QCD factorization and Parton distribution functions on lattice

Yan-Qing Ma

Brookhaven National Laboratory

Based on works done with: Jian-Wei Qiu

In preparation

Lunch Seminar

BNL, NY, Mar. 27, 2014

Outline

- **Introduction**
  - Parton distribution functions
  - Difficulties in calculation
- **New idea for PDFs: quasi PDFs**
- **General conditions for quasi PDFs**
- **Factorization at one-loop level**
- **All order factorization**
- **Matching coefficients at one-loop level**
- **Summary and outlook**

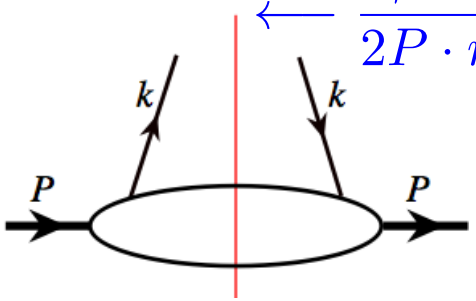
Definition of PDF

- **Lightcone quark distribution**

$$f_{q/p}(x,\mu^2) = \int \frac{d\xi_-}{4\pi} e^{-ix\xi_- P_+} \langle P|\overline{\psi}(\xi_-)\,\gamma_+ \exp\left\{-ig\int_0^{\xi_-} d\eta_- A_+(\eta_-)\right\}\psi(0)|P\rangle$$

- **Cut-vertex notation:**

$$\longleftarrow \frac{\gamma\cdot n}{2P\cdot n}\,\delta\left(x-\frac{k\cdot n}{P\cdot n}\right)\frac{d^4k}{(2\pi)^4}$$



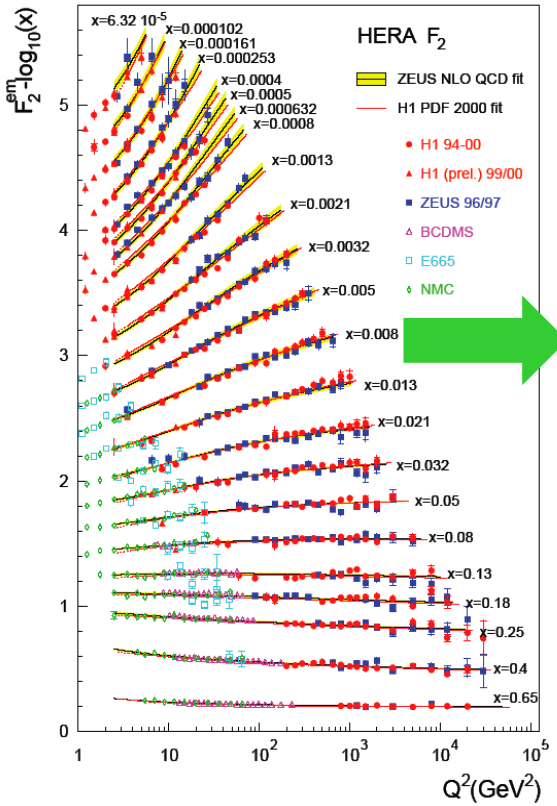
- **Boost invariant along n-direction**

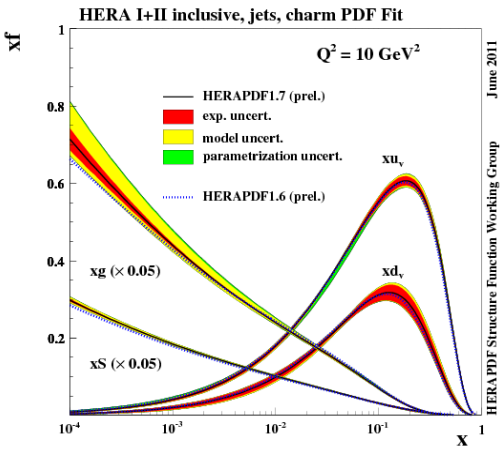
- **Parton interpretation emerges in $A_+ \equiv A\cdot n = 0$ gauge.**

PDF by fitting data

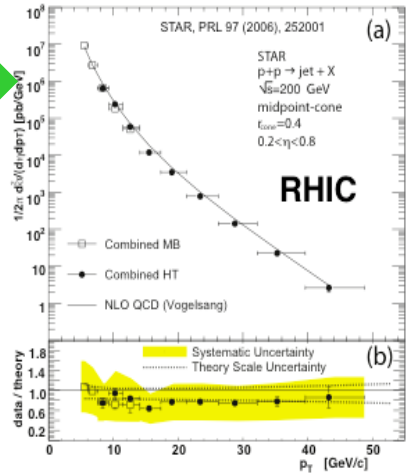
- **QCD@high energy: fit**

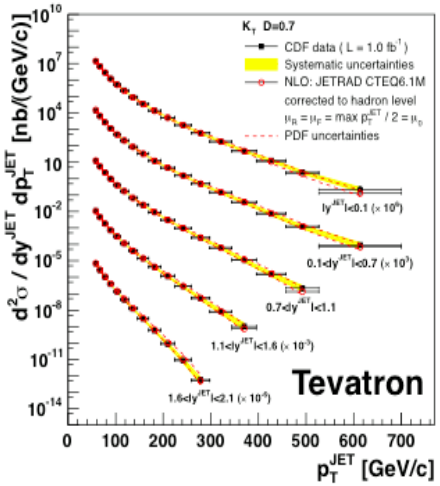
  NLO or NNLO DGLAP Evolution
  + NLO or NNLO calculations of partonic hard coeff.
  + a set of universal PDFs (CTEQ, MRST, NNPDF, …)

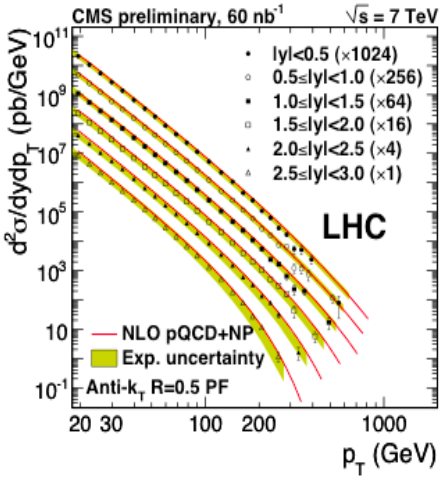




Measure e-p at 0.3 TeV (HERA)
Predict p-p and p-$\bar{p}$ at 0.2, 1.96, and 7 TeV





