Confinement and Chiral symmetry breaking from an Interacting Instanton-dyon ensemble for 2 colors and Nf flavors

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January 14. 2016

Overview

- Part 1: Instanton-dyons, confinement and holonomy
- Classical Interactions and holonomy dependent scaling
  - Parametrization of Classical interaction
  - Hard core and the scaling behavior
- Part 2: Dyon ensemble
  - Free energy of dyon ensemble
  - Setup on 3-sphere
  - Results for $N_f = 0$
- Part 3: Chiral symmetry breaking
  - Banks-Casher relation and random matrix theory
  - Chiral condensate from Eigenvalue distribution
  - Results for $N_f = 2$
- Summary

Part 1

- Instanton-dyons, confinement and holonomy
- Classical Interactions and holonomy dependent scaling
  - Parametrization of Classical interaction
  - Hard core and the scaling behavior

Instanton-Dyons

- **Calorons** with non-trivial Polyakov loop found by Kran, van Baal and Lee, Lu
- Seen to be composed of N_c **monopoles** or **dyons**
- Work with dyons as the fundamental degrees of freedom in $SU(2)$
- Amount of M and L dyons not the same
- Dyons have non-zero expectation value of $< A_4^3 >= 2\pi T\nu$, holonomy ν

$$P = \frac{1}{2}\mathrm{Re}\left[< Tr(L) >\right] = \cos(\pi\nu) \tag{1}$$

- Confined phase is $P = 0$ and $\nu = 0.5$
- Deconfined phase is $P = 1$ and $\nu = 0$

- The dyon solution for the M dyons is

$$
\begin{aligned}
A_4^a &= \pm \hat{r}_a \left(\frac{1}{r} - v \coth(vr) \right) \\
A_i^a &= \epsilon_{aij} \hat{r}_j \left(\frac{1}{r} - \frac{v}{\sinh(vr)} \right),
\end{aligned}
\tag{2}
$$

- $v = 2\pi T \nu$
- L dyons, is same solution, but with opposite sign and a gauge transformation of 2π
- Topological charge is ν for M and $1 - \nu$ for L dyons

	M	$\bar{M}$	L	$\bar{L}$
Sg^2	$8\pi^2\nu$	$8\pi^2\nu$	$8\pi^2(1-\nu)$	$8\pi^2(1-\nu)$
e	1	1	-1	-1
m	1	-1	-1	1

Zero Modes

- Dyons have 4 zero modes
  - 3 position and an U(1) phase

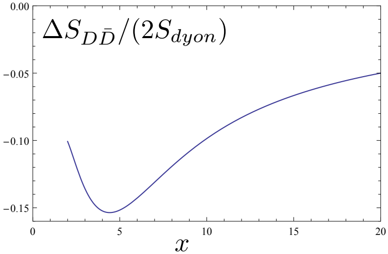
- M dyons have a periodic fermionic zero-mode
- L dyons have a anti-periodic fermionic zero-mode
- Only L dyons couple therefore to fermionic zero-modes

- Symmetry between ν and $1-\nu$ for no fermions
- This symmetry is broken by fermions

Classical interaction

- The classical interaction was found and parameterized using the streamline approach

$$\begin{aligned} \Delta S_{D\bar{D}} &= -2\frac{8\pi^2\nu}{g^2}(\frac{1}{x} - 1.632e^{-0.704x}) \\ x &= 2\pi\nu rT \end{aligned} \tag{3}$$



- Interaction for other combinations

$$\begin{aligned} \Delta S &= \frac{8\pi^2\nu}{g^2}\left(-e_1e_2\frac{1}{x} + m_1m_2\frac{1}{x}\right) \\ x &= 2\pi\nu rT \end{aligned} \tag{4}$$

Core and Scaling

- A lack of states at small distances modeled as a hard core

$$\Delta S_{Core} = \frac{\nu V_0}{1 + \exp\left[\sigma T(x - x_0)\right]} \quad (5)$$

$$x = 2\pi\nu r T \quad (6)$$

- Only for $x < x_0$
- $x_0 = 2$ chosen for simulation
- A change of coordinates show that the core should be proportional to ν and the size should scale as $1/\nu$

Debye Mass

- We simulate the Debye Mass as a cut off on our long range interactions

$$\begin{aligned} \Delta S &= \frac{8\pi^2\nu}{g^2}\left(-e_1 e_2 \frac{1}{x} + m_1 m_2 \frac{1}{x}\right) e^{-M_D r T} \\ x &= 2\pi\nu r T \end{aligned} \tag{7}$$

