# Geometrical scaling –
# – a window to saturation

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BNL 14.5.2015.

# message to take away:

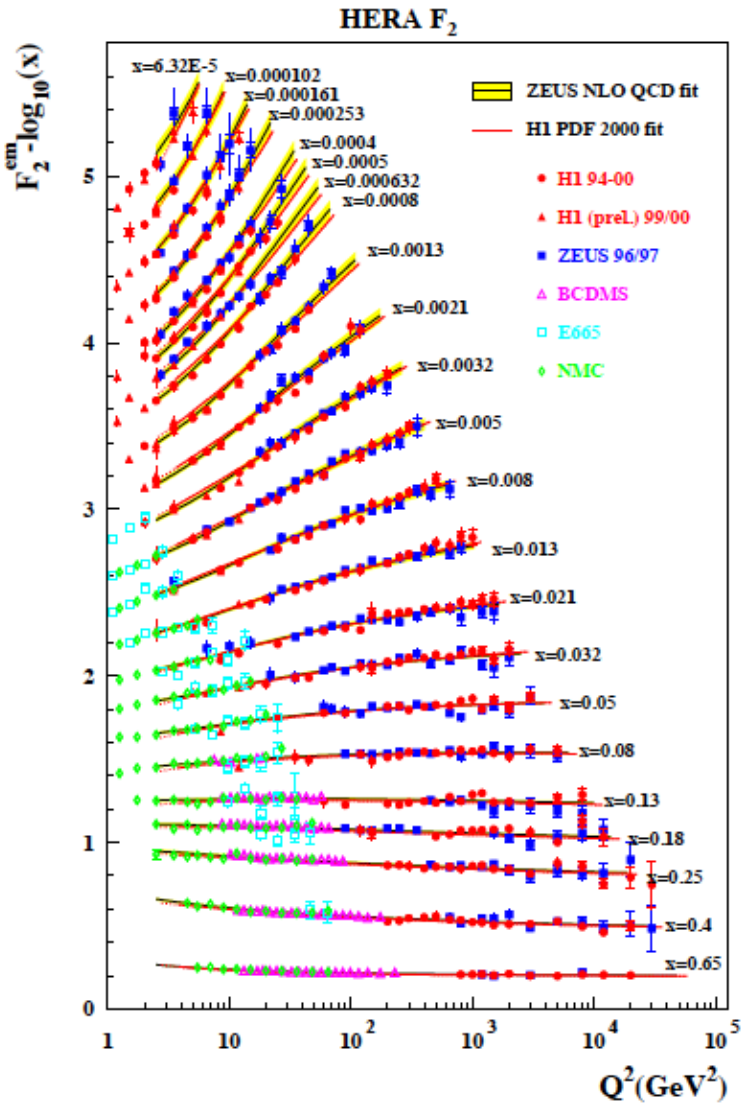
There exits an intermediate energy scale, called *saturation scale*, that, by dimensional arguments, deterimines inclusive and semi--inclusive observables in kinematical regions where no other energy (momentum) scales exist.

# message to take away:

**How does this scale emerge?**

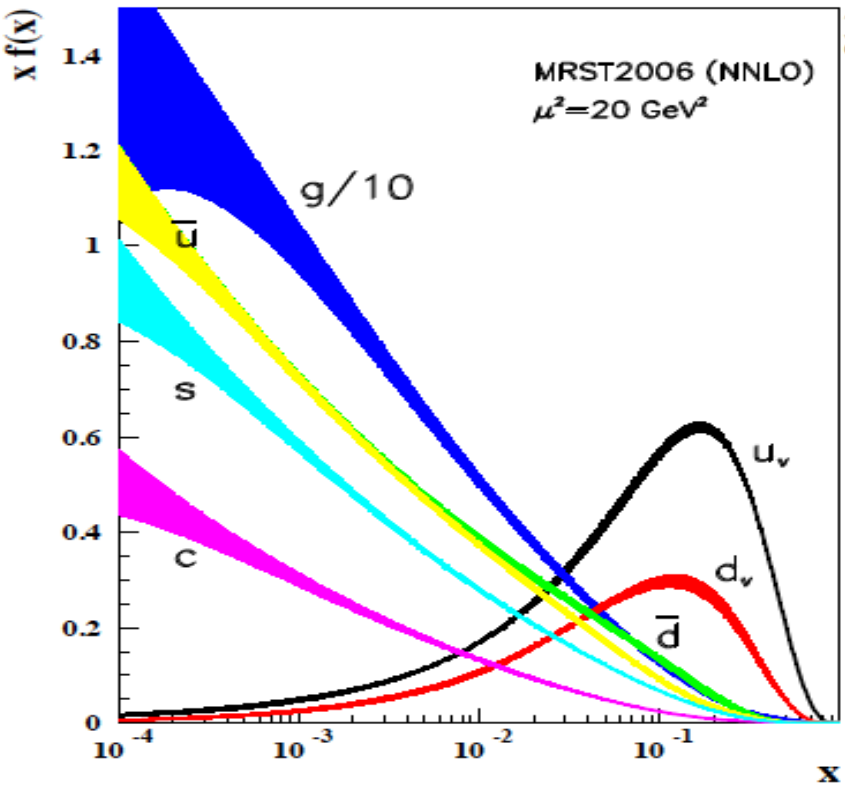
There exits an intermediate energy scale, called *saturation scale*, that, by dimensional arguments, deterimines inclusive and semi--inclusive observables in kinematical regions where no other energy (momentum) scales exist.

# DIS @ HERA

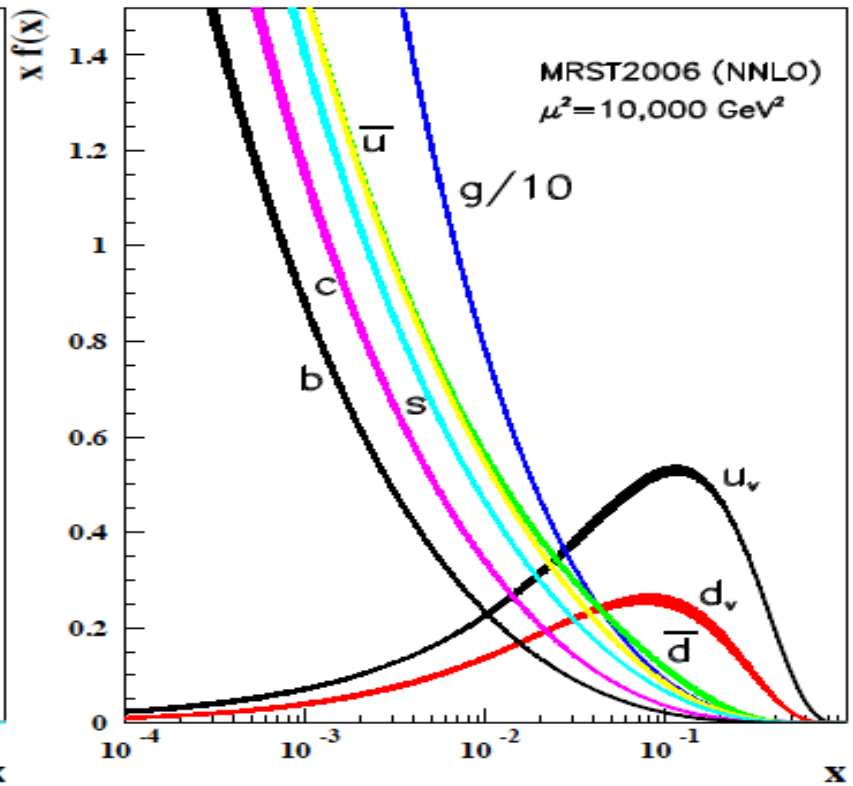

HERA $F_2$
$F_2^{em}$-$\log_{10}(x)$
$Q^2(GeV^2)$
ZEUS NLO QCD fit
H1 PDF 2000 fit
H1 94-00
H1 (prel.) 99/00
ZEUS 96/97
BCDMS
E665
NMC
x=6.32E-5
x=0.000102
x=0.000161
x=0.000253
x=0.0004
x=0.0005
x=0.000632
x=0.0008
x=0.0013
x=0.0021
x=0.0032
x=0.005
x=0.008
x=0.013
x=0.021
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.25
x=0.4
x=0.65

# DGLAP & BFKL Evolution

$Q^2 = 20\ \mathrm{GeV}^2$

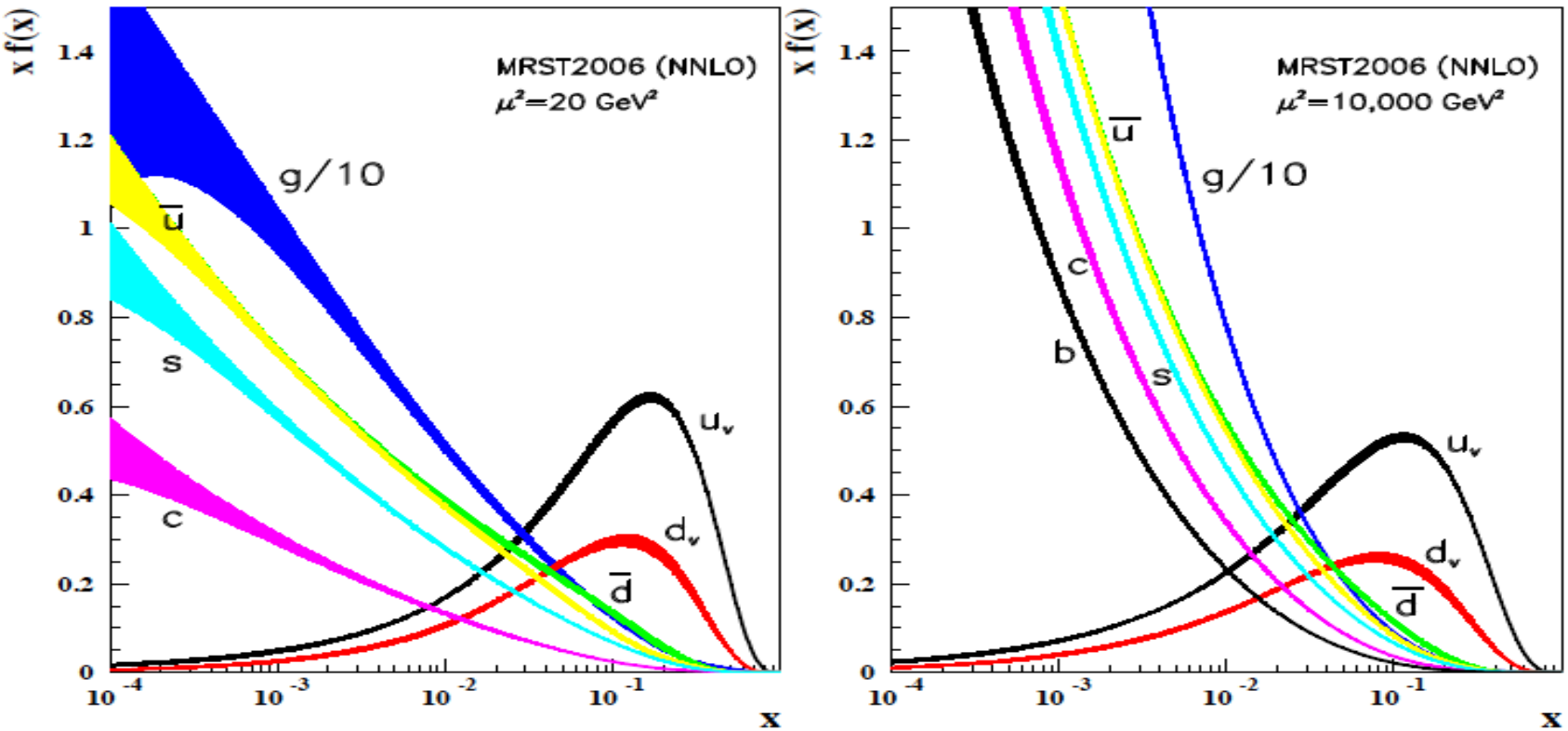



$Q^2 = 10\ 000\ \mathrm{GeV}^2$



# DGLAP & BFKL Evolution

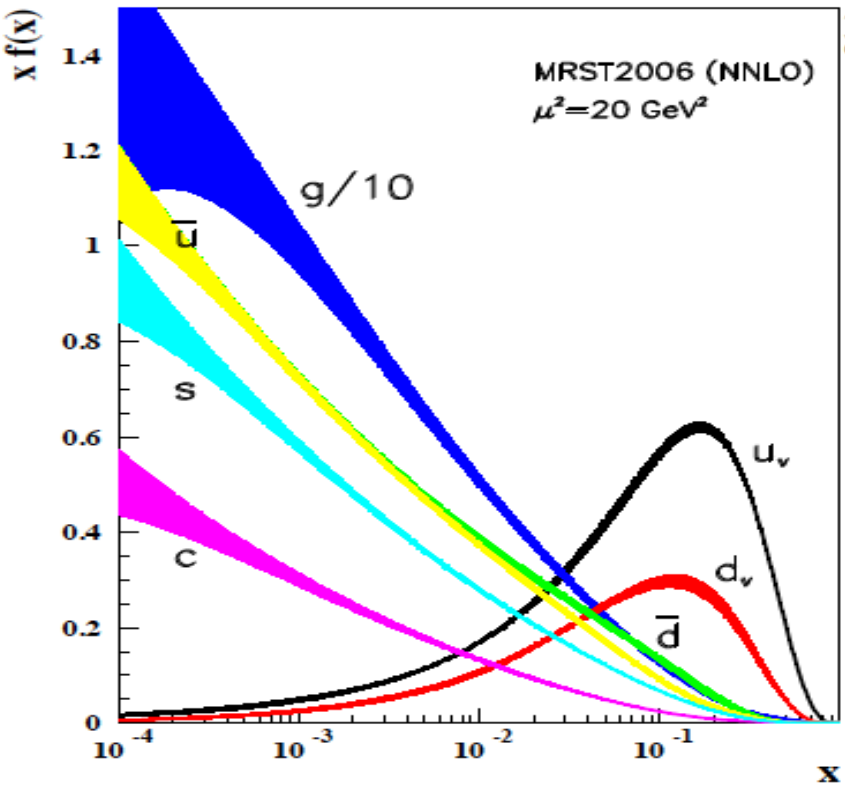
$Q^2 = 20\ \mathrm{GeV}^2 \xrightarrow{\text{DGLAP}} Q^2 = 10\,000\ \mathrm{GeV}^2$