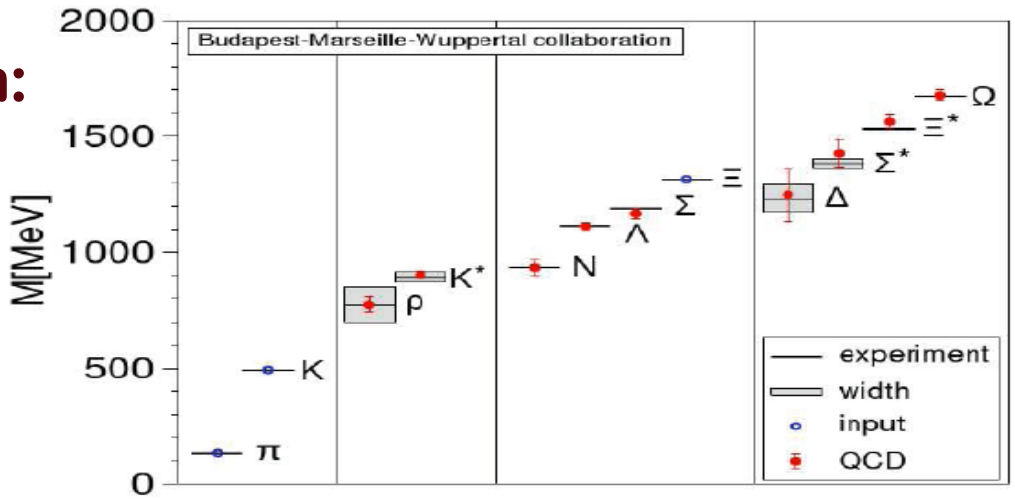


Lattice QCD

- **The main non-perturbative approach to solve QCD**

- **Hadron mass spectrum:**

  Predict the spectrum with limited inputs



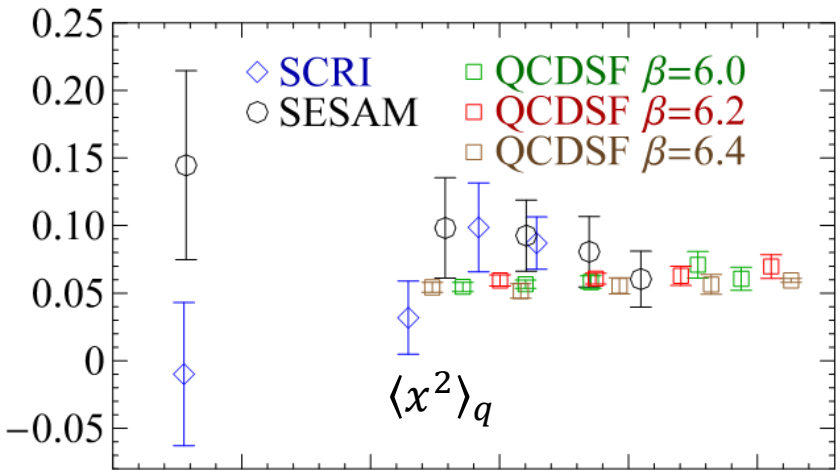
- **An intrinsically Euclidean approach**
  - "time" is Euclidean $\tau = i\,t$
  - No direct implementation of physical time

- **Cannot calculate PDFs directly, whose operators are time dependent**

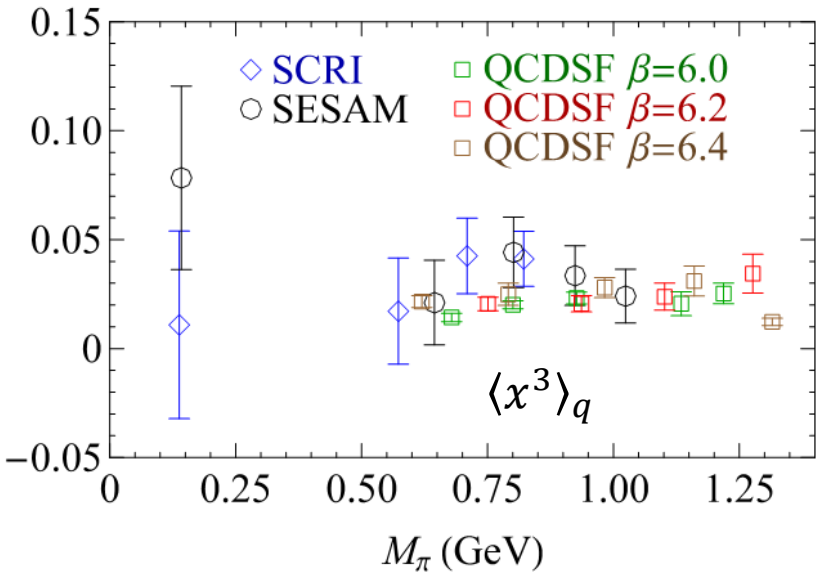
PDF from lattice

- **Only moments of PDFs can be calculated at present: local operator**
  - $\langle x^n(\mu^2)\rangle_q \equiv \int_0^1 dx\, x^n f_{q/p}(x,\mu^2)$

