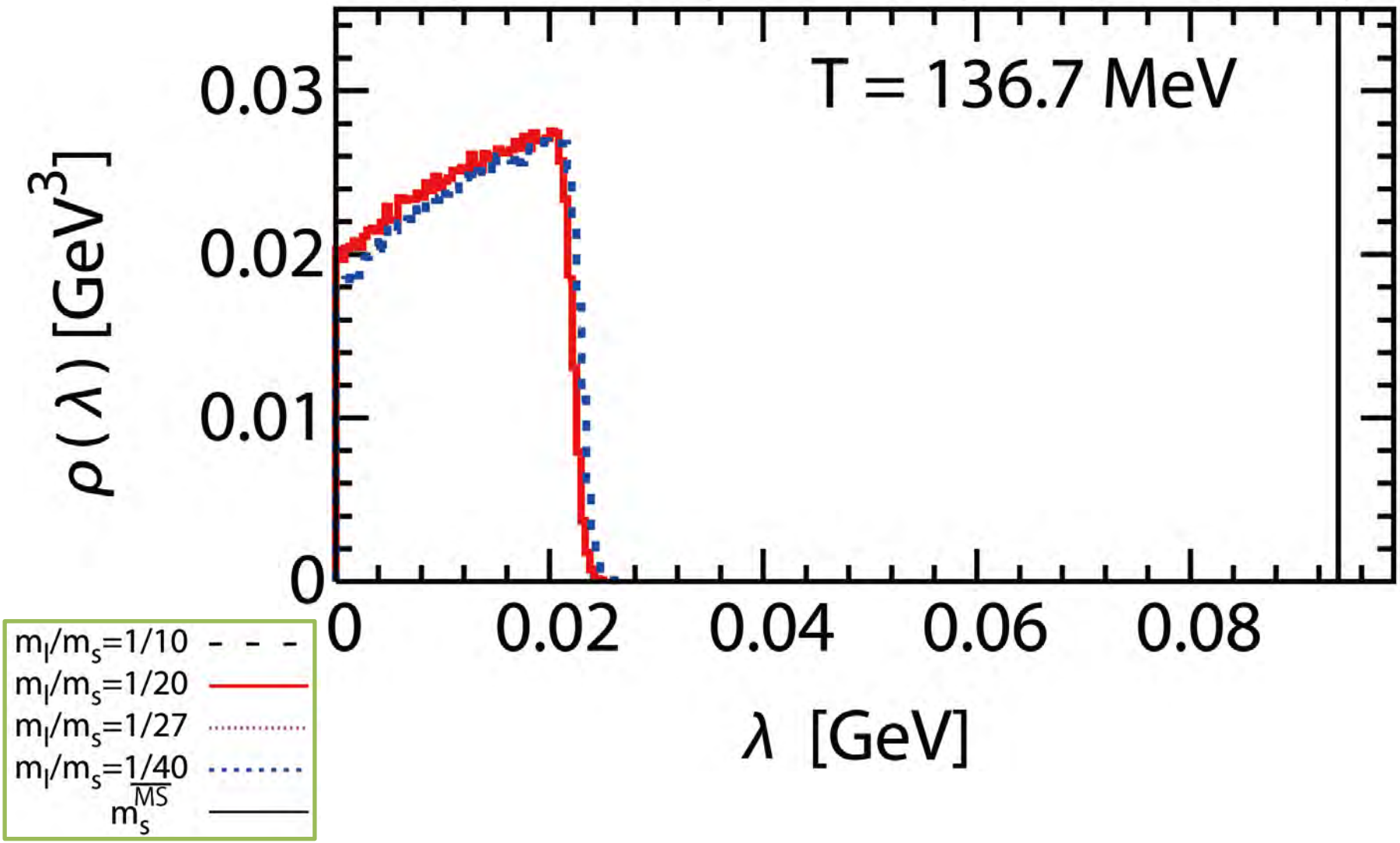

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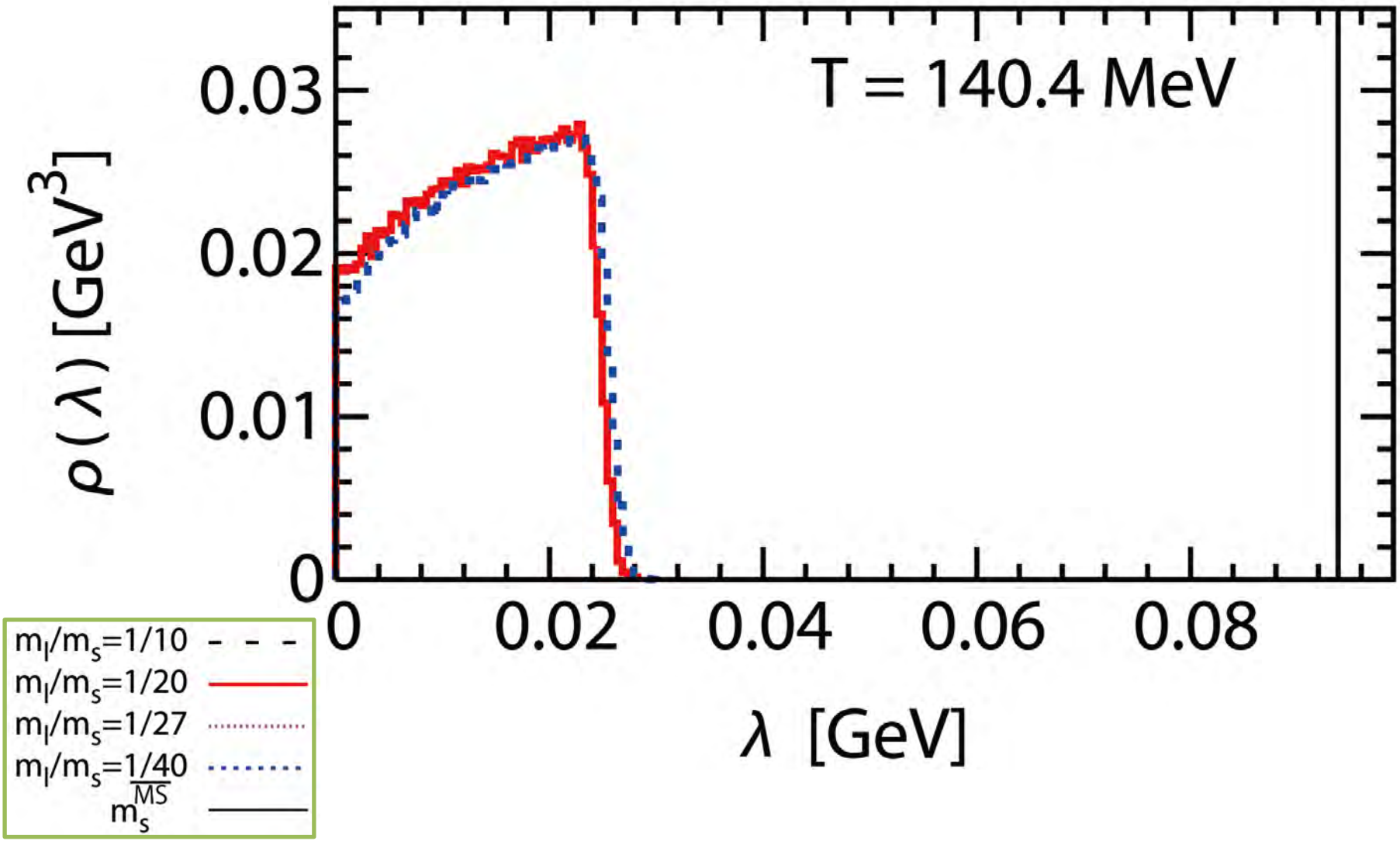

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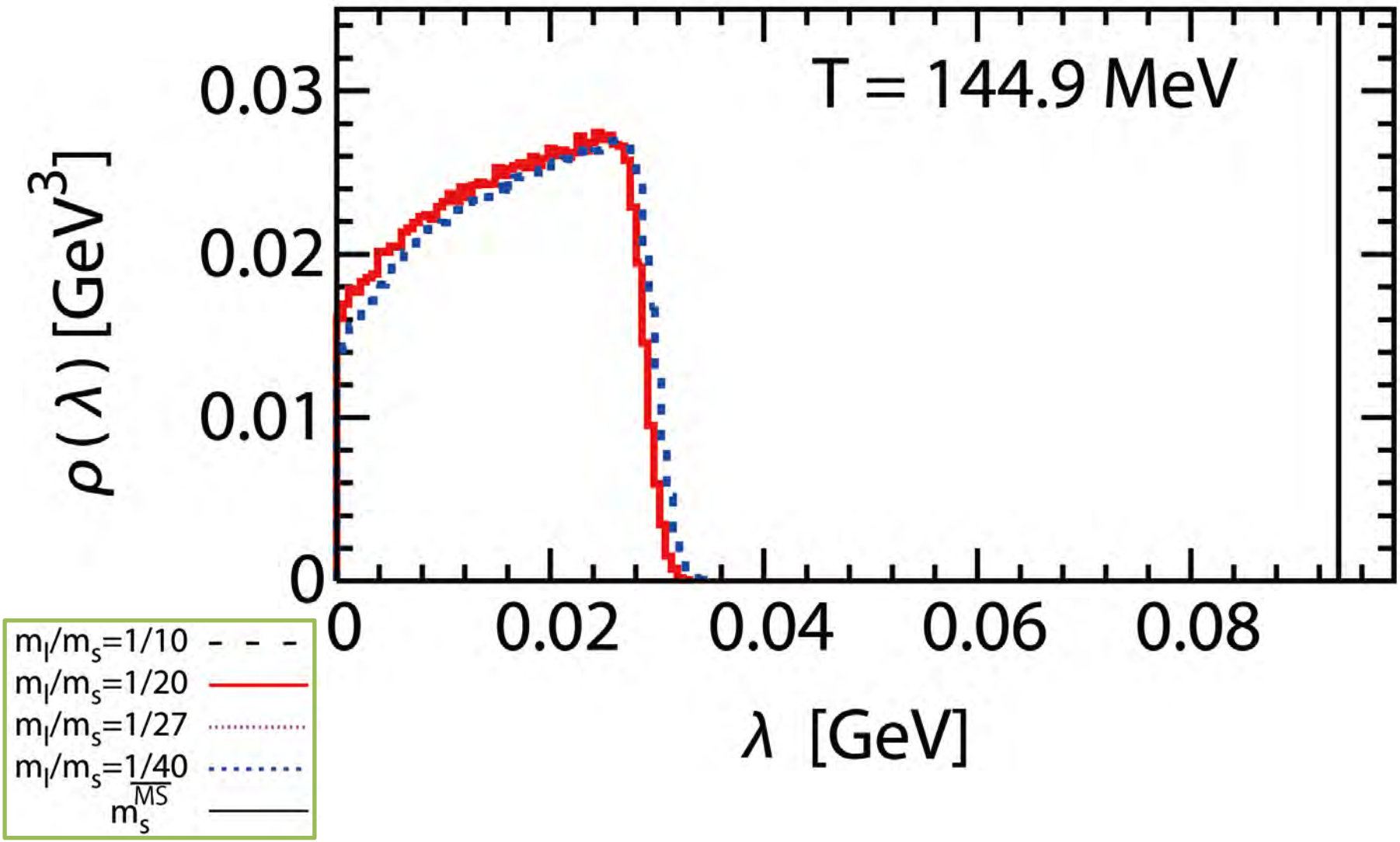