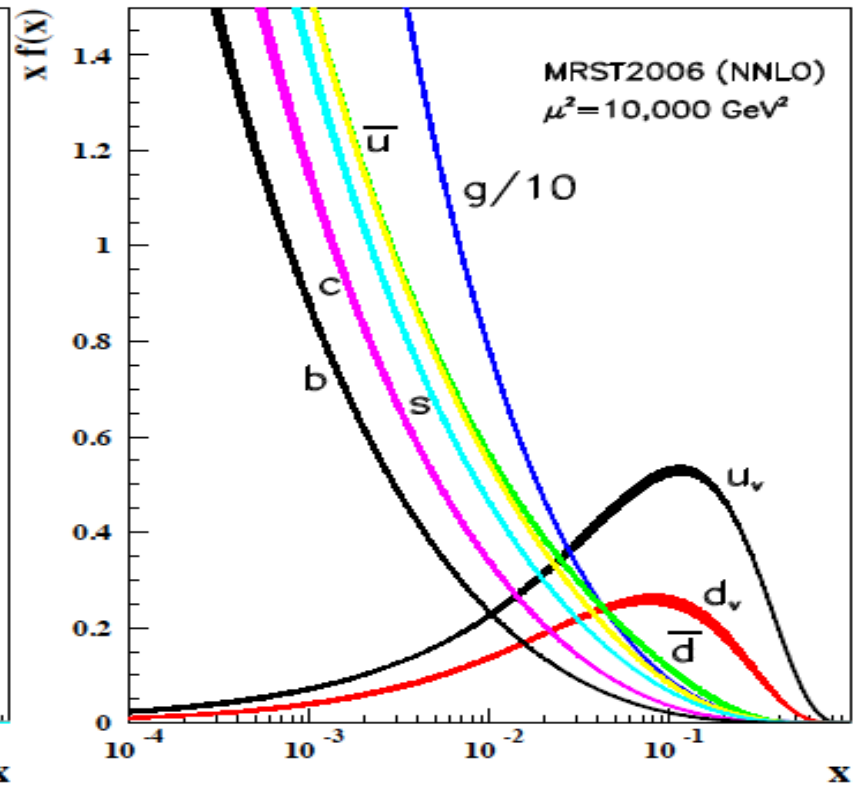


# DGLAP & BFKL Evolution

DGLAP

$Q^2 = 20\ \mathrm{GeV}^2$ → $Q^2 = 10\ 000\ \mathrm{GeV}^2$



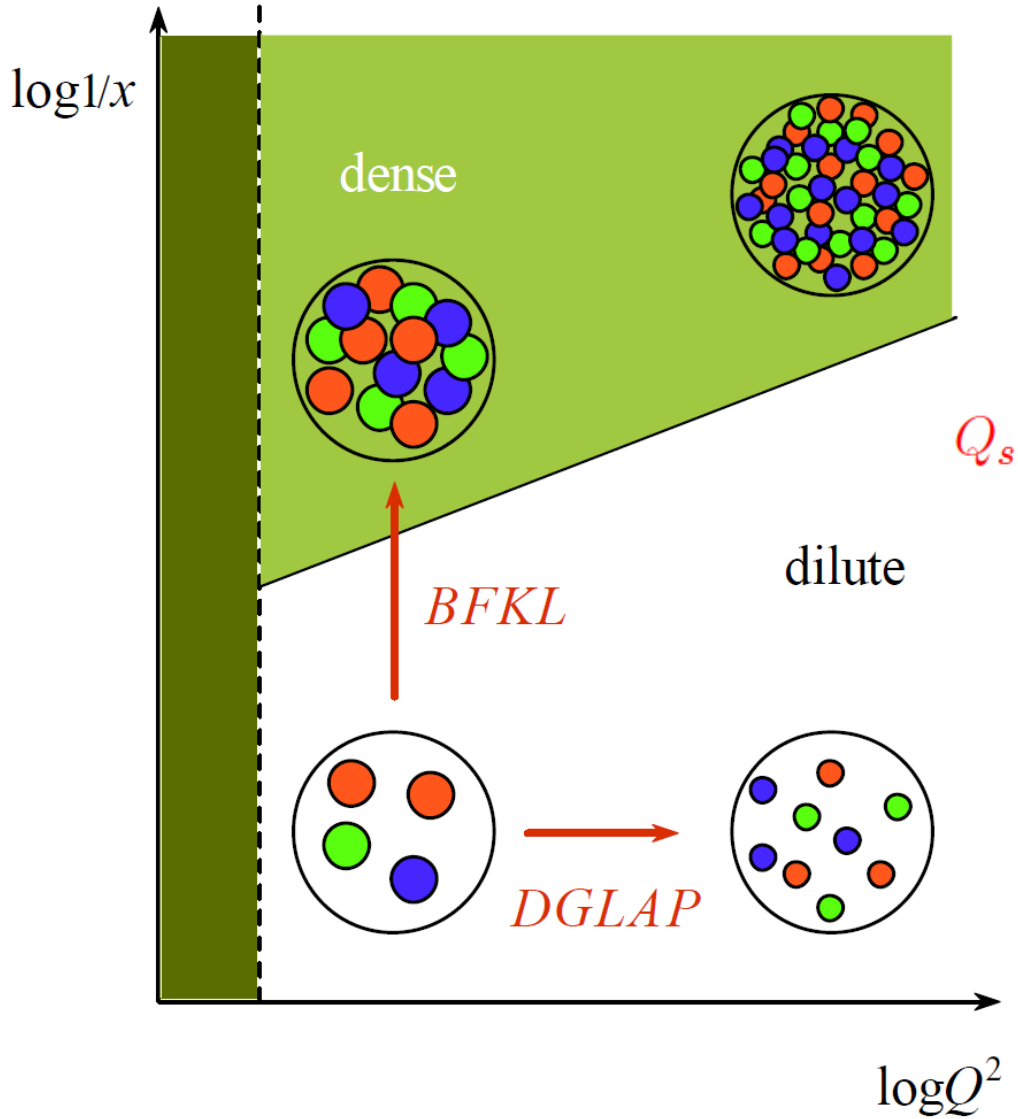

← BFKL

← BFKL

# DGLAP vs BFKL Evolution

small $x$
large $W$

Balitsky, Fadin, Kuraev, Lipatov




$$Q_s(x) = Q_0 \left(\frac{x_0}{x}\right)^{\lambda/2}$$

large $x$
small $W$

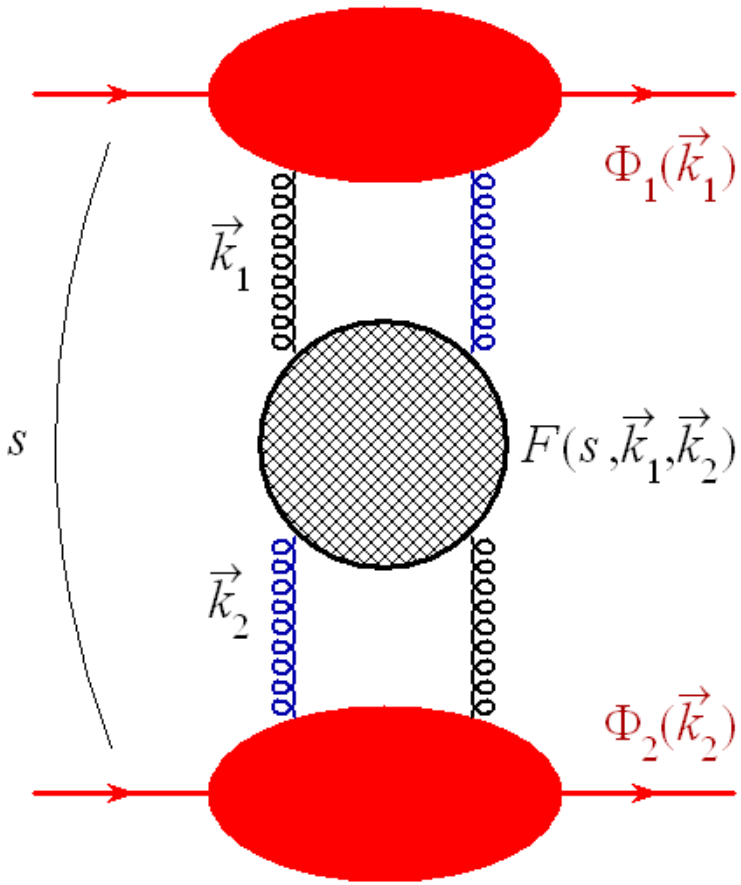
# Forward BFKL Amplitude

$$\sigma_{\text{tot}} = \frac{C_R}{(2\pi)^4} \int \frac{d^2\vec{k}_1}{\vec{k}_1^{\,2}} \frac{d^2\vec{k}_2}{\vec{k}_2^{\,2}} \, \Phi_1(\vec{k}_1)\Phi_2(\vec{k}_2) \, F(s, \vec{k}_1, \vec{k}_2)$$

transverse momenta

process dependent impact factors

energy dependent universal amplitude

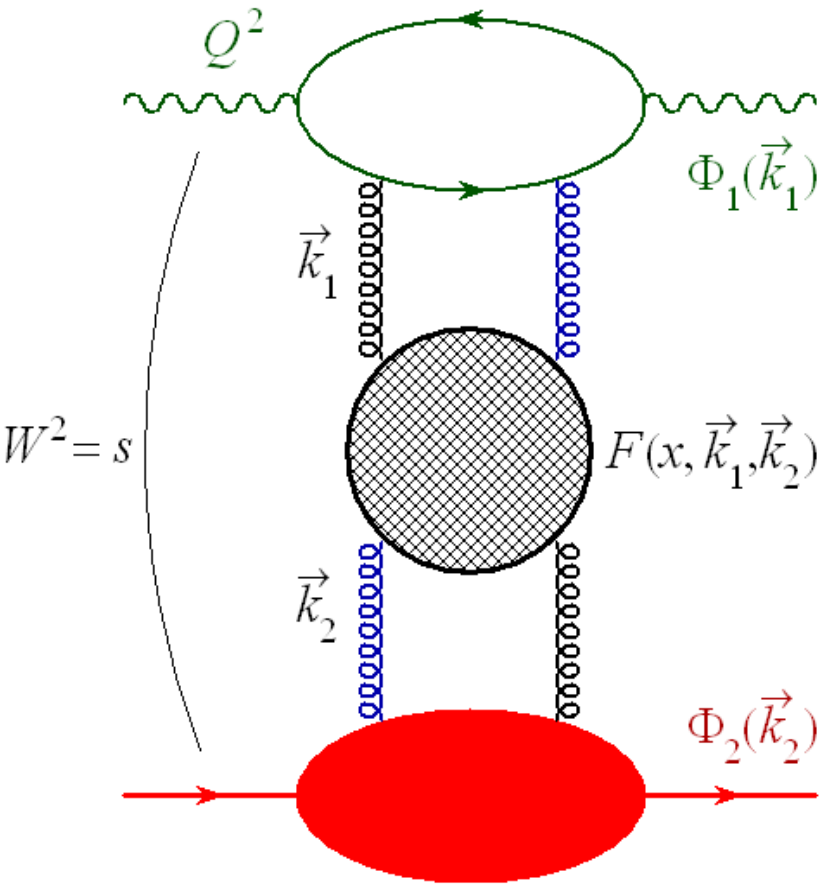



zeroth order „initial condition”:

$$F(s_0, \vec{k}_1, \vec{k}_2) \sim \delta^{(2)}(\vec{k}_1 - \vec{k}_2)$$

# Forward BFKL Amplitude

can be applied in DIS:




$$x = \frac{Q^2}{W^2 + Q^2} \simeq \frac{Q^2}{W^2} = \frac{s_0}{s}$$

# Solution of BFKL equation

$$F(s,\vec{k}_1,\vec{k}_2) = \frac{1}{\pi}\frac{1}{\sqrt{\vec{k}_1^{\,2}\vec{k}_2^{\,2}}}\left(\frac{s}{s_0}\right)^{\overline{\alpha}_s 4\ln 2}\frac{1}{\sqrt{\pi\Delta(s)}}\exp\left(-\frac{\ln^2(\vec{k}_1^{\,2}/\vec{k}_2^{\,2})}{\Delta(s)}\right)$$

$$\Delta(s) = 56\overline{\alpha}_s\zeta(3)\ln(s/s_0)$$

# BK Equation

$Y = \ln(\ 1/x\ )$

$$\tilde{N}(\vec{k}, Y) \ \sim \ \alpha_s \int F(x, \vec{k}, \vec{l}) \Phi(\vec{l}) \frac{d^2\vec{l}}{\vec{l}^2}$$

$$\frac{\partial}{\partial Y} \tilde{N}(k, Y) \ = \ \overline{\alpha}_s \, \chi(-\partial/\partial \ln k^2) \, \tilde{N}(k, Y) - \overline{\alpha}_s \, \tilde{N}^2(k, Y)$$

here $\chi$ is a BFKL characteristic function related to the kernel $K(k_1, k_2)$

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)$$

there exists a theorem from the '30 (Fisher, Kolomogorov, Petrovsky, Piscounov) that non-linear equations of this sort have asymptotically travelling wave solutions

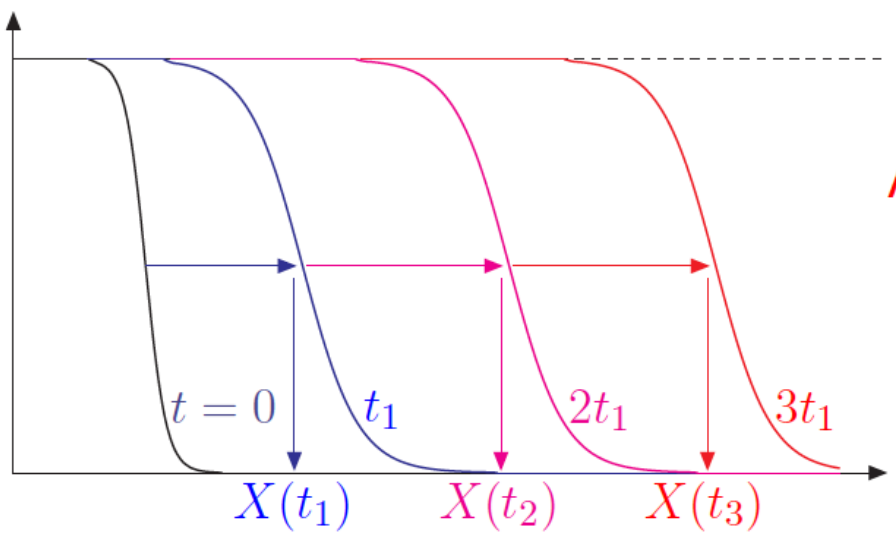
S. Munier, R.B. Peschanski
PRL 91 (2003) 232001
PRD 69 (2004) 034008

# Travelling waves

identify time : $t = Y$, position : $x = \ln k^2$

$$\boxed{\partial_t u(x,t) = \partial_x^2 u(x,t) + u(x,t) - u^2(x,t)}$$

Fisher-Kolmogorov-Petrovsky-Piscounov (F-KPP) 1937




Asymptotic solution:
travelling wave

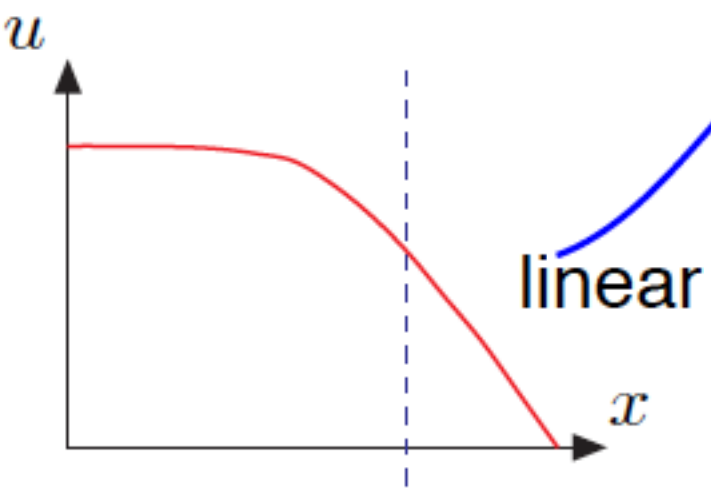
$$u(x,t) = u(x - v_c t)$$

Position: $X(t) = X_0 + v_c t$

© G. Soyez

# Travelling waves

$$\boxed{\partial_t u(x,t) = \partial_x^2 u(x,t) + u(x,t) - u^2(x,t)}$$




$$\partial_t u = \partial_x^2 u + u = \omega(-\partial_x) u$$

$$\omega(\gamma) = \gamma^2 + 1$$ same as $\overline{\alpha}_s \chi$

initial contition

velocity: $v = \dfrac{\omega(\gamma)}{\gamma}$

$$\Rightarrow u(x,t) = \int \frac{d\gamma}{2i\pi} u_0(\gamma) \exp[\omega(\gamma)t - \gamma x]$$

© G. Soyez

# Travelling waves

$$\partial_t u(x,t) = \partial_x^2 u(x,t) + u(x,t) - u^2(x,t)$$

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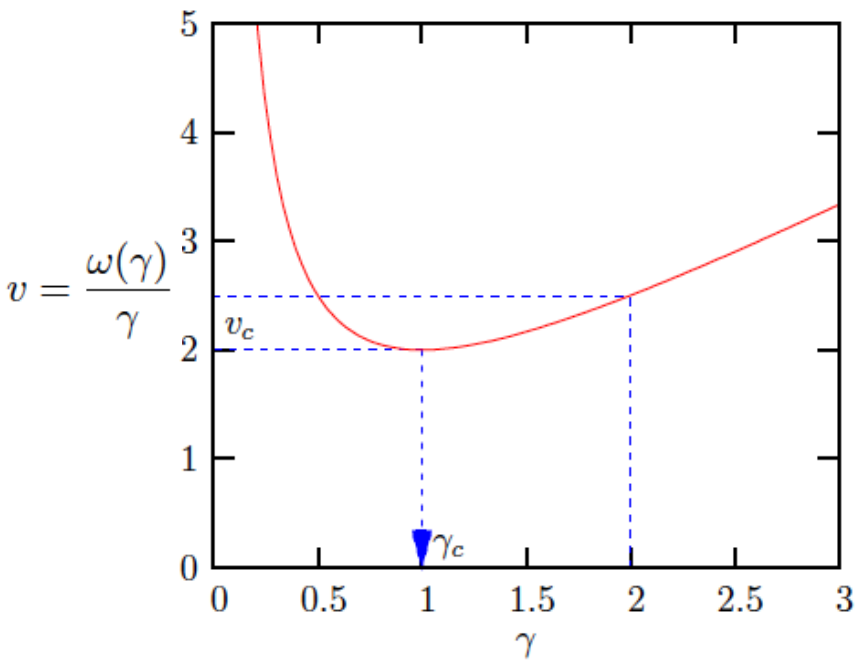


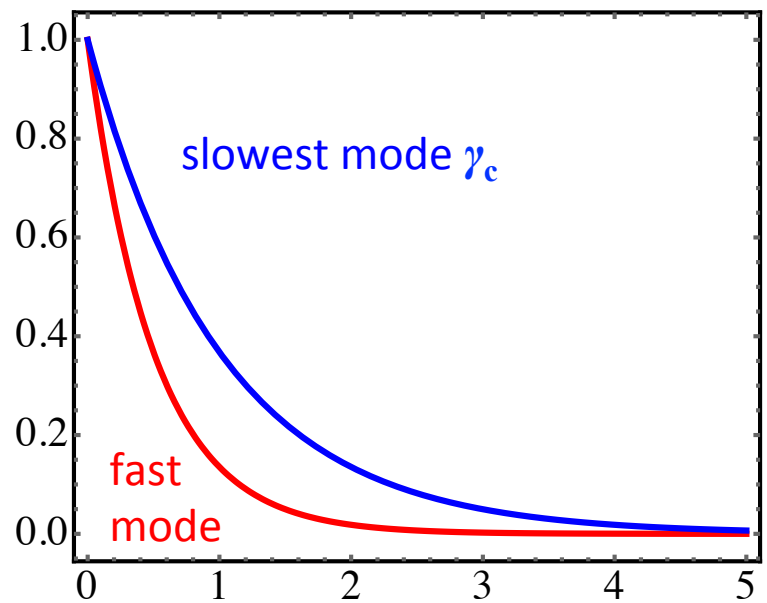


solution is driven by the slowest mode

# Travelling waves in QCD

$Y = \ln(\ 1/x\ )$

$$\frac{\partial}{\partial Y}\tilde{N}(k,Y) \;=\; \overline{\alpha}_s\,\chi(-\partial/\partial\ln k^2)\,\tilde{N}(k,Y) - \overline{\alpha}_s\,\tilde{N}^2(k,Y)$$

