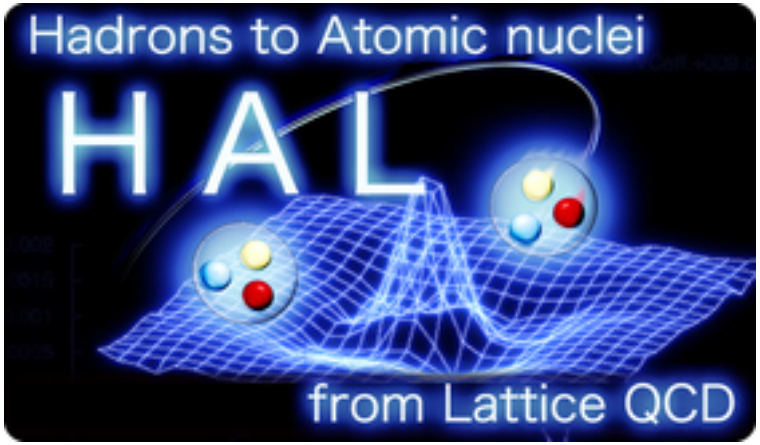
Baryon interactions from lattice QCD by Lüscher's finite volume method and HAL QCD method

Takumi Iritani (Stony Brook University)
for HAL QCD Collaboration

December 17 (Thu), 2015 @ BNL

Ref. TI for HAL QCD Coll., PoS(Lattice2015), arXiv:1511.05246 [hep-lat].



- S. Aoki, S. Gongyo, T. Miyamoto (YITP)
- T. Doi, T. Hatsuda, Y. Ikeda (RIKEN)
- T. Inoue (Nihon Univ.)
- N. Ishii, K. Murano (RCNP Osaka Univ.)
- H. Nemura, K. Sasaki (Univ. Tsukuba)
- F. Etminan (Univ. Birjand)

1. QCD to Nuclear Physics: Inconsistency of 2 Lattice QCD Approaches
2. Lattice Formulations for Hadron Interaction
3. Baryon Interaction from Lattice QCD
   - Lüscher's Method: Energy Shift in Finite Volume
   - HAL QCD Method: Potential
   - Energy Eigenvalues in Finite Volume
4. Beyond The Ground State
5. Summary and Outlook

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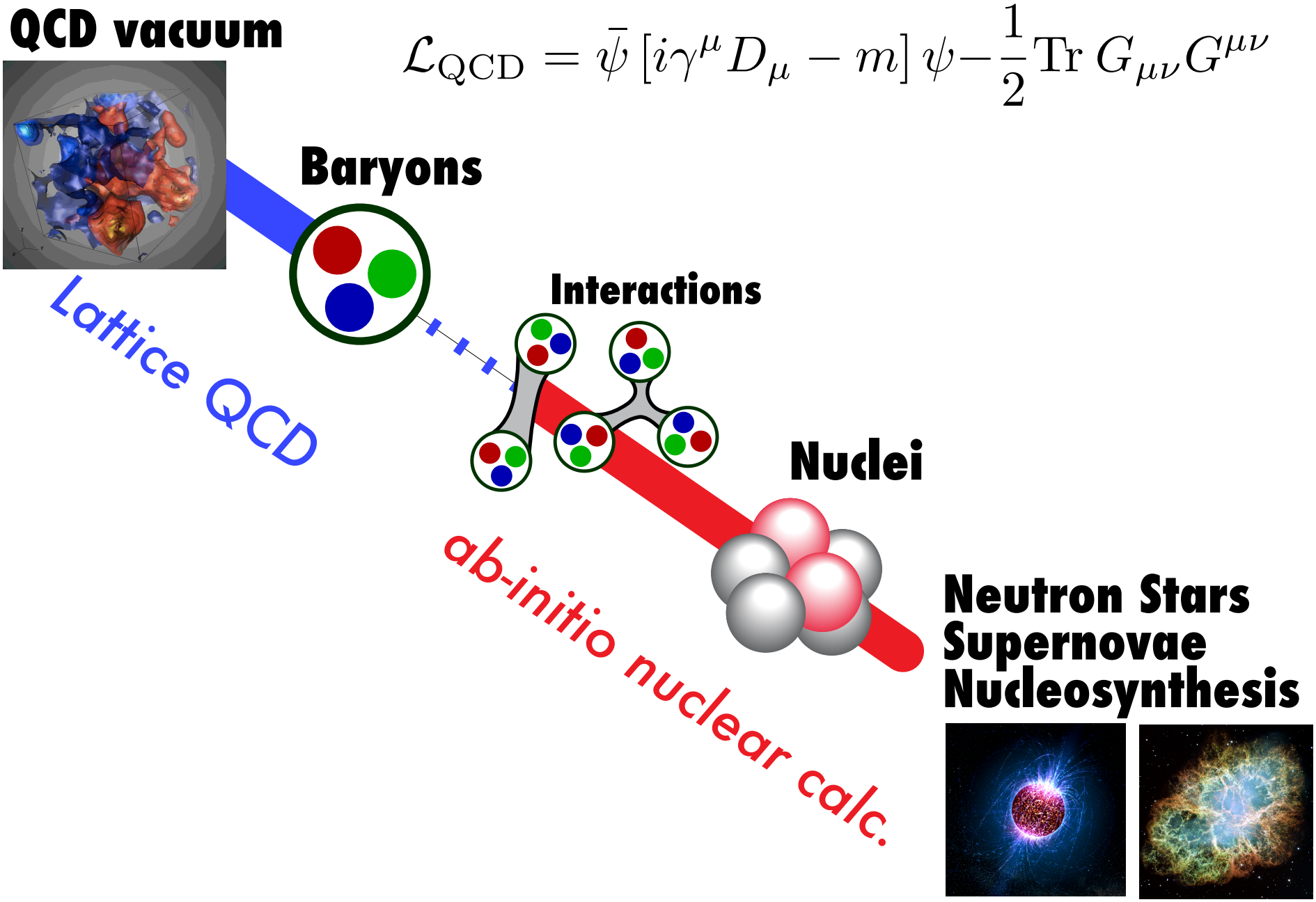
Dynamics of QCD: From Quarks to Universe

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}\left[i\gamma^{\mu}D_{\mu} - m\right]\psi - \frac{1}{2}\mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu}$$



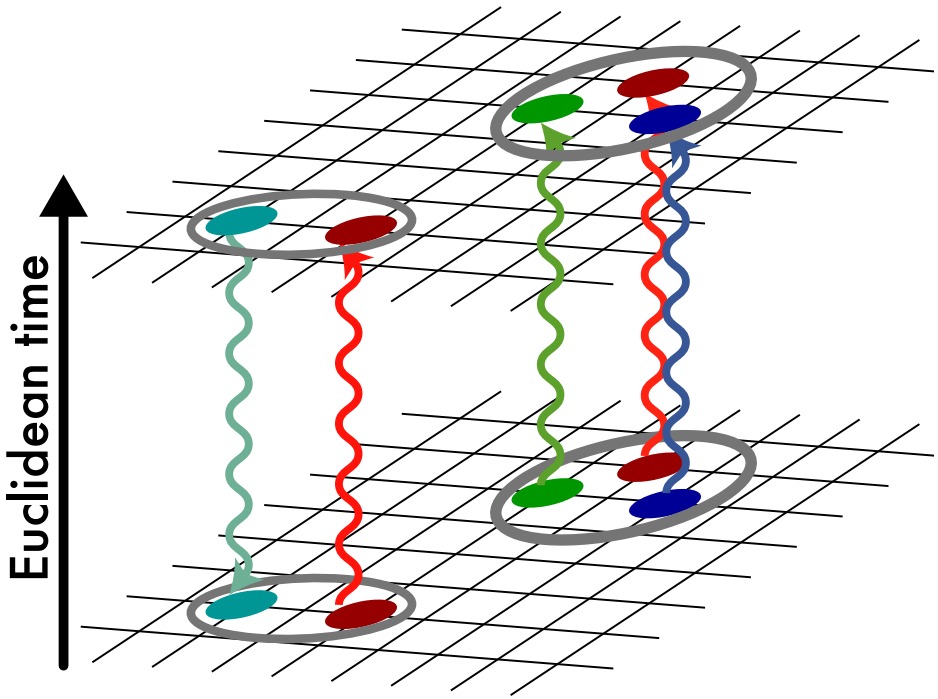
Lattice QCD: 1st Principle Calculation of QCD

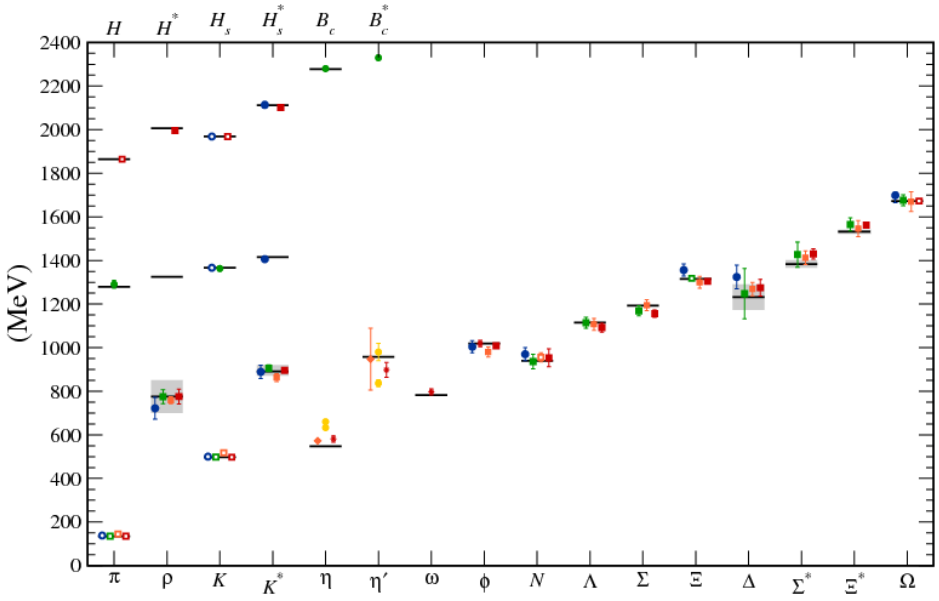
Formulate QCD on lattice & Path integral ▶ Monte Carlo calculation

$$\langle \mathcal{O} \rangle = \frac{\int DU D\bar{\psi} D\psi \mathcal{O}(U) e^{-S_{\rm QCD}}}{\int DU D\bar{\psi} D\psi e^{-S_{\rm QCD}}} \simeq \frac{1}{N} \sum_i^N \mathcal{O}(U_i)$$

ex. **hadron mass**

$$G(t) = \left\langle \mathcal{O}_H(t) \overline{\mathcal{O}}_H(0) \right\rangle = \sum_{n=0} c_n \exp\left(-m_n t\right) \simeq c_0 \exp\left(-m_0 t\right) \quad \text{for large } t$$





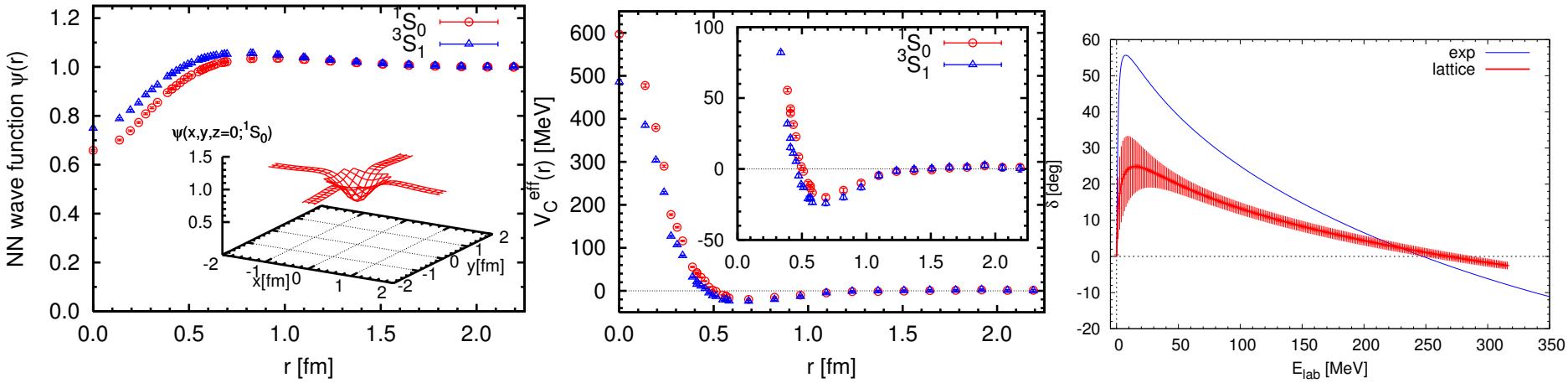
hadron spectrum (Fig. from Kronfeld '12)

2 Methods for Hadron Interaction in Lattice QCD

1. **Lüscher's finite volume method** — Lüscher '86, '91
   1. energy shift of two-particle system in **"box"** ▶ 2. phase shift

$$\Delta E_L = 2\sqrt{k^2+m^2} - 2m \implies k\cot\delta(k) = \frac{1}{\pi L}\sum_{\boldsymbol{n}\in\mathbb{Z}^3}\frac{1}{|\boldsymbol{n}|^2-(kL/2\pi)^2}$$

2. **HAL QCD method** — Ishii-Aoki-Hatsuda '07
   1. NBS wave function ▶ 2. potential ▶ 3. phase shift



Consistency of Lüscher and HAL — $I = 2$ $\pi\pi$ scattering

$\Rightarrow$ good agreement

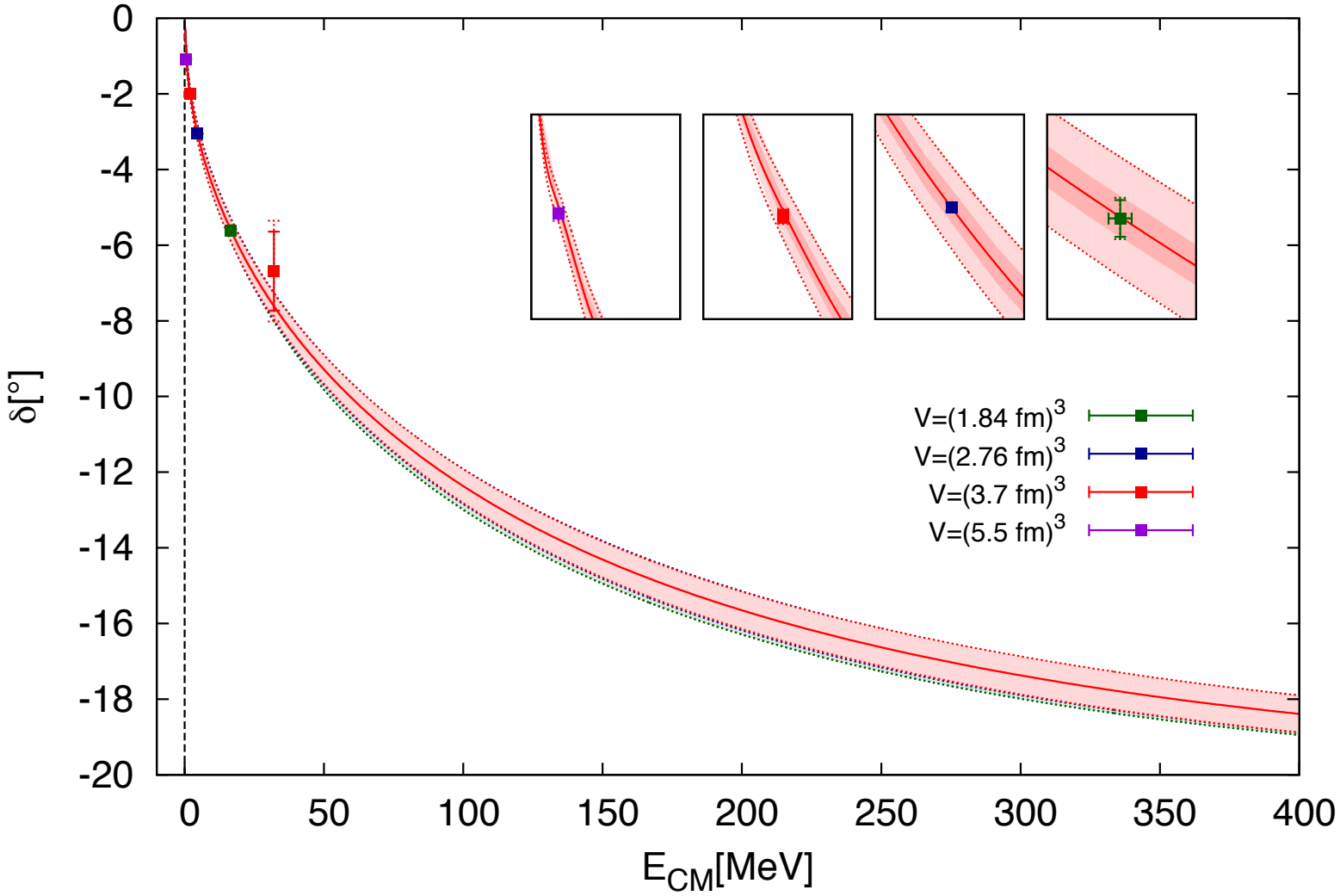
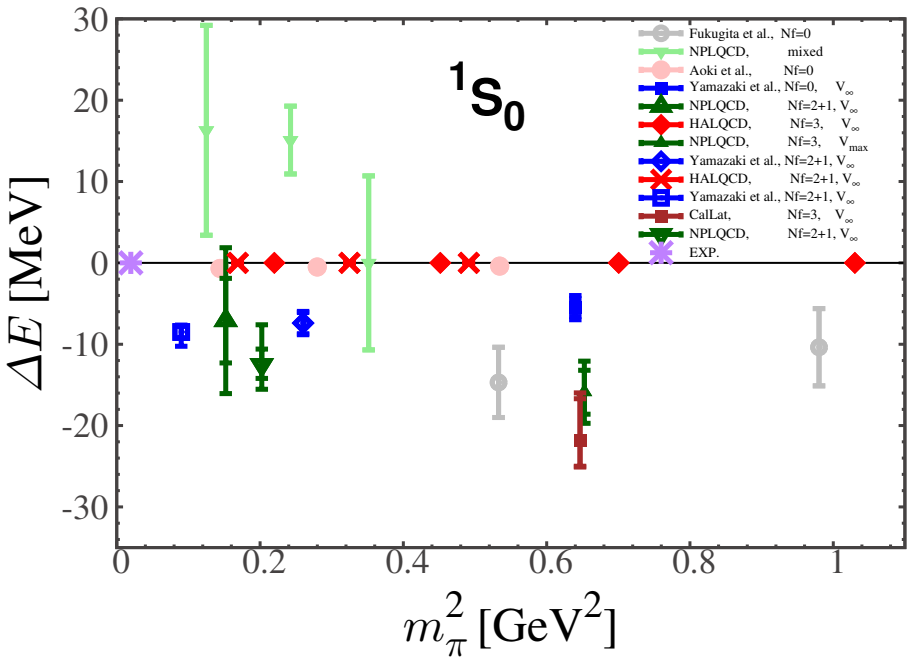


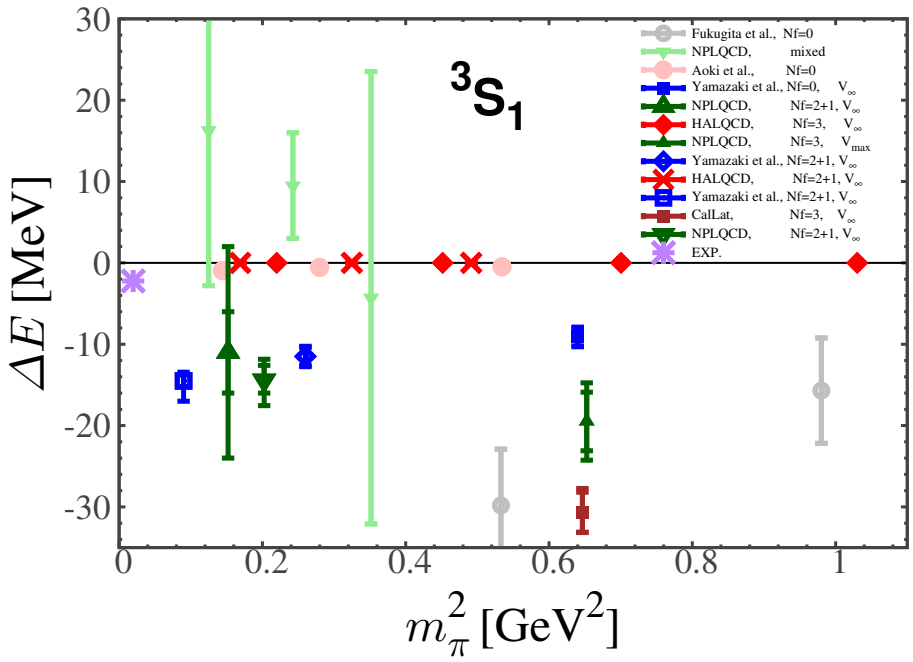
Figure: Kurth-Ishii-Doi-Aoki-Hatsuda '13

NN Interactions from Lattice QCD

	Lüscher		HAL QCD	phys. point
$^1S_0$	**bound**	$\Leftrightarrow$	**unbound**	**unbound**
$^3S_1$	**bound**	$\Leftrightarrow$	**unbound**	**bound**

- systematic deviations
  - unphysical pion mass ?
  - something problems with these methods or analyses ?