- We require that the used value obey

$$\frac{g^2}{2V}\frac{\partial^2 F}{\partial^2 v} = M_D^2 \tag{8}$$

- Such that the simulation is self consistent, within accuracy

Part 2

- Dyon ensemble
  - Free energy of dyon ensemble
  - Setup on 3-sphere
  - Results for $N_f = 0$

Ensemble

- We treat the dyons as point particles
- Cordinates is the position of the dyons
- We have the free gas contribution

$$
\begin{aligned}
Z_{unchanged} &= \sum_{N_M,N_L} \exp(-\tilde{V}_3 \frac{4\pi^2}{3}\nu^2\bar{\nu}^2) \\
&\times \left[\frac{1}{N_M!}\left(\tilde{V}_3 d_\nu\right)^{N_M}\right]^2 \left[\frac{1}{N_L!}\left(\tilde{V}_3 d_{\bar{\nu}}\right)^{N_L}\right]^2 \\
d_\nu &= \Lambda\left(\frac{8\pi^2}{g^2}\right)^2 e^{-\frac{\nu 8\pi^2}{g^2}} \nu^{\frac{8\nu}{3}-1}/(4\pi)
\end{aligned}
\tag{9}
$$

- and the interaction contribution

$$
\begin{aligned}
Z_{changed} &= \frac{1}{\tilde{V}_3^{2(N_L+N_M)}} \int D^3x \det(G) \exp(-\Delta S(x)) \\
\Delta f &\equiv -\log(Z_{changed})/\tilde{V}_3
\end{aligned}
\tag{10}
$$

Free Energy Density

- In the infinite volume limit the dominating configuration is the parameters that minimizes free energy density

$$f = \frac{4\pi^2}{3}\nu^2\bar{\nu}^2 - 2n_M \ln\left[\frac{d_\nu e}{n_M}\right] - 2n_L \ln\left[\frac{d_{\bar{\nu}} e}{n_L}\right] + \Delta f_{Interactions} \quad (11)$$

- Free energy density contains 3 items
  - The **GPY potential** that prefer **trivial** Holonomy
  - The **entropy** due to the dyons moving around
  - ($\Delta f_{Interactions}$) **Corrections** to the energy due to the interactions of the dyons
- $\Delta f_{Interactions}$ contains
  - Classical 2-point interaction, with a core
  - Diakonov determinant to describe the moduli space

- Metropolis algorithm used to find the expectation value of the action

$$
\begin{aligned}
e^{-F(\lambda)} &= \int Dx \exp(-\lambda S(x)) &(12)\\
\frac{\partial F}{\partial \lambda} &= \langle S \rangle. &(13)
\end{aligned}
$$

- One can the integrate up to get the free energy

$$
F(1) = \int_0^1 d\lambda < S(\lambda) > +F(0) \qquad (14)
$$

- Entropy is obtained from steep curve around $\lambda = 0$
- More points needed around $\lambda = 0$

Temperature

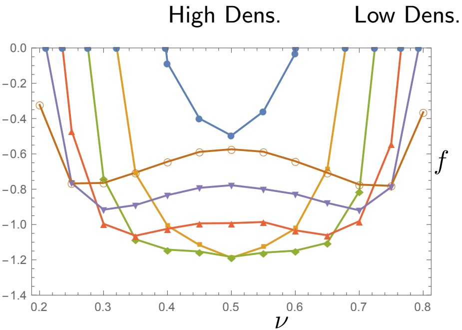
- We relate action to temperature through the running coupling constant

$$S(T) = \frac{8\pi^2}{g^2(T)} = b \cdot ln\left(\frac{T}{\Lambda}\right),\ b = \frac{11}{3}N_c - \frac{2}{3}N_F \tag{15}$$

- We show all results as both function of temperature and action of one instanton $8\pi^2/g^2$

Finding the dominating Configuration

- We minimize free energy in the following parameters:
  - Density of M dyons n_M and L dyons n_L
  - Holonomy ν
  - Debye mass describing the fall off of the fields



- The minimization is done as a function of Temperature T or Action $8\pi^2/g^2$
- High T: Low density of dyons prefer small holonomy ν such that action is low, but M dyons are large
- Low T: High density of dyons prefer symmetric case where M and L dyons are same size. Maximizes entropy

Ensemble on 3-sphere

- Simulation done on 3-sphere with 64 dyons

- Size of M dyons scales as $\frac{1}{\nu}$
- Size of L dyons scales as $\frac{1}{1-\nu}$
- Action of M dyons are $\nu 8\pi^2/g^2$
- Action of L dyons are $(1-\nu)8\pi^2/g^2$