Mellin transform: $$\tilde{N}(k,Y) = \int_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2i\pi} n_0(\gamma)\,\exp\left[\overline{\alpha}_s\,\chi(\gamma)\,Y - \gamma\ln k^2\right]$$

minimal velocity: $$v = \min\frac{\overline{\alpha}_s\,\chi(\gamma)}{\gamma} \quad\rightarrow\quad \gamma_c\chi'(\gamma_c) = \chi(\gamma_c) \qquad \begin{aligned}\gamma_c &= 0.6275\\ v_c &= 4.8834\bar{\alpha}\end{aligned}$$

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travelling wave condition: $\text{time}: \quad t = Y, \quad \text{position}: \quad x = \ln k^2$

$$\overline{\alpha}_s\chi(\gamma_c)Y - \gamma_c\ln(k^2/k_0^2) \;=\; -\gamma_c\ln\left[\left(\frac{1}{x}\right)^{-v_c}\frac{k^2}{k_0^2}\right] = -\gamma_c\ln\left[\frac{k^2}{Q_s^2(x)}\right]$$

saturation scale: $$Q_s^2(x) \;=\; k_0^2\left(\frac{1}{x}\right)^{v_c}$$

scaling variable

# Travelling waves in QCD imply Geometrical Scaling

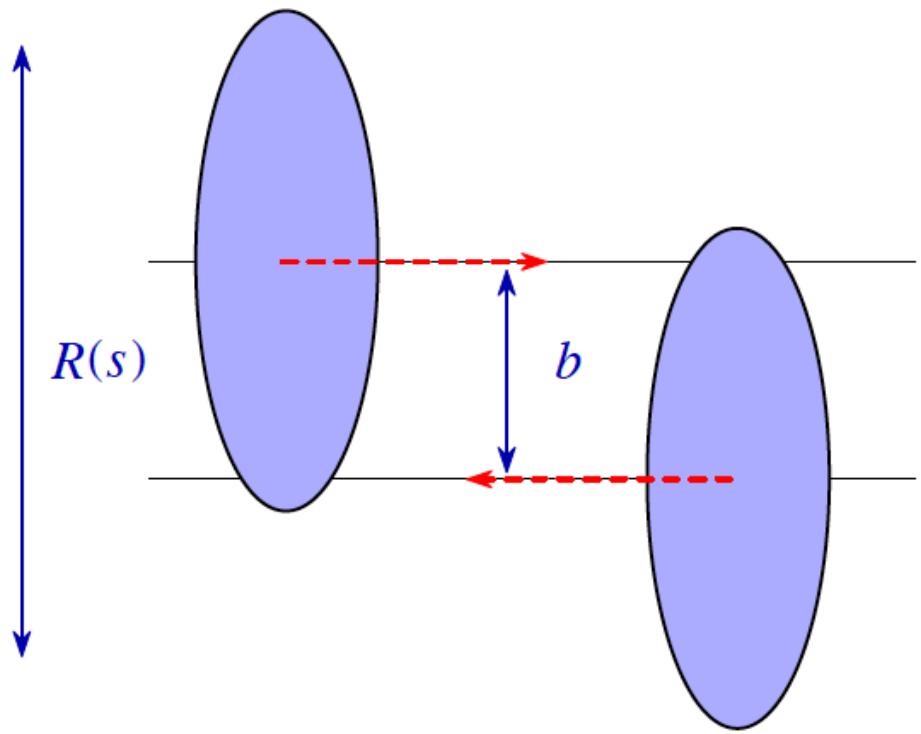
$$f(x, k^2) = \mathcal{F}\left(\frac{k^2}{Q_s^2(x)}\right)$$

$$Q_s(x) = Q_0 \left(\frac{x_0}{x}\right)^{\lambda/2}$$

# „Geometrical Scaling"

J. Dias de Deus, Nucl. Phys. B 59 (1973) 231;
A.J. Buras, J. Dias de Deus, Nucl.Phys. B 71 (1974) 481;
J. Dias de Deus, P. Kroll, J. Phys. G 9 (1983) L81;
J. Dias de Deus, Acta Phys. Polon. B 6 (1975) 613.

$$A(b,s) = A(b/R(s))$$



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J. Dias de Deus, Nucl. Phys. B 59 (1973) 231;
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*Geometric scaling for the total* $\gamma^*$ p *cross-section in the low* $x$ *region.*
A.M. Stasto, K. J. Golec-Biernat , J. Kwiecinski
Phys.Rev.Lett. 86 (2001) 596-599

$$\sigma_{\gamma^* p} \sim \frac{F_2(x, Q^2)}{Q^2} = \sigma_0 \mathcal{F}\left(\frac{Q^2}{Q^2_{\rm sat}(x)}\right)$$

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J. Dias de Deus, Nucl. Phys. B 59 (1973) 231;
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J. Dias de Deus, P. Kroll, J. Phys. G 9 (1983) L81;
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Phys.Rev.Lett. 86 (2001) 596-599

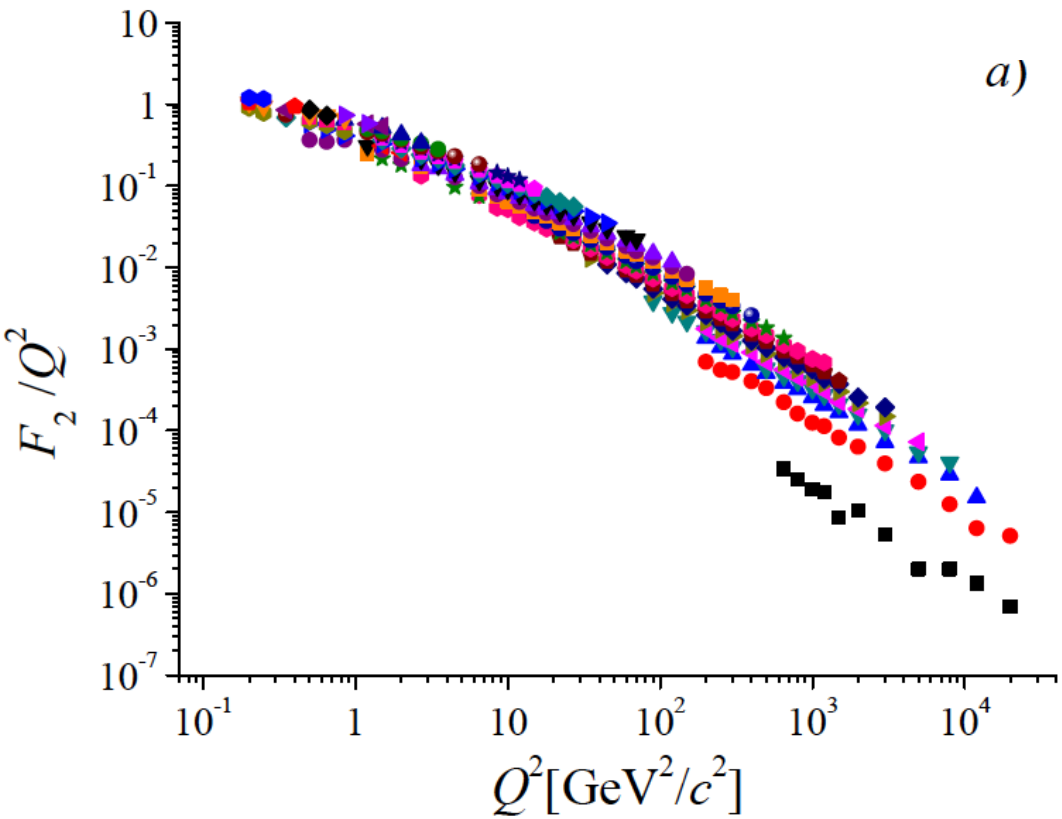
L. McLerran, M. Praszalowicz: Acta Phys.Polon.B41:1917,2010, B42:99,2011
M. Praszalowicz: Phys.Rev.Lett.106:142002,2011
M. Praszalowicz: Acta Phys.Polon. B42 (2011) 1557-1566
M. Praszalowicz, T. Stebel: JHEP 1303 (2013) 090

$$\frac{dN_{\rm ch}}{d\eta dp_{\rm T}^2}(s,p_{\rm T}) \quad = \quad \frac{1}{Q_0^2}\mathcal{F}\left(\frac{p_{\rm T}^2}{Q_{\rm sat}^2(s)}\right)$$

# Deep Inealstic Scattering

# Saturation scale: energy and $x$ dependence

$$Q^2_{\mathrm{sat}}(x) = Q^2_0 \left(\frac{x}{x_0}\right)^{-\lambda}$$

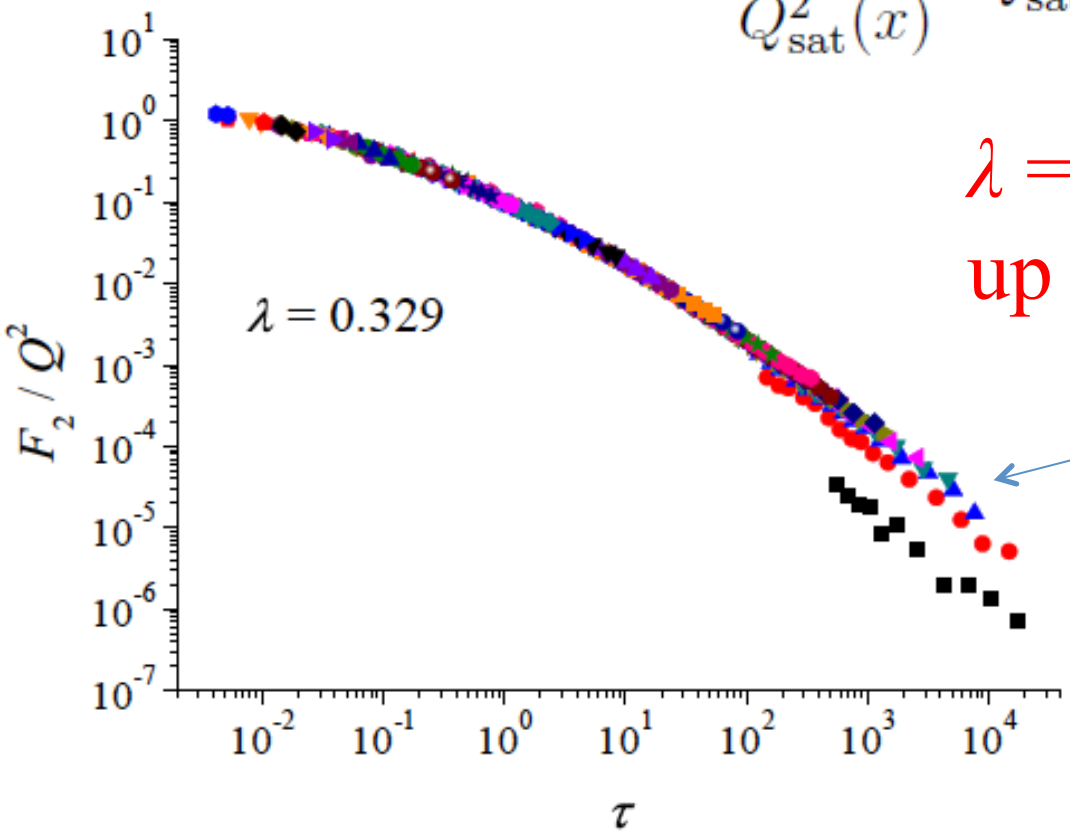



A.M. Stasto, K. J. Golec-Biernat, J. Kwiecinski
PRL 86 (2001) 596-599

M.Praszalowicz and T.Stebel
JHEP 1303, 090 (2013)
arXiv:1211.5305 [hep-ph]
and
JHEP 1304, 169 (2013)
arXiv:1302.4227 [hep-ph]

# Saturation scale: energy and $x$ dependence

$$\tau = \frac{Q^2}{Q^2_{\rm sat}(x)} \quad Q^2_{\rm sat}(x) = Q_0^2 \left(\frac{x}{x_0}\right)^{-\lambda}$$




$\lambda = 0.329 \pm 0.005$

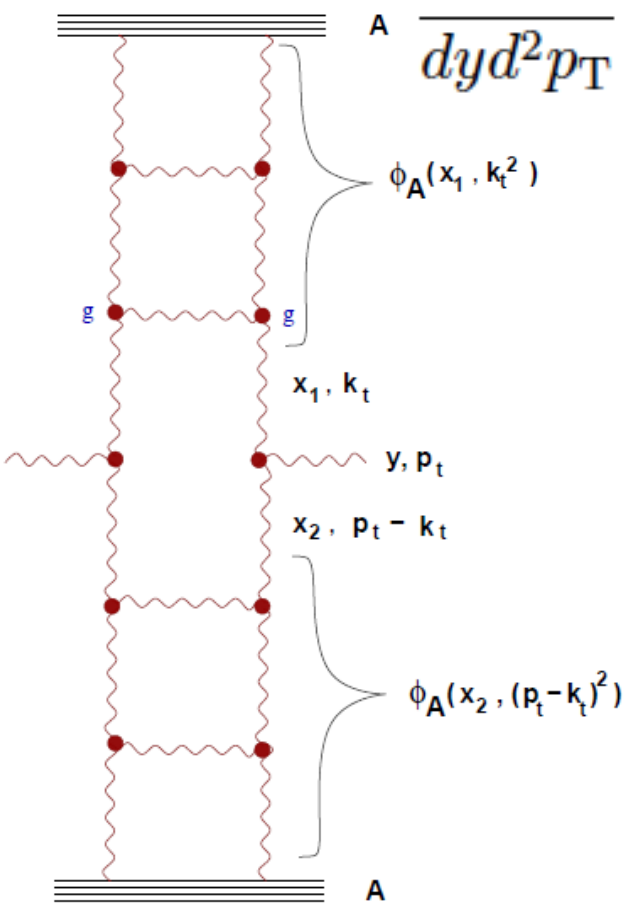
up to $x = 0.08$ (!)

large $x$

# proton-proton @ LHC

# Basics of geometrical scaling

Gribov, Levin Ryskin, *High $p_T$ Hadrons In The Pionization Region In QCD*. Phys.Lett.B100:173-176,1981.




$$\frac{d\sigma}{dy d^2 p_T} = \frac{3\pi\alpha_s}{2p_T^2} \int d^2\vec{k}_T \, \varphi_1(x_1, \vec{k}_T^2) \varphi_2(x_2, (\vec{k} - \vec{p})_T^2)$$

$$x_{1,2} = \frac{p_T}{\sqrt{s}} e^{\pm y}$$

gluon distribution unintegrated glue

$$xG(x, Q^2) = \int^{Q^2} dk_T^2 \, \varphi(x, k_T^2)$$

Kharzeev, Levin
Phys.Lett.B523:79-87,2001.

# Basics of geometrical scaling

gluon distribution unintegrated glue

$$xG(x,Q^2) = \int^{Q^2} dk_T^2\, \varphi(x,k_T^2)$$

Golec-Biernat – Wuesthoff (DIS)

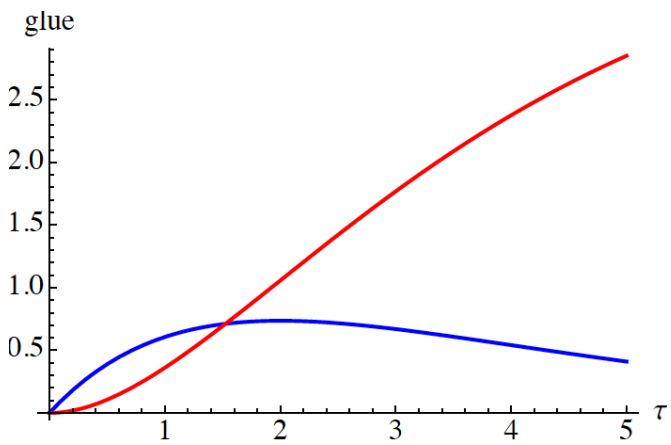
$$\varphi(x,k_T^2) = S_\perp \frac{3}{4\pi^2}\frac{k_T^2}{Q_s(x)^2}\exp\left(-k_T^2/Q_s(x)^2\right)$$

$$S_\perp = \sigma_0$$

Kharzeev – Levin (AA)

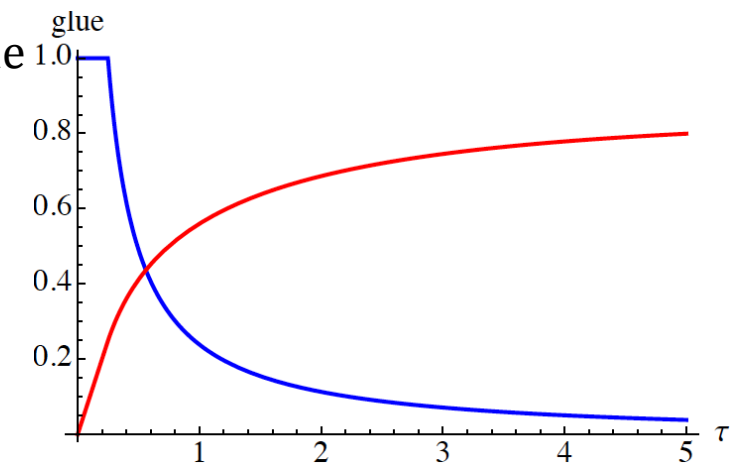
$$\varphi(x,k_T^2) = S_\perp \begin{cases} 1 & \text{for} \quad k_T^2 < Q_s(x)^2 \\ Q_s(x)^2/k_T^2 & \text{for} \quad Q_s(x)^2 < k_T^2 \end{cases}$$

