Effective Theories with Heavy Quarks on the Lattice

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Motivation

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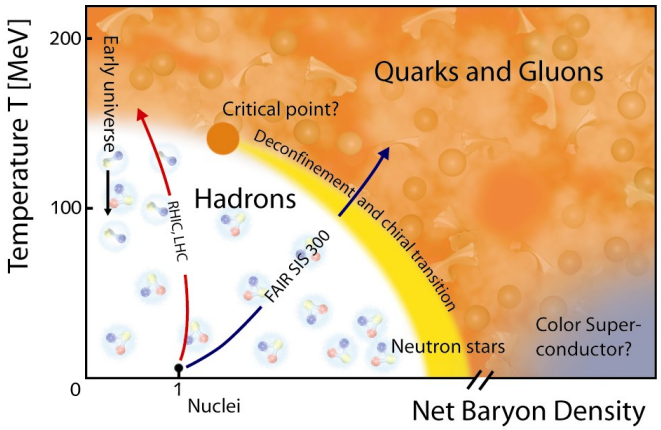
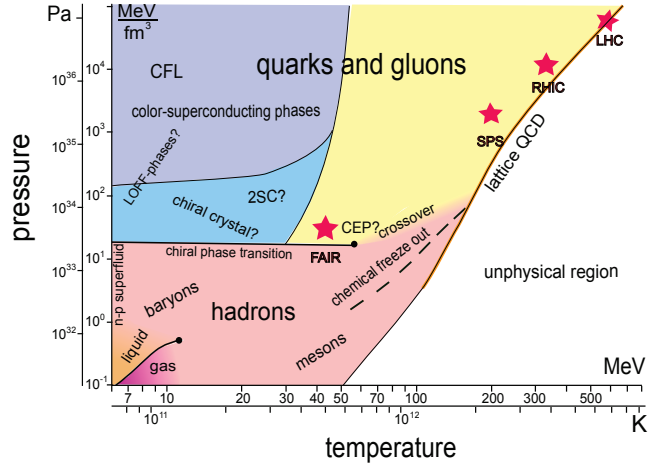
- Exploration of the QCD phase diagram at finite μ
- On the Lattice: sign problem!
- Use functional and effective models, e.g. PQM, PNJL
- Here: Effective Polyakov loop model on the lattice
- Detour: Polyakov loop model for two-color QCD





Wambach, Heckmann, Buballa [1111.5475]

Effective Polyakov Loop Theories

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- 3d SU(N) spin models sharing universal behavior at deconfinement transition with underlying gauge theory

  B.Svetitsky and L.G.Yaffe, Nucl. Phys. B210 (1982) 423

$$L = \mathrm{Tr} \prod_{t=1}^{N_\tau} U_o(\vec{x}, t)$$

- Less computational cost, especially with dynamical fermions
- SU(3) at finite density: Circumvent sign problem by using Worm algorithm or complex Langevin simulations

Different Aproaches

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- Wilson's gauge action and an effective Polyakov loop model for fermion action

  Seiler, Sexty, Stamatescu, Phys.Lett. B723 (2013); Sexty, QM 2014

- Find effective action by inverse Monte-Carlo methods

  Heinzl et al., Phys. Rev. D. 72 (2005)

- Find effective action by relative weights method

  Greensite, Langfeld, Phys. Rev. D. 87 (2013)

- Use combined strong coupling and hopping expansion

  Fromm, Langelage, Lottini, Philipsen, JHEP 1201 (2012)

Effective Action

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most general form:

$$S_{\text{eff}} = \sum_{ij} L_i \, K^{(2)}(i,j) \, L_j + \sum_{ijkl} L_i L_j \, K^{(4)}(i,j,k,l) \, L_k L_l + \ldots + \sum_i h^{(1)} \, L_i + \ldots$$

can also contain loops in adjoint or higher representations.

B. Svetitsky, Phys. Rept. 132 (1986), Heinzl et al., Phys. Rev. D. 72 (2005)

Find a way to calculate Kernels $K^{(2n)}$ and effective fermions couplings $h^{(n)}$:

- e.g. by using the Inverse Monte-Carlo method

$$\exp(-S_{\text{eff}}[L]) = \int \mathcal{D}U \; \delta(L - L[U]) \; \exp(-S[U])$$

- using combined strong coupling and hopping expansion

Strong Coupling Expansion

Yang-Mills Effective Action

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- Integrate out spatial degrees of freedom

$$-S_{\text{eff}} = \ln \int [dU_i] \, \exp\left[\frac{\beta}{2N_c} \sum_p \text{Tr}\, U_P + \text{Tr}\, U_p^\dagger\right] = \lambda_1 S_1 + \lambda_2 S_2 + \ldots$$

- expansion of $\exp[-S_p]$ in irreducible characters $\chi_r(U) = \text{Tr}\, D_r(U)$

$$-S_{\text{eff}} = \ln \int [dU_i] \prod_p \left[1 + \sum_{j \neq 0} d_j a_j(\beta) \chi_j(U_p)\right]$$

- higher dimensional representations go with higher orders in β

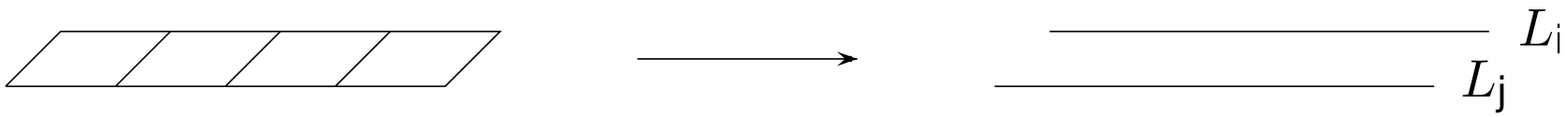
Strong Coupling Expansion

Yang-Mills Effective Action

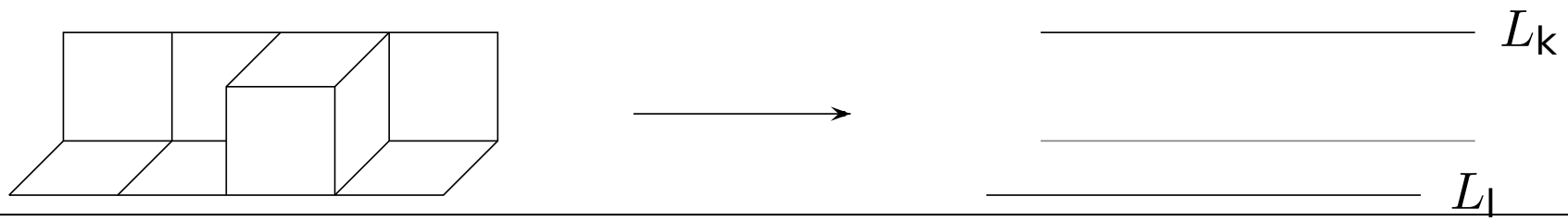
- Use cluster expansion to count all contributions from all graphs
- Integration rule:

$$\int dU\, \chi_r(V_1 U)\chi_{r'}(V_2 U^\dagger) = \delta_{rr'} \frac{1}{d_r} \chi_r(V_1 V_2)$$

Leading order: $\lambda_1 S_1 = \sum_{\langle ij \rangle} u^{N_\tau} L_i L_j$



Next-to-leading order: $\lambda_2 S_2 = N_\tau (N_\tau - 1) \sum_{[kl]} u^{2N_\tau + 2} L_k L_l$



Strong Coupling Expansion

Yang-Mills Effective Action

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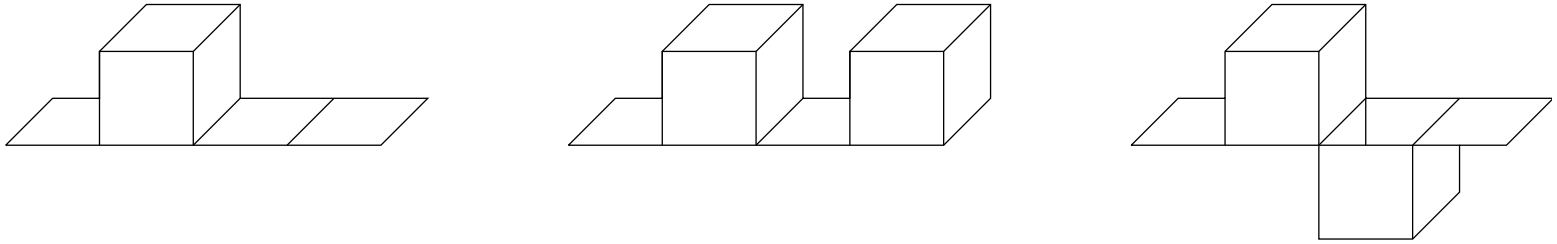
- Other graphs yield corrections to effective couplings:



$$\lambda_1(u, N_\tau \geq 5) = u^{N_\tau} \exp\left[N_\tau \left(4u^4 - 4u^6 + \frac{140}{3}u^8 - \frac{36044}{405}u^{10}\right)\right]$$

- Resum graphs winding around the Lattice multiple times

$$\sum_{<ij>} \left(\lambda_1 L_i L_j - \frac{1}{2}(\lambda_1 L_i L_j)^2 + \frac{1}{3}(\lambda_1 L_i L_j)^3 - \ldots\right) = \sum_{<ij>} \ln(1 + \lambda_1 L_i L_j)$$

Langelage, Lottini, Philipsen, JHEP 1102 (2011)

Heavy Fermions and Hopping Expansion

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$$\det[D] = \det[1 - \kappa H] = \exp\left(-\sum_{i=1}^{\infty} \frac{1}{i} \kappa^i \operatorname{Tr}[H^i]\right)$$

Leading order:

$$S_f^{(LO)} = -2 \sum_{\vec{x}} \left[(2\kappa e^{a\mu})^{N_\tau} L(\vec{x}) + (2\kappa e^{-a\mu})^{N_\tau} L(\vec{x}) \right]$$

Corrections from decorations:

$$h(u, \kappa, N_\tau \geq 3) = (2\kappa e^{a\mu})^{N_\tau} \exp\left[6 N_\tau \kappa^2 \frac{1 - u^{N_\tau}}{1 - u} \right]$$

Heavy Fermions and Hopping Expansion

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Similar to pure gauge: Sum up generalized Polyakov loops

$$\exp\left[-2\sum_{\vec{x}}\sum_{n=1}^{\infty}\left(\frac{(-1)^n}{n}h\,\mathrm{Tr}(W_{\vec{x}}^n)\right)\right] = \prod_{\vec{x}}\det\left[1+hW_{\vec{x}}\right]^2$$

Pietri, Feo, Seiler, Stamatescu, Phys. Rev. D. 76 (2007)

Partition function of the effective theory:

$$\boxed{Z = \int[dU_0]\prod_{<ij>}\left[1+\lambda_1 L_i L_j\right]\prod_{\vec{x}}\det\left[(1+hW_{\vec{x}})(1+\bar{h}W_{\vec{x}}^\dagger)\right]^2}$$

Test and Applications

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- Good agreement between eff. theory and YM theory for critical coupling $\Delta\beta_c < 4\%$
- Calculation of effective Polyakov loop potentials
  - $\mu = 0$: test of the effective theory
  - $\mu > 0$: input for functional methods
- Investigation of the phase diagram
  - Deconfinement transition at $\mu > 0$
  - BEC of diquarks

Effective Polyakov Loop Potentials from two-color QCD

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- Per-site probability distribution $P(L)$ via histogram-method from QC_2D simulations with 2 flavors of staggered quarks
- Per-site constrained potential