Aim of This Project

unfortunately, systematic discrepancies are reported between

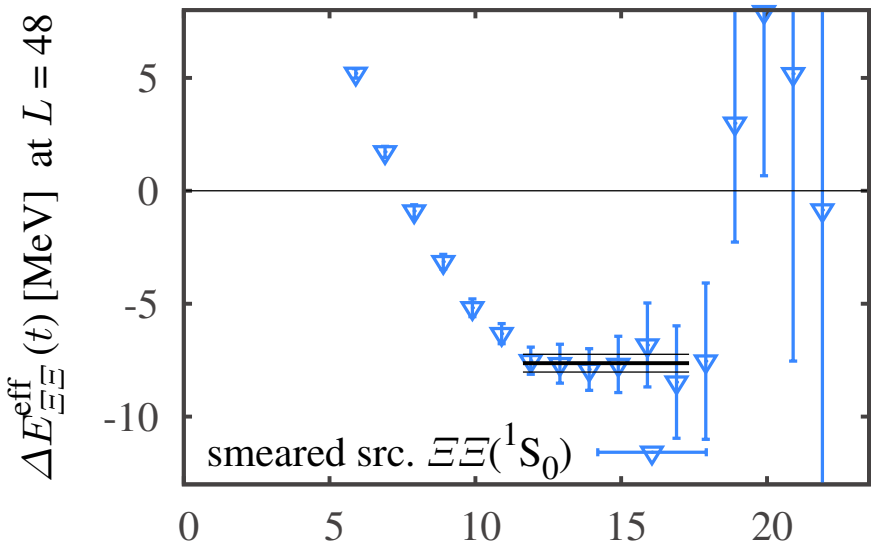
Lüscher's finite volume method and **HAL QCD method**

but those works used **different** quark mass, source (smeared and wall), ...

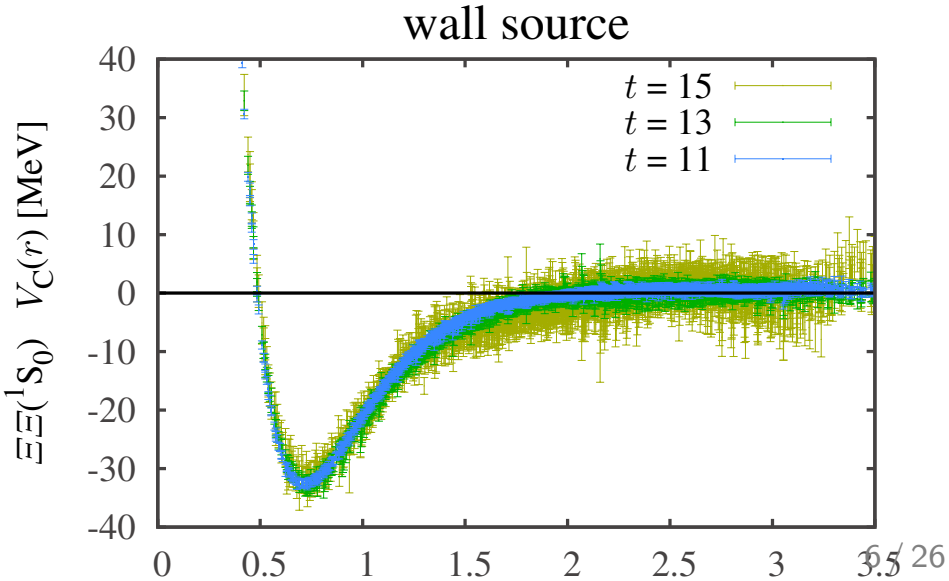
☐ *someone* has to **check baryon interactions**
from **both methods** with **the same setup**

☐ **an essential step** from **QCD** to **nuclear physics**,
which is correct/better?

■ measure energy shift



■ analyze potential



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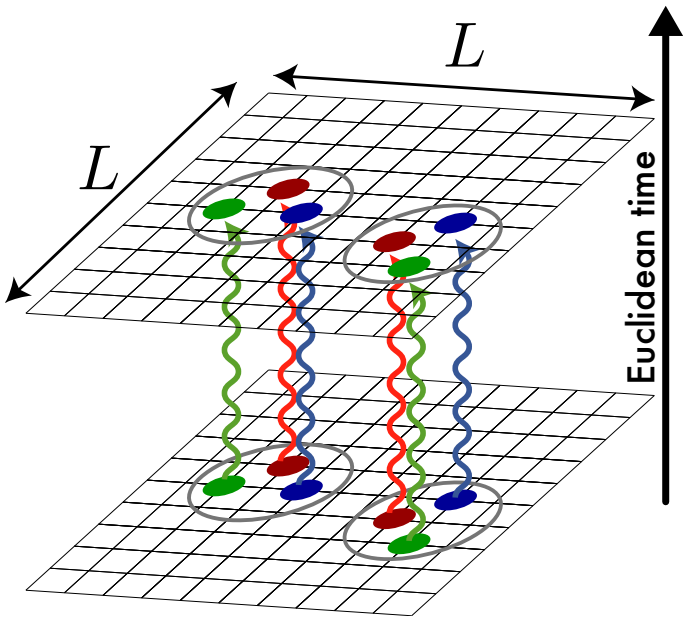
Lüscher's Finite Volume Method

- "**energy shift**" **in finite box** L^3

$$\Delta E_L = E_{BB} - 2m_B = 2\sqrt{k^2 + m_B^2} - 2m_B$$

$\Rightarrow$ **phase shift** $\delta(k)$

$$k \cot \delta(k) = \frac{1}{\pi L} \sum_{\boldsymbol{n} \in \mathbb{Z}^3} \frac{1}{|\boldsymbol{n}|^2 - (kL/2\pi)^2}$$



- **measurement**: plateau in **effective mass**

$$\Delta E_{\text{eff}}(t) = \log \frac{R(t)}{R(t+1)} \to \Delta E_L$$

$$R(t) = \frac{G_{BB}(t)}{\{G_B(t)\}^2} \to \exp\left[-\left(E_{BB} - 2m_B\right) t\right]$$

with $G_{BB}(t)(G_B(t))$: BB(B) correlators

- effective mass plot

$\Longrightarrow$ **standard method** in "single particle state"

- NN(^{1}S$_0$) energy shift

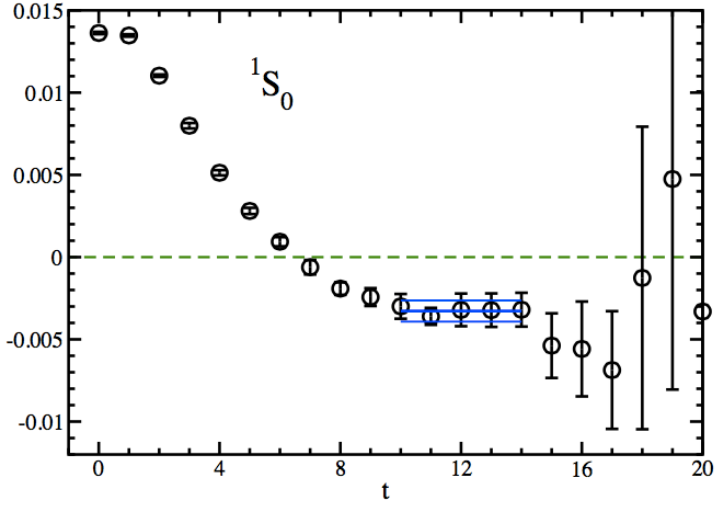
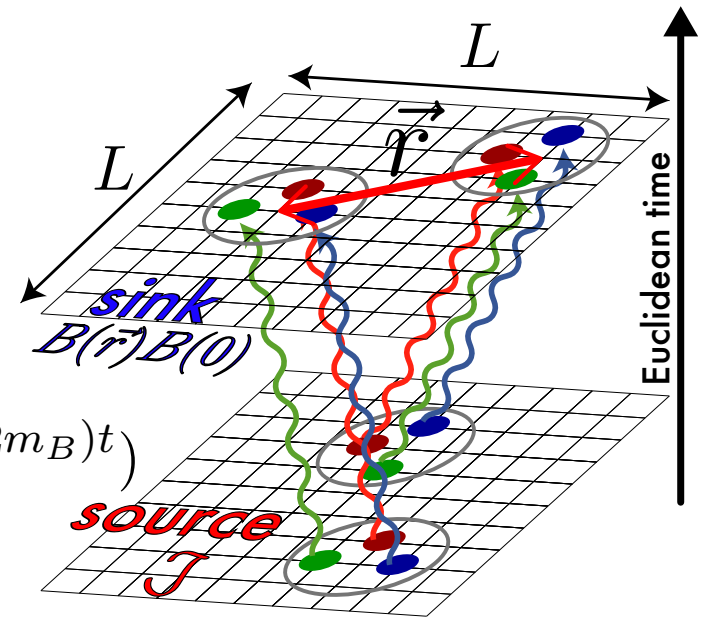


Fig. Yamazaki et al. '12

Time-dependent HAL QCD Method



- Nambu-Bethe-Salpeter correlation function

$$R(\vec{r},t) \equiv \frac{\langle 0|T\{B(\vec{x}+\vec{r},t)B(\vec{x},t)\}\bar{\mathcal{J}}(0)|0\rangle}{\{G_B(t)\}^2}$$
$$= \sum_n A_n \psi_n(\vec{r}) e^{-(E_n-2m_B)t} + \mathcal{O}(e^{-(E_{\rm th}-2m_B)t})$$

▶ in Lüscher's method: $\sum_{\vec{r}} R(\vec{r},t) = R(t) \Longrightarrow \exp\left[-(E_{BB}-2m_B)t\right]$

□ each $\psi_n(\vec{r})e^{-E_n t} \equiv \langle 0|T\{B(\vec{x}+\vec{r},t)B(\vec{x},t)\}|2B,n\rangle$ satisfies

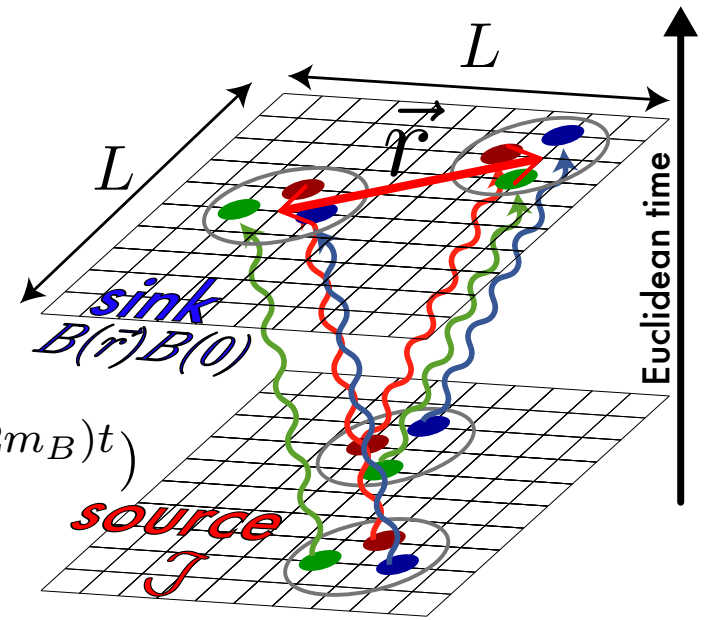
$$\left[\frac{k_n^2}{m_B} - H_0\right]\psi_n(\vec{r}) = \int d\vec{r'} U(\vec{r},\vec{r'})\psi_n(\vec{r'})$$

with non-local interaction kernel $U(\vec{r},\vec{r'})$

- R-corr. satisfies t-dep. Schrödinger-like eq. with **elastic** saturation

$$\left[\frac{1}{4m_B}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0\right] R(\vec{r},t) = \int d\vec{r'} U(\vec{r},\vec{r'}) R(\vec{r'},t)$$

Time-dependent HAL QCD Method



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  - **"potential"** using velocity expansion $U(r,r') \simeq V(r)\delta(r-r')$

$$V(\vec{r}) = \frac{1}{4m_B}\frac{(\partial/\partial t)^2 R(\vec{r},t)}{R(\vec{r},t)} - \frac{(\partial/\partial t)R(\vec{r},t)}{R(\vec{r},t)} - \frac{H_0 R(\vec{r},t)}{R(\vec{r},t)}$$

  - **This method does not require the ground state saturation.**

Difficulties in Multi-Baryons

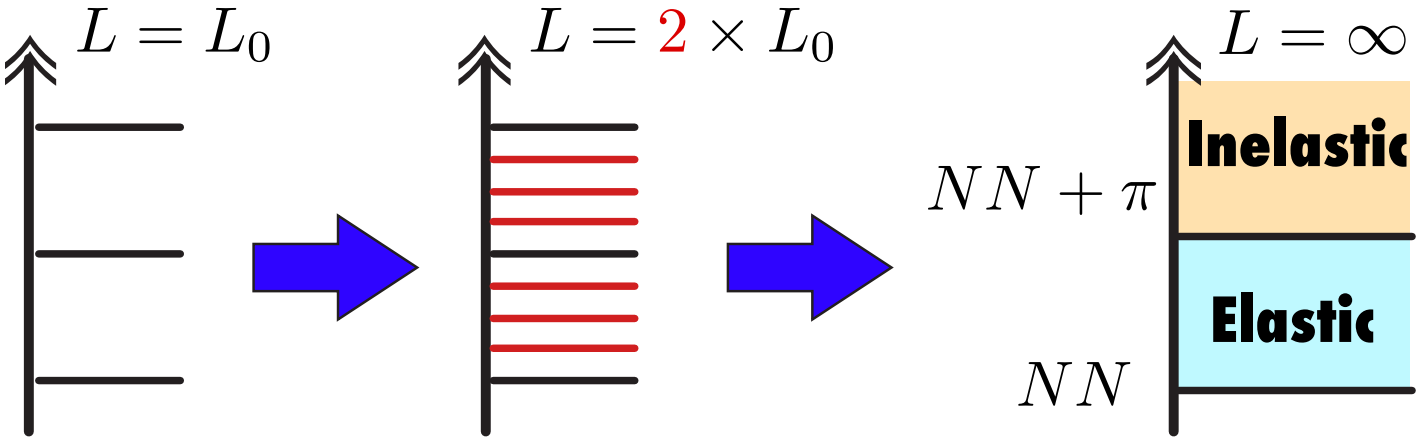
- Lüscher's method requires **ground state saturation**

$$G_{NN}(t) = c_0 \exp(-E_0^{(NN)} t) + c_1 \exp(-E_1^{(NN)} t) + \cdots \simeq c_0 \exp(-E_0^{(NN)} t)$$

  - S/N becomes worse as [mass number A] $\times$ [light quark] $\times$ [$t \to \infty$]

$$S/N \sim \exp\left[-A \times \left(m_N - \frac{3}{2} m_\pi\right) \times t\right]$$

  - smaller gaps: $\Delta E \sim \vec{p}^{\,2}/m \sim \mathcal{O}(1/L^2)$



- **HAL QCD method**
  - only **elastic states saturation** — E-indep potential

$$\left[\frac{1}{4m_B}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0\right] R(\vec{r}, t) = \int d\vec{r'} U(\vec{r}, \vec{r'}) R(\vec{r'}, t)$$

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Lattice Setup

- $2+1$ improved Wilson + Iwasaki gauge†

  lattice spacing $a = 0.08995(40)$ fm, $a^{-1} = 2.194(10)$ GeV

  $m_\pi = 0.51$ GeV, $m_N = 1.32$ GeV, $m_K = 0.62$ GeV, $m_\Xi = 1.46$ GeV

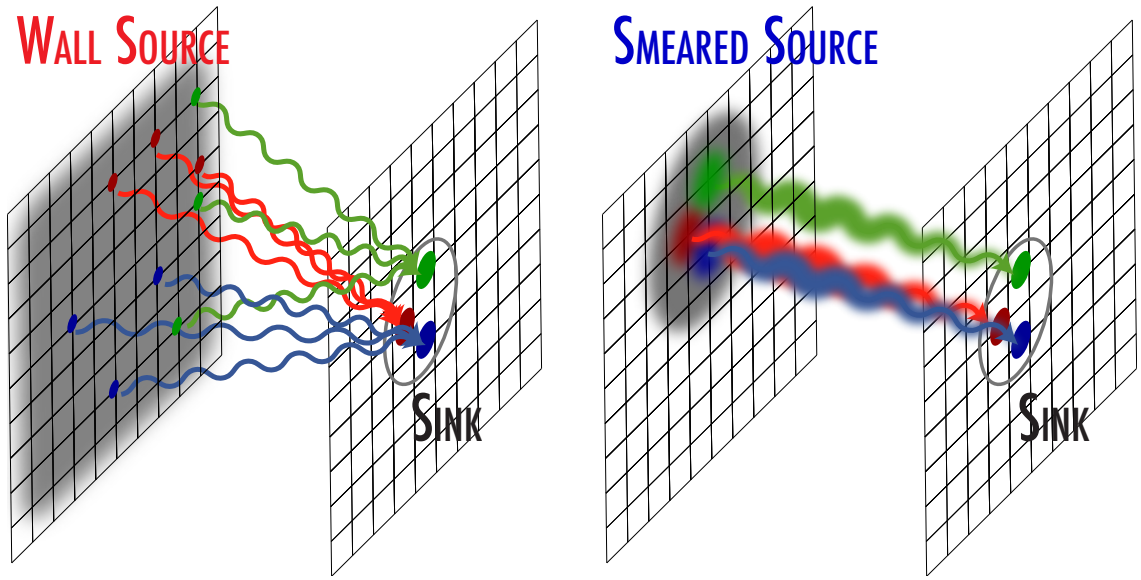
- **2-type quark sources**
  - **wall source**

    standard of HAL QCD
  - **smeared source**

    the same as Yamazaki *et al*†.

    → **NN int.** by Lüscher

    & **Helium** binding energy



□ analyze **$\Xi\Xi(^1S_0)$-channel** — the same rep. as NN($^1S_0$) and better S/N with high stat. — ex. 48^4: (#smeared src.)/(*Yamazaki et al.*) $\simeq 5$

volume	# conf.	# smeared src.	# wall src.
$40^3 \times 48$	200	512	48
$48^3 \times 48$	200	4 × 256	4 × 48
$64^3 \times 64$	327	256	4 × 32

† Yamazaki-Ishikawa-Kuramashi-Ukawa, arXiv:1207.4277.

