Real-time lattice simulations of fermion fields and Quark production in heavy-ion collisions

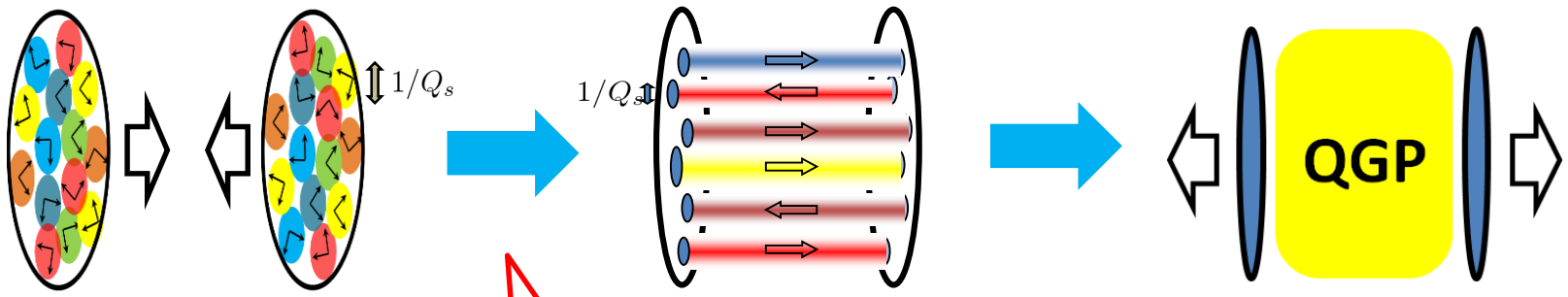
Naoto Tanji

RIKEN/BNL

collaboration with F. Gelis

RIKEN/BNL Lunch Time Talk
06/05/2014

Initial stage of heavy-ion collisions



color glass condensate

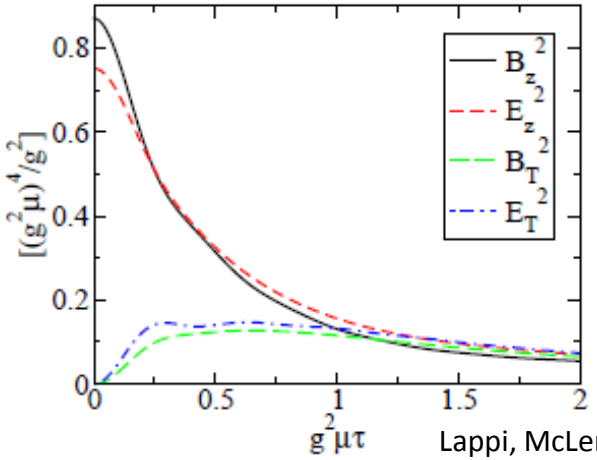
$$\mathbf{E}^a_{\mathrm{T}} \perp \mathbf{B}^a_{\mathrm{T}}$$

Glasma

$$\mathbf{E}^a_{\mathrm{L}} \parallel \mathbf{B}^a_{\mathrm{L}}$$

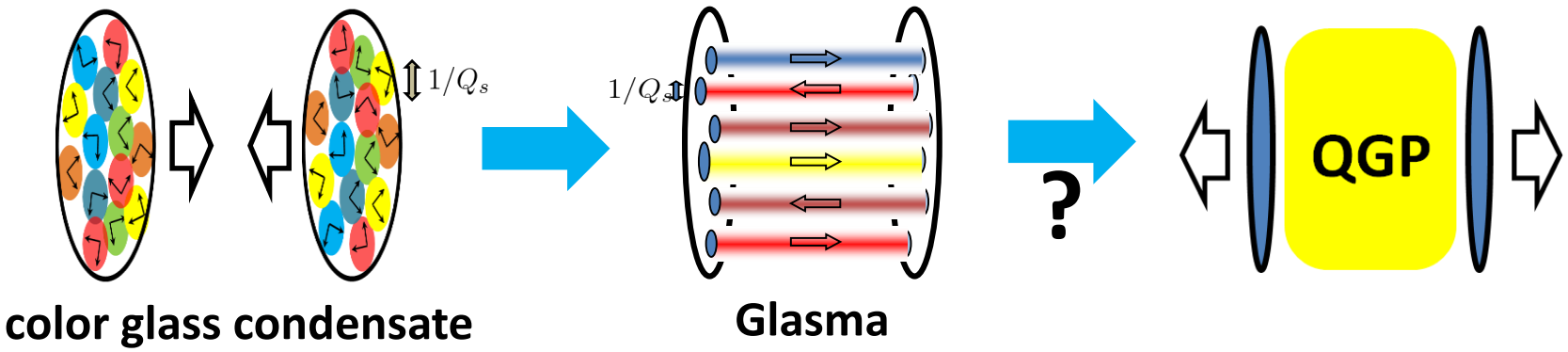
non-Abelian Gauss's laws

$$\nabla \cdot \mathbf{E}^a = -g f^{abc} \mathbf{A}^b \cdot \mathbf{E}^c$$
$$\nabla \cdot \mathbf{B}^a = -g f^{abc} \mathbf{A}^b \cdot \mathbf{B}^c$$



Lappi, McLerran (2006)

