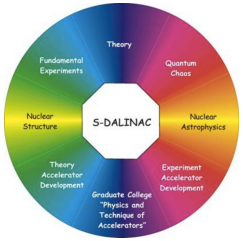
The Functional Renormalization Group Method and Delayed Magnetic Catalysis

Stefan Rechenberger
BNL, May 12, 2016



TECHNISCHE UNIVERSITÄT DARMSTADT

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Outline

- The Functional Renormalization Group Idea
- The Functional Renormalization Group Formalism
- The Functional Renormalization Group Applications
- Introduction External Magnetic Field
- The Functional Renormalization Group Analysis
- Summary

The Functional Renormalization Group Idea

Starting point is the QFT's generating functional:

$$Z[J] = \frac{1}{\mathcal{N}} \int_{p^2 \leq \Lambda^2} \mathcal{D}\varphi \exp\left\{ -S_\Lambda[\varphi] + \int \varphi \cdot J \right\} .$$

Alternatives are the Schwinger functional

$$\mathcal{W}[J] = \ln Z[J]$$

or its Legendre transform, the effective action

$$\Gamma[\phi] = \sup_J \left\{ \int \mathrm{d}^d x \, J(x) \, \phi(x) - \mathcal{W}[J] \right\} , \qquad \phi(x) = \langle \varphi(x) \rangle .$$

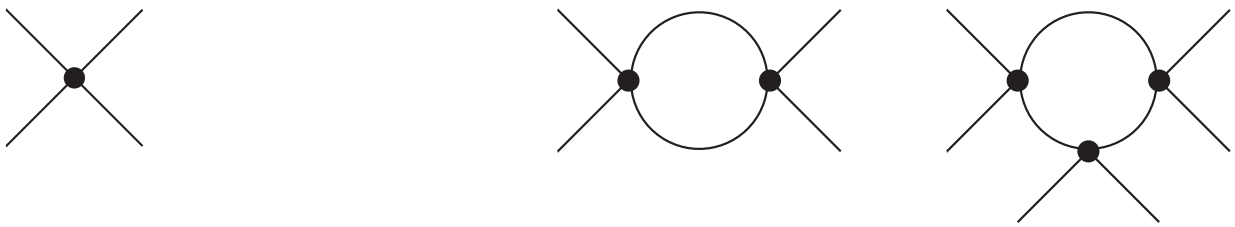
The Functional Renormalization Group Idea

Integrate out the "hard" modes:

$$Z[J] = \frac{1}{\mathcal{N}} \int_{p^2 \leq \Lambda'^2} \mathcal{D}\varphi \exp\left\{ -S_{\Lambda'}[\varphi] + \int \varphi \cdot J \right\}, \qquad \Lambda' < \Lambda\,.$$

Note that $S_{\Lambda'} \neq S_{\Lambda}$.

$$\text{e.g. } S_{\Lambda} \sim \int \lambda_4 \varphi^4 \quad \Rightarrow \quad S_{\Lambda'} \sim \int (\lambda_4' \varphi^4 + \lambda_6' \varphi^6 + \ldots)\,.$$



The Functional Renormalization Group Formalism

Define a scale dependent generating functional

$$Z_k[J] = \int \mathcal{D}\varphi \exp\left[-S[\varphi] - \Delta S_k[\varphi] + \int \mathrm{d}^d x' \, J(x') \, \varphi(x')\right]$$

with a regulator insertion acting as a scale dependent mass

$$\Delta S_k[\varphi] = \frac{1}{2} \int \frac{\mathrm{d}^d p}{(2\pi)^d} \, \varphi(p) \, R_k(p^2) \, \varphi(-p)$$

depending on the regulator R_k.

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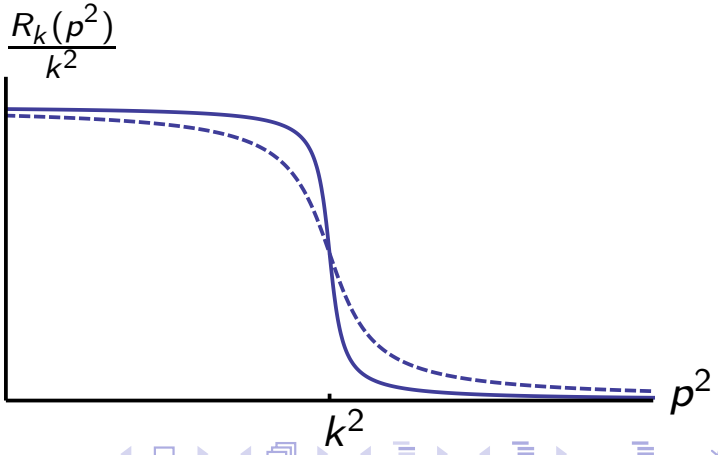
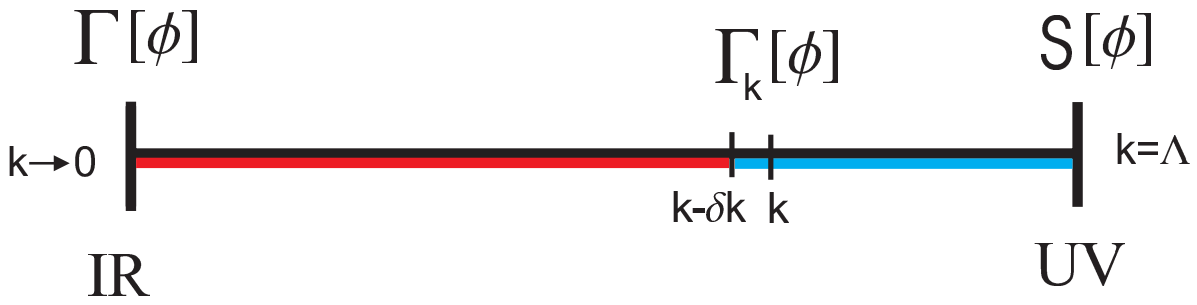