$S_\perp$ is the transverse size given by geometry




scaling variable

$$\tau = \frac{p_T^2}{Q_s^2(x)}$$




Michal Praszalowicz

# Basics of geometrical scaling

for $y \sim 0$ (central rapidity) *i.e.* for $x_1 \sim x_2 = x$ and for symmetric systems

$$\frac{d\sigma}{dy d^2 p_{\mathrm{T}}} = \frac{3\pi\alpha_{\mathrm{s}}}{2} \frac{Q_s^2(x)}{p_{\mathrm{T}}^2} \int \frac{d^2\vec{k}_{\mathrm{T}}}{Q_s^2(x)} \varphi_1\left(\vec{k}_{\mathrm{T}}^2/Q_s^2(x)\right) \varphi_2\left((\vec{k}-\vec{p})_{\mathrm{T}}^2/Q_s^2(x)\right)$$

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$$\frac{d\sigma}{dyd^2p_{\mathrm{T}}} = S_\perp^2\mathcal{F}(\tau) \quad \tau = \frac{p_{\mathrm{T}}^2}{Q_s^2(x)} \qquad Q_s(x) = Q_0\left(\frac{x_0}{x}\right)^{\lambda/2}$$

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$$\frac{d\sigma}{dyd^2p_{\mathrm{T}}} = S_\perp^2\mathcal{F}(\tau) \quad \tau = \frac{p_{\mathrm{T}}^2}{Q_s^2(x)} \qquad dp_{\mathrm{T}}^2 = \frac{2}{2+\lambda}\bar{Q}_{\mathrm{s}}^2(W)\,\tau^{-\lambda/(2+\lambda)}d\tau$$

$$\bar{Q}_s(W) = Q_0\left(\frac{W}{Q_0}\right)^{\lambda/(2+\lambda)}$$

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$$\frac{d\sigma}{dy} = S_\perp^2\int\mathcal{F}(\tau)d^2p_{\mathrm{T}} = S_\perp^2\bar{Q}_s^2(W)\int\mathcal{F}(\tau)\ldots d\tau = \frac{1}{\kappa}S_\perp^2\bar{Q}_s^2(W)$$

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$$\frac{d\sigma}{dy} = S_\perp\frac{dN}{dy} = \frac{1}{\kappa}S_\perp^2\bar{Q}_s^2(W) \rightarrow \bar{Q}_s^2(W) = \frac{\kappa}{S_\perp}\frac{dN}{dy}$$

# Basics of geometrical scaling

for $y \sim 0$ (central rapidity) *i.e.* for $x_1 \sim x_2 = x$ and for symmetric systems

$$\frac{d\sigma}{dy d^2 p_T} = \frac{3\pi\alpha_s}{2} \frac{Q_s^2(x)}{p_T^2} \int \frac{d^2\vec{k}_T}{Q_s^2(x)} \varphi_1\left(\vec{k}_T^2/Q_s^2(x)\right) \varphi_2\left((\vec{k}-\vec{p})_T^2/Q_s^2(x)\right)$$

$$\frac{d\sigma}{dy d^2 p_T} = S_\perp^2 \mathcal{F}(\tau) \quad \tau = \frac{p_T^2}{Q_s^2(x)} \qquad dp_T^2 = \frac{2}{2+\lambda} \bar{Q}_s^2(W)\, \tau^{-\lambda/(2+\lambda)} d\tau$$

$$\frac{d\sigma}{dy} = S_\perp^2 \int \mathcal{F}(\tau) d^2 p_T = S_\perp^2 \bar{Q}_s^2(W) \int \mathcal{F}(\tau) \ldots d\tau = \frac{1}{\kappa} S_\perp^2 \bar{Q}_s^2(W)$$

$$\frac{d\sigma}{dy} = S_\perp \frac{dN}{dy} = \frac{1}{\kappa} S_\perp^2 \bar{Q}_s^2(W) \rightarrow \boxed{\bar{Q}_s^2(W) = \frac{\kappa}{S_\perp} \frac{dN}{dy}}$$

saturation scale = gluon density per transverse area

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parton – hadron duality: power-like growth of particle multiplicity

saturation scale = gluon density per transverse area

# Geometrical scaling of $p_T$ distribution

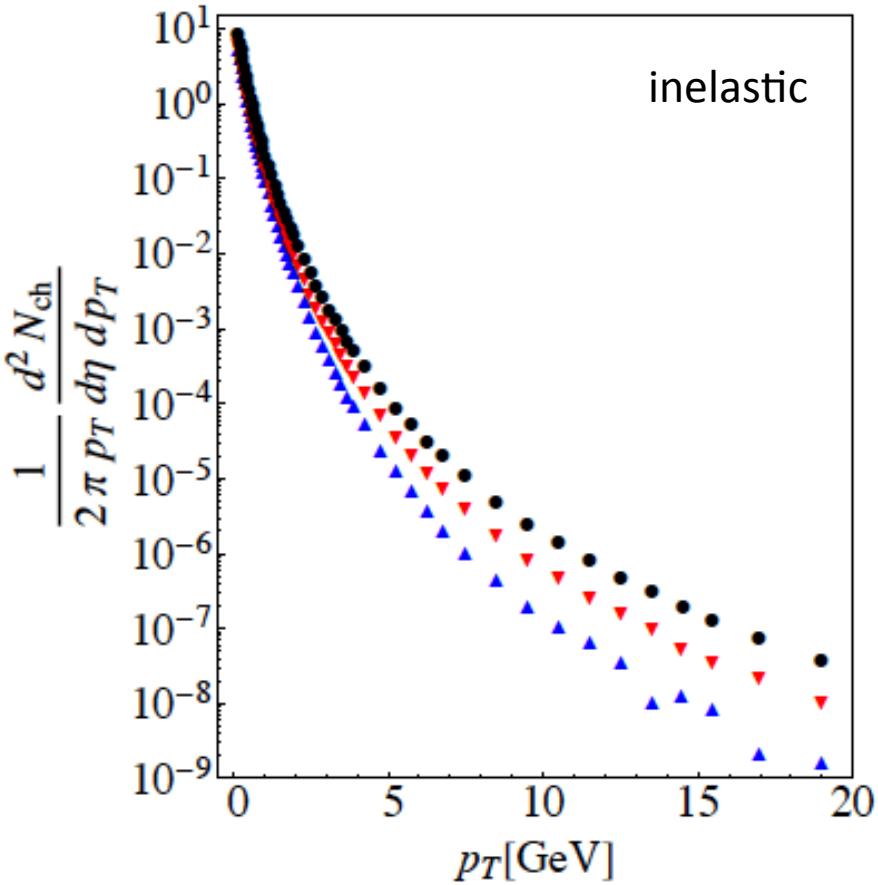
L. McLerran, M. P. Acta Phys.Polon.B41:1917,2010, B42:99,2011
M. P. Phys.Rev.Lett.106:142002,2011, Acta Phys.Pol. B42 (2011) 1557-1566
Phys.Rev. D87 (2013) 071502(R)

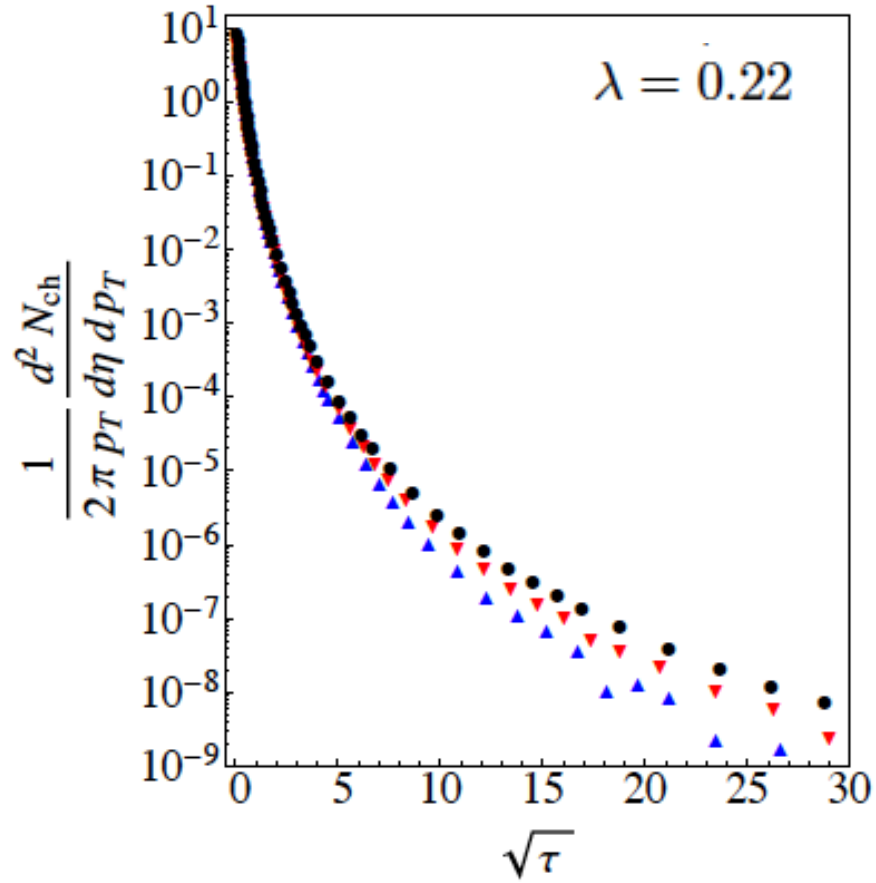
$$\tau = \frac{p_{\mathrm{T}}^2}{Q_{\mathrm{sat}}^2(p_{\mathrm{T}}/\sqrt{s})} = \frac{p_{\mathrm{T}}^2}{1\,\mathrm{GeV}^2}\left(\frac{p_{\mathrm{T}}}{\sqrt{s}\times 10^{-3}}\right)^{\lambda}$$

# Determination of lambda

$$\frac{dN_{\rm ch}}{dy d^2 p_{\rm T}} = S_\perp \mathcal{F}(\tau) \qquad \tau = \frac{p_{\rm T}^2}{Q_{\rm sat}^2(p_{\rm T}/\sqrt{s})} = \frac{p_{\rm T}^2}{1\,{\rm GeV}^2}\left(\frac{p_{\rm T}}{\sqrt{s}\times 10^{-3}}\right)^\lambda$$

ALICE 1307.1093 [nucl-ex], Eur.Phys.J C73 (2013) 2662



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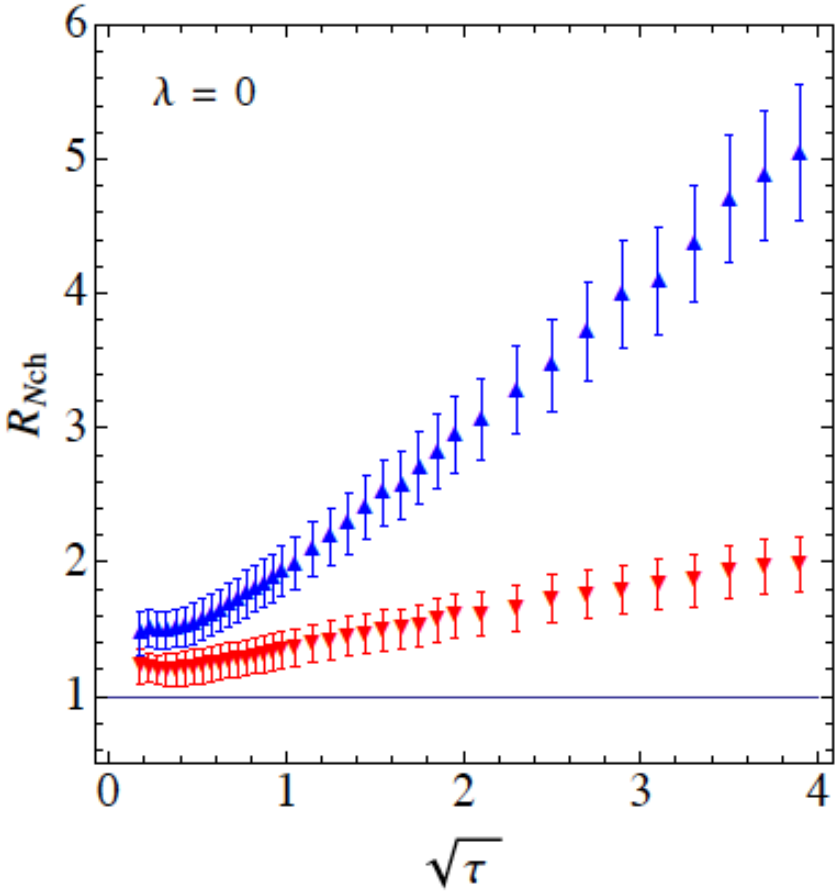
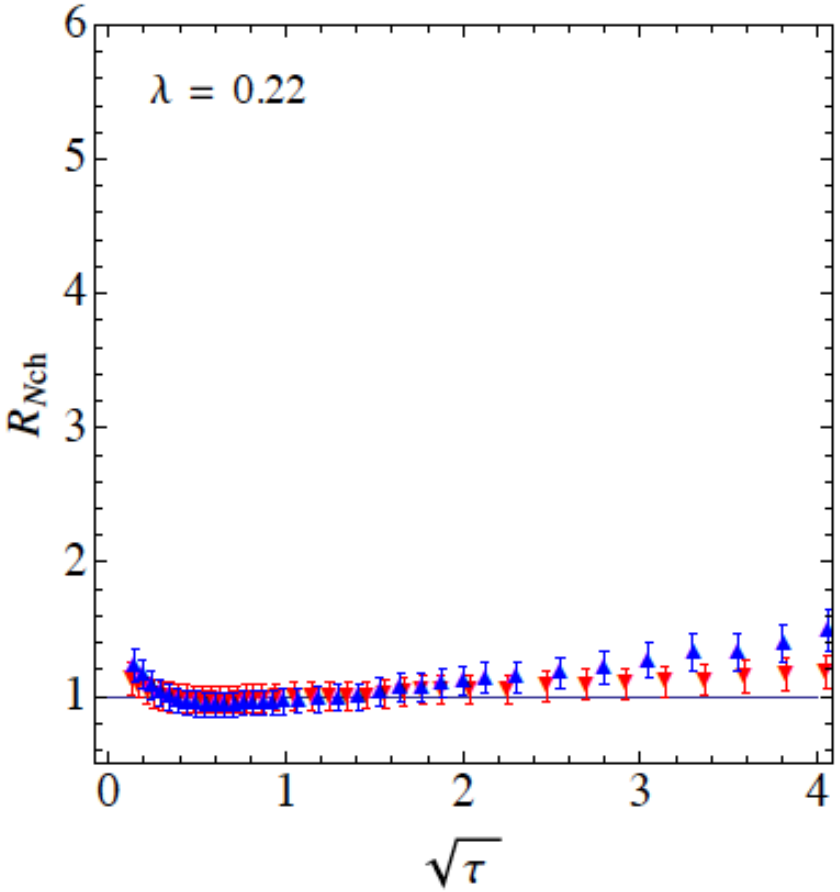
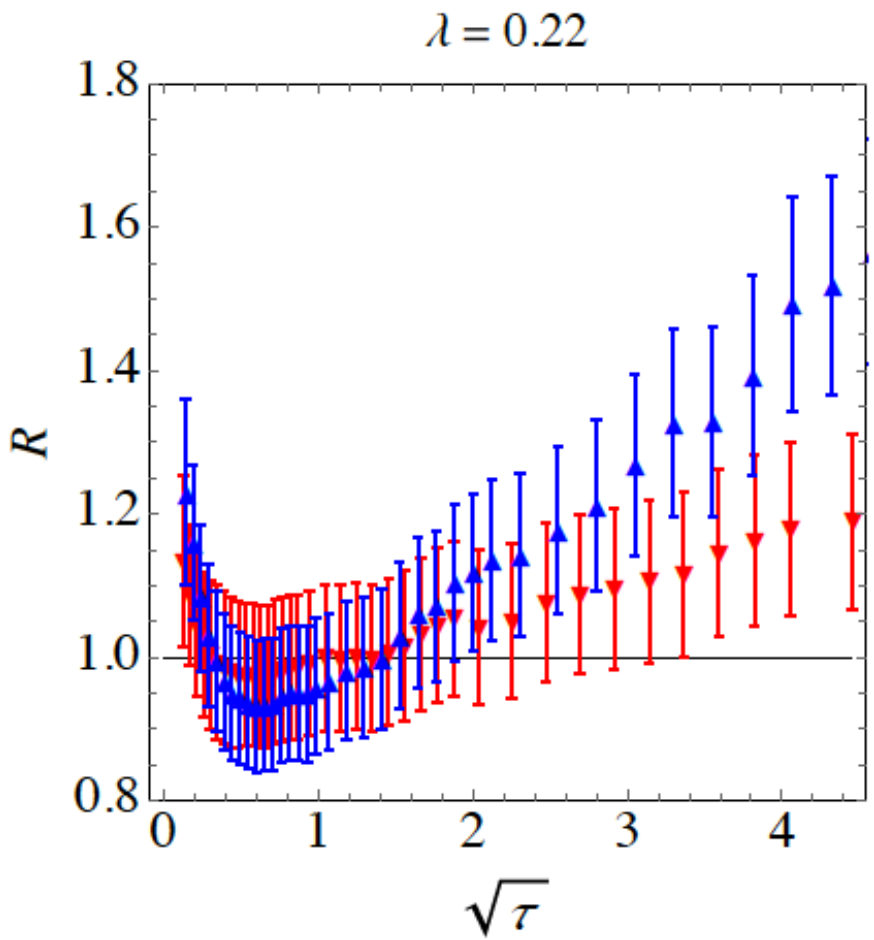
ALICE 1307.1093 [nucl-ex], Eur.Phys.J C73 (2013) 2662