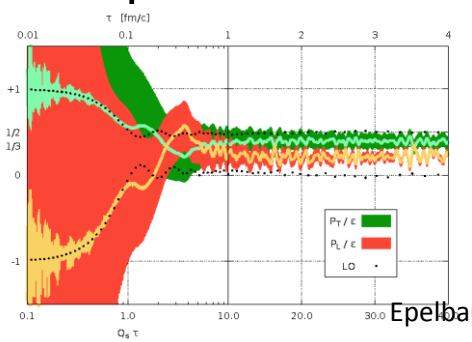
Initial stage of heavy-ion collisions



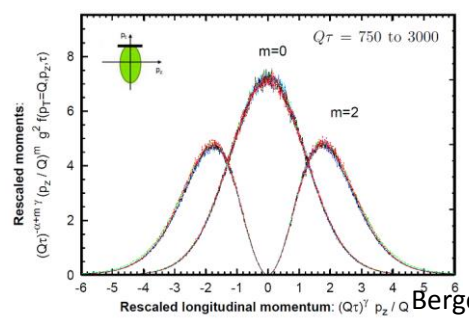
- **How is the system isotropized and thermalized?**

  weak coupling $g \ll 1$

  large occupation number $f \sim 1/g^2$ → classical dynamics



Epelbaum,Gelis (2013)



Berges et. al. (2013)

Classical statistical simulations of gluodynamics

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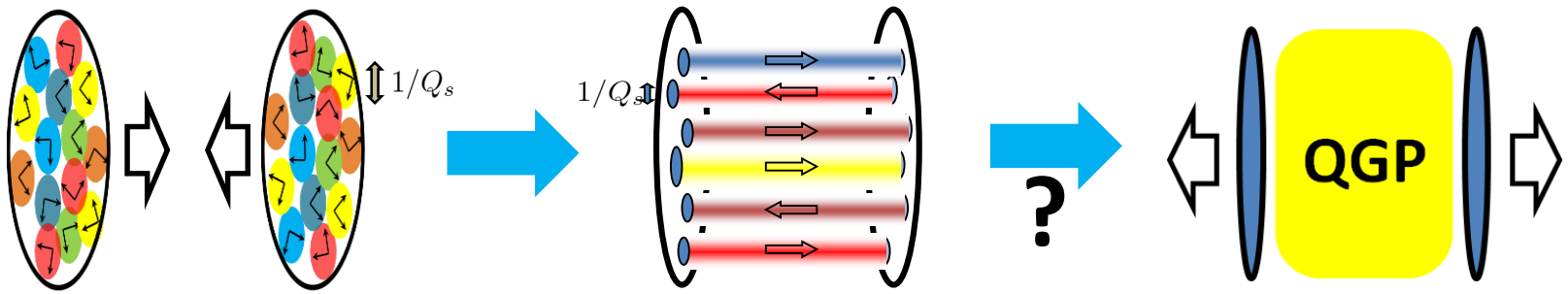

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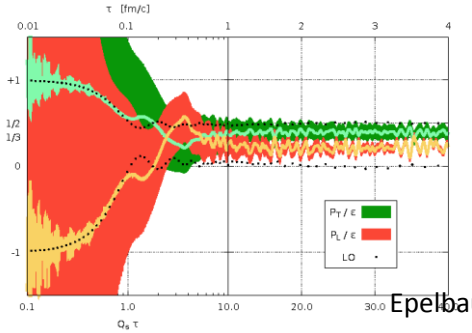

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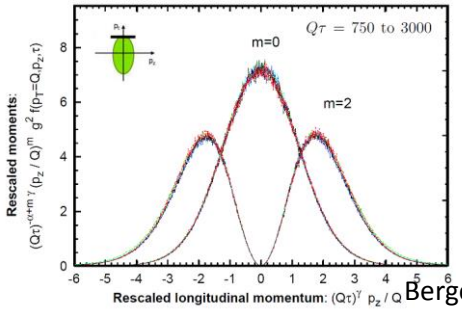


Berges et. al. (2013)

Classical statistical simulations of gluodynamics

How about quark production?

Quark production

CGC and glasma are almost purely gluonic matter.

How does the system reaches chemical equilibrium between light quarks and gluons?

Earlier works by F. Gelis, K. Kajantie and T. Lappi

PRC71, 024904(2005)
PRL96, 032304(2006)

- Limitation from numerical costs
- Treatment of the boost-invariance

There have been theoretical and technical advances on

- classical statistical method for over-occupied bosonic fields
- real-time lattice simulations of fermionic fields
- treatment of a boost-invariant system

Contents

- Derivation of the classical statistical method for over-occupied gluon fields
- Fermions in the classical statistical method
- Monte Carlo method for fermion fields
- Quark production from colliding color glass condensates

Derivation of the classical statistical method

a) Diagrammatically sum up unstable modes. K. Dusling *et al.* Nucl.Phys. A850 (2011) 69

b) From the Schwinger-Keldysh path-integral formalism.

S. Jeon, PRC72, 014907 (2005); arXiv:1308.0263

Compute $\langle 0_{\rm in}|\mathcal{O}|0_{\rm in}\rangle$ → the Schwinger-Keldysh (CTP) formalism

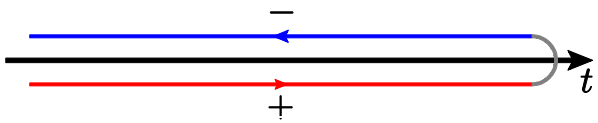
☐ gauge part

- Generating functional

$$Z[J_+,J_-]=\int\left[dA^{i+}dA^{i-}\right]\rho[A^{i+},A^{i-}]\int\mathcal{D}A^+\mathcal{D}A^-e^{i\int d^4x\left[\mathcal{L}_{\rm SK}+J_+A^+-J_-A^-\right]}$$