- **Works, but, hard and limited moments:**



Dolgov et al., hep-lat/0201021

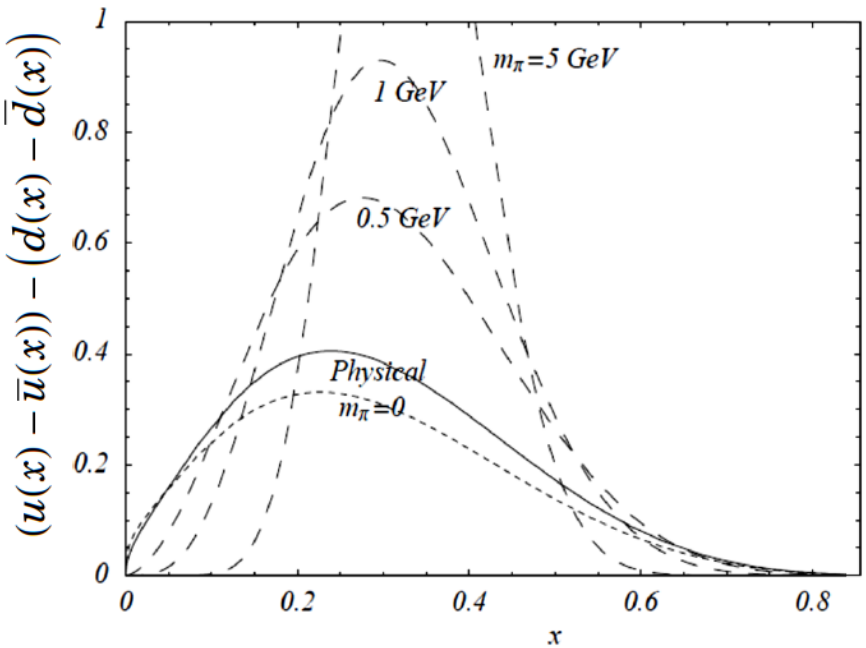


Gockeler et al., hep-ph/0410187

PDF from lattice

- **How to get x-dependent PDFs with a limited moments?**
  - Assume a smooth functional form with some parameters
  - Fit the parameters with the lattice calculated moments

$$x f_{q/p}(x, \mu_0^2) = a\, x^b (1-x)^c (1 + \epsilon \sqrt{x} + \gamma\, x)$$



W. Dermold et al., Eur.Phys.J.direct C3 (2001) 1-15

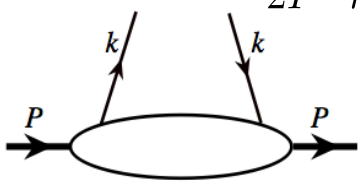
- Can not distinguish valence quark contribution from sea quarks

Ji's new idea

- **"Quasi" quark distribution (spin-averaged):** Ji, 1305. 1539

$$\tilde{f}_{q/p}(x,\mu^2,P_z) = \int \frac{d\xi_z}{4\pi} e^{-ix\xi_z P_z} \langle P|\overline{\psi}(\xi_z)\,\gamma_z \exp\left\{-ig\int_0^{\xi_z} d\eta_z A_z(\eta_z)\right\}\psi(0)|P\rangle$$

- **Cut-vertex notation** $\longleftarrow \dfrac{\gamma\cdot n_z}{2P\cdot n_z}\,\delta\left(x-\dfrac{k\cdot n_z}{P\cdot n_z}\right)\dfrac{d^4k}{(2\pi)^4}$



- **Features**
  - The matrix element depends on the momentum P along z-direction
  - Quark fields separated along the z-direction
  - The gauge-link along the z-direction
  - This distribution can be calculated using standard lattice method
- **No apparent parton interpretation**

Large P_z limit

Ji, 1305. 1539

- **Stated:**
  - The "quasi" quark distribution becomes the standard quark distribution at the limit of $P_z \to \infty$ after renormalizing all UV divergence
  - The leading-order Altarelli-Parisi evolution was derived in this way

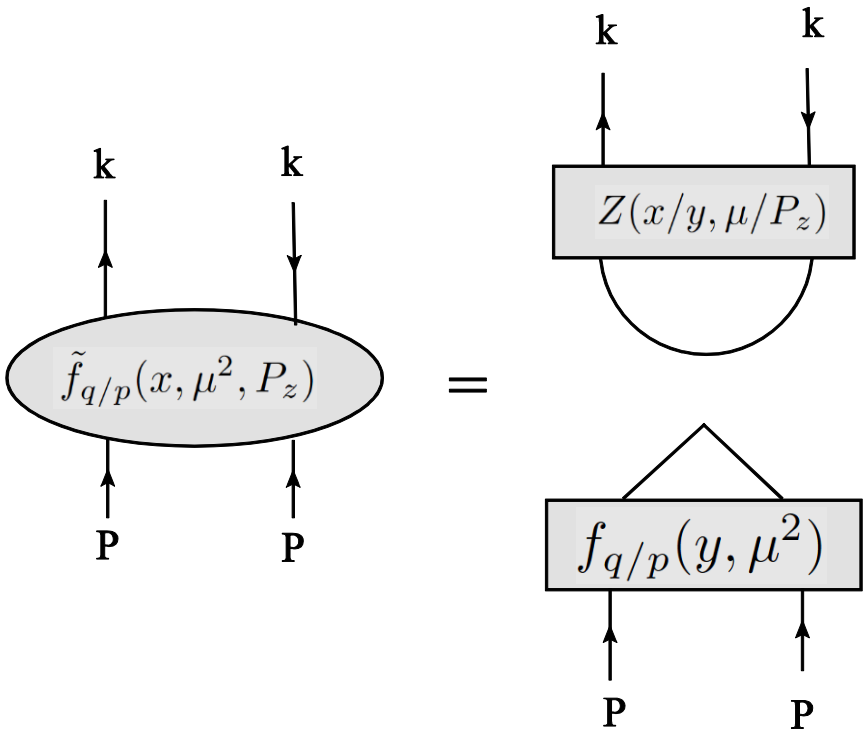
- **Finite but large P_z:**
  - It is potentially calculable in lattice QCD
  - Since it differs from the normal quark distribution by simply an infinite P_z limit, it should have the same IR physics
  - It should be related to the normal quark distribution by a matching condition

Conjecture: factorization

Ji, 1305. 1539

- **Stated:**
  - It is possible to have the following factorization up to power corrections:

$$\tilde{f}_{q/p}(x,\mu^2,P_z) = \int_x^1 dy/y\, Z(x/y,\mu/P_z) f_{q/p}(y,\mu^2)$$



Key questions

- Is there an all order factorization for finite but large P_z?
- How large of the P_z do we need?

