Flow fluctuations and multi-particle eccentricities in p+A and A+A collisions

Adam Bzdak

RIKEN BNL Research Center,
Brookhaven National Laboratory

based on:
AB, P. Bozek, L. McLerran, arXiv:1311.7325
AB, V. Skokov, arXiv:1312.7349

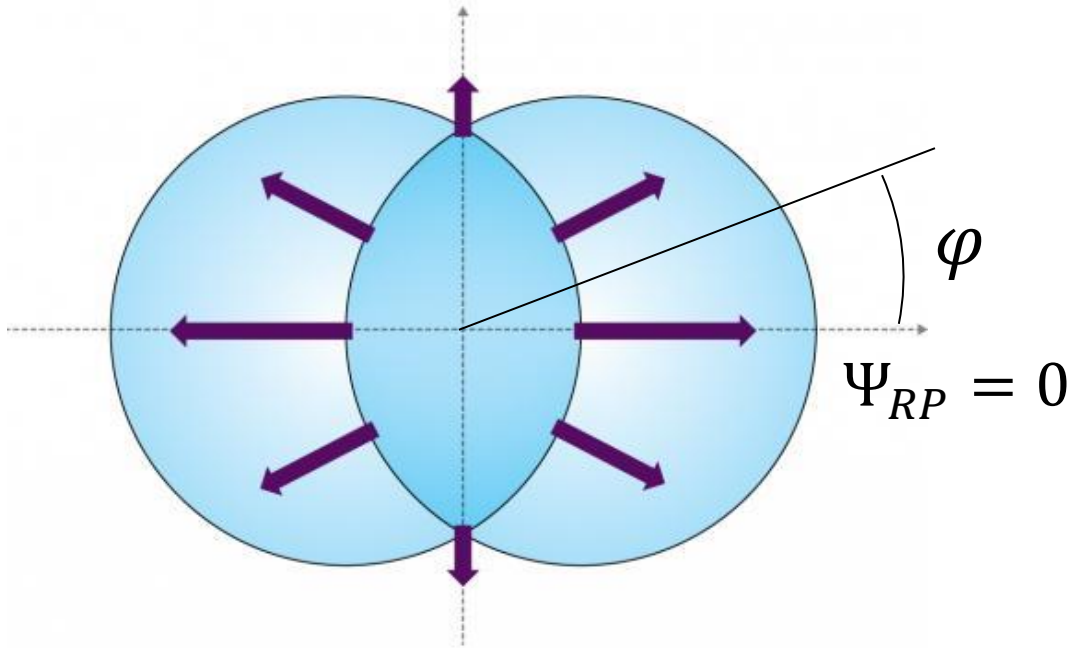
Outline

- introduction
- multi-particle cumulants
- A+A collisions
- p+A collisions
- conclusions

Elliptic flow, ellipticity

J.-Y. Ollitrault, PRD 46 (1992) 229

transverse plain of a heavy-ion collision

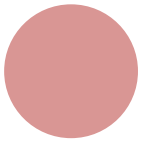


azimuthal angle

$$\frac{dN}{d\varphi} \sim 1 + 2v_2 \cos(2\varphi - 2\Psi_{RP}) + \cdots$$

Hydrodynamics:

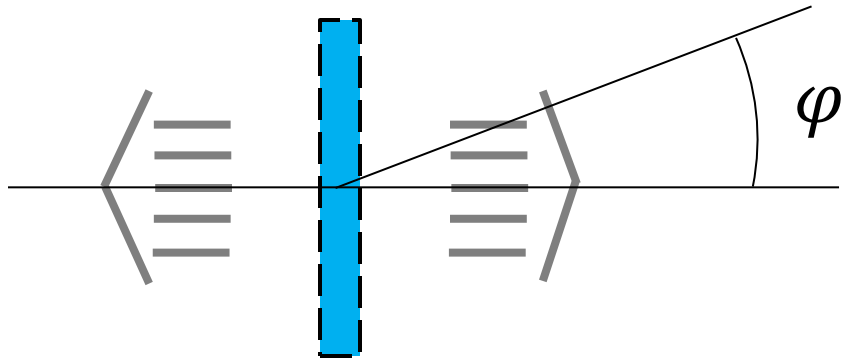
$$v_2^2 \sim \epsilon_2^2 = \frac{\left[\sum_i r_i^2 \cos(2\varphi_i)\right]^2 + \left[\sum_i r_i^2 \sin(2\varphi_i)\right]^2}{\left[\sum_i r_i^2\right]^2}$$



$\epsilon_2 \approx 0$

$\epsilon_2 \approx 1$

Let's consider an extreme case, particles only at 0 and π



$\Psi_{RP} = 0$

There are 2-, 4-,…, N-particle correlations

after integrating over Ψ_{RP}

$\varphi_1 - \varphi_2 = 0$ or $\varphi_1 - \varphi_2 = \pi$

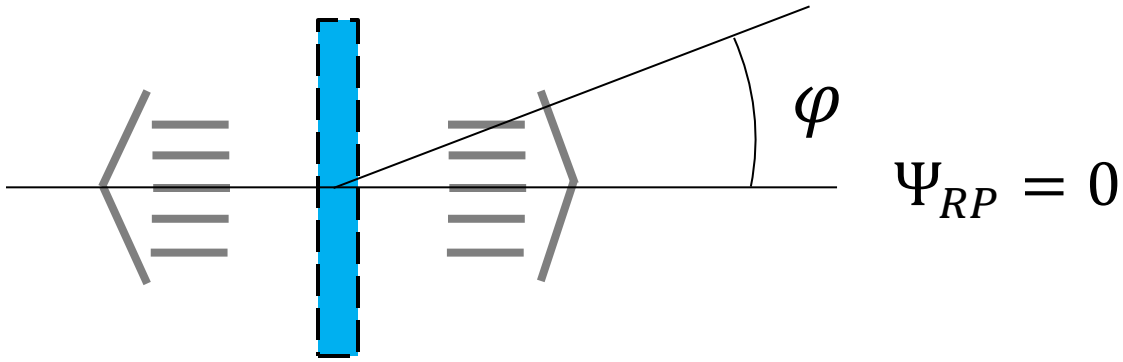
2-particle correlation function is peaked at 0 and π and has minimum at $\pi/2$. For real v_2 it is $\cos(2[\varphi_1 - \varphi_2])$

$\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4 = 0, \pi$

4-particle correlation function is non-zero, For real v_2, $\cos(2[\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4])$

Flow gives N-particle azimuthal correlations.

At a given reaction plane there are no correlations !



$$f_2(\varphi_1, \varphi_2; \Psi_{RP}) = f(\varphi_1; \Psi_{RP}) * f(\varphi_2; \Psi_{RP})$$

$$1 + 2v_2 \cos(2\varphi_1 - 2\Psi_{RP}) + \cdots$$

$$1 + 2v_2 \cos(2\varphi_2 - 2\Psi_{RP}) + \cdots$$

when integrated over $\Psi_{RP} \rightarrow$

$$\int d\Psi_{RP}\, f_2(\varphi_1, \varphi_2; \Psi_{RP}) \sim 1 + 2v_2^2 \cos(2\varphi_1 - 2\varphi_2)$$

$$\int d\Psi_{RP}\, f_4(\varphi_1, \varphi_2, \varphi_3, \varphi_4; \Psi_{RP}) \sim$$

$$1 + 2v_2^2 \cos(2\varphi_1 - 2\varphi_2) + \cdots$$

$$+ 2v_2^4 \cos(2\varphi_1 + 2\varphi_2 - 2\varphi_3 - 2\varphi_4) + \cdots$$

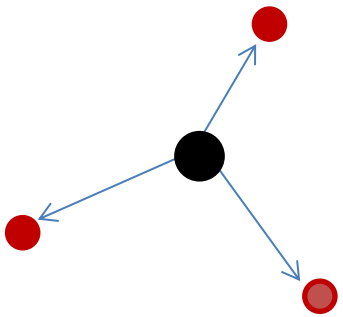
Particles are correlated <u>through the reaction plane</u>

Strengths of 2-, 4-, …, N-particle correlations are the same!

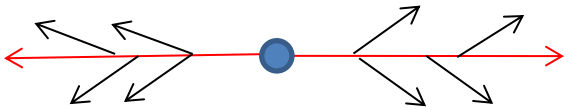
$$f_4(p_1, p_2, p_3, p_4; \Psi_{RP}) = f(p_1; \Psi_{RP}) * f(p_2; \Psi_{RP}) * f(p_3; \Psi_{RP}) * f(p_4; \Psi_{RP})$$

Non-flow contributions

- resonance decays

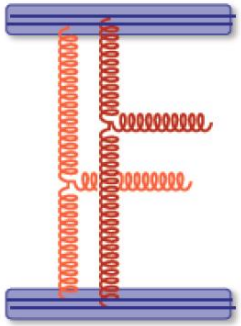


- jets



$$\varphi_1 - \varphi_2 \sim \pi$$
$$\varphi_1 - \varphi_2 \sim 0$$

- CGC



$$\varphi_1 - \varphi_2 \sim \pi$$
$$\varphi_1 - \varphi_2 \sim 0$$

- transverse momentum conservation, $\cos(\varphi_1 - \varphi_2)$

- etc.

Rapidity gap helps to reduce certain non-flow correlations.