- SK Lagrangian

$$\mathcal{L}^{\rm SK}_{\rm gauge}=\mathcal{L}_{\rm gauge}[A^+]-\mathcal{L}_{\rm gauge}[A^-]$$



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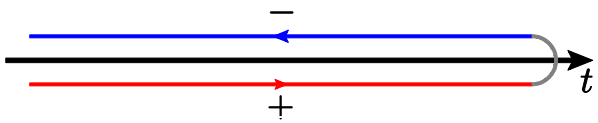

$$\mathcal{L}^{\rm SK}_{\rm gauge}=\mathcal{L}_{\rm gauge}[A^+]-\mathcal{L}_{\rm gauge}[A^-]$$

$$=\left([D_\mu,F^{\mu\nu}]-J^\nu_{\rm ext}\right)^a\eta^a_\nu+\frac{ig}{4}[D_\mu,\eta_\mu]^a[\eta^\mu,\eta^\nu]^a$$

Change of variables

$$A^\mu=\frac{A^\mu_++A^\mu_-}{2}\qquad\eta^\mu=A^\mu_+-A^\mu_-$$

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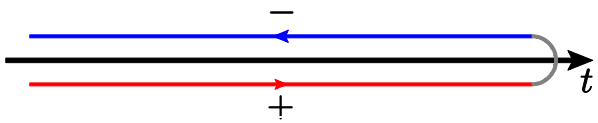

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classical field equation

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Derivation of the classical statistical method

- Classical approximation

Strong field → $A \gg \eta$

→ Neglect higher order terms in $\eta^\mu = A^\mu_+ - A^\mu_-$

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classical field equation

- The path integration over η gives

$$\delta\left([D_\mu, F^{\mu\nu}] - J^\nu_{\rm ext}\right)$$

which constraints the field trajectory to the classical path.

- The integrations over initial fields contain some quantum effects (vacuum fluctuations)

Fermions and the classical statistical method

- quark part

Fermion's occupation number ≤ 1 → Fermions are always quantum.

$$\mathcal{L}_{\text{matter}} = \bar{\psi}\left(i\gamma^{\mu}D_{\mu} - m\right)\psi$$

$$\boxed{D_{\mu} = \partial_{\mu} + igA_{\mu}}$$

Fermions and the classical statistical method

☐ quark part

Fermion's occupation number ≤ 1 ➡ Fermions are always quantum.

$$\mathcal{L}_{\rm matter} = \bar{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi$$

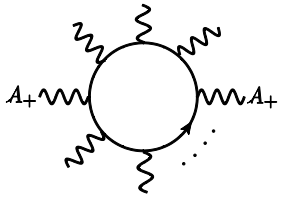
Quadratic fields can be integrated out.

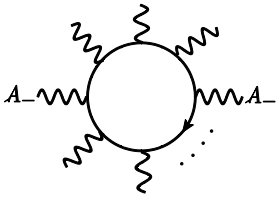
$$D_\mu = \partial_\mu + igA_\mu$$

$$\int \mathcal{D}[\bar{\psi}_+, \bar{\psi}_-, \psi_+, \psi_-] e^{i\int d^4x \mathcal{L}^{SK}_{\rm matter}} = {\rm Det}\left[i(S_{\rm F}^+)^{-1}\right] {\rm Det}\left[i(S_{\rm F*}^-)^{-1}\right]]$$

$S_{\rm F}^+(x,y)$: time-ordered propagator dressed by the gauge field A_+

$S_{\rm F*}^-(x,y)$: anti-time-ordered propagator dressed by the gauge field A_-





- Expand the Dirac determinants w.r.t. η^μ

$$\mathrm{Det}\left[i(S_{\mathrm{F}}^{+})^{-1}\right]\mathrm{Det}\left[i(S_{\mathrm{F}*}^{-})^{-1}\right]$$
$$= \mathrm{Det}\left[i(S_{\mathrm{F}})^{-1}\right]\mathrm{Det}\left[i(S_{\mathrm{F}*})^{-1}\right] e^{-i\frac{g}{2}\mathrm{Tr}[(S_{\mathrm{F}}+S_{\mathrm{F}*})\gamma_\mu]\eta^\mu+\mathcal{O}(\eta^2)}$$

Propagator dressed by the averaged gauge field A^μ

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Propagator dressed by the averaged gauge field A^μ

- Integrate over η^μ

Classical Yang-Mills equation which couples to the quark current

$$\boxed{[D_\mu, F^{\mu\nu}] = J^\nu_{\mathrm{ext}} + J^\nu_{\mathrm{quark}}}$$

$$J^\nu_{\mathrm{quark}}(x) = i\frac{g}{2}\mathrm{Tr}[(S_{\mathrm{F}} + S_{\mathrm{F}*})\gamma^\nu]$$

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$$J^\nu_{\mathrm{quark}}(x) = i\frac{g}{2}\mathrm{Tr}[(S_{\mathrm{F}} + S_{\mathrm{F}*})\gamma^\nu]$$
$$= \frac{g}{2}\langle\Omega_{\mathrm{in}}|\left[\hat{\bar{\psi}}(x), \gamma^\nu\hat{\psi}(x)\right]|\Omega_{\mathrm{in}}\rangle$$

