The origin of axial anomaly and the high temperature phase of QCD

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January 15, 2015

Reference:

V. Dick, F. Karsch, E. Laermann, S. Mukherjee and S.Sharma, Proceedings of Lattice 2013[arxiv:1311.3943] & in preparation

Outline

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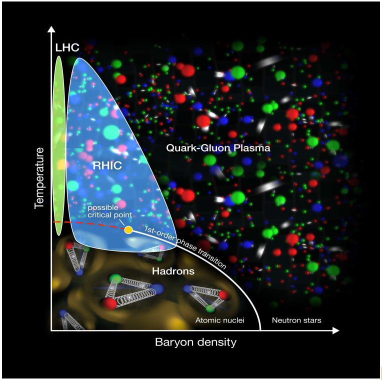
Phase diagram of QCD: what we know

- The phase diagram of strongly interacting matter is largely unknown
- Confinement and chiral symmetry breaking observed but mechanism still not understood
- Existence of critical end-point associated with chiral symmetry restoration$\rightarrow$ first hints from experiments already coming up![STAR collaboration, PRL, 14]



[http://www.bnl.gov/rhic/news]

Symmetries of QCD

- The physics described by Quantum Chromodynamics

$$\mathcal{L} = -\frac{1}{4} F_{a\mu\nu} F_a^{\mu\nu} + \sum_{f=1}^{N_f} \bar{\psi}_f (\not{D} + m_f) \psi_f \ ,$$

- $m_f = 0$, $\mathcal{L}$ invariant under
  $U_L(N_f) \times U_R(N_f) \equiv SU(N_f)_V \times SU(N_f)_A \times U_B(1) \times U_A(1)$
- At T=350 MeV, $m_{c,b}$ may not affect thermodynamics
- $m_s \sim \Lambda_{QCD}$, $m_u, m_d << \Lambda_{QCD}$,
  The approximate symmetry: $SU(2)_V \times SU(2)_A \times U_B(1) \times U_A(1)$
- QGP$\rightarrow$ hadron transition results in light pions[chiral symmetry breaking]
  $SU(2)_V \times SU(2)_A \times U_B(1) \rightarrow SU(2)_V \times U_B(1)$
- Symmetries determine the order parameter: $\langle \bar{\psi}_l \psi_l \rangle$.
- $U_A(1)$ $\rightarrow$not a symmetry yet influences the chiral phase transition

[Pisarski & Wilczek, 84]

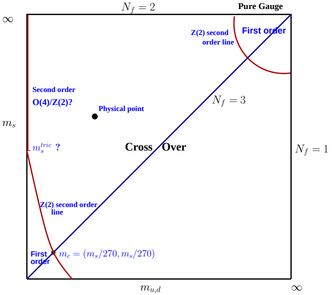
What is known about the $U_A(1)$

- Perturbative RG studies on sigma models with same symmetries as N_f flavour QCD:

  In 3D, $N_f \geq 3$: 1st order phase transition independent of $U_A(1)$.

  $N_f = 2$: If $U_A(1)$ effectively restored $\Rightarrow$ 1st or 2nd order with $U_L(2) \times U_R(2) \rightarrow U_V(1)$ criticality

  [Pisarski & Wilczek, 84, Butti, Pelissetto & Vicari, 03,13]



Important to get insight from lattice studies.

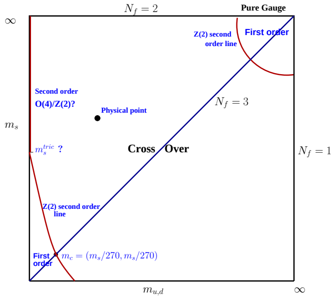
What is known about the $U_A(1)$

- Perturbative RG studies on sigma models with same symmetries as N_f flavour QCD:

  In 3D, $N_f \geq 3$: 1st order phase transition independent of $U_A(1)$.

  $N_f = 2$: If $U_A(1)$ broken $\Rightarrow$ 2nd order transition with $O(4)$ critical exponents.

  [Pisarski & Wilczek, 84, Butti, Pelissetto & Vicari, 03,13]



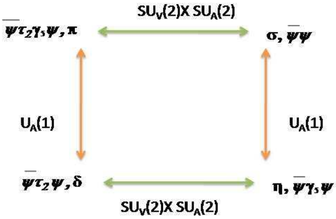
Important to get insight from lattice studies.

Implications for critical point

- The existence of critical point crucially depends on number of light quarks.
- Associated with chiral symmetry restoration $\rightarrow$ important to use fermions with exact chiral symmetry at finite μ. [Narayanan & Sharma, 11 , Gavai & Sharma, 12]

- Carefully study the effects of the axial anomaly?
- At $T = 0$, anomaly effects related to instantons [t'Hooft, 76]. What happens at finite T? First suggestions already exist [Shuryak, 82, Gross, Pisarski & Yaffe, 84]. Comprehensive lattice studies still lacking.

The $U_A(1)$ breaking



Not an exact symmetry$\rightarrow$ what observables to look for?