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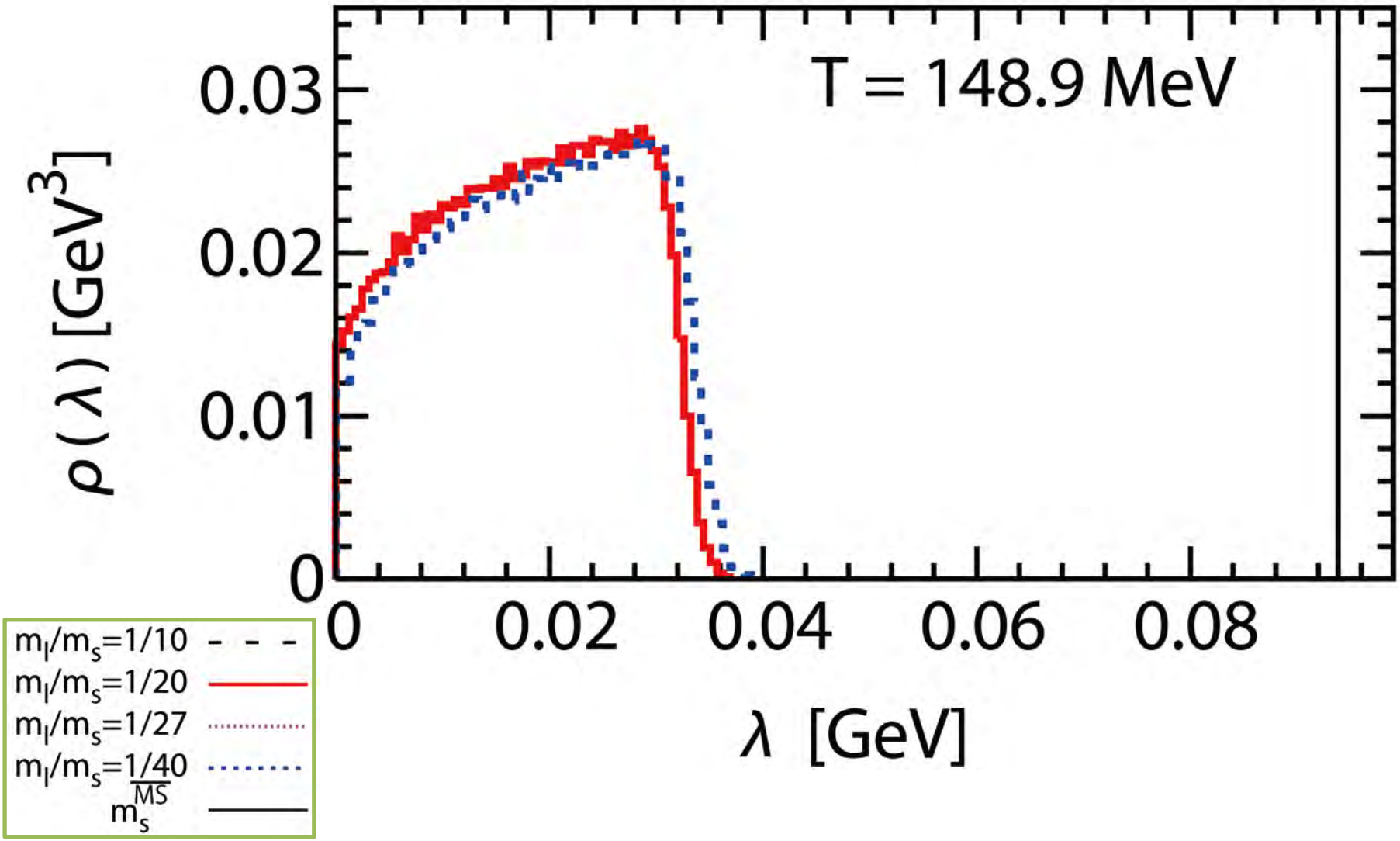

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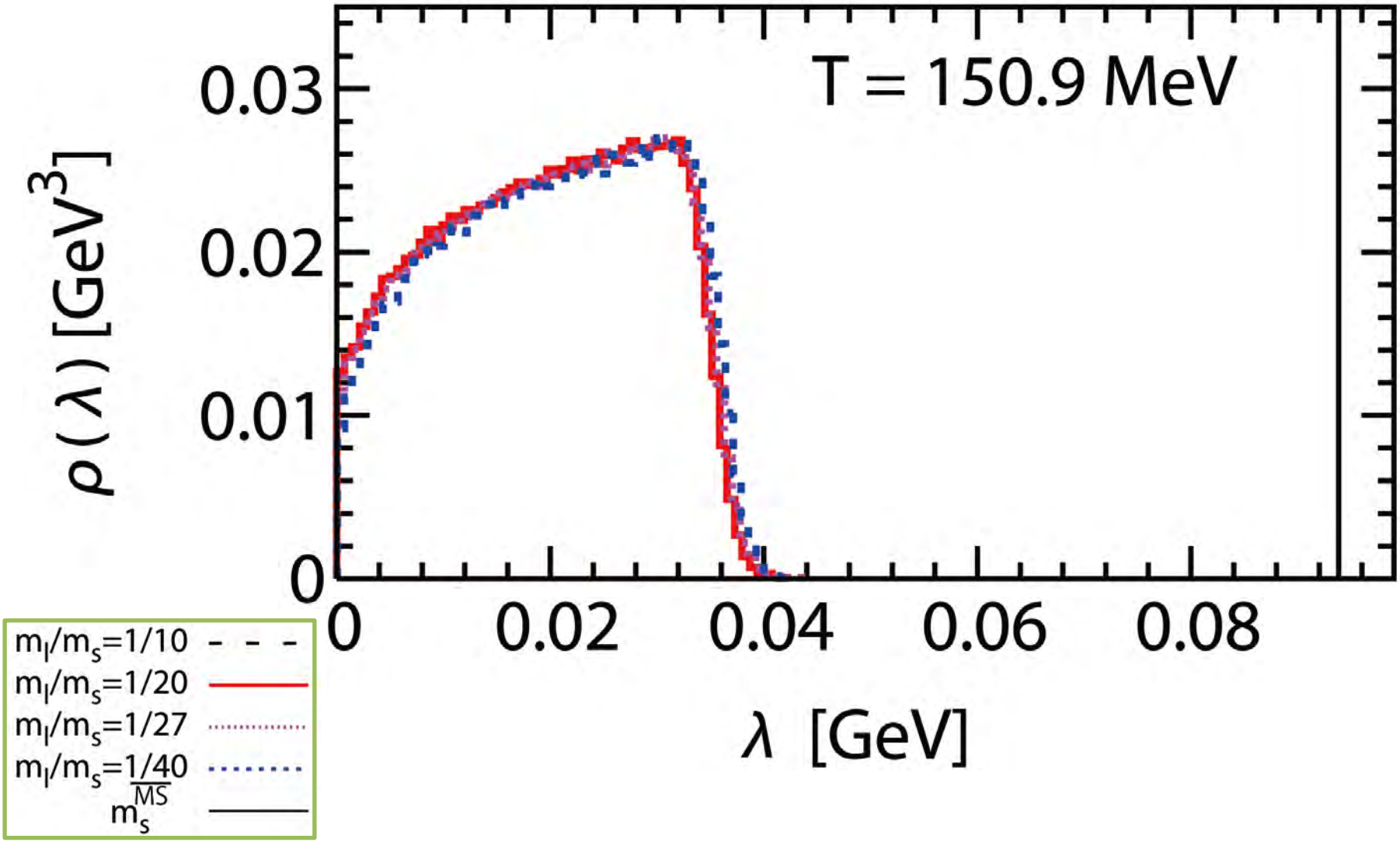

