RIKEN/Nuclear BNL Lunch Seminar, May 5th, 2016

Vorticity in heavy-ion collisions and cold atoms

Xu-Guang Huang

Fudan University, Shanghai

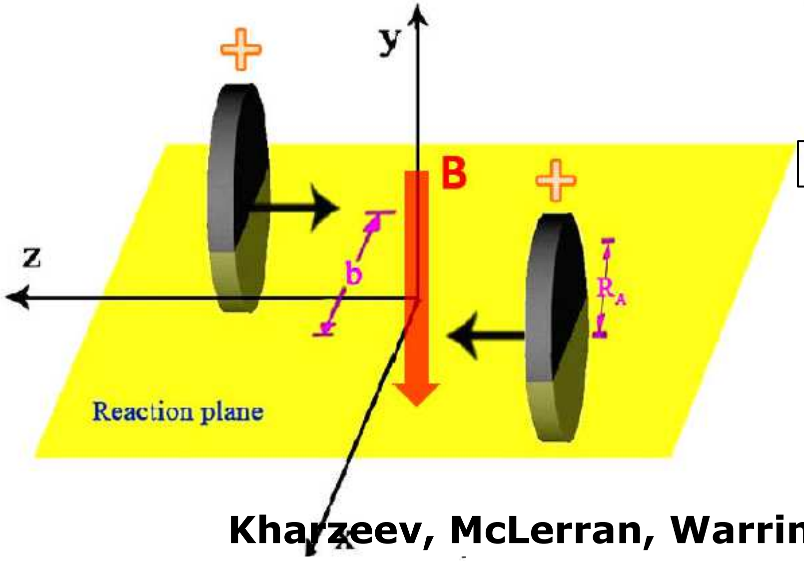
Outline

- Introduction
- Vorticity in heavy-ion collisions
- A vorticity induced anomalous collective mode: chiral vortical wave
- Stir a cup of cold Fermi gas with spin-orbit coupling: desktop simulator of chiral magnetic effect
- Summary

Introduction

Introduction

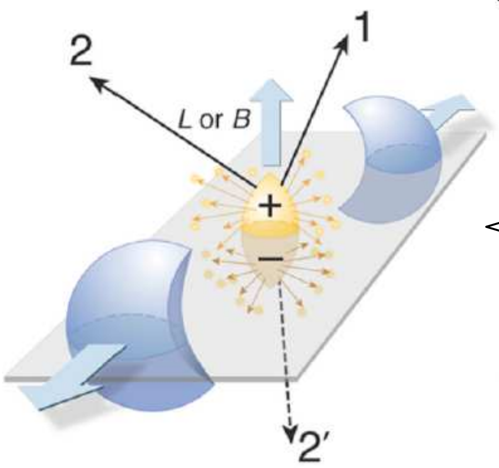
Consider a non-central heavy-ion collision



generate magnetci field

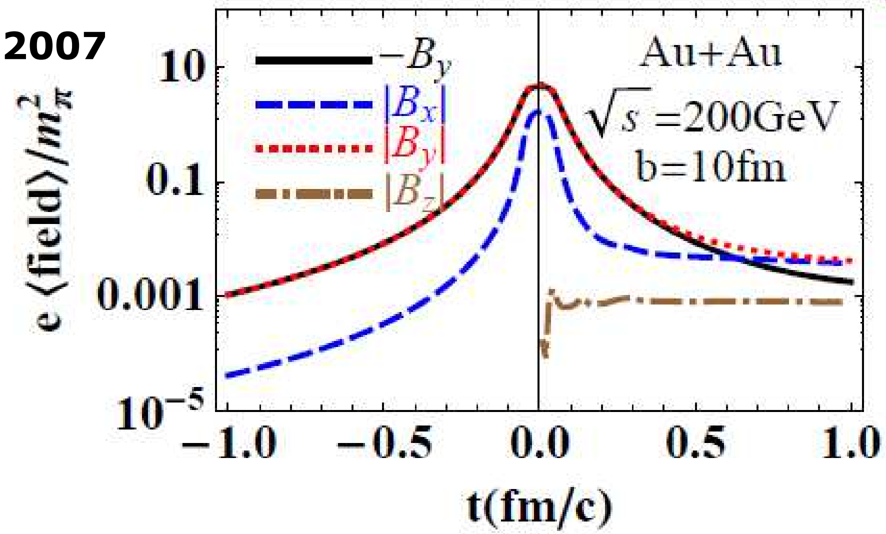
Actually, B field is very strong but short lived

Kharzeev, McLerran, Warringa 2007



Chiral magnetic effect

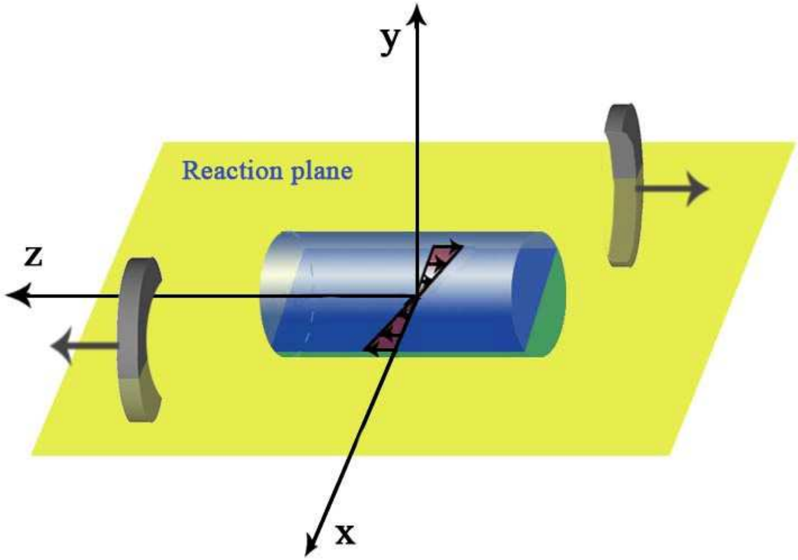
$$\mathrm{J}_V = \frac{N_c e}{2\pi^2}\mu_A \mathrm{B}$$



Skokov et al 2009, 2012, Deng and XGH 2012, 2015 … …

Introduction

Consider a non-central heavy-ion collision



Finite angular moment (AM)

⇩

Manifested as flow shear*

⇩

Finite vorticity(local rotation)

$$\vec{\omega} = \vec{\nabla} \times \vec{v}$$

$$J_0 \sim Ab\sqrt{s}/2 \quad \Longrightarrow \quad J \sim \int d^3x I(x)\omega(x)$$

$I(x) \sim [x^2 - (x\cdot\hat{\omega})^2]\varepsilon(x)$ is the moment of inertia density

J_0 is about 10^6 for RHIC Au+Au @ 200 GeV,
system volume is ~ fm^3, very large AM density

*For low energy collision, the system after collision may be globally rotating

Introduction

Such vorticity can bring interesting phenomena

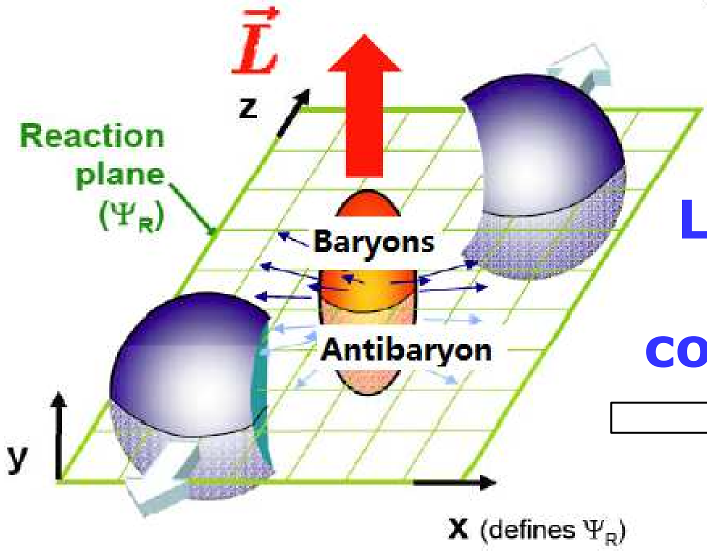
Chiral vortical effect: vorticity + chiral anomaly

(Kharzeev, Zhitnitsky 2007, Erdmenger etal 2009, Son, Surowka 2009, Banerjee etal 2011, Landsteiner etal 2011)