Conditions for quasi PDFs

Ma, Qiu, In preparation

- **Supposing we have another set of PDFs, which can give an equally good decomposition of any differential cross section:**

$$d\sigma(Q, x_F) = \sum_i \int_0^1 dx\, d\hat{\sigma}_i(Q, x_F, x, \mu^2)\, f_{i/p}(x, \mu^2)$$

$$d\sigma(Q, x_F) = \sum_i \int d\tilde{x}\, d\hat{\tilde{\sigma}}_i(Q, x_F, \tilde{x}, \{\nu^2\})\, \tilde{f}_{i/p}(\tilde{x}, \{\nu^2\})$$

- **Necessary condition:** Free of soft divergence, same collinear divergence, UV renormalizable (if not cut-off)
- **Necessary and sufficient condition:**

$$\tilde{f}_{i/p}(\tilde{x}, \{\nu^2\}) = \sum_j \int_0^1 \frac{dx}{x}\, Z_{ij}\left(\frac{\tilde{x}}{x}, \{\frac{\nu^2}{\mu^2}\}\right) f_{j/p}(x, \mu^2)$$

$$f_{i/p}(x, \mu^2) = \sum_j \int \frac{d\tilde{x}}{\tilde{x}}\, \tilde{Z}_{ij}\left(\frac{x}{\tilde{x}}, \{\frac{\nu^2}{\mu^2}\}\right) \tilde{f}_{j/p}(\tilde{x}, \{\nu^2\})$$

..... (1)

- Z and $\tilde{Z}$ are finite, free of UV and IR divergence
- Sufficient condition: obviously
- Necessary condition: choosing special cross section, which can either mimic normal PDF or quasi PDF

Nonuniqueness of PDFs

- **There are a lot of quasi PDFs that can satisfy Eq.(1)**
  - Perturbative expansion: $Z = \sum_i Z^{(i)}$
  - Any $Z_{ij}(x)$ that satisfy the following condition is sufficient to define a set of qualified quasi PDFs: $Z^{(0)}_{ij}(x) = \delta_{ij}\delta(1-x)$

Eq.(1) gives: $$\delta_{ij}\delta(1-x) = \sum_{i'} \int \frac{dx'}{x'} \tilde{Z}_{ii'}(x/x') Z_{i'j}(x')$$

Solution: $$\tilde{Z}^{(0)}_{ij}(x) = \delta_{ij}\delta(1-x),$$

$$\tilde{Z}^{(n)}_{ij}(x) \overset{n\geq 0}{=} -\sum_{m=1}^{n} \sum_{i'} \int \frac{dx'}{x'} \tilde{Z}^{(n-m)}_{ii'}(x/x') Z^{(m)}_{i'j}(x')$$

- **Necessary and sufficient condition for Z: unique solution for X of equation:**

$$\sum_{i'} \int \frac{dx'}{x'} X_{ii'}(x/x') Z^{(0)}_{i'j}(x') = Y_{ij}(x)$$

Matching coefficients

- **Once we choose an operator definition for quasi PDFs , Z and $\tilde{Z}$ can be determined by matching condition**

$$\tilde{f}^{(n)}_{i/i'}(\tilde{x},\{\nu^2\}) = \sum_{m=0}^{n}\sum_{i}\int_0^1 \frac{dx}{x}\, Z^{(n-m)}_{ij}\Big(\frac{\tilde{x}}{x},\{\frac{\nu^2}{\mu^2}\}\Big)\, f^{(m)}_{j/i'}(x,\mu^2)$$

Suppose: $f^{(0)}_{i/j}(x,\mu^2) = \tilde{f}^{(0)}_{i/j}(x,\{\nu^2\}) = \delta_{ij}\delta(1-x)$

Solution: $Z^{(0)}_{ij}\Big(x,\{\frac{\nu^2}{\mu^2}\}\Big) = \delta_{ij}\delta(1-x),$

$$Z^{(n)}_{ij}\Big(x,\{\frac{\nu^2}{\mu^2}\}\Big) \overset{n\geq 0}{=} \tilde{f}^{(n)}_{i/j}(x,\{\nu^2\}) - f^{(n)}_{i/j}(x,\mu^2) - \sum_{m=1}^{n-1}\sum_{k}\int_0^1 \frac{dx'}{x'}\, Z^{(n-m)}_{ik}\Big(\frac{x}{x'},\{\frac{\nu^2}{\mu^2}\}\Big) f^{(m)}_{k/j}(x',\mu^2)$$

- **Large x behavior of quasi PDF constrained by small x behavior of normal PDF**

QCD is IR logarithmic divergent: $f^{(m)}_{k/j}(x',\mu^2) \overset{x'\to 0}{=} O(1/x')$

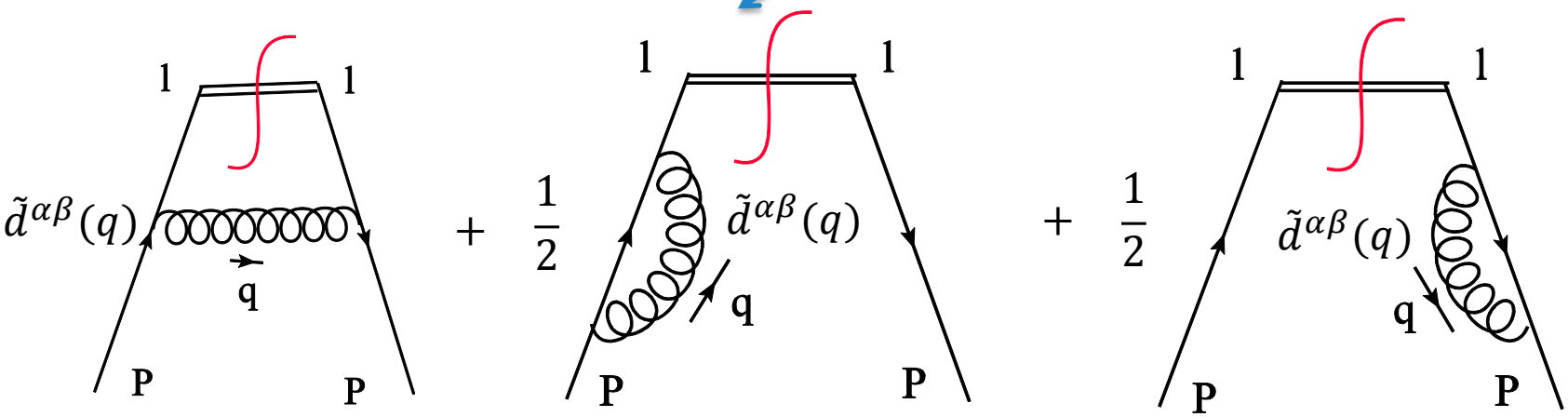
Integration is finite needs: $Z^{(n)}_{ij}(x,\{\nu^2/\mu^2\}) \overset{x\to\pm\infty}{=} O(1/x^2)$ ..... **(2)**

