Bose-Einstein Condensation, Isotropization, and Thermalization in Overpopulated Systems

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OUTLINE

• Overpopulated Glasma & Bose-Einstein Condensation
• The Dynamical Onset of BEC
• Thermalization in Overpopulated Scalar System
• Isotropization in Overpopulated Scalar System
• Summary

References:
Blaizot, JL, McLerran, Nucl. Phys. A920, 58(2013);
Approach to Hydro Onset: How?

\[ Q_s \sim 10 \Lambda_{QCD} \]

it should be amenable to a weakly coupled description

\[ A \sim 1/g \]

initially dominated by strong classical field

\[ T_{max} \sim 2 \Lambda_{QCD} \]

it is plausibly a strongly coupled plasma

\[ f \sim 1 \]

dominated by quanta

How thermalization happens?
And quickly??
Overpopulated Glasma

The precursor of a thermal quark-gluon plasma, known as glasma, is born as a gluon matter with **HIGH OVERPOPULATION**:

Very large occupation number

\[ f \sim \frac{1}{\alpha_s} \]

\[ E \sim \Lambda \]

\[ M_D \sim (gf^{1/2})\Lambda \]

\[ \Lambda_s \sim (g^2 f)\Lambda \]

Key observations:
- scale separation;
- \( O(1) \) scattering rate

\[ f \star f \star \alpha_s^2 \sim O(1) \]
Unexpected “Detour”: BEC

We started out to derive a kinetic equation and solve it for verifying our expected thermalization via scaling solution…

\[
D_t f(p) = \xi \left( \Lambda_s^2 \Lambda \right) \nabla \cdot \left[ \nabla f(p) + \frac{p}{\Lambda_s} \left( \frac{\alpha_s}{\Lambda_s} \right) f(p) [1 + f(p)] \right]
\]

Two important scales:
- hard scale Lambda
- soft scale Lambda_s

The numerical evolution kept blowing up despite months’ struggle of finding any potential error …

At some point we finally realized:

THE OVERPOPULATED SYSTEM IS DRIVEN TO A TRUE PHYSICAL SINGULARITY WHERE BEC OCCURS!
Strong Evidence of BEC from Scalar Field Theory Simulations

Absolutely true for pure elastic scatterings; True, in transient sense, for systems with inelastic processes

From: Berges & Sexty 1201.0687

From: Epelbaum & Gelis 1107.0668
Overpopulation: Thermodynamic Consideration

Our initial gluon system is highly OVERPOPULATED:

\[ f(p) = f_0 \theta(1 - p/Q_s), \]

\[ \epsilon_0 = f_0 \frac{Q_s^4}{8\pi^2}, \quad n_0 = f_0 \frac{Q_s^3}{6\pi^2}, \quad n_0 \epsilon_0^{-3/4} = f_0^{1/4} \frac{2^{5/4}}{3\pi^{1/2}}, \]

This is to be compared with the thermal BE case:

\[ n \epsilon^{-3/4} |_{SB} = \frac{30^{3/4} \zeta(3)}{\pi^{7/2}} \approx 0.28 \]

Overpopulation occurs when: \[ f_0 > f_0^c \approx 0.154 \]

Identifying \( f_0 \rightarrow 1/\alpha_s \), even for \( \alpha_s = 0.3 \), the system is highly overpopulated!!

Overpopulation \( \longrightarrow \) BEC
Quantum Coherence implies OVERPOPULATION:

$$\frac{\Lambda dB}{d} \sim \left(n\epsilon^{-3/4}\right) \alpha \sim \hat{O}(1)$$
BEC in The Very Cold

Brilliant evaporative cooling: precisely to achieve OVERPOPULATION

Cooling procedure: kick out fast atoms (truncating UV tail); then let system relax toward new equilibrium; relaxation via IR particle cascade & UV energy cascade.

It took ~70 years to achieve OVERPOPULATION, thus BEC in ultra-cold bose gases.

\[ n \cdot \epsilon^{-3/4} > \hat{O}(1) \text{ threshold} \]
# BEC in the Very Hot!

## Temperature

<table>
<thead>
<tr>
<th>$10^{-8}K$</th>
<th>$10^0K$</th>
<th>$10^1K$</th>
<th>$10^2K$</th>
<th>$\sim$</th>
<th>$10^{12}K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>cold atomic gas</td>
<td>liquid helium;</td>
<td>magnon</td>
<td>cavity photon;</td>
<td>overpopulated glasma!</td>
<td></td>
</tr>
<tr>
<td>cosmic axion?</td>
<td></td>
<td></td>
<td>magnon</td>
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</tbody>
</table>
Another example: idea of overcooled pion gas in heavy ion collisions.

Key point: under suitable conditions, non-conserved particles may become effectively or transiently conserved.
Can Kinetic Theory Describe BEC?