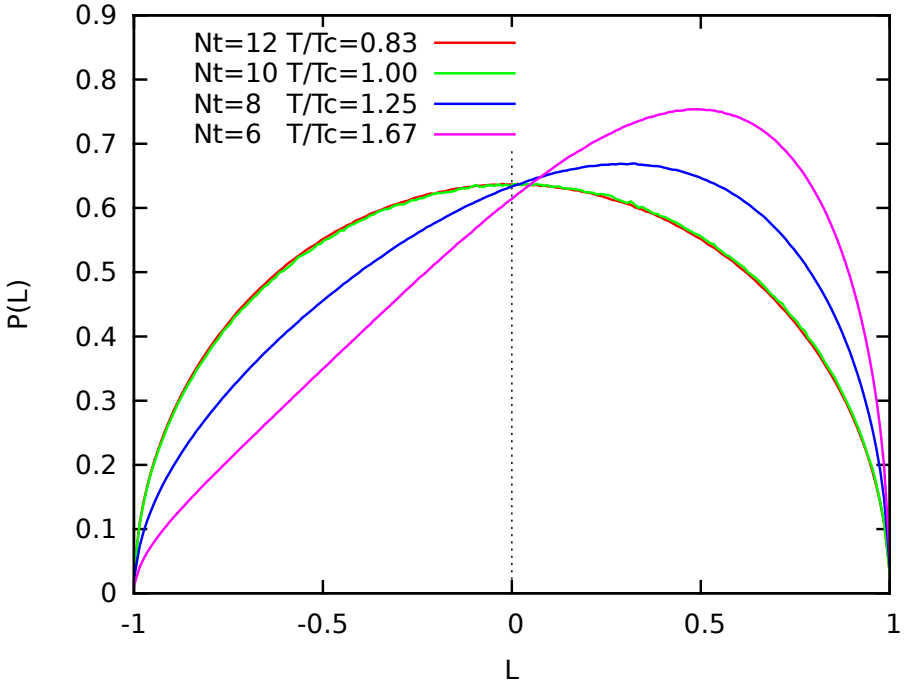
$$V_0(L) = -\log P(L)$$

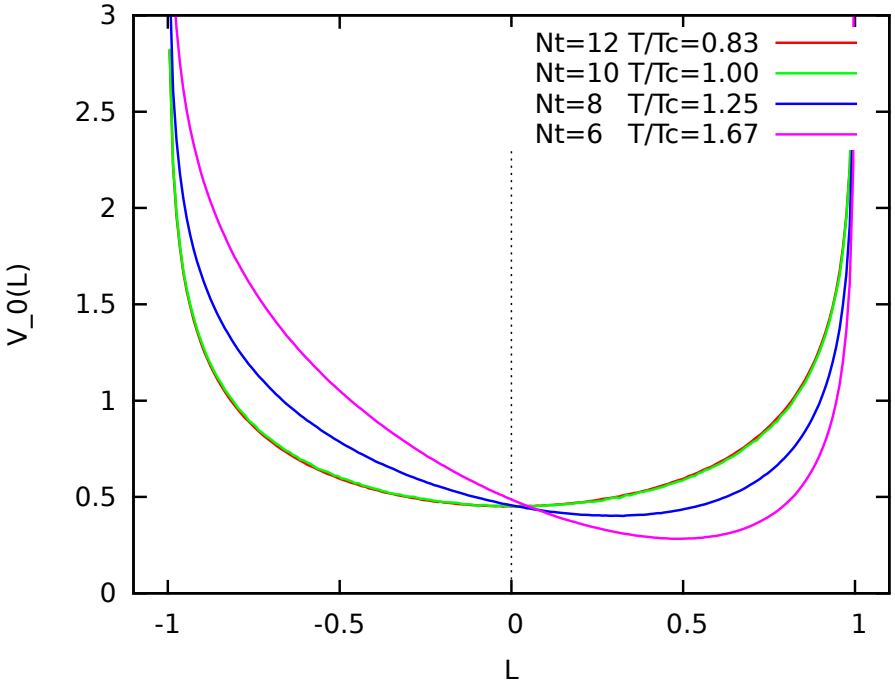
- Obtain actual per-site effective potential by Legendre transform

$$W(h) = \log \int dL \; \exp(-V_0(L) + hL)$$

$$V_{\text{eff}}(\hat{L}) = \sup_h (\hat{L}h - W(h))$$

Polyakov Loop Distributions and Effective Potentials at β = 2.577856





by Smith, Dumitru, Pisarski, von Smekal, Phys. Rev. D. 88 (2013)

- Pure gauge results in fixed scale approach

Polyakov Loop Distributions and Effective Potentials at β = 2.577856

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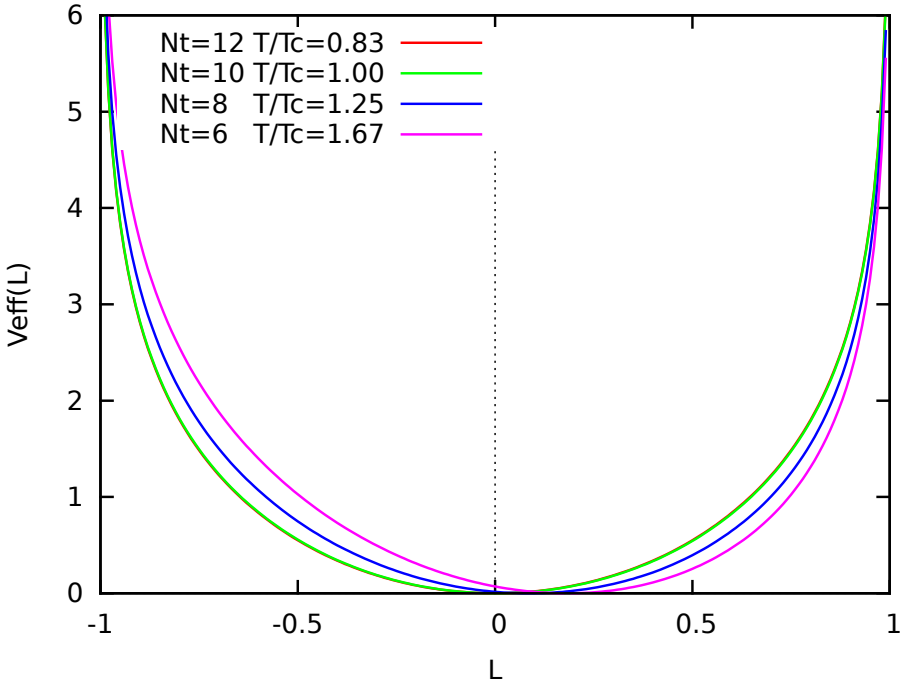
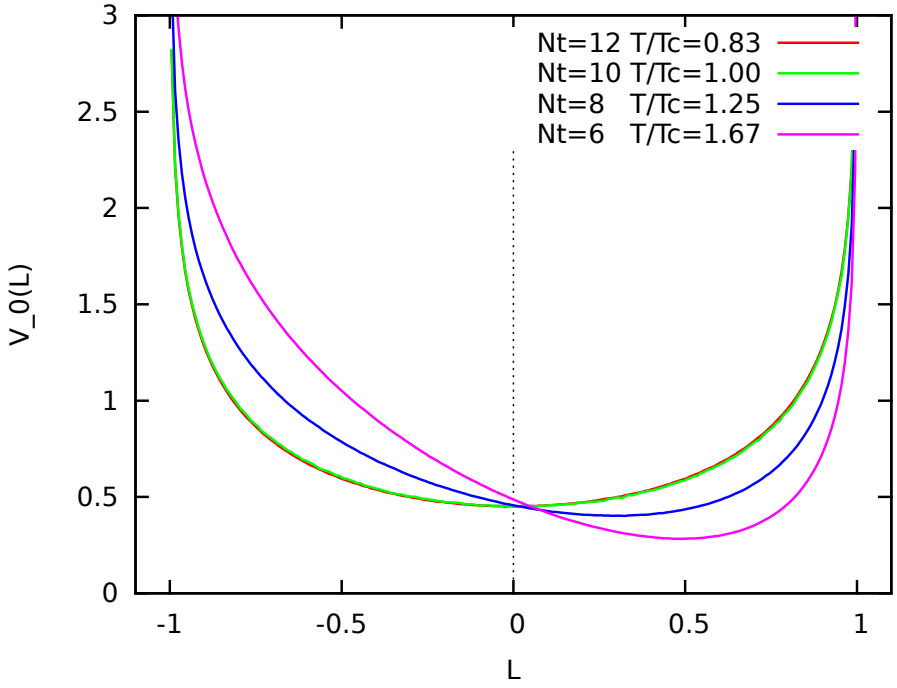
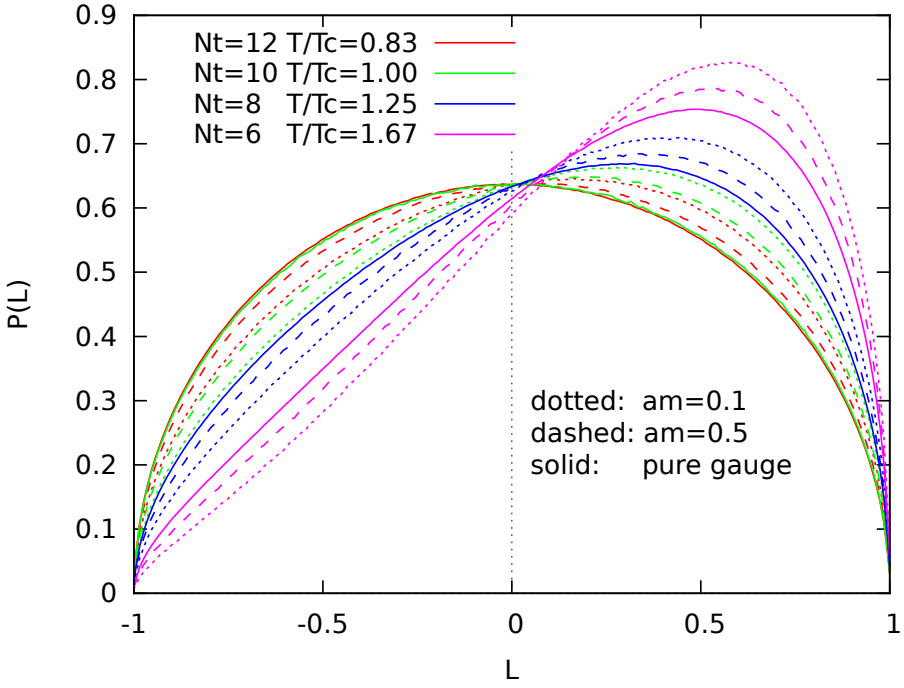


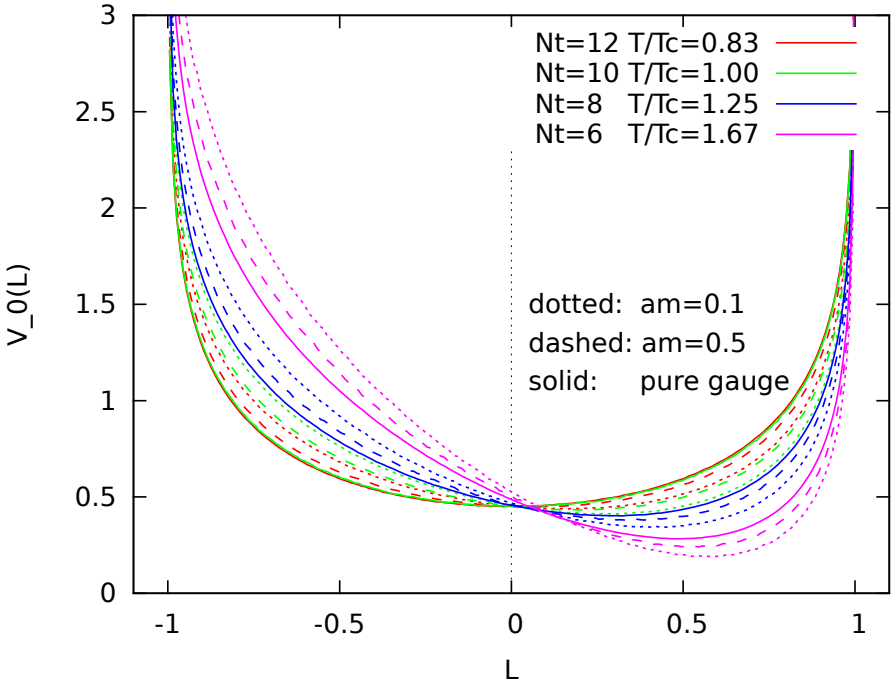


by Smith, Dumitru, Pisarski, von Smekal, Phys. Rev. D. 88 (2013)