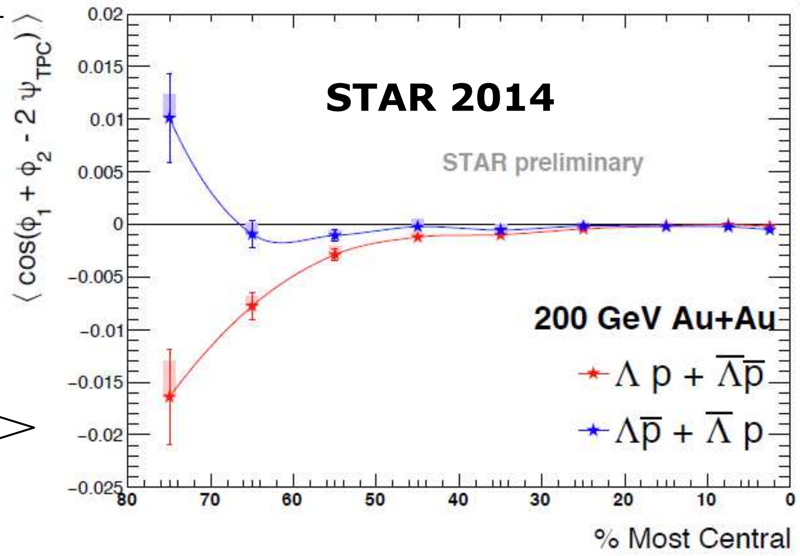
$$j = \chi\omega, \quad \chi = N_c\mu\mu_5/(2\pi^2)$$

$$j_5 = \chi_5\omega, \quad \chi_5 = N_c[T^2/12 + (\mu^2+\mu_5^2)/(4\pi^2)]$$

Phenomenology: baryon-antibaryon separation w.r.t reaction plane



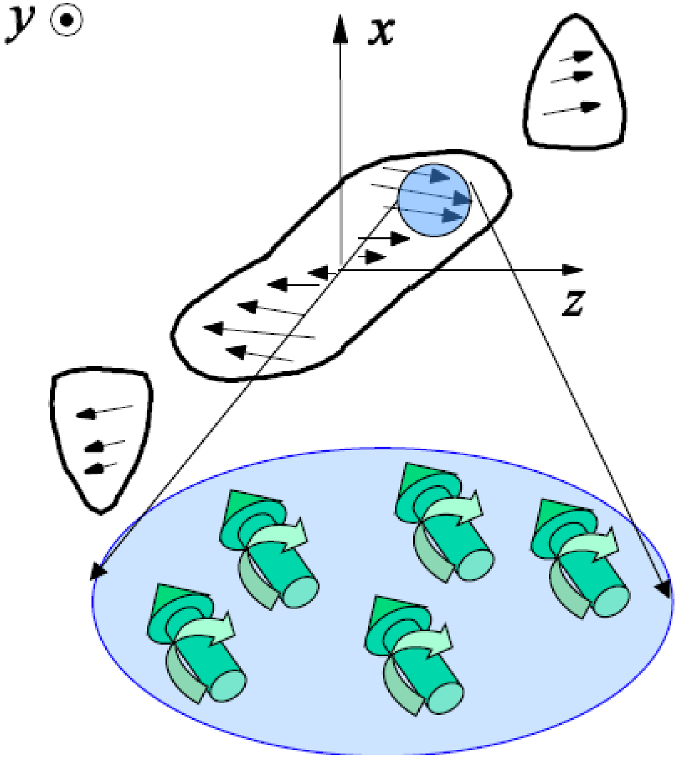
Lambda-proton correlation



Introduction

Such vorticity can bring interesting phenomena

Global spin polarization of quarks due to spin-orbit coupling (Liang, Wang 2005)



$$P \equiv \frac{\Delta\sigma}{\sigma} \equiv \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow} = -\frac{\pi \mu p}{2E(E+m)}$$

Hadrons can inherit the spin polarization of quarks

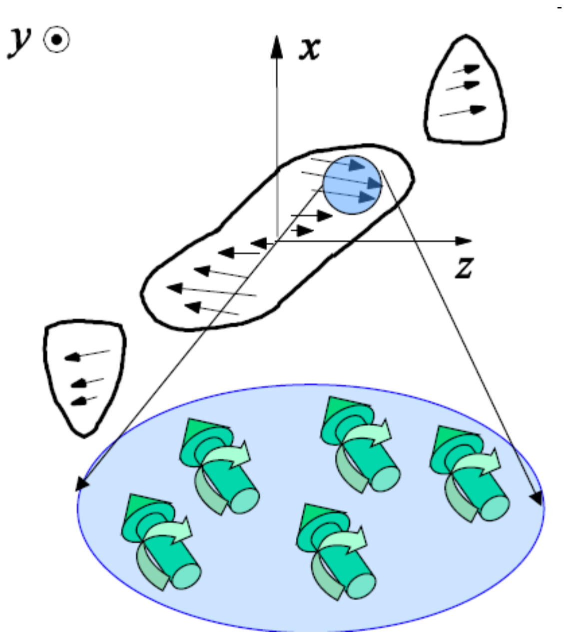
E.g., Lambda polarization

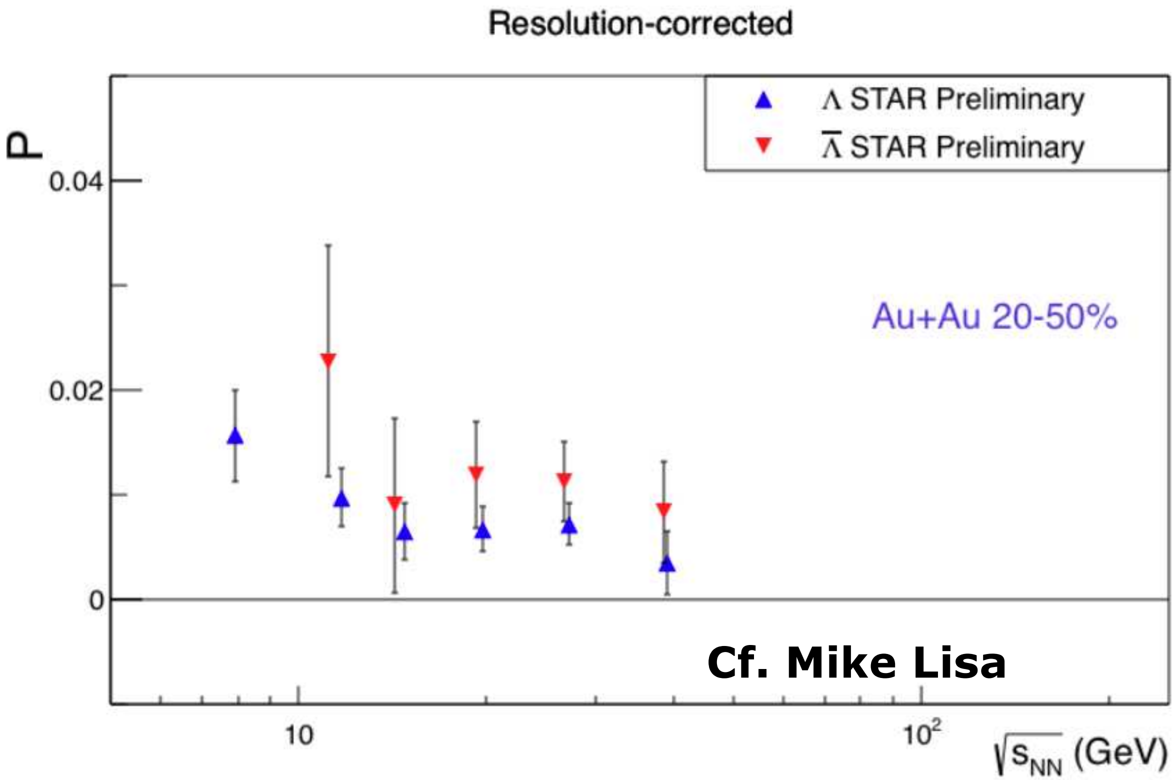
Introduction

Such vorticity can bring interesting phenomena

Global spin polarization of quarks due to spin-orbit coupling

E.g., Lambda polarization





Cf. Mike Lisa

Vorticity in heavy-ion collisions

Deng and XGH, arXiv: 1603.06117

Vorticity in HICs

As vorticity play a key role in CVE and Lambda polarization, we now study vorticity itself in detail

Event-by-event generation of vorticity in HICs by using HIJING model

Definition of velocity field

$$v_1^a(x) = \frac{1}{\sum_i \Phi(x, x_i)} \sum_i \frac{p_i^a}{p_i^0} \Phi(x, x_i) = \frac{J^a}{J^0}$$ **~ Particle flow velocity**

$$v_2^a(x) = \frac{\sum_i p_i^a \Phi(x, x_i)}{\sum_i [p_i^0 + (p_i^a)^2 / p_i^0] \Phi(x, x_i)} = \frac{T^{0a}}{T^{00} + T^{aa}}$$ **~ Energy flow velocity**

Smearing function Phi

$$\Phi_G(x, x_i) = \frac{K}{\tau_0 \sqrt{2\pi\sigma_\eta^2} 2\pi\sigma_r^2} \exp\left[-\frac{(x - x_i)^2 + (y - y_i)^2}{2\sigma_r^2} - \frac{(\eta - \eta_i)^2}{2\sigma_\eta^2} \right]$$

Parameters are so chosen that with hydro, it is consistent with data (Pang, Wang, Wang 2012)

Vorticity in HICs

As vorticity play a key role in CVE and Lambda polarization, we now study vorticity itself in detail

Event-by-event generation of vorticity in HICs by using HIJING model

Definition of velocity field

$$v_1^a(x) = \frac{1}{\sum_i \Phi(x, x_i)} \sum_i \frac{p_i^a}{p_i^0} \Phi(x, x_i) = \frac{J^a}{J^0}$$ **~ Particle flow velocity**

$$v_2^a(x) = \frac{\sum_i p_i^a \Phi(x, x_i)}{\sum_i [p_i^0 + (p_i^a)^2 / p_i^0] \Phi(x, x_i)} = \frac{T^{0a}}{T^{00} + T^{aa}}$$ **~ Energy flow velocity**