Degeneracy of the correlators with specific quantum numbers in meson channels [Shuryak, 94]

The $U_A(1)$ breaking

- The difference of the integrated correlators is zero as $U_A(1)$ is restored.

$$\chi_\pi - \chi_\delta = \int d^4x \, \left[\langle i\pi^+(x) i\pi^-(0)\rangle - \langle \delta^+(x)\delta^-(0)\rangle\right]$$

- In terms of the eigenvalue density $\rho(\lambda, m_f)$ of Dirac operator,

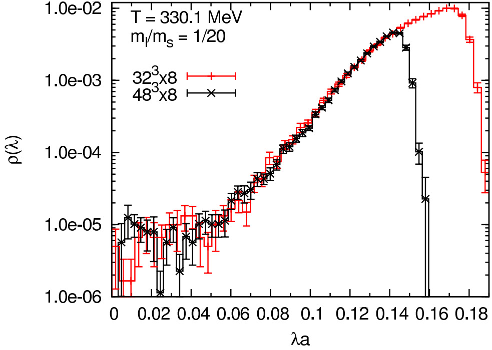
$$\chi_\pi - \chi_\delta \overset{V\to\infty}{\to} \int_0^\infty d\lambda \frac{4m_f^2 \, \rho(\lambda, m_f)}{(\lambda^2 + m_f^2)^2} \; , \; \langle\bar{\psi}\psi\rangle \overset{V\to\infty}{\to} \int_0^\infty d\lambda \frac{2m_f \, \rho(\lambda, m_f)}{(\lambda^2 + m_f^2)}$$

- If chiral symmetry restored: $\langle\bar{\psi}\psi\rangle = 0 \Rightarrow \lim_{m_f\to 0} \lim_{V\to\infty} \rho(0, m_f) \to 0$.
- A gap in the infrared spectrum $\Rightarrow$ $U_A(1)$ restored
- chiral symmetry restored + $U_A(1)$ broken if:
  $\lim_{\lambda\to 0} \rho(\lambda, m_f) \to \delta(\lambda) m_f^\alpha \; , 1 < \alpha < 2$

Outline

Motivation for our work

The earlier results on $U_A(1)$ are not conclusive



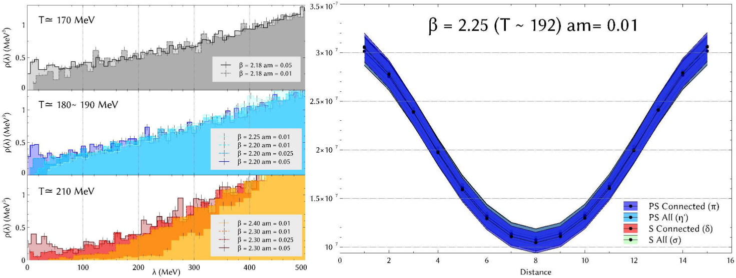
Dynamical (improved)Staggered fermions: Large $32^3 \times 8$ lattice $\rightarrow$ $U_A(1)$ broken [Ohno et. al. 12]

Same observation noted earlier for smaller lattice[Chandrasekharan & Christ 96]

Issues with lattice artifacts?

Exact chiral invariance lost

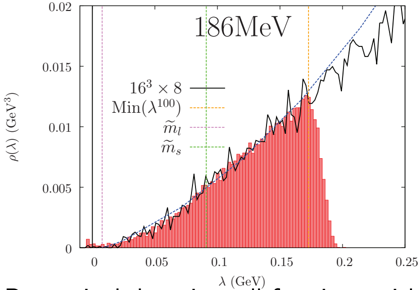
Motivation for our work



Dynamical overlap fermions with exact chiral symmetry on lattice→ $U_A(1)$ restored [Cossu et. al, JLQCD collaboration, 11, 12]

Pion mass 220 MeV! Effects of fixing the topology? Thermodynamic equilibrium?

Motivation for our work



Dynamical domain wall fermions with better chiral symmetry $\rightarrow U_A(1)$ broken [Buchoff et. al. 13] Low statistics in the lower end of spectrum?

Optimal domain wall fermions: On small lattice $\rightarrow U_A(1)$ restored [Chiu et. al. 13]

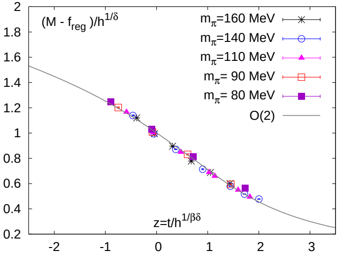
Motivation for our work

Issues need to be addressed

- Sufficient number of instantons ?
- Importance of volume effects
- Sufficient control on the lattice artifacts?

Motivation for our work

- At present improved versions of staggered fermions used extensively for QCD thermodynamics. Continuum limits for T_c, χ_2 are in agreement. [Budapest-Wuppertal collaboration 10, HotQCD collaboration, 11].
- Highly improved staggered quarks(HISQ): minimal taste symmetry breaking$\Rightarrow$ lattice artifacts reduced.
- Hints about the $O(4)$ scaling from these configurations $\rightarrow$ effects of anomaly? [Bielefeld-BNL collaboration, 09, HotQCD collaboration, 11]



- No index theorem for staggered fermions makes these studies difficult!

Motivation for our work

- Another motivation: understanding the microscopic constituents of the QCD medium and the mechanism of confinement and χ_{SB}.
- The origin of the χ_{SB} explained through the Instanton Liquid Model. [Shuryak, 82]
- Around T_c transition from liquid phase of disordered instantons-antiinstantons (IA) to a phase of IA molecules [Schaefer, Shuryak & Verbaarschot, 95]
- Instantons cannot alone account for confinement.
- A Coulomb gas of dyons has confining features [Martemyanov & Molodtsov, 97].
- Signatures of dyons found out in pure gauge theory [Ilgenfritz et. al ,06,13].
- Important to understand from an independent study on the lattice with dynamical fermions.
- Microscopic origin of $U_A(1)$ anomaly at finite T?

Outline

Technique used till now

- Look for $U_A(1)$ restoration in 2+1 flavour QCD configurations generated with HISQ fermions.
- To measure the localized topological structures $\rightarrow$ remove the ultraviolet fluctuations
- Two known methods Cooling & Smearing
- amounts to substituting the gauge links successively by averaged/smeared adjacent links
- May cause disappearence of small instantons, density$\propto \rho^{-5}$.