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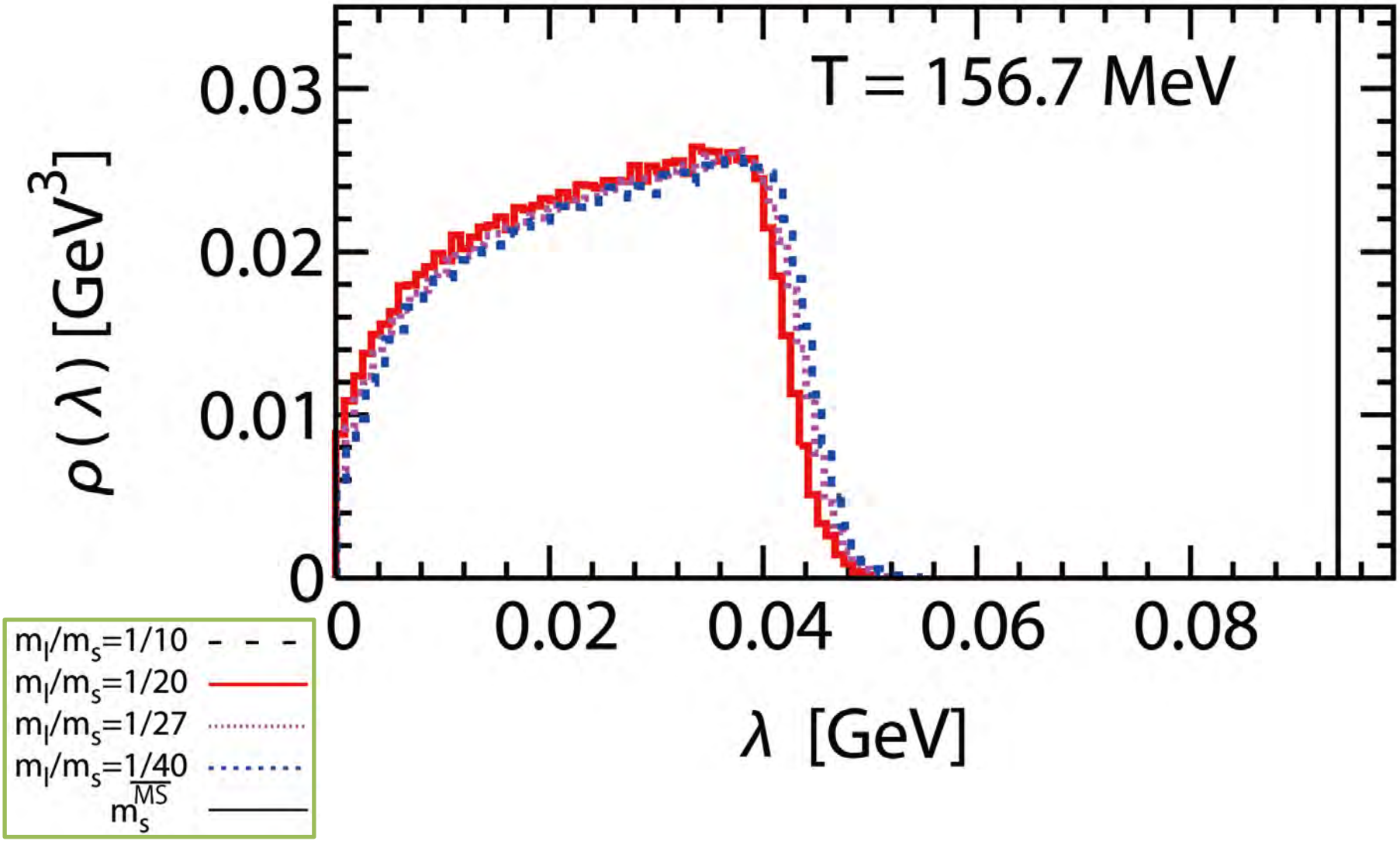

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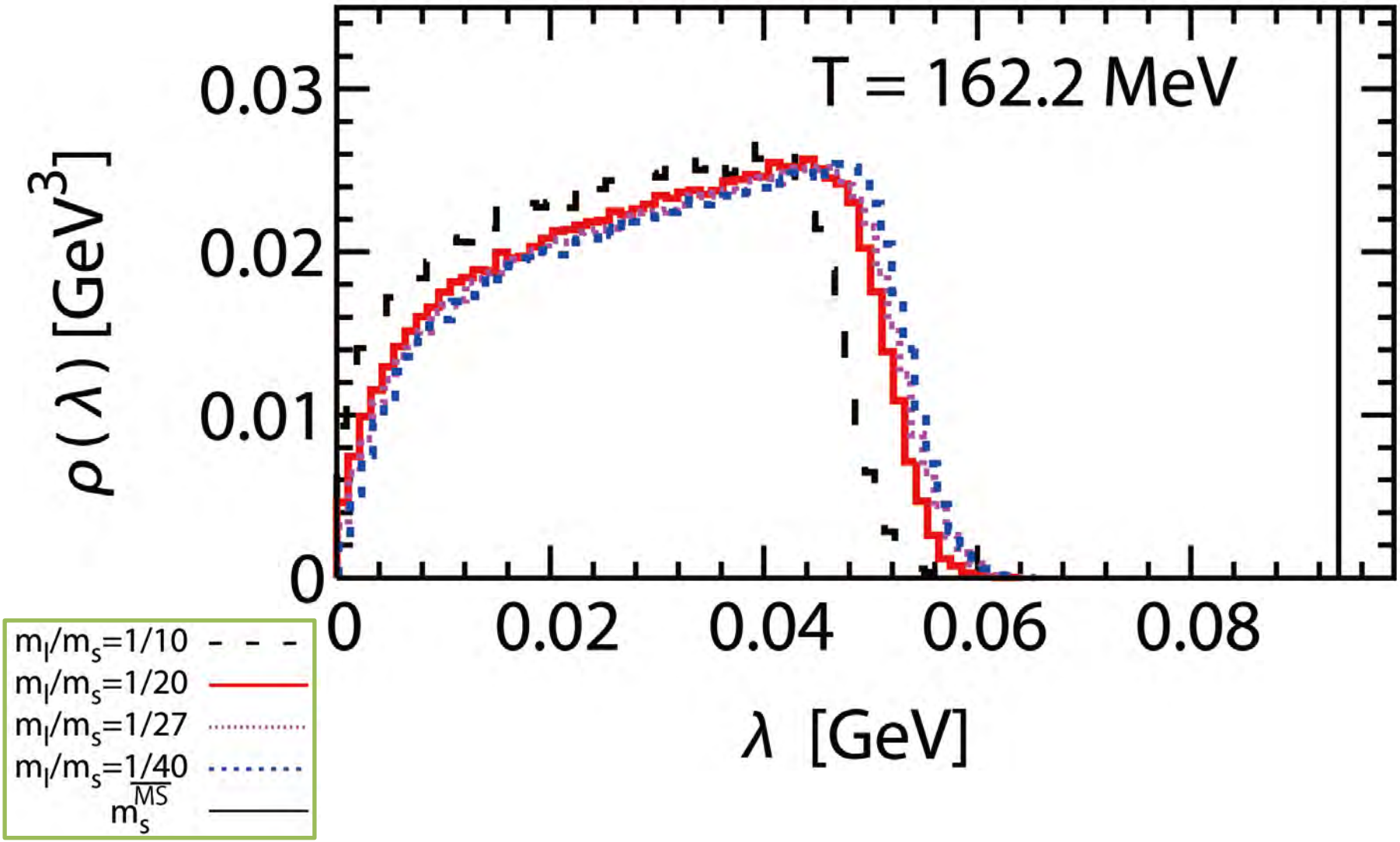

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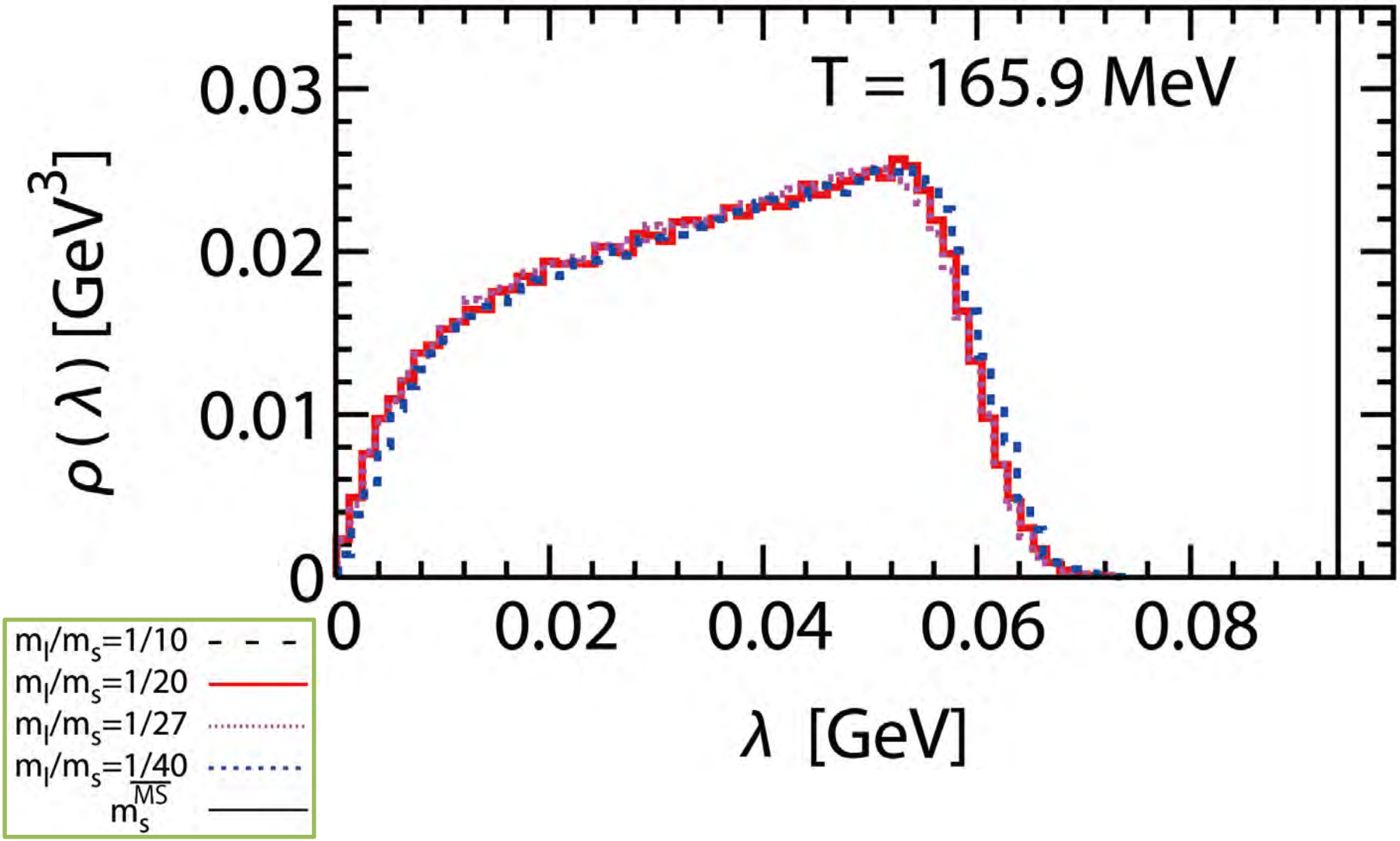
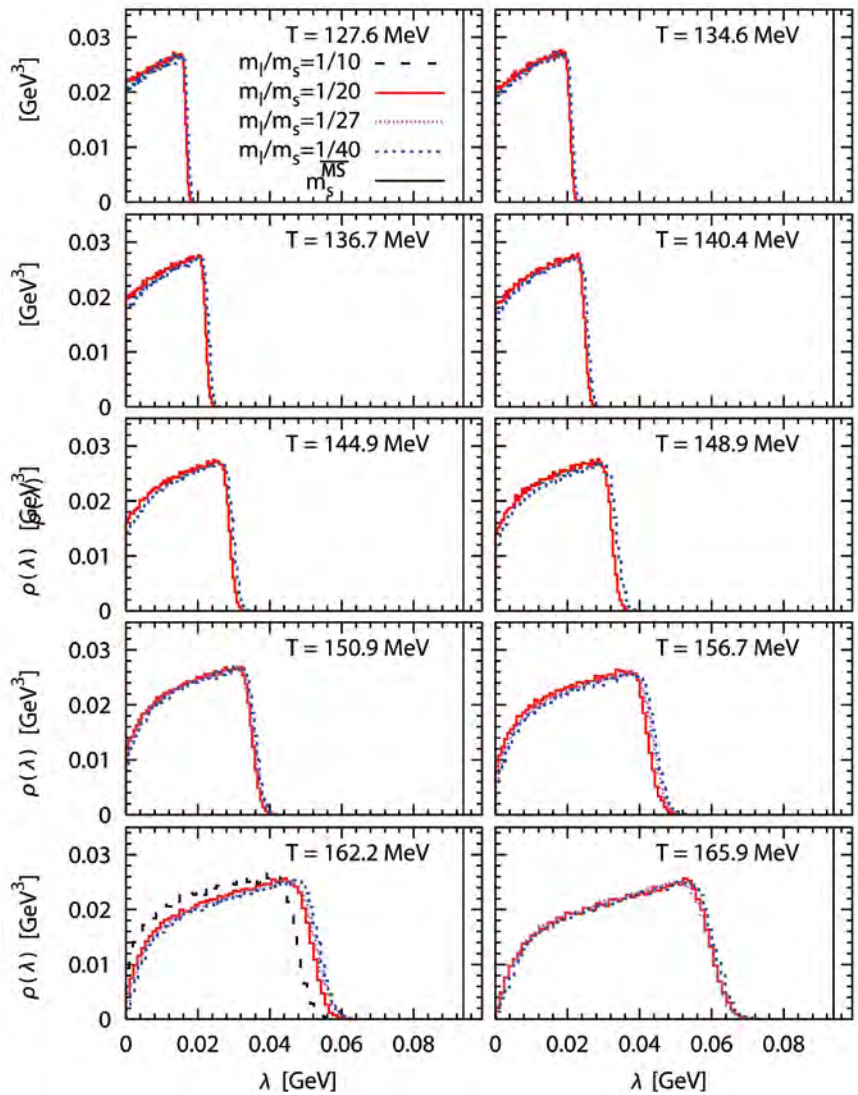