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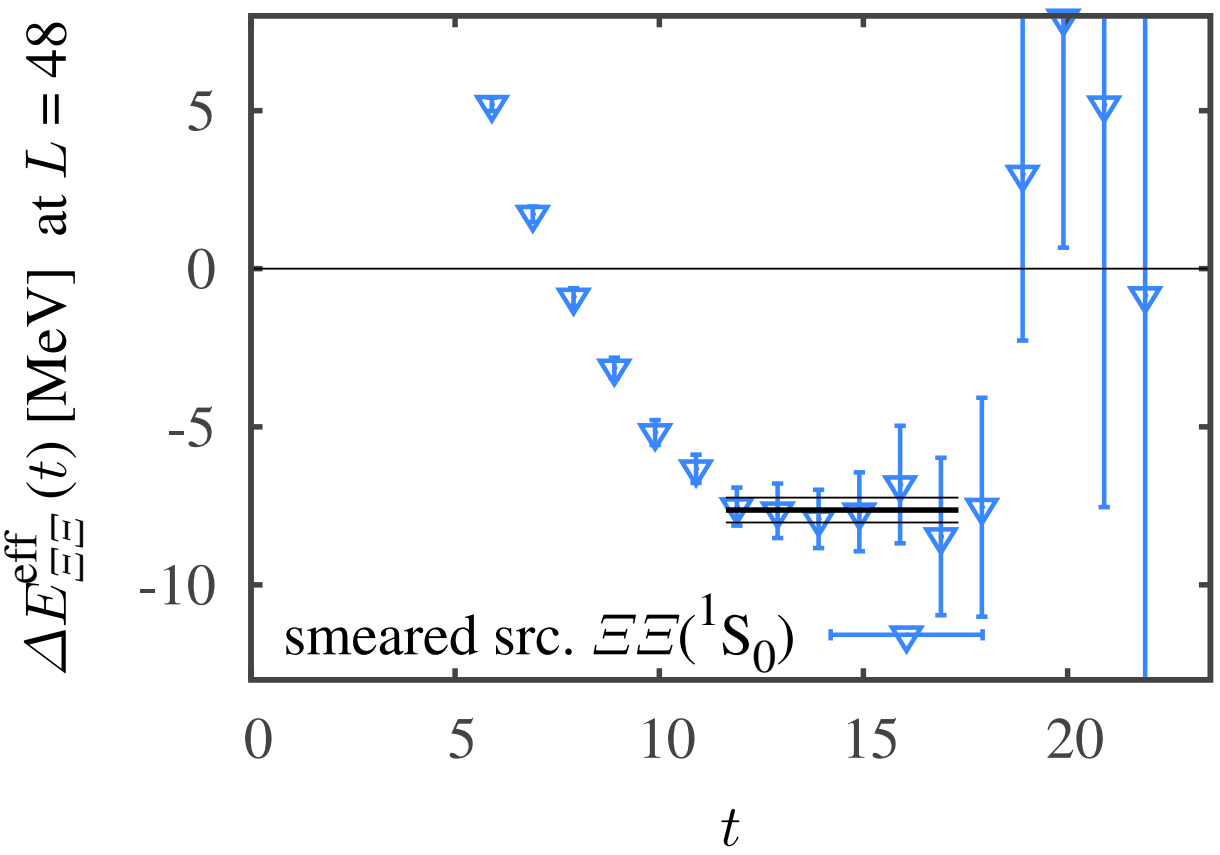
$\Xi\Xi(^1S_0)$ Effective Energy Shift Plot

effective mass plot

$$\Delta E^{\rm eff}_{\Xi\Xi}(t) = \log R(t)/R(t+1) \to E_{\Xi\Xi} - 2m_\Xi$$

with $R(t) \equiv G_{\Xi\Xi}(t)/\{G_\Xi(t)\}^2$

- **smeared source**
  $\Delta E_L \sim -8$ MeV



$\Xi\Xi(^1S_0)$ Effective Energy Shift Plot

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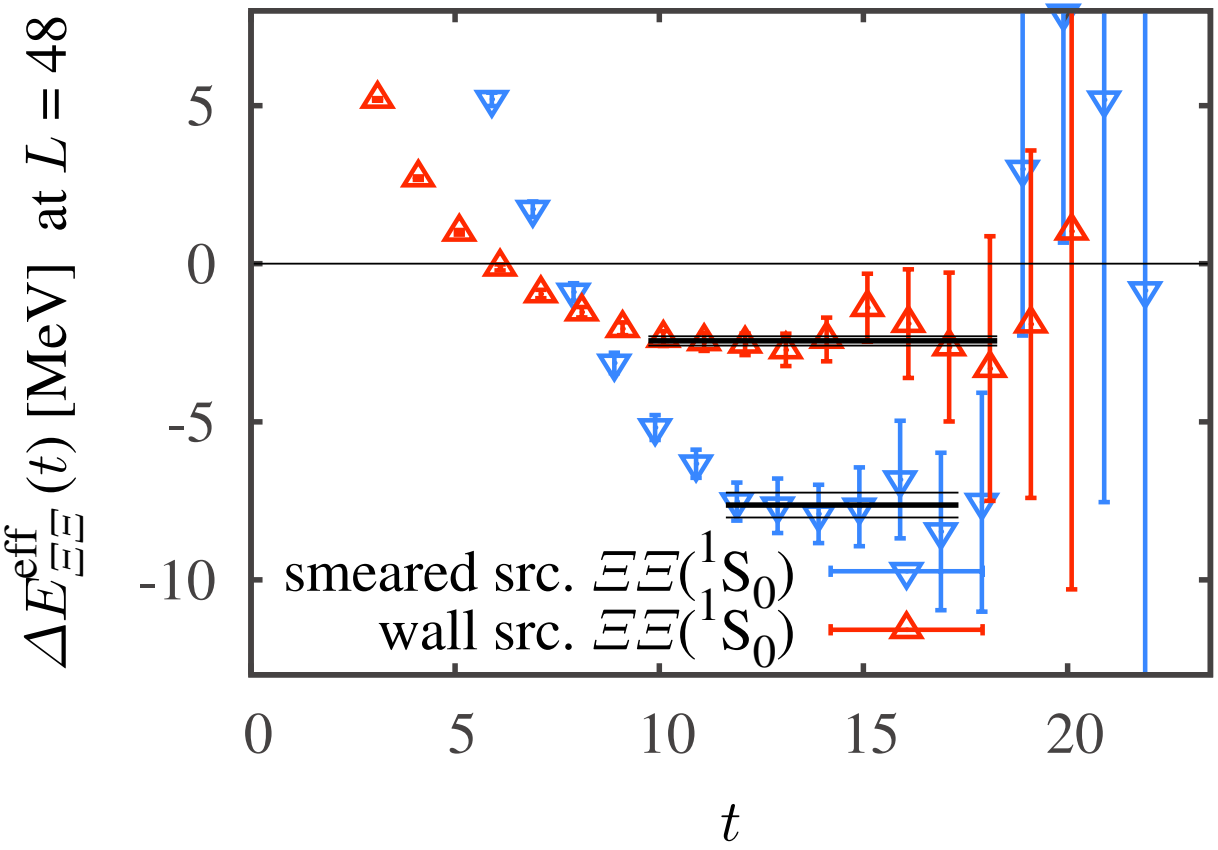
$$\Delta E^{\rm eff}_{\Xi\Xi}(t) = \log R(t)/R(t+1) \to E_{\Xi\Xi} - 2m_\Xi$$

with $R(t) \equiv G_{\Xi\Xi}(t)/\{G_\Xi(t)\}^2$

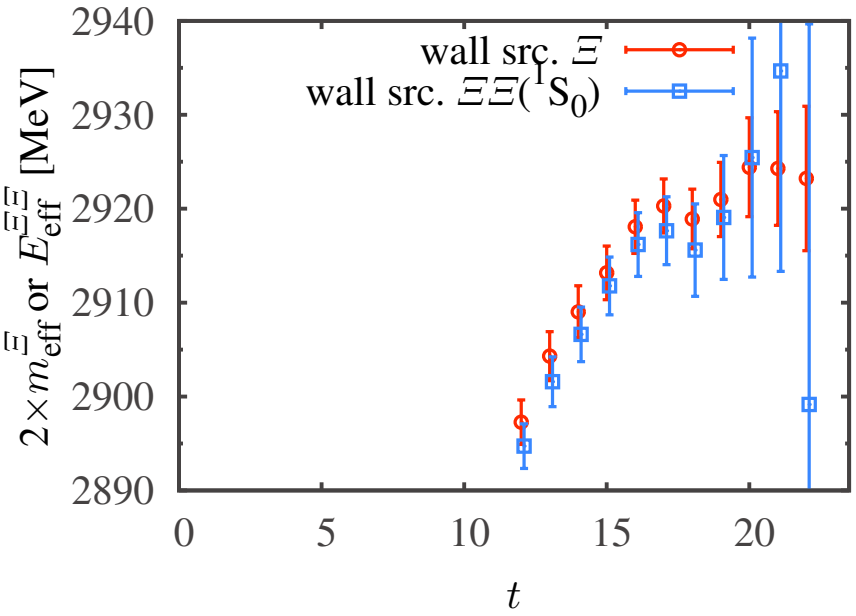
- **wall source**
  $\Delta E_L \sim -3$ MeV
- **smeared source**
  $\Delta E_L \sim -8$ MeV
- **plateau** depends on quark source

  ???

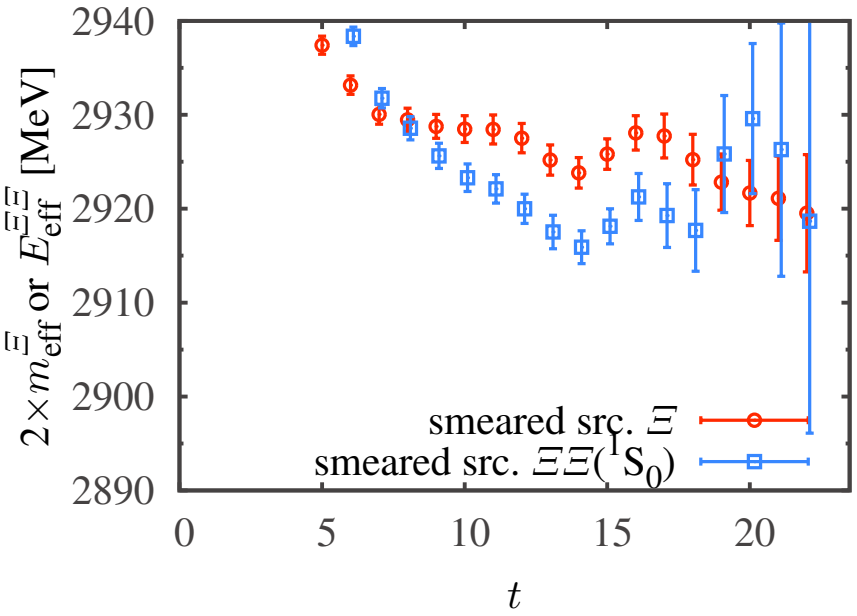


Effective Masses of ΞΞ and Ξ

wall source



smeared source

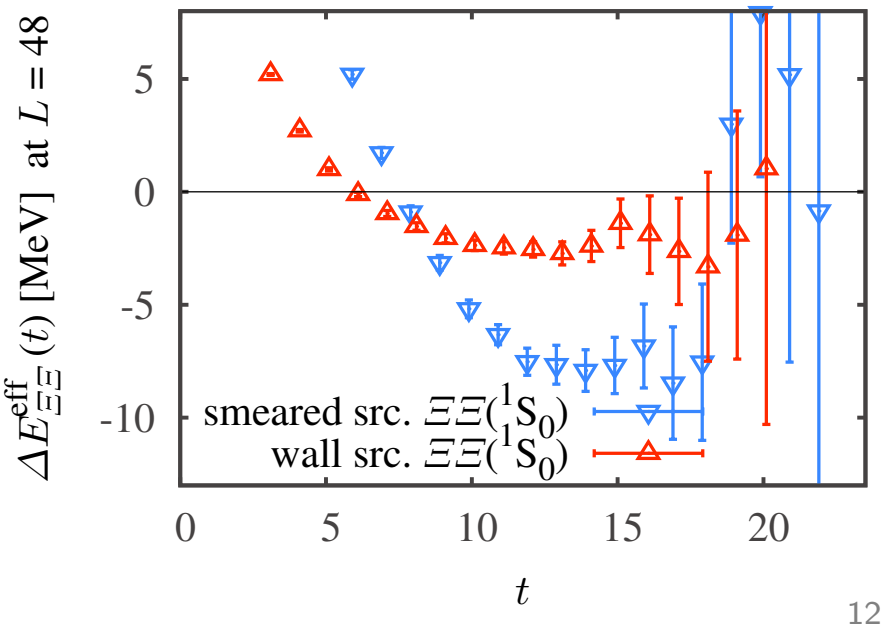


- Be careful with effective mass plot **w/o plateaux** in $E^{\rm eff}_{\Xi\Xi}(t)$ & $m^{\rm eff}_{\Xi}(t)$,

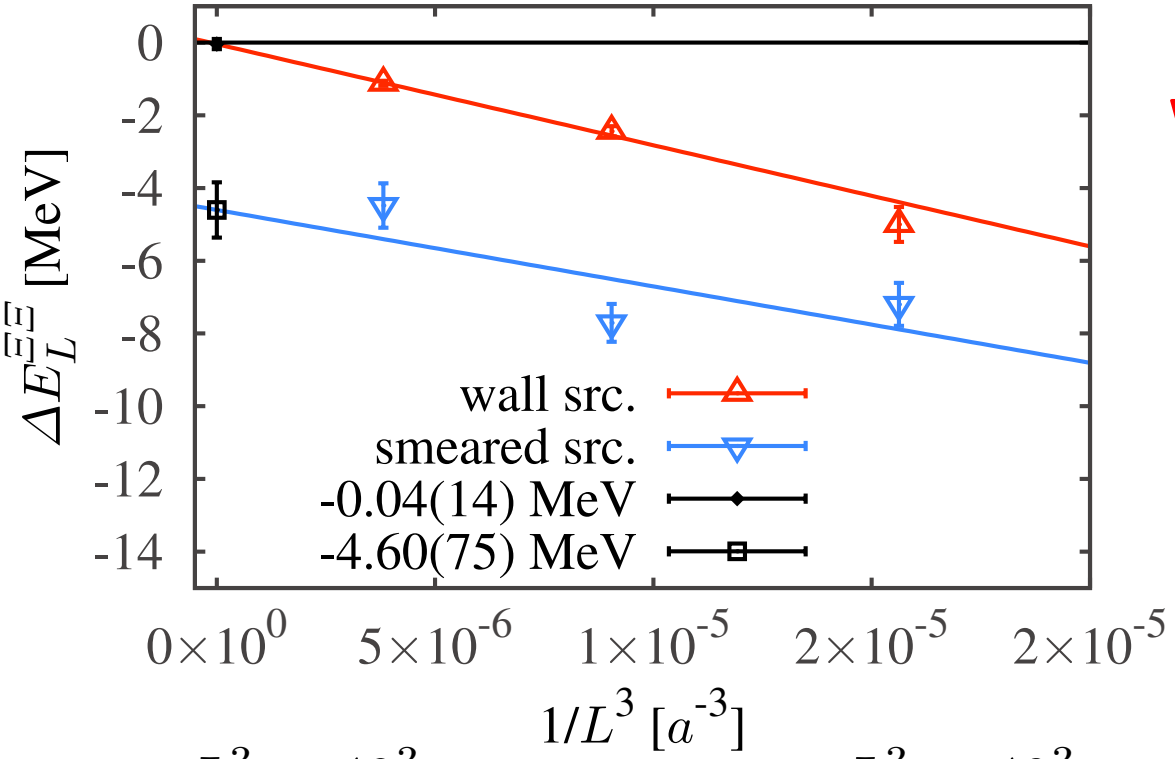
$$\Delta E_{\rm eff}(t) = E^{\Xi\Xi}_{\rm eff}(t) - 2m^{\Xi}_{\rm eff}(t)$$

shows a **fake plateau** by cancellation

▶ we need much larger t,
and much more statistics



Fake Plateaux $\Longrightarrow$ Wrong Conclusion



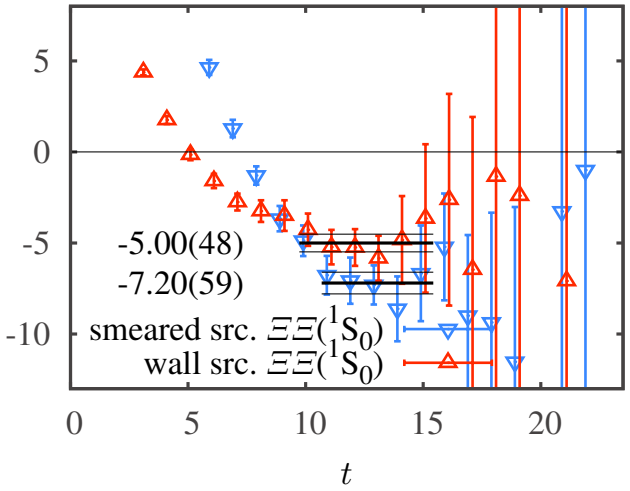
- **"direct method"**

wall src. vs **smeared src.**

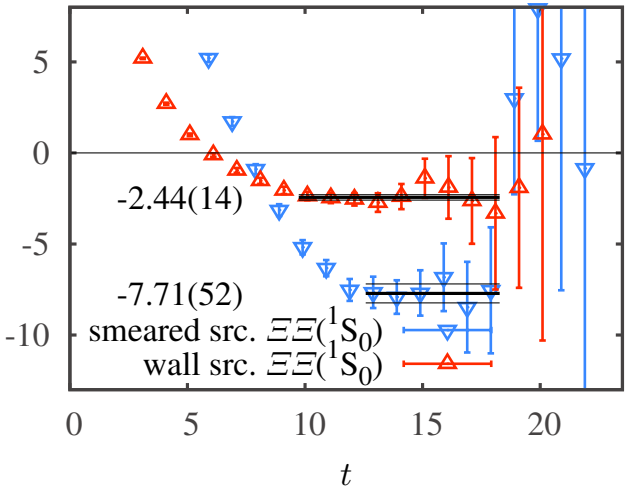
- different plateaux
- different conclusions **unbound** or **bound**