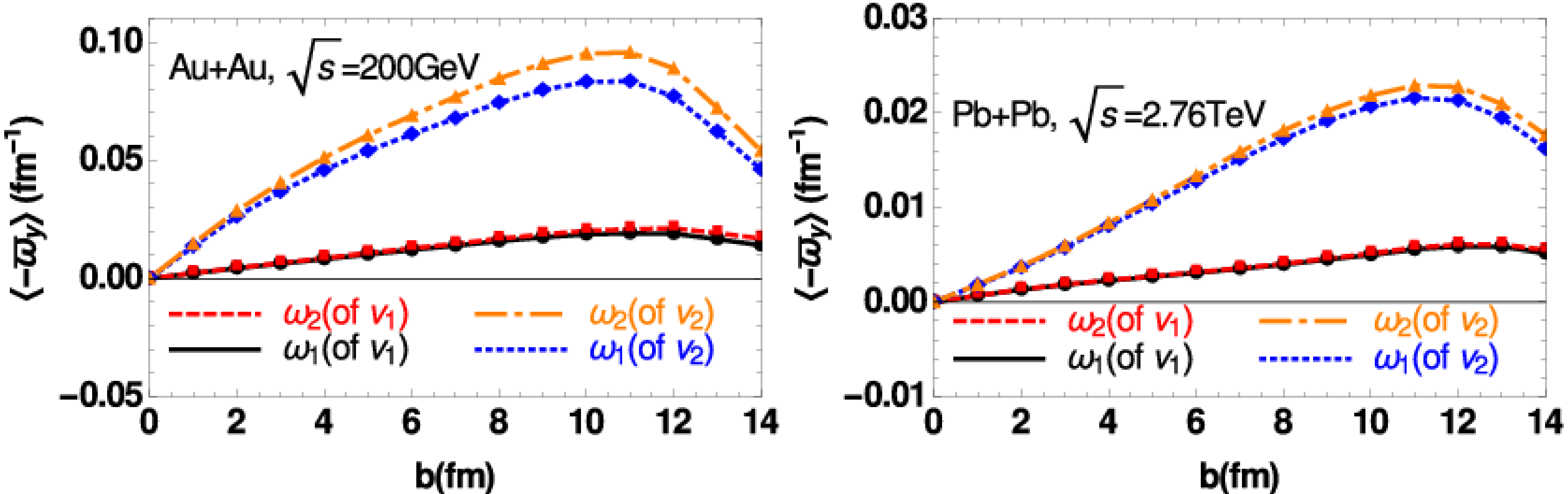
Definition of vorticity field (for each definition of v)

$$\omega_1 = \nabla \times v,$$ **~ nonrelativistic definition**

$$\omega_2 = \gamma^2 \nabla \times v.$$ **~ relativistic definition with Lorentz correction**

Vorticity in HICs

Impact parameter dependence



•**Showed is the vorticity at zero rapidity at initial time averaged over the reaction zone and averaged over 10^5 events.**

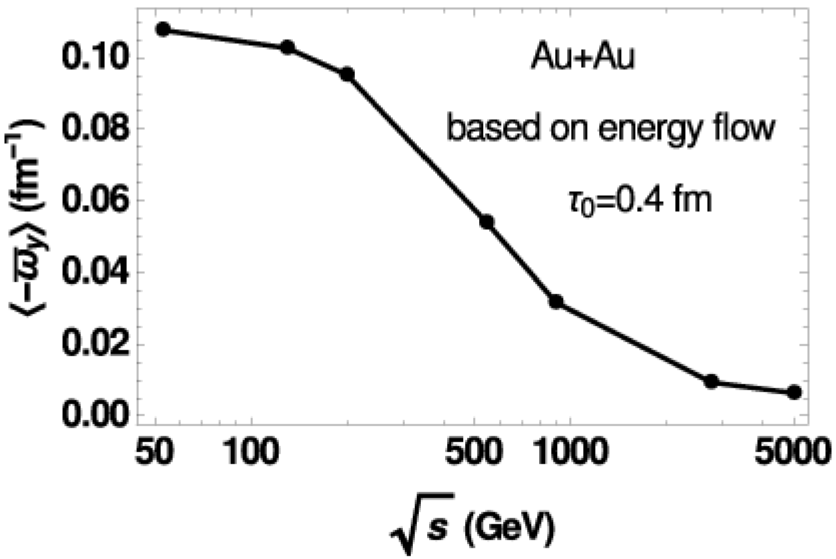
•**RHIC: Take T~300 MeV, T^2*vorticity~10^6 MeV^3 comparable to magnetic field effect via CSE. But at LHC, initial vortical effect is smaller than B effect**

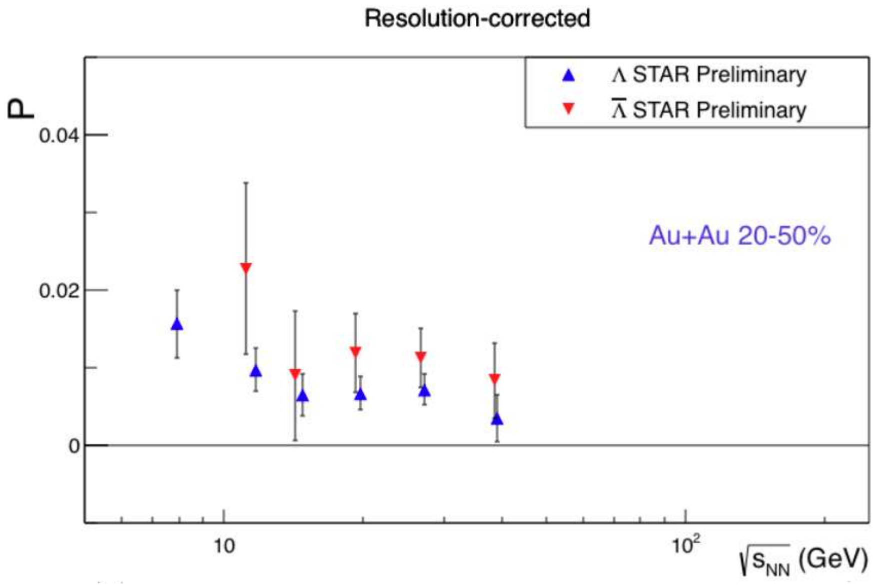
•**At b<2R_A, increase with b; then drops. Anglular momentum has a similar behavior.**

•**Energy flow has a larger vorticity**

Vorticity in HICs

Collision energy dependence





•Consistent with the Lambda polarization result of STAR

•Total angular momentum increases with energy, but vorticity at zero rapidity decreases with energy. Reason: with energy grows, moment of inertia increases quickly; more AM carried by finite rapidity particles

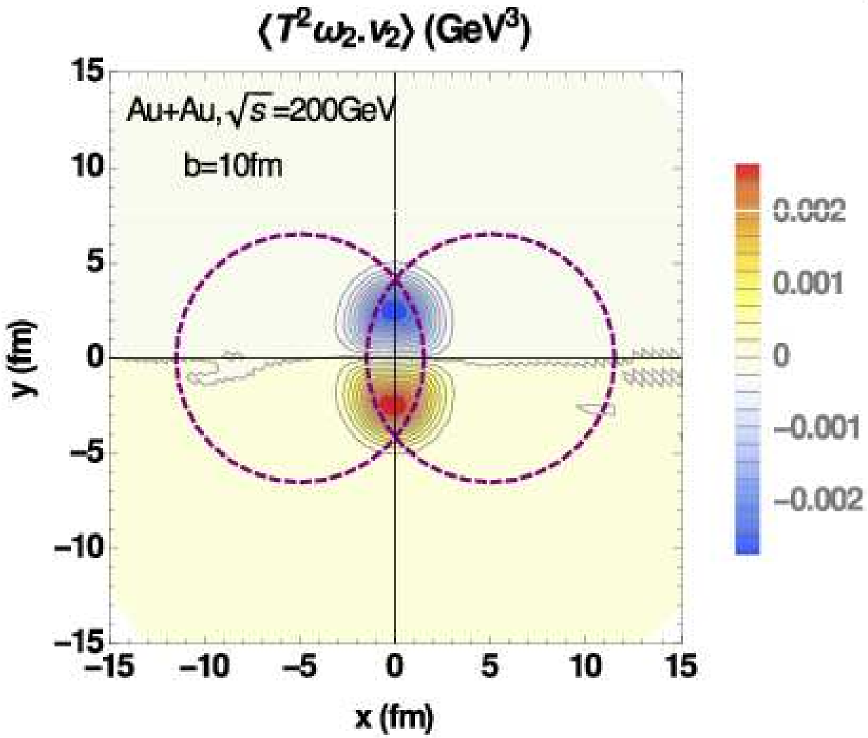
•Higher collision energy, closer to Bjorken, thus smaller vorticity

•Indicates stronger chiral vorticty effect at lower energy

•(Jiang, Lin, Liao 2016 found similar energy dependence)