The Functional Renormalization Group Formalism

Define a scale dependent **effective average action**

$$\Gamma_k[\phi] = \sup_J \left\{ \int \mathrm{d}^d x \, J(x)\, \phi(x) - \mathcal{W}_k[J] \right\} - \Delta S_k[\phi] \,, \qquad \mathcal{W}_k = \ln Z_k \,.$$



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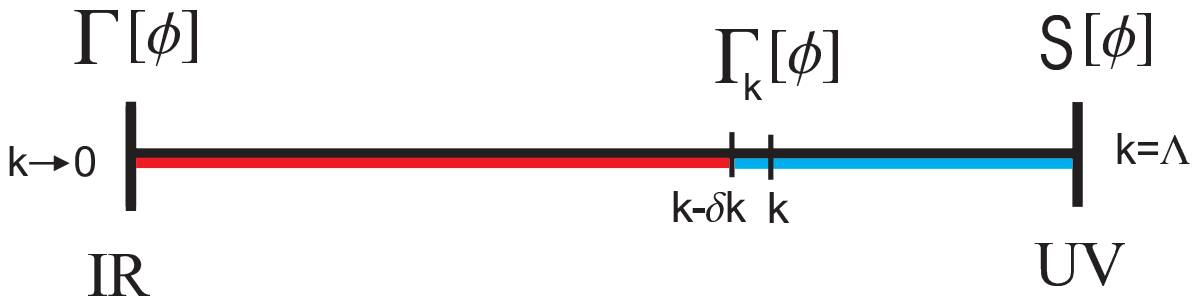

Next one can derive a differential equation for Γ_k a.k.a. ERGE / FRGE / Wetterich equation (C. Wetterich, Phys. Lett. B **301**, 90 (1993))

$$k\partial_k \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \frac{k\partial_k R_k}{\Gamma_k^{(2)} + R_k} \,.$$

The Functional Renormalization Group Formalism

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- FRGE is an exact equation
- Γ_k depends on R_k
  $\Gamma_{k\to 0}$ does not
- $\Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta\phi\,\delta\phi}$
- Tr $\to$ momentum integral
  and many more traces
- IR and UV finite

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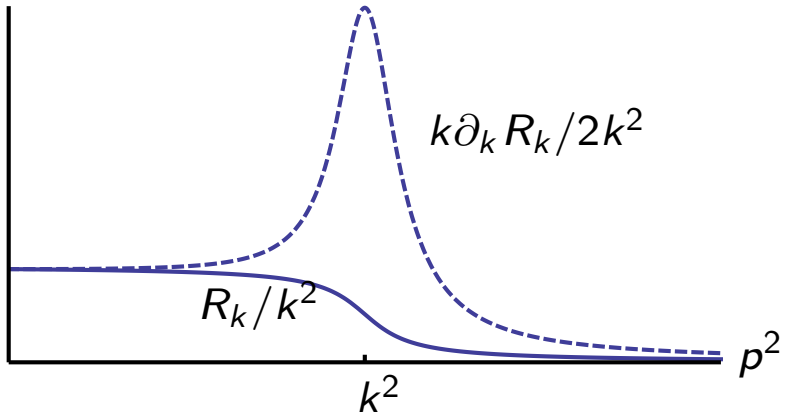

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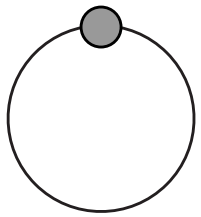

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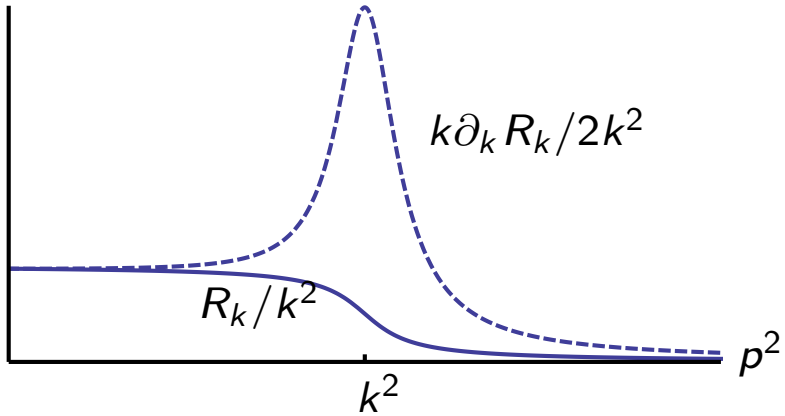