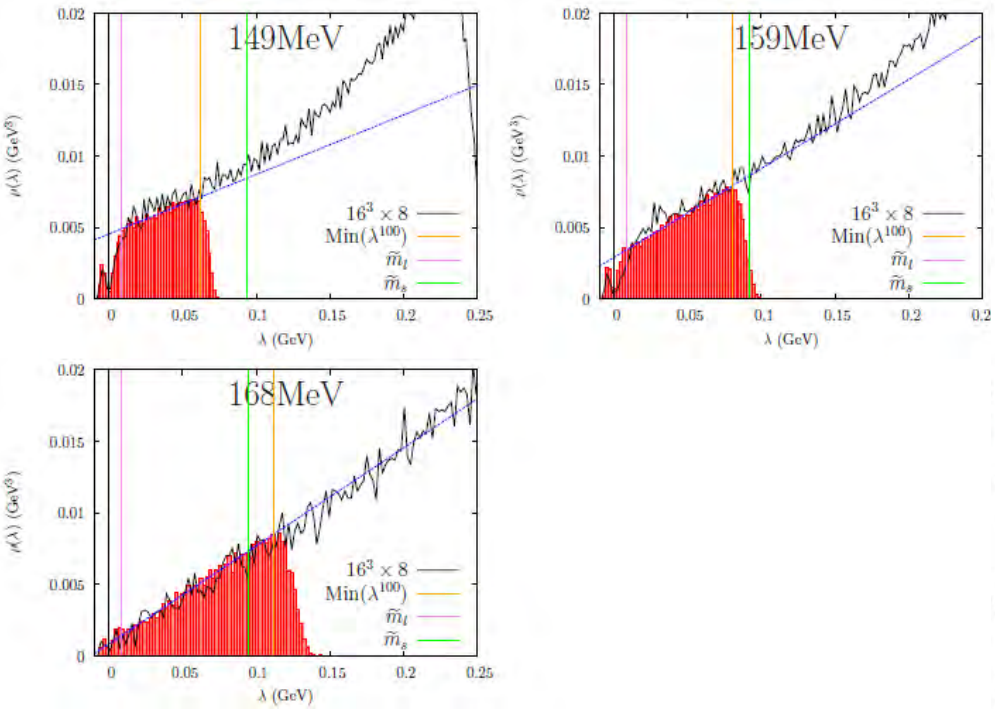
Results: eigenvalue distribution (low *T*)

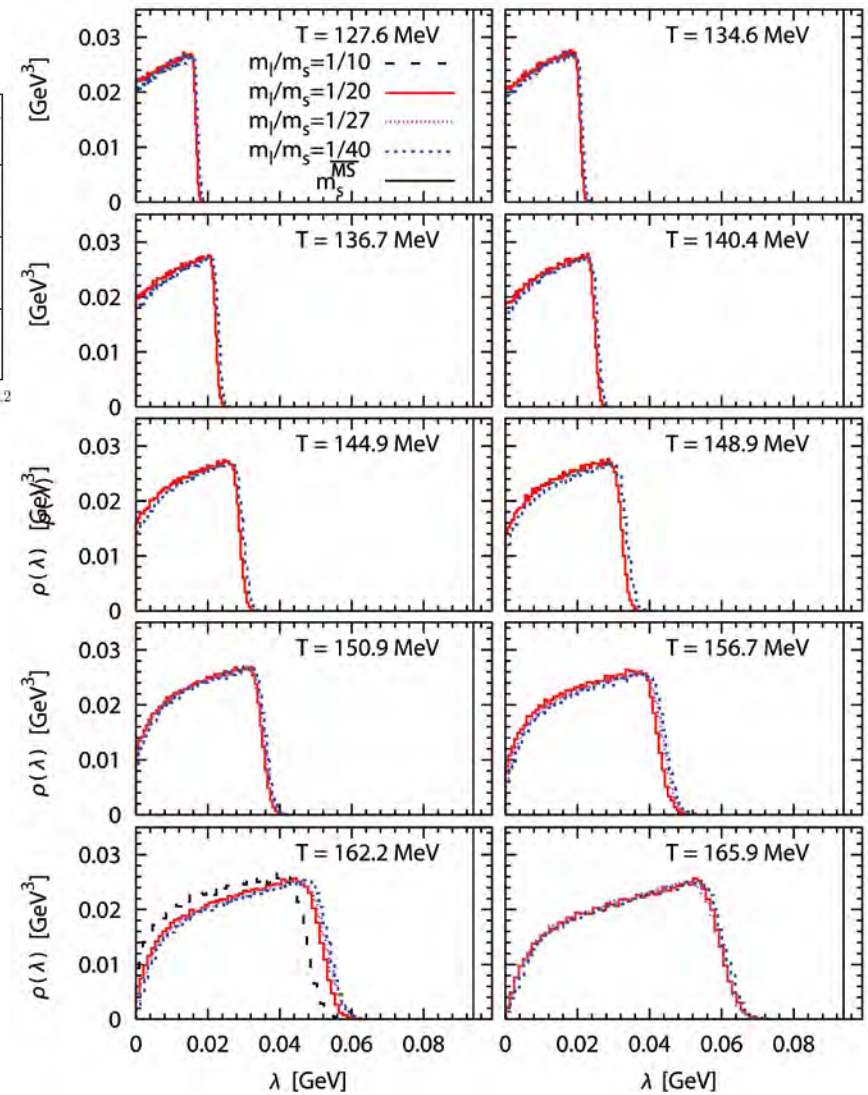
- The intercept of $\rho(\lambda)$ becomes smaller as temperature increases.
- Smaller quark mass has smaller intercept.
- The intercept becomes almost 0 at T = 165.9 MeV
- Temperature dependence is consistent with what is expected from the restoration of the chiral symmetry.



Results: eigenvalue distribution (low *T*)