- Alternative: Use the index theorem somehow!

Index theorem on the lattice

- It is impossible to define chiral fermions on lattice which are (ultra)local. [Nielsen & Ninomiya, 82]
- Overlap fermions [Narayanan & Neuberger, 94, Neuberger, 98] have exact chiral symmetry on the lattice.

$$D_{ov} = M(1 + \gamma_5\ \mathbf{sgn}(\gamma_5 D_W(-M)))\ ,\ \mathbf{sgn}(A) = A/\sqrt{A.A}.$$

- It satisfies the Ginsparg-Wilson relation $\{\gamma_5, D_{ov}\} = a D_{ov}\gamma_5 D_{ov}$ [Ginsparg & Wilson, 82]
- Under the transformations $\delta\bar{\psi} = i\alpha\bar{\psi}\gamma_5$ and $\delta\psi = i\alpha\gamma_5(1 - D_{ov})\psi$, the QCD action remains invariant [Luscher, 98].
- D_{ov} has an exact index theorem like in the continuum $\Rightarrow$ the zero modes of D_{ov} related to topological structures of the underlying gauge field. [Hasenfratz, Laliena & Niedermeyer, 98]

Index theorem on the lattice

- Use the overlap as valence quarks or probe
- The Highly improved staggered quark(HISQ) configurations form the sea quarks in background
- We look at the eigenvalue distribution of D_{ov} on the HISQ ensembles.
- Zero modes of D_{ov} related to topological structures of HISQ sea
- Infrared part of eigenvalue distribution gives us idea about the χ_{SB} , $U_A(1)$, the topological structures that contribute to them

Ensembles used in our work

- Lattice size: 4D hypercube with $N = 32,\ 24$ sites along each spatial dim and $N_\tau = 8,\ 6$ sites along temporal dim.
- Volumes, $V = N^3 a^3$, Temperature, $T = \frac{1}{N_\tau a}$, a is the lattice spacing.
- Box size: $m_\pi\ V^{1/3} > 3$
- 2 light+1 heavy flavour
- Input m_s physical ≈ 100 MeV and $m_s/m_l = 20 \Rightarrow m_\pi = 160$ MeV.
- To study the chiral limit we have another set of $N_\tau = 6$ configurations T_c with lighter than physical $m_\pi = 110$ MeV.

Numerical details

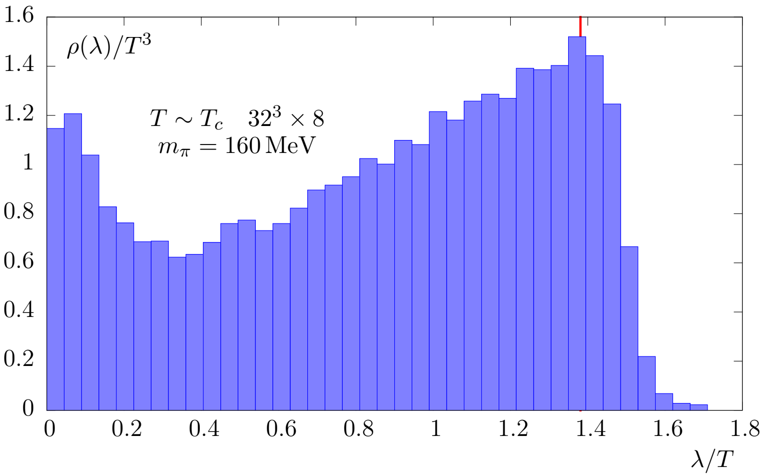
Implementing the overlap operator

- Matrix sign function non-trivial!
- For the lowest modes sign function was computed explicitly.
- For the higher modes, sign function approximated as a Zolotarev Rational Polynomial with 15 terms.
- The sign function is computed as precise as 10^{-10}.

Eigenvalue computation

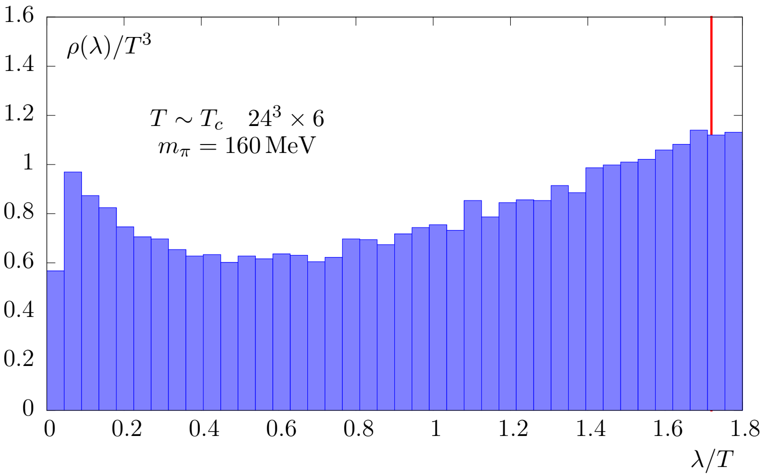
- The Kalkreuter-Simma Ritz algorithm for eigenvalues of $D_{ov}^{\dagger} D_{ov}$.
- Convergence criterion: $\epsilon^2 < 10^{-8}$.
- The GPU cards used in video games were used for such expensive computations

Our results: Eigenvalue density of D_{ov} in the scaling region



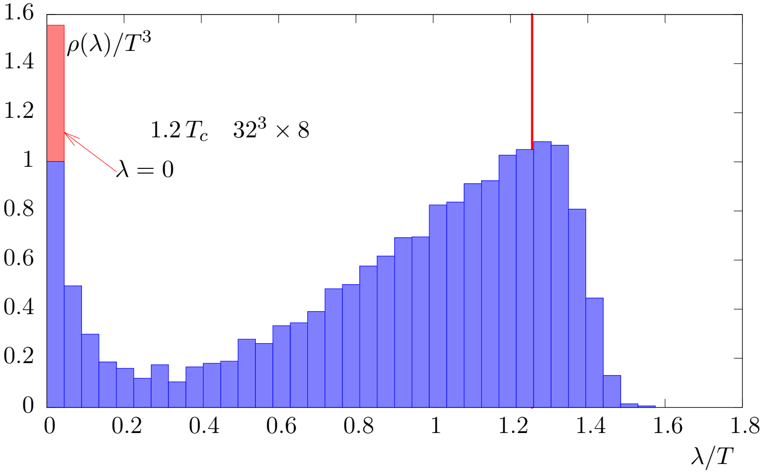
A large pile of near zero modes-NO GAP SEEN!