The Functional Renormalization Group
Applications

For practical applications we have to truncate Γ_k. Considering e.g. the $O(N)$ model one can use the vertex expansion

$$\Gamma_k[\phi] = \sum_n \frac{1}{n!} \int \mathrm{d}^d x_1 \ldots \mathrm{d}^d x_n \, \Gamma_k^{(n)} \, \phi^n$$

to get a coupled system of flow equations

$$k\partial_k \Gamma_k^{(2)}$$

$$k\partial_k \Gamma_k^{(4)}$$

$$\vdots$$

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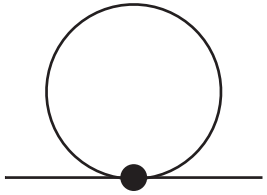

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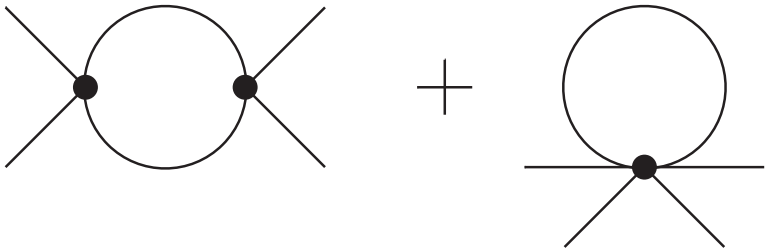

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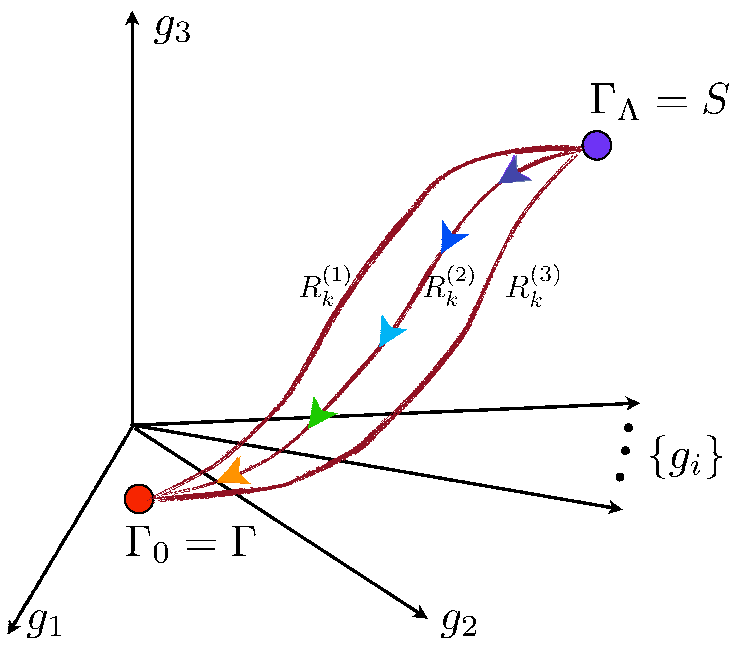
The Functional Renormalization Group
Applications

Illustration of flow in theory space



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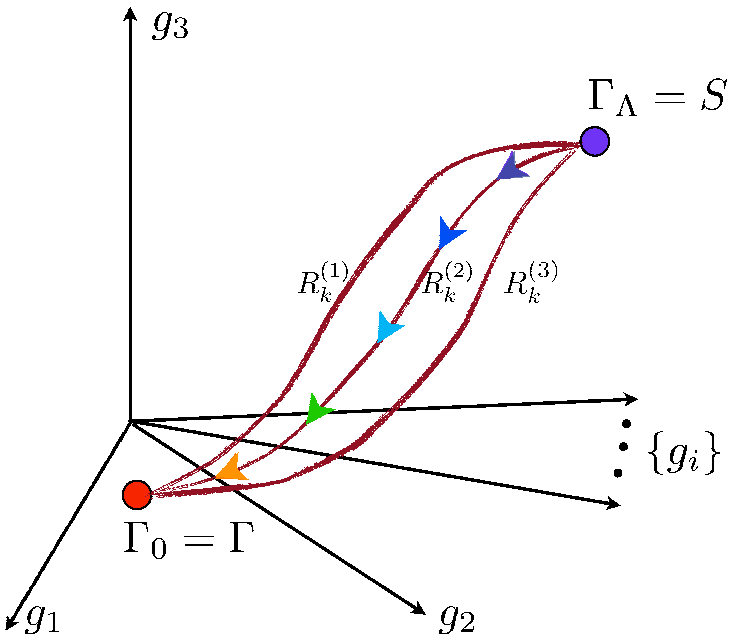