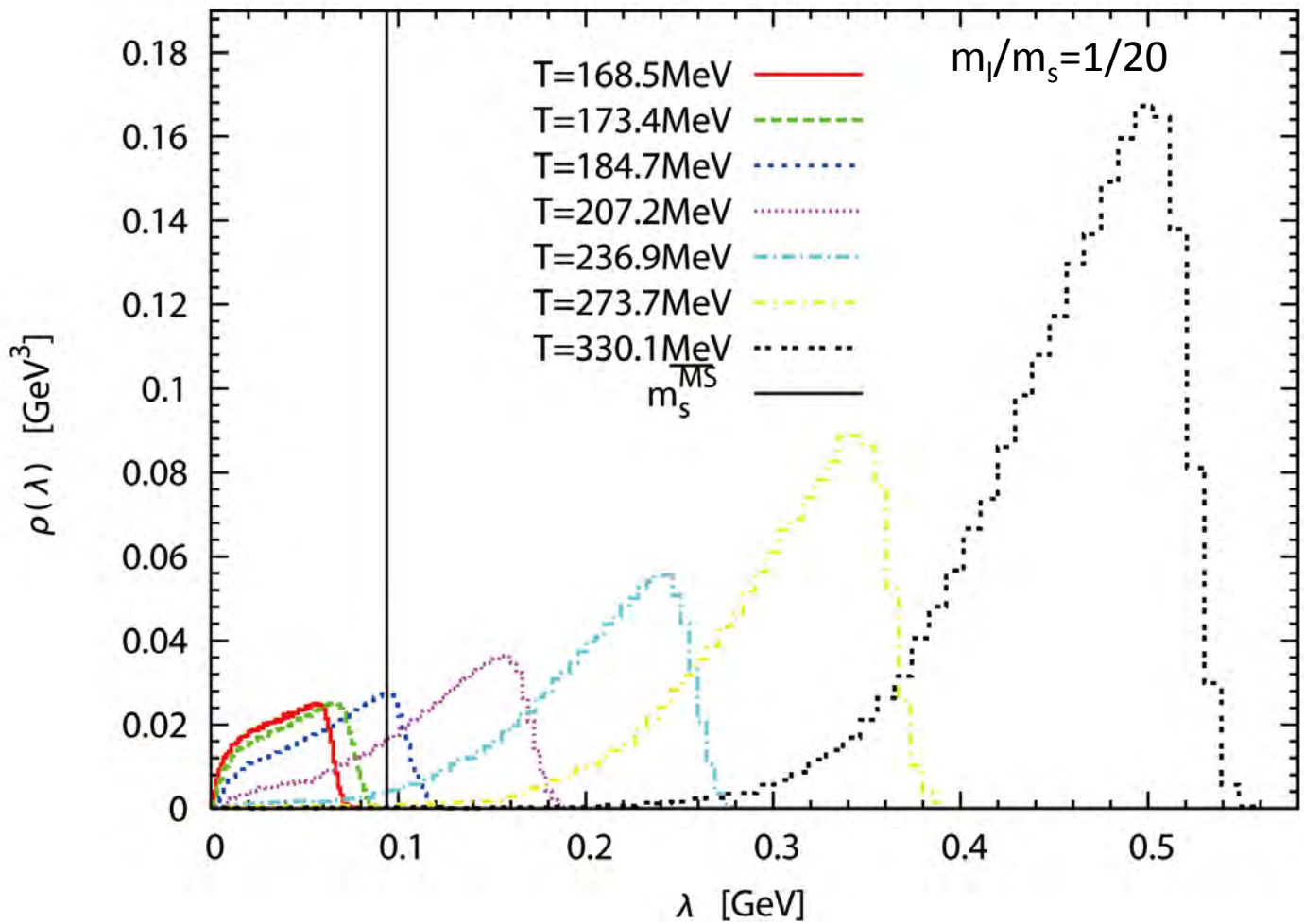
M. I. Buchoff *et al*. [LLNL/RBC Collaboration] (2013)





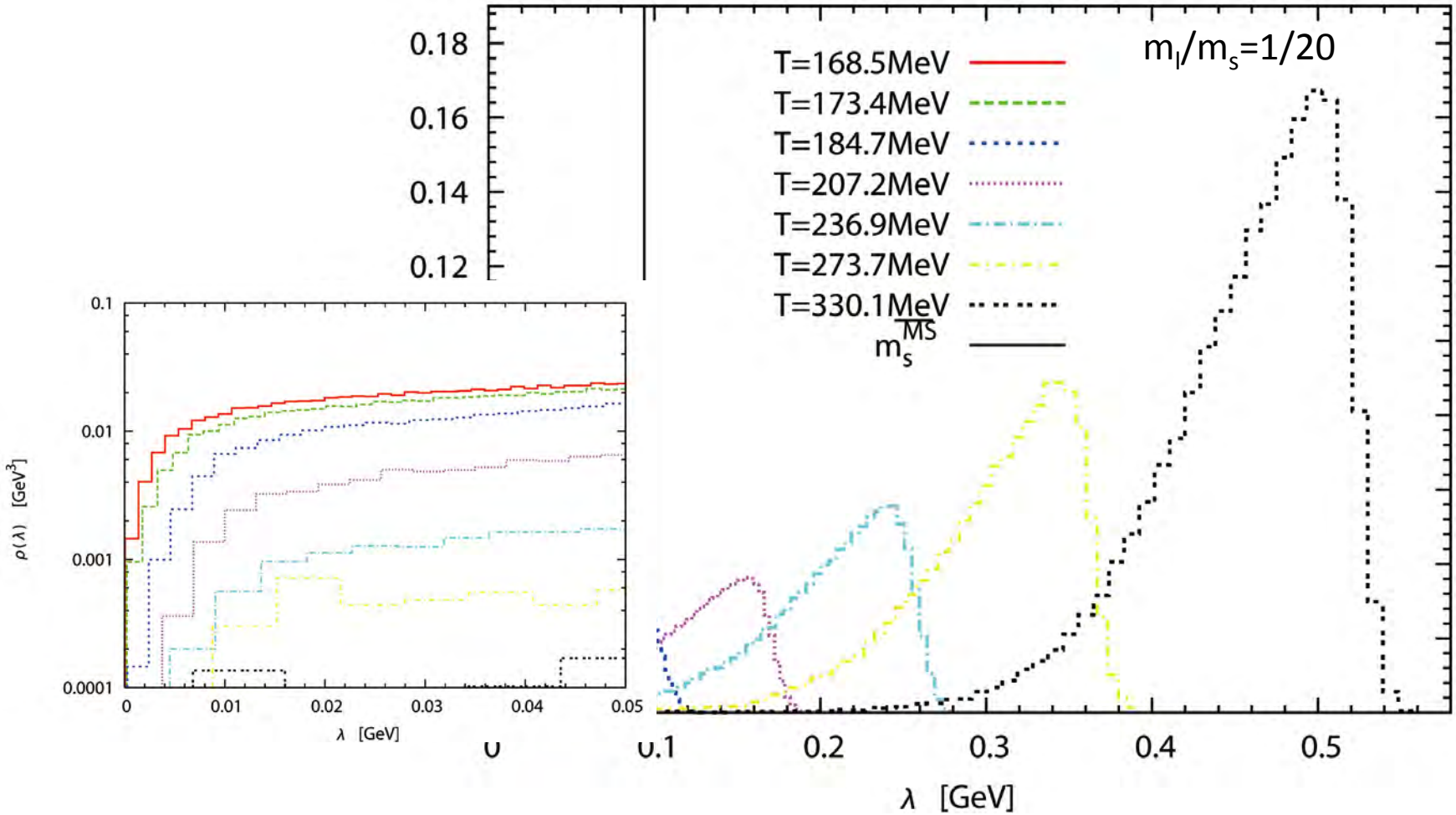
Our result is similar to the domain wall case.

Results: eigenvalue distribution (high *T*)



$\rho(\lambda)$ has a tail approaching the origin.

Results: eigenvalue distribution (high *T*)



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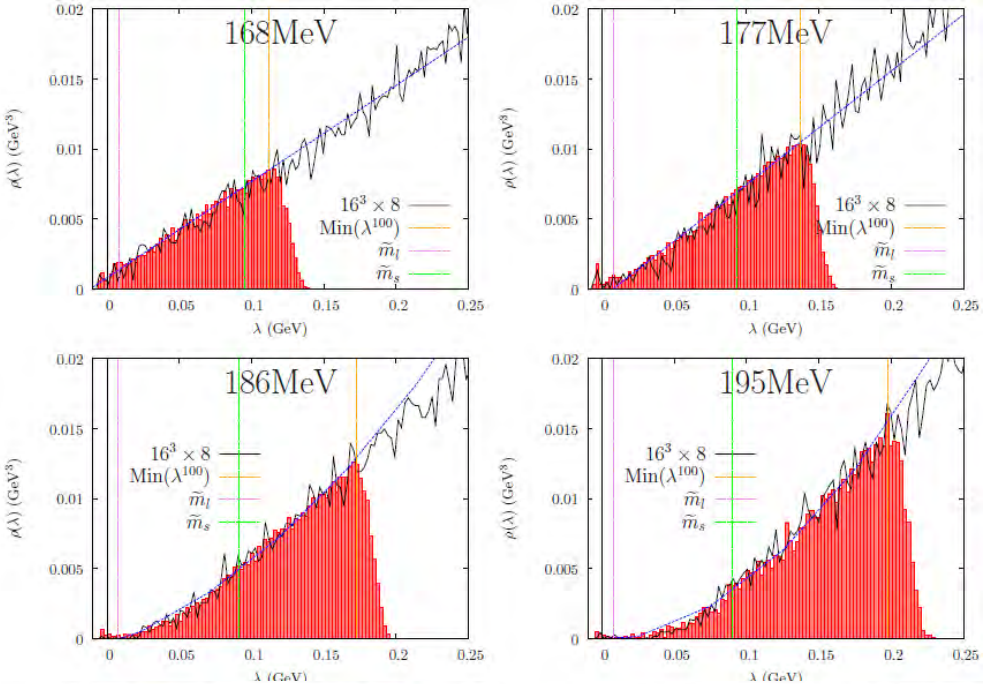
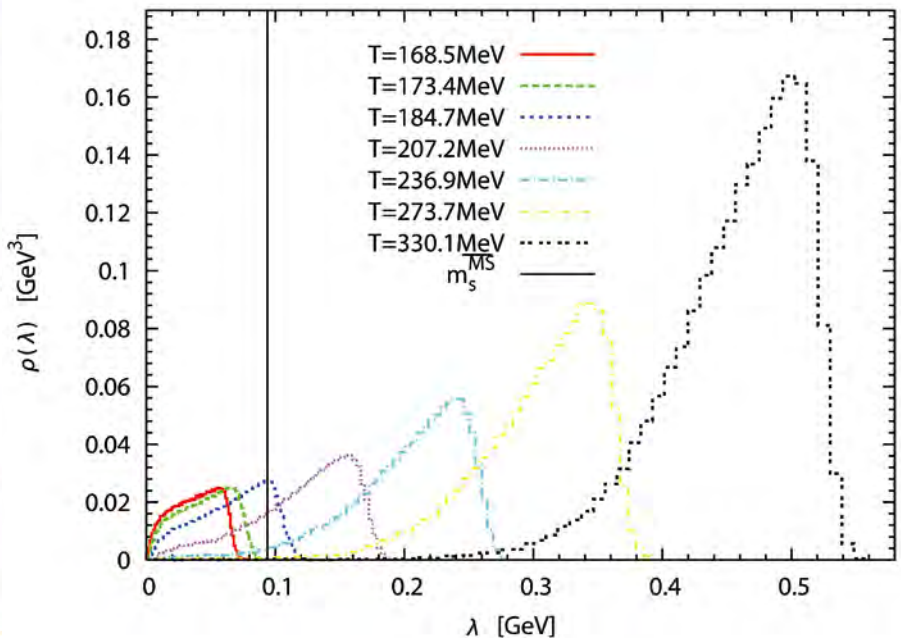
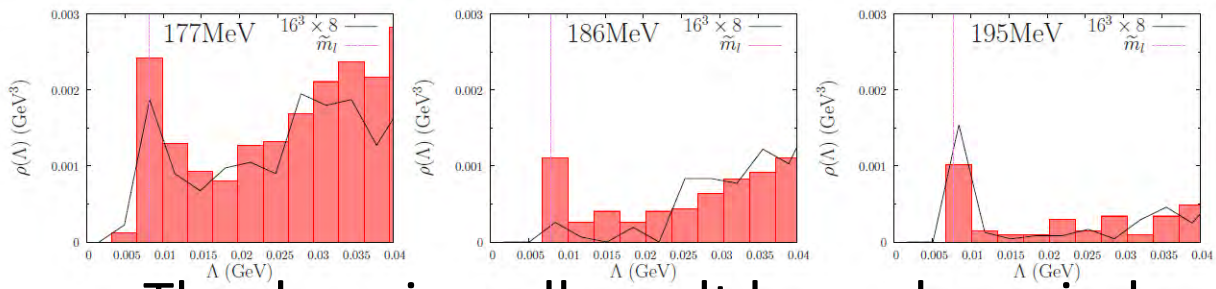