Vorticity in HICs

Flow Helicity distribution



- **Helicity separation w.r.t the reaction plane**

- **Without anomaly, under ideal relativistic hydro. Eq.:**

$$\frac{d}{dt}\int d^3\mathbf{x}T^2\vec{v}\cdot\vec{\omega}_2 = 0$$

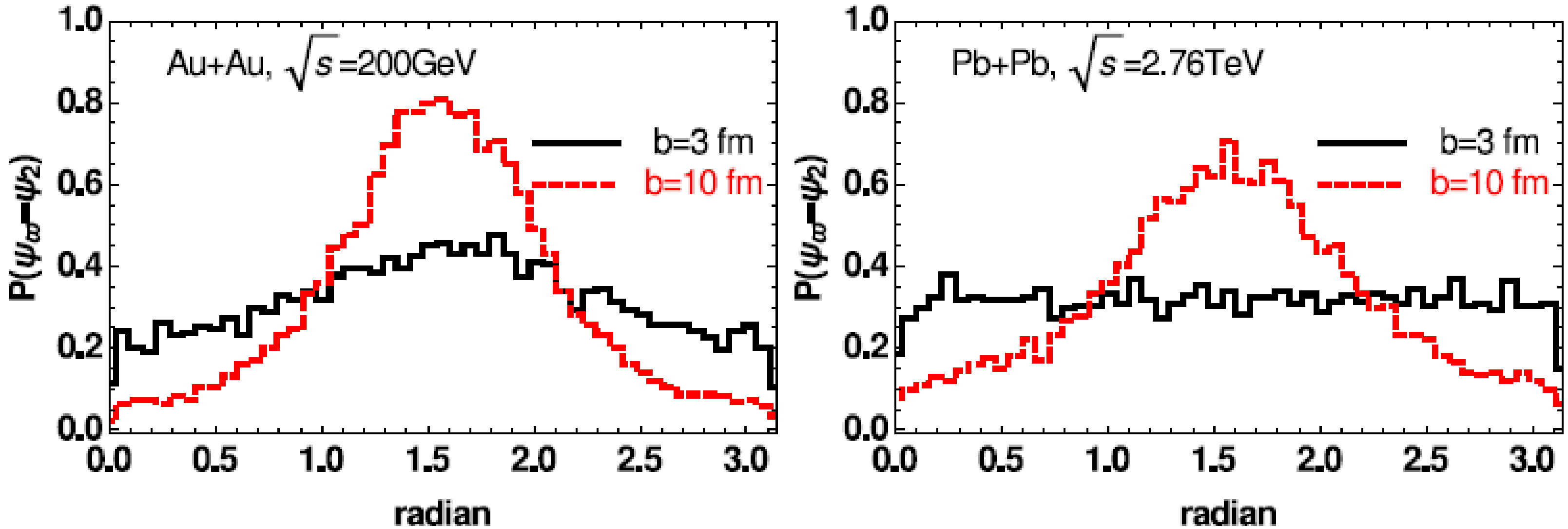
- **In anomalous hydro:**

$$\frac{d}{dt}\int d^3\mathbf{x}T^2\vec{v}\cdot\vec{\omega}_2 = \frac{12}{N_c}\frac{d}{dt}\int d^3\mathbf{x}n_5$$

- **A mechanism to generate fermion chirality**

Vorticity in HICs

Event-by-event fluctuation of vorticity orientation



- Shown is histogram of azimuthal angle of vorticity relative to participant plane (PP)
- Clear event-by-event fluctuation in vorticity orientation
- For small b, fluctuation so strong that correlation with PP is lost
- Large b, Gaussian around pi/2

Vorticity in HICs

Such fluctuation can strongly influence vorticity driven effects, e.g., chiral vortical effect.

Consider the experimental measured correlation:

$$\gamma_{\alpha\beta} = \langle \cos(\phi_\alpha + \phi_\beta - 2\psi_2) \rangle$$

CVE induced two particle distribution:

$$f_{\alpha\beta}^{\mathrm{CVE}} \propto \boldsymbol{\omega}^2 \cos(\phi_\alpha - \psi_\omega) \cos(\phi_\beta - \psi_\omega)$$

$$\Longrightarrow \quad \gamma_{\alpha\beta} \propto \langle \boldsymbol{\omega}^2 \cos[2(\psi_\omega - \psi_2)] \rangle$$

If no fluctuation:

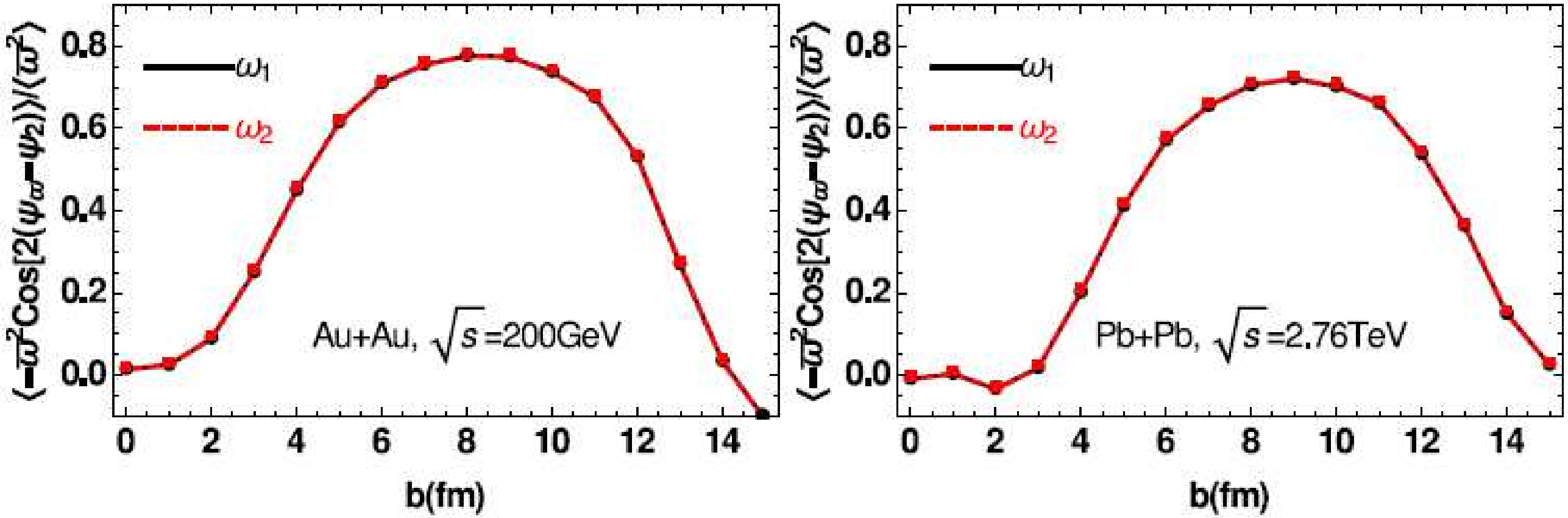
$$\gamma_{\alpha\beta} \propto \langle \boldsymbol{\omega}^2 \rangle$$

Thus this correlation quantifies azimuthal fluctuation:

$$R_2 = \frac{1}{\langle \bar{\omega}^2 \rangle} \langle \bar{\omega}^2 \cos[2(\psi_\omega - \psi_2)] \rangle$$

Vorticity in HICs

Azimuthal correlation between vorticity and participants



- **Vorticity of particle flow and energy flow show same correlation**
- **At very central and very peripheral, small correlation. Reason: either vorticity or participant angle fluctuates strongly.**
- **Very little dependence on collision energy, as it is geometry dominated.**
- **Strongest correlation at b~8-9 fm. Suppression factor ~ 0.8.**

Vorticity in HICs

Time evolution (qualitative argument)

(Nonrelativistic) Vorticity equation:

$$\frac{\partial \omega}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{\omega}) + \nu \nabla^2 \boldsymbol{\omega}$$

with kinematic shear viscosity:

$$\nu = \eta/(\varepsilon + P) = T^{-1}(\eta/s)$$

- **Reynolds number:** $\mathrm{Re} = UL/\nu$

- **If Re<<1 with initial profile** $\boldsymbol{\omega}(0, \boldsymbol{x}) = \omega_0 e^{-x_\perp^2/\sigma_r^2}$

$$\frac{\partial \omega}{\partial t} = \nu \nabla^2 \omega \Longrightarrow \boldsymbol{\omega}(t, \boldsymbol{x}) = \omega_0 \frac{\sigma_r^2}{\sigma_r^2 + 4\nu t} \exp\left(-\frac{x_\perp^2}{\sigma_r^2 + 4\nu t}\right)$$

- **Decay slowly for t < sigma^2/4v**

Vorticity in HICs

Time evolution (qualitative argument)

- If Re>>1 with initial profile

$$\frac{\partial \omega}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{\omega})$$

- Vortex line is frozen in the fluid. Vorticity will decay due to QGP expansion.
- Suppose longitudinal Bjorken expansion and transverse Gaussian initial entropy distribution caused transverse expansion

$$s(\boldsymbol{x}_\perp) = s_0 \exp\left(-\frac{x^2}{2a_x^2} - \frac{y^2}{2a_y^2}\right)$$

- Vorticity decays:

$$\omega_y(t, \boldsymbol{x}) = \frac{t_0}{t} \exp\left[-\frac{c_s^2}{2a_x^2}(t^2 - t_0^2)\right] \omega_y(t_0, \boldsymbol{x}_0)$$

- For t< 7 fm, inversely proportional to t
- Realistic viscous hydrodynamic simulations is desirable.