$\hat{\psi}(x)$ is a field operator satisfying the Dirac equation under the gauge field:

$$\boxed{[i\gamma^\mu(\partial_\mu + igA_\mu) - m]\,\hat{\psi}(x) = 0}$$

1. **Generate an ensemble of the initial gauge fields according to the Wigner function**

$$A^\mu(t_0, \mathbf{x}) = A^\mu_{\text{coherent}}(t_0, \mathbf{x}) + \sum_n \left[a^\mu_n(t_0, \mathbf{x}) c_n + a^{\mu\,*}_n(t_0, \mathbf{x}) c^*_n\right]$$

Gaussian random number $\langle c_n c^*_{n'} \rangle_{\text{ens}} = \frac{1}{2}\delta_{n,n'}$

2. **For each gauge configuration, solve the CYM equation and the Dirac equation as associated equations**

$$[D_\mu, F^{\mu\nu}] = J^\nu_{\text{ext}} + J^\nu_{\text{quark}}$$

$$[i\gamma^\mu(\partial_\mu + igA_\mu) - m]\,\hat{\psi}(x) = 0$$

3. **Take the ensemble average and the expectation values**

$$\langle \mathcal{O}(A) \rangle_{\text{ens}} \qquad \langle \Omega_{\text{in}} | \hat{\mathcal{O}}(\psi) | \Omega_{\text{in}} \rangle$$

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pure classical approximation for gauge fields as a first step

2. **For each gauge configuration, solve the CYM equation and the Dirac equation as associated equations**

$$[D_\mu, F^{\mu\nu}] = J^\nu_{\text{ext}} + \cancel{J^\nu_{\text{quark}}}$$

Neglecting the back-reaction

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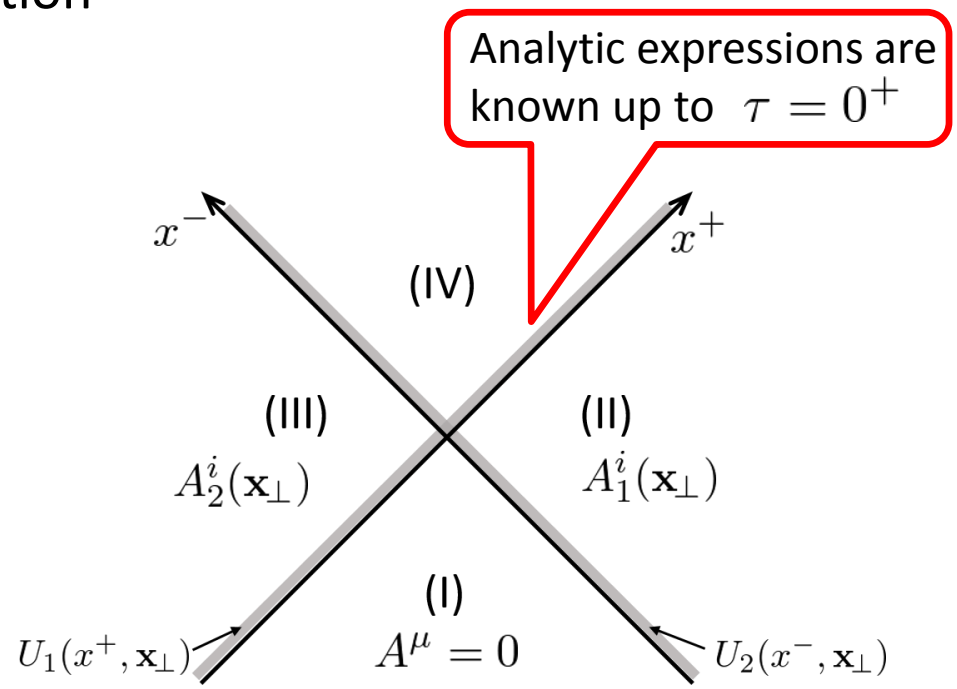
Setup

☐ gauge fields

- SU(2)
- boost-invariant classical fields without fluctuations
- generated by colliding CGCs with the McLerran-Venugopalan model
- in the forward light cone, solve the classical YM eq. numerically
- no back reaction



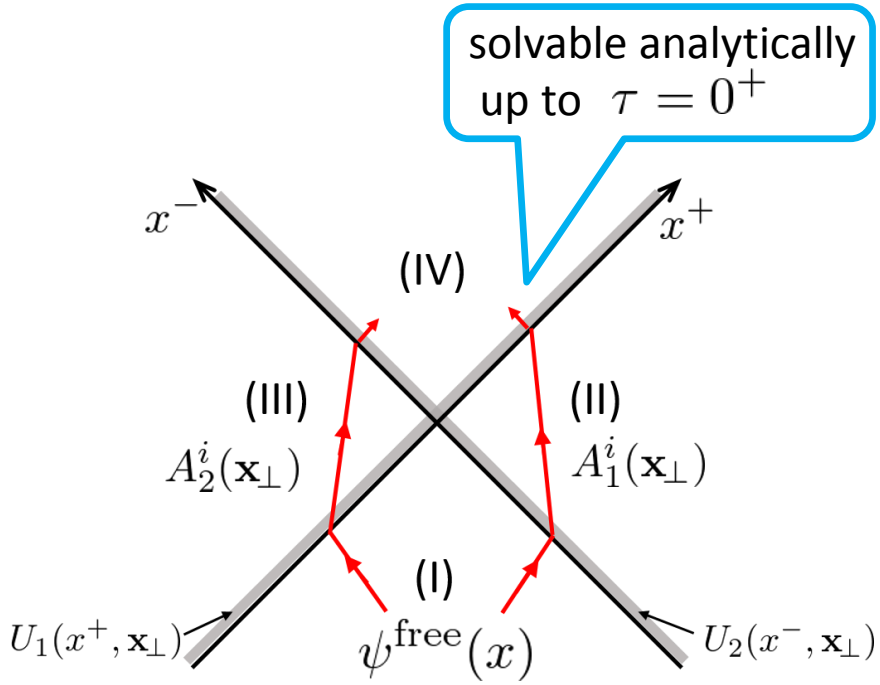
Setup

☐ quark fields

- obey the Dirac equation

$$[i\gamma^\mu(\partial_\mu + igA_\mu) - m]\,\hat{\psi}(x) = 0$$

- free fields in the region (I)
- in the forward light cone, solve the Dirac eq. numerically