Equivalently: $\tilde{f}^{(n)}_{i/j}(x,\{\nu^2\}) \overset{x\to\pm\infty}{=} O(1/x^2)$

One loop example: q→q

- **Diagrams: $A_z = 0$ gauge**

$$\frac{\gamma \cdot n_z}{2P \cdot n_z}\delta\left(x - \frac{l \cdot n_z}{P \cdot n_z}\right)\frac{d^4 l}{(2\pi)^4}$$



- **Polarization: double pole!!!**

$$\tilde{d}^{\alpha\beta}(q) = -g^{\alpha\beta} + \frac{q^\alpha n_z^\beta + n_z^\alpha q^\beta}{q_z} - \frac{n_z^2 q^\alpha q^\beta}{q_z^2}$$

Integrand

- **Real+Virtual**

$$y = l_z/P_z \text{ and } \lambda^2 = l_\perp^2/P_z^2$$

$$\tilde{f}^{b(1)}_{q/q}(x,\tilde{\mu}^2,P_z^2) = C_F\frac{\alpha_s}{2\pi}N_\epsilon\int_0\frac{dl_\perp^2}{l_\perp^{2+2\epsilon}}\int_{-\infty}^{+\infty}\frac{dl_z}{P_z}\times\frac{1}{2}\left[\delta(y-x)-\delta(1-x)\right]\left\{\frac{1+y^2-\epsilon(1-y)^2}{1-y}\right.$$
$$\left.\times\left[\frac{y}{\sqrt{\lambda^2+y^2}}+\frac{1-y}{\sqrt{\lambda^2+(1-y)^2}}\right]+\frac{(1-\epsilon)y\lambda^2}{[\lambda^2+y^2]^{3/2}}+\frac{\lambda^2}{(1-y)\sqrt{\lambda^2+y^2}}+\frac{y\lambda^2}{(1-y)^2\sqrt{\lambda^2+(1-y)^2}}\right\}$$

- **Vector current conservation:** $\int_{-\infty}^{+\infty}dx\ \tilde{f}^{0(1)}_{q/q}(x,\tilde{\mu}^2,P_z^2)=0$
- **Soft divergence:** $y \to 1$
- **Collinear divergence:** $\lambda^2 \to 0$
- **Upper limit of $l_\perp^2$ is not specified. Different choice corresponds to different regularization/renormalization scheme.**

Cancellation of soft divergence

➢ Cancellation of double pole

- $\delta(y-x)-\delta(1-x) \sim O(1-y)$ as $y \to 1$. Only the last term is possible to be soft divergent:

$$[\delta(y-x)-\delta(1-x)]/(1-y)^2 \overset{y\to 1}{\sim} 1/(1-y)$$

- Integration: $y \in (-\infty, +\infty)$

➢ Regulator

- Feynman gauge: pole at $y \to 1$ is regularized by infinite small imaginary part. The correct expressions for double pole:

$$1/[(1-y)^2+\varepsilon^2]$$

- Integration of double pole is finite: odd function around the pole. Only principle value contributes

Cancellation of collinear divergence

$$\tilde{f}^{b(1)}_{q/q}(x,\tilde{\mu}^2,P_z^2) = C_F\frac{\alpha_s}{2\pi}N_\epsilon\int_0\frac{dl_\perp^2}{l_\perp^{2+2\epsilon}}\int_{-\infty}^{+\infty}\frac{dl_z}{P_z}\times\frac{1}{2}\left[\delta(y-x)-\delta(1-x)\right]\left\{\frac{1+y^2-\epsilon(1-y)^2}{1-y}\right.$$

$$\left.\times\left[\frac{y}{\sqrt{\lambda^2+y^2}}+\frac{1-y}{\sqrt{\lambda^2+(1-y)^2}}\right]+\frac{(1-\epsilon)y\lambda^2}{[\lambda^2+y^2]^{3/2}}+\frac{\lambda^2}{(1-y)\sqrt{\lambda^2+y^2}}+\frac{y\lambda^2}{(1-y)^2\sqrt{\lambda^2+(1-y)^2}}\right\}$$

- **Decomposition**

$$\frac{y}{\sqrt{\lambda^2+y^2}}+\frac{1-y}{\sqrt{\lambda^2+(1-y)^2}} = 2\theta(0<y<1)-\left[\mathrm{Sgn}(y)\frac{\sqrt{\lambda^2+y^2}-|y|}{\sqrt{\lambda^2+y^2}}+\mathrm{Sgn}(1-y)\frac{\sqrt{\lambda^2+(1-y)^2}-|1-y|}{\sqrt{\lambda^2+(1-y)^2}}\right]$$

  - **Terms in brackets $O(\lambda^2)$, no collinear divergence**
  - **Only the first term is collinear divergent: $0<y<1$, the same as collinear divergence in normal PDF**

- **Collinear divergence cancel in matching coefficients!**

Matching beyond one-loop

- **What is needed?**

$$\tilde{f}_{i/p}(\tilde{x},\{\nu^2\}) = \sum_j \int_0^1 \frac{dx}{x} Z_{ij}\left(\frac{\tilde{x}}{x},\{\frac{\nu^2}{\mu^2}\}\right) f_{j/p}(x,\mu^2)$$

$$f_{i/p}(x,\mu^2) = \sum_j \int \frac{d\tilde{x}}{\tilde{x}} \tilde{Z}_{ij}\left(\frac{x}{\tilde{x}},\{\frac{\nu^2}{\mu^2}\}\right) \tilde{f}_{j/p}(\tilde{x},\{\nu^2\})$$

  - Need to show that the "quasi PDFs" are free of soft divergence, like the normal PDFs, order-by-order to all orders
  - Need to show that the "quasi" PDFs share the same collinear divergence, perturbatively to all orders, as the normal PDFs