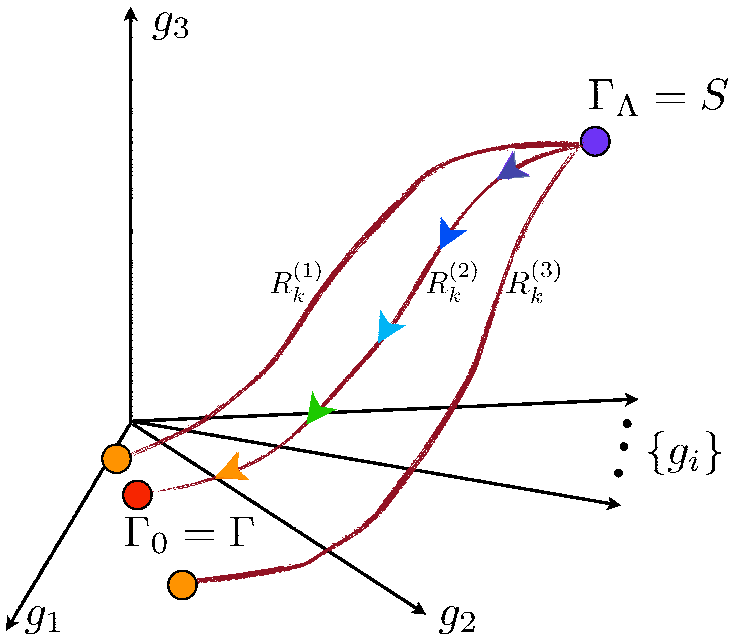

The Functional Renormalization Group
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There is a wide range of applications for this powerful non-perturbative tool:

- critical phenomena
- solid state physics
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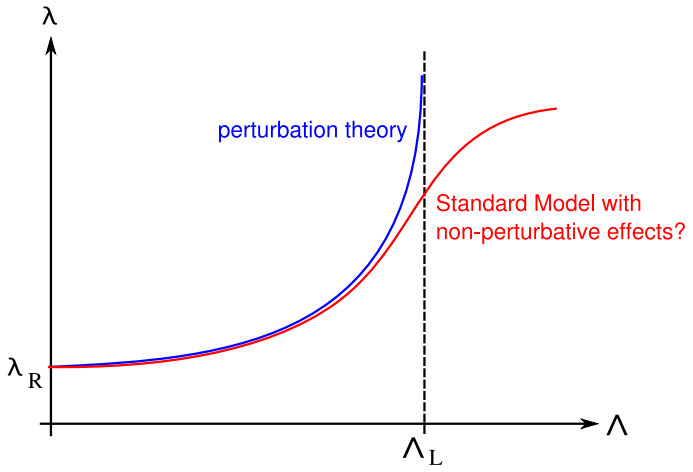

(H. Gies, S.R. and M. M. Scherer, Eur. Phys. J. C **66**, 403 (2010))
(H. Gies, S.R., M. M. Scherer and L. Zambelli, Eur. Phys. J. C **73**, 2652 (2013)))

The Functional Renormalization Group Applications

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- critical phenomena
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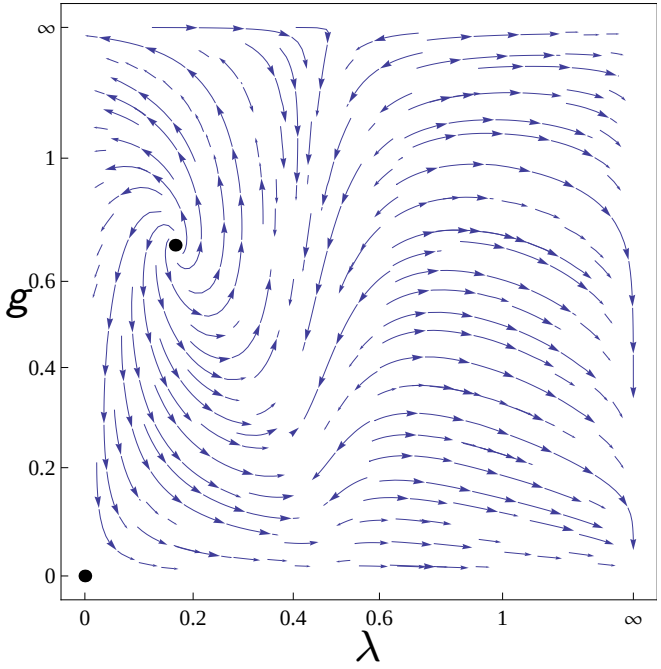
- triviality in Higgs sector
- Quantum Gravity



(Ph.D. thesis S.R. (2013))

The Functional Renormalization Group Applications

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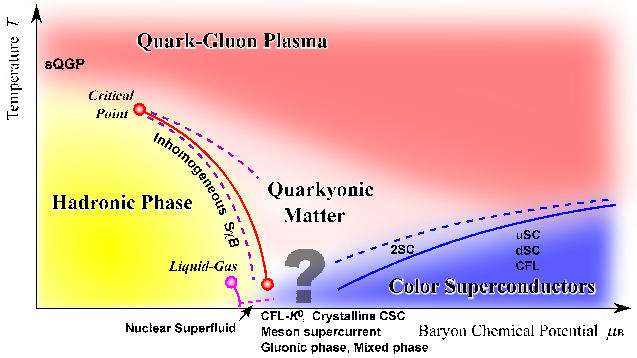
- QCD phase diagram

⋮



(K. Fukushima and T. Hatsuda, Rept. Prog. Phys. **74**, 014001 (2011))

QCD in an External Magnetic Field

Strong magnetic fields are relevant e.g.