	M	$\bar{M}$	L	$\bar{L}$
e	1	1	-1	-1
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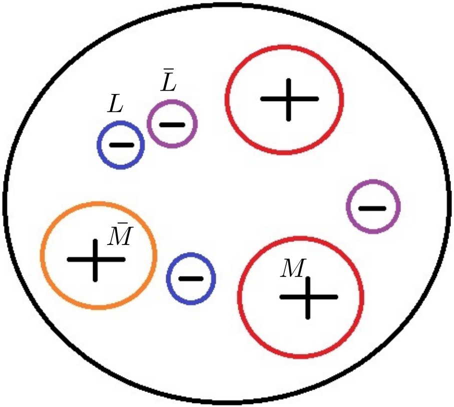
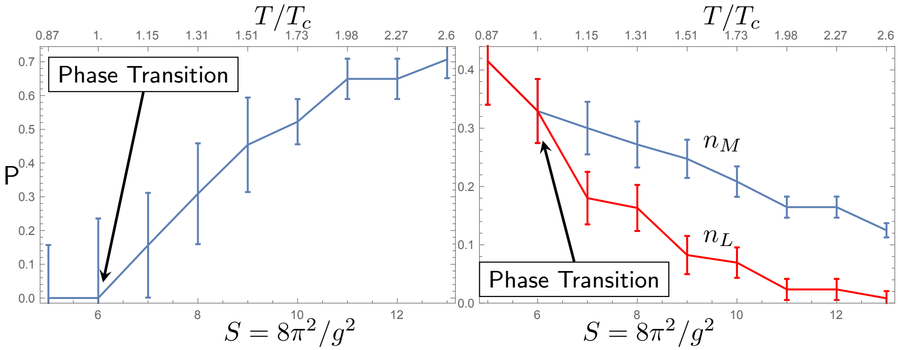


- Holonomy ν related to expectation value of Polyakov loop

$$P = \frac{1}{2} < Tr(L) >= \cos(\pi\nu) \tag{16}$$

Pure Gauge: Polyakov Loop

- Density of dyons increases at lower temperature
- Increased density forces M dyons to become smaller
- The **Polyakov loop** and the **dyons density** as a function of action/temperature



The ensemble of dyons gives confinement for $T < T_c$

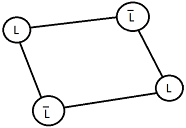
Part 3

- Chiral symmetry breaking
  - Banks-Casher relation and random matrix theory
  - Chiral condensate from Eigenvalue distribution
  - Results $N_f = 2$

Fermions

- L dyons have fermionic **zero modes** for anti-periodic fermions
- The determinant of the Dirac operator = closed loops of hopping over L's

$$Det \begin{vmatrix} 0 & T_{ij} \\ T_{ji} & 0 \end{vmatrix} = \sum_{\text{All combinations}}$$



- Actual shape of T_{ij} is chosen from overlap of fermionic zero-modes

$$T_{ij} \quad = \quad \bar{v}c' \exp\left(-\sqrt{11.2 + (\bar{v}r/2)^2}\right) \tag{17}$$

- Shape is postulated from overlap of specific direction of Dirac string
- New Interaction becomes

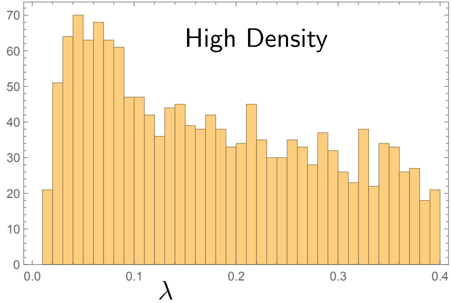
$$\begin{aligned} Z_{changed} \quad &= \quad \frac{1}{\tilde{V}_3^{2(N_L+N_M)}} \int D^3x \det(G) \exp(-\Delta S(x)) \\ &\times \prod_i \lambda_i^{N_f} \end{aligned} \tag{18}$$

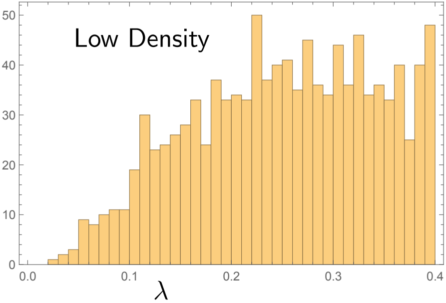
Chiral Condensate

- The Banks-Casher relation for the chiral condensate tells us that

$$|<\bar{\psi}\psi>| \quad = \quad \pi\rho(\lambda)_{\lambda\to 0, m\to 0, V\to\infty} \tag{19}$$