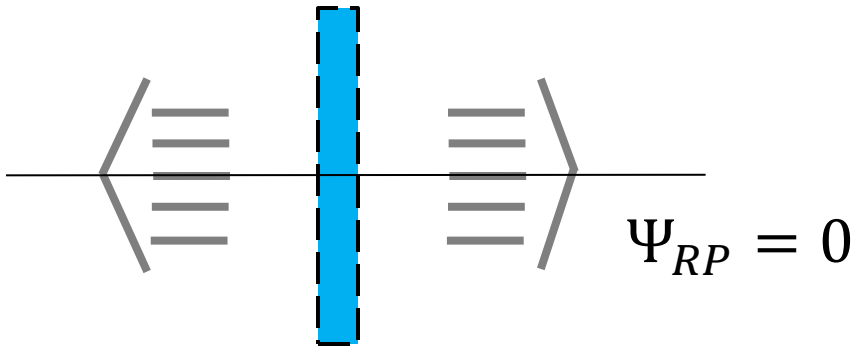
Flow from correlations

N.Borghini, P.M.Dinh, J.-Y.Ollitrault, PRC 63 (2001) 054906

$$(v_2\{2\})^2 = \left\langle e^{i2(\varphi_1-\varphi_2)} \right\rangle$$

$$= \left\langle v_2^2 \right\rangle + c_2$$

c_2 is non-flow



$$f_2(\varphi_1, \varphi_2; \Psi_{RP}) =$$

$$f(\varphi_1; \Psi_{RP}) * f(\varphi_2; \Psi_{RP}) * [1 + C_2(\varphi_1, \varphi_2)]$$

Flow from correlations

N.Borghini, P.M.Dinh, J.-Y.Ollitrault, PRC 63 (2001) 054906

$$(v_2\{2\})^2 = \left\langle e^{i2(\varphi_1-\varphi_2)} \right\rangle$$

c_m is non-flow

$$= \left\langle v_2^2 \right\rangle + c_2$$

to subtract c_2

$$(v_2\{4\})^4 = -\left\langle e^{i2(\varphi_1+\varphi_2-\varphi_3-\varphi_4)} \right\rangle + 2\left\langle e^{i2(\varphi_1-\varphi_2)} \right\rangle^2$$

$$= -\langle v_2^4 \rangle + 2\left\langle v_2^2 \right\rangle^2 + c_4$$

etc.

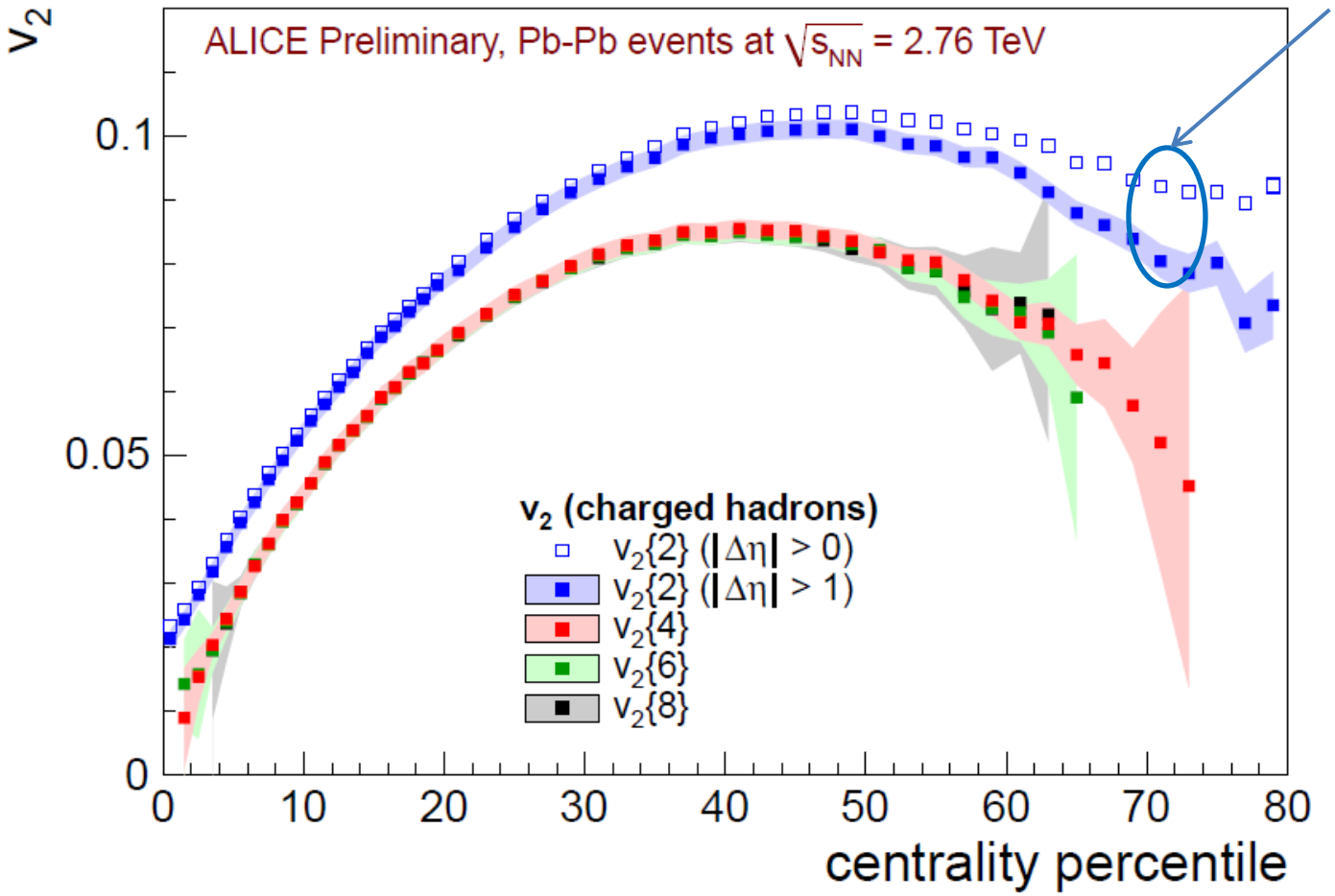
If in each event $v_2 = \overline{v_2}$:

$$v_2\{2\} = \overline{v_2} + c_2$$

$$v_2\{m\} = \overline{v_2} + c_m$$

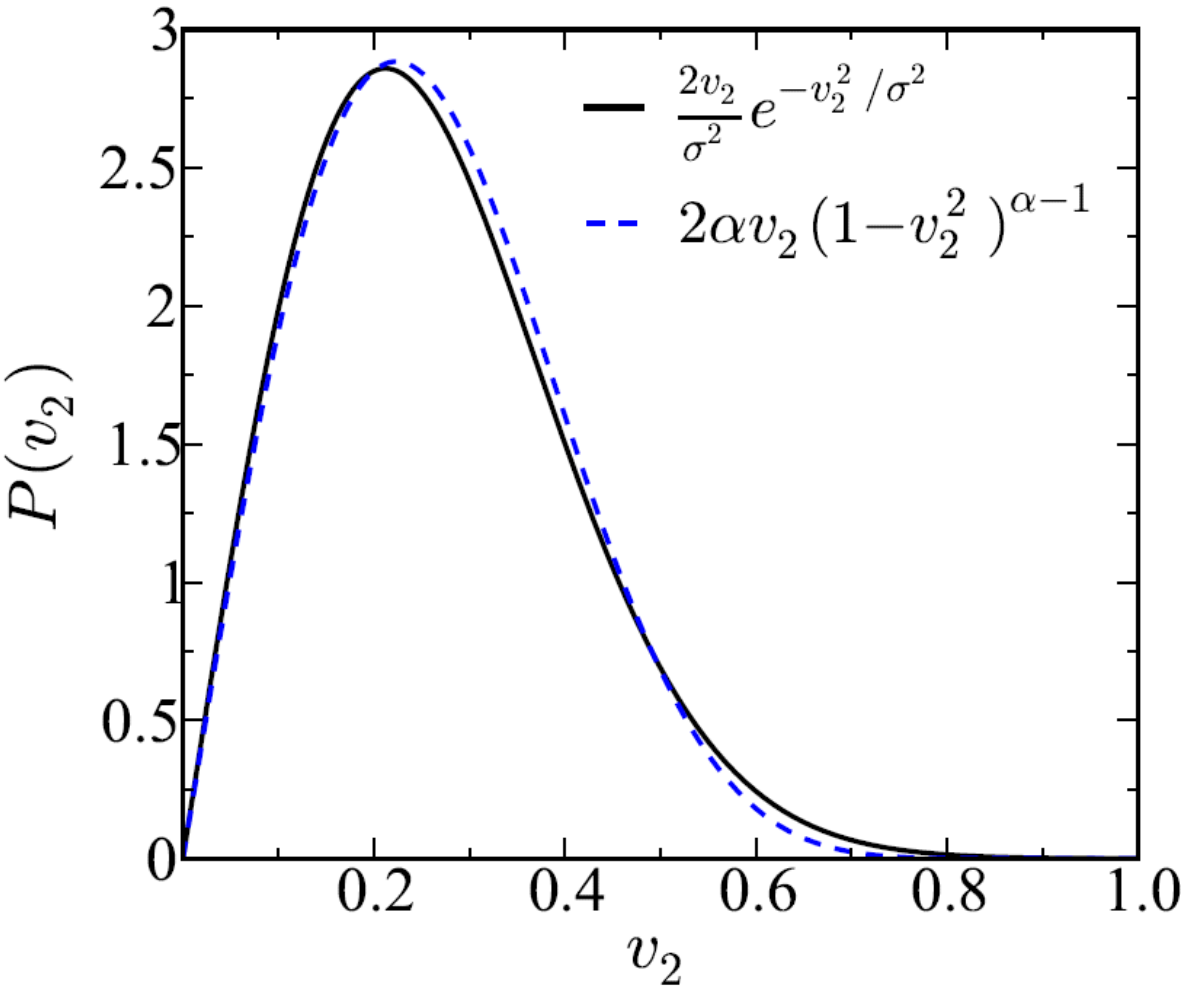
Pb-Pb, ALICE

this difference is due to c_2



difference between $v_2\{2\}$ and $v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$ is due to flow fluctuations …