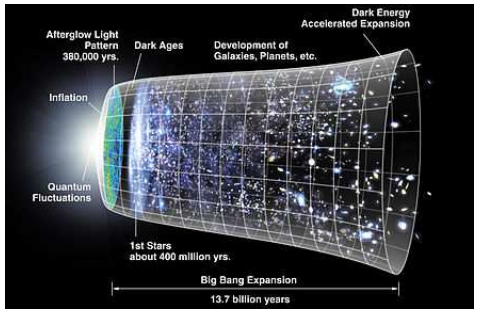
- in magnetars,

(R. C. Duncan and C. Thompson, Astrophys. J. **392**, L9 (1992))

(sci.esa.int)

- in the early universe,

(Giovannini and Shaposhnikov, Phys. Rev. D **57**, 2186 (1998))
(Kajantie et al, Nucl. Phys. B **544**, 357 (1999))
(De Simone et al, JCAP **1110**, 030 (2011))
. . .



(en.wikipedia.org)

QCD in an External Magnetic Field

Strong magnetic fields are relevant e.g.

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(Kharzeev et al, Nucl. Phys. A **803**, 227 (2008))
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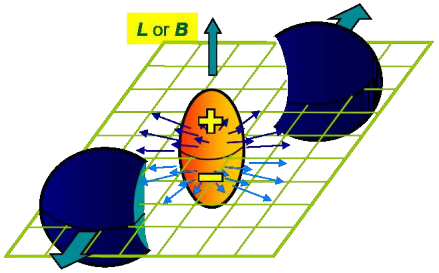

(STAR collaboration)

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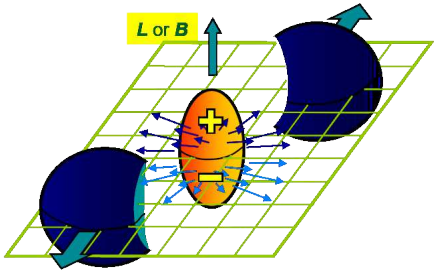

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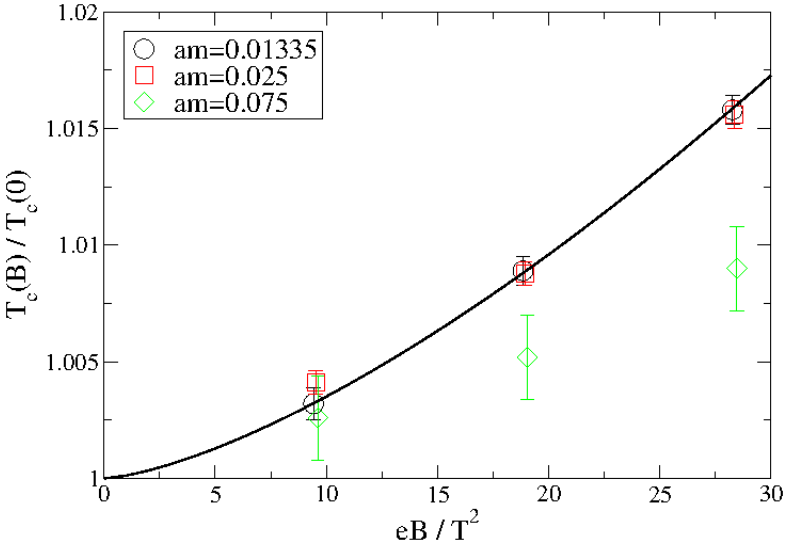
How does the magnetic field influence strongly interacting matter?

$T_\chi(B)$ on the Lattice

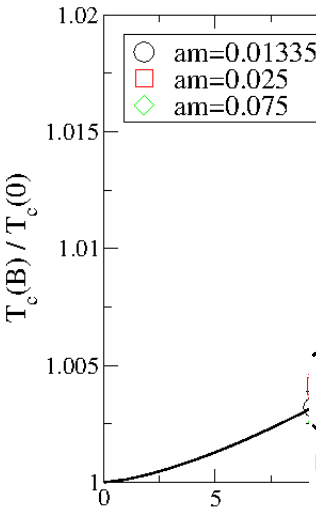
magnetic catalysis



(D'Elia et al, Phys. Rev. D **82**, 051501 (2010))