- **where is the true ΔE_L**

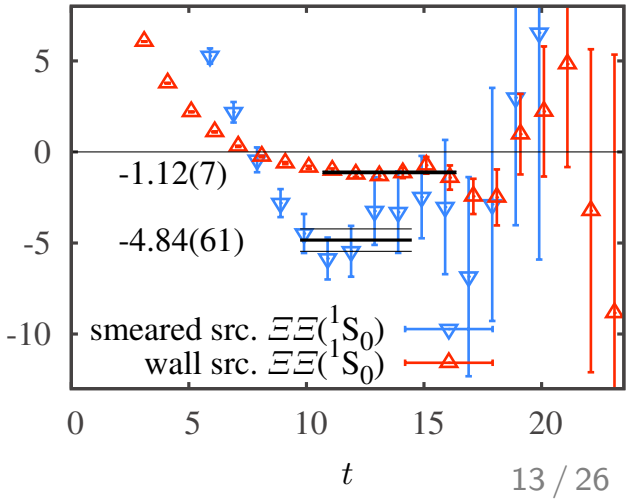
$L^3 = 40^3$



$L^3 = 48^3$

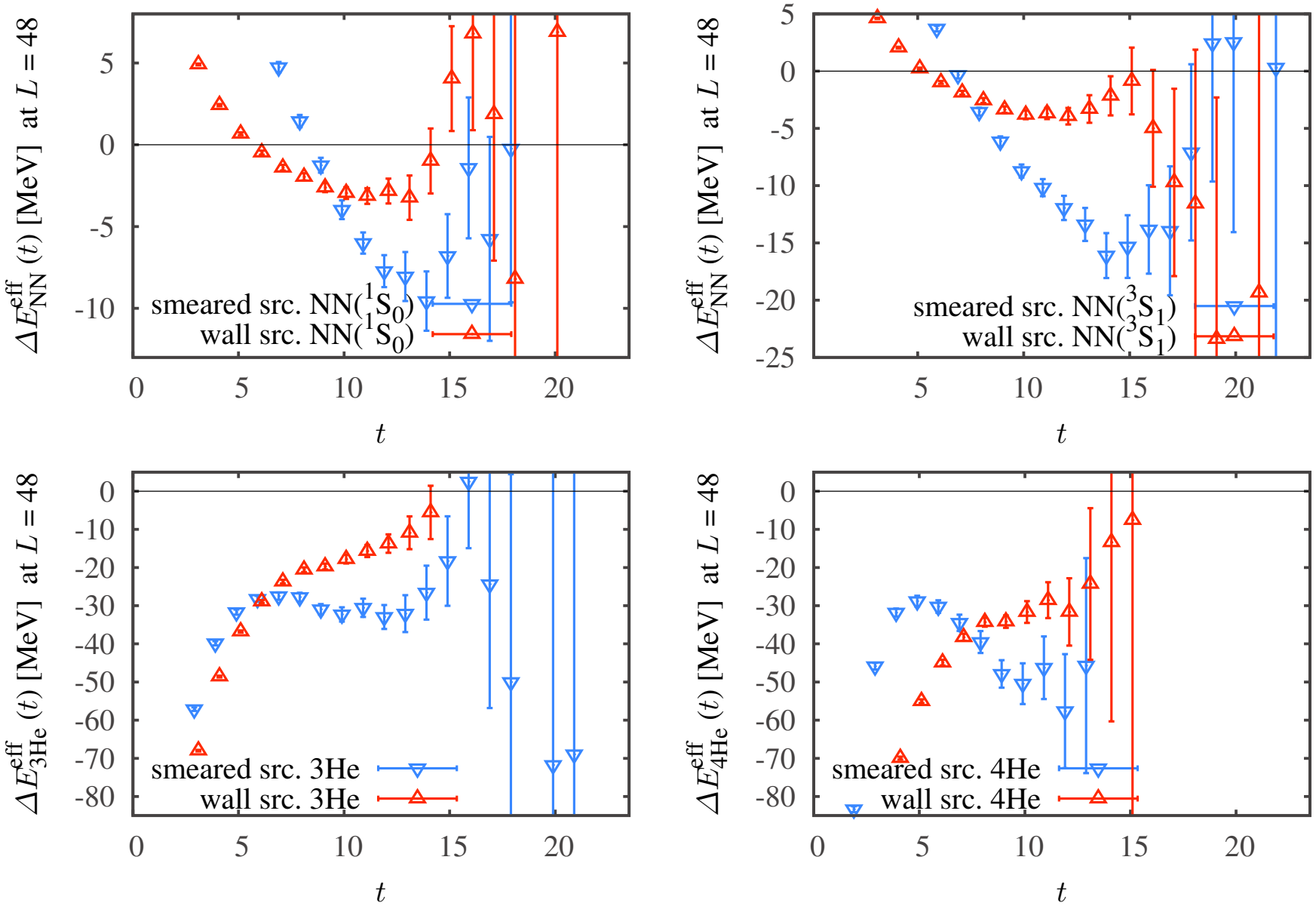


$L^3 = 64^3$



NN(^{1}S$_0$), NN(^{3}S$_1$), Triton and Helium

Do not trust plateaux.



- speed up for ^{3}He/^{4}He calc. — unified contraction algorithm Doi-Endres '13

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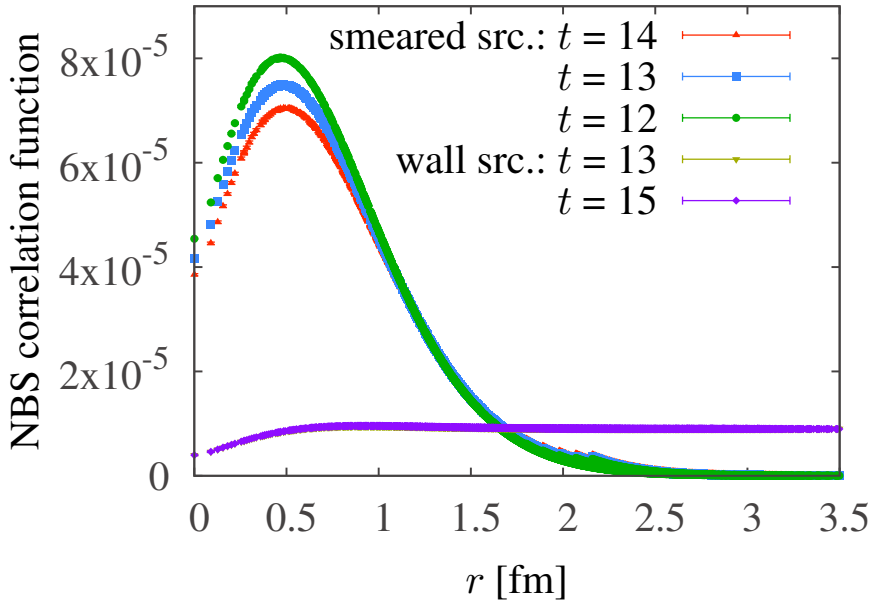
NBS Wave Function and $\Xi\Xi(^1S_0)$ Central Potential $V_c(\vec{r})$

- **wall src.** — weak t-dep.
- **smeared. src.** — strong t-dep.
  $\Longrightarrow$ $\sim \mathcal{O}(100)$ MeV cancellation
- t-dep. HAL method works well!!

$$V_c(\vec{r}) = -\frac{H_0 R}{R} - \frac{(\partial/\partial t)R}{R} + \frac{(\partial/\partial t)^2 R}{4mR}$$



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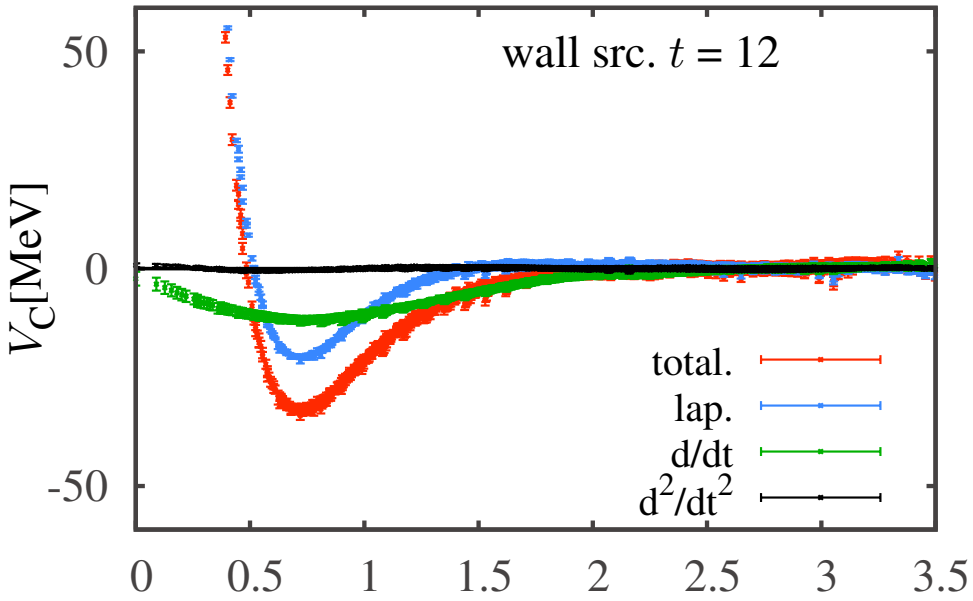
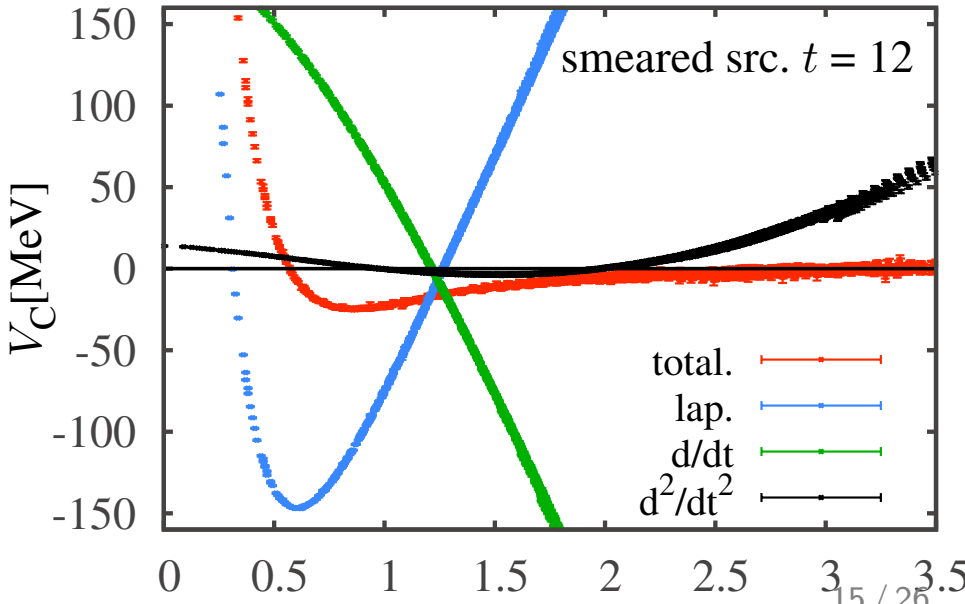


□ **smeared src.**



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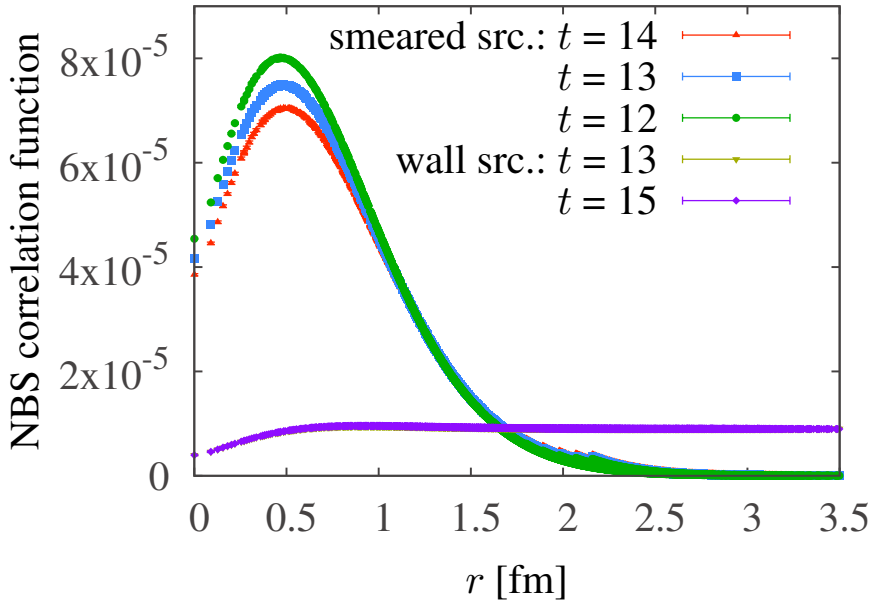

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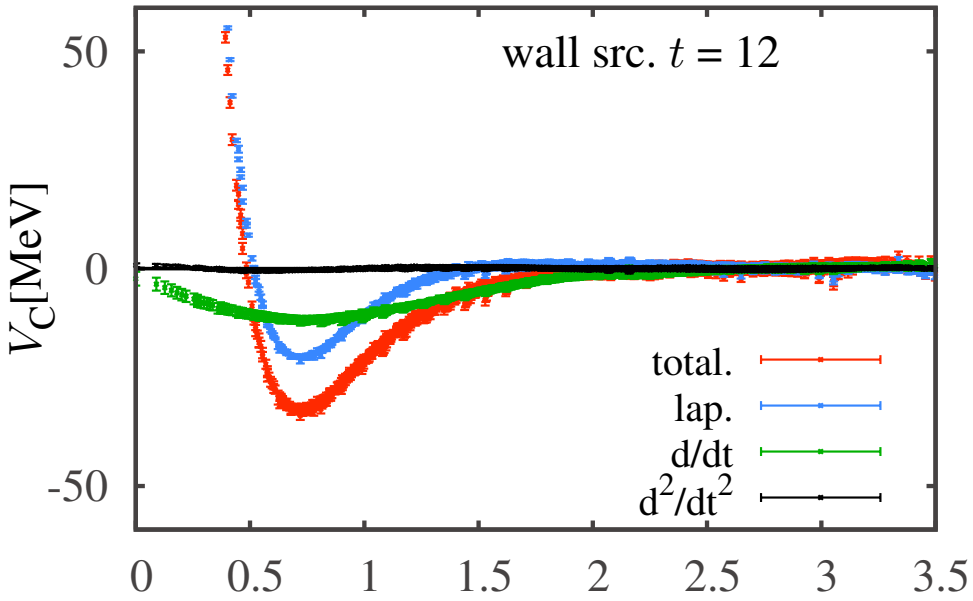
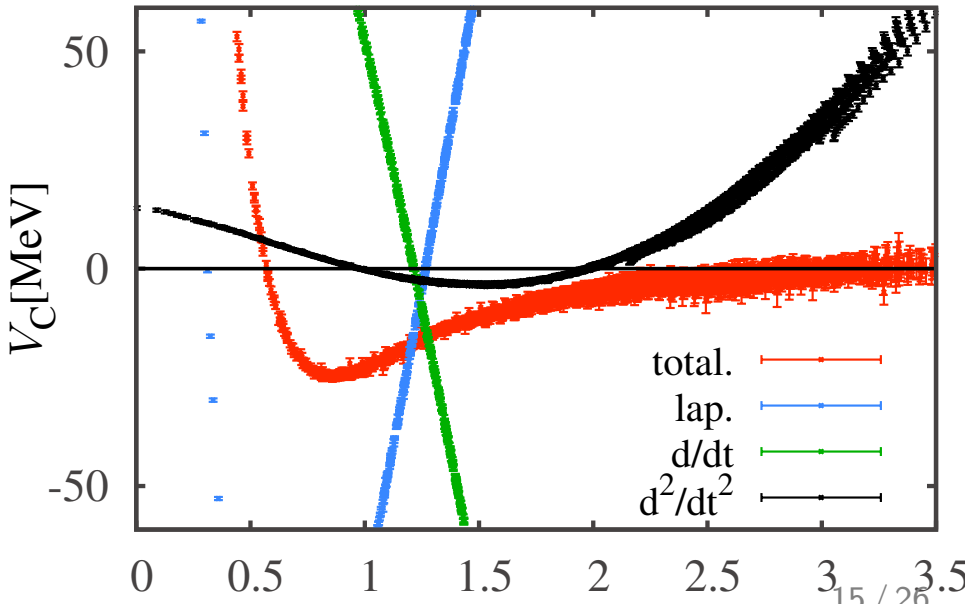
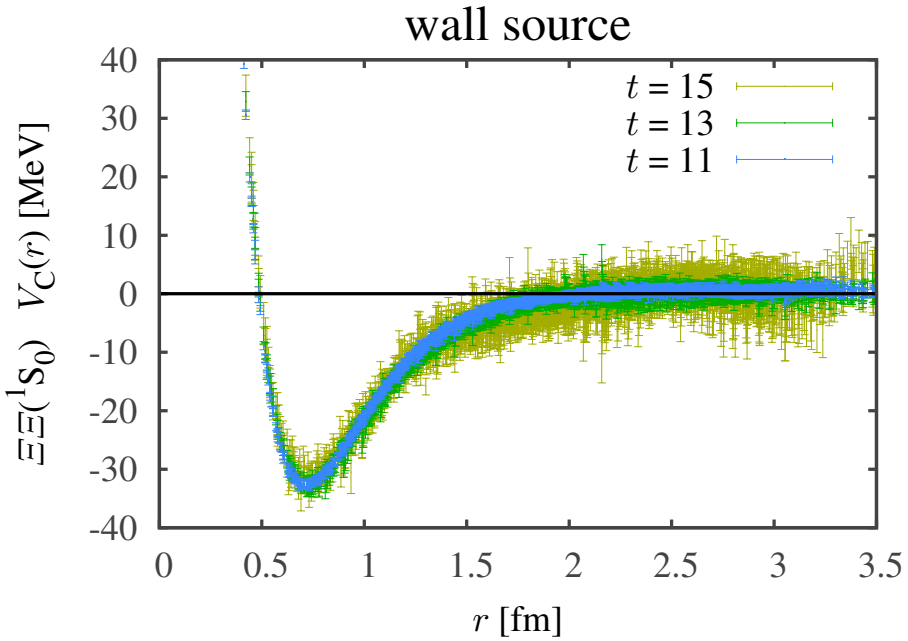


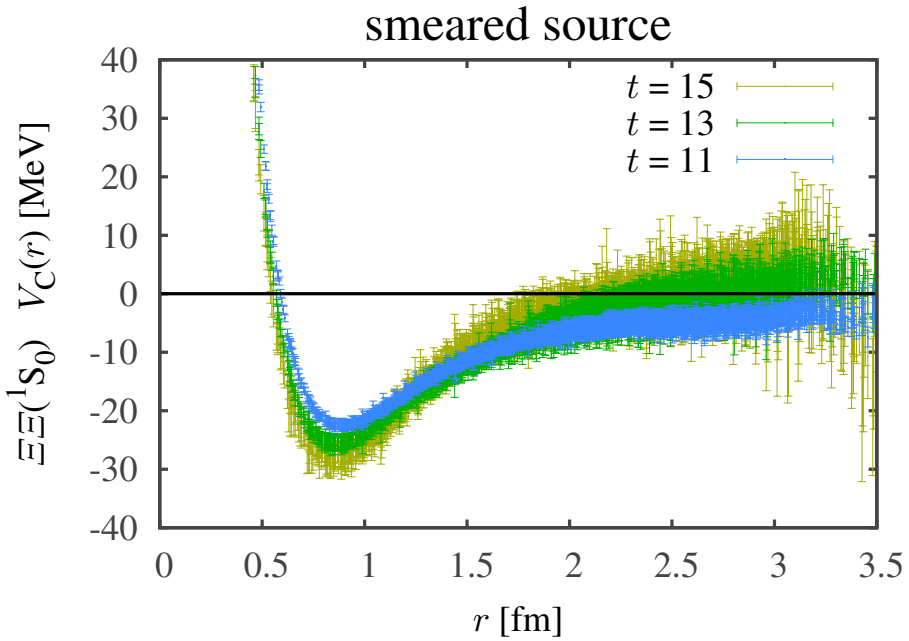
□ **smeared src.**



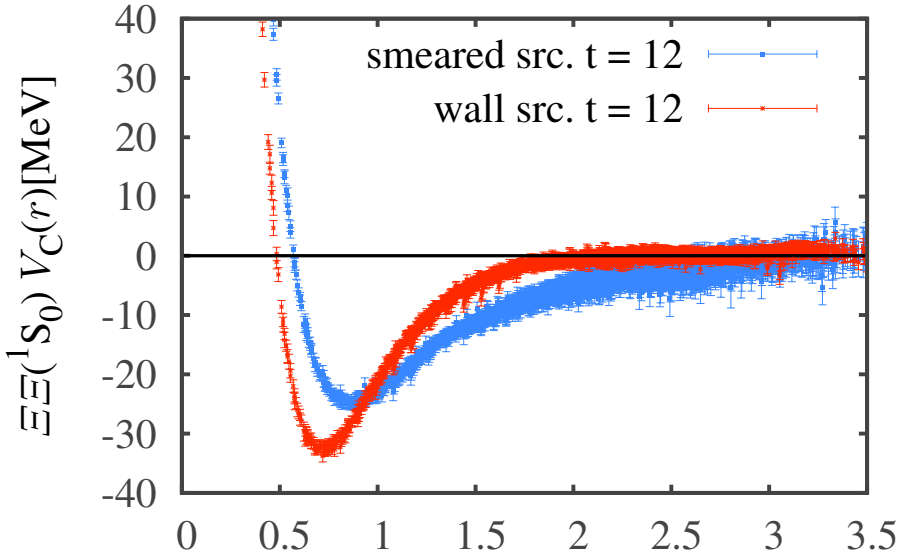
Comparison of $\Xi\Xi(^1S_0)$ potential

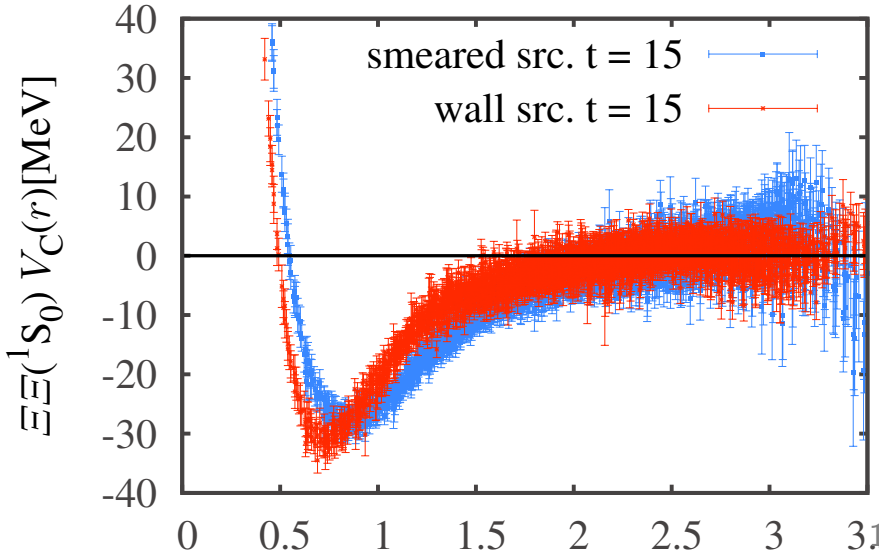
- **wall src.** "stable" $\Longleftrightarrow$ **smeared src.** t-depend