Our results: Eigenvalue density of D_{ov} in the scaling region



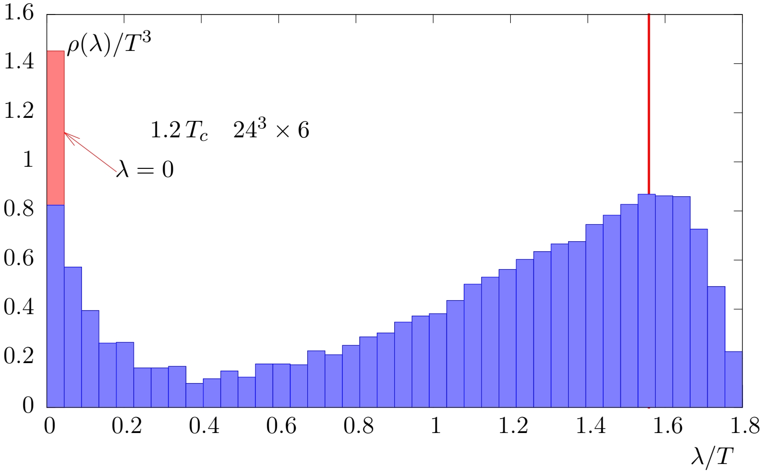
The result is independent of lattice cut-off effects

Eigenvalue density of D_{ov} at higher temperatures



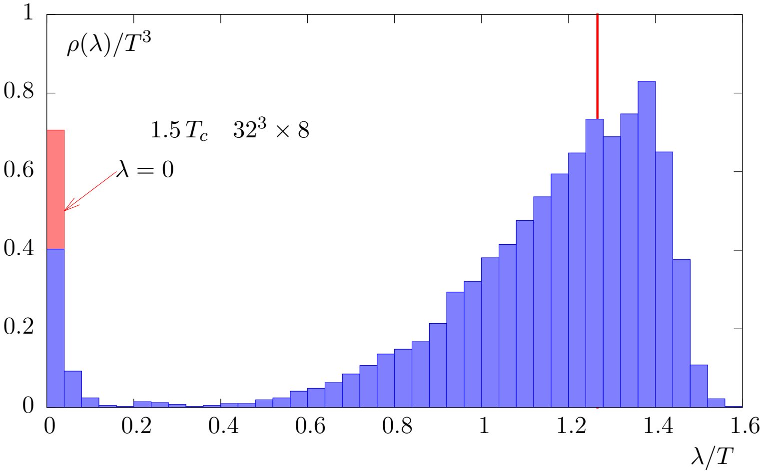
Near zero mode contribution persists at $1.2\,T_c$.

Eigenvalue density of D_{ov} at higher temperatures



The eigenvalue distribution is independent of the cut-off effects

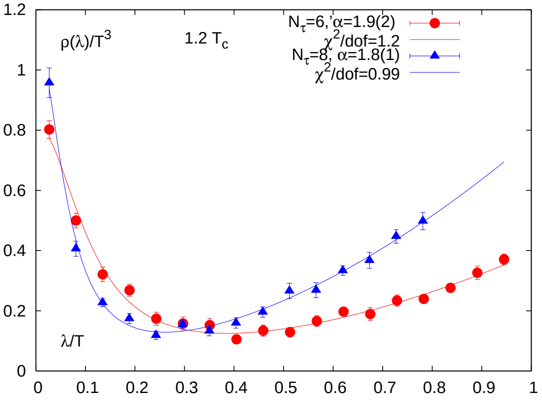
Eigenvalue density of D_{ov} at higher temperatures



Near zero modes contribution small but no gap observed even at $1.5\ T_c$.

Eigenvalue distribution at high T

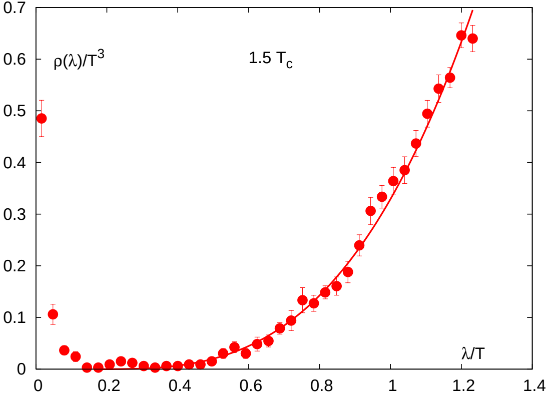
Fit the eigenvalue density to ansatz: $\rho(\lambda) = \frac{A\epsilon}{\lambda^2 + A} + B\lambda^\gamma$



Near zero mode reminiscent of $m^\alpha \delta(\lambda)$. Shows little cut-off dependence.