multi-particle moments are not trivial



Gauss:

$$v_2\{2\} = \sigma$$
$$v_2\{4\} = 0$$
$$v_2\{6\} = 0$$
$$v_2\{8\} = 0$$

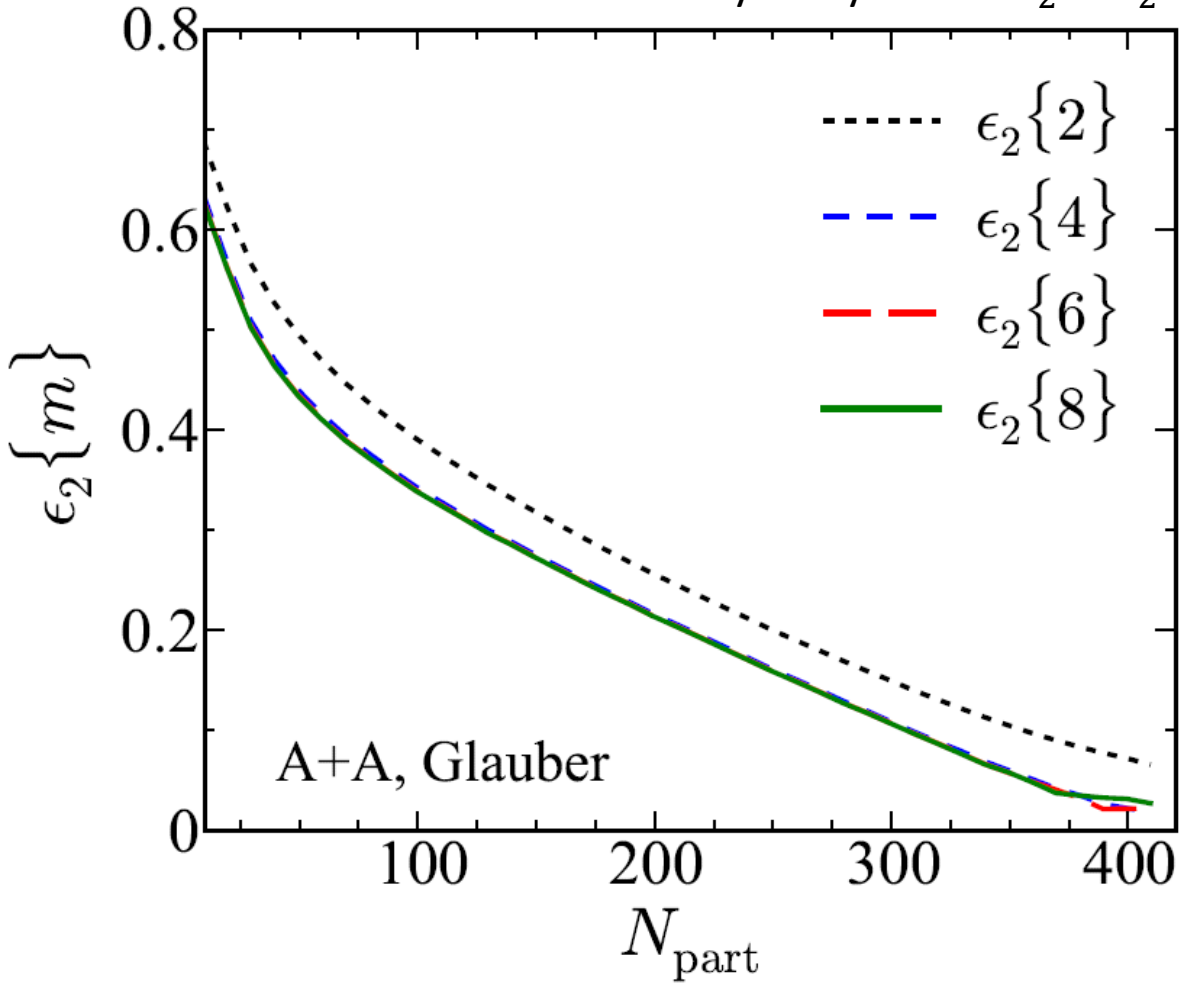
Power:

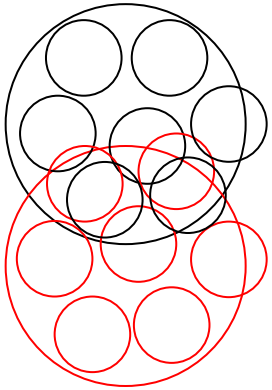
$$v_2\{2\} \approx 0.295$$
$$v_2\{4\} \approx 0.186$$
$$v_2\{6\} \approx 0.169$$
$$v_2\{8\} \approx 0.164$$

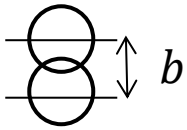
In this plot: $\sigma = 0.3,\ \alpha = 10.5$

Monte-Carlo calculation for ellipticities (so called Glauber MC)

in hydrodynamics $v_2 \sim \epsilon_2$







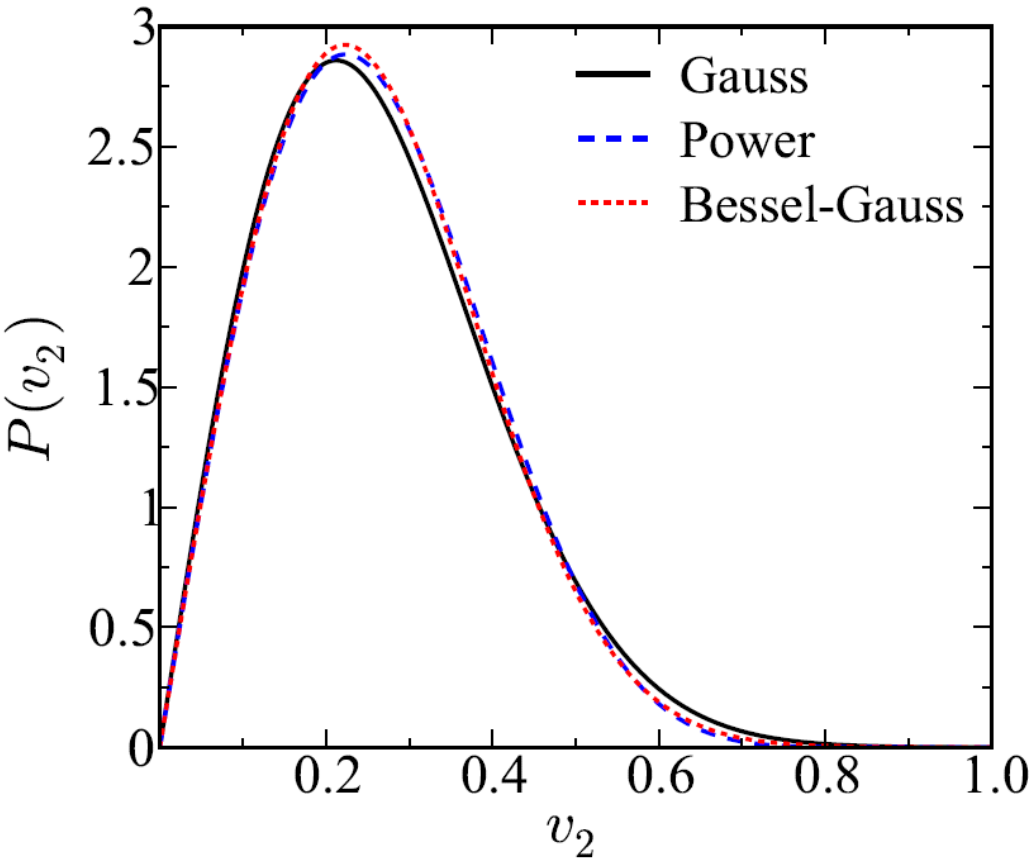
$$\sigma_{in}(b) \sim e^{-b^2/\gamma}$$

inelastic cross-section

AB, P. Bozek, L. McLerran, arXiv:1311.7325

In A+A collisions $P(\epsilon_2)$ is well described by Bessel-Gaussian distribution

$$P(\epsilon_2) = \frac{2\epsilon_2}{\sigma^2} I_0\left(\frac{2\epsilon_2 b}{\sigma^2}\right) e^{-(\epsilon_2^2+b^2)/\sigma^2}$$



$$v_2\{2\} = \sqrt{\sigma^2 + b^2}$$
$$v_2\{4\} = b$$
$$v_2\{6\} = b$$
$$v_2\{8\} = b$$

In this plot:

$v_2\{2\} = 0.295$

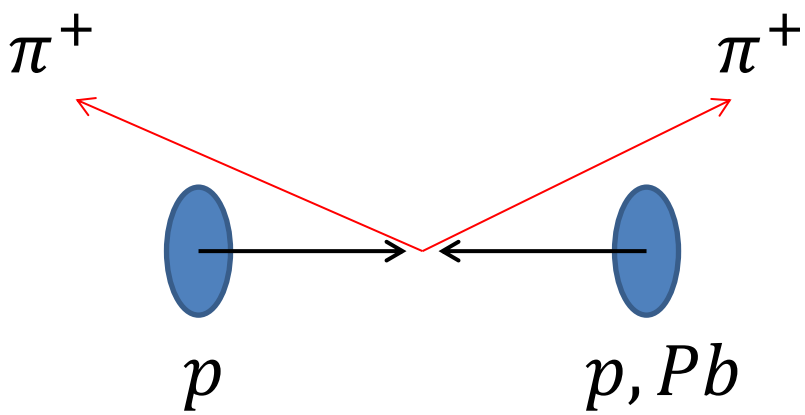
$b = 0.17$

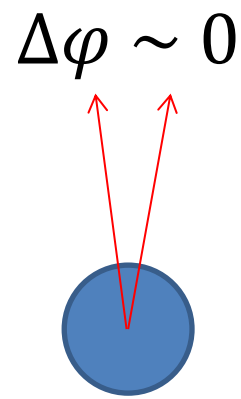
S.A. Voloshin, A.M. Poskanzer, A. Tang, G. Wang, PLB 659 (2008) 537

AB, P. Bozek, L. McLerran, arXiv:1311.7325

What about p+A?