Computing fermions on lattice

- Mode functions approach Aarts, Smit 1998

$$[i\gamma^\mu(\partial_\mu + igA_\mu) - m]\,\hat{\psi}(x) = 0$$

linear in the Dirac field

$$\hat{\psi}(x) = \sum_s \int d^3p\,[\psi^+_{\mathbf{p},s}(x)a_{\mathbf{p},s} + \psi^-_{\mathbf{p},s}(x)b^\dagger_{\mathbf{p},s}]$$

$\psi^\pm_{\mathbf{p},s}(x)$: mode functions, c-number solutions of the Dirac eq.

$$[i\gamma^\mu(\partial_\mu + igA_\mu) - m]\,\psi^\pm_{\mathbf{p},s}(x) = 0 \qquad \lim_{t\to-\infty}\psi^\pm_{\mathbf{p},s}(x) = \psi^{\mathrm{free},\pm}_{\mathbf{p},s}(x)$$

$$J^\mu_{\mathrm{quark}} = \frac{g}{2}\langle 0|\left[\hat{\bar{\psi}}(x), \gamma^\mu\hat{\psi}(x)\right]|0\rangle$$

$$= -\frac{g}{2}\sum_s \int d^3p\,\{\bar{\psi}^+_{\mathbf{p},s}(x)\gamma^\mu\psi^+_{\mathbf{p},s}(x) - \bar{\psi}^-_{\mathbf{p},s}(x)\gamma^\mu\psi^-_{\mathbf{p},s}(x)\}$$

With the c-number mode functions, expectation values can be computed.

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Numerical cost

$$N_t \times N^2_{\mathrm{latt}}$$

expensive in 3+1dim.

With the c-number mode functions, expectation values can be computed.

Monte Carlo method with male and female stochastic fields

Borsanyi, Hindmarsh 2009

- Male and female fields
  - initial condition

$$\psi_{\mathrm{M}}(t_0, \mathbf{x}) = \sum_s \int d^3p \left[\psi^+_{\mathbf{p},s}(t_0, \mathbf{x}) c_{\mathbf{p},s} + \psi^-_{\mathbf{p},s}(t_0, \mathbf{x}) d_{\mathbf{p},s}\right]$$

$$\psi_{\mathrm{F}}(t_0, \mathbf{x}) = \sum_s \int d^3p \left[\psi^+_{\mathbf{p},s}(t_0, \mathbf{x}) c_{\mathbf{p},s} - \psi^-_{\mathbf{p},s}(t_0, \mathbf{x}) d_{\mathbf{p},s}\right]$$

  - evolution equation

$$\left[i\gamma^\mu(\partial_\mu + igA_\mu) - m\right]\psi_{\mathrm{M,F}}(x) = 0$$

- Gaussian random numbers

$$\langle c_{\mathbf{p},s} c^*_{\mathbf{p}',s'}\rangle = \langle d_{\mathbf{p},s} d^*_{\mathbf{p}',s'}\rangle = \frac{1}{2}\delta_{s,s'}\delta^3(\mathbf{p} - \mathbf{p}')$$

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The expectation value can be reproduced by male-female ensemble average.

$$-g\langle\bar\psi_{\mathrm{M}}(x)\gamma^\mu\psi_{\mathrm{F}}\rangle = -\frac{g}{2}\sum_s\int d^3p\left\{\bar\psi^+_{\mathbf{p},s}(x)\gamma^\mu\psi^+_{\mathbf{p},s}(x) - \bar\psi^-_{\mathbf{p},s}(x)\gamma^\mu\psi^-_{\mathbf{p},s}(x)\right\}$$

$$= J^\mu_{\mathrm{quark}}$$

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$$= J^\mu_{\mathrm{quark}}$$

The male-female combination is necessary to get this minus sign which originates in anti-commutativity.

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Numerical cost

$$N_{\mathrm{config}} \times N_{\mathrm{latt}}^2$$

  - evolution equation

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The expectation value can be reproduced by male-female ensemble average.

$$-g\langle\bar\psi_{\mathrm{M}}($$

$$\psi^-_{\mathbf{p},s}(x)\}$$

Total cost

$$N_{\mathrm{config}} \times N_{\mathrm{latt}} \times (N_{\mathrm{latt}} + N_t) \ll N_t \times N_{\mathrm{latt}}^2$$