$T_\chi(B)$ on the Lattice

magnetic catalysis / anti-catalysis



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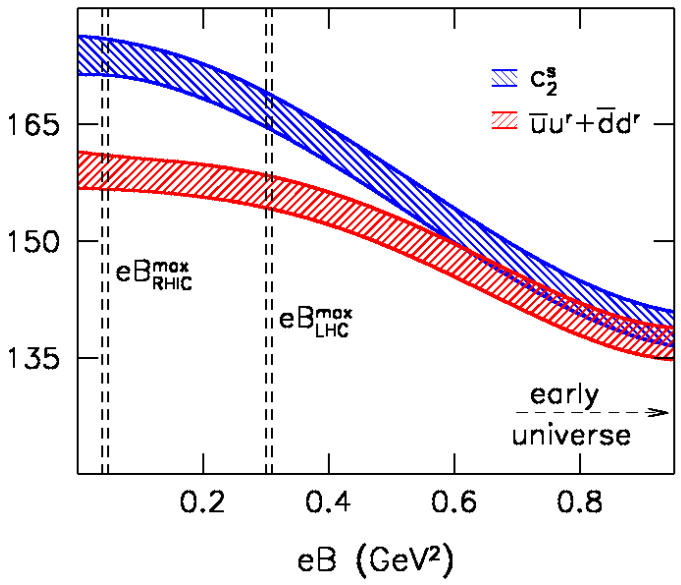
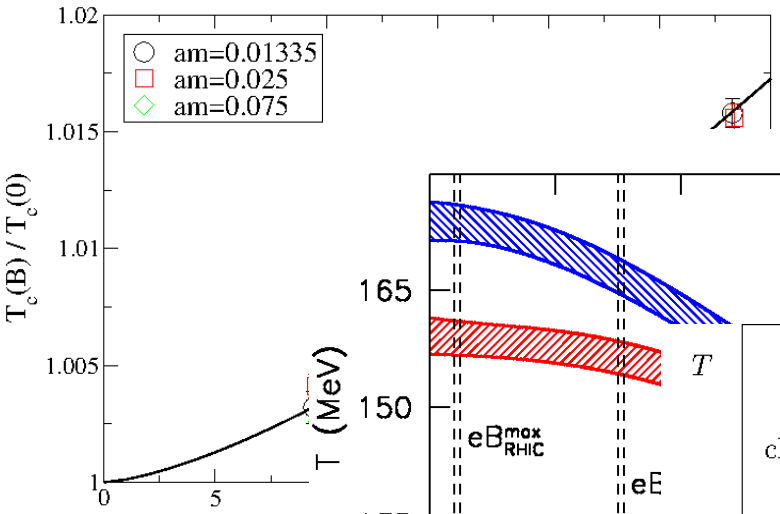


(Bali et al, JHEP **1202**, 044 (2012))

$T_\chi(B)$ on the Lattice

magnetic catalysis / anti-catalysis / delayed catalysis



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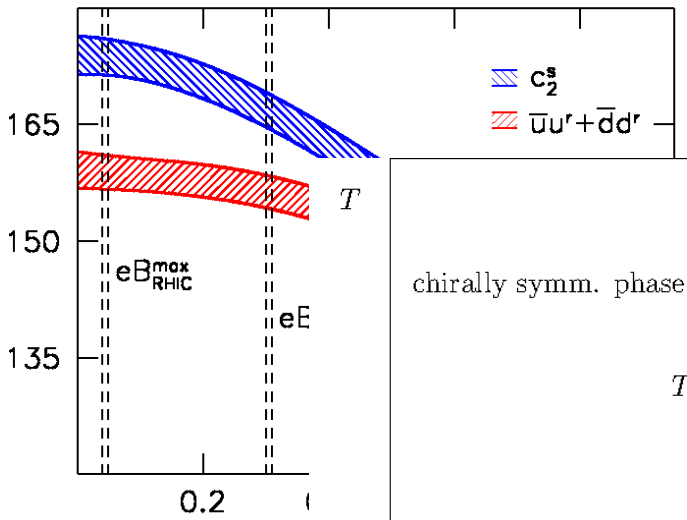

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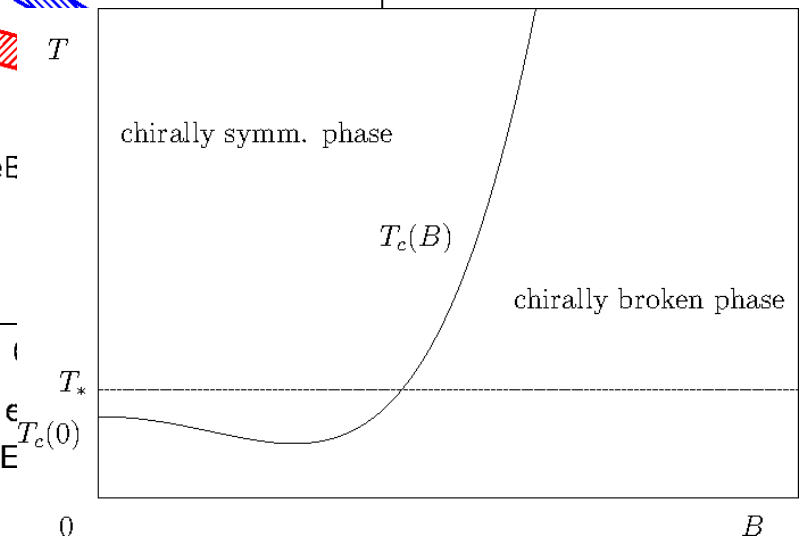
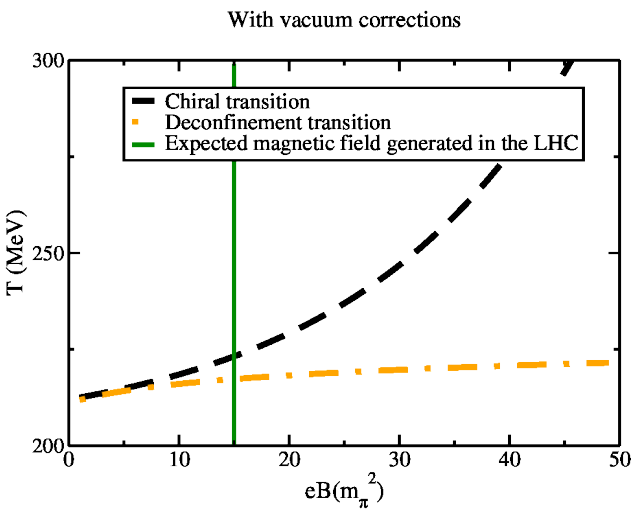
conjecture