Chiral vortical wave

(Jiang, XGH, Liao, PRD 92, 071501 (2015))

Chiral magnetic wave

Recall: Chiral magnetic effect and chiral separation effect

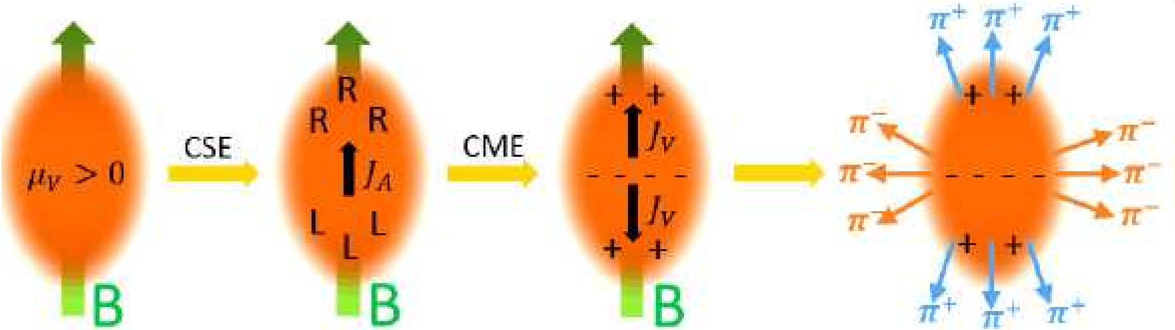
$$\mathbf{J}_V = \frac{N_c e}{2\pi^2}\mu_A \mathbf{B} \qquad \mathbf{J}_A = \frac{N_c e}{2\pi^2}\mu_V \mathbf{B}$$

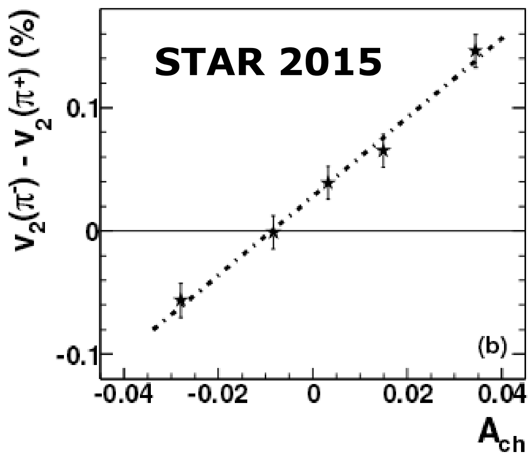
net μ_V $\rightarrow$ CSE $\rightarrow$ chirality separation $\rightarrow$ CME $\rightarrow$ charge separation $\rightarrow$ CSE $\rightarrow$ $\cdots \Rightarrow$ Chiral magnetic wave (CMW)

$$\partial_t J^0_{R/L} + \nabla \cdot J_{R/L} = 0, \qquad J_R = \frac{1}{2}(J_V + J_A) \qquad J_L = \frac{1}{2}(J_V - J_A)$$

- **Once substituting CME and CSE, obtain wave equations: one wave propagating along B and another opposite to B** **(Kharzeev, Yee 2011)**

- **Experimental implication: charged pion v_2 splitting** **(Burnier etal 2011)**

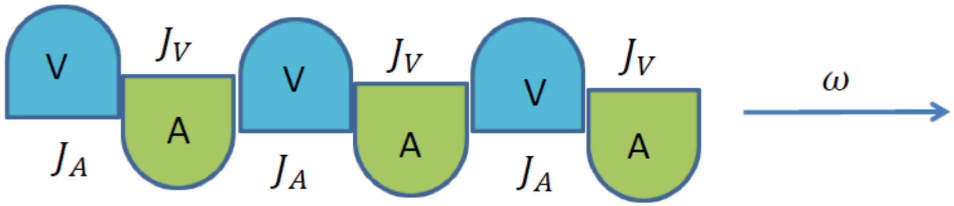




Chiral vortical wave

The vortical analogue of chiral magnetic wave*

$$\vec{J}_A = \left(\frac{T^2}{6} + \frac{\mu^2 + \mu_5^2}{2\pi^2}\right)\vec{\omega}, \qquad \vec{J}_V = \frac{\mu\mu_5}{\pi^2}\vec{\omega}$$



- **A new collective mode. To reveal its dispersion we use continuity eq.**

$$\partial_t n_{L,R} + \nabla \cdot \vec{J}_{L,R} = 0$$

- **Substitute CVE currents. Obtain Burgers wave equation which is linearized to normal wave equation**

$$\partial_t n_{L,R} = \pm \frac{\omega\alpha^2}{\pi^2}\partial_x(n_{L,R}^2) \Longrightarrow \pm \boxed{\frac{2\omega\alpha^2}{\pi^2} n_0}\,\partial_x(n_{L,R})$$

$$\alpha = \frac{\partial\mu}{\partial n}$$ **~ inverse baryon susceptiblity**

CVW velocity

*** The vorticity is define as half the one we defined in the first part of the talk**

Chiral vortical wave

chiral vortical wave from chiral kinetic equation

(Sthepanov, Yin 2012, Son, Yamamoto 2012, Wang etal 2012)

$$\partial_t f_\pm + \dot{\vec{x}} \cdot \partial_{\vec{x}} f_\pm + \dot{\vec{p}} \cdot \partial_{\vec{p}} f_\pm = C_\pm[f_+, f_-]$$

$$\sqrt{G_\pm}\dot{\vec{x}} = {}^{\vec{p}}/_{p} \pm {}^{\vec{\omega}}/_{p}$$

- **Berry monopole at p=0**

$$\sqrt{G_\pm}\dot{\vec{p}} = 2\vec{p} \times \vec{\omega}$$

$$\sqrt{G_\pm} = 1 \pm {}^{\vec{p}\cdot\vec{\omega}}/_{p^2}$$

- **Berry curvature modified phase pace measure**

- **Seek for wave mode**

$$f_\pm(t, \vec{x}, \vec{p}) = f_{0\pm}(p) + \delta f_\pm(t, \vec{x}, \vec{p})$$

$$\delta f_\pm(t, \vec{x}, \vec{p}) = \pm\partial_p f_{0\pm}(p) \int d\nu\, d^3k\, e^{i(\nu t - \vec{k}\cdot\vec{x})} h(\nu, \vec{k}, \vec{p})$$

Chiral vortical wave

chiral vortical wave from chiral kinetic equation

- The leading equation for h at low frequency and momentum

$$\nu \boxed{\int_{\vec p} [\partial_p f_{0+}(p) + \partial_p f_{0-}(p)]} h(\nu, \vec k, \vec p) = \pm \vec k \cdot \boxed{\int_{\vec p} \frac{\vec \omega}{p} [\partial_p f_{0+}(p) - \partial_p f_{0-}(p)]} h(\nu, \vec k, \vec p)$$

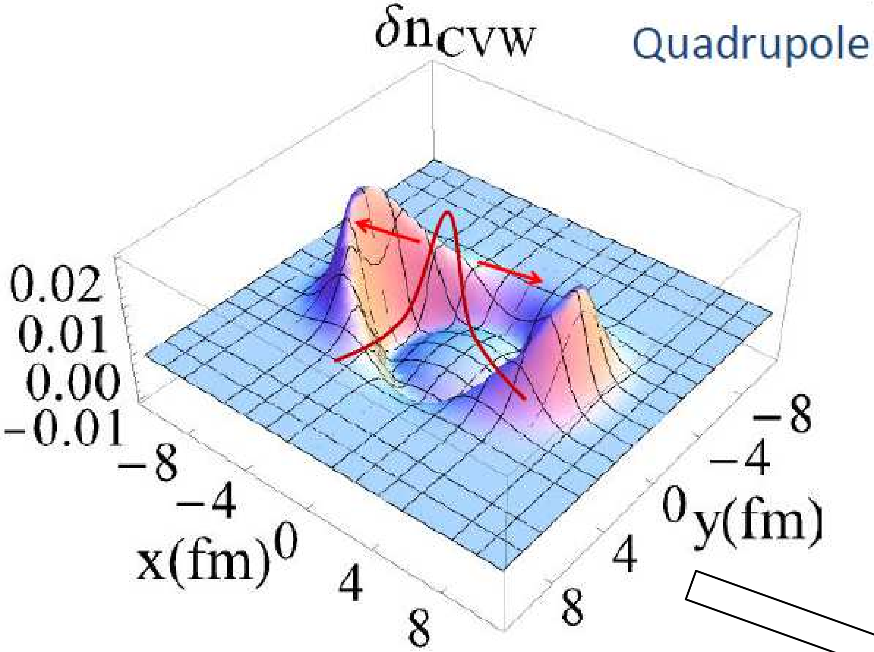
$$= \partial n / \partial \mu = \alpha^{-1}$$

$$\overset{F.D.}{\Longrightarrow} -\frac{\mu_0}{2\pi^2} = \frac{-\alpha n_0}{2\pi^2}$$

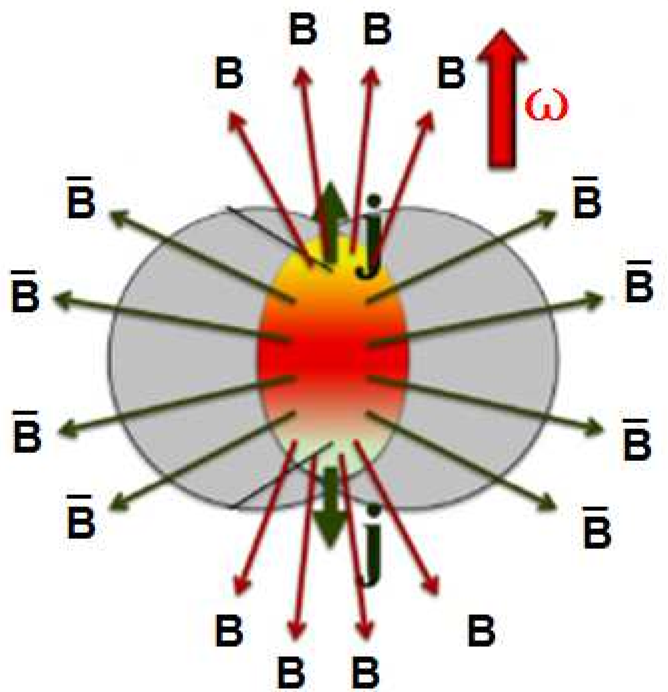
- Gives the same wave mode as before
- CVW needs a net background baryon density n_0