if $N_{\mathrm{config}} \ll N_{\mathrm{latt}}$ and $N_{\mathrm{config}} \ll N_t$

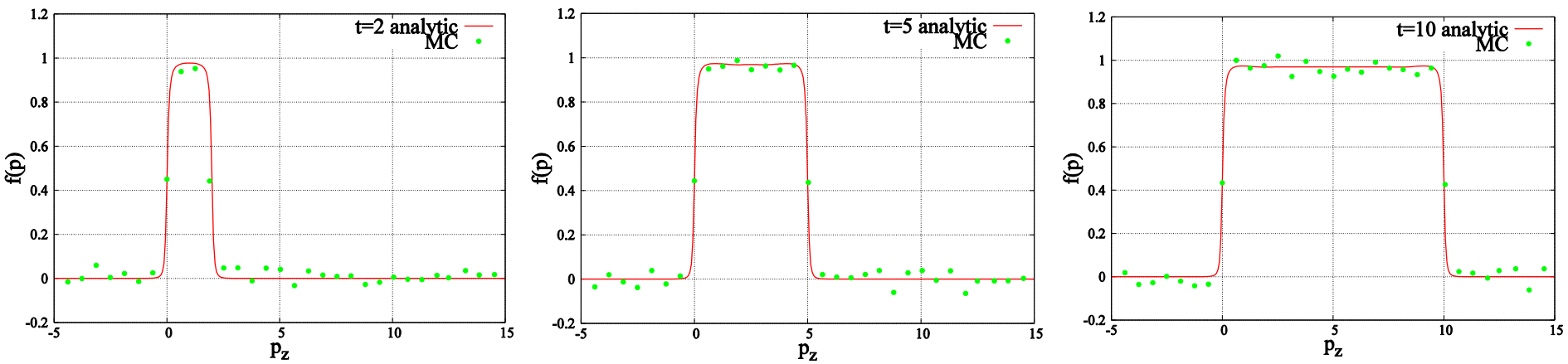
necessary to get this minus sign which originates in anti-commutativity.

Benchmark --- QED uniform and constant electric field

Schwinger mechanism particle pair production

$$m = 0.1$$
$$N_x = N_y = 32$$
$$N_z = 128$$
$$N_{\text{config}} = 256$$
$$L_x = L_y = L_z = 10.$$

scaled by eE



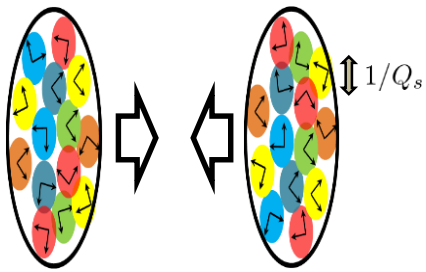
Comparison between the analytic and MC results

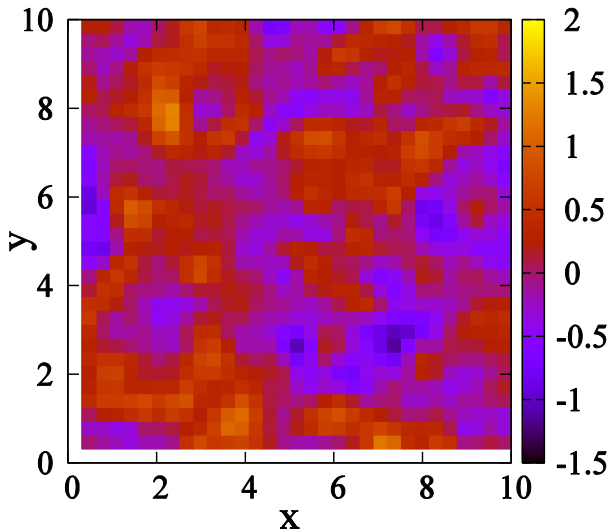
The MC method well reproduces the analytic results.

McLerran-Venugopalan initial condition

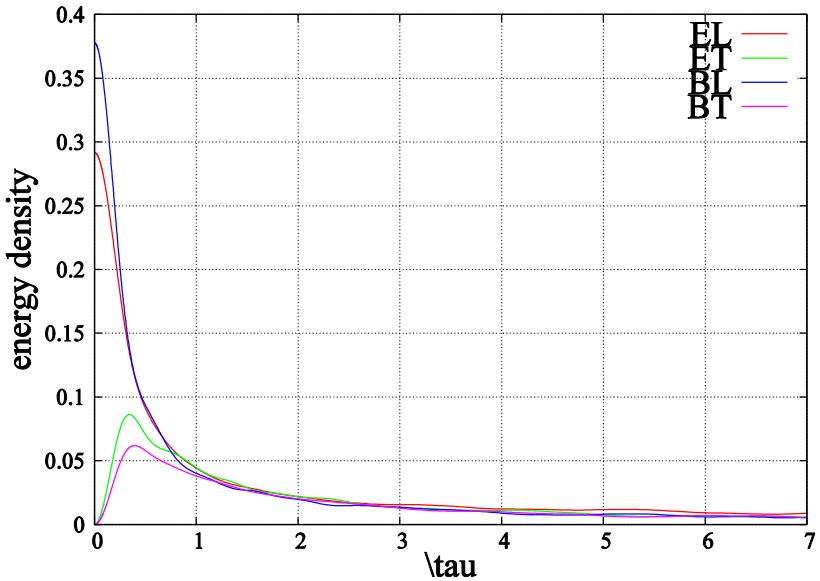
Random color sources

$$\langle \rho^{(n)}(x_\perp)\rho^{(m)}(x'_\perp)\rangle = g^2\mu^2\delta^{nm}\delta(x_\perp - x'_\perp)$$