(Ilgenfritz et al, Phys. Rev. D **89**, 054512 (2014))

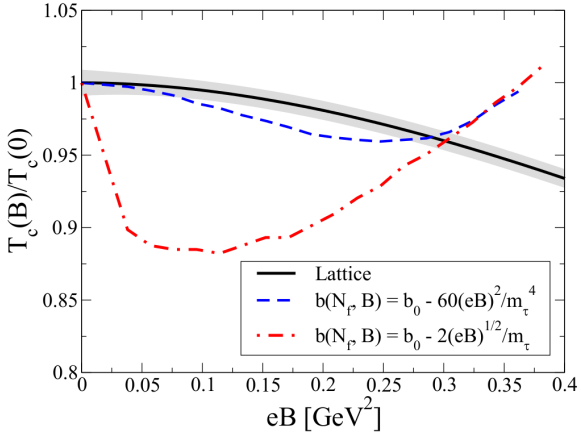
$T_\chi(B)$ in Model Calculations

magnetic catalysis



(Mizher et al, Phys. Rev. D **82**, 105016 (2010))
(Gatto and Ruggieri, Phys. Rev. D **82**, 054027 (2010))
(V. Skokov, Phys. Rev. D **85**, 034026 (2012))
(Fukushima and Pawlowski, Phys. Rev. D **86**, 076013 (2012))
(Kamikado and Kanazawa, JHEP **1403**, 009 (2014))
. . .

magnetic anti-catalysis



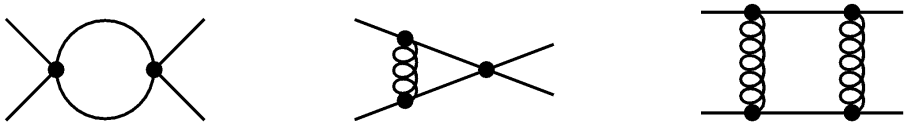
(Fraga et al, Phys. Lett. B **731**, 154 (2014))
(Ferreira et al, Phys. Rev. D **89**, 116011 (2014))
(Ferrer et al, arXiv:1407.3503)
(Farias et al, Phys. Rev. C **90**, 2, 025203 (2014))
. . .

The Functional Renormalization Group Analysis

$$\Gamma_k = \int \mathrm{d}^4x \left\{ \frac{1}{4} F^i_{\mu\nu} F^i_{\mu\nu} + \bar{\psi}\, \imath \not{D} \psi - \frac{1}{2\xi} A^i_\mu \partial_\mu \partial_\nu A^i_\nu \right.$$
$$\left. + \frac{\bar{\lambda}_\sigma}{2} \left[\left(\bar{\psi}\psi\right)^2 - \left(\bar{\psi}\tau_\chi \gamma_5 \psi\right)^2 \right] \right\}$$

- exponential regulator
- Feynman gauge ($\xi = 1$)
- $N_\mathrm{f} = 2$ flavours and $N_\mathrm{c} = 3$ colors
- $\lambda_\sigma^{\mathrm{UV}} = 0 \quad \Rightarrow \quad$ only a single input parameter (gauge coupling at large momentum scale)
- criterion for χSB: $k^2 \bar{\lambda}_\sigma = \lambda_\sigma \to \infty$ for $k \to k_{\mathrm{crit}}$