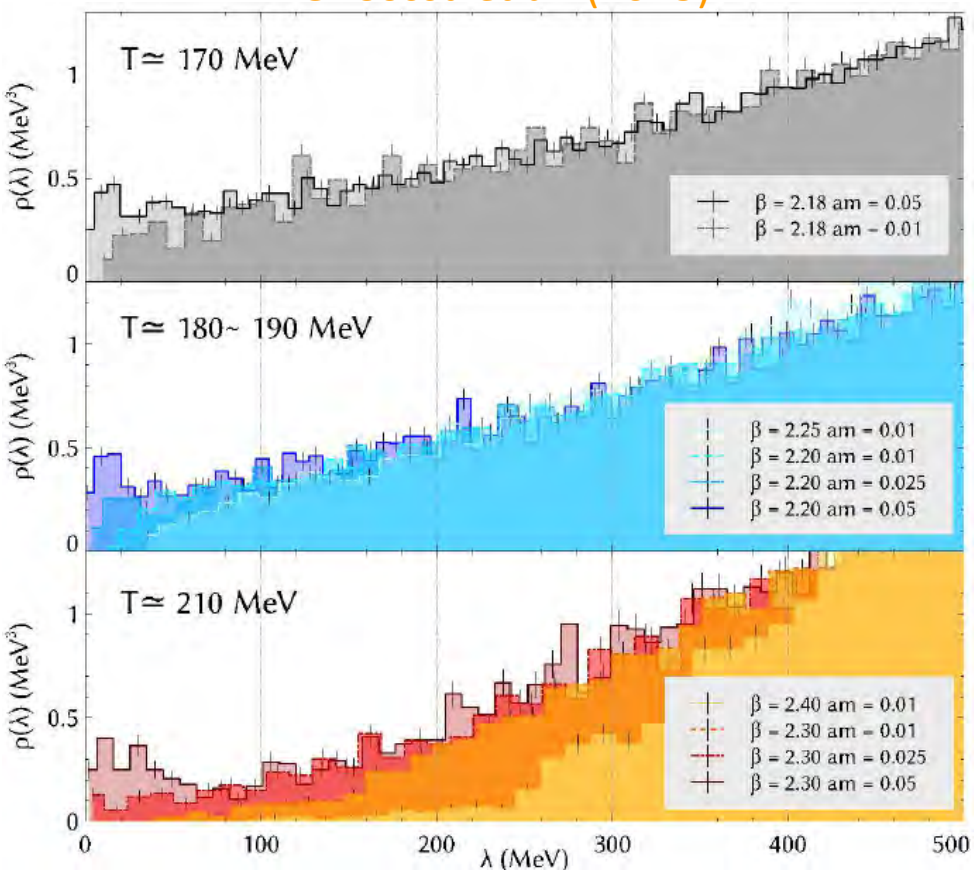


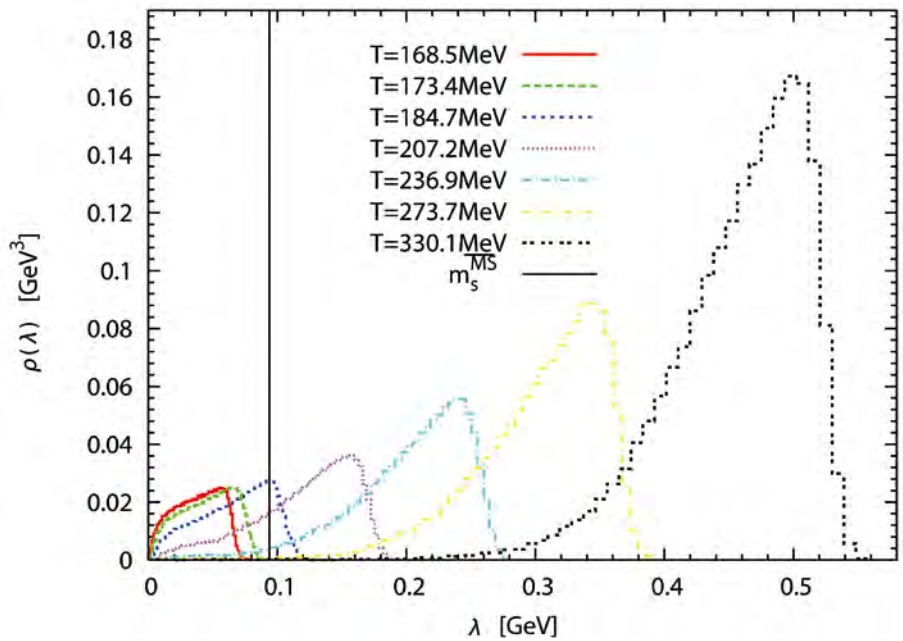
M. I. Buchoff *et al.* [LLNL/RBC Collaboration] (2013)

The domain wall result has volume independent peak near zero, which may correspond to the dilute instanton gas contribution, while our result doesn't have such near zero modes.

Results: eigenvalue distribution (high *T*)

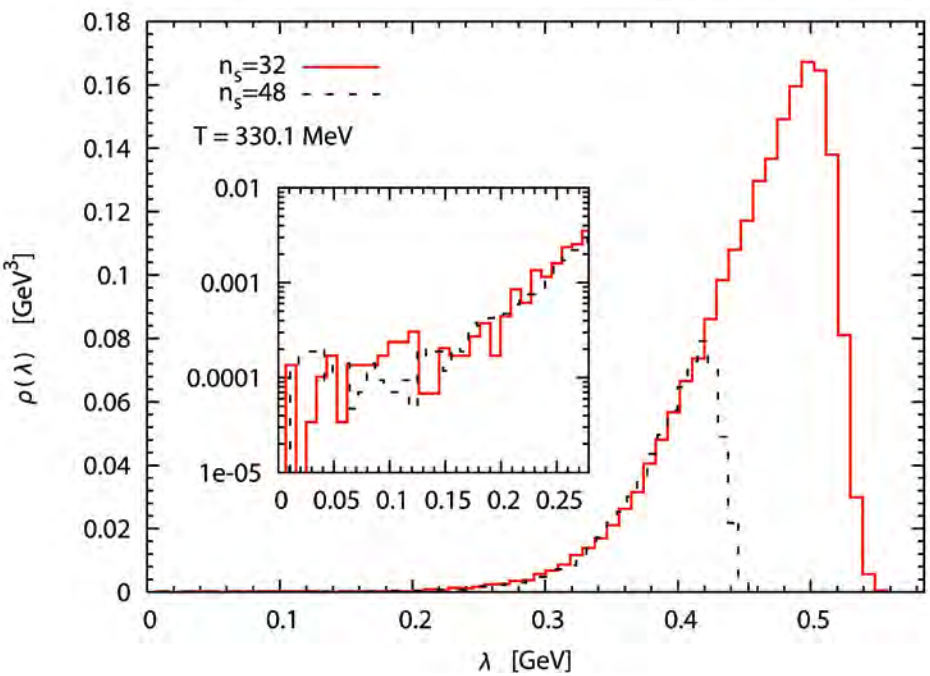
G. Cossu *et al*. (2013)

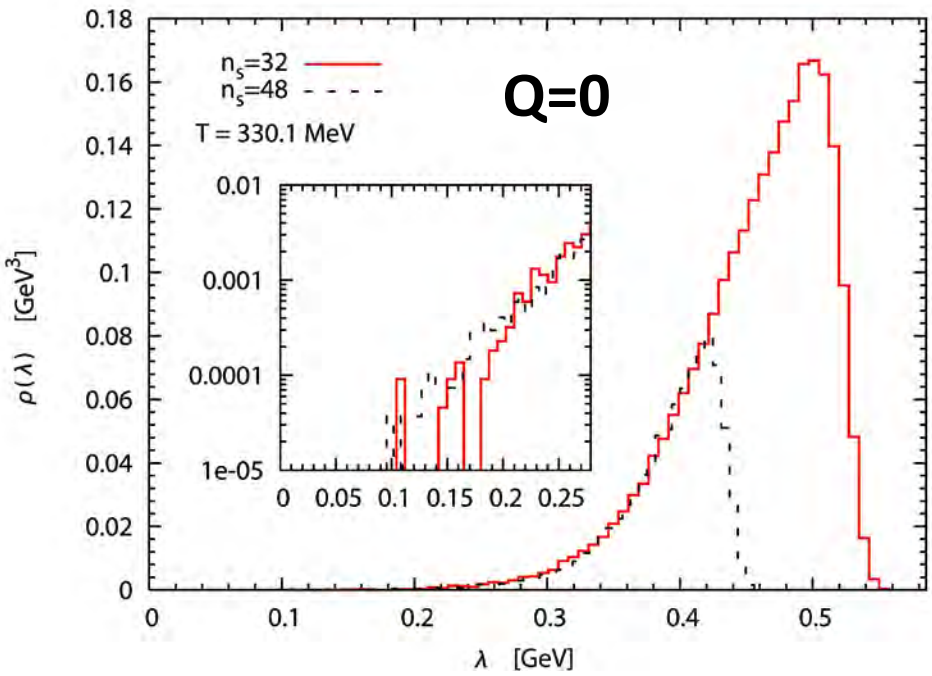




A gap start developing in the overlap spectra as temperature increases and the quark mass decreases in contrast to our result.