Initial electric field configuration



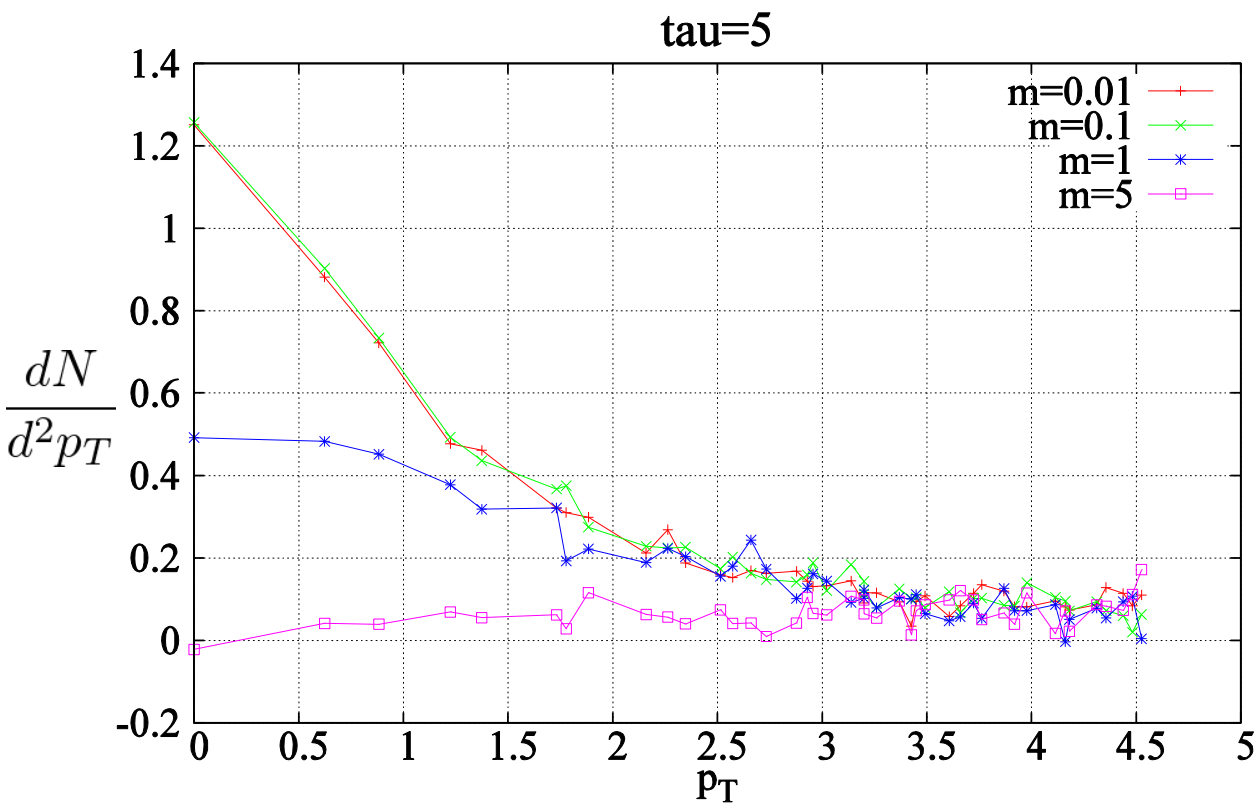
The time-evolution of energy density

All dimensionful quantities are scaled by $g^2\mu$.

Transverse spectra of produced quarks

$$N_x = N_y = 32$$
$$N_\eta = 128$$
$$N_{\text{config}} = 256$$
$$L_x = L_y = 10.$$
$$L_\eta = 20.$$

