Viscous Velocity Gradient Correction to Thermal Photon Emission Rate at Strong Coupling

based on
PRD 91,086011,2015 (1409.7674) KM and Ho-Ung Yee

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6. The Total Thermal Photon Emission Rate Including the Viscous Correction

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- replacing $f_0(k)$ by $f(k) \equiv f_0(k) + \delta f(k)$ in AMY's calculation of equilibrium thermal photon emission rate at weak coupling, one can get a viscous velocity gradient correction to the equilibrium thermal emission rate, which can be written as [Shen, Paquet, Heinz, and Gale, 2015]

$$\frac{d\Gamma}{d^3k} = \left(\Gamma_0 + \frac{\pi^{\mu\nu}\hat{k}_\mu\hat{k}_\nu}{2(\epsilon+\mathcal{P})}\Gamma_1\right)\,,$$

where $\pi^{ij} = \eta\sigma^{ij}$

- our motivation is to calculate Γ_1 at strong coupling using AdS/CFT

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[Son and Starinets'02; Iqbal and Liu'09]

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- therefore, it's enough to calculate the retarded Green function G^R in order to calculate the thermal photon emission rate including its viscous velocity gradient correction

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$$\begin{aligned}\delta G^R_{\mu\nu}(k) &= \frac{i}{32\pi G_5}\sigma_{ij}\int_{r_H}^{\infty} dr\,\frac{S(r)}{r}\Bigg[\left(\partial_r \mathcal{G}^R_{i\nu}(k,r)\right)\left(\partial_r \mathcal{G}^R_{\mu j}(k,r)\right) g^{rr} \\ &+ \left(k_i \mathcal{G}^R_{\rho\nu}(k,r) - k_\rho \mathcal{G}^R_{i\nu}(k,r)\right)\left(k_j \mathcal{G}^R_{\mu\sigma}(k,r) - k_\sigma \mathcal{G}^R_{\mu j}(k,r)\right) g^{\rho\sigma}\Bigg]\end{aligned}$$

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$$\frac{d\Gamma^{\rm shear}}{d^3\vec{k}}(\epsilon^\mu) = \frac{e^2}{(2\pi)^3 2\omega}\epsilon^\mu(\epsilon^\nu)^*\delta G^<_{\mu\nu}(k) = \frac{e^2}{(2\pi)^3 2\omega}2n_B(\omega){\rm Re}\left[\delta G^{(R)}(k;\epsilon)\right] ,$$

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and

$$\begin{aligned} \Gamma^{(1)}(\omega) &= \frac{1}{(2\pi)^3 2\omega}2n_B(\omega)\frac{N_c^2 T}{16\pi^2} \\ &\times\ {\rm Im}\left[\frac{1}{\phi_0(k)^2}\int_{r_H}^{\infty}dr\left[\partial_r\left(\frac{S(r)}{r^2}\right)r^3 f(r)H(r)\partial_r H(r) + \omega^2\frac{S(r)H(r)^2}{r^3}\right]\right] \end{aligned}$$

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$$\begin{aligned}\Gamma^{(0)}(\omega) &= \frac{1}{(2\pi)^3\omega}\, n_B(\omega)\frac{N_c^2 T\omega}{32\pi} \\ &\times \left|{}_2F_1\left(1 - \frac{1}{2}(1+i)\frac{\omega}{2\pi T}, 1 + \frac{1}{2}(1-i)\frac{\omega}{2\pi T}; 1 - i\frac{\omega}{2\pi T}; -1\right)\right.\end{aligned}$$

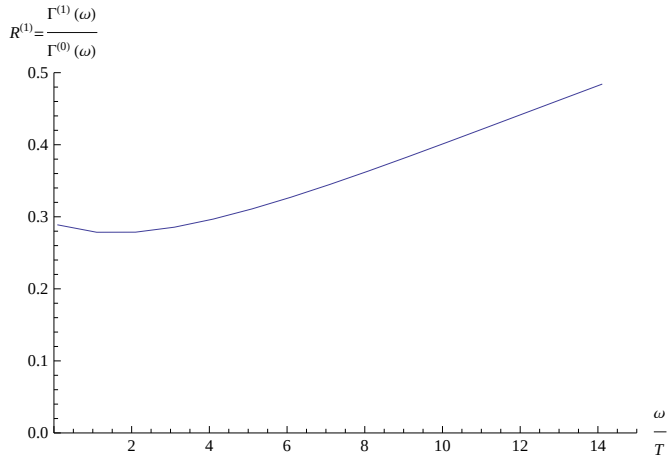

Figure: at strong coupling [KM, and Yee, 2015]

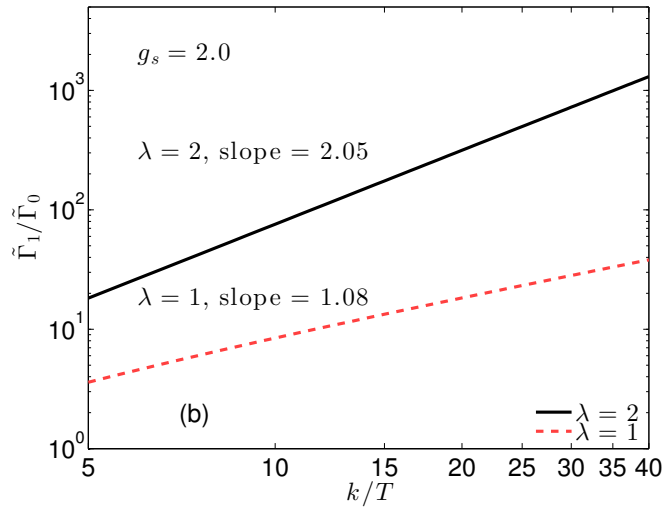

Figure: at weak coupling for $2 \to 2$ scattering [Shen, Paquet, Heinz, and Gale, 2015]

Thank You!