- Pure gauge results in fixed scale approach

Unquenched Polyakov Loop Distributions and Effective Potentials at β = 2.577856

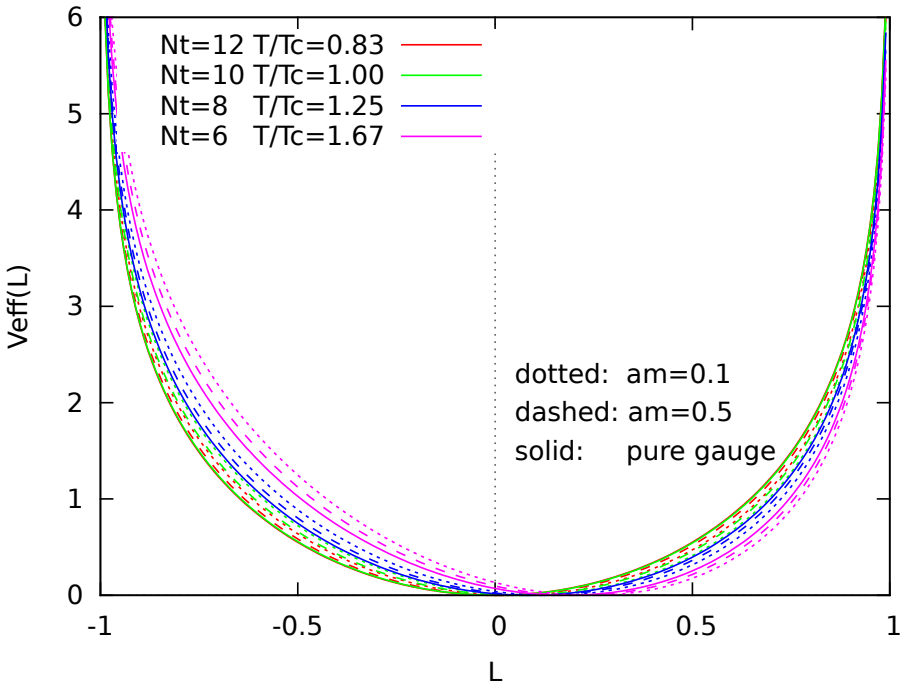


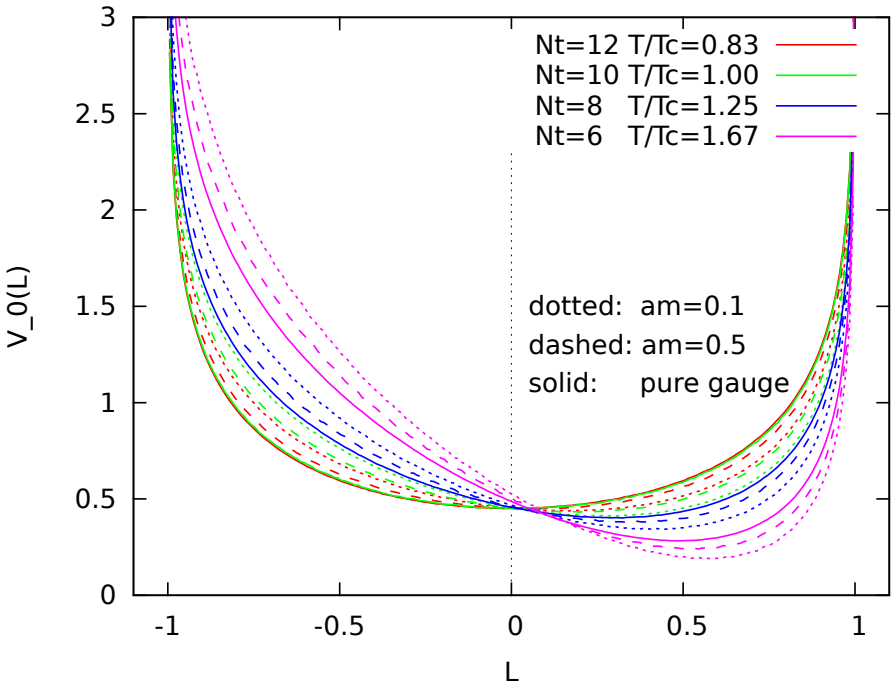


Scheffler, PS, Smith, von Smekal in preparation

- With N_f = 2 staggered quarks, neglect scale change through quark masses

Unquenched Polyakov Loop Distributions and Effective Potentials at β = 2.577856





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Polyakov Loop Distributions, Effective Theory

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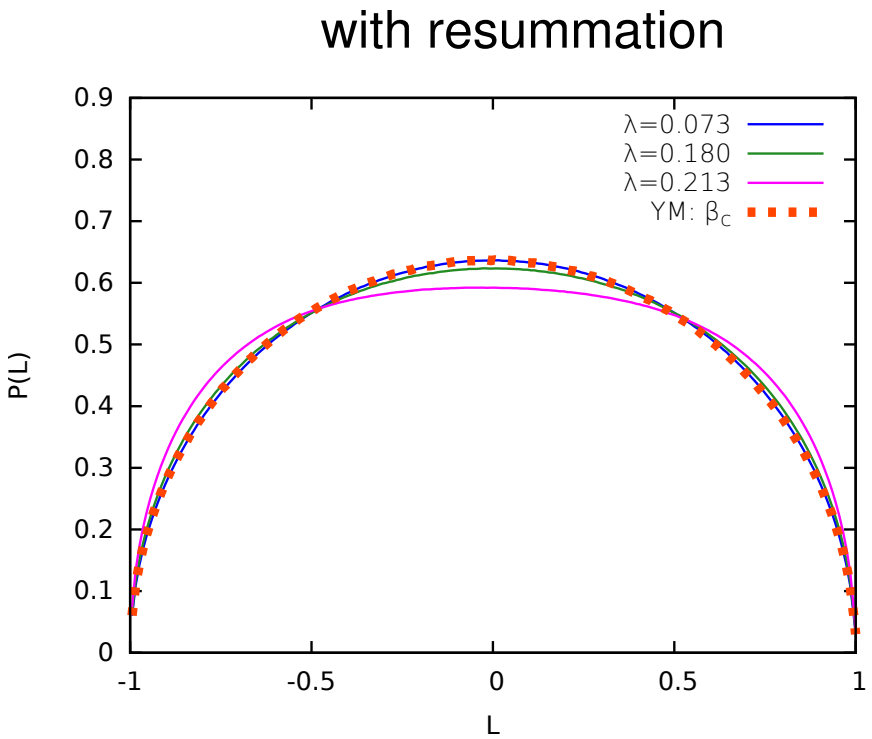

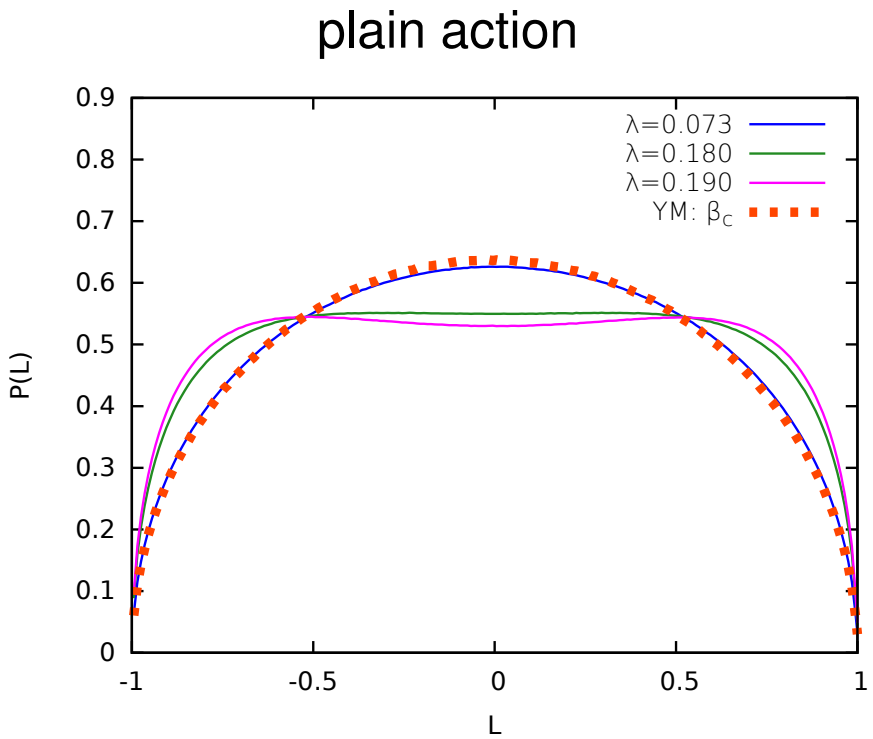

- Strong coupling limit: distributions match pure gauge theory
- $\lambda \to \lambda_c$: distributions get deformed, resummed theory reproduces pure gauge theory quite well

Polyakov Loop Distributions, Effective Theory

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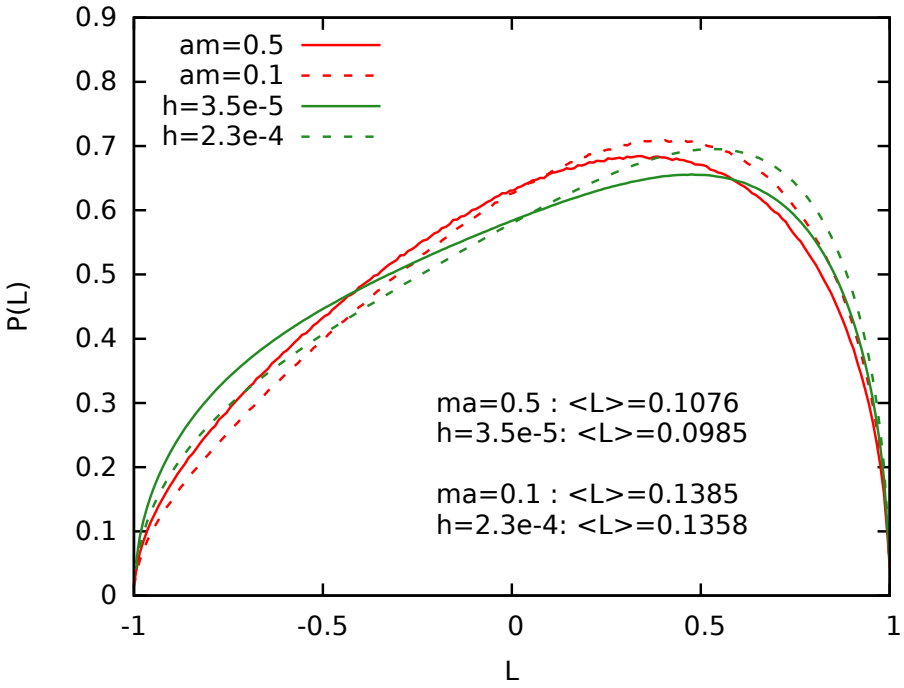
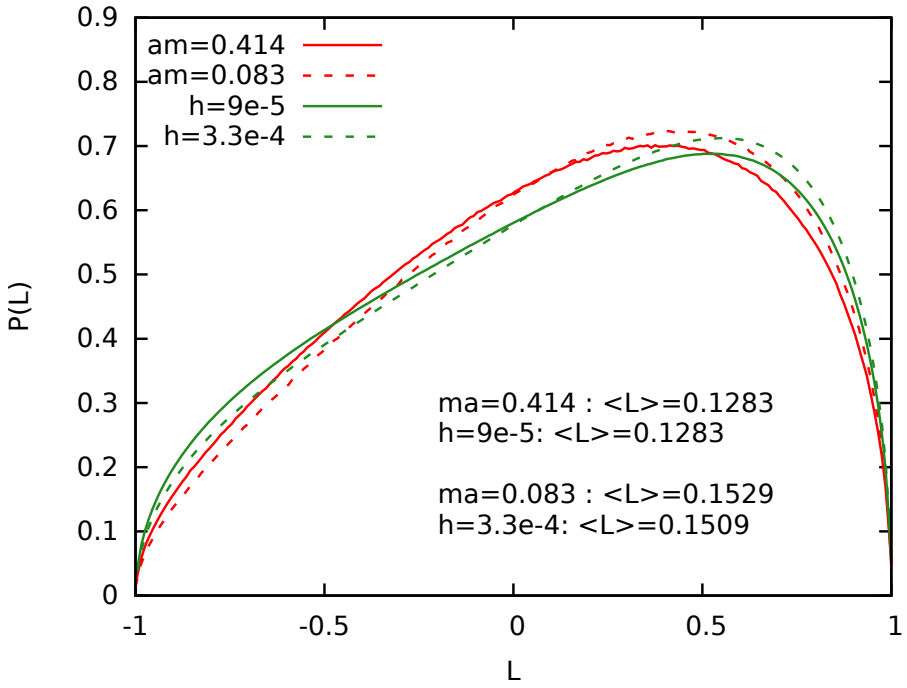
$T/T_c = 1.25$