Eigenvalue distribution at high T

Fit the eigenvalue density to ansatz: $\rho(\lambda) = \frac{A\epsilon}{\lambda^2 + A} + B\lambda^\gamma$

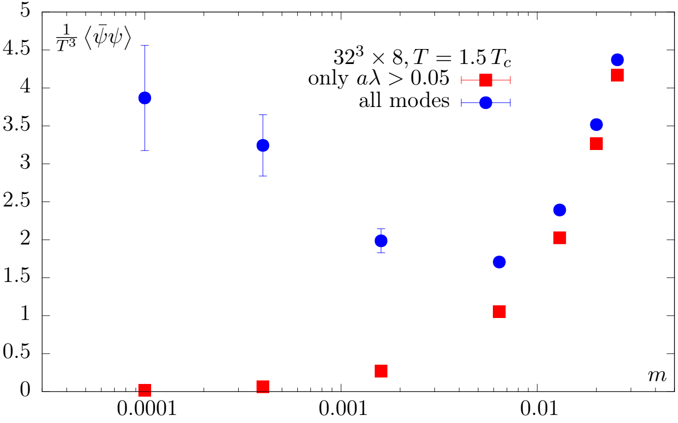


At higher T, the bulk and the near zero peak decouples

Bulk rises as λ^3.

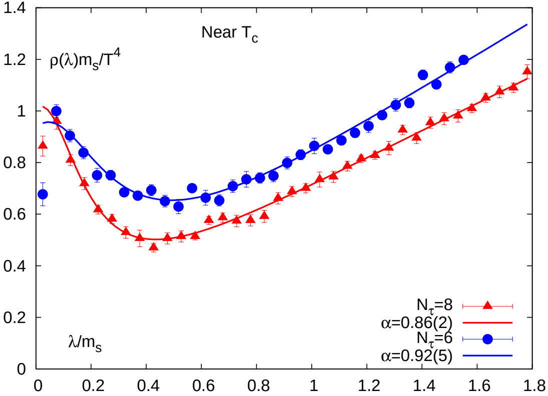
Eigenvalue distribution at high T

Fit the eigenvalue density to ansatz: $\rho(\lambda) = \frac{A\epsilon}{\lambda^2 + A} + B\lambda^\gamma$



If near zero: $\rho(\lambda) \propto m_{sea}^{\alpha}\delta(\lambda) \Rightarrow \langle\bar{\psi}\psi\rangle \sim m_{sea}^{\alpha}/m_{val}$.
Bulk: $\rho(\lambda) \propto \lambda^3 \Rightarrow \langle\bar{\psi}\psi\rangle \sim m_{val}$

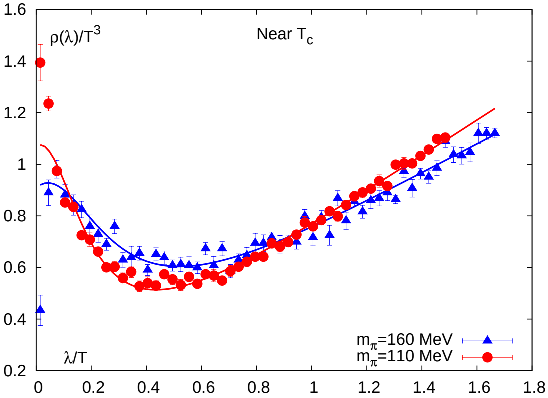
If $\rho(\lambda, m)$ analytic in $m^2 \Rightarrow$ signature of $U_A(1)$ restoration as chiral symmetry restored: $\rho \propto \lambda^3$ [Aoki, Fukaya & Taniguchi, 12]



$\sim T_c$ Bulk part rises linearly as λ.

Near zero peak sharper with finer lattice

If $\rho(\lambda, m)$ analytic in $m^2 \Rightarrow$ signature of $U_A(1)$ restoration as chiral symmetry restored: $\rho \propto \lambda^3$ [Aoki, Fukaya & Taniguchi, 12]

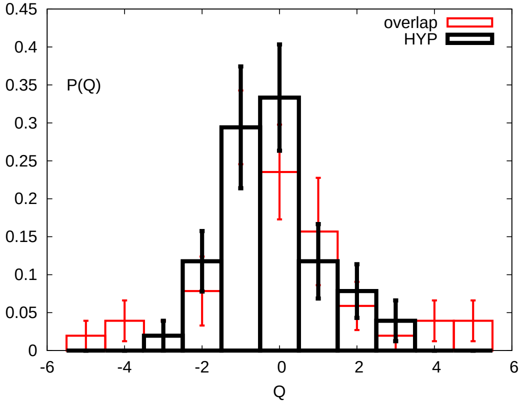


Near zero peak becomes sharper in the chiral limit $\Rightarrow$ signal of criticality?

Distribution of topological charge

Q computed from zero modes of D_{ov} compared to from the $F\tilde{F}$ on smoothened (smeared) configurations [Ohno et. al., 11]

HYP smearing results with higher statistics!



Occurrence of higher Q modes observed in both the methods.

Distribution of topological charge

Q computed from zero modes of D_{ov} compared to from the $F\tilde{F}$ on smoothened (smeared) configurations [Ohno et. al., 11]

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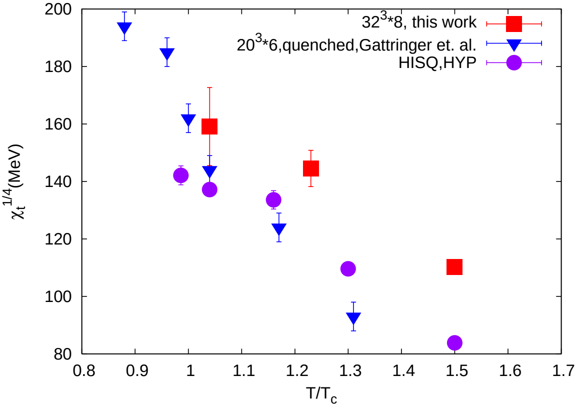


Resultant values of χ_t larger than quenched result or smearing.