- **Above factorization is nontrivial!**
  - The condition $\tilde{f}_{i/p}(x,\mu^2,P_z) \to f_{i/p}(x,\mu^2)$ in the limit $P_z \to \infty$ is not enough
  - There may be terms infrared divergence at $O\left(\frac{\mu^2}{P_z^2}\right)$
  - The condition in Eq.(2) is needed to check

Soft cancellation

- **Factorize quasi PDFs by normal PDFs**
- **One –loop case:**

  Soft pole cancel between real diagrams and loop diagrams

- **All order**

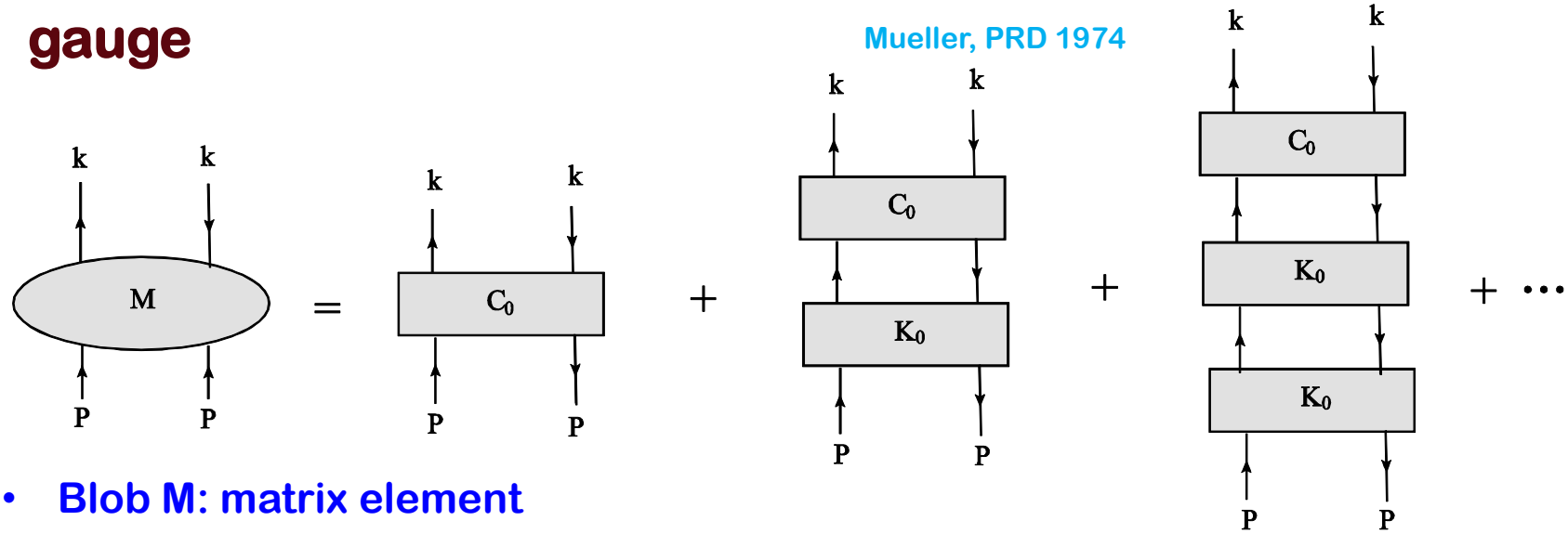
  - Summing over all intermediate states, soft pole cancelled by unitarity.
  - Vector current conservation at one-loop level is correct to all order.

$$\int_{-\infty}^{+\infty} dx\ \tilde{f}^{0(1)}_{q/q}(x, \tilde{\mu}^2, P_z^2) = 0.$$

Decomposition for collinear divergence

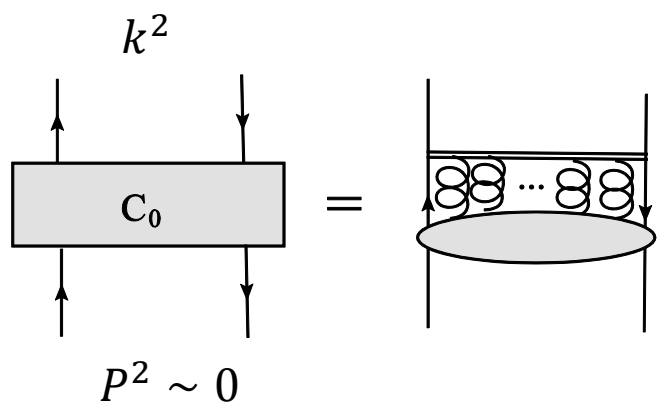
- **Generalized ladder diagrams decomposition in a physical gauge**

Mueller, PRD 1974



- Blob M: matrix element
- C_0, K_0: 2PI kernels
- Ordering in virtuality $p^2 \ll k^2 \sim \mu^2$

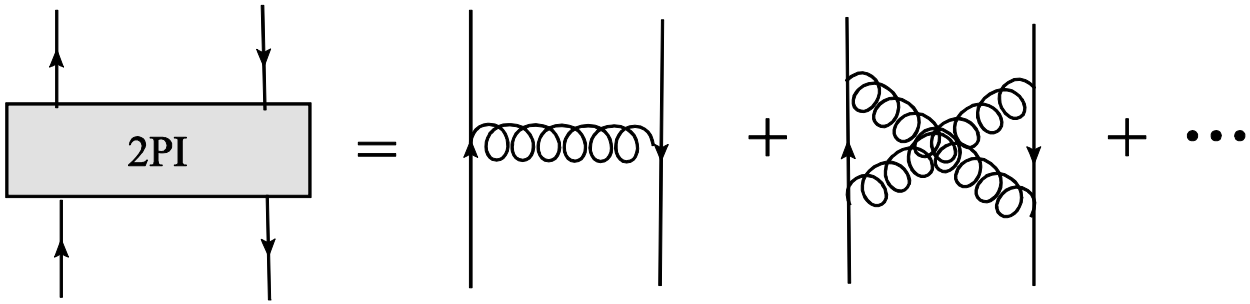
- **General axial gauge**



- Double line: gauge link

2PI kernels

- **Diagrams**



- **Using physical gauge, 2PI diagrams are finite**

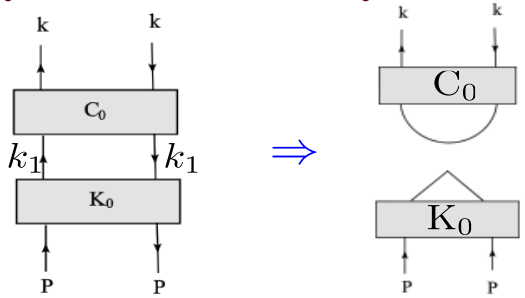
Ellis, Georgi, Machacek, Politzer, Ross, 1978, 1979