- $t \nearrow\nearrow$, **"smeared src."** results approach to **"wall src."**





Next Leading Order of Derivative Expansion

Derivative expansion: $U(r,r') = \{V_0(r) + V_1(r)\nabla^2\}\delta(r-r')$ (for $^1S_0$)

$$\left[\frac{1}{4m}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0\right] R(r,t) = \int d^3r' U(r,r') R(r',t)$$

$$\therefore \frac{1}{4m}\frac{(\partial^2/\partial t^2)R}{R} - \frac{(\partial/\partial t)R}{R} - \frac{H_0 R}{R} = V_0(r) + V_1(r)\frac{\nabla^2 R(r,t)}{R(r,t)} \equiv \tilde{V}_{\rm total}(r,t)$$

- Now, we have $R^{\rm smear}$ and $R^{\rm wall}$

$$\begin{cases} V_0(r) + V_1(r)\nabla^2 R^{\rm smear}/R^{\rm smear} = \tilde{V}^{\rm smear}_{\rm total}(r, t_{\rm smear}) \\ V_0(r) + V_1(r)\nabla^2 R^{\rm wall}/R^{\rm wall} = \tilde{V}^{\rm wall}_{\rm total}(r, t_{\rm wall}), \end{cases}$$

- LO $V_0(r)$ and NLO $V_1(r)$ potentials are given by

$$V_1(r) = \frac{\tilde{V}^{\rm smear}_{\rm total}(r, t_{\rm smear}) - \tilde{V}^{\rm wall}_{\rm total}(r, t_{\rm wall})}{\nabla^2 R^{\rm smear}/R^{\rm smear} - \nabla^2 R^{\rm wall}/R^{\rm wall}}$$

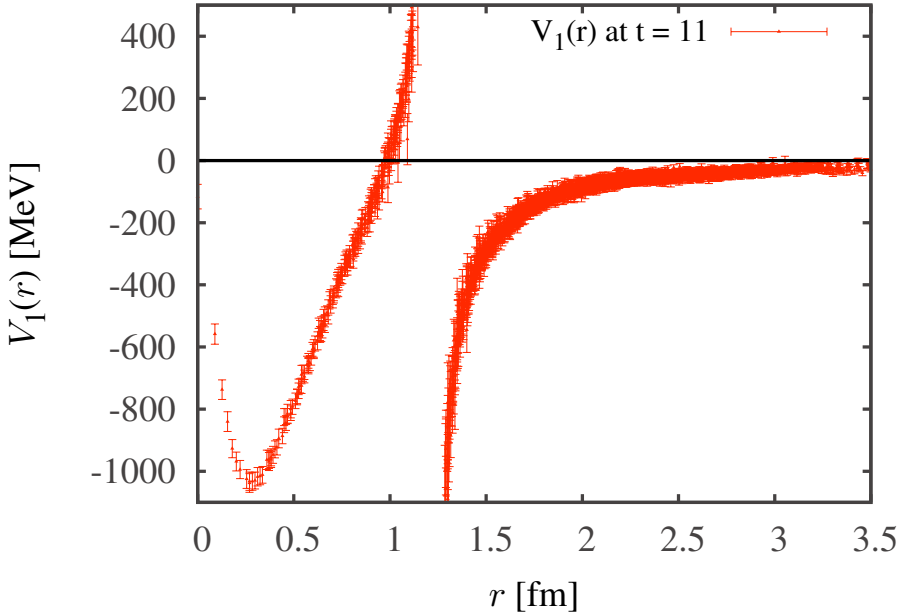
$$V_0(r) = \tilde{V}^{\rm smear}_{\rm total}(r, t_{\rm smear}) - V_1(r)\frac{\nabla^2 R^{\rm smear}}{R^{\rm smear}}.$$

- $\nabla^2 R/R - \nabla^2 R/R$ in denominator

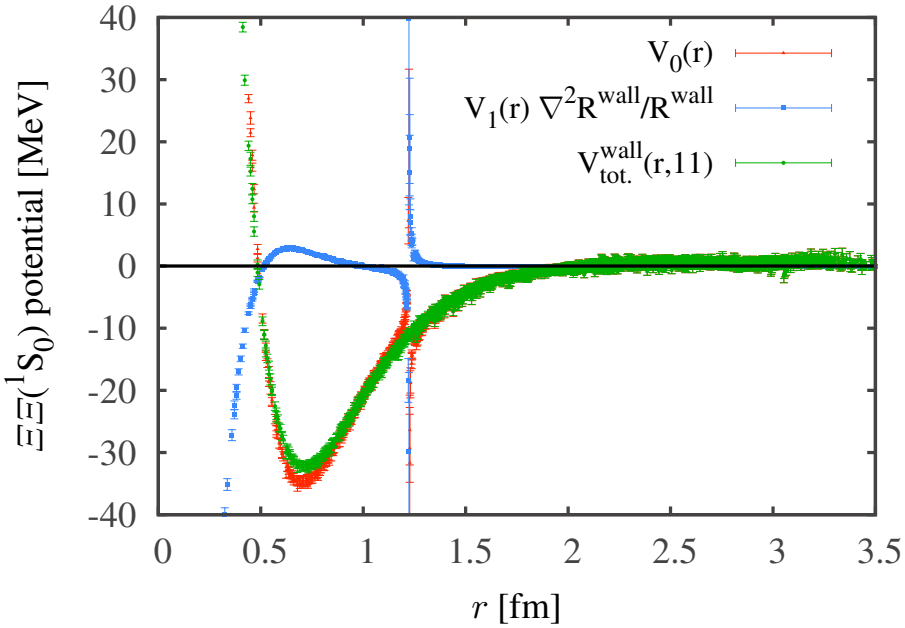
Results: NLO Potential

- results of HAL QCD method
  - source independent

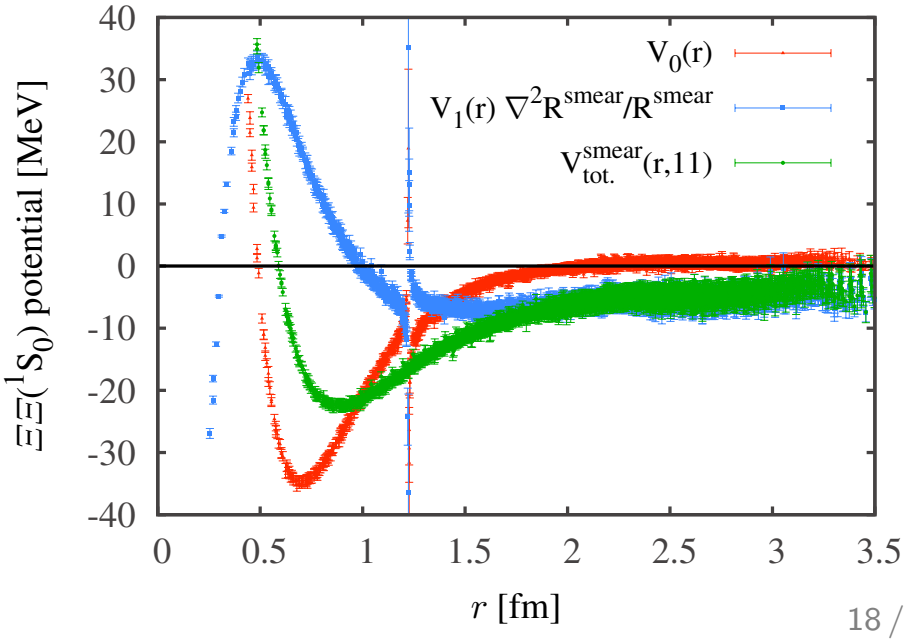
$$V_{\text{total}}(r,t) = V_0(r) + V_1(r)\frac{\nabla^2 R(r,t)}{R(r,t)}$$



- **wall source**
  - Good convergence of **LO pot.**



- **smeared source**
  - **with NLO pot. correction**



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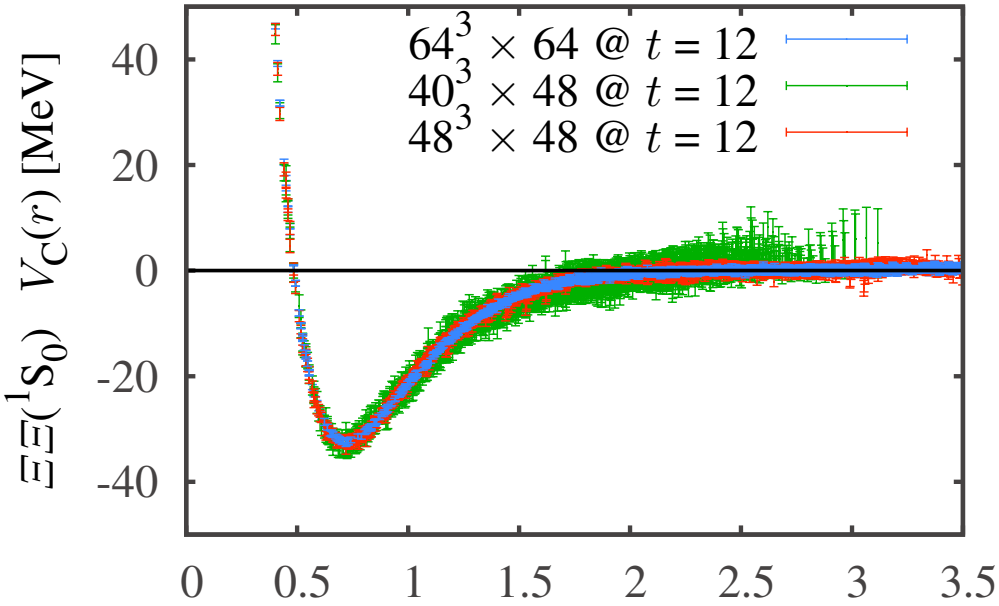
HAL QCD Method: Potential $\longrightarrow$ Energy Shift Eigenvalue

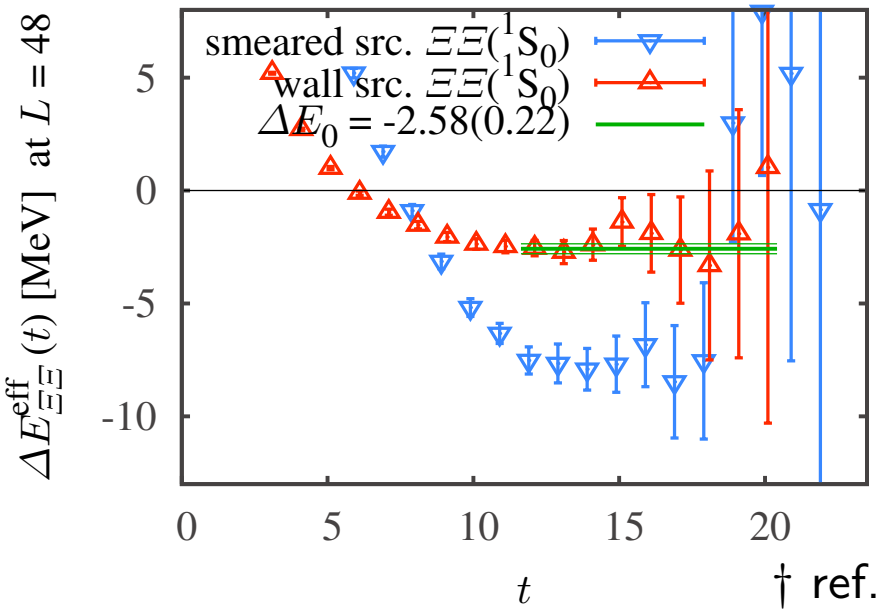
- HAL QCD gives reliable interaction **w/o g.s. saturation**
  quark source, time, and volume independent
- what is the true **"energy shift"** in finite volume

1. **INPUT**: potential $V(\vec{r})$
2. **SOLVE**: eigenvalue problem†

$$[H_0 + V]\,\psi = \Delta E \psi$$

in finite volume L^3





vol.	g.s. [MeV]	1st [MeV]
40^3	**-4.55(1.18)**	75.63(1.31)
48^3	**-2.58(22)**	52.87(33)
64^3	**-1.13(9)**	28.71(9)

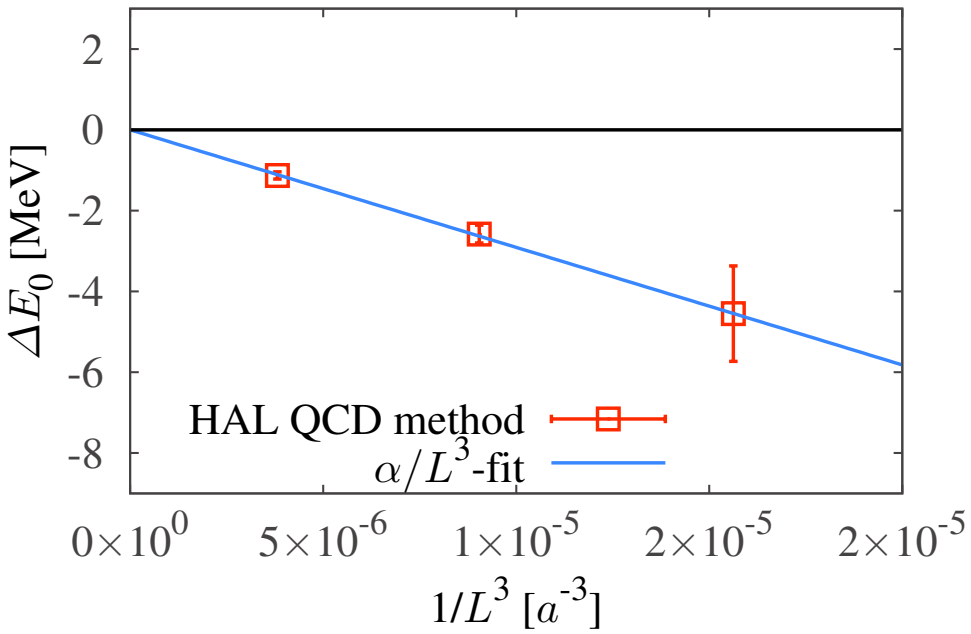
eigenvalues using $V_c(\vec{r}, t = 12)$

† ref. B. Charron for HAL Coll. arXiv:1312.1032.

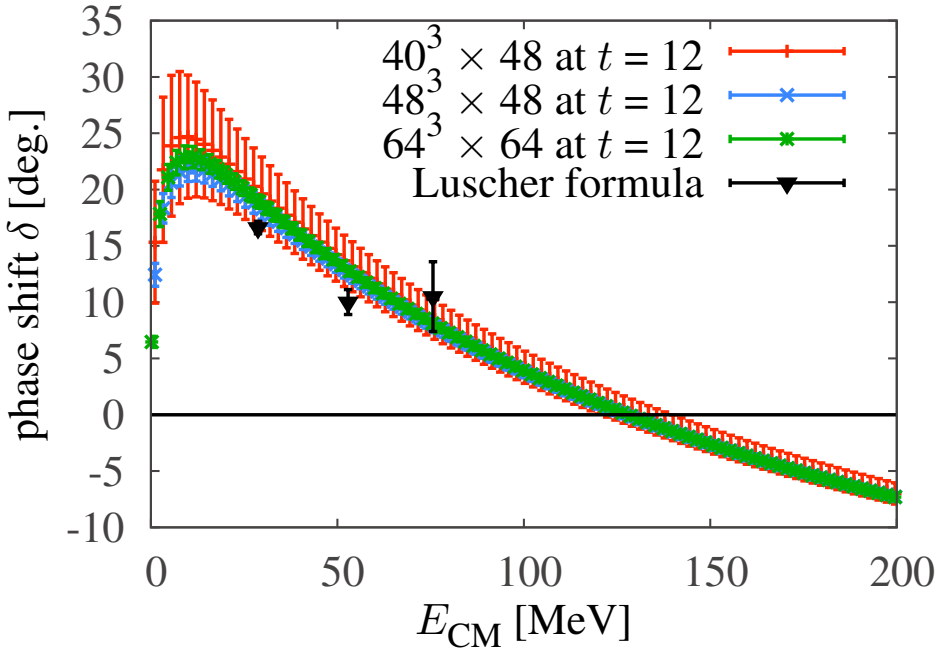
$\Xi\Xi(^1S_0)$ is Unbound

$$k\cot\delta(k) = \frac{1}{\pi L}\sum_{n\in\mathbf{Z}^3}\frac{1}{|n|^2-(kL/2\pi)^2}, \qquad \Delta E = 2\sqrt{m^2+k^2}-2m$$

volume dep. of ΔE_0



phase shift δ



- wall src. pot. $\Longrightarrow$ solve eigenvalue & Lüscher formula $\Longrightarrow$ phase shift
- wall src. pot. $\Longrightarrow$ fit & solve Schrödinger eq. $\Longrightarrow$ phase shift
  - using 2 Gaussians + (Yukawa)^2 ansatz for $V_c(\vec{r})$ fit

Short Summary: Consistency of 2 Methods

- **ground state saturation** of multi-baryon is extremely difficult
- **even without g.s. saturation** there may appear **plateau**
  $\Longrightarrow$ it should be checked by other source or variational method
  fake plateaux $\Longrightarrow$ **wrong conclusion**
- HAL QCD method gives source indep. results **w/o g.s. saturation**.

✓ ~~"Lüscher vs HAL"~~ $\Longrightarrow$ "Lüscher(**smeared**) vs Lüshcer(**wall**)"

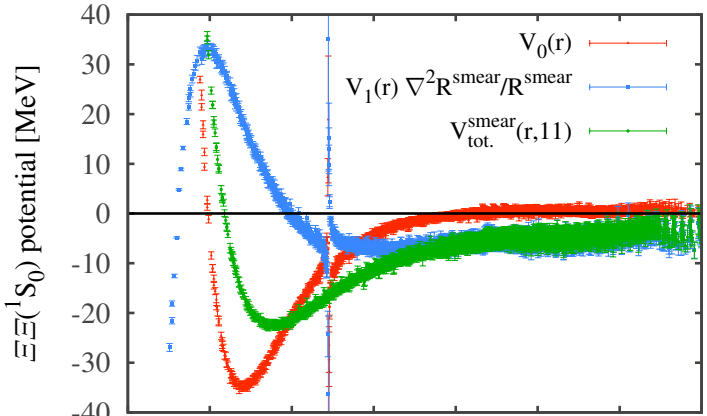
finite vol. ⊿E agree.