Discovery at the LHC: proton-proton (-lead) collisions at $\sqrt{s} = 7000$ (5020) GeV

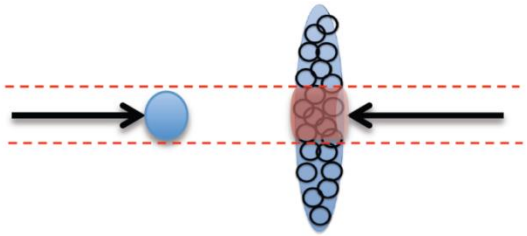


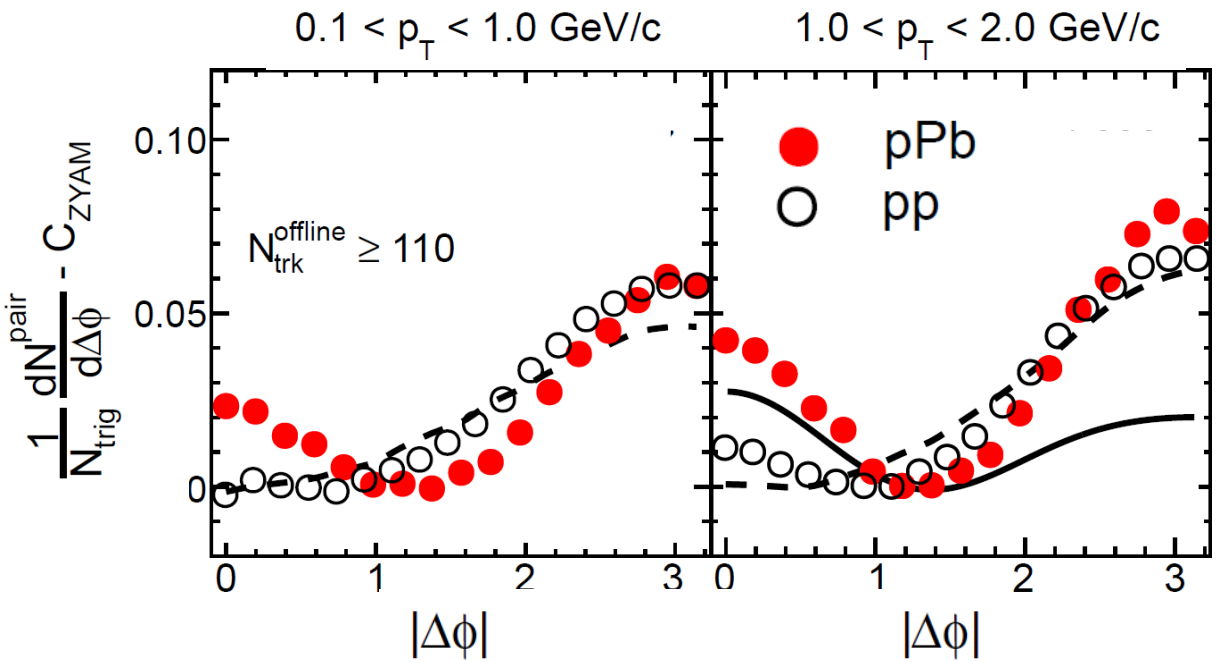


Particles with large rapidity separation prefer to have similar azimuthal angles.

Also back-to-back contribution is present (jets).

Sample of the CMS data:

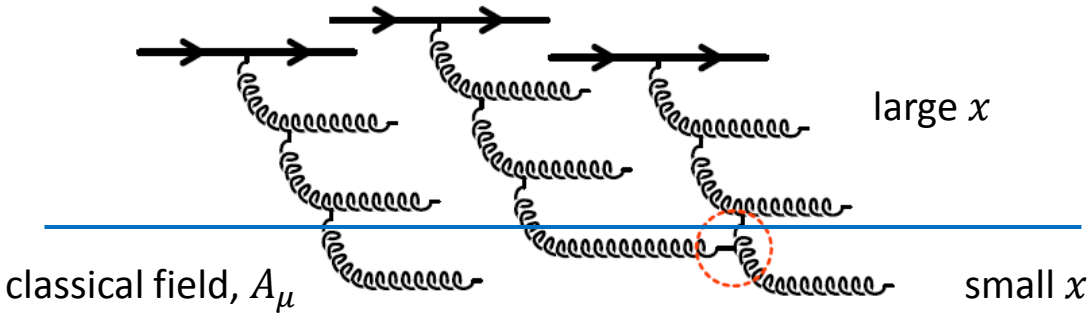


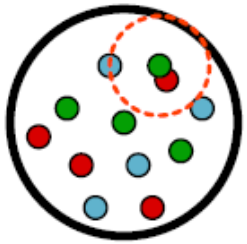


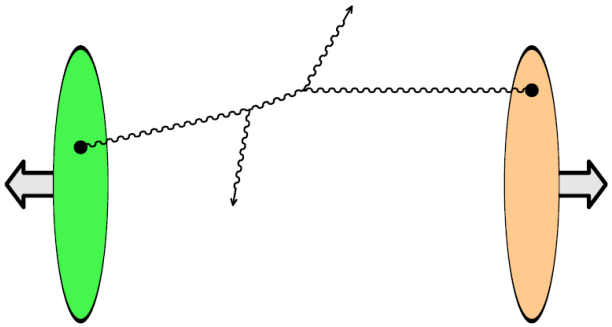
Hydrodynamics ?
Color Glass Condensate ?
CGC + hydro ?
Something else ?

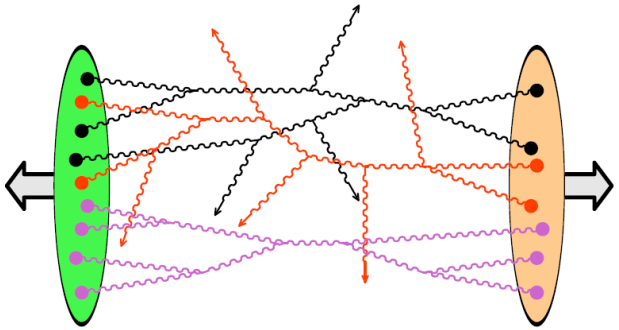
CMS Coll., JHEP 1009 (2010) 091; PLB 718 (2013) 795
ALICE Coll., PLB 719 (2013) 29
ATLAS Coll., PRL 110, 182302 (2013)
PHENIX Coll., PRL 111, 212301 (2013), d+Au at 200 GeV

CGC in a nutshell:





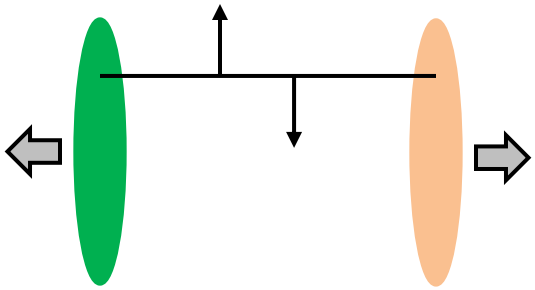




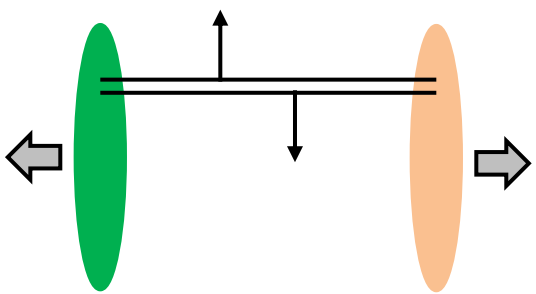
dilute system,
Feynman diagrams

dense system,
multi-parton interactions

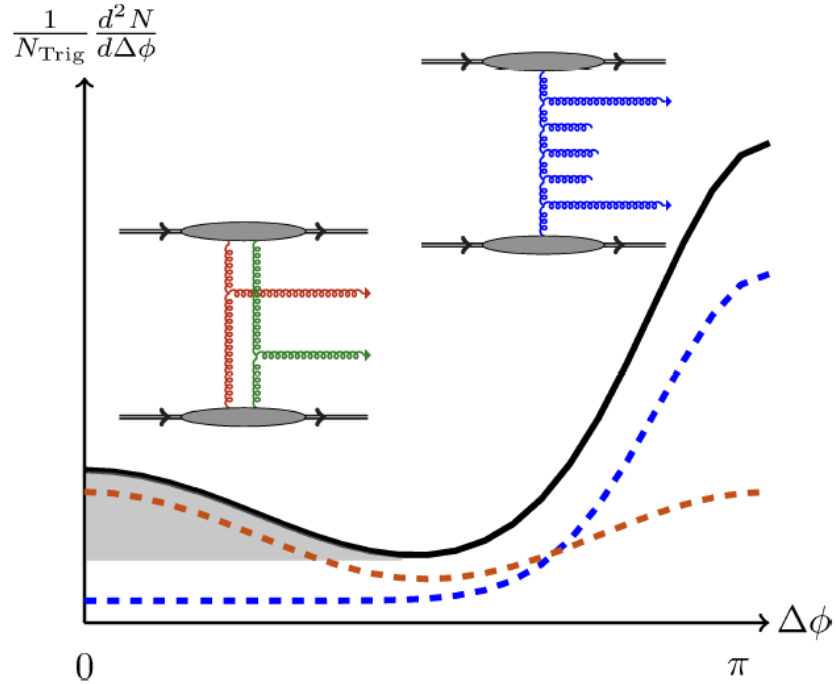
Relevant diagrams:



two back-to-back jets

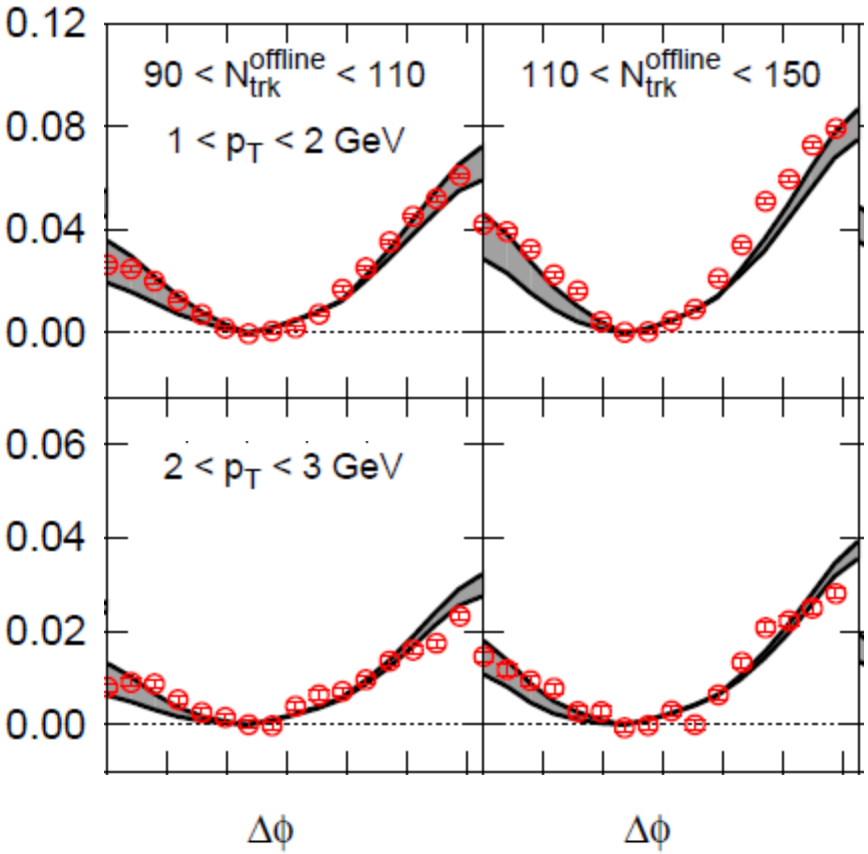


collimation “diagram”

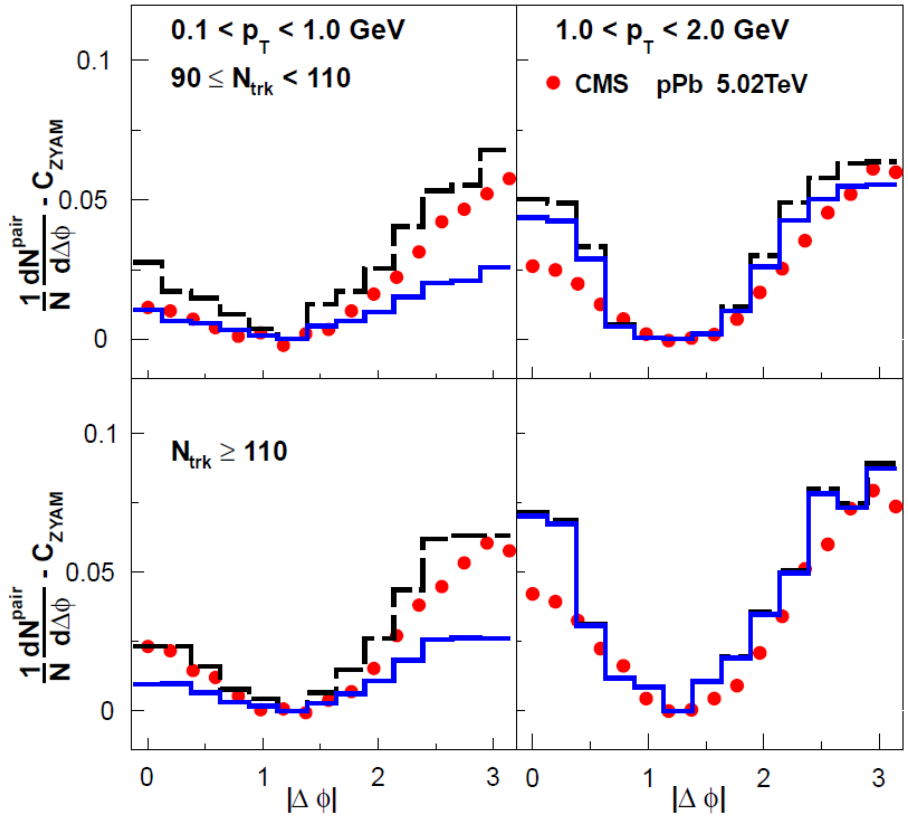


K.Dusling, R.Venugopalan, PRD 87 (2013) 094034; PRL 108 (2012) 262001

CGC



Hydrodynamics

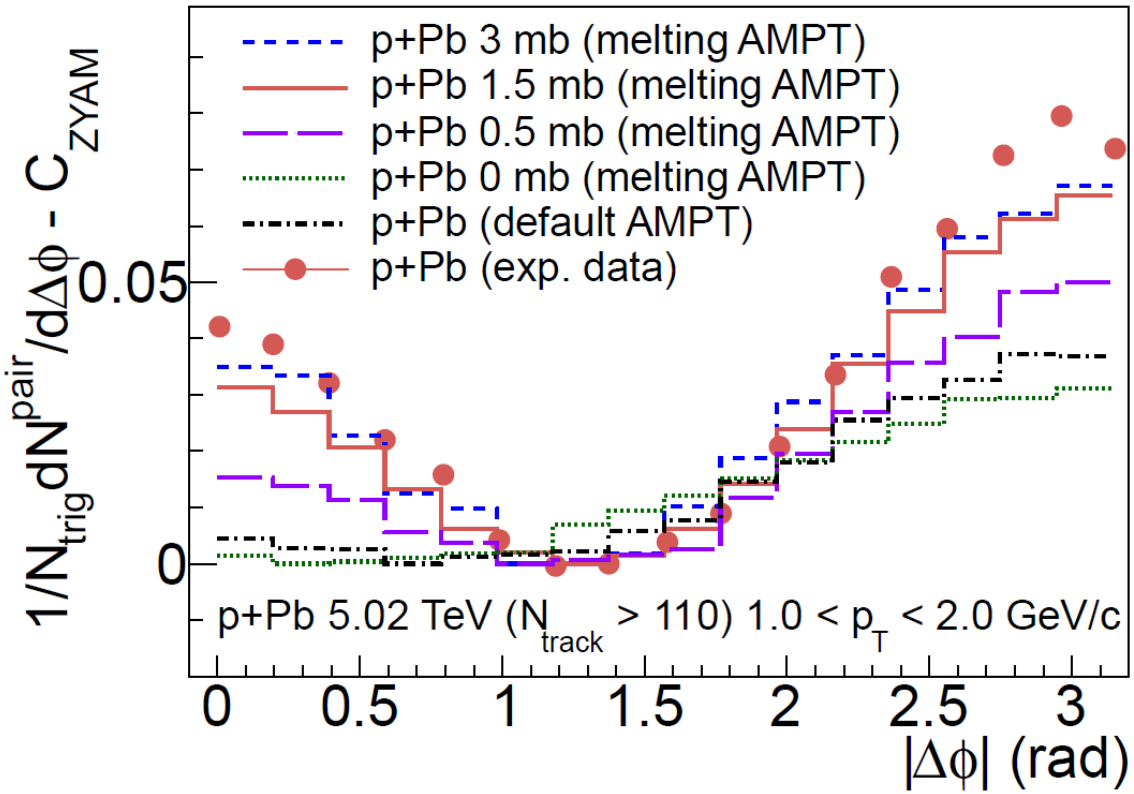


K. Dusling, R. Venugopalan, PRL 108 (2012) 262001, PRD 87 (2013) 094034

P. Bozek, PRC 85 (2012) 014911
P. Bozek, W. Broniowski, PLB 718 (2013) 1557

Parton scatterings (AMPT)