$T/T_c = 1.5$



Scheffler, PS, Smith, von Smekal in preparation

- Couplings λ and h are matched to reproduce the correct $\langle L \rangle$

Polyakov Loop Distributions, Effective Theory

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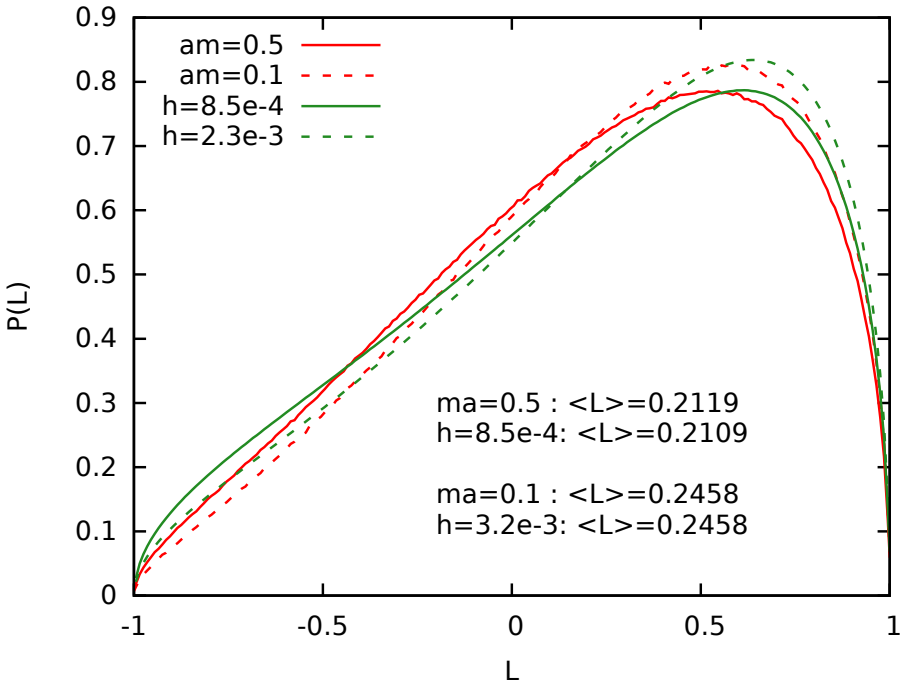
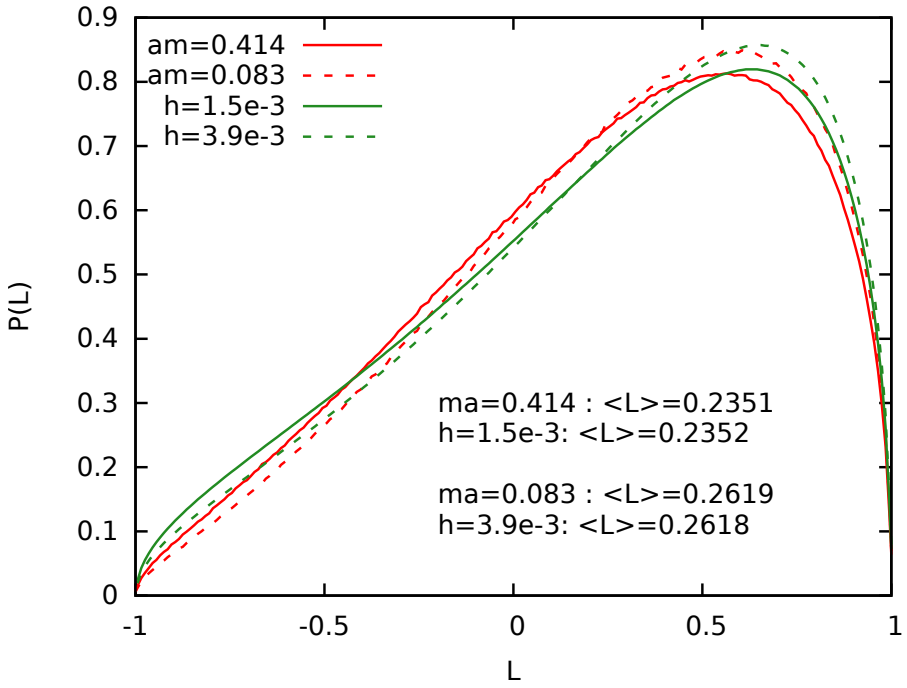
$T/T_c = 1.67$



$T/T_c = 2.0$



Scheffler, PS, Smith, von Smekal in preparation

- Couplings λ and h are matched to reproduce the correct $\langle L \rangle$

Effective Polyakov Loop Potentials, Effective Theory

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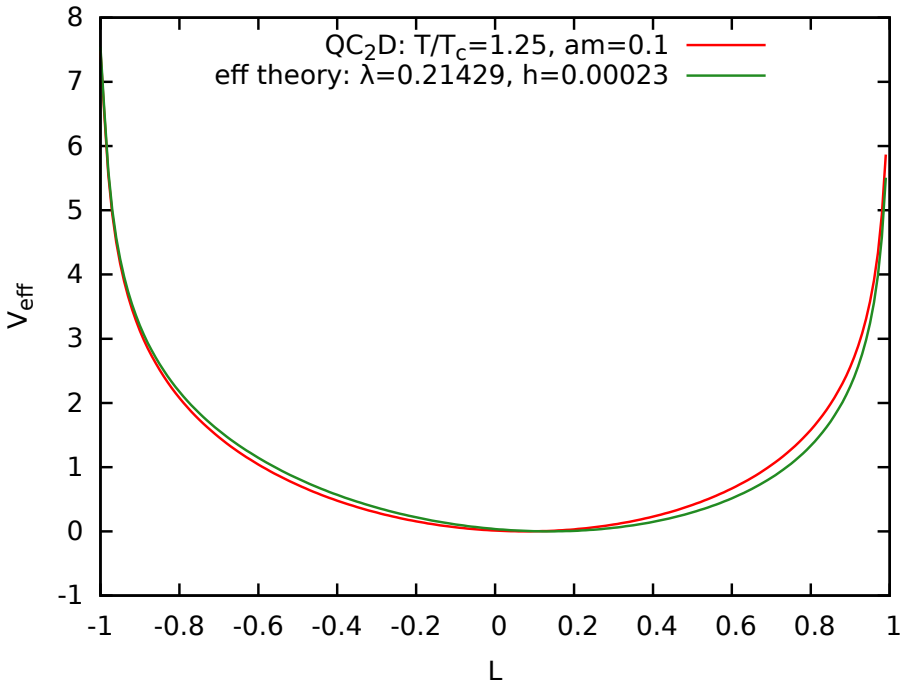
$T/T_c = 1.25$



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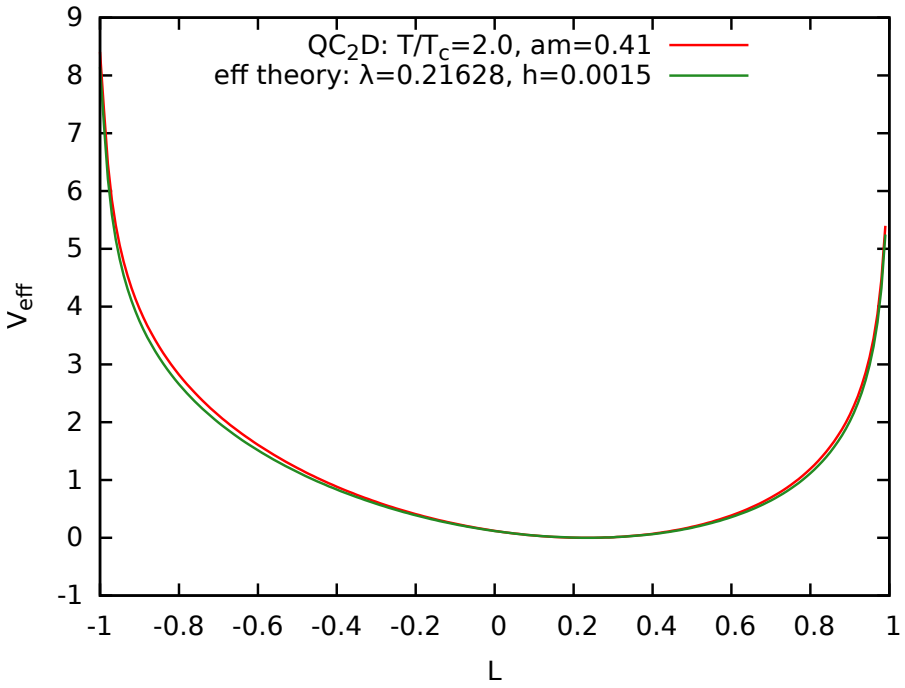

Scheffler, PS, Smith, von Smekal in preparation

- very good agreement between effective theory and full two-color QCD

Effective Polyakov Loop Distributions at finite μ

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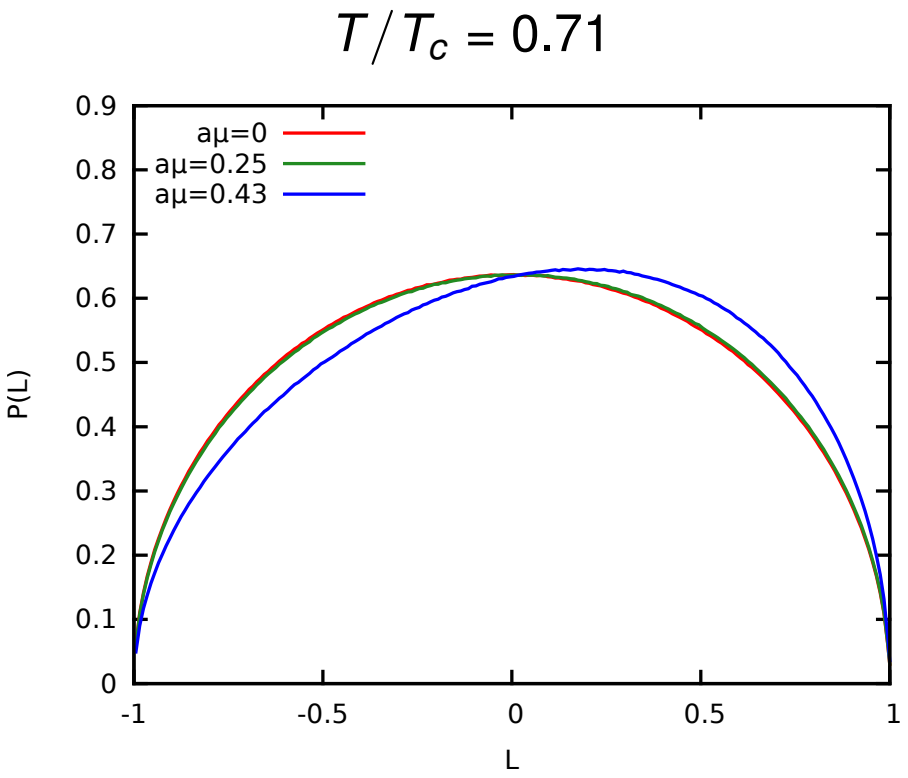