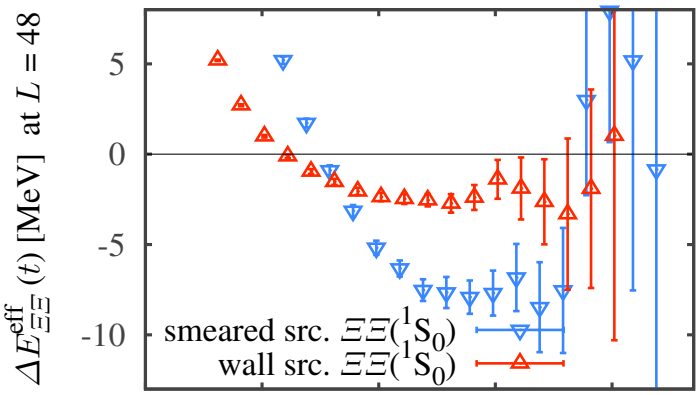
Pot. with **wall** src. ⟷ **Direct** with **wall** src.

NLO pot. ↕ ↕ **Fake plateaux**

Conflict

Pot. with **smear** src. ⟷ **Direct** with **smear** src.





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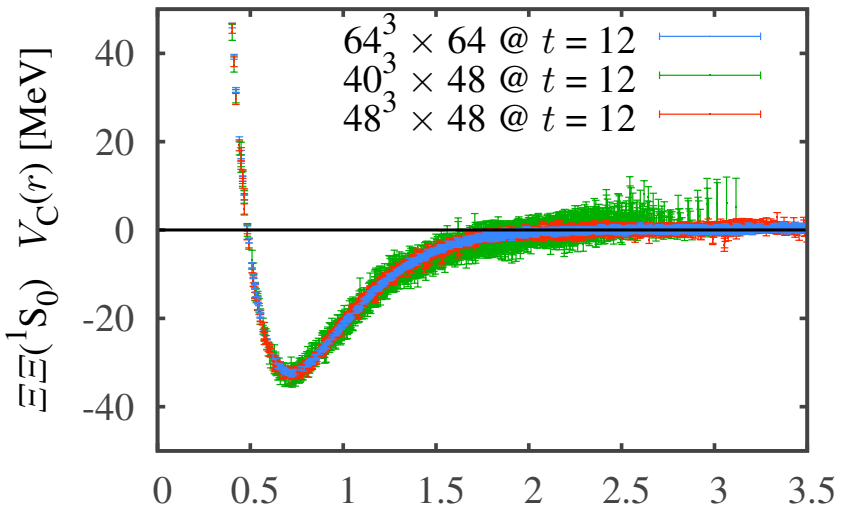
Potential $\longrightarrow$ Energy Eigenvalues & Eigenfunctions

1. **INPUT**: potential $V(\vec{r})$
2. **SOLVE**: eigenvalue problem

$$[H_0 + V]\,\psi = \Delta E \psi$$

in finite volume L^3



in addition to g.s. energy shift ΔE_0,

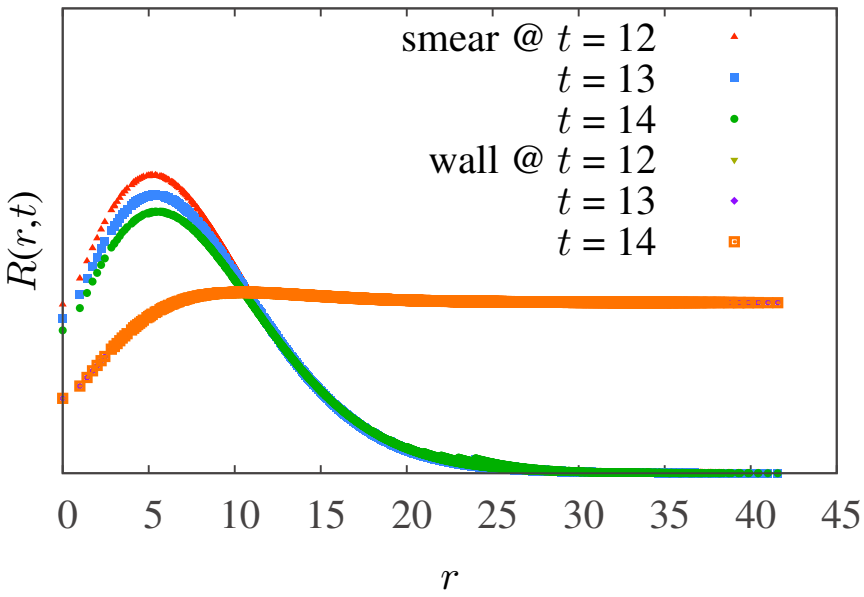
▶ **eigenvalues** ΔE_n and **eigenfunctions** $\Psi_n(\vec{r})$ of **excited states**

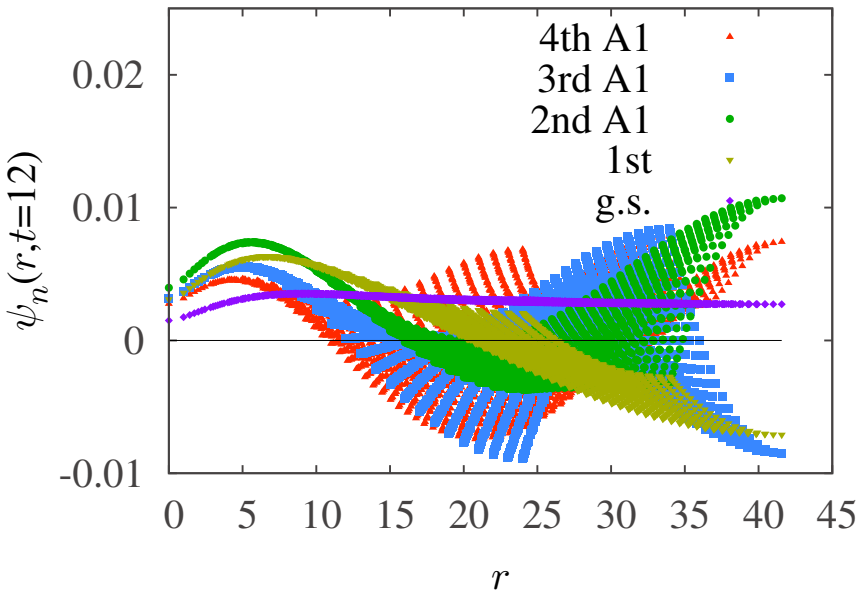
!!!

vol.	g.s.[MeV]	**1st [MeV]**
40^3	-4.55(1.18)	**75.63(1.31)**
48^3	-2.58(22)	**52.87(33)**
64^3	-1.13(9)	**28.71(9)**

eigenvalues using $V_c(\vec{r}, t = 12)$

Wavefunction and Eigenfunctions





n-th A1	ΔE_n [MeV]
0	-2.58
1	52.49
2	112.08
3	169.78
4	224.73

Table: 48^3 wall source @ $t = 12$

- **A1 states** (SO$(3, \mathbf{Z})$ rep. of $L = 0$) below inelastic threshold (48^3 vol.)
  - $\Xi + \Xi \to \Xi^* + \Xi$ @ 0.29 GeV
  - $\Xi + \Xi \to \Omega + \Sigma$ @ 0.29 GeV

cf. $m_\pi = 0.51$ GeV

Effective Mass from Eigenstate Decomposition

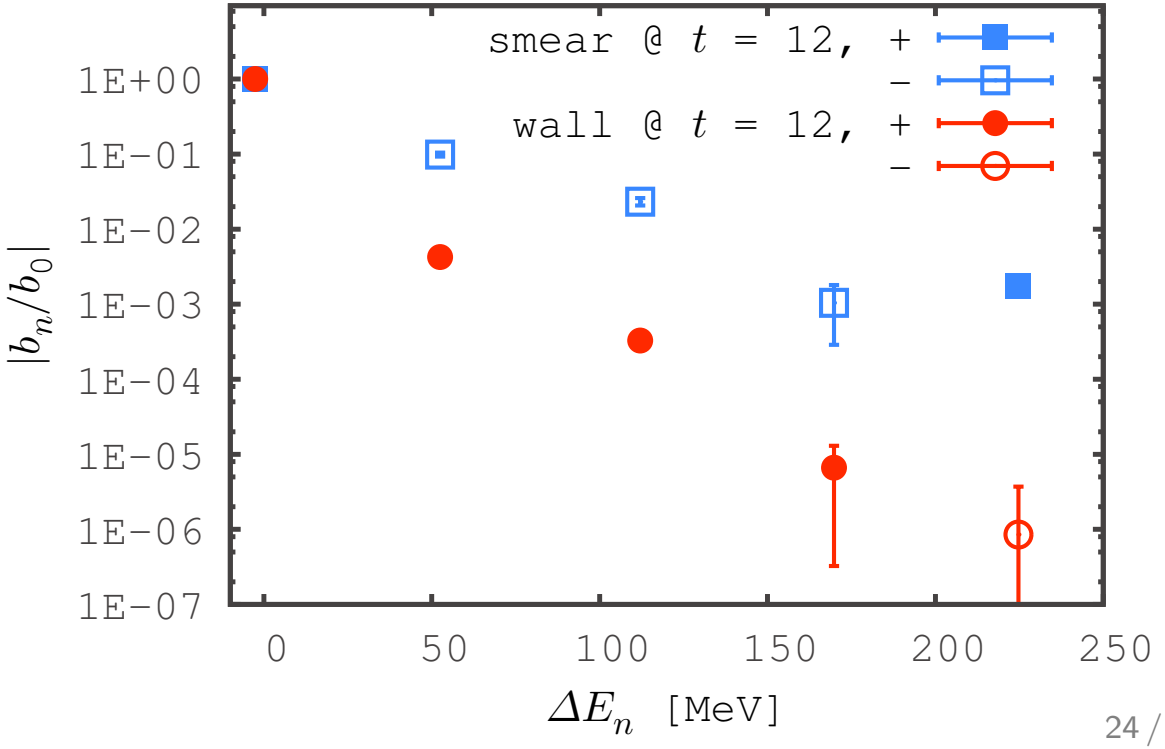
- R-corr. decomposition by energy eigenmodes

$$R(\vec{r},t) = \sum_n a_n \Psi_n(\vec{r},t) \exp\left(-\Delta E_n t\right)$$

$$R(\vec{p}=0,t) = \sum_n a_n \left[\sum_{\vec{r}} \Psi_n(\vec{r})\right] \exp\left(-\Delta E_n t\right) = \sum_n b_n \exp\left(-\Delta E_n t\right)$$

"contamination" of excited states b_n/b_0

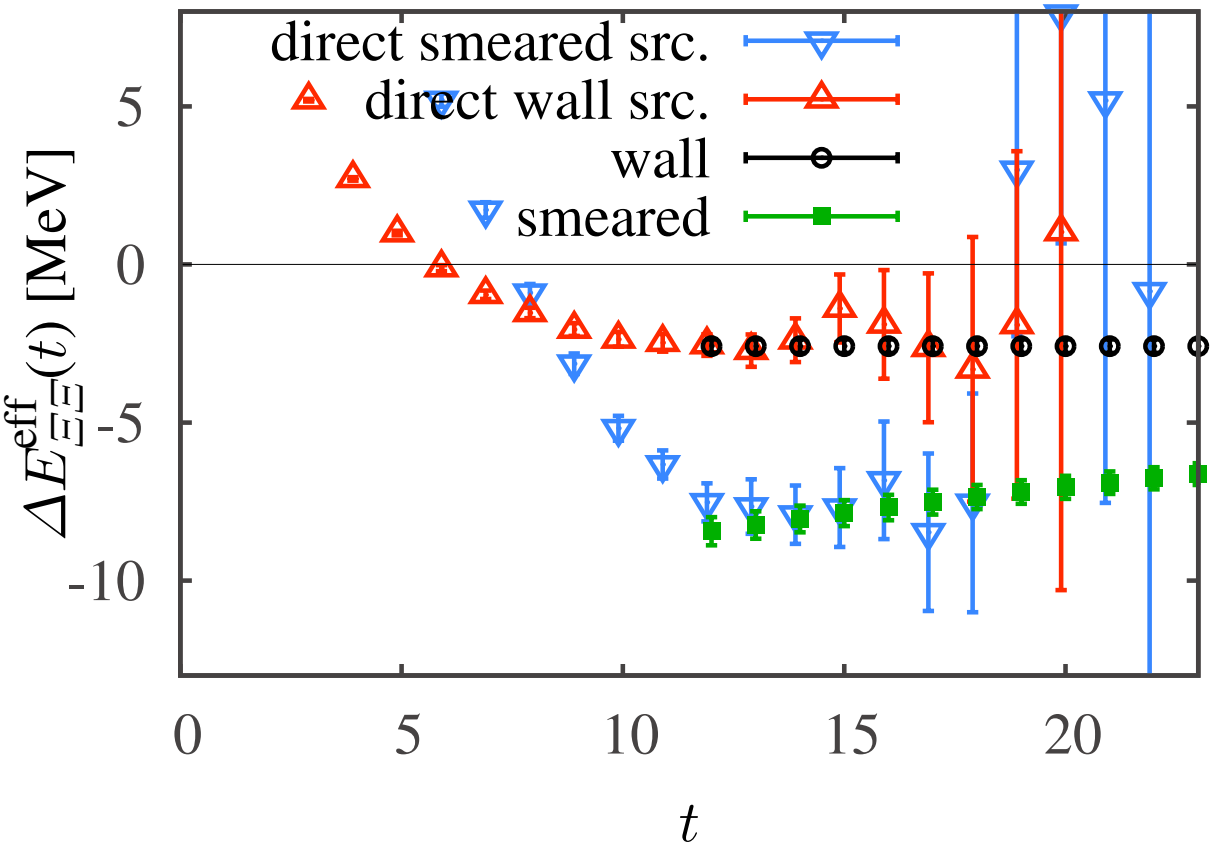
- ex. **1st excited state**
  - **wall source**
    $b_1/b_0 \ll 0.01$
  - **smeared source**
    $b_1/b_0 \simeq -0.1$
  - with energy gap
    $E_1 - E_0 \simeq 50$ MeV
    for $L^3 = 48^3$



Excited States Contribution to $\Delta E_{\rm eff}(t)$ and Fake Plateau

$$\Delta E_{\rm eff}(t) = \log \frac{\sum_n b_n \exp\left(-\Delta E_n t\right)}{\sum_n b_n \exp\left(-\Delta E_n (t+1)\right)}$$

- "direct measurement" — well reproduced by low-lying 3 eigenmodes†
  - **"fake plateau"** of **smeared src.** around $t = 12 - 18$

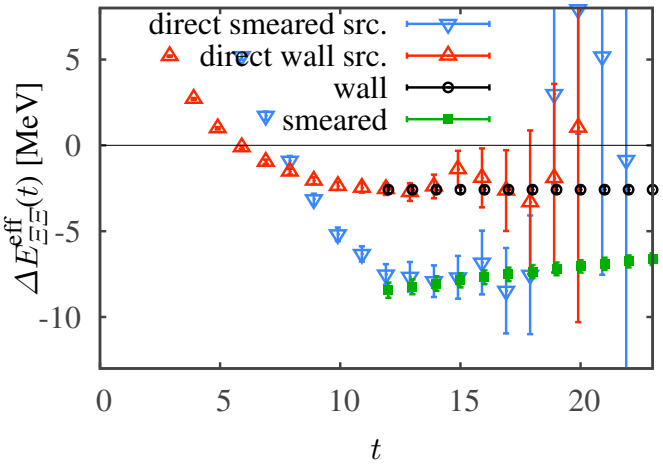


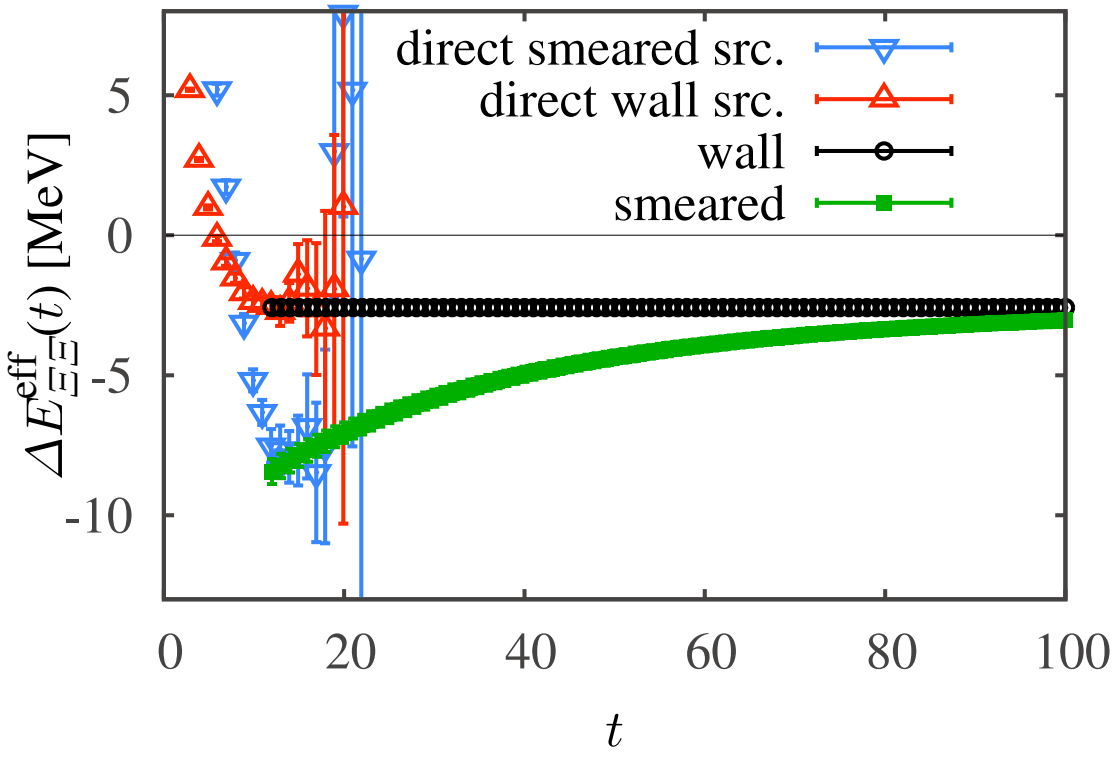
† eigenvalues ΔE_n, coefficients $b_n^{\rm smear/wall}$ for $n = 0, 1, 2$, at $t = 12$.