Measure of $U_A(1)$

- In terms of the eigenvalues of Dirac operator [Edwards, Heller & Narayanan, 98],

$$\chi_\pi - \chi_\delta = \frac{4T}{V} \sum_{|\lambda_k|>0} \langle \frac{m^2(4-\lambda_k^2)^2}{[\lambda_k^2(4-m^2)+4m^2]^2} + 2\frac{\langle |Q| \rangle}{m^2 V}$$

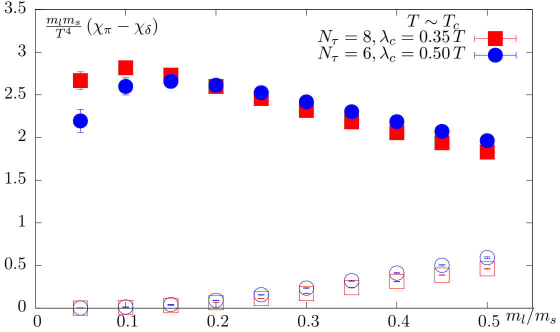
$$\langle \bar{\psi}\psi \rangle = \sum_{|\lambda_k|>0} \frac{2T}{V} \left\langle \frac{m(4-\lambda_k^2)}{[\lambda_k^2(4-m^2)+4m^2]} \right\rangle + 2\frac{\langle |Q| \rangle}{mV}$$

Measure of $U_A(1)$

- To understand physics of HISQ sea: tune valence overlap mass to HISQ sea
- m_s tuned by matching RG invariant combination $m_s^2 \frac{\frac{\langle \bar{\psi}\psi \rangle_s}{m_s} - \chi^s_{conn}}{T^4}$
- The zero modes do not affect tuning results at such large lattice volumes
- Once m_s tuned, look at the physical observables for a range of $m_l < m_s$.

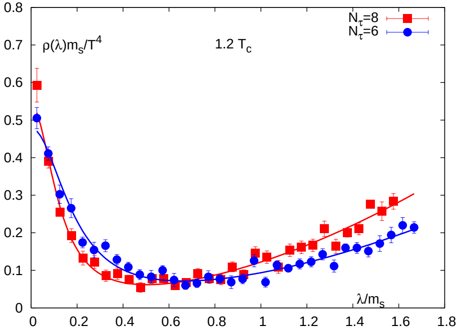
Measure of $U_A(1)$

- The renormalized $\chi_\pi - \chi_\delta$ has negligible cut-off dependence near T_c.
- Significant contribution comes from the near zero modes than the bulk $\Rightarrow$ near zero modes responsible for $U_A(1)$ breaking



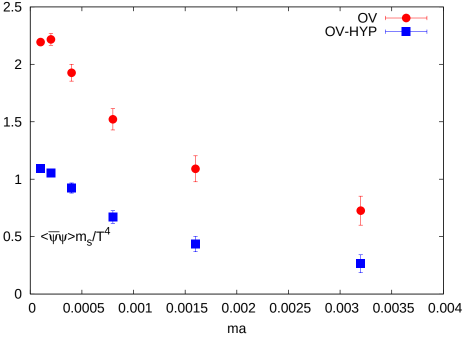
Robustness of near zero modes

- The infrared part could be affected by unphysical dislocations.
- Effect of partial quenching as well as rough configurations.
- These have smaller classical action than instantons.
- Sensitive to lattice cut-off effect
- We do not observe any significant cut-off dependence of the renormalized eigenspectrum at high T.



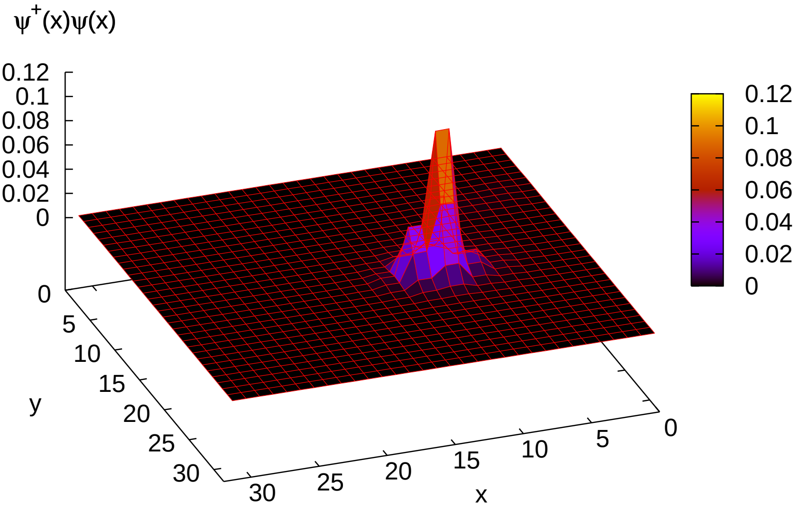
Robustness of near zero modes

- Dislocations have a smaller action that the instantons.
- HYP smearing [Hasenfratz & Knechtli, 02] eliminates such small localized structures.