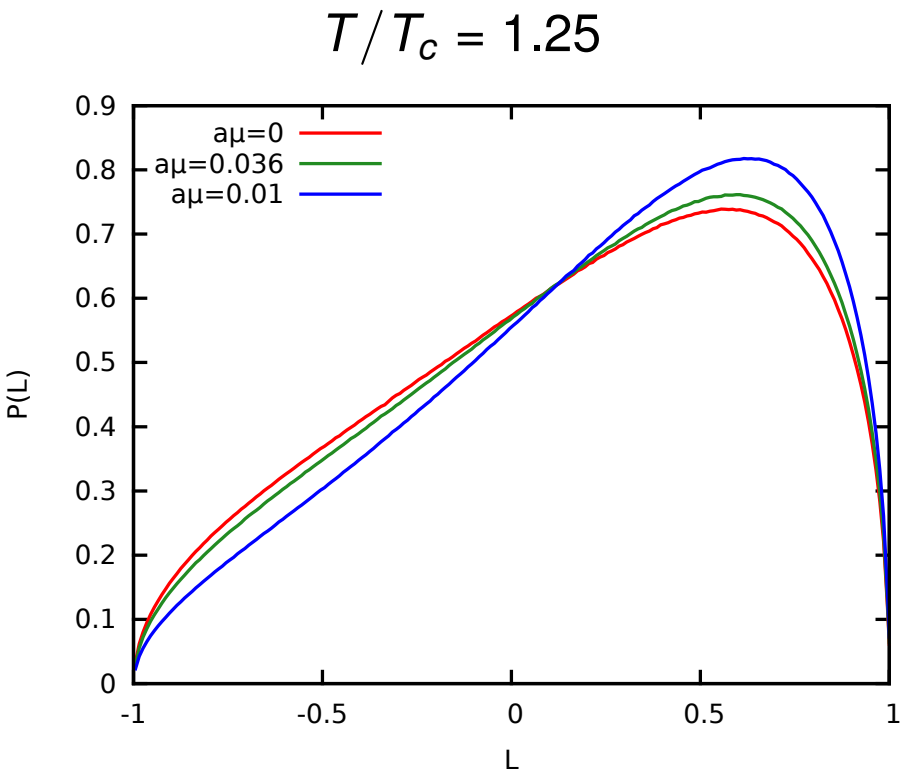

PRELIMINARY

Further Results

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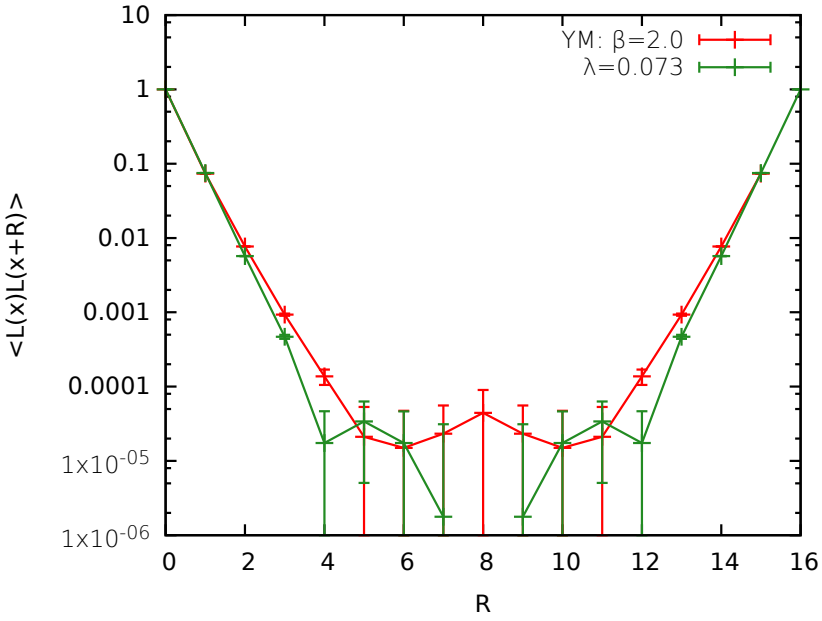
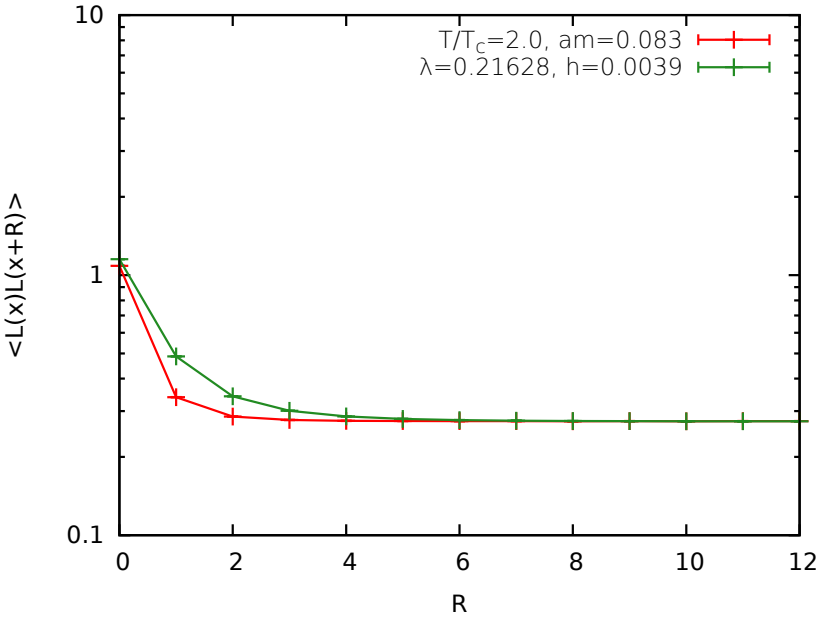
- Adjoint loops: strong fluctuations around λ_c, no benefit at $\lambda \neq \lambda_c$
- Correlation functions:





Deconfinement Transition

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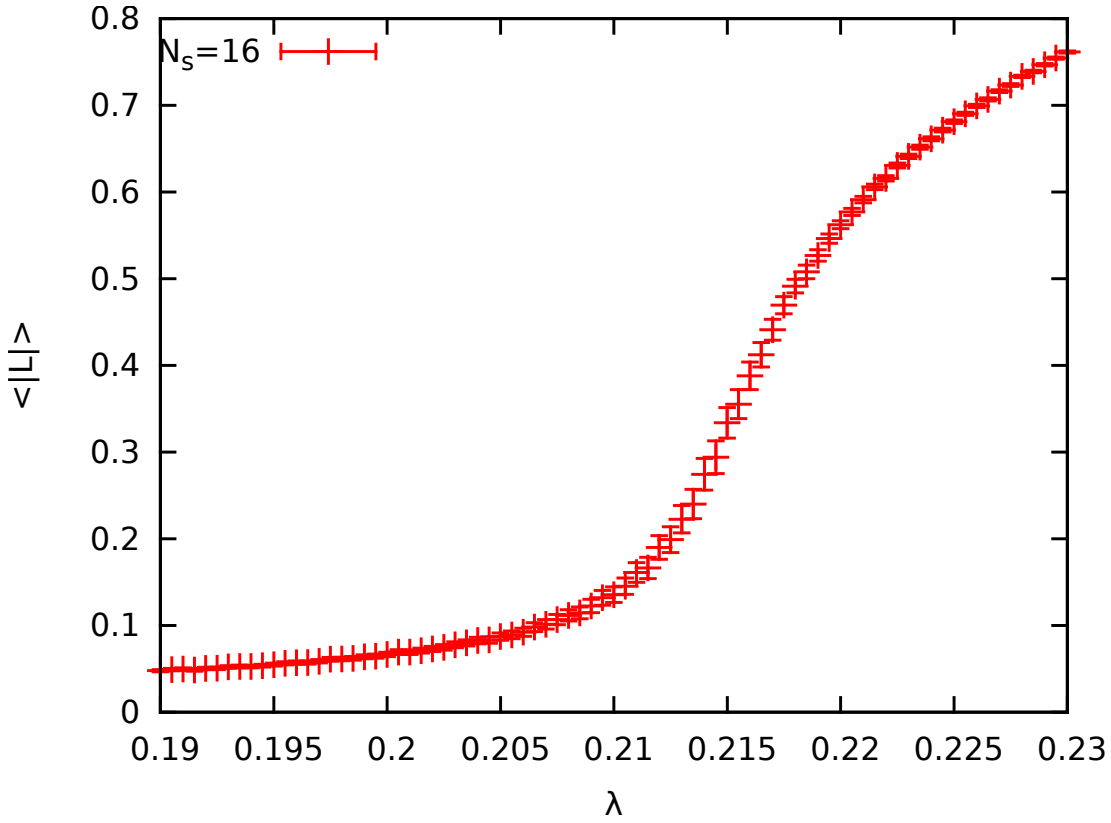
- $N_f = 1$, $N_s = 16$, $h = 2.14 \times 10^{-8}$



Phase Diagram

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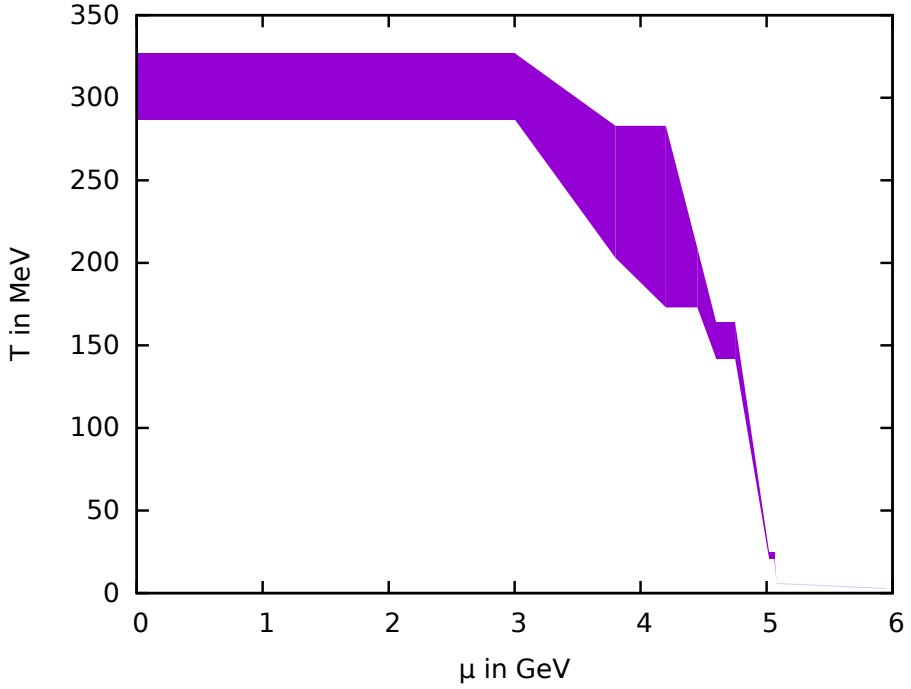
- m_D = 10 GeV, N_f = 1, N_s = 16, β = 2.4836
- Qualitative agreement with full QC_2D studies with smaller quark masses.



Cold and Dense QC_2D with Heavy Quarks

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- Use effective theory at low temperature and finite chemical potential with heavy quarks
- Combined strong coupling and hopping expansion to order $\mathcal{O}(\kappa^n, u^m)$, $n + m = 4$ compare: *Langelage, Neuman, Philipsen [1403.4162]*
- Low temperature, strong coupling $\longrightarrow \lambda \leq 10^{-16}$
- Fermionic partition function
- Calculate density:

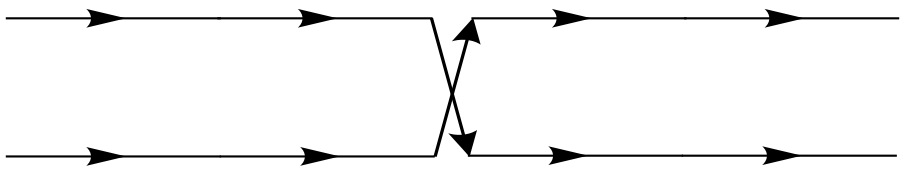
$$n = \frac{T}{V} \frac{\partial \log Z}{\partial \mu}$$

Hopping Expansion: Kinetic Part

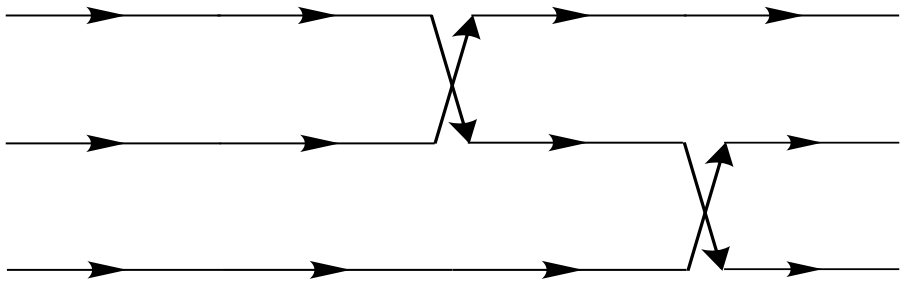
$$\det[D] = \det[1 - \kappa T] \det[1 - (1 - \kappa T)^{-1} S]$$

$\rightarrow$ Quark line interactions

Order κ^2:



Order κ^4:



Effective Action

$$-S_{\text{eff}} = \sum_{\vec{x}} \log(1 + h\mathrm{Tr}\, W_{\vec{x}} + h^2)^2$$

Effective Action

$$-S_{\text{eff}} = \sum_{\vec{x}} \log(1 + h\mathrm{Tr}\,W_{\vec{x}} + h^2)^2 - 2h_2 \sum_{\vec{x},i} \mathrm{Tr}\frac{hW_{\vec{x}}}{1 + hW_{\vec{x}}}\mathrm{Tr}\frac{hW_{\vec{x}+i}}{1 + hW_{\vec{x}+i}}$$

Effective Action

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$$
\begin{aligned}
-S_{\text{eff}} = & \sum_{\vec{x}} \log(1 + h\mathrm{Tr}\, W_{\vec{x}} + h^2)^2 - 2h_2 \sum_{\vec{x},i} \mathrm{Tr}\frac{hW_{\vec{x}}}{1 + hW_{\vec{x}}} \mathrm{Tr}\frac{hW_{\vec{x}+i}}{1 + hW_{\vec{x}+i}} \\
& + 2\frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i} \mathrm{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \mathrm{Tr}\frac{hW_{\vec{x}+i}}{(1 + hW_{\vec{x}+i})^2} \\
& + \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \mathrm{Tr}\frac{hW_{\vec{x}-i}}{1 + hW_{\vec{x}-i}} \mathrm{Tr}\frac{hW_{\vec{x}-j}}{1 + hW_{\vec{x}-j}} \\
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\end{aligned}
$$