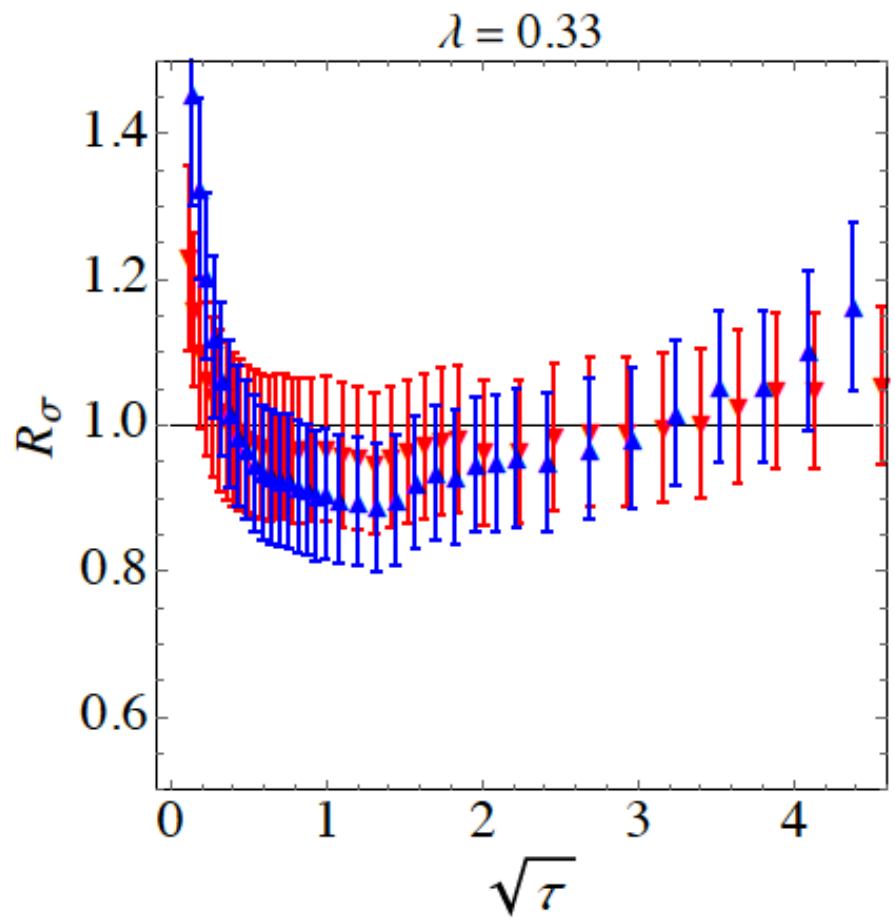



# What should scale in pp?

multiplicity




cross-section

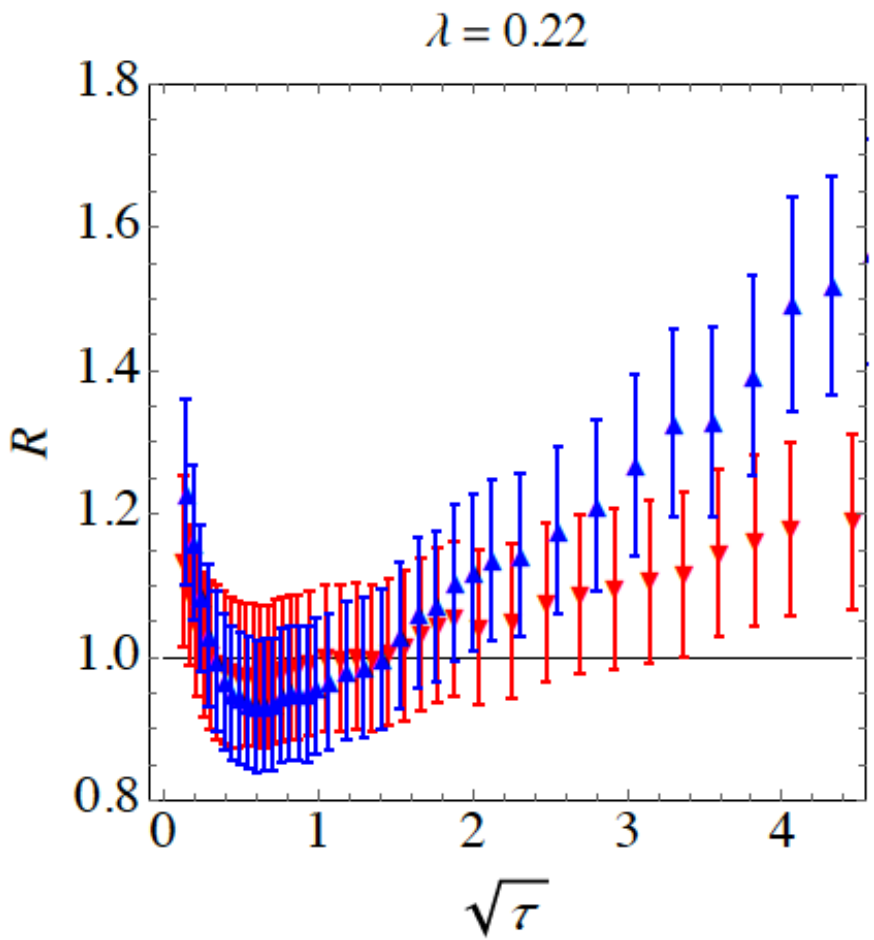


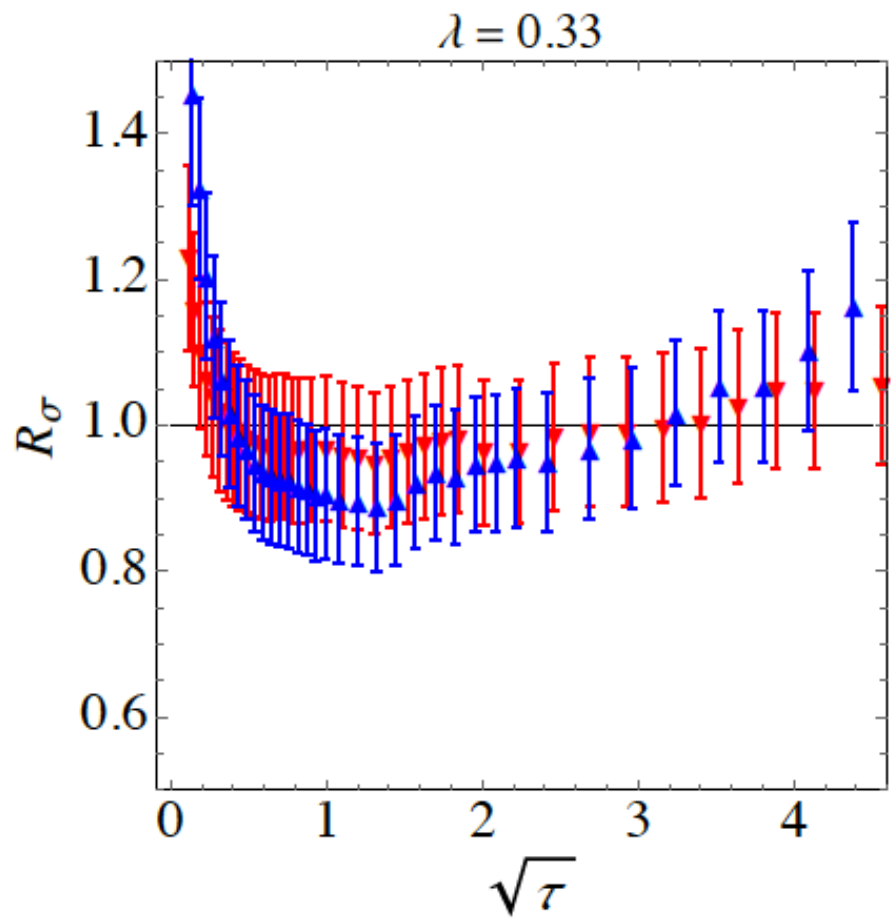
# What should scale in pp?

$$\frac{d\sigma}{dy} = S_\perp \frac{dN}{dy} = \frac{1}{\kappa} S_\perp^2 \bar{Q}_s^2(W)$$

multiplicity

cross-section



# Consequences of GS

$$\frac{dN_{\mathrm{ch}}}{dy\,dp_{\mathrm{T}}^2} = \frac{1}{Q_0^2} F(\tau) \quad \Rightarrow \quad \frac{dN_{\mathrm{ch}}}{dy} = \int \frac{dp_{\mathrm{T}}^2}{Q_0^2} F(\tau)$$

$$\tau = \frac{p_{\mathrm{T}}^2}{Q_0^2} \left(\frac{p_{\mathrm{T}}}{W}\right)^{\lambda/2}$$

$$W \sim \sqrt{s}$$

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$$W \sim \sqrt{s}$$

$$\frac{dN_{\rm ch}}{dy} = \int \frac{dp_{\rm T}^2}{Q_0^2} F(\tau)$$

integral over $d\tau$ is energy independent

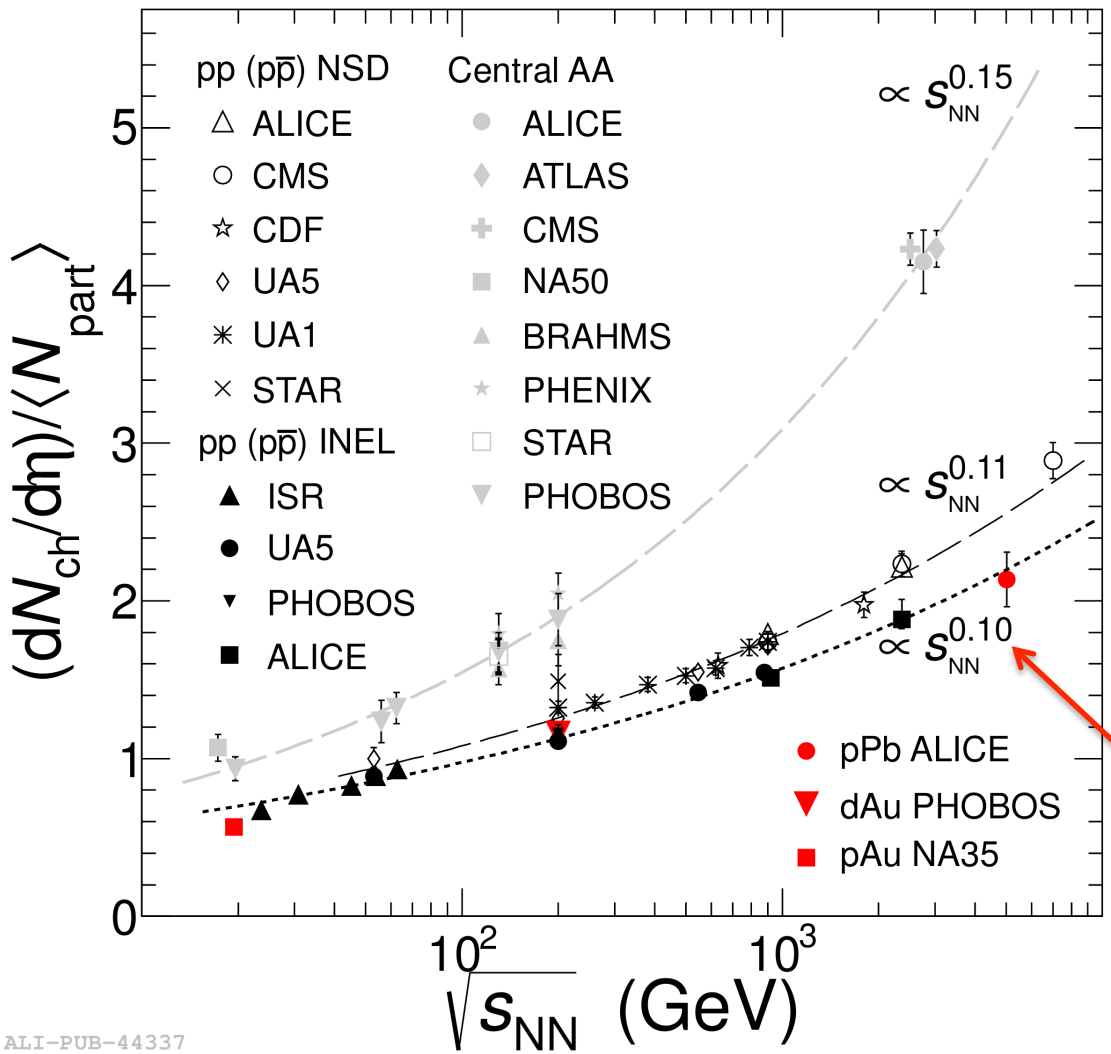
$$\frac{dp_{\rm T}^2}{Q_0^2} = \frac{2}{2+\lambda}\left(\frac{W}{Q_0}\right)^{\frac{2\lambda}{2+\lambda}} \tau^{-\frac{\lambda}{2+\lambda}} d\tau$$

effective growth of multiplicity is slower than $\lambda$

$$\lambda_{\rm eff} = \frac{2\lambda}{2+\lambda} < \lambda$$

# Power-like growth of multiplicity

http://th-www.if.uj.edu.pl/school/2014/talks/braun-munzinger_1.pdf




plot: P. Braun-Munzinger, 54 Cracow School of Theoretical Physics (from ALICE-PUB-44337)

$$\frac{dN_{ch}}{dy} \sim S_\perp \bar{Q}_s^2(W)$$

$$\sim S_\perp Q_0^2 \left(\frac{W^2}{Q_0^2}\right)^{\lambda/(2+\lambda)}$$

transverse area is energy indpendent

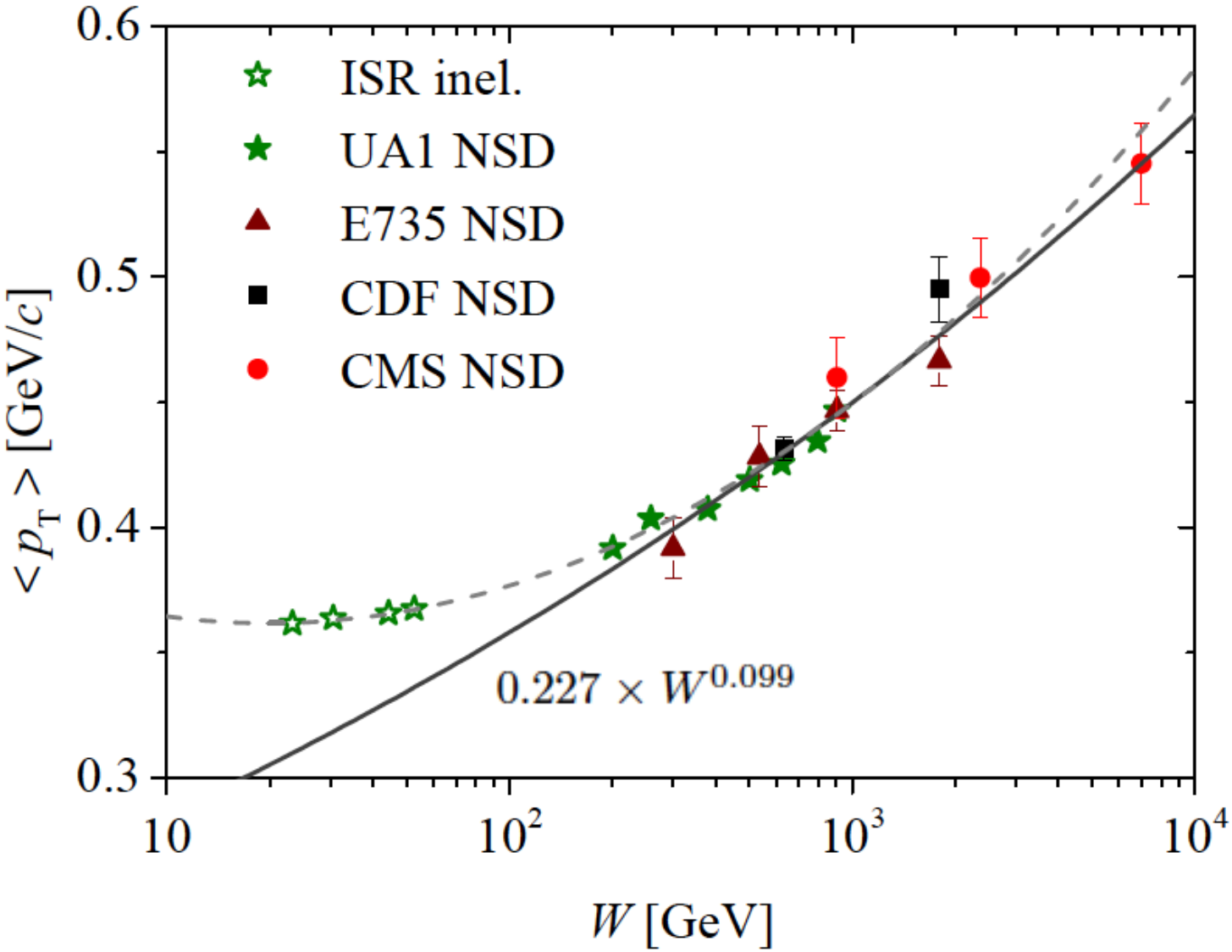
$$\lambda/(2+\lambda) \simeq 0.099$$

# Average transverse momentum

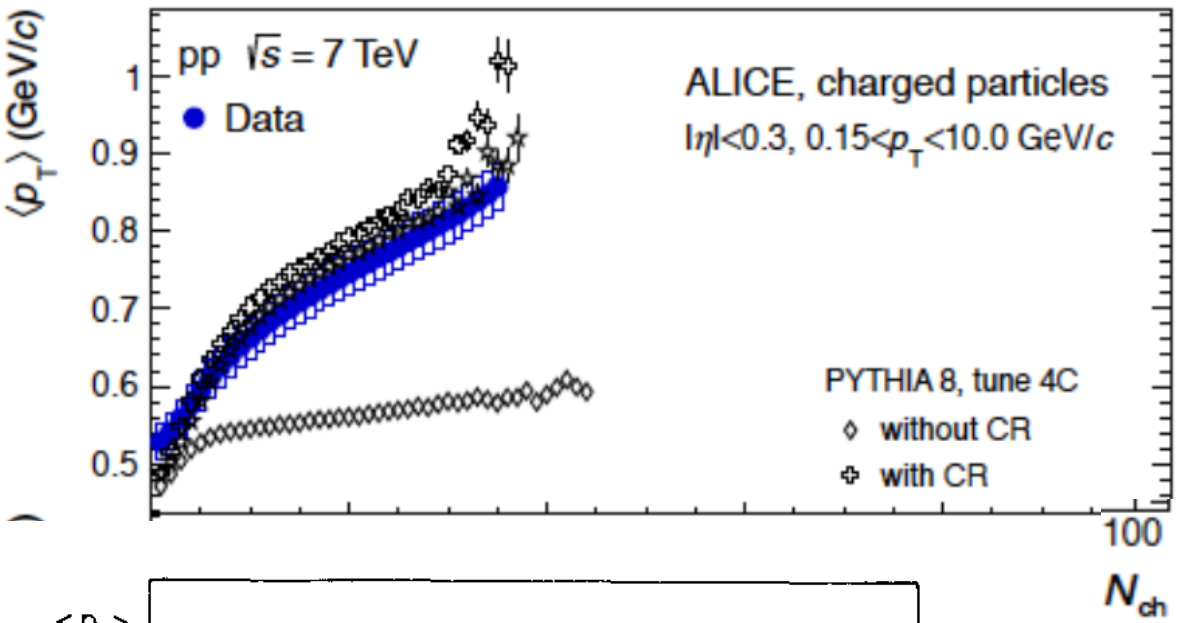
$$\frac{dN_{\rm ch}}{dy d^2 p_{\rm T}} = S_\perp \mathcal{F}(\tau)$$

$$\langle p_{\rm T} \rangle = \frac{\int p_T \frac{dN_{\rm g}}{dy d^2 p_{\rm T}} d^2 p_T}{\int \frac{dN_{\rm g}}{dy d^2 p_{\rm T}} d^2 p_T} \sim \bar{Q}_{\rm s}(W) \sim Q_0 \left(\frac{W}{Q_0}\right)^{\lambda/(2+\lambda)}$$