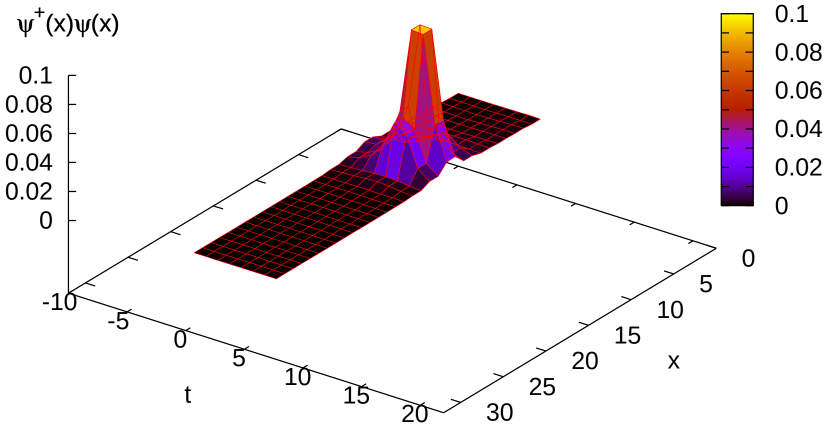


- Smearing does not completely eliminate the near zero modes.
- Difference? Smearing may suppress small instantons.

Localization properties of zero modes at 1.5 T_c



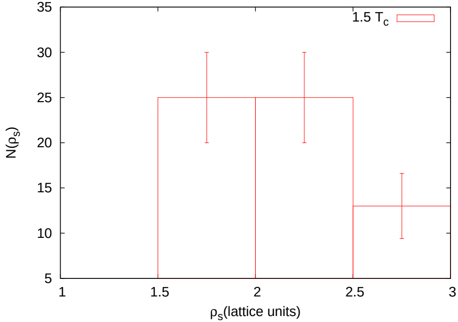
Localization properties of zero modes at 1.5 T_c



Localization properties of zero modes at $1.5\ T_c$

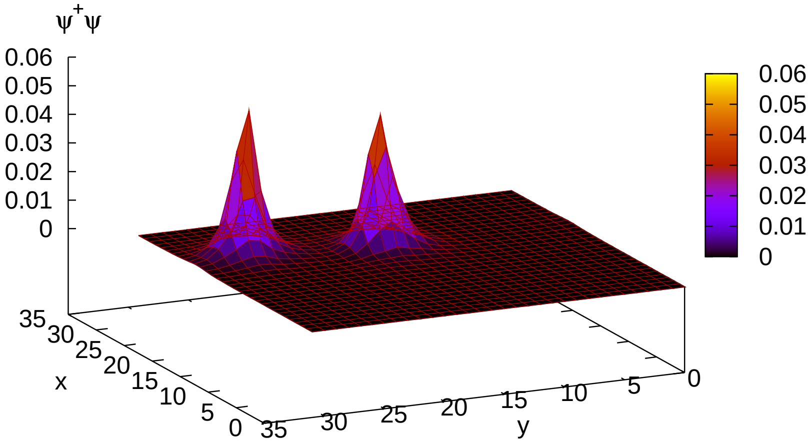
At $T = 0$, fermion zero modes associated with instantons give

$$\psi_0^\dagger(x)\psi_0(x) \simeq \frac{\rho^2}{\pi}\frac{1}{(x^2+\rho^2)^3}\ , \tag{1}$$

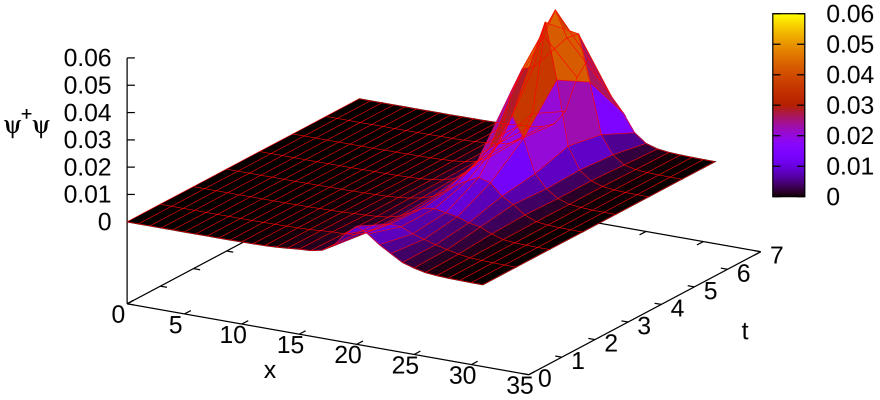


$\rho = 0.23(4)$ fm ¡ $L \sim 3.3$ fm and $1/T \sim 0.8$ fm .

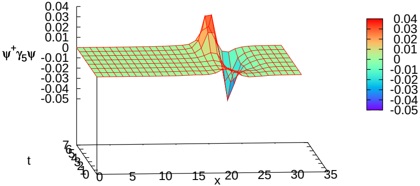
The near-zero modes

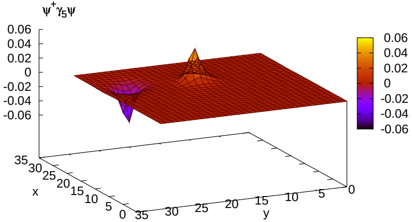


The near-zero modes



The near-zero modes





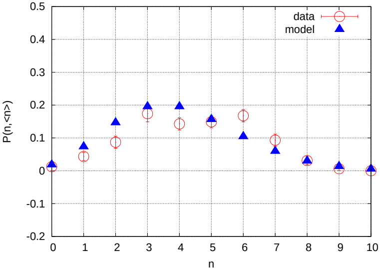
- The chiral density profiles gives us a hint that these are due to superposition of instanton-antiinstanton pairs.
- Consistent with finite T instanton solutions with trivial holonomy and radius $\rho < 1/T$. [Harrington & Shepard, 78]

The nature of the near zero modes at $1.5\,T_c$

- Presence expected from dilute instanton gas approximation(DIGA)? Observed for pure SU(3) gauge theory
  [Edwards, Heller, Kiskis & Narayanan, 99, Gavai & Gupta, 02].
  Is it expected in QCD where fermion interactions present?
- If n=total no. of instantons+antiinstantons, according to DIGA

$$P(n,\langle n\rangle) = \langle n\rangle^n \mathrm{e}^{-\langle \mathrm{n}\rangle}/\mathrm{n}!$$

- For $\lambda/T < 0.4$, the value of $\langle n\rangle = 4 = \langle n^2\rangle \Rightarrow$ density $\simeq 0.13 fm^{-4}$.