- Landau rule: infrared divergenceare associated with physical, kinematically allowed subprocesses

- 2PI diagrams: from interference terms (other than the lowest order one)

Factorization

- **Spinor decomposition –power counting:**

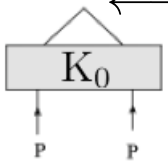


$$\longleftarrow \frac{1}{2}\gamma\cdot\hat{k}_1 = \frac{1}{2}\gamma\cdot(x_1 P)$$

$$\longleftarrow \frac{\gamma\cdot n}{2\hat{k}_1\cdot n} = \frac{1}{x_1}\frac{\gamma\cdot n}{2P\cdot n}\,\delta\left(x_1 - \frac{k_1\cdot n_z}{P\cdot n_z}\right)dx_1$$

- **Approximation**

$$n_z^\mu = \frac{1}{\sqrt{2}}\left[\frac{P^\mu}{P\cdot n} - n^\mu\right]$$



$$\longleftarrow \frac{\gamma\cdot n_z}{2\hat{k}_1\cdot n_z} = \frac{1}{x_1}\frac{\gamma\cdot n}{2P\cdot n} + \mathcal{O}\left(\frac{P^2}{k^2}\right)$$

Contact terms

Qiu, PRD 1990

- **Factorize the last kernel, and then recursively:**

- $\mathbb{P}$: extracts the singular part of integration

$$\begin{aligned} M =& C_0\sum_{i=0}^{\infty} K_0^i \\ =& C_0\left[1 + \sum_{i=1}^{\infty} K_0^{i-1}\mathbb{P}K_0 + \sum_{i=1}^{\infty} K_0^{i-1}(1-\mathbb{P})K_0\right] \\ =& C_0\left[1 + \sum_{i=1}^{\infty} K_0^{i-1}(1-\mathbb{P})K_0\right] + M\mathbb{P}K_0 \end{aligned}$$

$$M = \left(C_0\frac{1}{1-(1-\mathbb{P})K_0}\right)\left(\frac{1}{1-\mathbb{P}K_0}\right)$$

Divergence are included in the second term

Factorized form

- **Quasi PDFs can be factorized by normal PDFs.**
- **As proved above, the factorization is reversible:**

$$\tilde{f}_{i/p}(\tilde{x},\{\nu^2\}) = \sum_j \int_0^1 \frac{dx}{x} Z_{ij}\left(\frac{\tilde{x}}{x},\{\frac{\nu^2}{\mu^2}\}\right) f_{j/p}(x,\mu^2)$$

$$f_{i/p}(x,\mu^2) = \sum_j \int \frac{d\tilde{x}}{\tilde{x}} \tilde{Z}_{ij}\left(\frac{x}{\tilde{x}},\{\frac{\nu^2}{\mu^2}\}\right) \tilde{f}_{j/p}(\tilde{x},\{\nu^2\})$$

- The first equation: check the consistence between lattice calculation and experimental data
- The second equation: extract normal PDFs from lattice calculation directly!

- **Slightly different from the factorized from given by Ji:**

$$\tilde{f}_{q/p}(x,\mu^2,P_z) = \int_x^1 dy/y\, Z(x/y,\mu/P_z) f_{q/p}(y,\mu^2)$$

- Integration region, more flavors, reversed factorization

One loop matching coefficient

- **Result in cut-off regularization scheme**

$$Z_{qq}^{(1)} = C_F \frac{\alpha_s}{2\pi}\left[P_{qq}\ln\frac{\mu^2}{\tilde{\mu}^2} - (1-x) + \left(\frac{x\Lambda_{1-x}}{(1-x)^2} + \frac{\Lambda_x}{1-x} + \frac{\mathrm{Sgn}(x)\Lambda_x}{\Lambda_x + |x|} - P_{qq}\Delta_{\mathrm{cut}}\right)_N\right]$$

with

$$P_{qq} = \frac{1+x^2}{1-x} \qquad \Lambda_x = \sqrt{\tilde{\mu}^2/P_z^2 + x^2} - |x|$$

$$\Delta_{\mathrm{cut}} = \mathrm{Sgn}(x)\ln\left(1 + \frac{\Lambda_x}{2|x|}\right) + (x \to 1-x)$$

$$\int_{-\infty}^{+\infty} dx \left(g(x)\right)_N h(x) = \int_{-\infty}^{+\infty} dx\, g(x)\left[h(x) - h(1)\right]$$

- **Features**
  - As $x \to \pm\infty$, we have a correct behavior: $Z_{qq}^{(1)} \to O(1/x^2)$
  - Vanishes as $P_z \to \infty$ (if normal PDFs are also regularized by cut-off), expected by Ji's argue.
  - Finite terms: $O(\frac{\alpha_s}{2\pi}\frac{\mu^2}{P_z^2})$, good convergence!

- **Explicit calculation: same features for all other channels!**

Comparison with the work by Xiong et al.