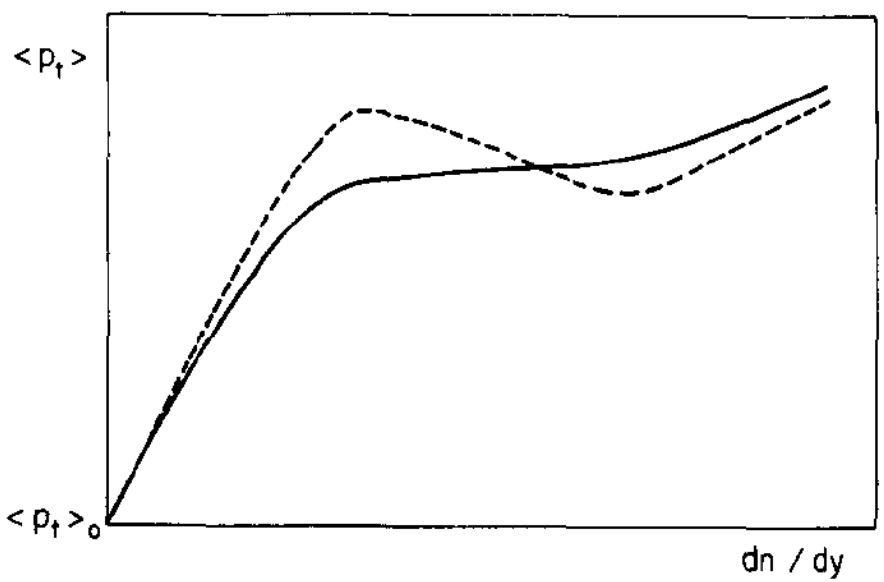
# Average transverse momentum



# Mean $p_T$ as a function of $N_{ch}$




- $\langle p_T \rangle (N_{ch})$ – correlations are sensitive to the fine details of dynamics
- difficult to describe by untuned MonteCarlos
- possible sign of phase transition




**MULTIPLICITY DEPENDENCE OF $p_t$ SPECTRUM AS A POSSIBLE SIGNAL FOR A PHASE TRANSITION IN HADRONIC COLLISIONS**

L. VAN HOVE
*CERN, Geneva, Switzerland*

Received 25 August 1982

# Mean $p_T$ as a function of $N_{ch}$

$$\langle p_{\mathrm{T}} \rangle \sim \bar{Q}_{\mathrm{s}}(W)$$

# Mean $p_T$ as a function of $N_{ch}$

$$\langle p_{\mathrm{T}} \rangle \sim \bar{Q}_{\mathrm{s}}(W) \sim \sqrt{\frac{dN/dy}{S_\perp}} \sim \frac{1}{R}\sqrt{\frac{dN}{dy}}$$

interaction radius

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interaction radius

phenomenological formula:

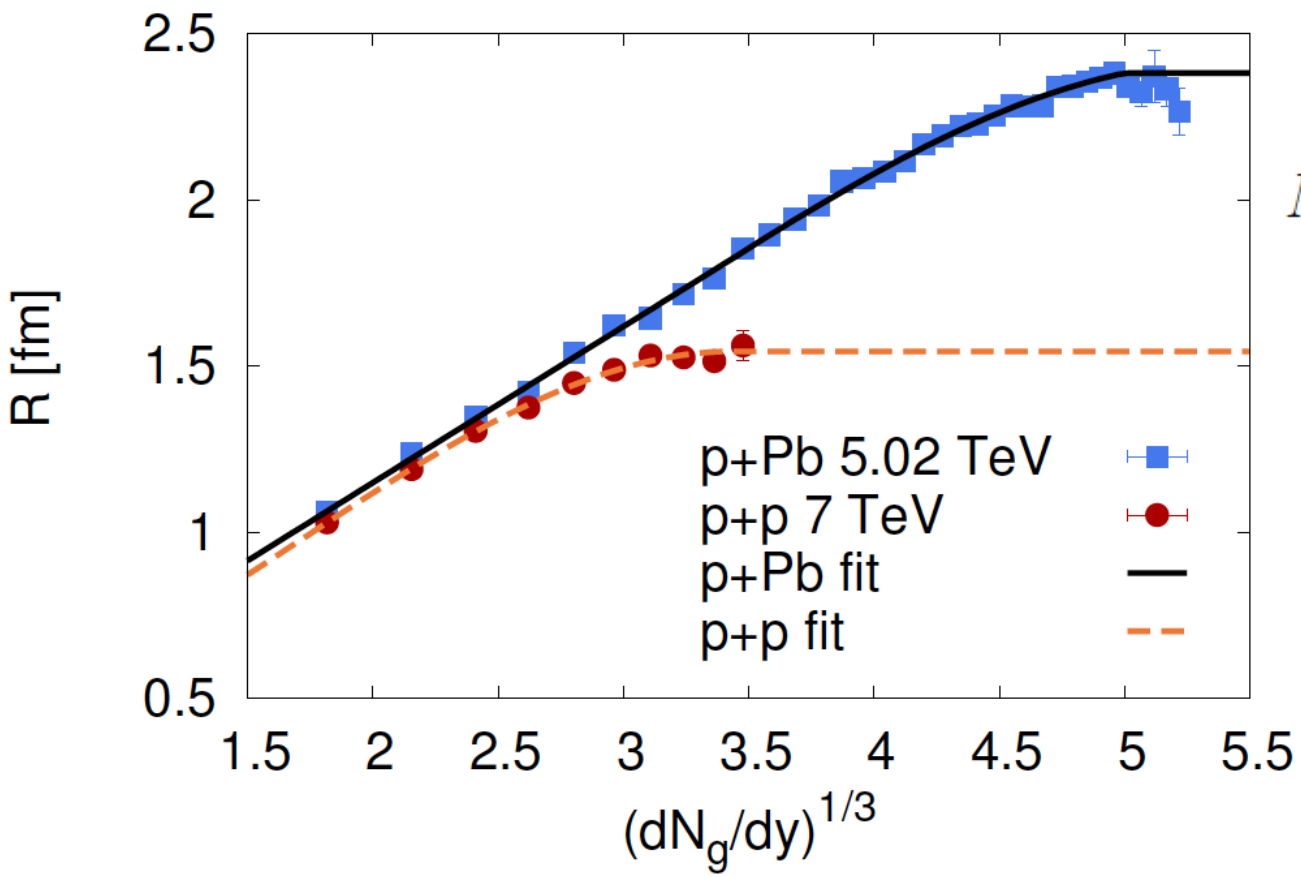
$$\langle p_{\mathrm{T}} \rangle = \alpha + \beta \frac{1}{R}\sqrt{\frac{dN}{dy}}$$

nonperturbaitive
coefficient

α, β do not depend on energy, do depend on particle species

# Interaction radius

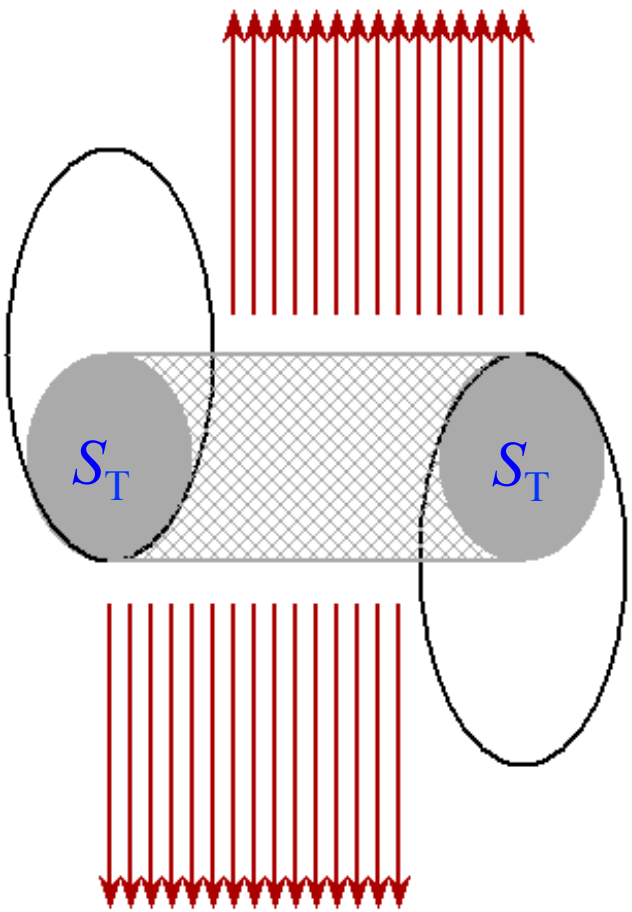
A. Bzdak, B. Schenke, P. Tribedy and R. Venugopalan,
*Initial state geometry and the role of hydrodynamics in proton-proton, proton-nucleus and deuteron-nucleus collisions*,
Phys. Rev. C 87 (2013) 064906, [arXiv:1304.3403 [nucl-th]].



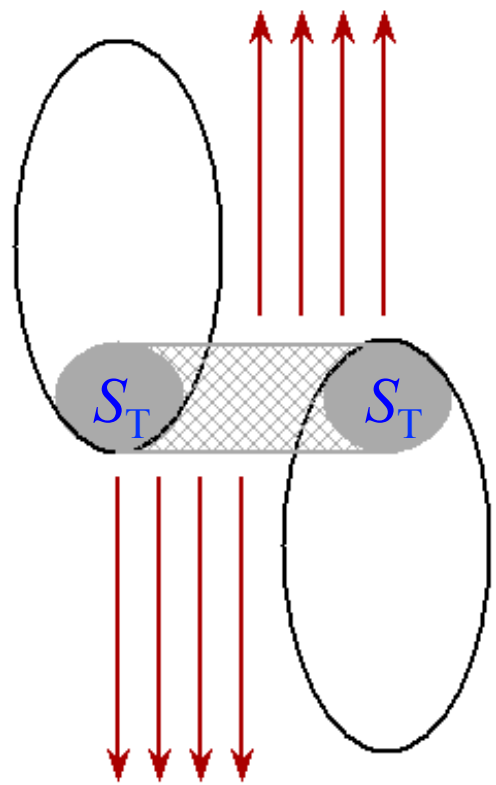

$$N_{\rm ch} = \frac{1}{\gamma \Delta y} \int_{\Delta y} \frac{dN_{\rm g}}{dy} dy$$

# Interaction radius

Transverse size and expansion time (longitudinal size) are proportional for fixed multiplicity




large multiplicity




small multiplicity

similar effect in multipomeron model, where string tension is growing with multiplicity

M. A. Braun, C. Pajares
Phys. Lett. B 287, 154(1992)
Nucl. Phys. B 390 , 542, 559 (1993)
N. Armesto, D.A. Derkach, G.A. Feofilov
Phys. of At. Nuclei 71, 2087 (2008)

# Scaling of mean $p_T$

$$\langle p_{\mathrm{T}} \rangle = \alpha + \beta \frac{\sqrt{N_{\mathrm{ch}}}}{R(\gamma N_{\mathrm{ch}})}$$

parton-hadron duality

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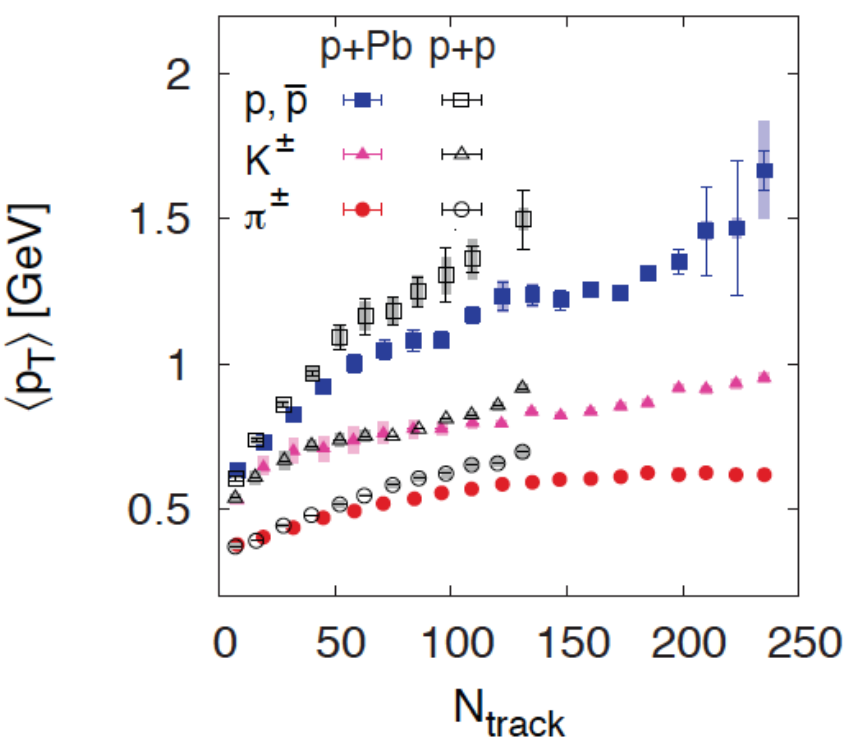
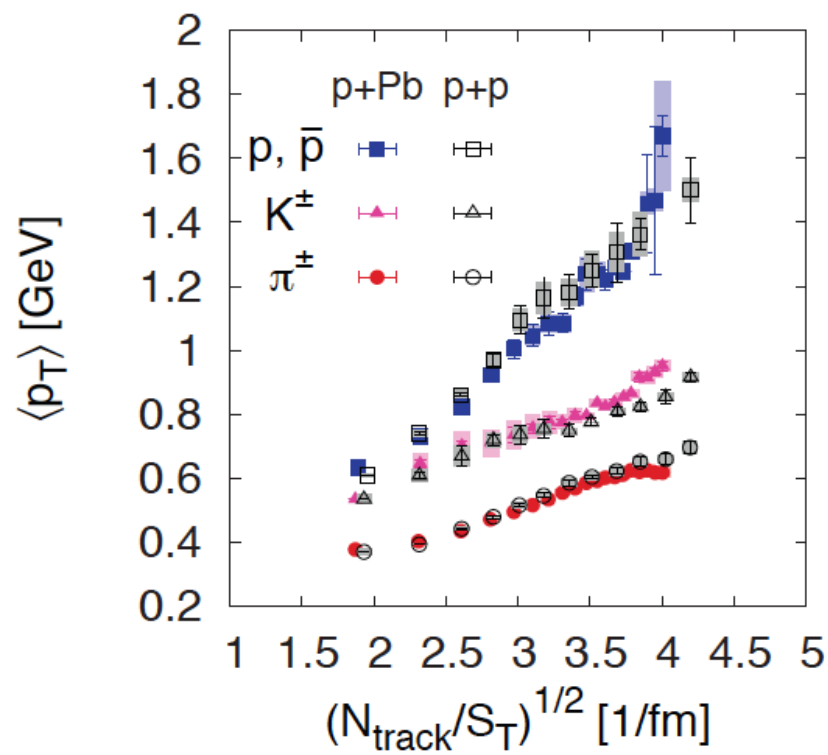
$$\langle p_T \rangle = \alpha + \beta \frac{\sqrt{N_{ch}}}{R(\gamma N_{ch})}$$