Chiral vortical wave

Experimental implication: baryon charge quadrupole



- More baryon charges at the tips of the fireball, more antibaryon charges at the center



- Stronger in-plane radial expansion lets antibaryons get larger elliptic flow than baryons

Chiral vortical wave

Lambda-anti-Lambda v_2 splitting

•**Chemical potential shift of quark of flavor f (leading order in q):**

$$\delta\mu_f \propto 2q^f_\Omega \cos(2\phi_s)$$

$$q^f_\Omega = [\int dxdy(\delta n_f)\cos(2\phi_s)]/[\int dxdy(\delta n_f)]$$ **~ quadrupole moment**

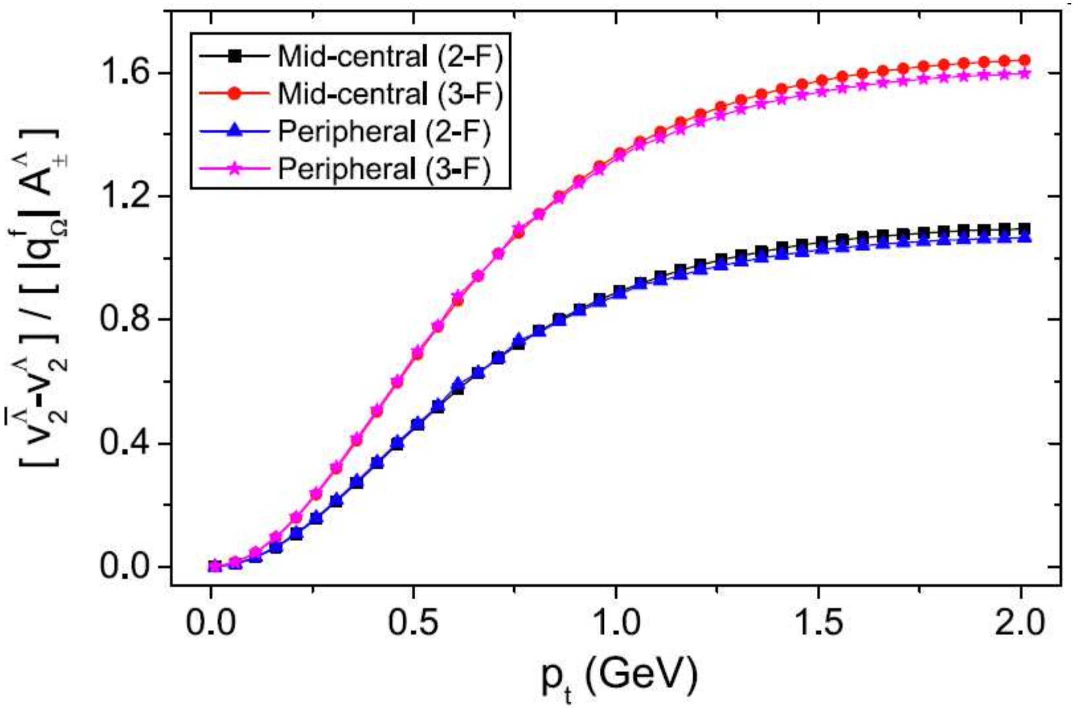
•**Lambda (carries baryon charge but no electric charge: responses to CVW but not CMW)**

$$\delta\mu_\Lambda \propto 2(q^u_\Omega + q^d_\Omega + q^s_\Omega)\cos(2\phi_s)$$

$$\Downarrow$$

$$\Delta v_2 = v_2^{\bar\Lambda} - v_2^{\Lambda} \propto |q^f_\Omega| A^\Lambda_\pm$$

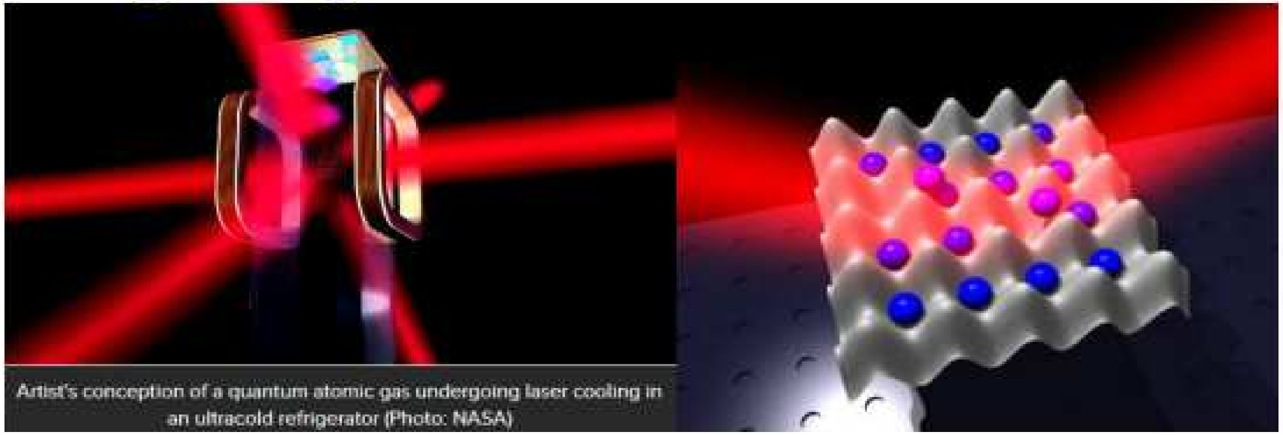
$$A^\Lambda_\pm = (N^\Lambda - N^{\bar\Lambda})/(N^\Lambda + N^{\bar\Lambda})$$



Anomalous transport in cold atomic gases

Motivation

- The CME/CSE/CVE etc are masked by various backgrounds in HICs, it is hard to pin down and to explore their properties in HICs.
- Question: Is there any system that exhibits anomalous transport in a controllable way?
- Answer: Yes! One example is the Dirac or Weyl semimetal **(Li, et al, 1412.6543 and many other recent experimental progresses).**
- Here we propose another possibility: The cold atomic gases.
- Atomic gases experiments. $10^5 - 10^6$ atoms put in magnetic trap or optical trap, and cooled down to nano Kelvin by using laser cooling or evaporating cooling



- A lot of exciting low-temperature phenomena have been observed: superfluidity, Bose-Einstein condensation, BCS-BEC crossover, novel superfluid, polaron gases, ferromagnetisim,......

Spin-orbit coupled atomic gases

- In 2011, a new type of cold Bose gases generated in which the spin is coupled to the orbital motion of the atoms **(Spielman et al 2011)**.
  The single-particle Hamiltonian(Rashba-Dresselhaus SOC):

$$\mathcal{H} = \frac{\mathbf{p}^2}{2m} - \lambda\sigma_x p_y$$

- In 2012, same type spin-orbit coupling (SOC) for Fermi gases produced in MIT **(Zwierlein group 2012)** and in Shanxi**(Zhang group 2012)**.
- Other types of SOC also possible, e.g., the Weyl SOC: **(Spielman et al 2012)**

$$\mathcal{H} = \frac{\mathbf{p}^2}{2m} - \lambda\sigma \cdot \mathbf{p}$$

- Now we show: there are CME and CSE in Weyl spin-orbit coupled Fermi gases.

Semiclassical equations of motion

- Consider the Weyl SOC, $\lambda\sigma \cdot \mathbf{p}$, in single atom Hamiltonian

$$\mathcal{H} = \frac{\mathbf{p}^2}{2m} - \lambda \mathbf{p} \cdot \sigma$$

- Along $\mathbf{p}$, the spin has two projection which defines two helicities (we will call them chiralities as well) , right-hand (project along $\mathbf{p}$) and left-hand (project along $-\mathbf{p}$).
- Consider atoms in a harmonic trap and let them rotate.

$$\begin{aligned} \mathcal{H} &= \frac{[\mathbf{p} - \mathbf{A}(\mathbf{x})]^2}{2m} - \lambda[\mathbf{p} - \mathbf{A}(\mathbf{x})] \cdot \sigma + A_0(\mathbf{x}) \\ A_0(\mathbf{x}) &= V(\mathbf{x}) - (m/2)(\omega \times \mathbf{x})^2 - \mu \\ \mathbf{A}(\mathbf{x}) &= m\omega \times \mathbf{x} \end{aligned}$$

- Integrate out the spin degree of freedom and at $O(\hbar)$ level: the semiclassical EOM **(Niu 1998-)**

$$\begin{aligned} \sqrt{G_c}\dot{\mathbf{x}} &= \nabla_{\mathbf{k}}\varepsilon_c + c\hbar \mathbf{E} \times \Omega + c\hbar(\Omega \cdot \nabla_{\mathbf{k}}\varepsilon_c)\mathbf{B}, \\ \sqrt{G_c}\dot{\mathbf{k}} &= \mathbf{E} + \nabla_{\mathbf{k}}\varepsilon_c \times \mathbf{B} + c\hbar(\mathbf{E} \cdot \mathbf{B})\Omega \end{aligned}$$

where $\mathbf{k} = \mathbf{p} - \mathbf{A}$ is the kinetic momentum, $\sqrt{G_c} = 1 + c\hbar\mathbf{B} \cdot \Omega$, $\mathbf{E} = -\nabla V(\mathbf{x})$— effective E-field, $\mathbf{B} = 2m\omega$—effective B-field, Ω—Berry curvature. $c = \pm$ for right- or left-hand.