Kinetic description is widely used for BEC phenomena (trapped atoms, hard sphere gas, polaritons, cosmic scalars, ...)
Kinetic Equations with Small Angle Scatterings

\[ \mathcal{D}_t f(p) = \xi (\Lambda^2_s \Lambda) \nabla \cdot \left[ \nabla f(p) + \frac{\vec{p}}{p} \left( \frac{\alpha_s}{\Lambda_s} \right) f(p) [1 + f(p)] \right] \]

UV & IR cascade

\[ \mathcal{D}_t f = -\nabla \cdot J \]

f_0=0.1 (underpopulated)  
f_0=1 (overpopulated)

Blaizot, JL, McLerran, 1305.2119, NPA2013
How Thermalization Proceeds

Initial glasma: \( \Lambda \sim \Lambda_s \sim Q_s \)

Equilibrium Distribution (with the same Energy density)

Initial gluon distribution

Separation of two scales toward thermalization

Saturation Scale \( Q_s \sim 1 \text{ GeV} \) or larger, weakly coupled

Particle cascade toward IR

Energy cascade toward UV

Thermalized weakly-coupled QGP:

\[ \Lambda \sim T \]

\[ \Lambda_s \sim \alpha_s \times T \]

\[ \frac{\Lambda_s}{\Lambda} \sim \alpha_s \]
How BEC Onset Occurs Dynamically?

A crucial step: rapid IR local thermalization

Very strong particle flux toward IR, leading to rapid growth and almost instantaneous local thermal distribution of very soft modes

What happens next depends on INITIAL CONDITION: underpopulation v.s. overpopulation

Blaizot, JL, McLerran, 1305.2119, NPA2013
In underpopulated case, the system thermalizes to thermal BE distribution.
Overpopulated Case: How Onset of BEC Develops?

Before it could reach equilibrium, onset of BEC occurs! A critical IR distribution develops, i.e. $\mu^*$ vanishes. (In thermal BEC: global distribution must be critical.)

\[ f(p \to 0) \to \frac{T^*}{p - \mu^*} \]

$\mu^* \to 0$

\[ f_0 = 1 \]
Overpopulated Case: How Onset of BEC Develops?

\[ f \xrightarrow{p \ll M\mu^*} \frac{T^*}{|\mu^*|} \]

\[ f(p \to 0) \to \frac{T^*}{p - \mu^*} \]

\[ f \xrightarrow{p \gg M\mu^*} \frac{T^*}{p} \]
Overpopulated Case: How Onset of BEC Develops?

\[ f \xrightarrow{p \gg \mu^*} \frac{T^*}{p} \]

\[ f \rightarrow \frac{T^*}{|\mu^*|} \]

\[ f(p \rightarrow 0) \rightarrow \frac{T^*}{p - \mu^*} \]
Overpopulated Case: How Onset of BEC Develops?

\[
f \rightarrow \frac{T^*}{|\mu^*|} \quad \text{for} \quad p << \mu^*
\]

\[
f \rightarrow \frac{T^*}{p} \quad \text{for} \quad p >> \mu^*
\]

\[
f(p \rightarrow 0) \rightarrow \frac{T^*}{p - \mu^*}
\]
Overpopulated Case: How Onset of BEC Develops?

\[
f \rightarrow \frac{T^*}{\mu^*} \\
\mu^* \rightarrow 0
\]

proceed in a self-similar scaling way

\[
f(p \rightarrow 0) \rightarrow \frac{T^*}{p - \mu^*}
\]

\[ p << \mu^* \]

\[ p >> \mu^* \]
Onset of Dynamical BEC

Onset of dynamical (out-of-equilibrium) BEC:
* occurring in a finite time
* local $\mu^*$ vanishes with a scaling behavior
* persistence of particle flux toward zero momentum

\[ |\mu^*| = C(\tau_c - \tau)^\eta. \]
\[ \eta \approx 1 \]

For different $f_0 = 0.2, 0.3, 0.5, 0.8, 1, 2, 5$

Blaizot, JL, McLerran, 1305.2119, NPA2013
Effects of Finite Masses

Interesting issues when there is finite external mass:

* Onset changes, $\mu^*$ --> Mass
* Deep IR dispersion changes, $\sim p^2$ (NR) instead of $\sim p$ (UR)

Interesting issues when there is finite screening mass: no more enhancement of small angle scatterings.

Very similar onset dynamics as in the massless case!
Including the Inelastic

An inelastic kernel including 2<-->3 processes
(Gunion-Bertsch, under collinear and small angle approximation)

\[ D_t f_p = C_{2\leftrightarrow 2}^{\text{eff}}[f_p] + C_{1\leftrightarrow 2}^{\text{eff}}[f_p], \]

\[ C_{1\leftrightarrow 2}^{\text{eff}} = \xi \alpha_s^2 R \frac{I_a}{I_b} \left\{ \int_0^{z_c} \frac{dz}{z} \left[ g_p f_{(1-z)p} f_{zp} - f_p g_{(1-z)p} g_{zp} \right] \right. \]

\[ \left. + \int_0^{z_c} \frac{dz}{(1-z)^4} \left[ g_p g_{zp/(1-z)} f_p/(1-z) - f_p f_{zp/(1-z)} g_p/(1-z) \right] \right\} \]

Huang & JL, arXiv:1303.7214

A number of features:
* fixed point: BE distribution with zero chemical potential
* always positive at very small momentum
* purely inelastic case --- correctly thermalize to BE

The question changes now:
no condensate in thermal states,
but dynamical BEC while still far from being thermal.
Effects from the Inelastic

Local effect: enhance IR growth, accelerate the onset

Global effect: reduce number density, enhance entropy growth

R: ratio of the inelastic to the elastic kernel

Huang & JL, arXiv:1303.7214
The “Fuller” Picture

What we find: the inelastic process catalyzes the onset of dynamical (out-of-equilibrium) BEC. It might sound contradicting with common wisdom ... but it is NOT.