Excited States Contribution to $\Delta E_{\rm eff}(t)$ and Fake Plateau

$$\Delta E_{\rm eff}(t) = \log \frac{\sum_n b_n \exp\left(-\Delta E_n t\right)}{\sum_n b_n \exp\left(-\Delta E_n (t+1)\right)}$$

- "direct measurement" — well reproduced by low-lying 3 eigenmodes†
  - ▶ **"fake plateau"** of **smeared src.** around $t = 12 - 18$
- ☐ **g.s. saturation** of smeared source — **100 lattice units $\sim$ 10 fm !!!**





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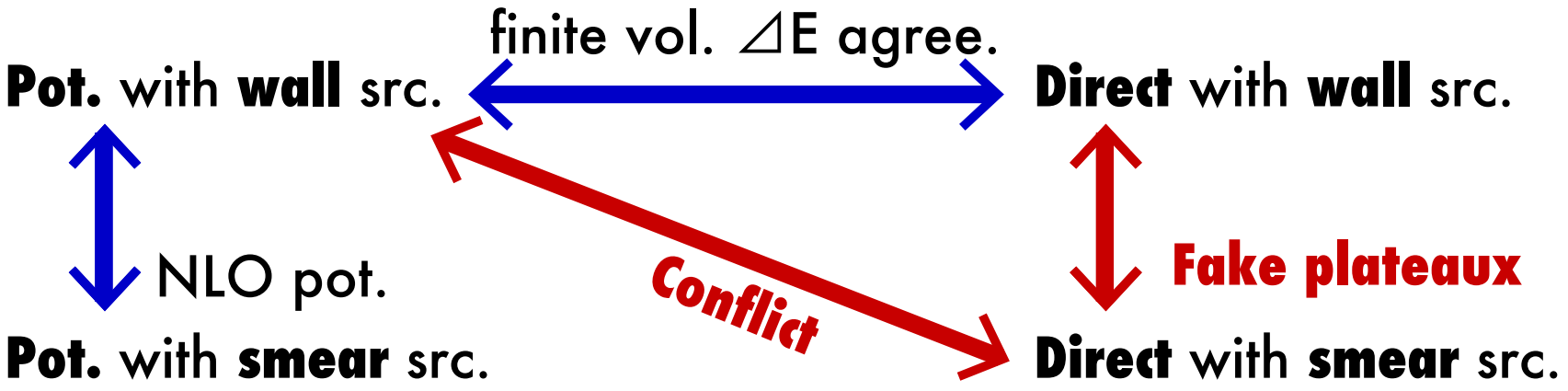
Summary: Lüscher ~~vs~~ and HAL QCD

Lüshcer and **HAL QCD** methods are **consistent**

$\Longrightarrow$ **Lattice QCD can calculate reliable hadron interactions.**

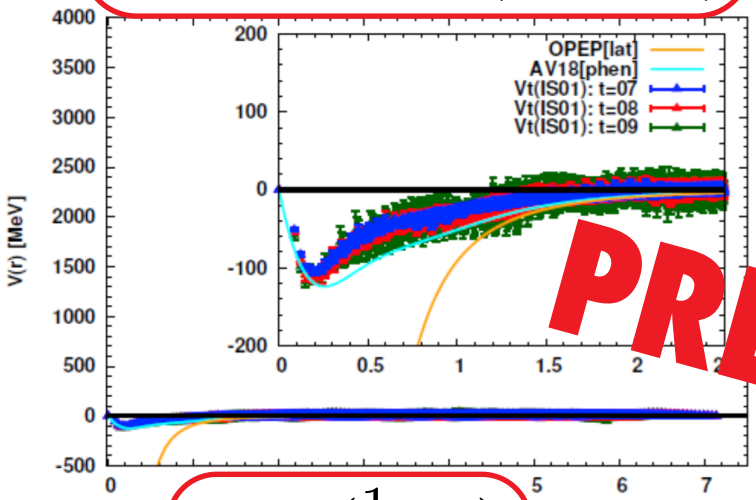
- "direct method" — **ground state saturation** is extremely difficult
  $\Longrightarrow$ *do not trust plateau*, it may be a **wrong signal!**
  ✓**check** other sources or variational method
- HAL QCD works well **without g.s. saturation**.
- **NBS corr.** and **"potential"** clarify excited states and **fake plateau**.

▶ Let's move on **"physical point"** straight and steady.

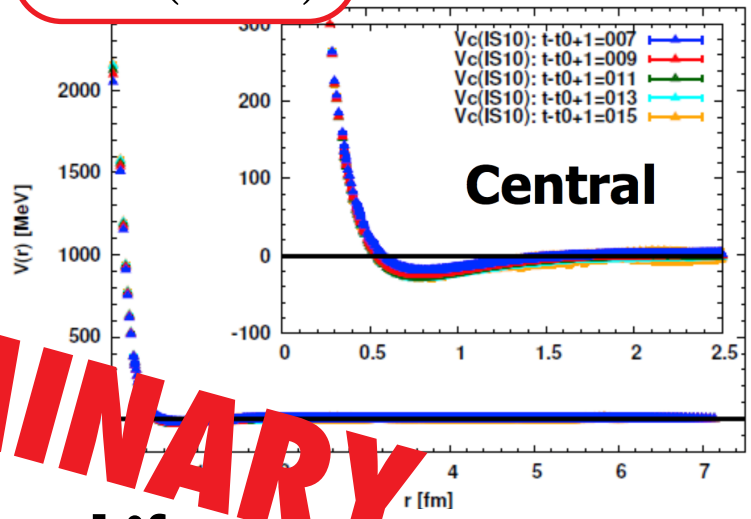


HAL QCD Coll. at physical point is now going on.

NN-Potential (tensor)

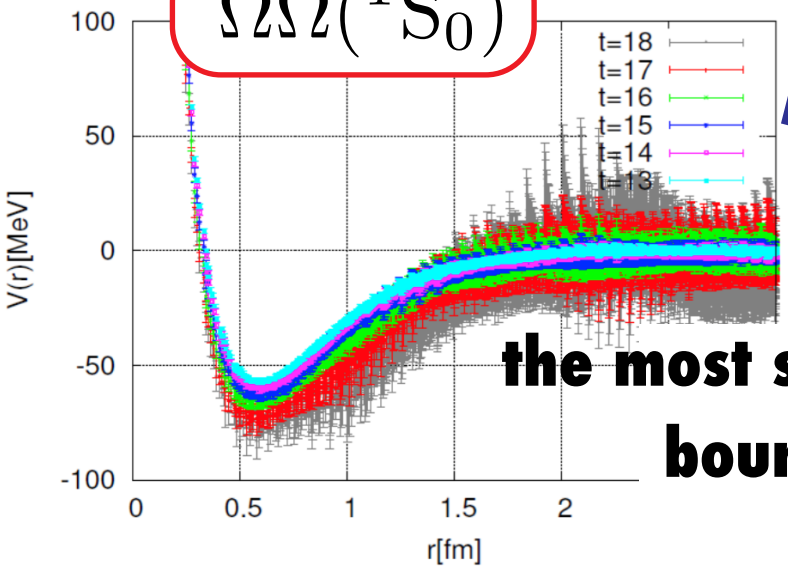


$\Xi\Xi(^1S_0)$

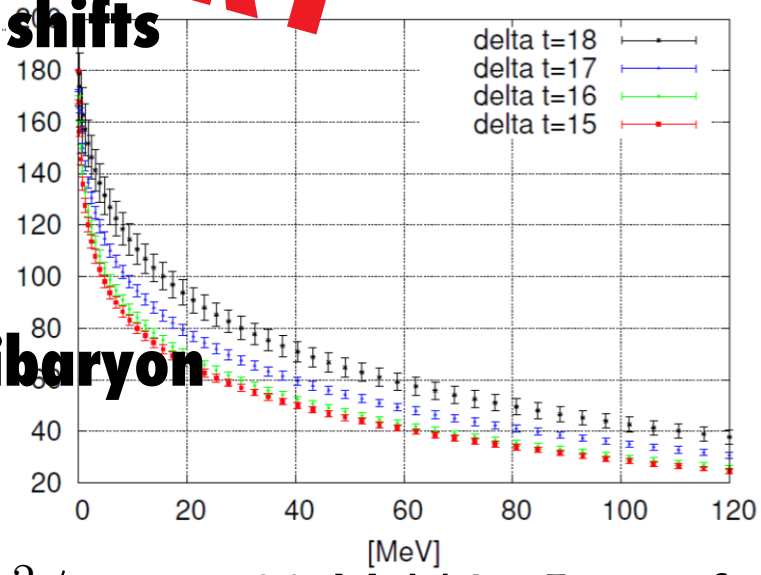


PRELIMINARY

$\Omega\Omega(^1S_0)$



phase shifts



the most strange dibaryon bound state?

- **"direct measurement"**: $1/t \sim \Delta E \sim p^2/m_N \sim$ 20 MeV in $L = 8$ fm

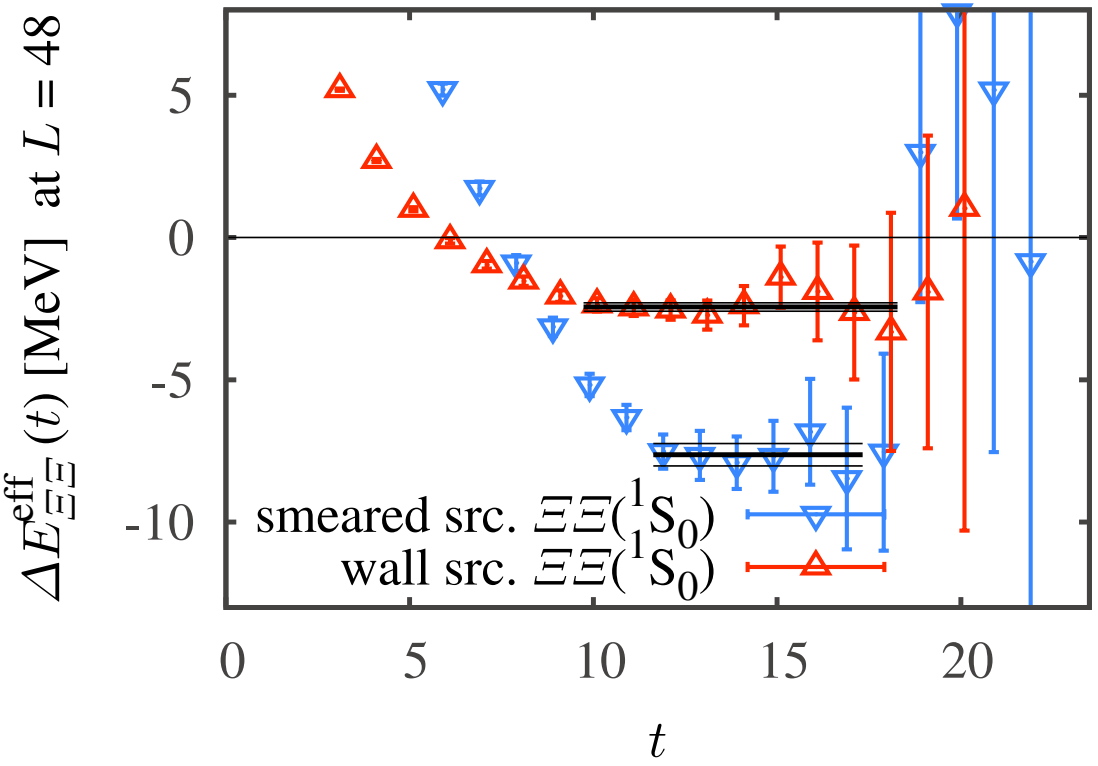
$$\Longrightarrow \quad S/N \sim \exp\left[-A \times \left(m_N - \tfrac{3}{2}m_\pi\right) \times t\right] \sim 10^{-25}$$

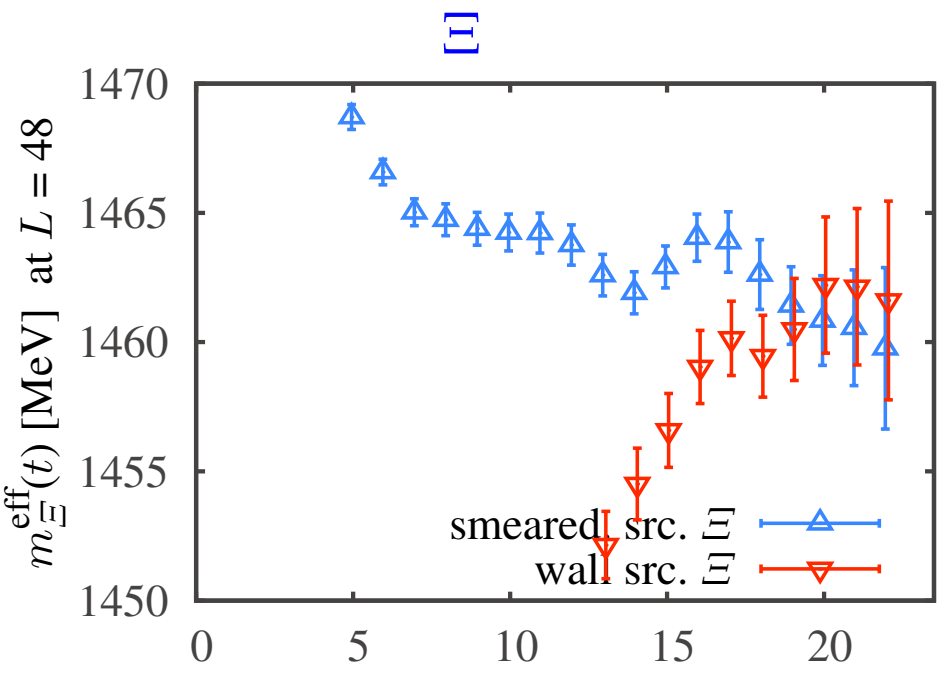
6 Appendix

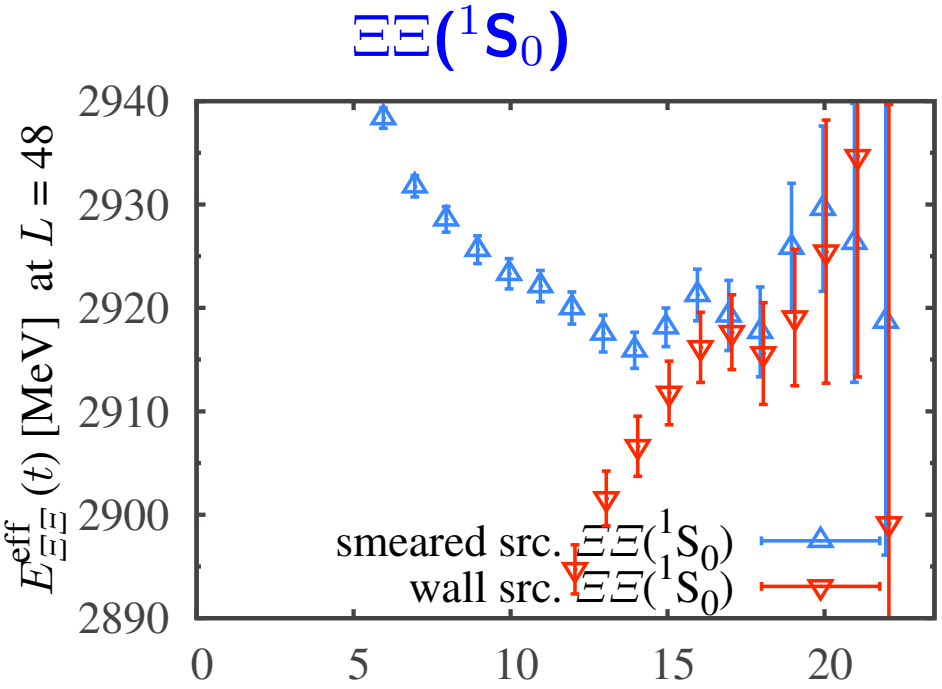
Effective Masses Plot

$$\Delta E^{\text{eff}}_{\Xi\Xi}(t) = E^{\text{eff}}_{\Xi\Xi}(t) - 2m^{\text{eff}}_{\Xi}(t)$$

shows plateau
even without plateaux
in $m^{\text{eff}}_{\Xi}(t)$ or $E^{\text{eff}}_{\Xi\Xi}(t)$







Results:NLO Potential (1) $V_0(r)$ and $V_1(r)$

$$V_0(r) = \tilde{V}^{\rm exp.}_{\rm total} - V_1(r)\frac{\nabla^2 R^{\rm exp.}}{R^{\rm exp.}}, \quad V_1(r) = \frac{\tilde{V}^{\rm exp.}_{\rm total} - \tilde{V}^{\rm wall}_{\rm total}}{\nabla^2 R^{\rm exp.}/R^{\rm exp.} - \nabla^2 R^{\rm wall}/R^{\rm wall}}$$

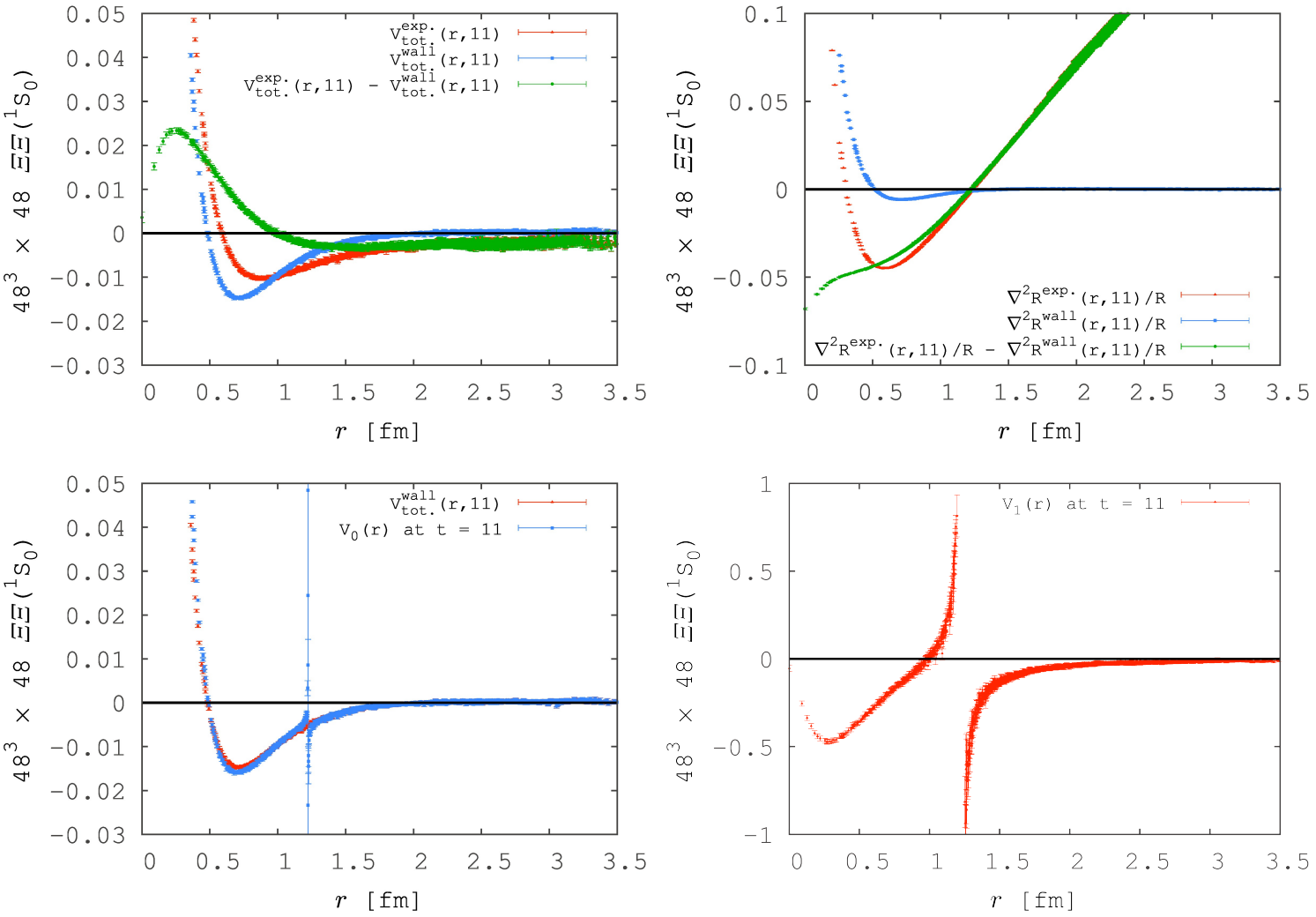


Figure: (a) diff. of $\tilde{V}_{\rm total}$ (b) diff. of $\nabla^2 R/R$ (c) $V_0(r)$ and $\tilde{V}^{\rm wall}$ (d) $V_1(r)$