Chiral anomaly

- The kinetic equation reads **(Son and Yamamoto 2012, Stephanov and Yin 2012, Gao,Wang,Pu,Chen,Wang 2012)**

$$\partial_t f_c + \dot{\mathbf{x}} \cdot \nabla_{\mathbf{x}} f_c + \dot{\mathbf{k}} \cdot \nabla_{\mathbf{k}} f_c = I[f_c]$$

- Direct calculation gives the $U(1)$ chiral anomaly in current of chirality c:

$$\partial_t n_c + \nabla_{\mathbf{x}} \cdot \mathbf{j}_c = c(\mathbf{E} \cdot \mathbf{B}) \int \frac{d^3\mathbf{k}}{(2\pi)^3} f_c \nabla_{\mathbf{k}} \cdot \Omega = c f_c(\mathbf{k}_0) \frac{W}{4\pi^2} \mathbf{E} \cdot \mathbf{B}$$

where W is the winding number of the Berry curvature.

- Write down the current $\mathbf{j}_c$ explicitly:

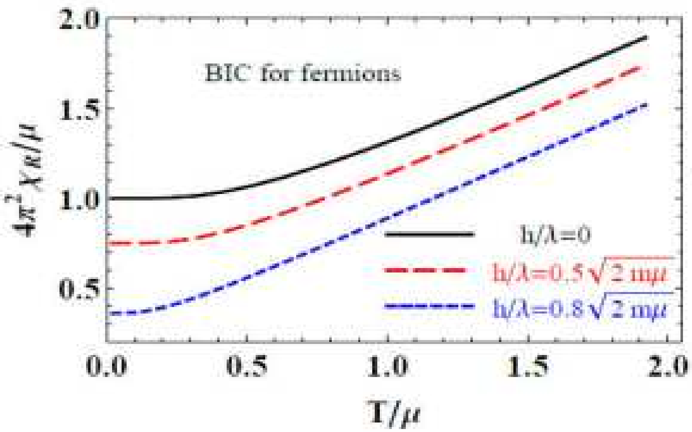
$$\begin{aligned} \mathbf{j}_c &= \int \frac{d^3\mathbf{k}}{(2\pi)^3} f_c \nabla_{\mathbf{k}} \varepsilon_c + c\mathbf{E} \times \int \frac{d^3\mathbf{k}}{(2\pi)^3} \Omega f_c \\ &\quad + c\mathbf{B} \int \frac{d^3\mathbf{k}}{(2\pi)^3} (\Omega \cdot \nabla_{\mathbf{k}} \varepsilon_c) f_c. \end{aligned}$$

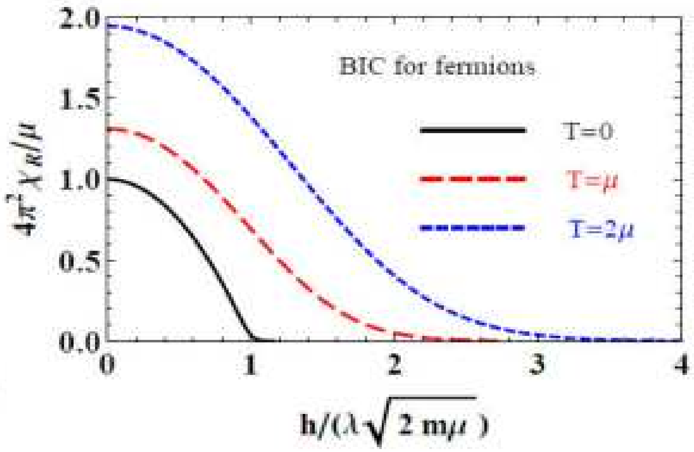
- The third term is $\mathbf{B}$-induced currents:

$$\mathbf{j}_c^{\mathbf{B}-\mathrm{ind}} = \chi_c \mathbf{B}, \quad \chi_c = c \int \frac{d^3\mathbf{k}}{(2\pi)^3} (\Omega \cdot \nabla_{\mathbf{k}} \varepsilon_c) f_c$$

Chiral magnetic/separation effects

- The $\mathbf{B}$-induced conductivity χ_c for Fermi gas **(XGH, Sci.Rep. 6, 20601 (2016))**





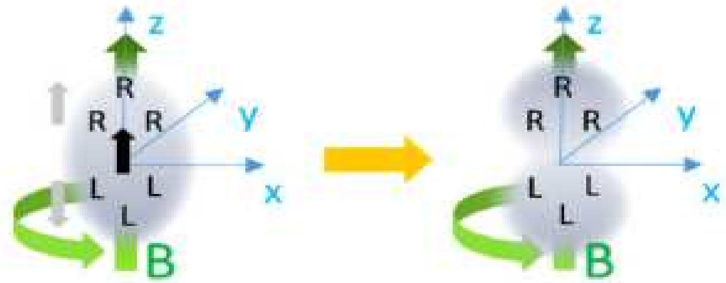
- *If there is parity-odd domains* in the Fermi gases $\Rightarrow$ $\mu_R = \mu + \mu_A, \mu_L = \mu - \mu_A \Rightarrow$

$$\mathbf{j}_V^{\mathbf{B}-\mathrm{ind}} \equiv \mathbf{j}_R^{\mathbf{B}-\mathrm{ind}} + \mathbf{j}_L^{\mathbf{B}-\mathrm{ind}} = \frac{\mu_A}{2\pi^2}\mathbf{B},$$
$$\mathbf{j}_A^{\mathbf{B}-\mathrm{ind}} \equiv \mathbf{j}_R^{\mathbf{B}-\mathrm{ind}} - \mathbf{j}_L^{\mathbf{B}-\mathrm{ind}} = \frac{\mu}{2\pi^2}\mathbf{B}$$

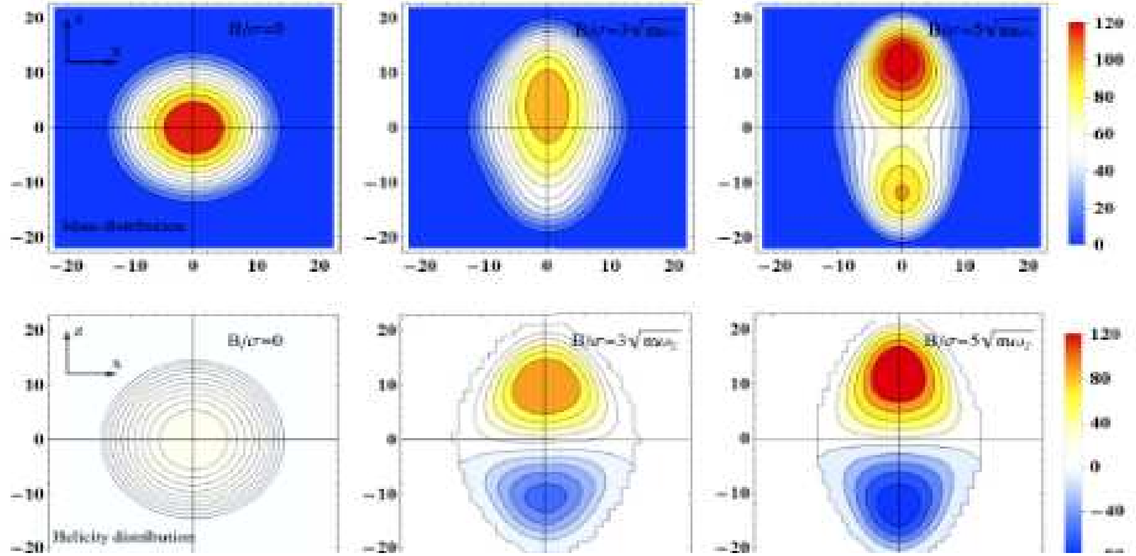
- These are exactly the chiral magnetic/speration effects!
- Question: how can produce parity-odd domins in Fermi gases?

Chiral dipole and mass quadrupole

- Very like what happen in QGP, the CMW exists in SOC atomic gases, which transport chirality and mass **(XGH, Sci.Rep. 6, 20601 (2016))**

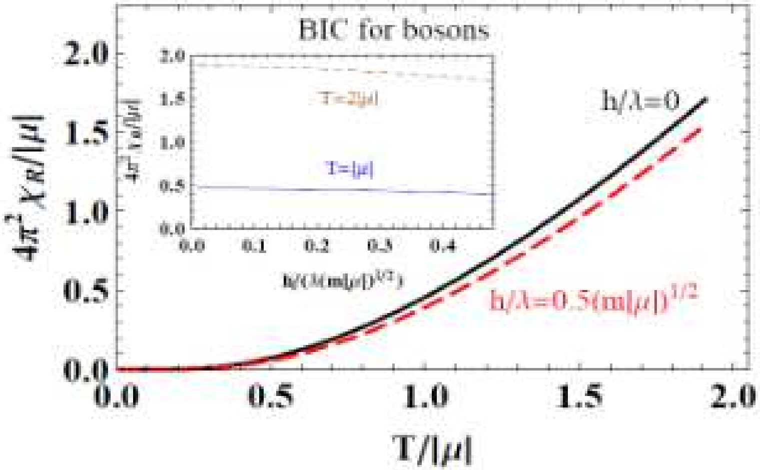


- Unlike in QGP, the presence of trap will finally stop these transport currents and system reaches a equilibrium configuration where appear a mass quadrupole and chiral dipole. The mass quadrupole may be tested by light absorption images technique.