Xiong et al. 1310.7471

- **Xiong et al. also calculated the one-loop matching coefficients for $q \to q$ channel, and claimed the factorization at one-loop level. Our results for $q \to q$ channel agree with their results at intermediate stage.**

$$\tilde{q}^{(1)}(x) = \frac{\alpha_S C_F}{2\pi} \begin{cases} \frac{1+x^2}{1-x} \ln \frac{x(\Lambda(x)-xP^z)}{(x-1)(\Lambda(1-x)+P^z(1-x))} + 1 - \frac{xP^z}{\Lambda(x)} + \frac{x\Lambda(1-x)+(1-x)\Lambda(x)}{(1-x)^2 P^z}, & x > 1, \\ \frac{1+x^2}{1-x} \ln \frac{(P^z)^2}{m^2} + \frac{1+x^2}{1-x} \ln \frac{4x(\Lambda(x)-xP^z)}{(1-x)(\Lambda(1-x)+(1-x)P^z)} - \frac{4x}{1-x} + 1 - \frac{xP^z}{\Lambda(x)} + \frac{x\Lambda(1-x)+(1-x)\Lambda(x)}{(1-x)^2 P^z}, & 0 < x < 1, \\ \frac{1+x^2}{1-x} \ln \frac{(x-1)(\Lambda(x)-xP^z)}{x(\Lambda(1-x)+(1-x)P^z)} - 1 - \frac{xP^z}{\Lambda(x)} + \frac{x\Lambda(1-x)+(1-x)\Lambda(x)}{(1-x)^2 P^z}, & x < 0 \end{cases}$$

$$\Lambda(x) = \sqrt{\mu^2 + x^2 (P^z)^2}$$

- **However, their final matching coefficient violates condition Eq.(2)**
  - Their results are obtained by taking the $\mu \to \infty$ limit
  - As $x \to \pm\infty$, their matching coefficient behaves as $Z^{(1)}_{q/q}(x) \sim O(1/x)$

$$\tilde{q}^{(1)}(x,\mu,P^z) = \frac{\alpha_S C_F}{2\pi} \begin{cases} \frac{1+x^2}{1-x} \ln \frac{x}{x-1} + 1 + \frac{\mu}{(1-x)^2 P^z}, & x > 1, \\ \frac{1+x^2}{1-x} \ln \frac{(P^z)^2}{m^2} + \frac{1+x^2}{1-x} \ln \frac{4x}{1-x} - \frac{4x}{1-x} + 1 + \frac{\mu}{(1-x)^2 P^z}, & 0 < x < 1, \\ \frac{1+x^2}{1-x} \ln \frac{x-1}{x} - 1 + \frac{\mu}{(1-x)^2 P^z}, & x < 0, \end{cases}$$

More recent development

Lin et al. 1402.1462

- **First lattice calculation is available**

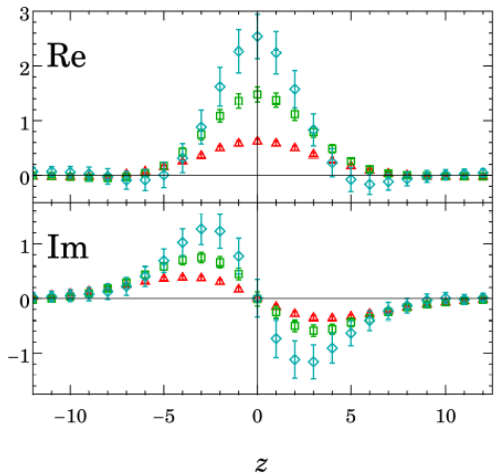


FIG. 1. The real (top) and imaginary (bottom) parts of the nonlocal isovector matrix element h of Eq. 3 computed on a lattice with the nucleon momentum P_z (in units of $2\pi/L$) = 1 (red triangles), 2 (green squares), 3 (cyan diamonds).

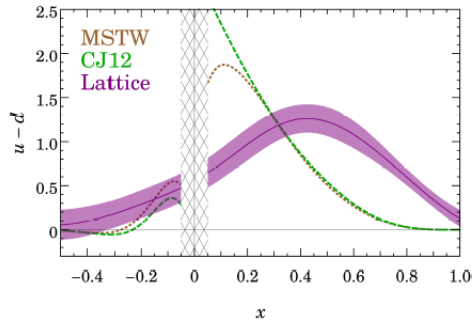


FIG. 2. The unpolarized isovector quark distribution $u(x) - d(x)$ computed on the lattice (purple band), compared with the global analyses by MSTW [13] (brown dotted line), and CTEQ-JLab (CJ12, green dashed line) [14] with medium nuclear correction near $(1.3\text{GeV})^2$. The negative x region is the sea quark distribution with $\overline{q}(x) = -q(-x)$.

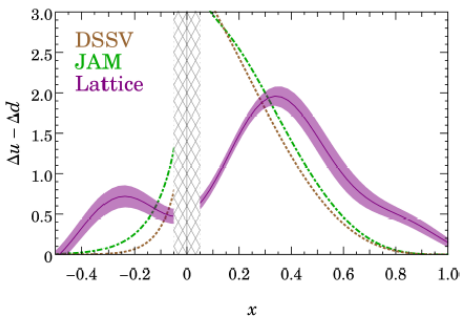


FIG. 3. (top) The isovector helicity distribution $\Delta u(x) - \Delta d(x)$ (purple band) computed on the lattice, along with selected global polarized analyses by JAM [19] (green dot-dashed) and DSSV09 [3] (brown dotted line). The corresponding sea-quark distributions are $\Delta\overline{q}(x) = \Delta q(-x)$.

Good convergence, but results are not consistent with experimental data

Matching coefficients?

Summary

- PDFs, FFs, and other correlation functions are fundamental in QCD, but, hard to calculate from first principle.
- Recent new idea: normal distributions can be obtained by corresponding lattice calculable quasi distributions by taking $P_z \to \infty$.
- We derived the general condition (matching relation) for quasi PDFs that can equally good factorize any cross section.
- We prove the factorization to all order in α_s for some specifically chosen quasi PDFs.
- Explicit one-loop calculation confirm the factorization.

Outlook

- **Additional matching:** Ishikawa, Qiu, Yoshita, in progress

$$f_{\rm M}(x,\mu^2) \longleftrightarrow \tilde{f}_{\rm M}(x,\tilde{\mu}^2,P_z^2)$$
$$\Updownarrow$$
$$\tilde{f}_{\rm E}(x,\tilde{\mu}^2,P_z^2) \longleftrightarrow \tilde{f}_{\rm E}^{\rm Lat}(x,1/a,P_z^2)$$

  - Intermediate matching is not needed as far as quasi PDFs have the same results in both Minkowski space and Euclidean Space

- **Quasi distributions for TMDs, GPDs….?**

  How to find a set of quasi distributions that satisfy a general condition, like Eq.(1)?

- **Lattice calculation is still hard…**

Thank you!