Increasing 
Inelastic

R increases

Elastic only
Evolution beyond Onset

* To evolve the system beyond onset, one needs a set of kinetic equations describing the co-evolution of condensate + gluons.
* It is difficult (at the moment) to do that for the gauge field system.
* We instead study the SCALAR SYSTEM to explore the interesting interplay between condensate and particles toward thermalization.

Kinetic equations for scalar system:  \[ |\mathcal{M}|^2 = \chi^2 \]  

Two types of fixed points from under-/over-populated initial conditions:

1. a Bose-Einstein distribution \[ g_{BE} = \frac{1}{e^{(E-\mu)/T} - 1} \] with any \( \mu \leq M \) and zero condensate \( n_c = 0 \);
2. a Bose-Einstein distribution \[ g_{BE} = \frac{1}{e^{(E-\mu)/T} - 1} \] with \( \mu = M \) and a nonzero condensate \( n_c > 0 \).
Evolution before Onset of BEC

Rapid growth of infrared occupation in a self-similar scaling fashion

\[ \epsilon = E(p) - M \]

Classical thermal @ deep IR

Intermediate IR

Self-similar scaling

UV tail
Self-Similar Scaling Analysis

Scaling from stationary cascade (c.f. Semikoz-Tkachev)

\[ f(\epsilon, \tau) = A^{-\alpha}(\tau)f_s(\epsilon/A(\tau)) \]
\[ f(0, \tau) \propto [(\tau_c - \tau)(\alpha - 1)]^{-\alpha/(2(\alpha - 1))} \]

We have found consistent scaling exponents in this case.

Note: S-T uses classical limit of kinetic equations, while we maintain full quantum factors.
Evolution after Onset of BEC

Two remarks:
* $f(p)$ gradually switches from $f \sim 1/p^{2}$ toward $f \sim 1/p$
* McLerran parameterization

$$f \rightarrow \frac{T^*}{E - M}$$

UV tail

$$f \rightarrow e^{-(E-M)/T^*}$$

classical thermal @ IR

$$(T^*)_{IR}$$

$$(T^*)_{UV}$$
Final Approach toward Thermalization

$T \to T_{th}^{h}(1 + ce^{-\tau/\tau_r})$

$\tau_r \simeq 0.254$

$n_c \to n_c^{th}(1 - ce^{-\tau/\tau_r})$

$\tau_r \simeq 0.271$

Pertinent time scale:

$t = \tau \times \frac{64\pi^3}{\lambda^2}$

$64\pi^3 \simeq 1984$

$f_0 \sim 8$

$t_{th} \sim \hat{O}(10^{3\sim4})$
Interesting questions:
* How anisotropy affects evolution, particularly BEC onset?
* How the system evolves toward isotropy?
[Note: static box for now, but anisotropic I.C.]
Evolution from Anisotropic I.C.

* IR part essentially maintains isotropy all the time
* Same IR self-similar scaling behavior before onset
* Same IR classical thermal after onset
* UV tails keep adjusting toward isotropy
Isotropization from Anisotropic I.C.

underpopulated case

\[ \tau_{iso} \sim \frac{64 \pi^3}{\chi^2 f_0^2} \]
We now study the overpopulated case: in particular the comparison between the classical limit and the full quantum.

**pressure isotropization**

The system appears to have difficulty with isotropization in the classical limit — WHY?

* Isotropization mostly concerns ~UV scale where occupation $f \sim O(1)$ or even less

* The classical approximation underestimates isotropizing scatterings:

$$f_L f_L (1 + f_T)(1 + f_T) - f_T f_T (1 + f_L)(1 + f_L) = (2 f_T f_L + f_L + f_T)(f_L - f_T)$$
Summary

* Initial gluon system at very early stage of a heavy ion collision is characterized by **high overpopulation**.

* Elastic process (alone) in highly overpopulated system can induce **very rapid growth of soft modes** and drive toward equilibration. This is a very robust feature and may lead to a transient **Bose-Einstein Condensate**.

* Dynamical onset of BEC in a scaling way is found to be a very robust feature despite many details.

* Inelastic processes may further enhance the rapid growth of soft modes and **catalyze the onset** of BEC (but will remove the condensate afterwards). The time window for a condensate could be sizable.

* We hope to be able to include longitudinal expansion, and to quantitatively compare kinetic results with other approach soon.