Effective Action

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$$
\begin{aligned}
-S_{\text{eff}} = & \sum_{\vec{x}} \log(1 + h\mathrm{Tr}\,W_{\vec{x}} + h^2)^2 - 2h_2 \sum_{\vec{x},i} \mathrm{Tr}\frac{hW_{\vec{x}}}{1 + hW_{\vec{x}}} \mathrm{Tr}\frac{hW_{\vec{x}+i}}{1 + hW_{\vec{x}+i}} \\
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& + 2\frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \mathrm{Tr}\frac{hW_{\vec{x}-i}}{1 + hW_{\vec{x}-i}} \mathrm{Tr}\frac{hW_{\vec{x}+j}}{1 + hW_{\vec{x}+j}} \\
& + \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \mathrm{Tr}\frac{hW_{\vec{x}+i}}{1 + hW_{\vec{x}+i}} \mathrm{Tr}\frac{hW_{\vec{x}+j}}{1 + hW_{\vec{x}+j}} \\
& + \kappa^4 N_\tau^2 \sum_{x,i} \frac{h^4}{(1 + h\mathrm{Tr}\,W_{\vec{x}} + h^2)(1 + h\mathrm{Tr}\,W_{\vec{x}+i} + h^2)} \,.
\end{aligned}
$$

Cold and Dense QC_2D with Heavy Quarks

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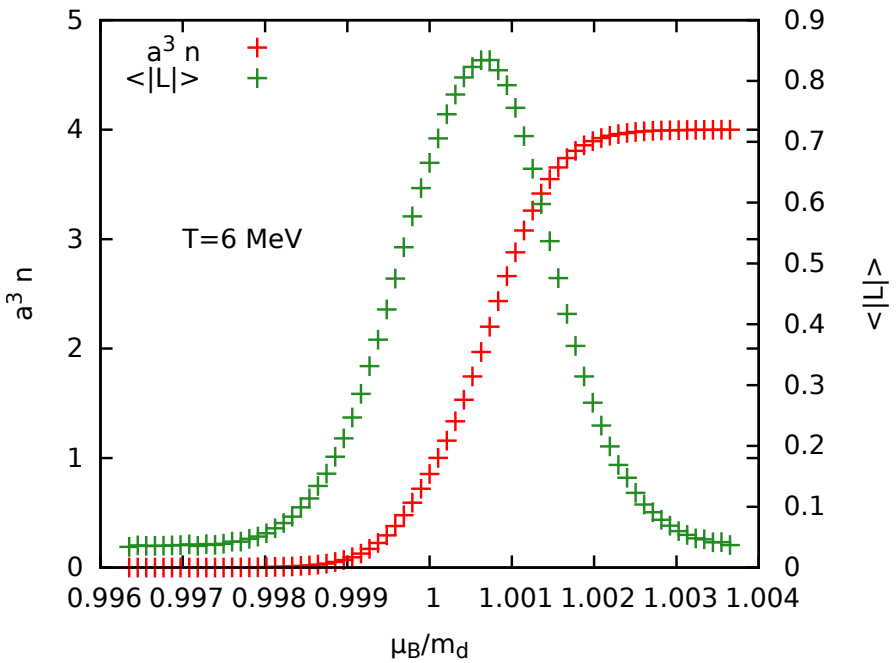
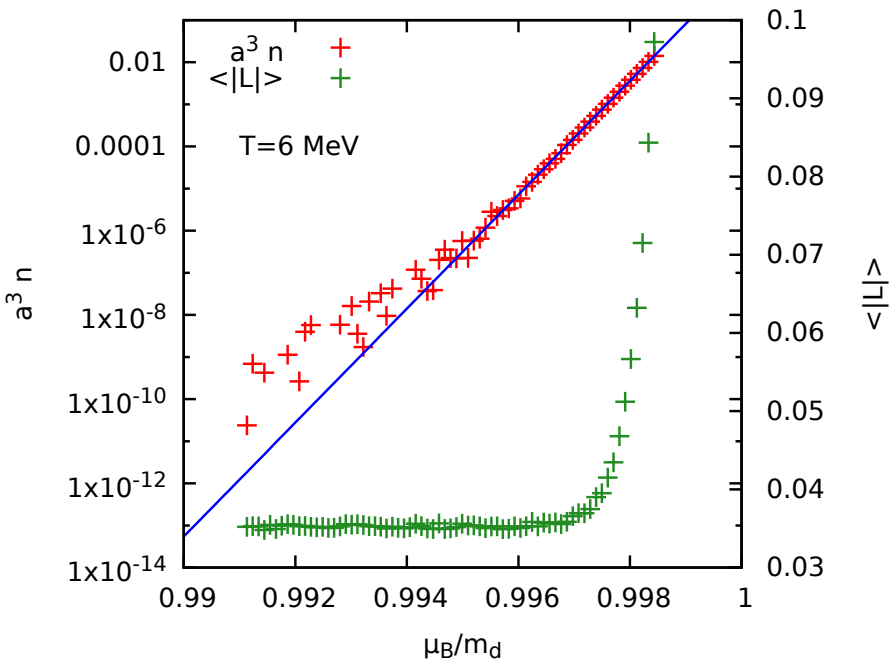
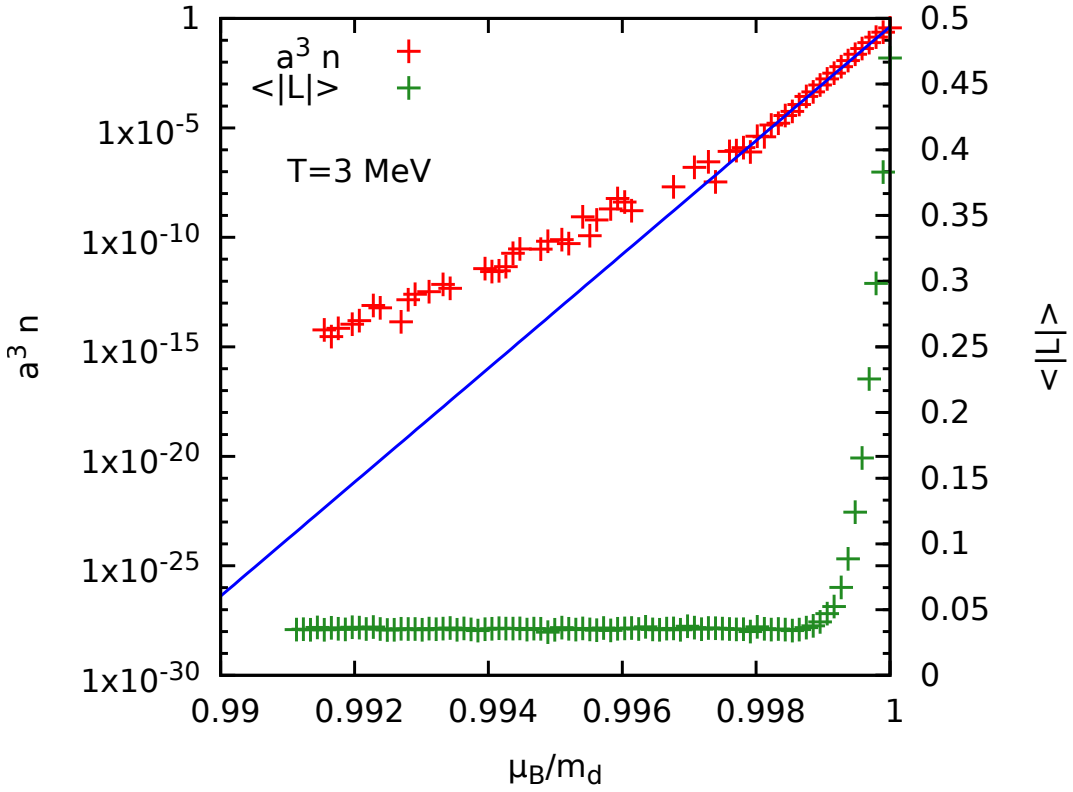
- Very heavy quarks: m_D = 19.17 GeV
- Deconfinement transition with unphysical lattice saturation



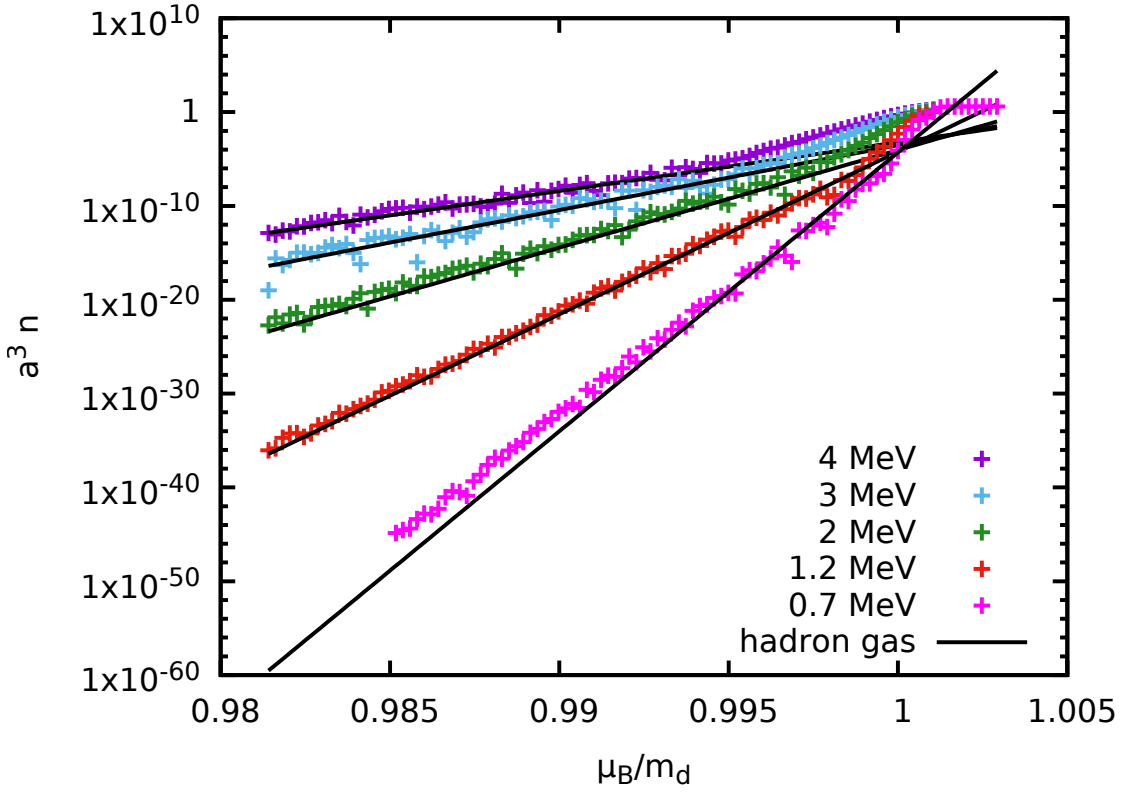


Cold and Dense QC_2D with Heavy Quarks



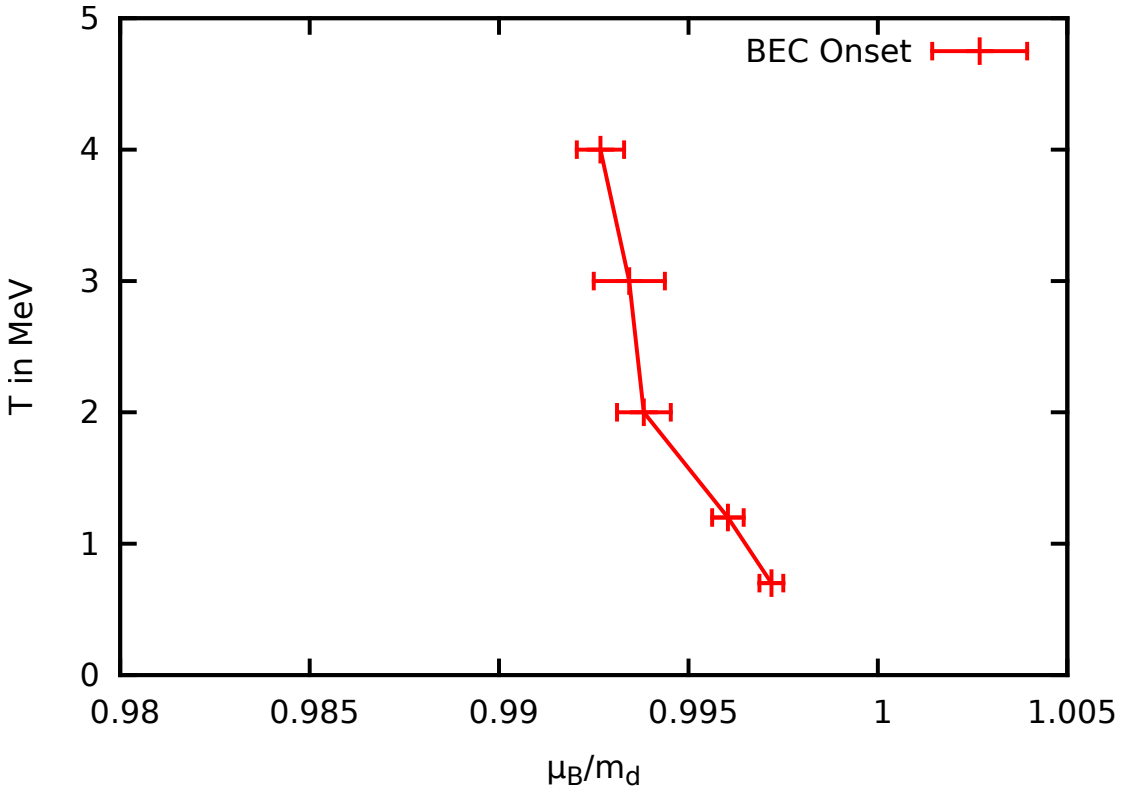
- Diquark BEC??