- For finite volume we need to look at eigenvalue distribution around 0
- Low density: Linear attraction makes $L\bar{L}$ pairs
- High density: Collectivizes into large clusters





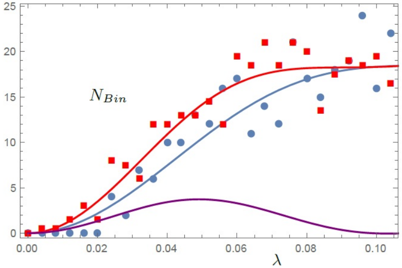
- Random matrix theory was fitted to these distributions for 2 flavors

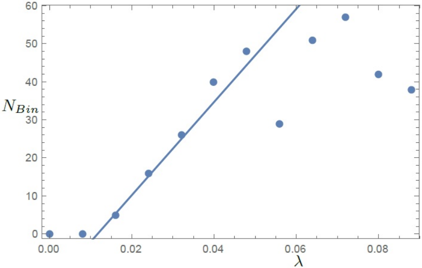
- Random matrix theory predict the volume scaling of the zero-mode zone

$$\begin{aligned} \rho(x) &= V\Sigma_2[\frac{x}{2}(J_2(x)^2 - J_1(x)J_3(x)) \\ &+\frac{1}{2}J_2(x)(1-\int_0^x dt J_2(t))] \end{aligned} \tag{20}$$

$$x = \lambda V \Sigma_1 \tag{21}$$

- Chiral condensate can be interpreted as the difference between two volume, one twice as large

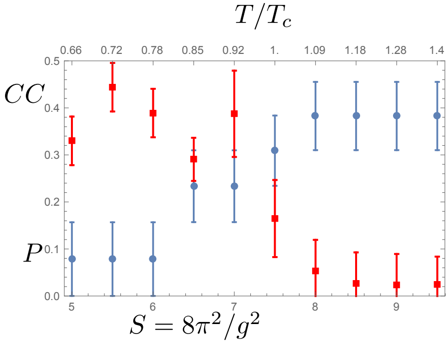


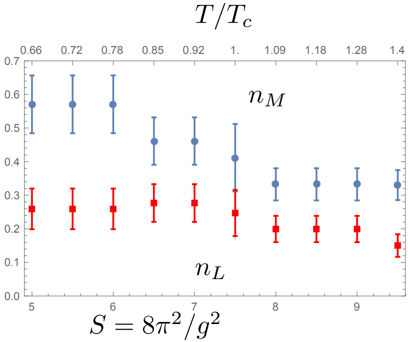


- Gap of eigenvalues another indicator of chiral symmetry breaking

Polyakov Loop and Chiral Condensate for $N_f = 2$

- The drop in the Polyakov loop increases the **effective density** of L dyons, creating a chiral condensate
- M and L dyons density not the same below T_c.

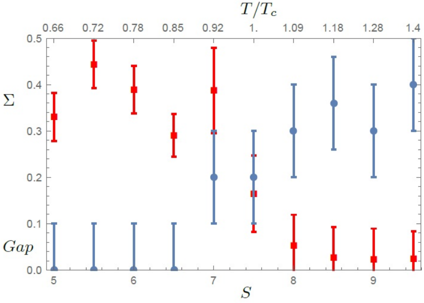


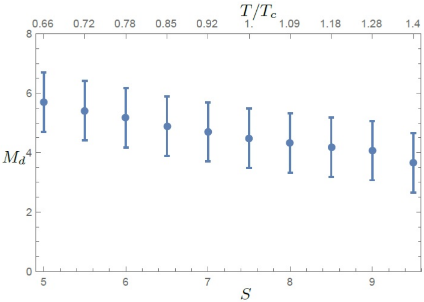


- The mechanisms behind confinement and chiral condensate are closely related. The increased density of dyons.

Gap and Debye mass

- Chiral condensate and gab of eigenvalues correspond within accuracy
- Debye mass seen to be a factor 2 too large





- Likely reason due to higher order corrections to GPY-potential and need for a larger core

Summary

1. Simulation of 64 dyons done
2. The free energy as a function of temperature, densities, holonomy and Debye mass has been found
3. High density of dyons pushes holonomy up and Polyakov loop down $\Rightarrow$ Confinement
4. Dirac determinant included through the overlap of fermionic zero-modes
5. At high dyon density the eigen modes collectivizes $\Rightarrow$ Broken chiral symmetry
6. We can now describe the phenomenon of chiral symmetry breaking and confinement through the increase in density of dyons