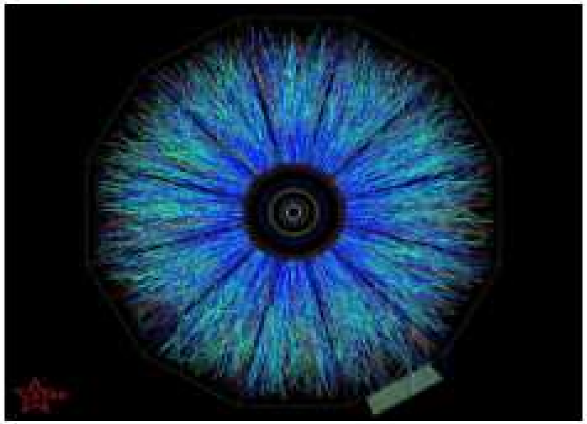


Link the hottest to the coldest

- The similar thing happens also in Bose gases, e.g., the BIC



- The CME/CSE initiated in the study of the hottest matter, the QGP, can possibly be realized in the coldest matter, the cold atoms.



Artist's conception of a quantum atomic gas undergoing laser cooling in an ultracold refrigerator (Photo: NASA)

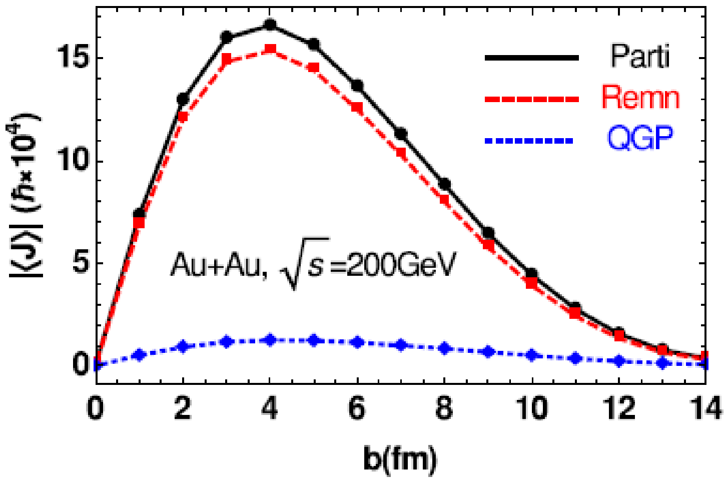
Summary

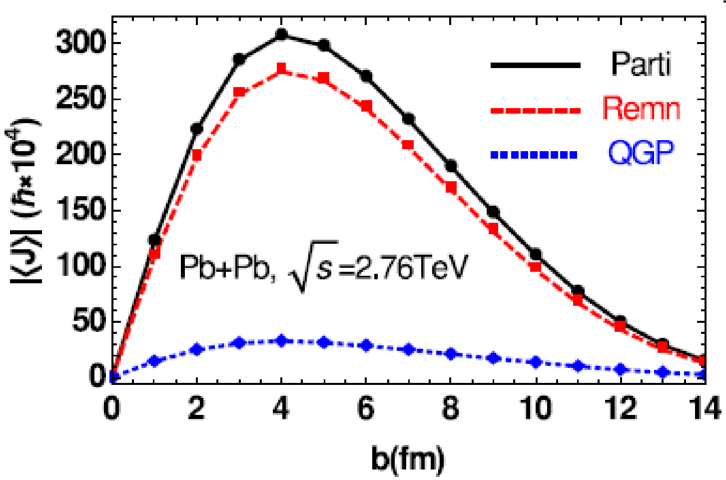
- Noncentral heavy-ion collisions generate flow shear and vorticity
- The vorticity increases with centrality but decreases with collision energy
- The vorticity orientation suffers from strong event-by-event fluctuation
- The vorticity can induce a new collective mode via CVE, the chiral vortical wave
- CVW can induce Lambda and antiLambda v_2 splitting
- Rotating cold atoms with Weyl spin-orbit coupling may simulate the CME/CSE

Thank you very much for your attention

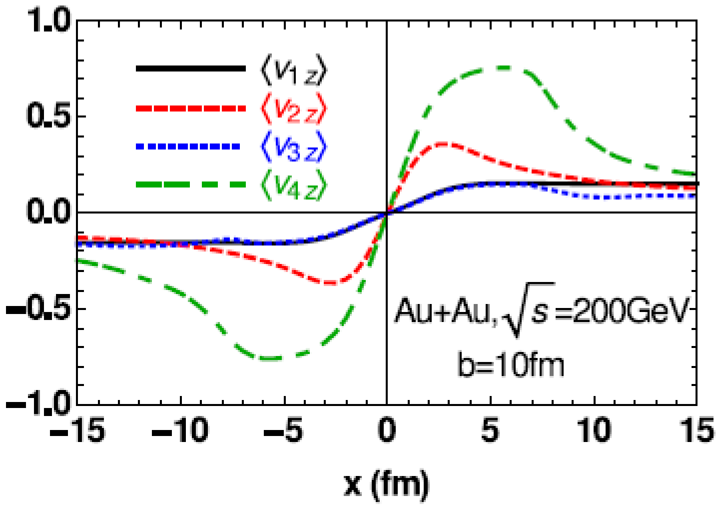
Backup

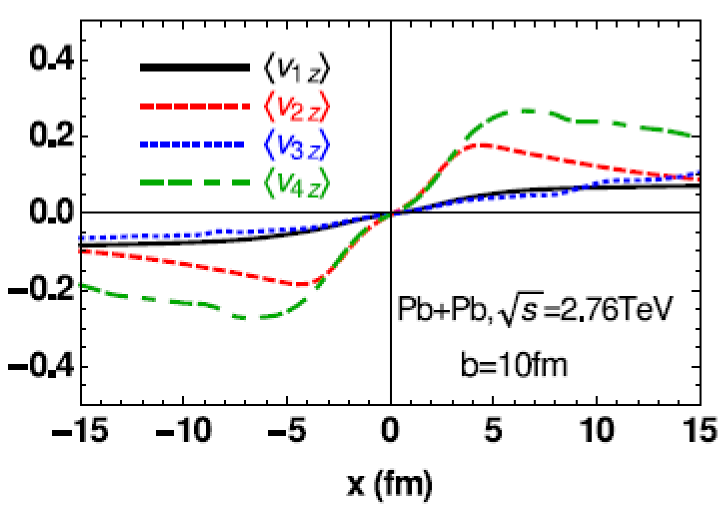
Angular momentum in overlapping region





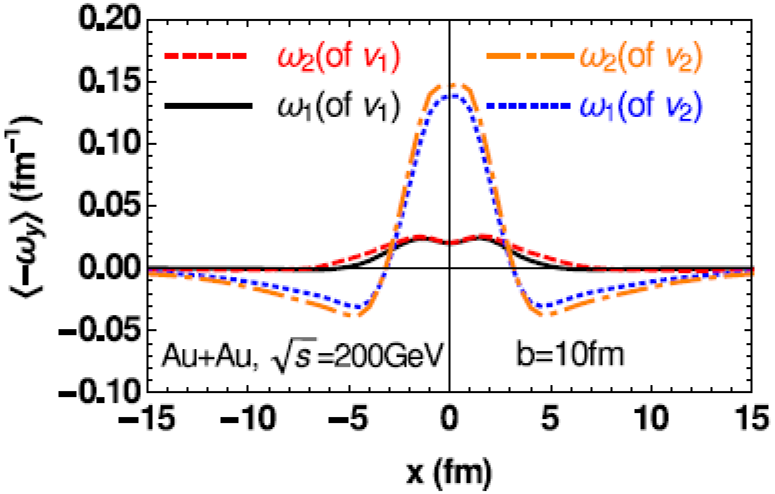
Velocity profile

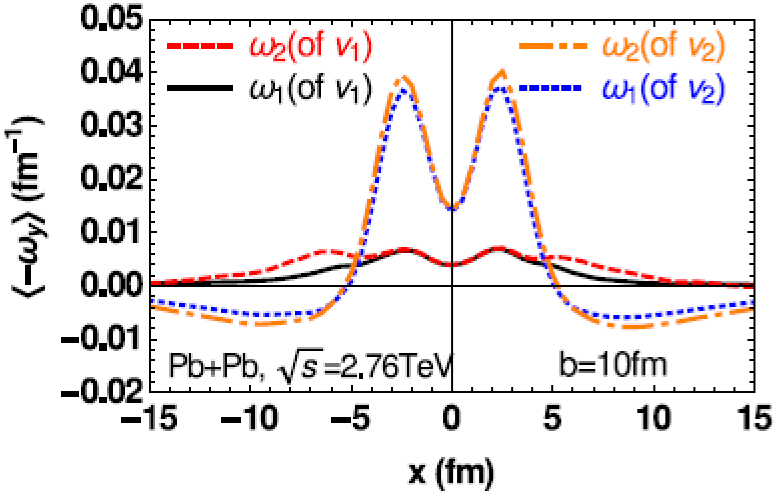




Backup

Spacial distribution





Rapidity dependence