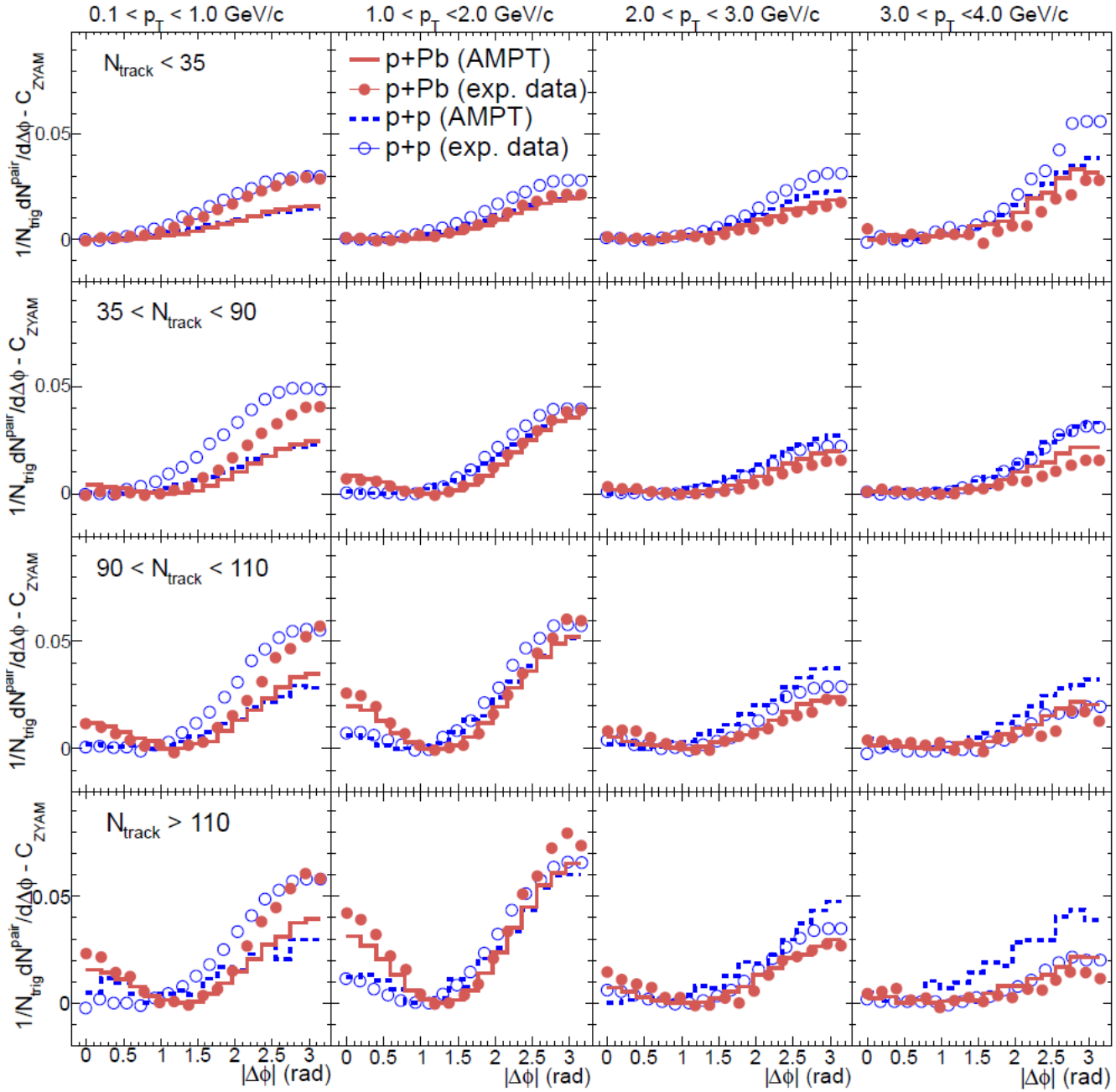
$\sigma = 1.5$ mb



Only elastic 2-to-2 scatterings.

Not clear to what degree a simple partonic picture is OK.

Gu-Liang Ma, AB, arXiv:1404.4129 (today)



Despite many problems, model works very well

Basically only one free parameter $\sigma = 1.5$ mb

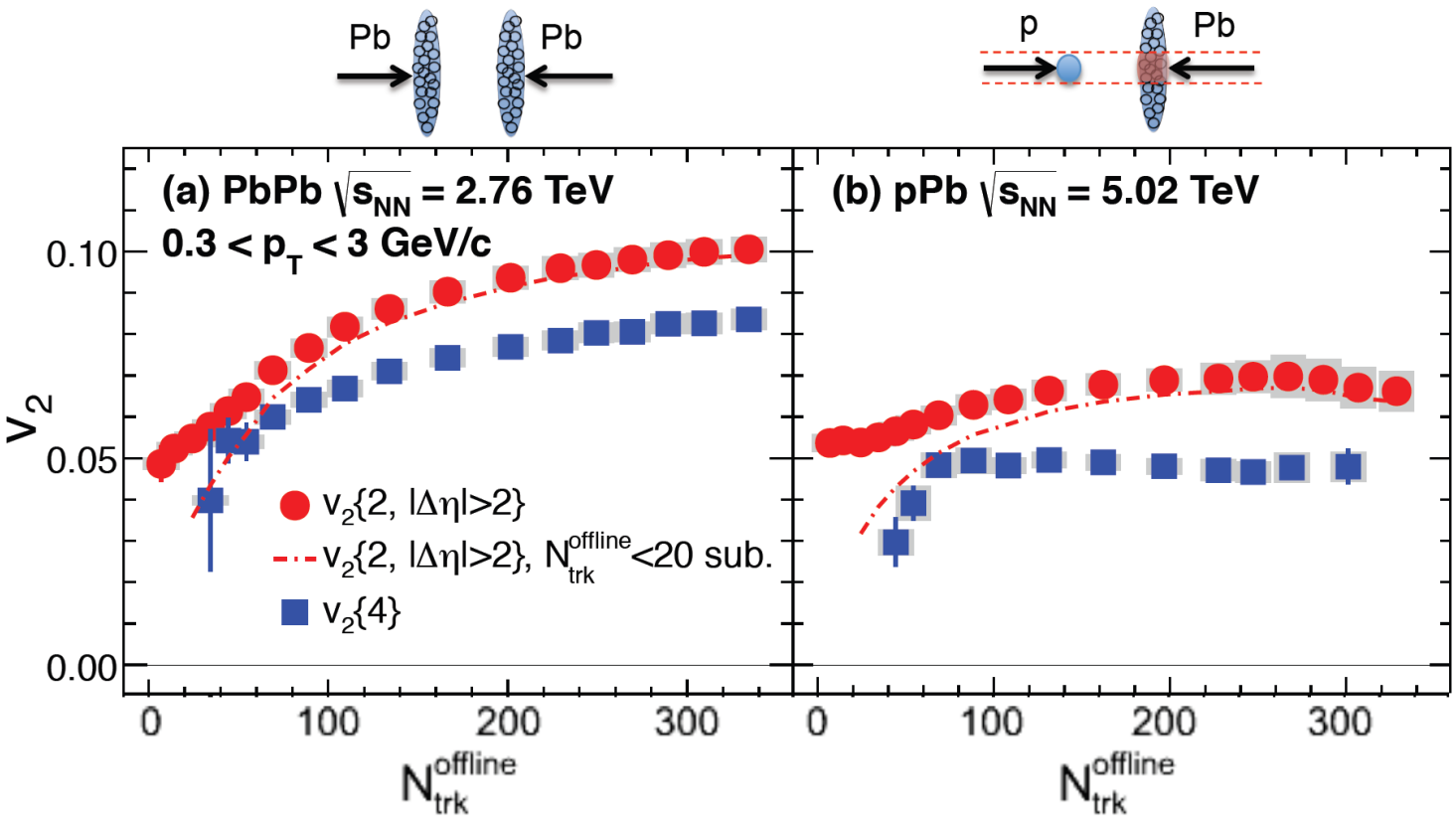
It combines best features of CGC and hydrodynamics

More results soon

Gu-Liang Ma, AB, arXiv:1404.4129 (today)

$v_2\{2\}$ and $v_2\{4\}$ in p+Pb

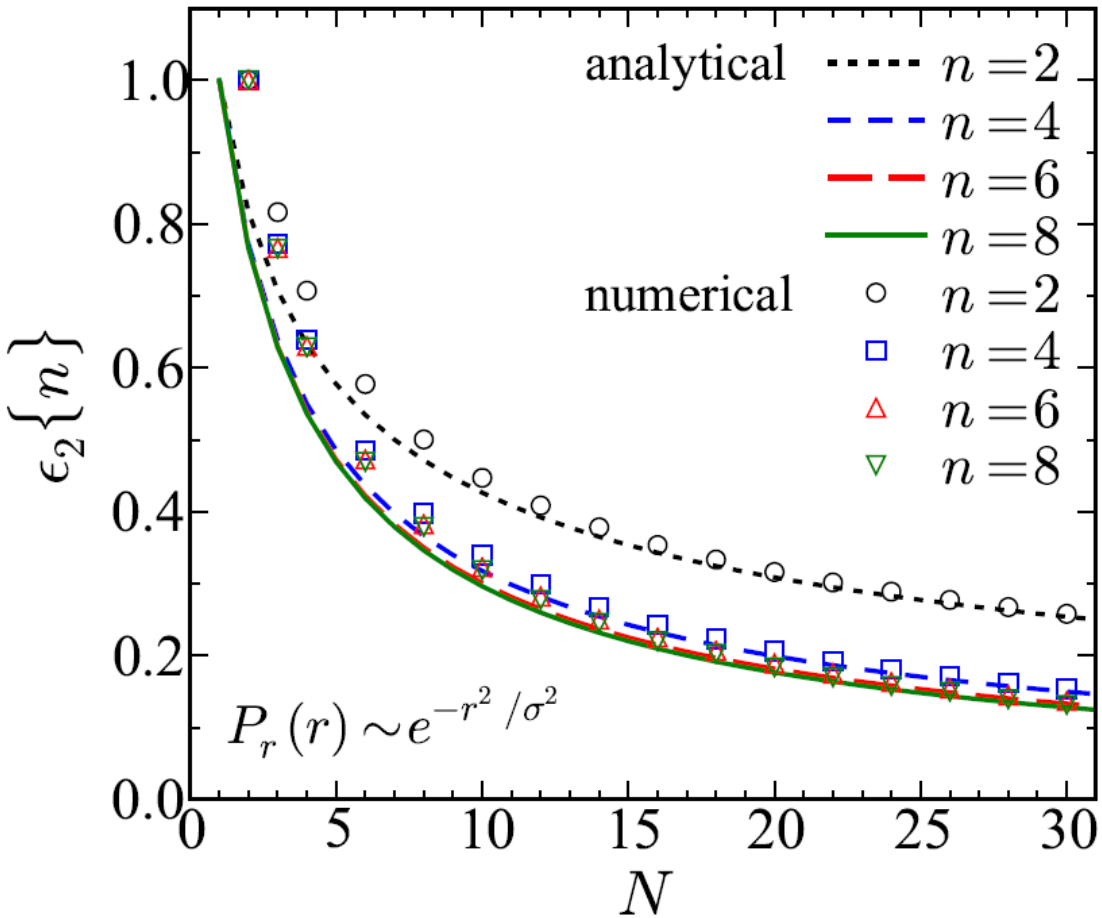
CMS Coll., PLB 724 (2013) 213

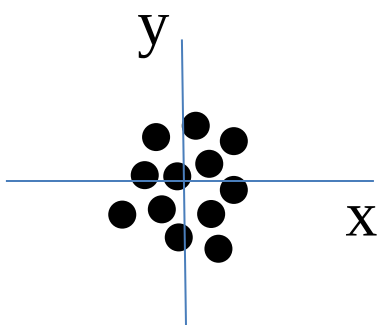


looks like A+A

What is the CGC answer? Is $v_2\{4\}$ positive ?
If not, it could be the decisive test.