Cold and Dense QC_2D with Heavy Quarks



- below μ_c: hadron gas $\propto \exp\left(\frac{\mu_B - m_d}{T}\right)$

BEC Onset



- What happens for smaller quark masses?

Summary

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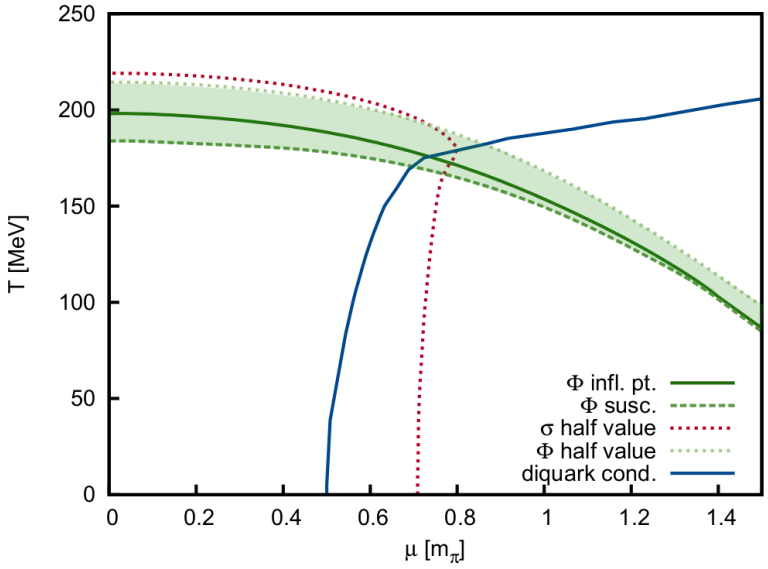
- Effective lattice theory, where sign problem can be avoided
- Unquenched Polyakov loop potentials from full QC_2D and effective theory
- Small T, finite μ: deconfiment transition with unphysical lattice saturation
- Silverblaze property
- Possible diquark BEC?? $\rightarrow$ difference between two and three colors?



Strodthoff, von Smekal, Phys. Lett. B 731 (2014)

Outlook

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- Effective Polyakov loop potentials at finite density
- Cold and dense regime: go to smaller quark masses $\rightarrow$ higher order in hopping expansion
- Map BEC transititon for higher T
- Full QC_2D at finite density

Backup Slides

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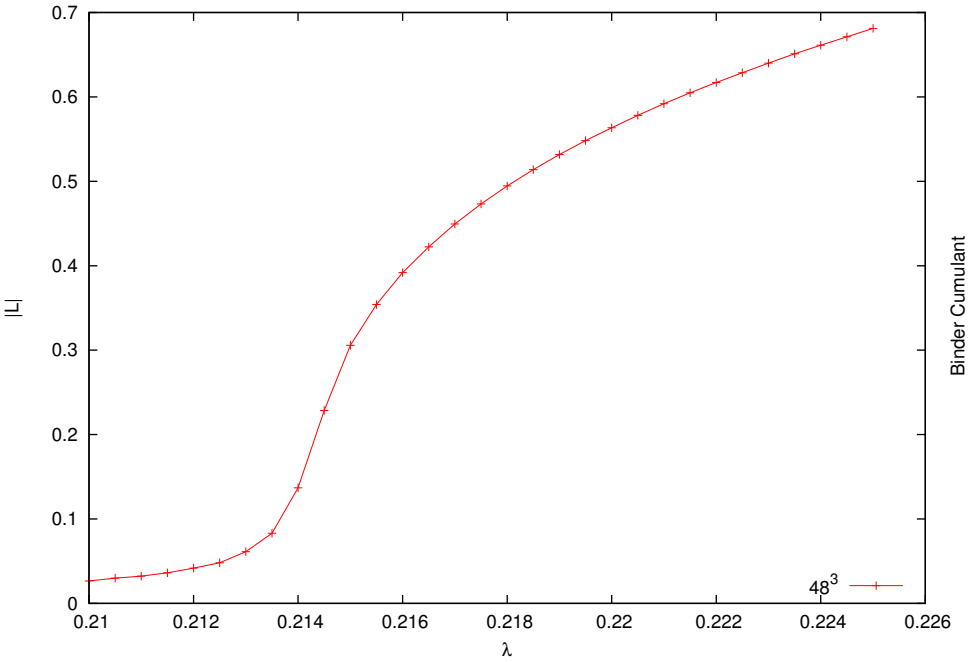
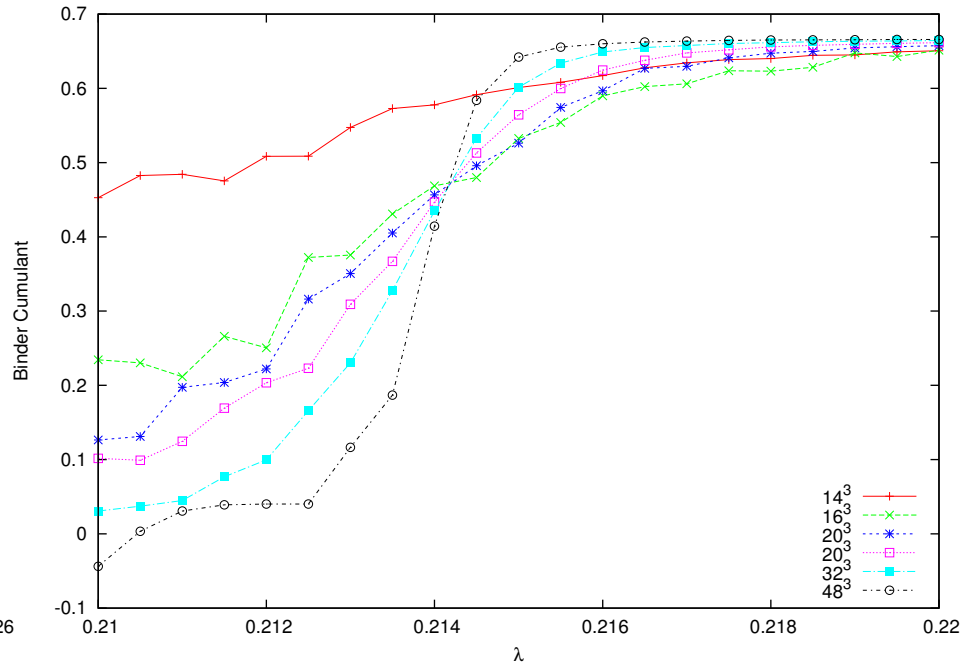
$$-S_{\text{eff}} = \ln \int [dU_i] \prod_p \left[1 + \sum_{j \neq 0} d_j a_j(\beta) \chi_j(U_p) \right] ,$$

$$-S_{\text{eff}} = \sum_{C = X_l^{n_l}} a(C) \prod_l \Phi(X_l; [W_j])^{n_l} ,$$

$$\Phi(X_l; [W_j]) = \int [dU_i] \prod_{p \in X_l} d_{r_p} a_{r_p} \chi_{r_p}(U_p)$$