scaling variable

parton-hadron duality

CMS Collaboration, Eur. Phys. J. C72 (2013) 2164, C74 (2014) 2847



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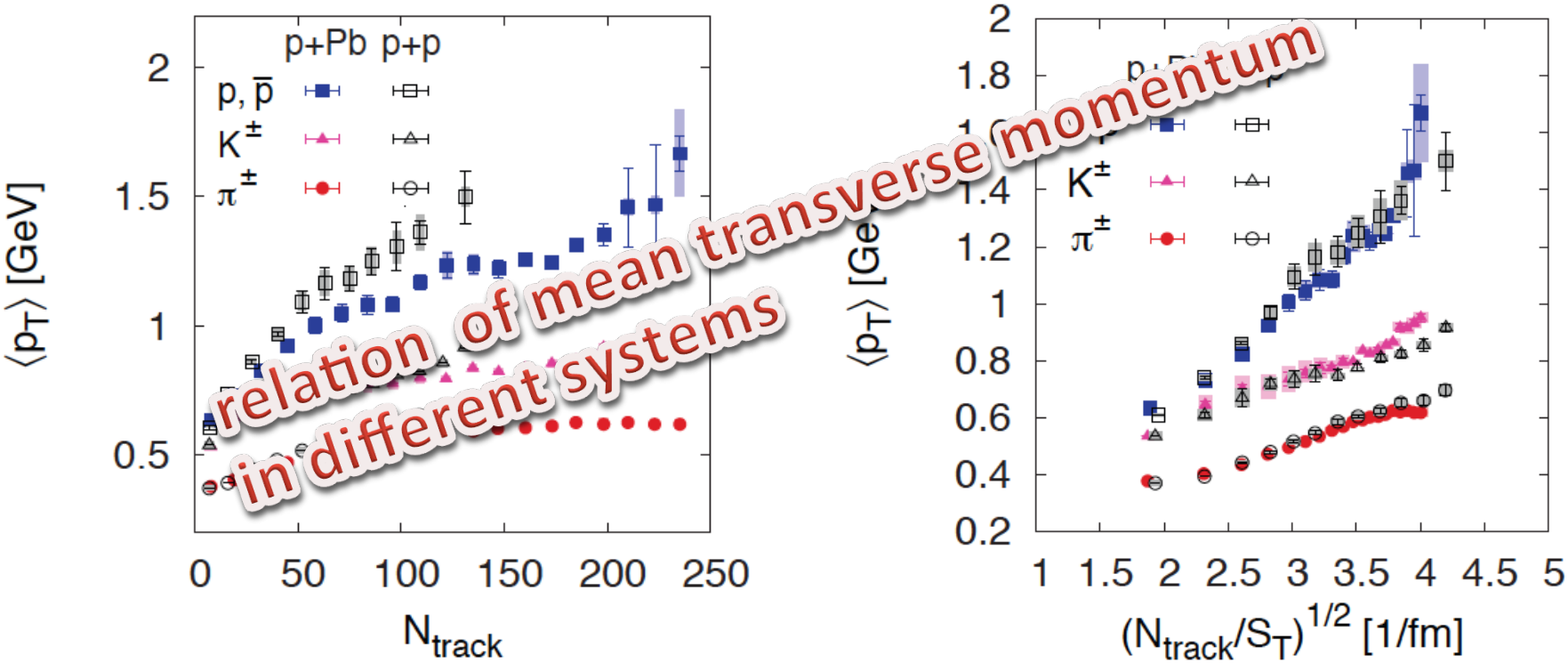
parton-hadron duality

CMS Collaboration, Eur. Phys. J. C72 (2013) 2164, C74 (2014) 2847



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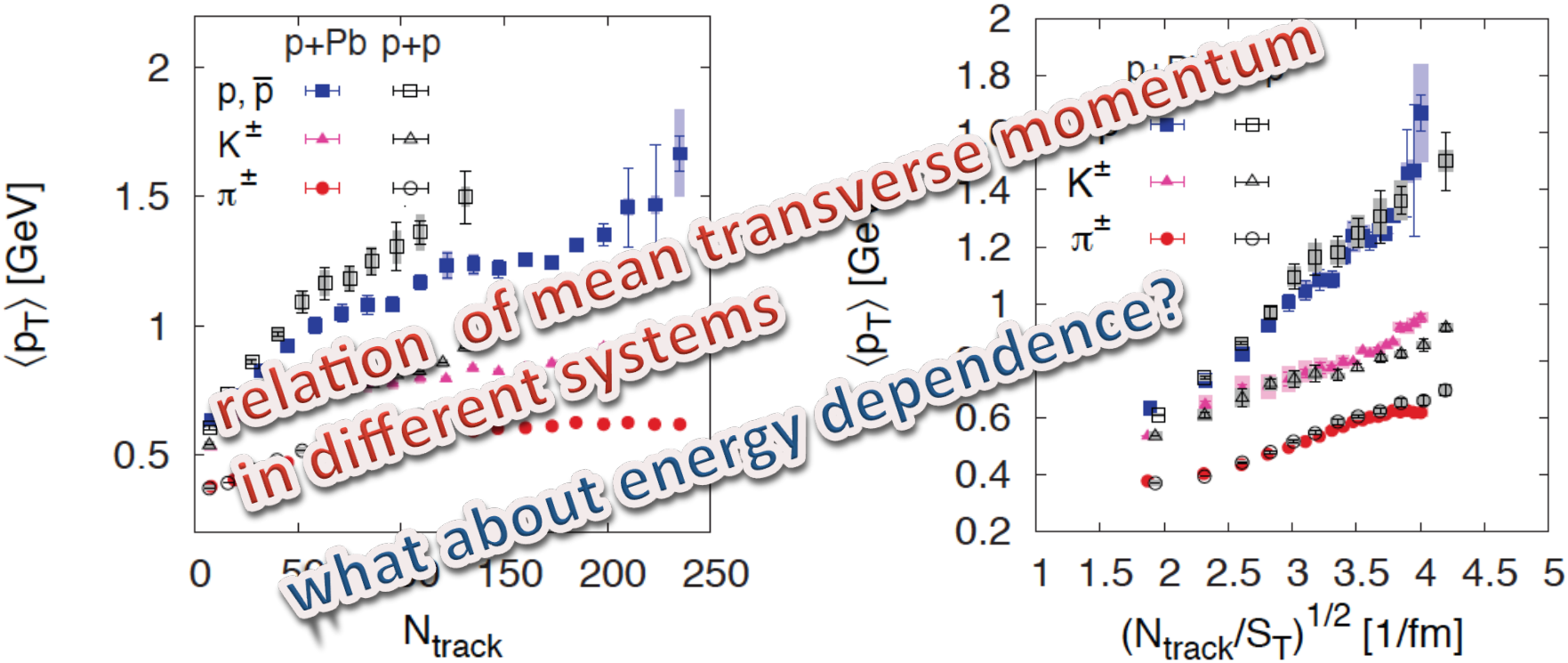
CMS Collaboration, Eur. Phys. J. C72 (2013) 2164, C74 (2014) 2847




relation of mean transverse momentum in different systems

what about energy dependence?

# Energy dependence of mean $p_T$ - apparent paradox?

$$\frac{dN_{\rm ch}}{dy} \sim S_\perp \bar{Q}_s^2(W)$$

$$\sim S_\perp Q_0^2 \left(\frac{W^2}{Q_0^2}\right)^{\lambda/(2+\lambda)}$$

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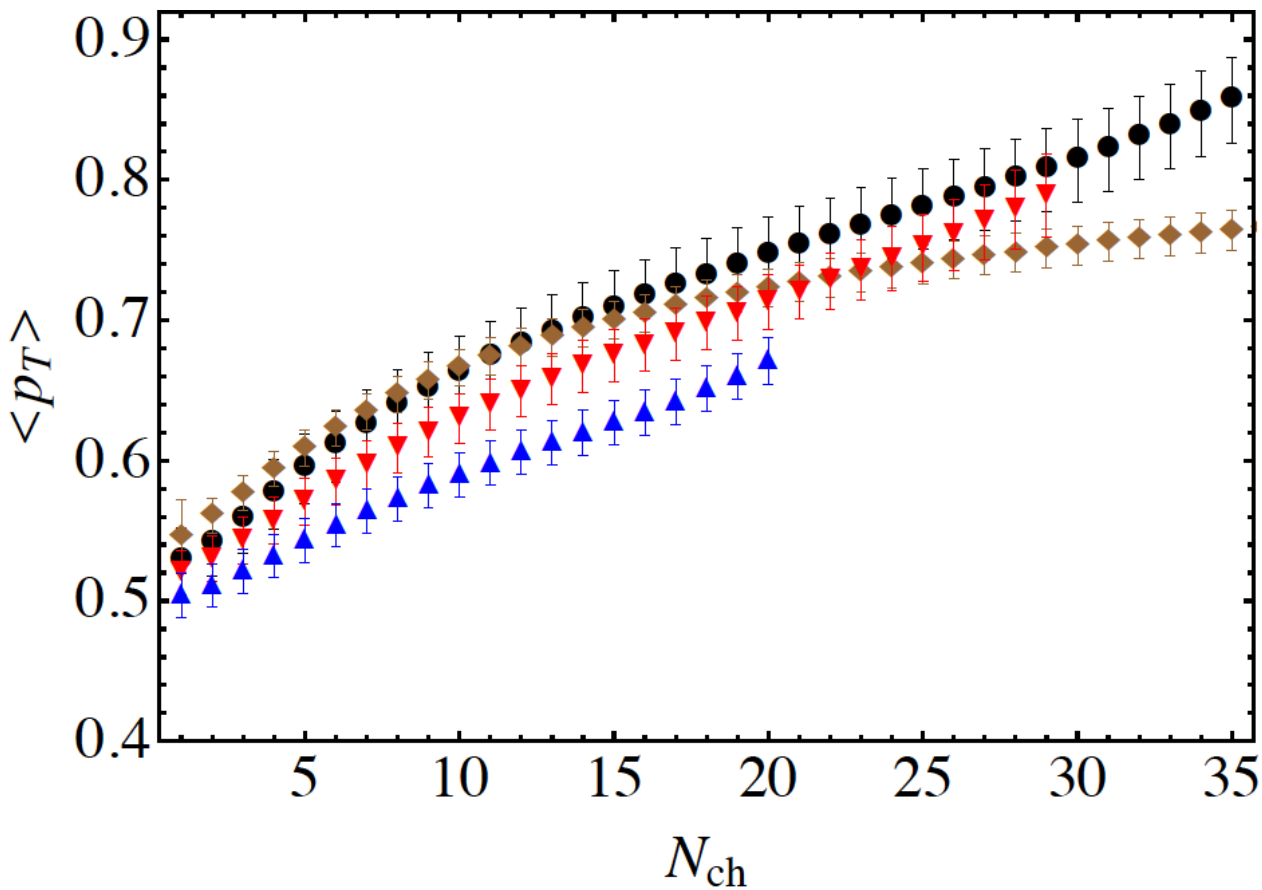
new scaling variable

$$\langle p_{\rm T}\rangle|_W = \alpha + \beta \frac{\sqrt{N_{\rm ch}}}{R(\sqrt[3]{\gamma N_{\rm ch}})|_W} = \alpha + \beta \left(\frac{W}{W_0}\right)^{\lambda/(2+\lambda)} \frac{\sqrt{N_{\rm ch}}}{R(\sqrt[3]{\gamma N_{\rm ch}})|_{W_0}}$$

# Mean $p_T$ scaling

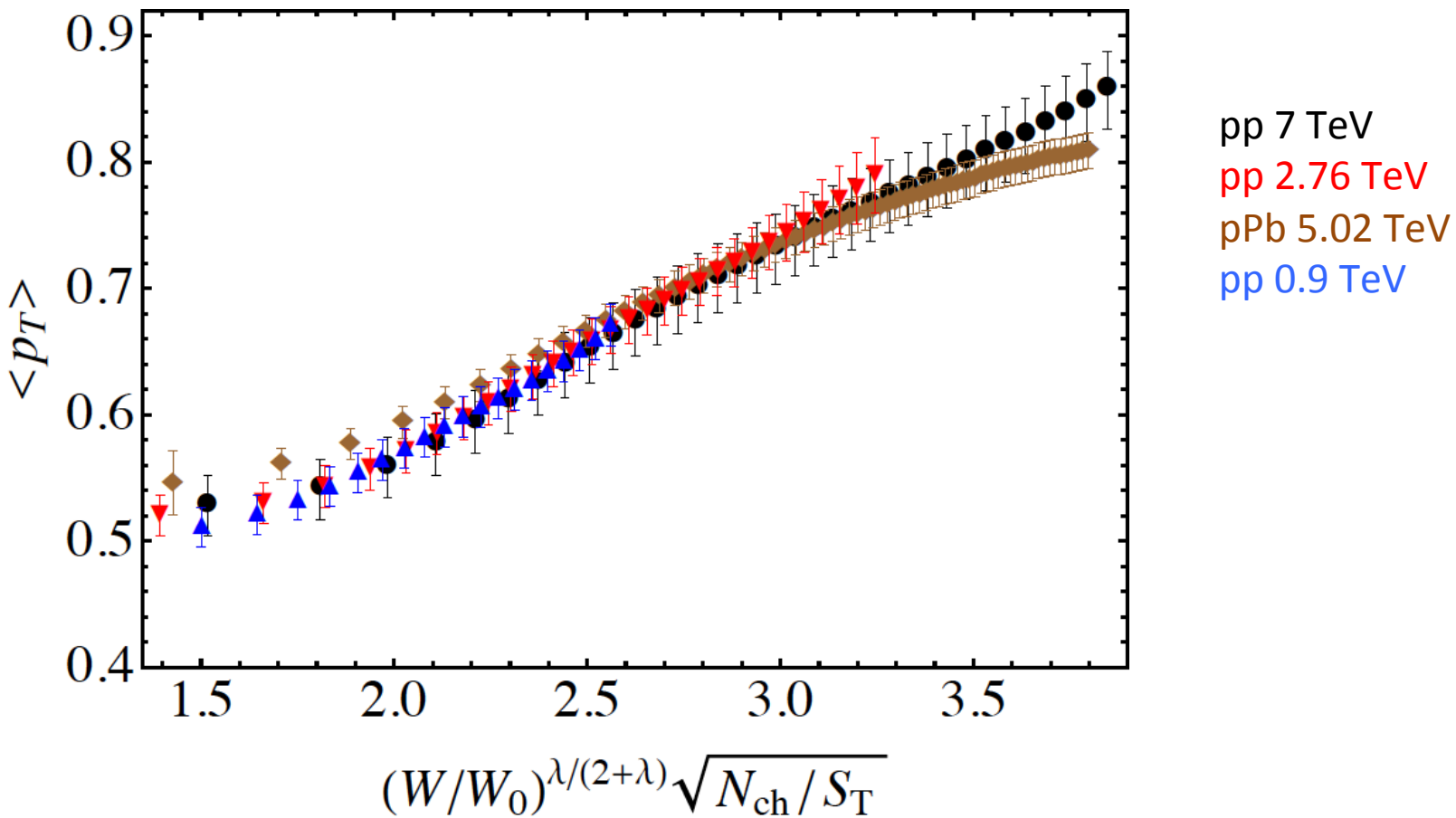
ALICE Collaboration, Phys. Lett. B727 (2013) 371 [arXiv:1307.1094 [nucl-ex]]




pp 7 TeV
pp 2.76 TeV
pPb 5.02 TeV
pp 0.9 TeV

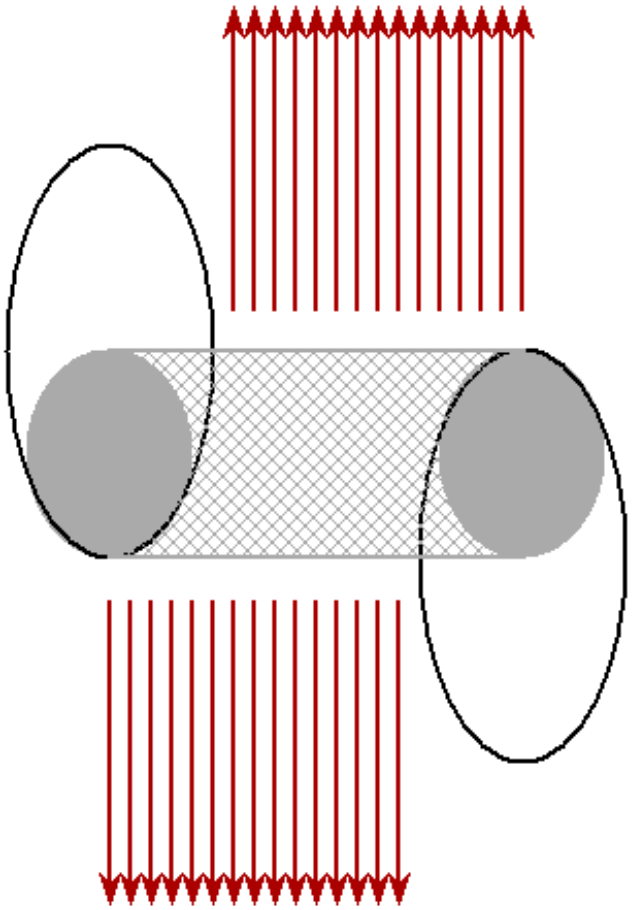
# Mean $p_T$ scaling

ALICE Collaboration, Phys. Lett. B727 (2013) 371 [arXiv:1307.1094 [nucl-ex]]