Results: eigenvalue distribution (highest *T*)



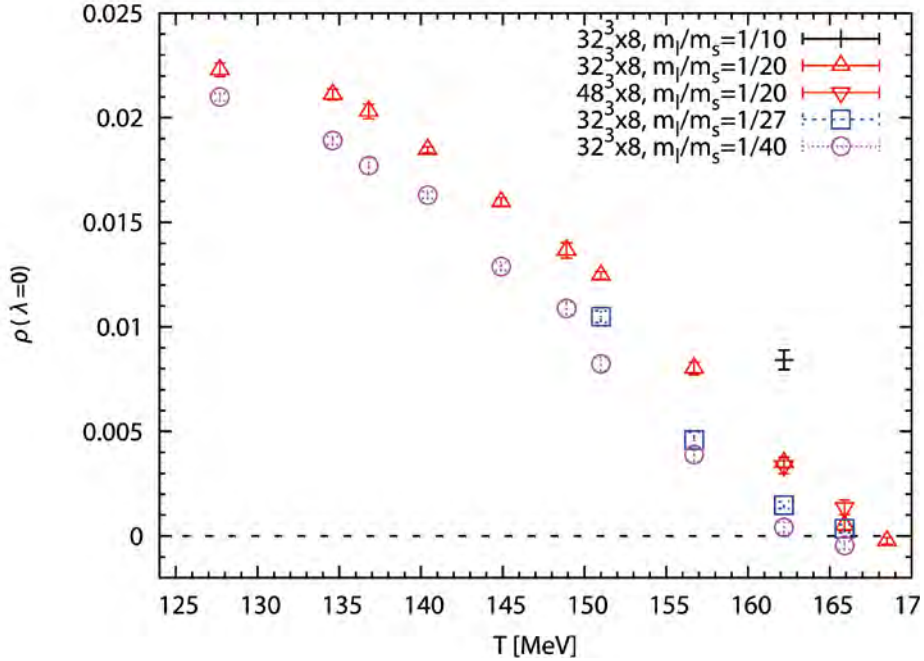


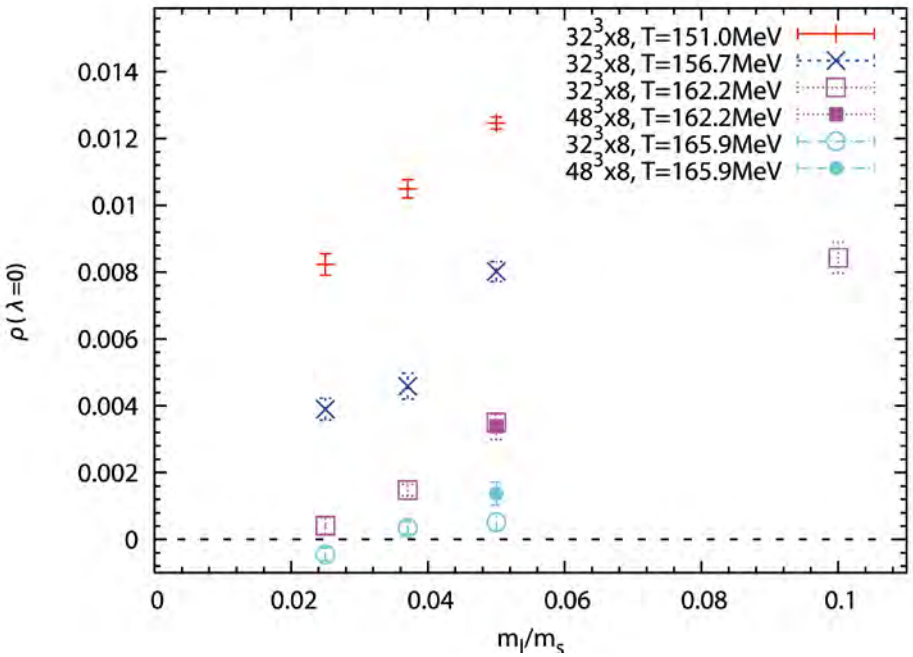
- There is small volume dependence.
- Near zero modes belongs to the non-trivial topological sector.
- The tail near zero may be just too small to see in our calculation.
- **There is no clear evidence of the gap.**

Results: quark mass dep. Of $\rho(\lambda = 0)$

- Fitting the spectral density to

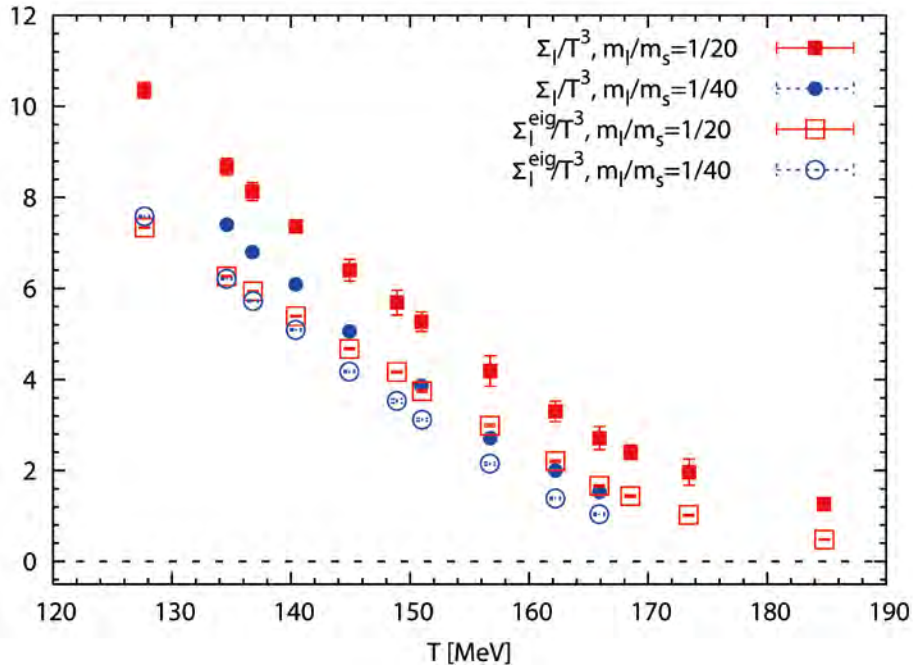
$$\rho(\lambda) = c_0 + c_1\lambda + c_2\lambda^2 + c_3\lambda^3 \Rightarrow c_0 \simeq \rho(0)$$

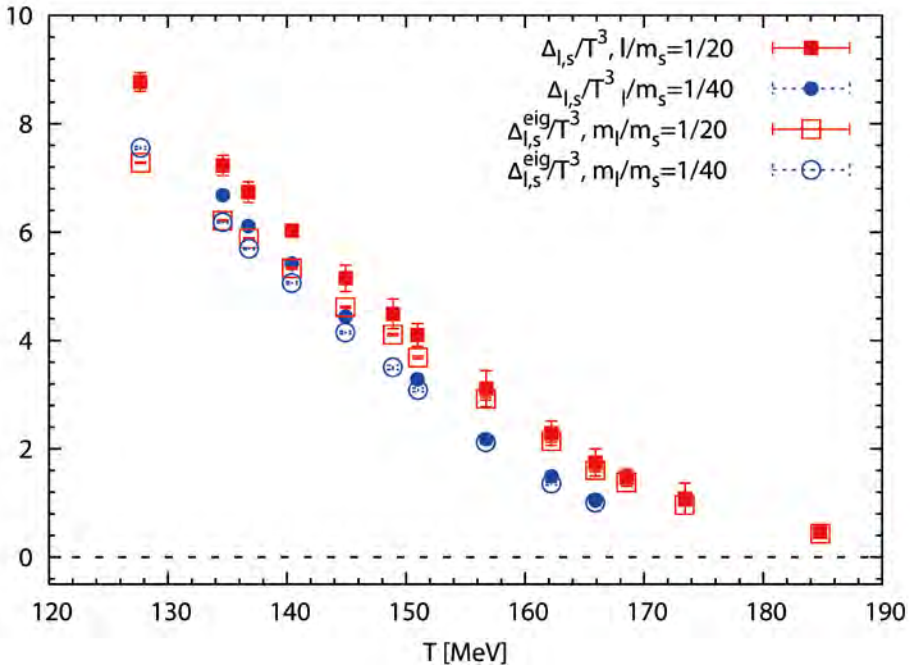




- $\rho(0)$ vanishes about 165-170 MeV.
- It is not clear whether $\rho(0) \sim m$ or not.