Pure Gauge

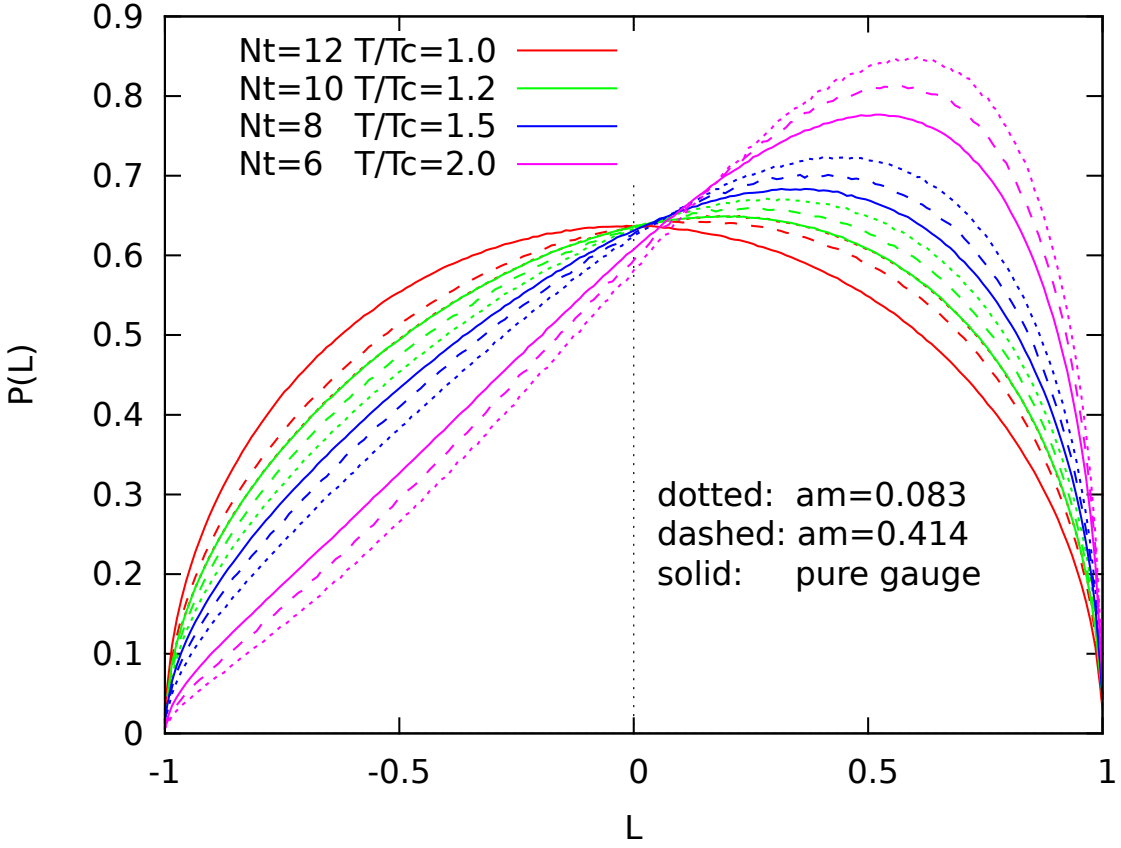




$$B(|L|) = 1 - \frac{\langle |L|^4 \rangle}{3\langle |L|^2 \rangle^2}$$

Polyakov Loop Distributions

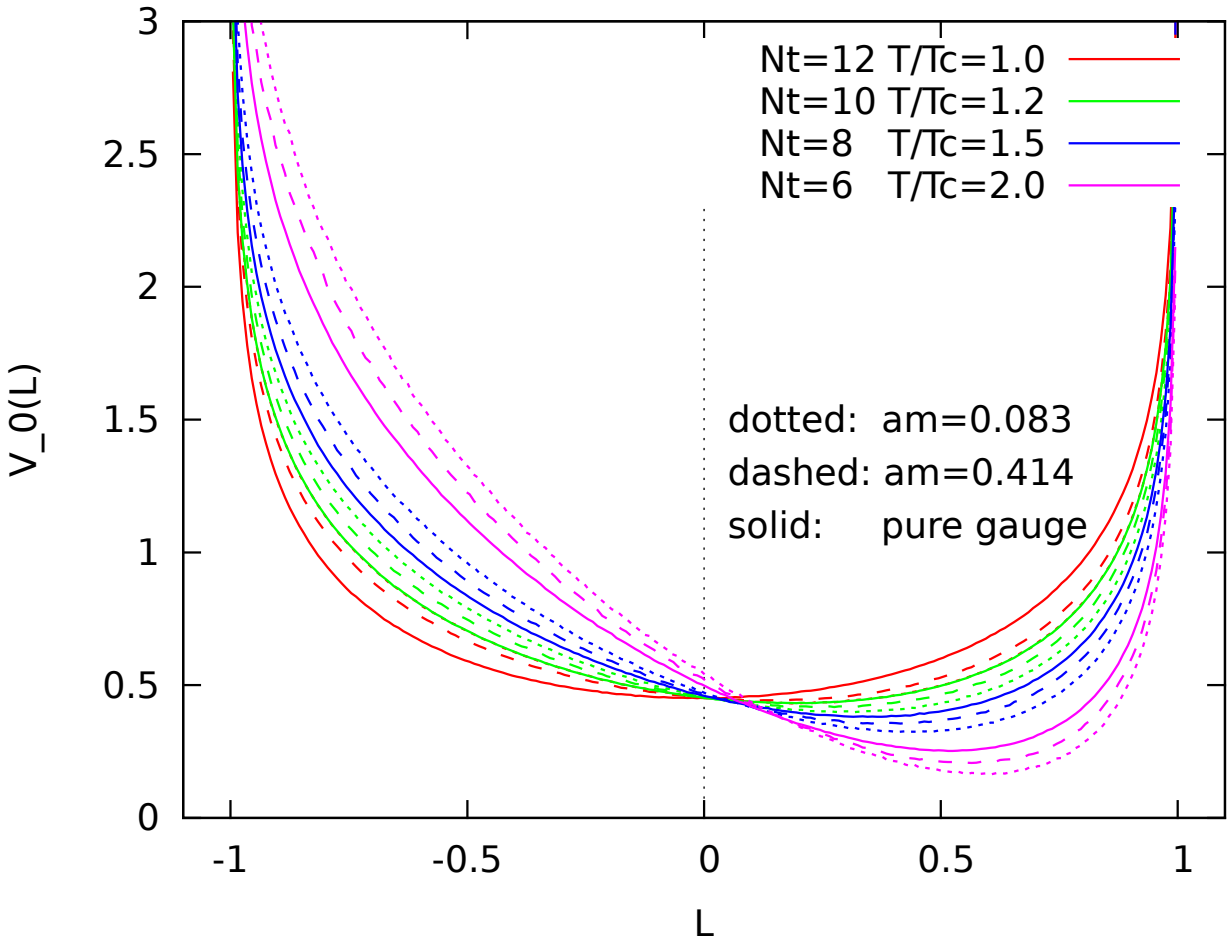
at β = 2.635365



◀ β = 2.577856

Polyakov Loop Effective Potential

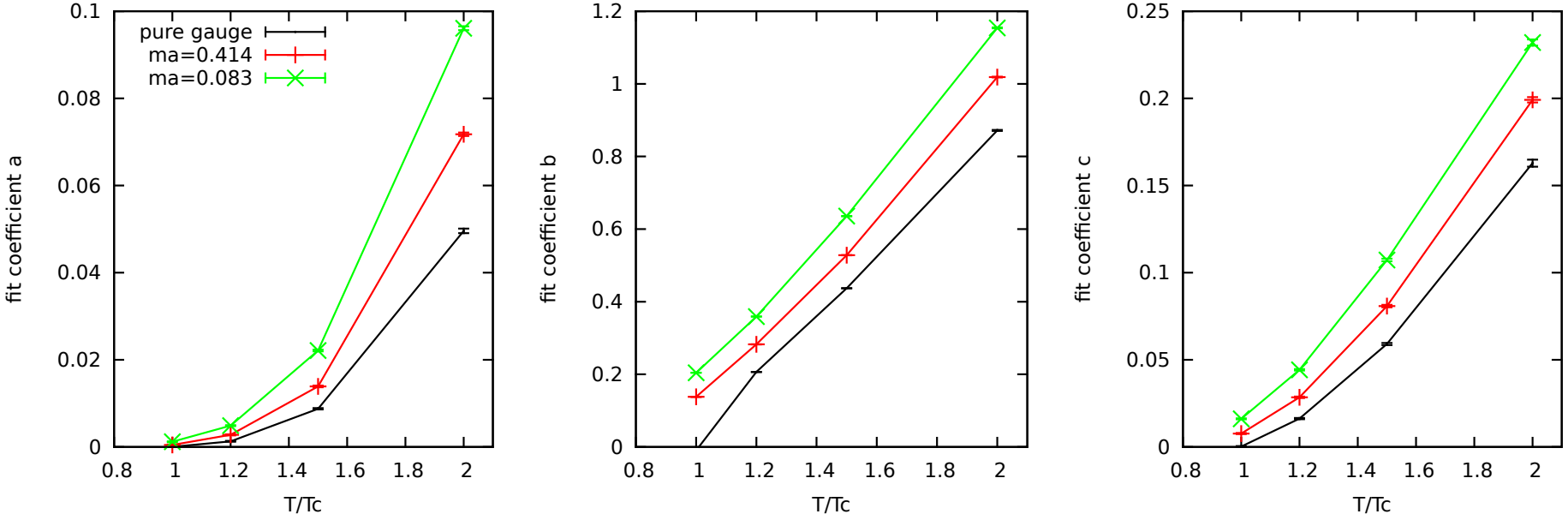
at β = 2.635365



◂ β = 2.577856

Fit Coefficients

at β = 2.635365

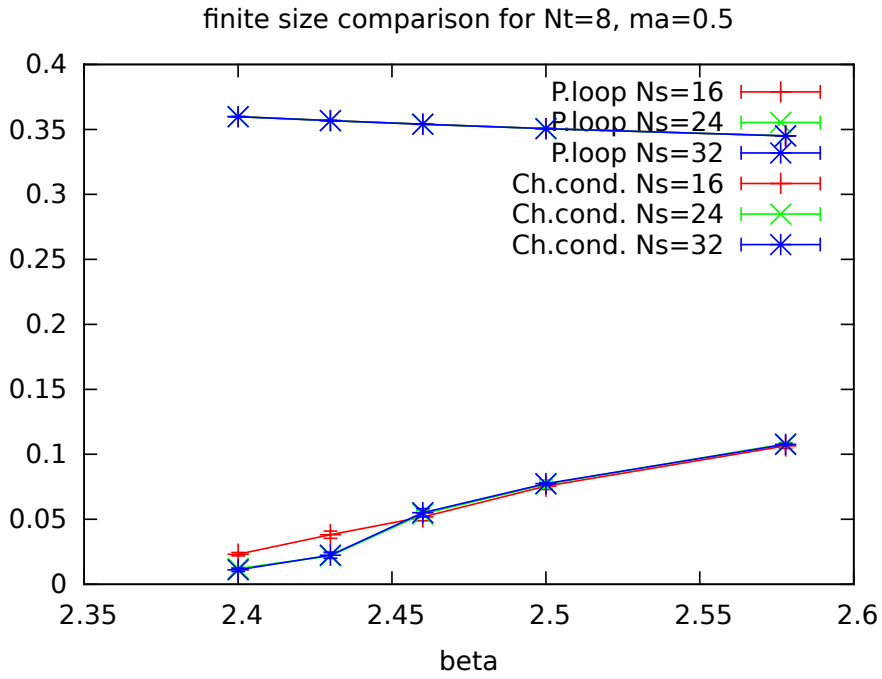


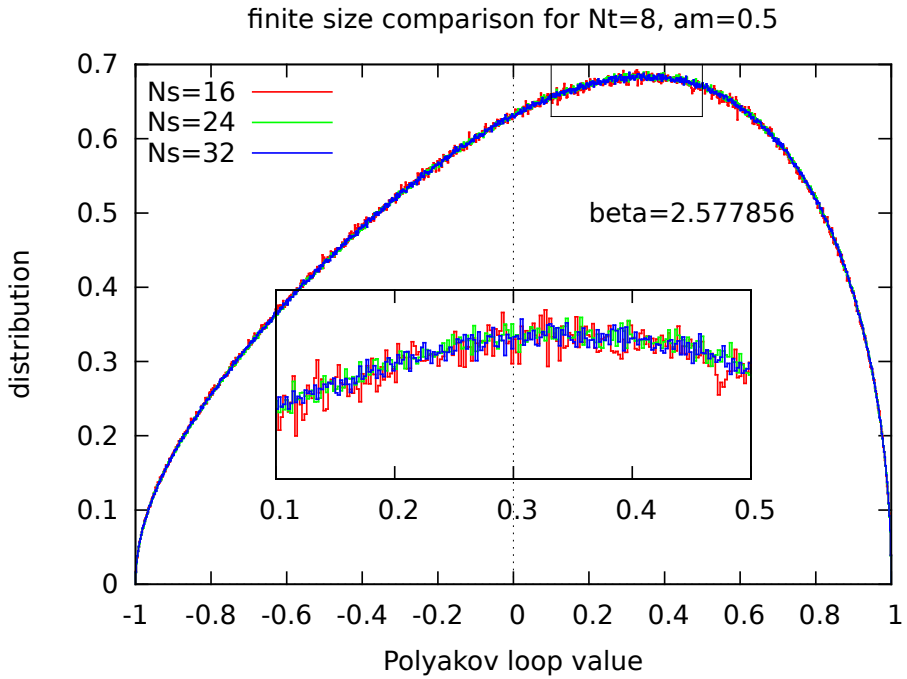
$$V_0(l) = V^{(T_c)}(l) + a(T) - b(T)l + c(T)l^2$$

◀ β = 2.577856

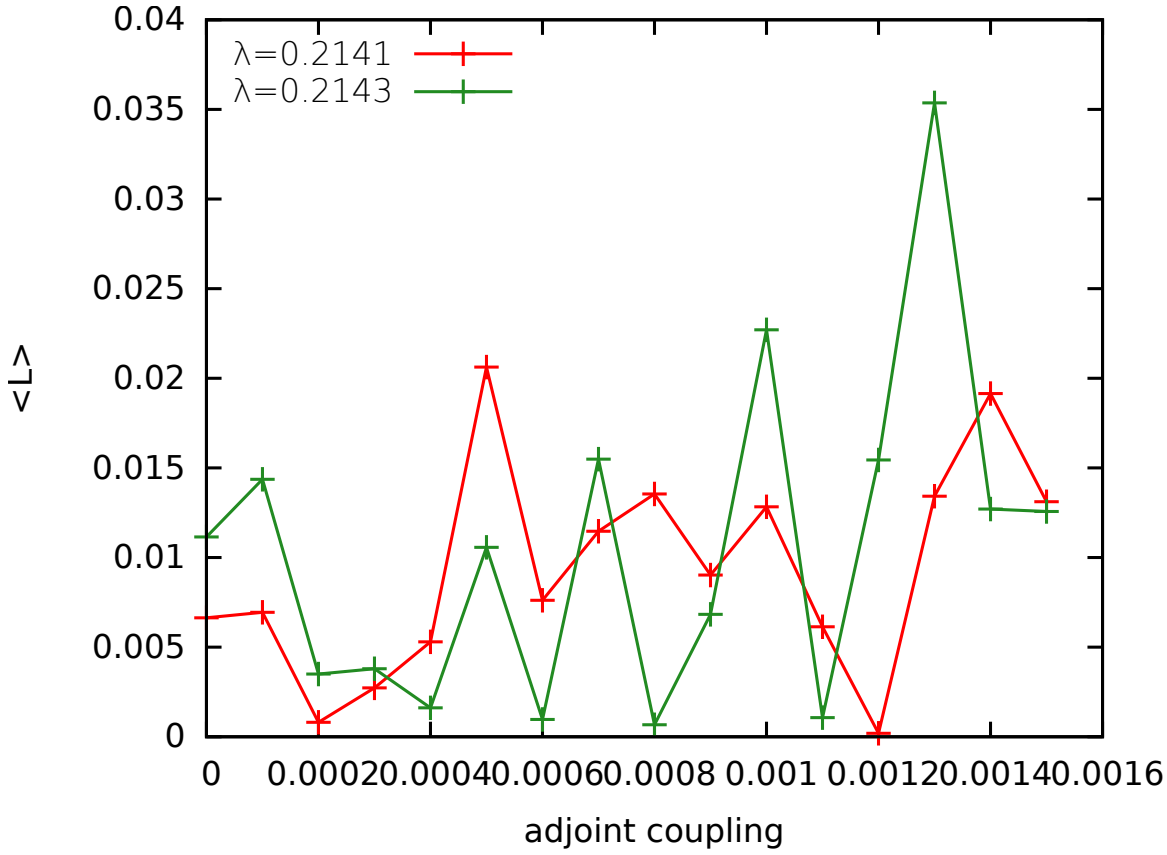
Finite Volume Test

$N_t = 8, am = 0.5$





Adjoint Coupling



Analytic relations for Cold and Dense regime

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$$h = \exp\left[N_\tau\left(a\mu + \ln 2\kappa + 6\kappa^2\frac{u - u^{N_\tau}}{1-u}\right)\right] ,$$

$$am_D = -2\ln(2\kappa) - 6\kappa^2 - 24\kappa^2\frac{u}{1-u} + 6\kappa^4 + \mathcal{O}(\kappa^4 u^2, \kappa^2 u^5) .$$