Let's start with a random model (p+p or p+A)





$$r = \sqrt{x^2 + y^2}$$

$$\left\langle \epsilon_2^{2n} \right\rangle = \frac{n! \lim\limits_{z\to 0} \frac{d^n}{dz^n} \left\langle I_0\left(2\sqrt{z} r^2\right)\right\rangle^N}{\lim\limits_{z\to 0} \frac{d^{2n}}{dz^{2n}} \left\langle e^{-r^2 z}\right\rangle^N}$$

AB, V. Skokov, arXiv:1312.7349
AB, P. Bozek, L. McLerran, arXiv:1311.7325

In p+A collisions $P(\epsilon_2)$ is best described by power law distribution

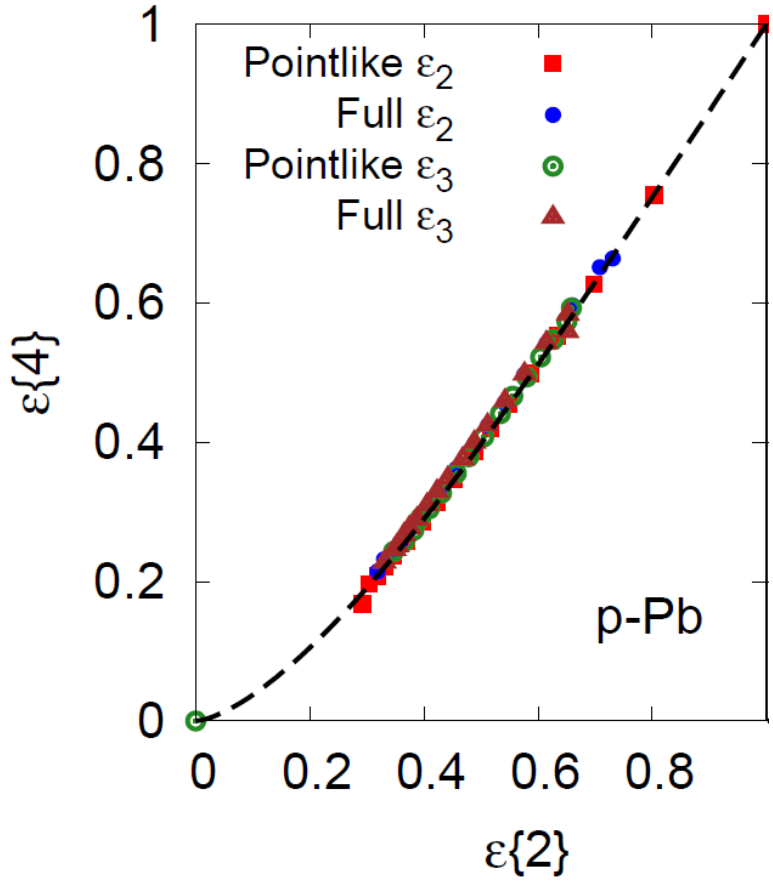
$$P(\epsilon_2) = 2\alpha\epsilon_2(1-\epsilon_2^2)^{\alpha-1}$$

for Gaussian distribution $P_r(r)$:

$$\alpha = (N-1)/2$$

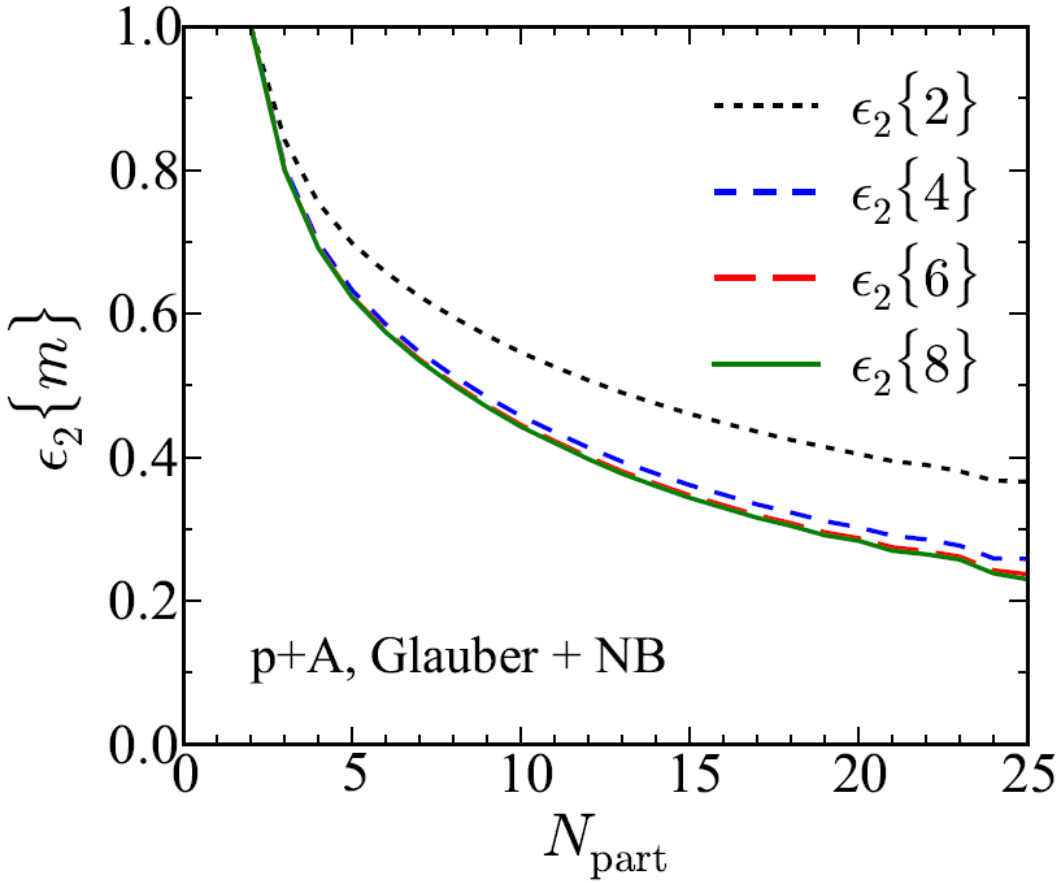
$$\epsilon_2\{4\} = \epsilon_2\{2\}^{3/2}\left(\frac{2}{1+\epsilon_2\{2\}^2}\right)^{1/4}$$

$$\frac{\epsilon_2\{4\}}{\epsilon_2\{2\}} = \left(\frac{2}{2+\alpha}\right)^{1/4}$$



L. Yan, J.-Y. Ollitrault, arXiv:1312.6555

Glauber in p+A with negative binomial distribution



Hydro calculations:

$$v_2\{2\} = 0.082 \pm 0.002$$
$$v_2\{4\} = 0.055 \pm 0.004$$
$$v_2\{6\} = 0.052 \pm 0.005$$

AB, P. Bozek, L. McLerran, arXiv:1311.7325

Comments:

- Perhaps $v_2\{n\}$ can help to understand what happens in small systems. If we see

$$v_2\{2\} > v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$$

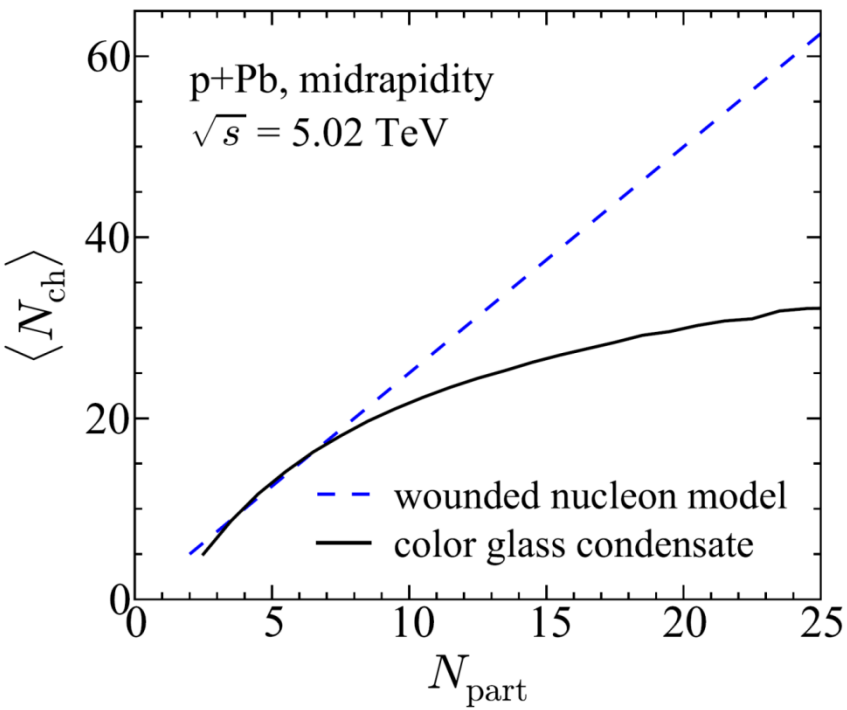
it will suggest importance of initial geometry (hydro?)