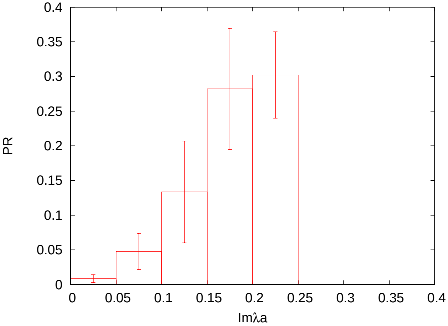


The measures of localization: PR

The Participation Ratio measures the degree of localization

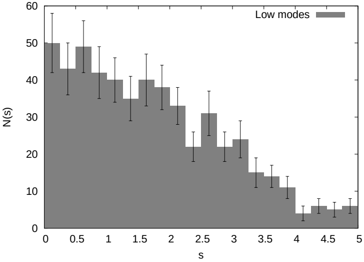
$$PR = \frac{1}{N^3 N_\tau} \left[\sum_x \left(\psi^\dagger(x)\psi(x) \right)^2 \right]^{-1} .$$

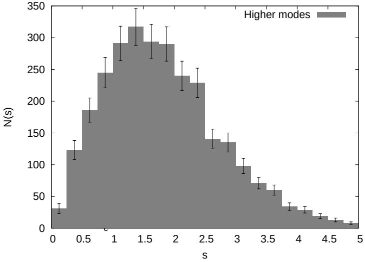
$PR = 1$ (delocalized), $PR = f$ (localized)
The low-lying modes are highly localized unlike the bulk $\rightarrow$ Anderson-Mott localization? [Garcia-Garcia & Osborn, 06, Kovacs et al, 12,13].



The characteristics of the eigenspectrum

- The level spacing between eigenvalues has a system dependent mean$+$ fluctuations which are universal
- Unfolding or removing the bias gives fluctuations consistent with Poissonian for $\lambda/T < 0.4$.
- Level spacing of large eigenvalues seem to be following predictions from Random Matrix Theory with Gaussian $\rightarrow$ delocalized.

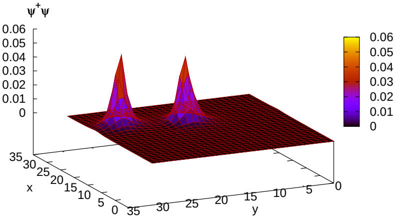


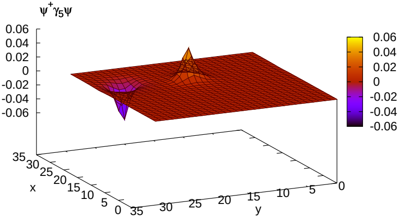


Outline

What we studied till now

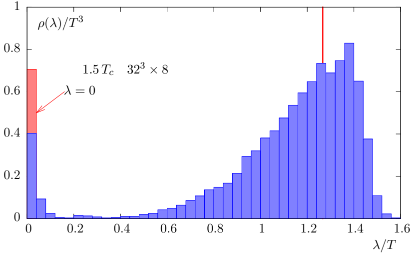
- We have studied the topological structure of large volume HISQ fermion configurations used for QCD thermodynamics.
- All these configurations have topological structures even at 1.5 T_c.

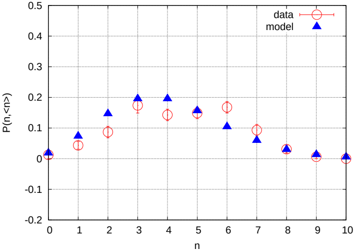




What we studied till now

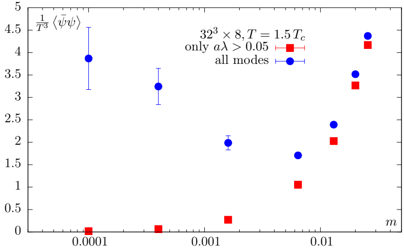
- Significant presence of near zero modes even at this temperature $\Rightarrow$ $U_A(1)$ is not restored.
- The high temperature phase can be described as dilute gas of instantons: av radius $\rho = 0.23(4)fm$, density $=0.13fm^{-4}$.

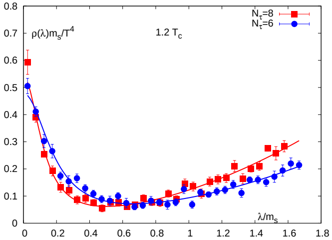




What we studied till now

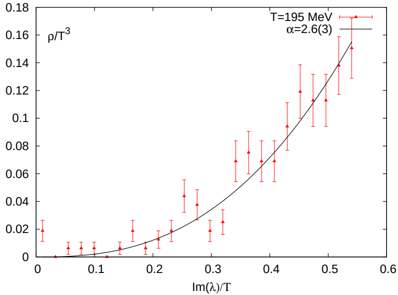
- The near zero peak at $T > T_c$ consistent with $\delta(\lambda)$ behaviour consistent with dilute instanton gas model.
- These are primarily responsible for $U_A(1)$ breaking





Future directions

- Use overlap on chiral domain wall fermion configurations. Much clearer interpretation of the topology issues.
- First results exciting! near zero peak persists near $1.2\ T_c$.



- Study microstructures or dyons in calorons with non-trivial holonomy.
- Understand implications of the presence of localized near-zero modes.