Results:NLO Potential (2) V_0 and V_1 in smeared and wall

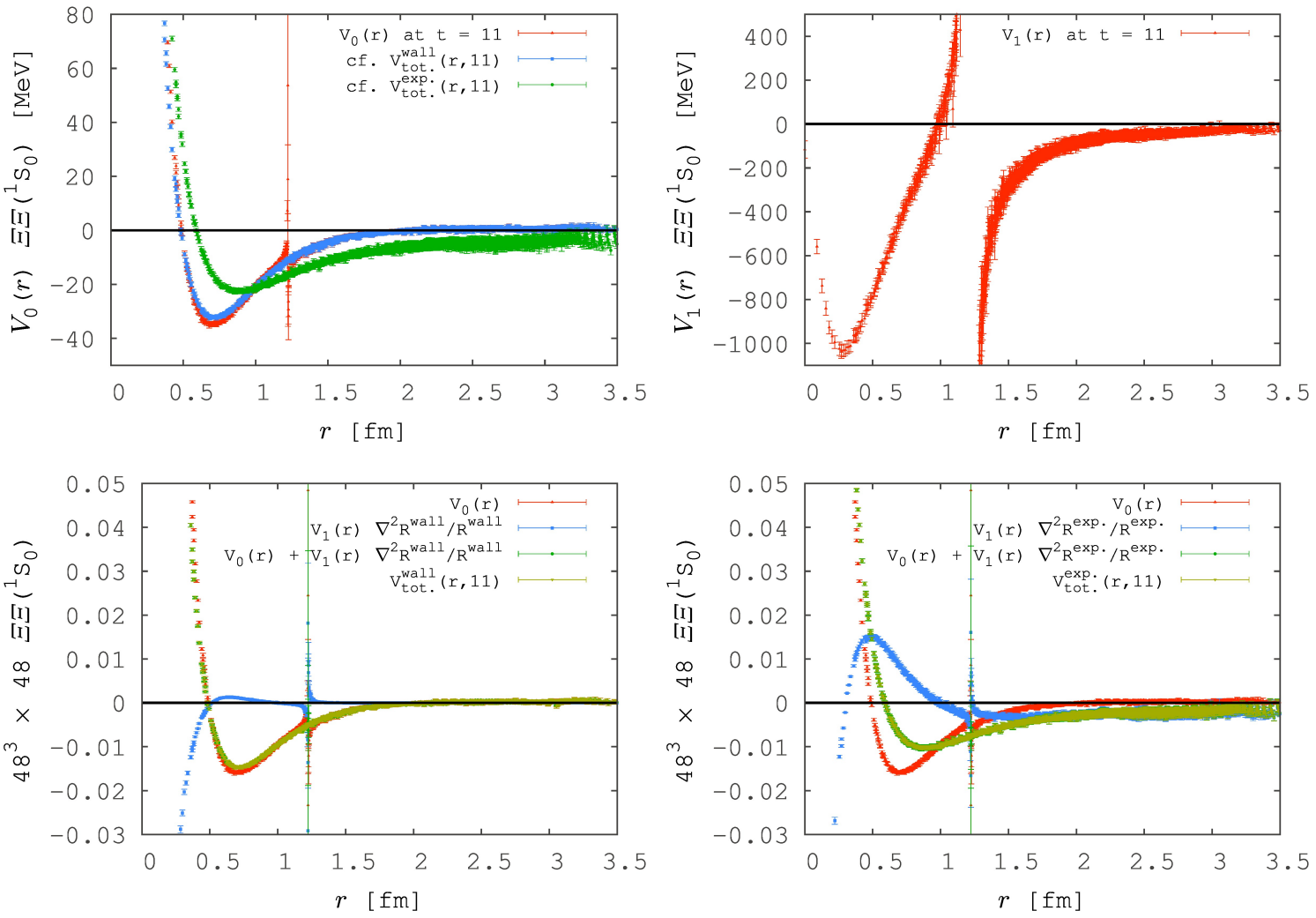


Figure: (a) $V_0(r)$ (b) $V_1(r)$ (c) wall src. summary (d) smeared src. summary

Results:NLO Potential (3) SVD

solving SVD with several t's

$$\begin{pmatrix} 1 & \nabla^2 R^{\text{exp.}}(r,t_1)/R^{\text{exp.}}(r,t_1) \\ 1 & \nabla^2 R^{\text{wall}}(r,t_1)/R^{\text{wall}}(r,t_1) \\ 1 & \nabla^2 R^{\text{exp.}}(r,t_2)/R^{\text{exp.}}(r,t_2) \\ 1 & \nabla^2 R^{\text{wall}}(r,t_2)/R^{\text{wall}}(r,t_2) \\ \vdots & \vdots \end{pmatrix} \begin{pmatrix} V_0(r) \\ V_1(r) \end{pmatrix} = \begin{pmatrix} \tilde{V}^{\text{exp.}}_{\text{total}}(r,t_1) \\ \tilde{V}^{\text{wall}}_{\text{total}}(r,t_1) \\ \tilde{V}^{\text{exp.}}_{\text{total}}(r,t_2) \\ \tilde{V}^{\text{wall}}_{\text{total}}(r,t_2) \\ \vdots \end{pmatrix}$$

$$A \begin{pmatrix} V_0(r) \\ V_1(r) \end{pmatrix} = B \longrightarrow U\Sigma V^t \begin{pmatrix} V_0(r) \\ V_1(r) \end{pmatrix} = B,$$

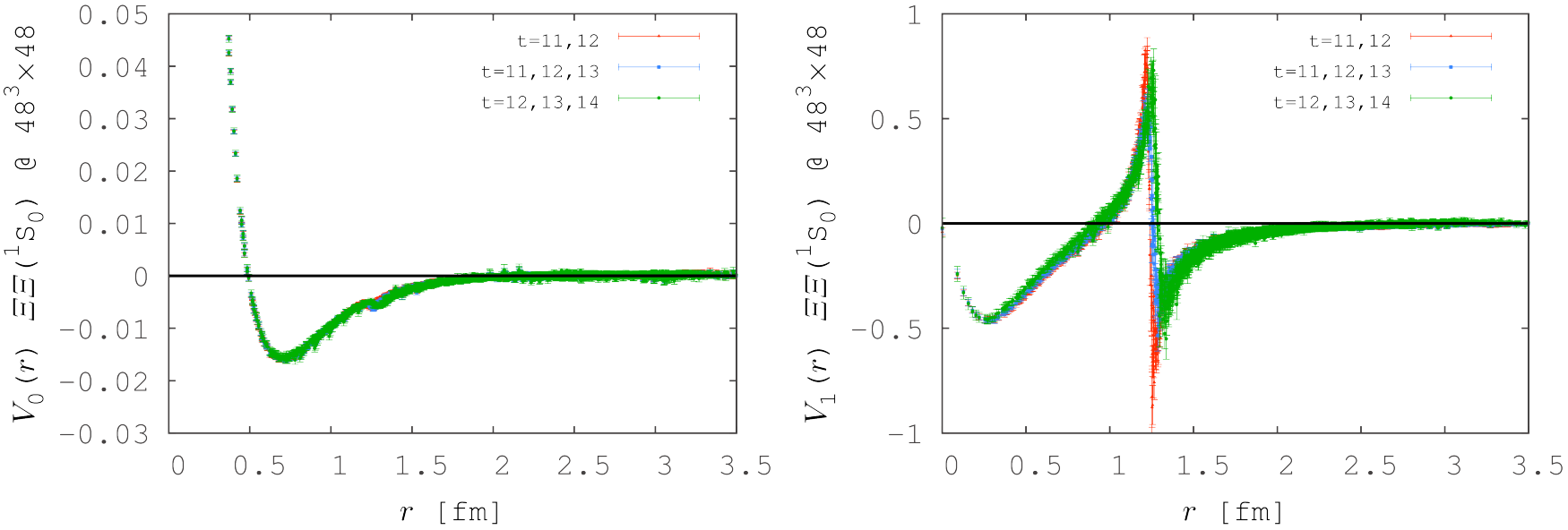


Figure: (a) $V_0(r)$ (b) $V_1(r)$ using SVD

Results:NLO Potential (4) Energy Eigenvalues

$\tilde{V}_{\rm tot}^{\rm wall}$, $V_0(r)$ and $V_0(r) + V_1(r)\nabla^2$
energy eigenvalues are consistent with each other
$\Longrightarrow$ wall source potential $\simeq$ LO

volume at $48^3 \times 48$	t	g.s. [MeV]	1st [MeV]
$\tilde{V}_{\rm total}^{\rm wall}(r)$	11	-2.30(20)	53.08(29)
	12	-2.58(22)	52.87(33)
	13	-2.70(30)	52.80(41)
$\tilde{V}_{\rm total}^{\rm exp.}(r)$	13	-5.33(59)	49.86(58)
	14	-4.76(55)	50.46(54)
$V_0(r)$	11,12,13	-2.47(23)	52.91(32)
	12,13,14	-2.62(28)	52.86(37)
$V_0(r) + V_1(r)\nabla^2$	11,12,13	-2.56(1.56)	53.07(2.73)
	12,13,14	-2.74(2.52)	51.58(3.77)

Table: The energy eigenvalues from "potential method".

Weight of Eigenstates in Wavefunction

$$R^{\text{wall/smear}}(\vec{x},t) = \sum_n a_n^{\text{wall/smear}} \Psi_n(\vec{x},t) e^{-\Delta E_n t}$$

with eigenfunctions Ψ_n and eigenvalues ΔE_n

- smeared $\Longrightarrow$ higher states are significant $|a_0^{\text{exp.}}| \sim |a_1^{\text{exp.}}| \sim |a_2^{\text{exp.}}|$
- wall $\Longrightarrow$ g.s. dominant $|a_0^{\text{wall}}| \gg |a_1^{\text{wall}}| \gg |a_2^{\text{wall}}|$

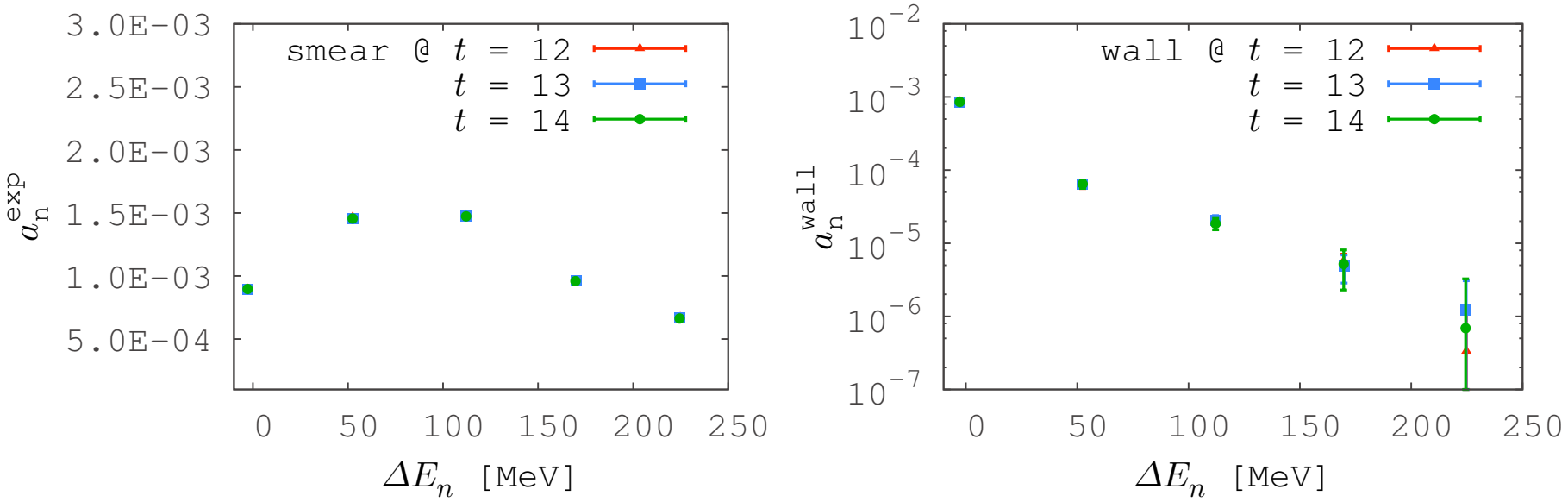


Figure: (a) smeared source. (b) wall source.

Projected Effective Energy Shift

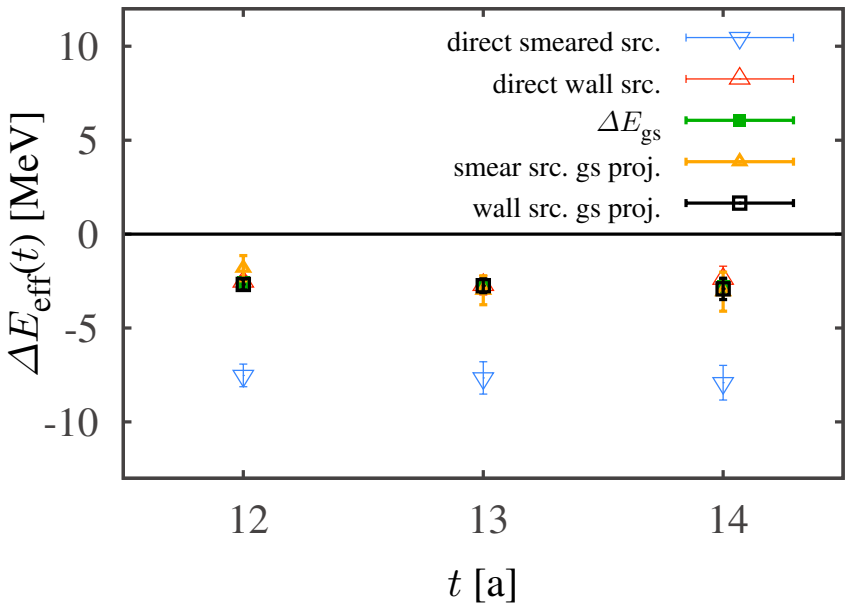
$$\tilde{R}^{(f)}(t) \equiv \sum_{\vec{r}} f(\vec{r}) \sum_{\vec{x}} \langle 0|B(\vec{r}+\vec{x},t)B(\vec{x})\bar{\mathcal{J}}(0)|0\rangle = \sum_{\vec{r}} f(\vec{r})R(\vec{r},t)$$

and

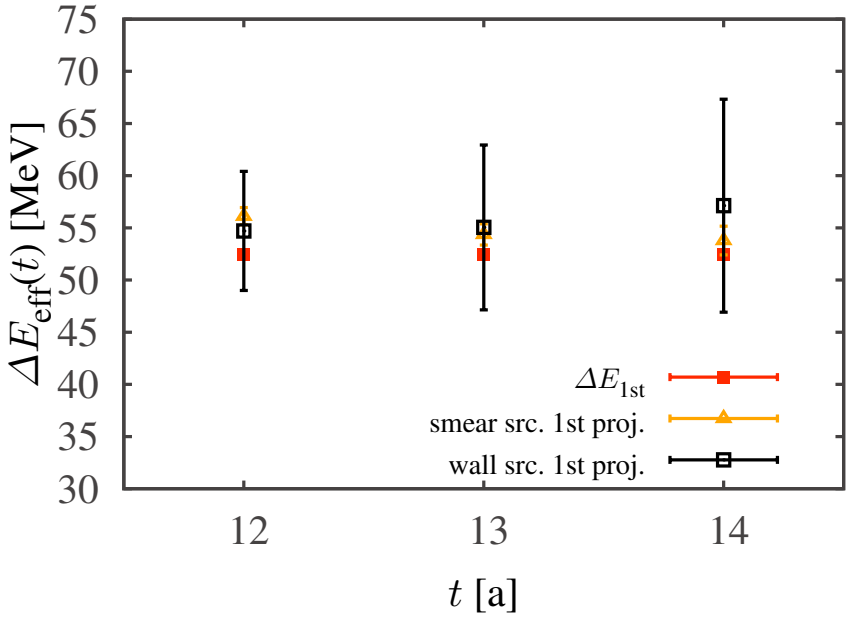
$$\Delta\tilde{E}^{(f)}_{\text{eff}}(t) = \log \frac{\tilde{R}^{(f)}(t)}{\tilde{R}^{(f)}(t+1)}$$

- ground state projection
- 1st excited state projection

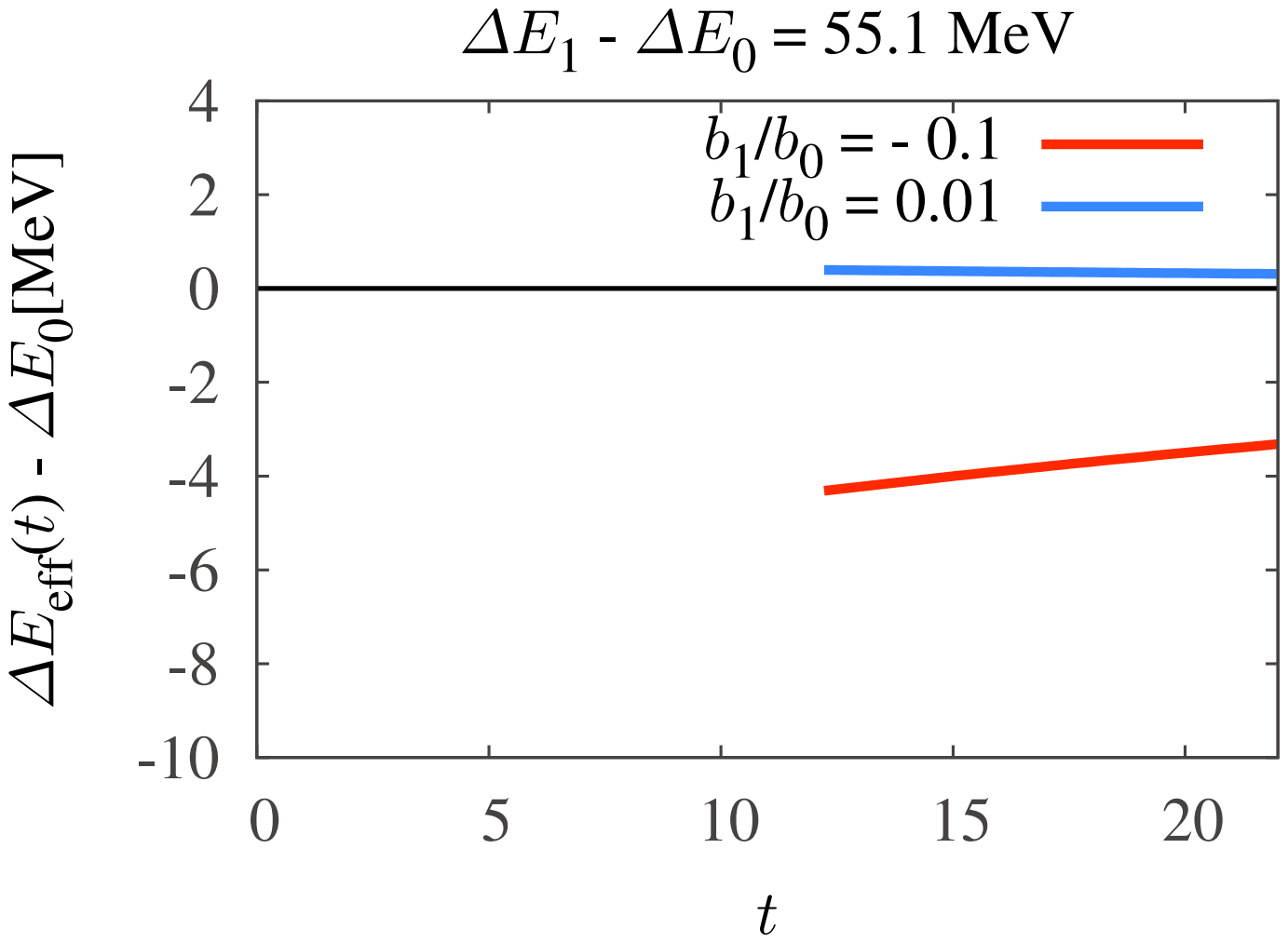
$$f(\vec{r}) = \Psi^{\dagger}_{\text{gs}}(\vec{r})$$



$$f(\vec{r}) = \Psi^{\dagger}_{1}(\vec{r})$$



Excited States Contamination and "Fake-Plateau"

ex. $R(t) = b_0 \exp(-\Delta E_0 t) + b_1 \exp(-\Delta E_1 t)$

- at $b_1/b_0 = -0.1$ — "plateau-like" behavior for $t = [12 : 20]$