- In CGC, $v_2\{4,6,8\}$ are not obvious, even sign is not clear, that is $(v_2\{4\})^4$ could be negative.

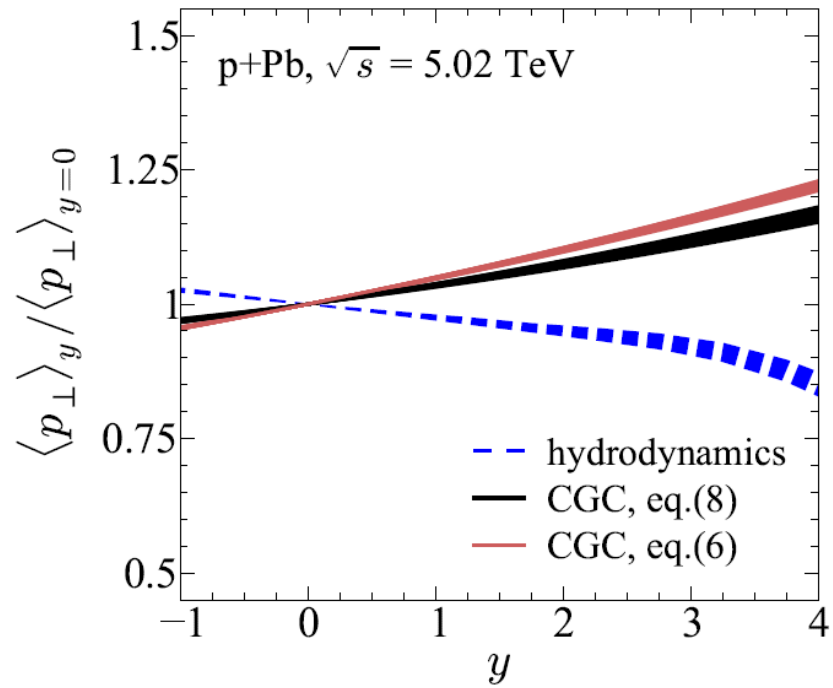
- We need other tests …

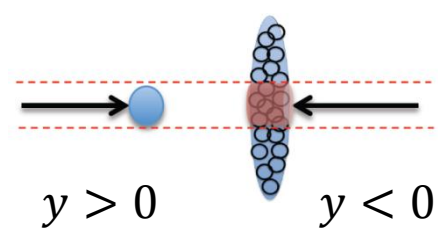
Other tests: (covered by V.Skokov some time ago)

Number of particles at $\eta = 0$ is $\sim \ln(N_{part})$ in CGC



$\langle p_T \rangle$ as a function of rapidity

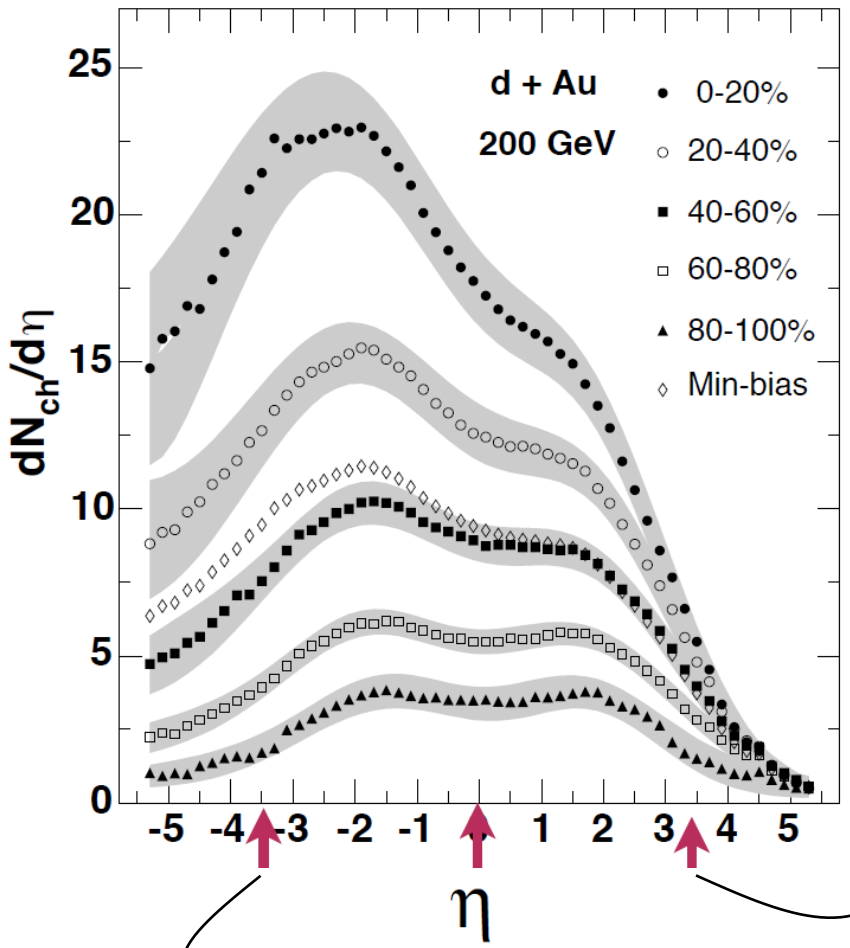


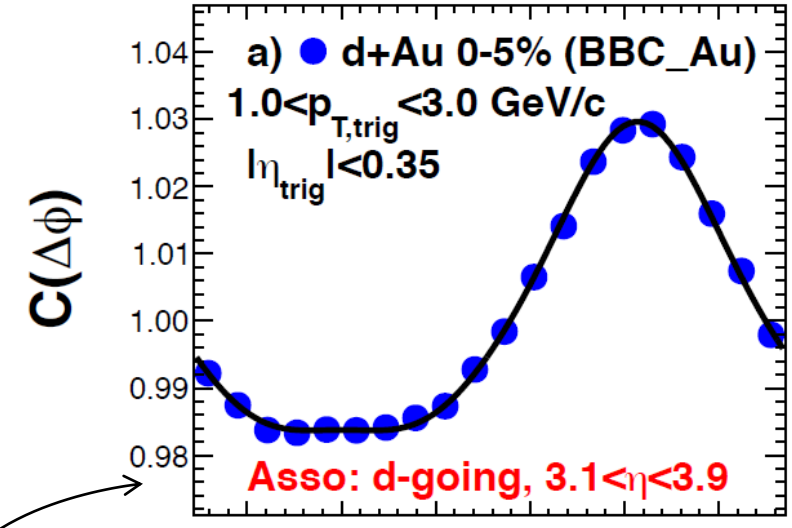


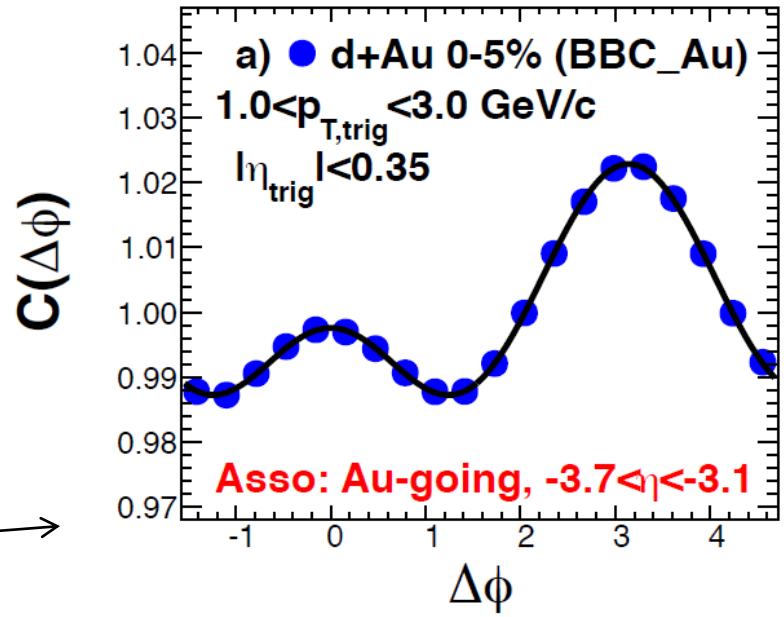
AB, V.Skokov, PRL 111 (2013) 182301
P.Bozek, AB, V.Skokov, PLB 728 (2014) 662

Interesting PHENIX result

PHOBOS, PRC 72 (2005) 031901







To be checked in all models.

Conclusions

- higher order flow cumulants are very sensitive to $P(\epsilon_2)$
- in both A+A and p+A collisions we observe (Glauber)

$$\epsilon_2\{2\} > \epsilon_2\{4\} \approx \epsilon_2\{6\} \approx \epsilon_2\{8\}$$

- hydro evolution can translate it into

$$v_2\{2\} > v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$$

- any physics which translates initial geometry into final anisotropy should have this property
- CGC expectations are not yet clear (sign)