scaled by $g^2\mu$

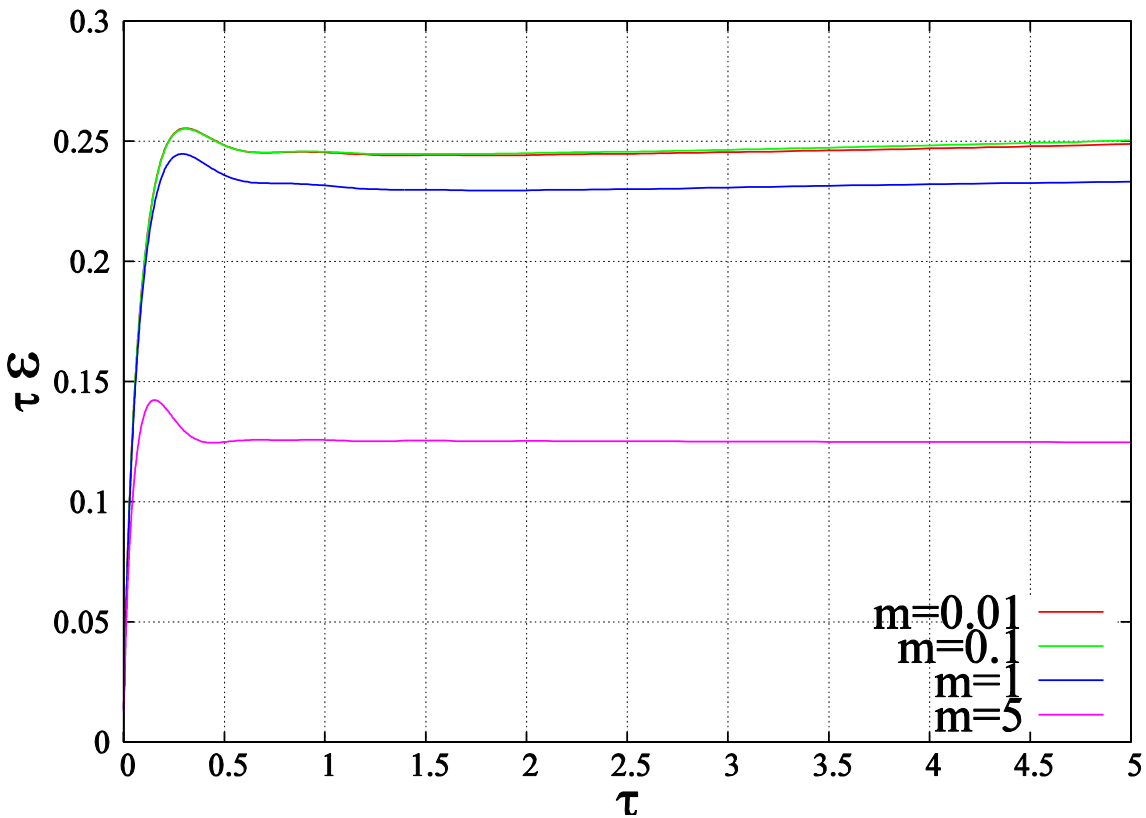


mass dependence of the transverse spectra

Energy density

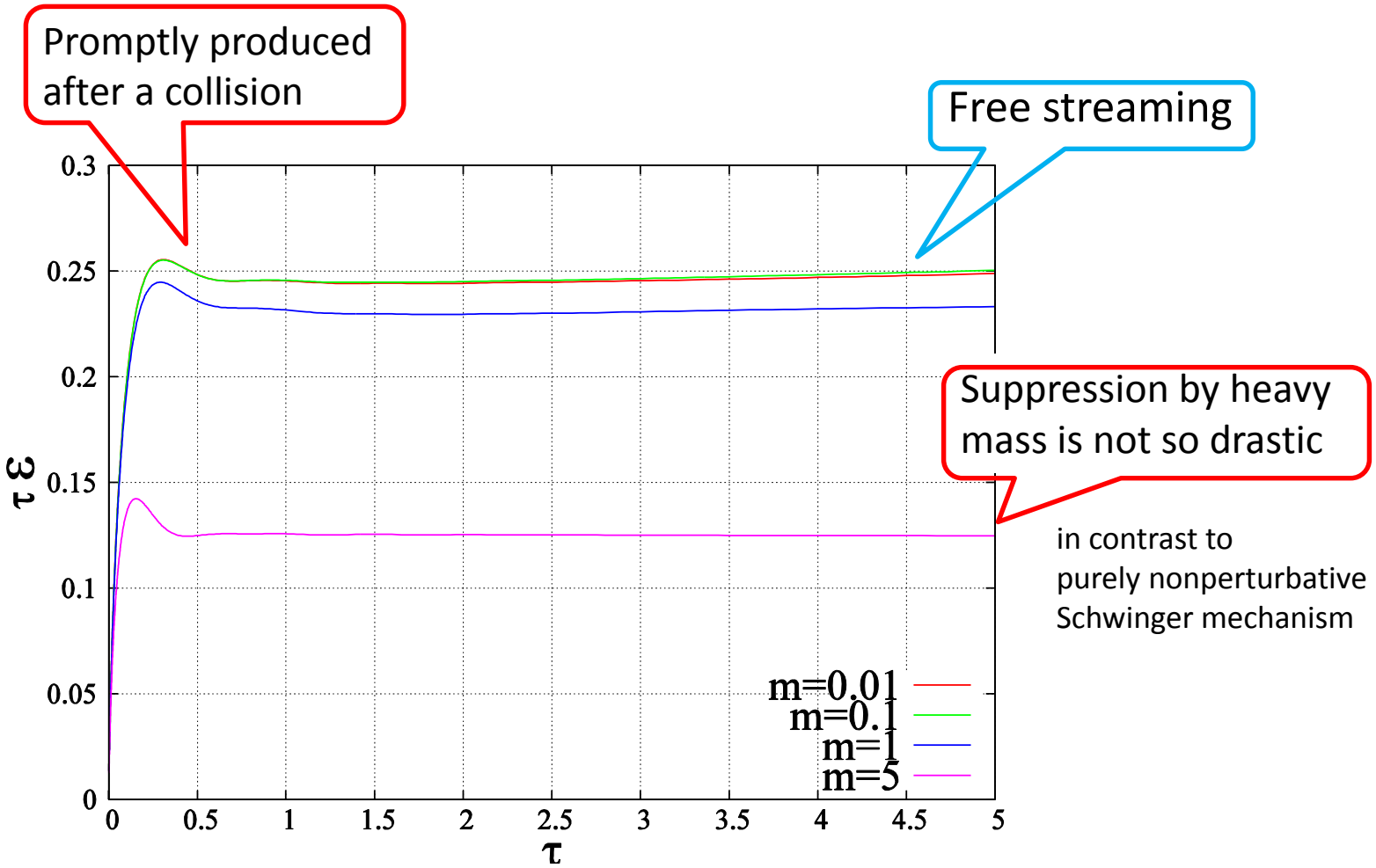
$$N_x = N_y = 32$$
$$N_\eta = 128$$
$$N_{\text{config}} = 256$$
$$L_x = L_y = 10.$$
$$L_\eta = 20.$$

scaled by $g^2\mu$



time evolution of $\tau\mathcal{E}$

Energy density



time evolution of $\tau\mathcal{E}$

Summary and outlook

- Fermion dynamics can be embedded in classical statistical simulations for over-occupied gluon fields.
- The MC method enables us to simulate fermion dynamics on lattice under inhomogeneous classical gauge fields.
- The quark production in expanding gauge fields with the MV initial condition is computed.
- Quarks are promptly produced after a collision of color glass condensates.

  - ✓ Gauge field fluctuations, instabilities
  - ✓ Real-time simulations of the chiral magnetic effects