Results: chiral condensate

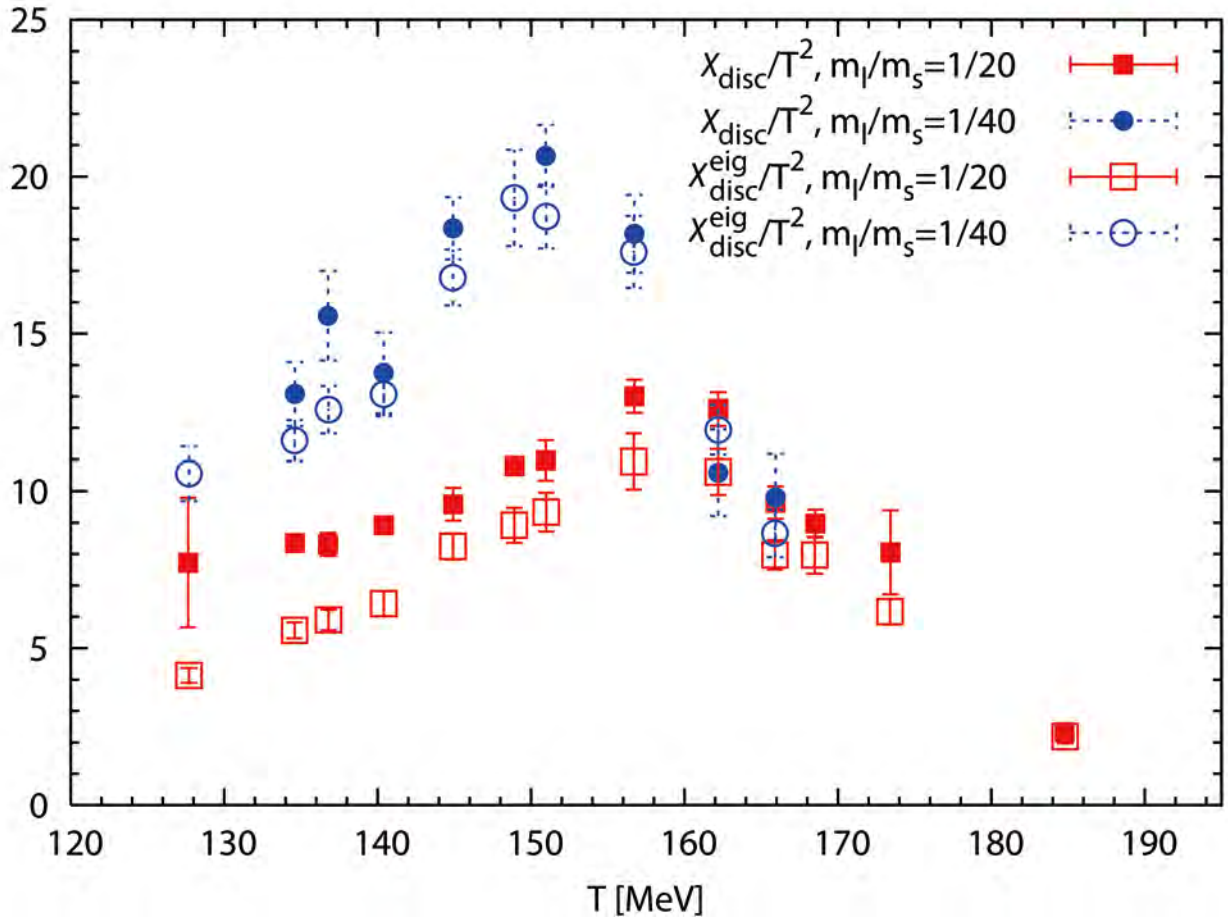




$$\Delta_{l,s} \equiv \langle\bar{\psi}\psi\rangle_l - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_s$$:subtracting additive UV divergence

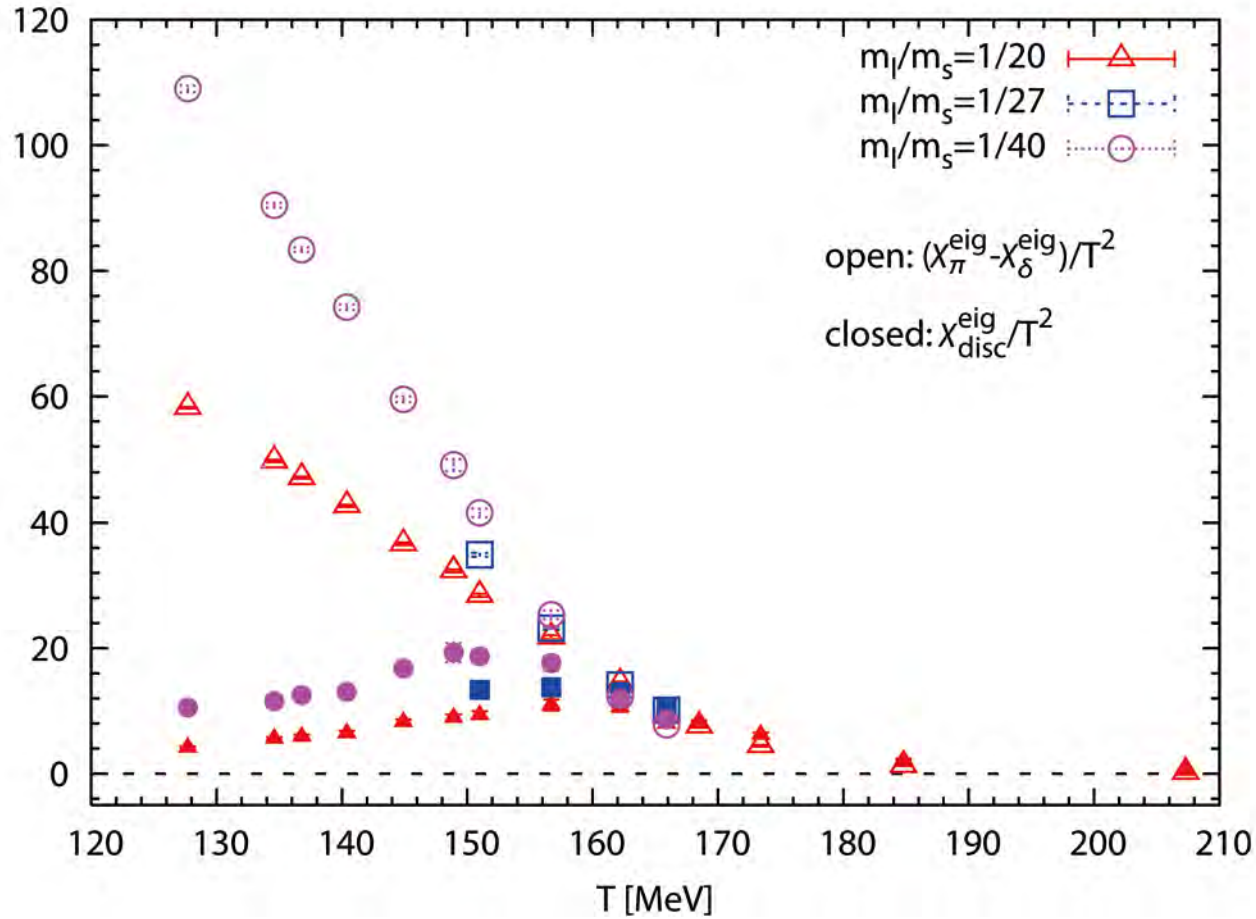
- The result given by eigenvalues is close to that calculated by ordinal way at high temperatures after the subtraction.

Results: susceptibilities



- The result given by eigenvalues has peak at the same position to the result calculated by ordinal way.

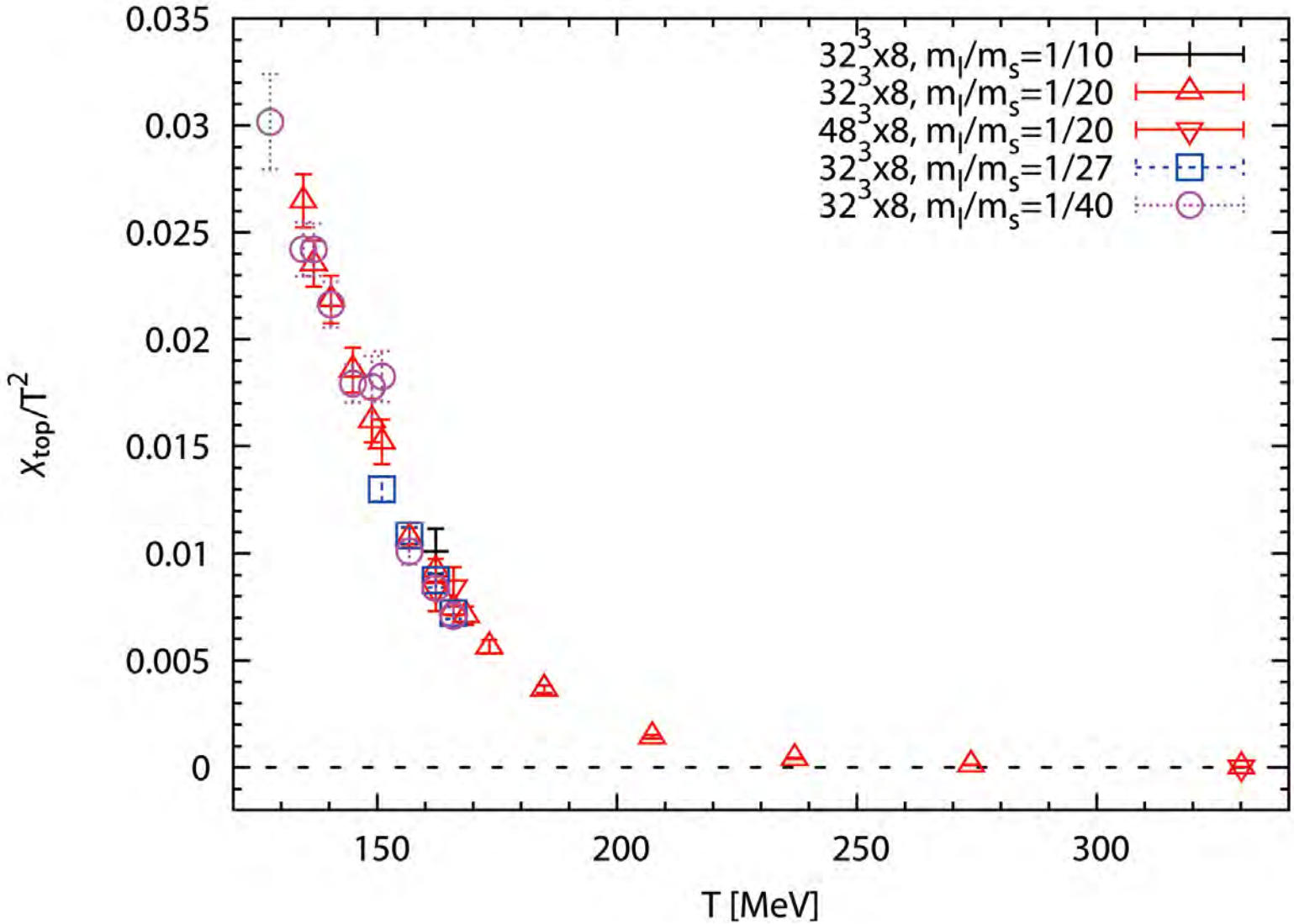
Results: susceptibilities



- The $U_A(1)$-violating susceptibility difference and the disconnected susceptibility merge with each other above T_c and still have non-zero value up to T=200 MeV.

Conclusions

- Dirac spectrum at finite temperature has been studied with lattice QCD simulations by using (2+1)-flavor dynamical HISQ.
- The intercept of the Dirac spectrum decreases as temperature increases and almost vanished at temperature about 165-170 MeV, which is consistent with what is expected from the chiral symmetry restoration.
- There is no clear evidence for a gap in the spectral density above T_c and it is difficult to say whether the intercept has a linear quark mass dependence or not.
- Chiral condensate can be reproduced well from the low-lying eigenvalues at high temperature once the additive UV divergence is subtracted.
- Disconnected susceptibility given by the eigenvalues has a same peak position to that calculated by the ordinal way.

End

Topological susceptibility