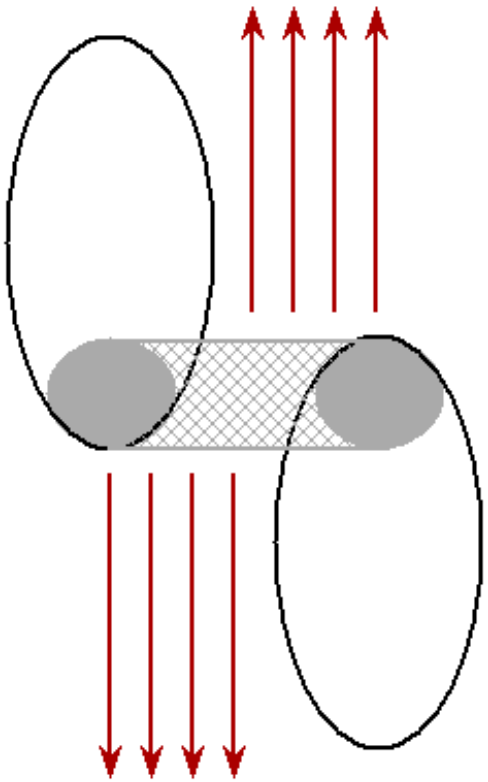


# Geometrical Scaling in Heavy Ion Collisions

# GS in HI: centrality



small centrality



large centrality

number of participants

$$N_{\text{part}} \sim V$$

# GS in HI: centrality dependence

$$S_\perp \sim N_{\rm part}^{2/3}$$
$$\frac{dN}{dy} \sim N_{\rm part}$$

*Geometrical Scaling of Direct-Photon Production in Hadron Collisions from RHIC to the LHC*
Christian Klein-Bösing, Larry McLerran arXiv:1403.1174

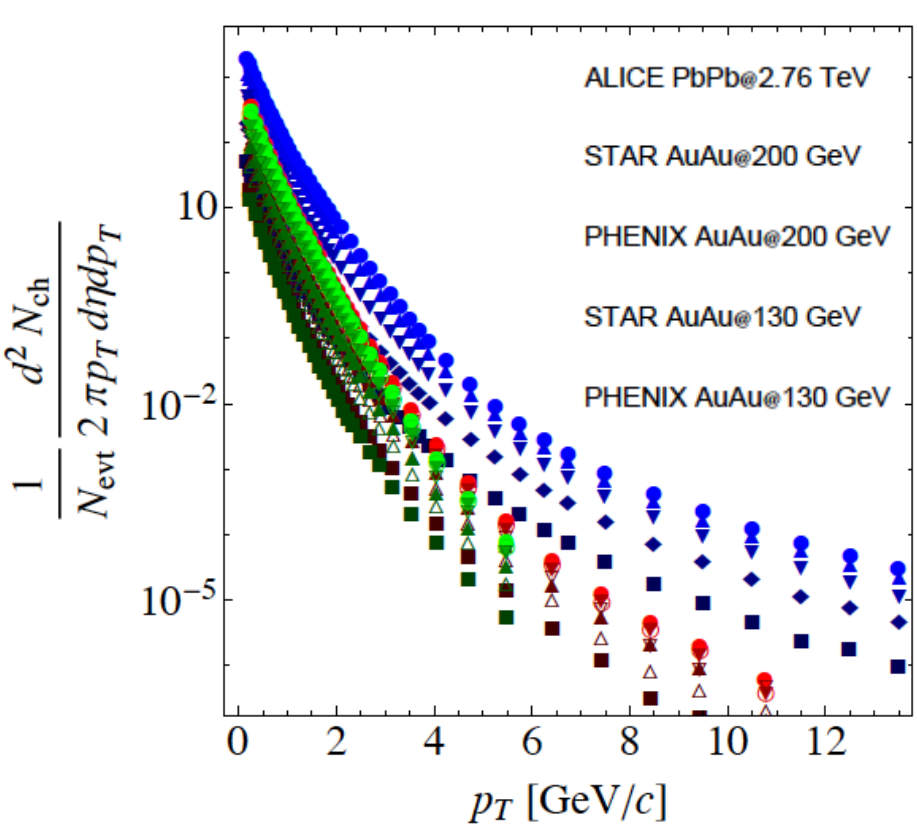
Scaling of the saturation scale:

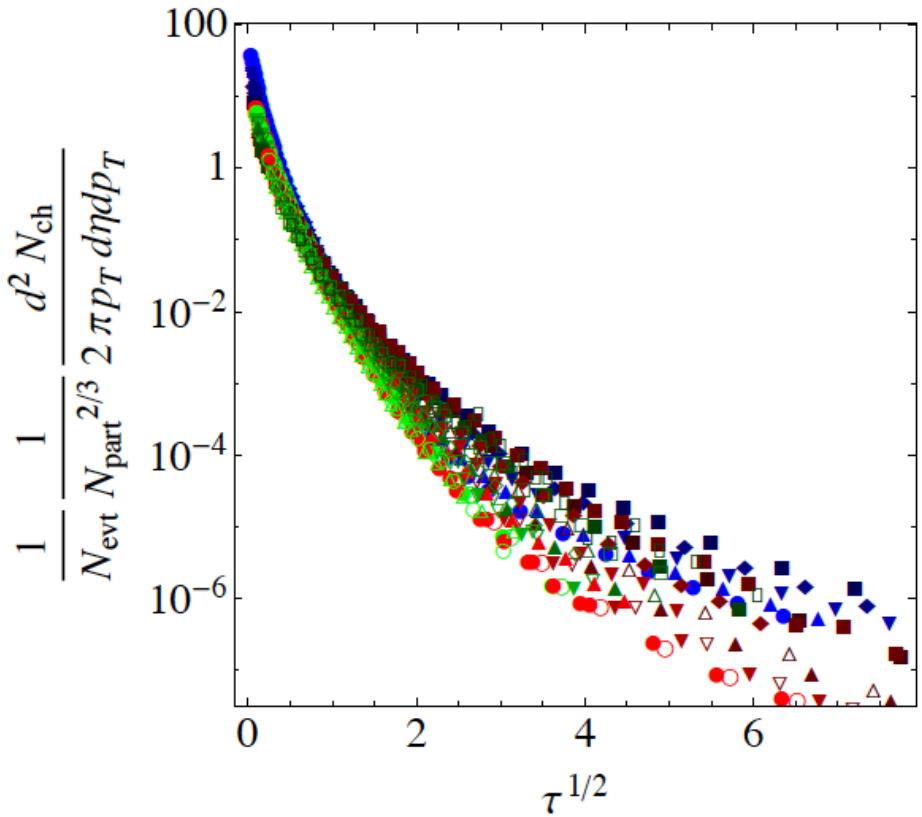
$$Q_s^2(x) = \frac{\kappa}{S_\perp}\frac{dN}{dy} \sim N_{\rm part}^{1/3}\left(\frac{\sqrt{s}}{p_{\rm T}}\right)^\lambda$$

$$\frac{Q_0^2}{N_{\rm part}^{2/3}}\frac{dN_{\rm ch}}{2\pi p_{\rm T}\, d\eta dp_{\rm T}} = F(\tau)$$

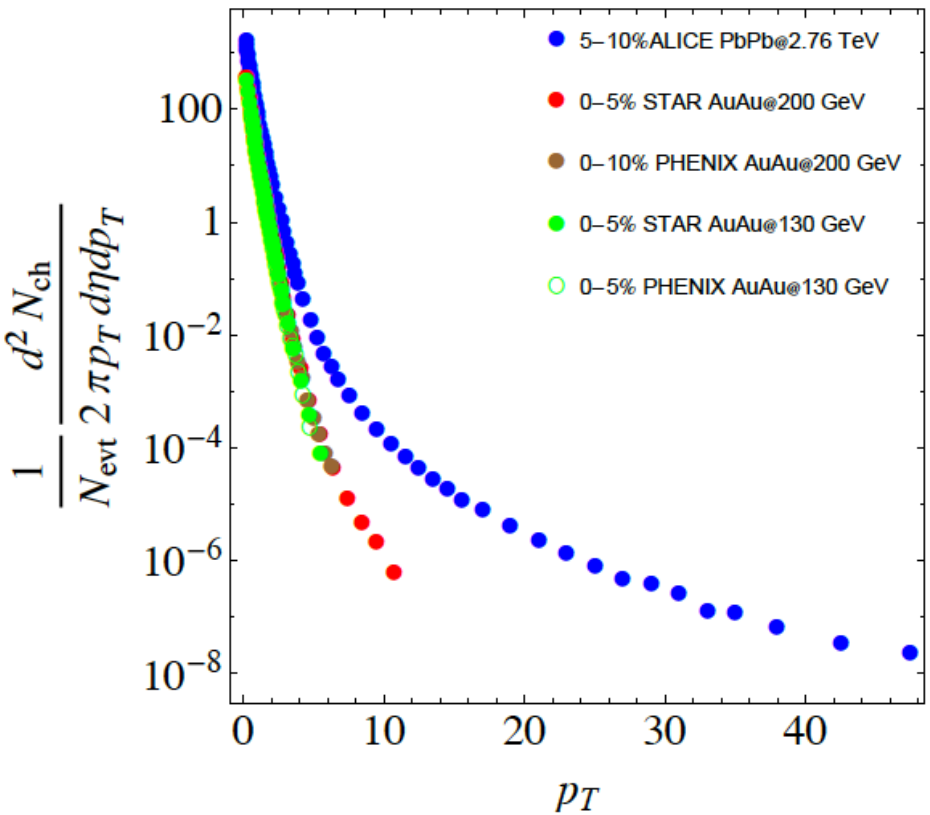
$$\tau = \frac{1}{N_{\rm part}^{1/3}}\frac{p_{\rm T}^2}{Q_0^2}\left(\frac{p_{\rm T}}{W}\right)^\lambda$$

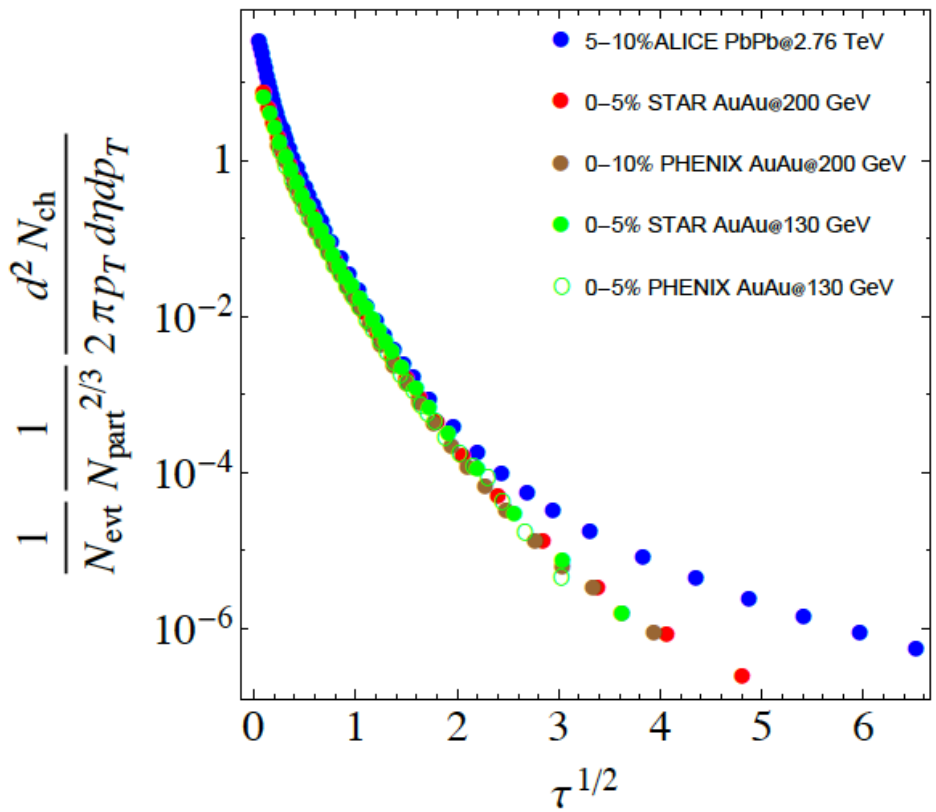
# GS in HI


ALICE PbPb@2.76 TeV
STAR AuAu@200 GeV
PHENIX AuAu@200 GeV
STAR AuAu@130 GeV
PHENIX AuAu@130 GeV
$\frac{1}{N_{evt}} \frac{d^2 N_{ch}}{2\pi p_T d\eta dp_T}$
$p_T$ [GeV/$c$]


$\frac{1}{N_{evt}} \frac{1}{N_{part}^{2/3}} \frac{d^2 N_{ch}}{2\pi p_T d\eta dp_T}$
$\tau^{1/2}$

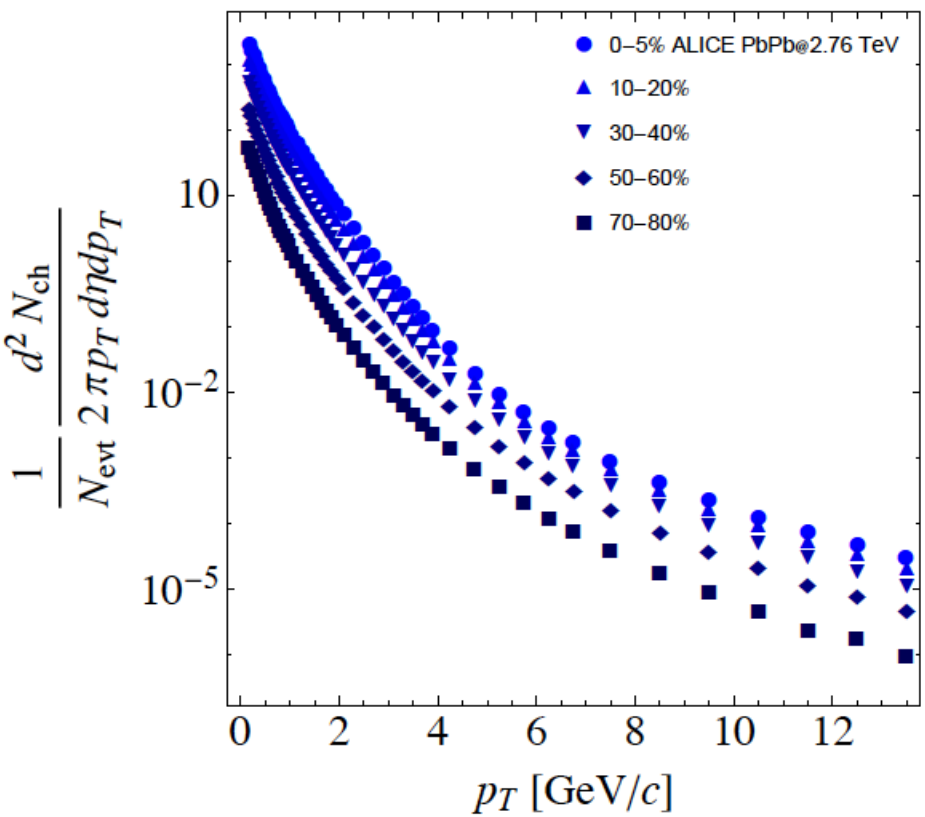
# Energy Scaling in HI

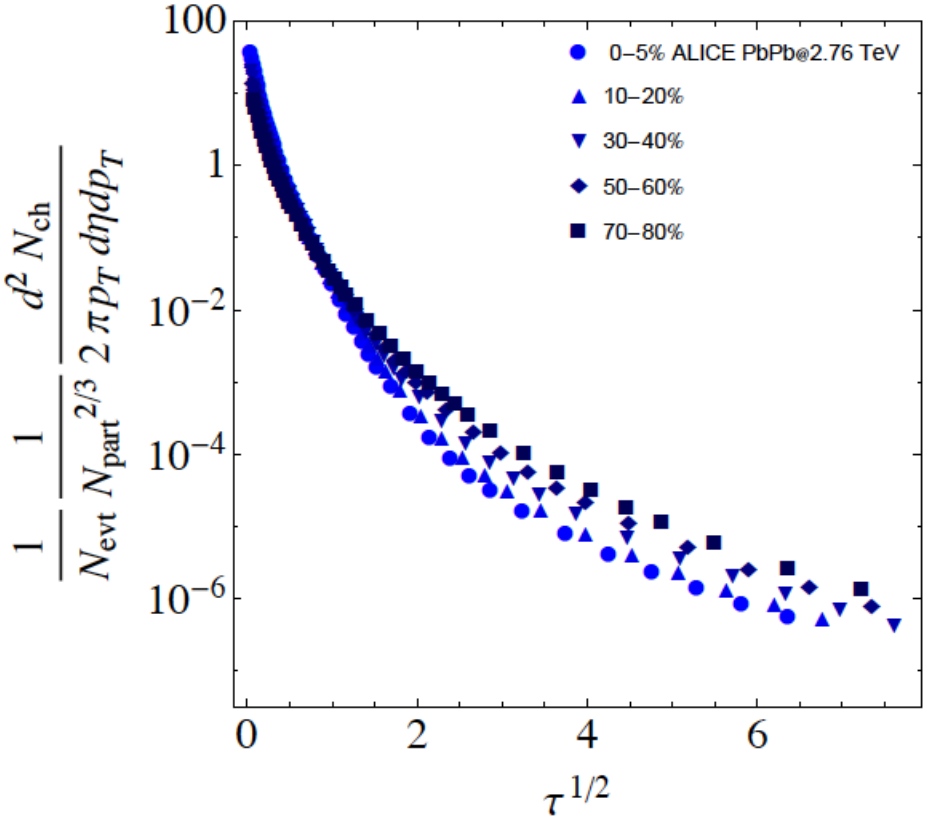



energy scaling works quite well, why?

# Centrality Scaling in HI



# Summary

- QCD evolution equations lead to overabundance of gluons
- Nonlinear evolution introduces new scale: *saturation momentum*
- GS should emerge if no other scales are present
- GS in DIS works for rather high Bjorken $x$
- GS works also for charged particles in pp
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- GS for mean $p_T$ and for $\langle p_T \rangle (N_{ch})$
- Energy and centrality dependence of GS in HI
- Is GS a real sign of saturation?
- Why in pp GS is not washed out by FSI?
- Why in HI hydro preserves (at least partially) GS?
- Nonuniversality: different values of $\lambda$, what should scale in pp?