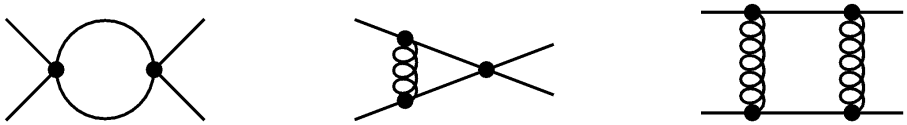
The Functional Renormalization Group Analysis



$$\partial_t \lambda_\sigma = 2\lambda_\sigma - a(\tfrac{T}{k}, \tfrac{B}{k^2})\, \lambda_\sigma^2 - b(\tfrac{T}{k}, \tfrac{B}{k^2})\, g^2 \lambda_\sigma - c(\tfrac{T}{k}, \tfrac{B}{k^2})\, g^4$$

$$t = \ln(k/\Lambda)\,, \qquad \partial_t = k\partial_k$$

The Functional Renormalization Group Analysis



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(H. Gies, J. Jaeckel and C. Wetterich, Phys. Rev. D **69**, 105008 (2004))
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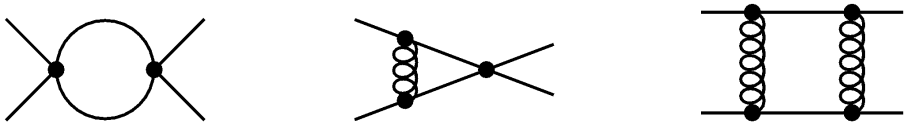
The Functional Renormalization Group Analysis



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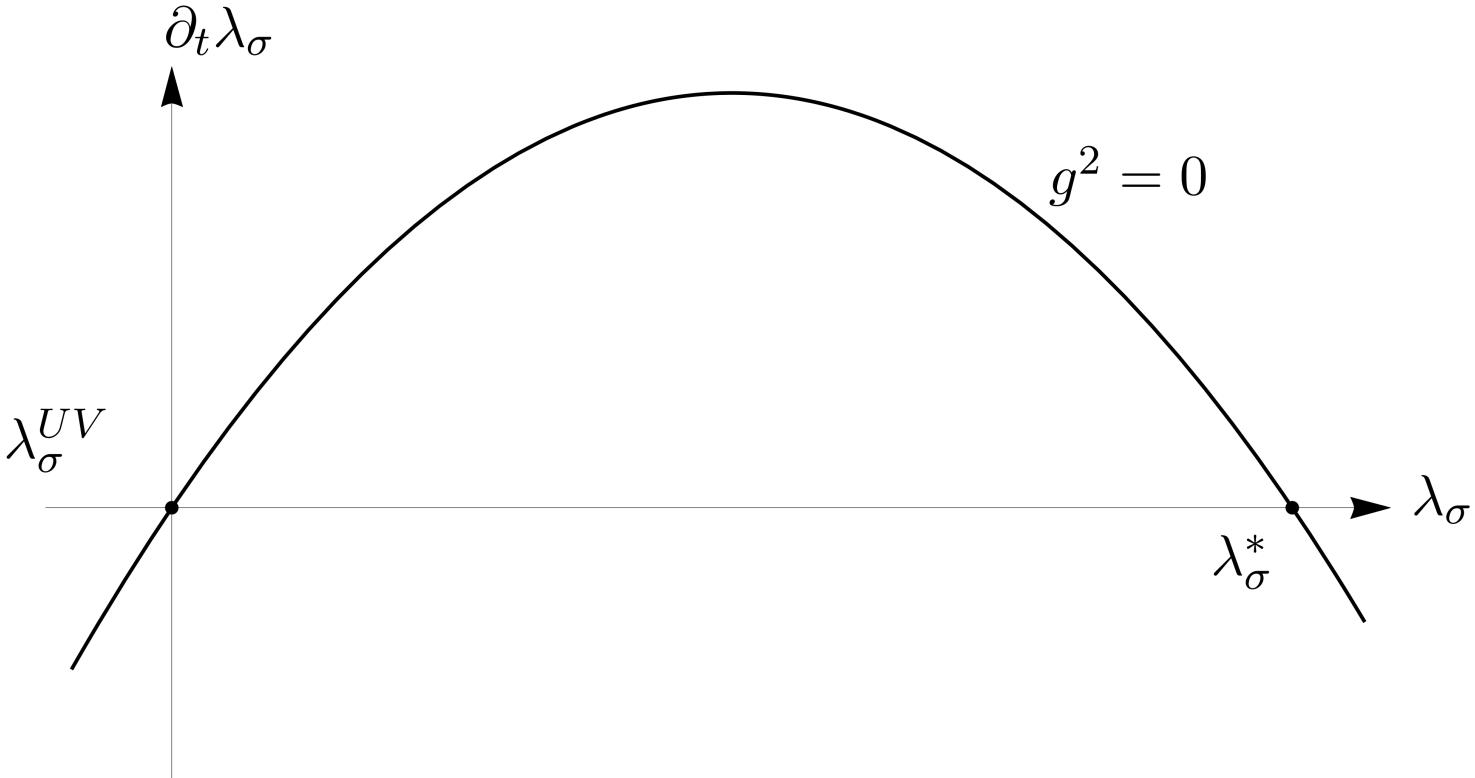

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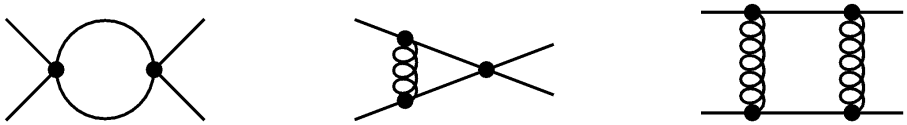

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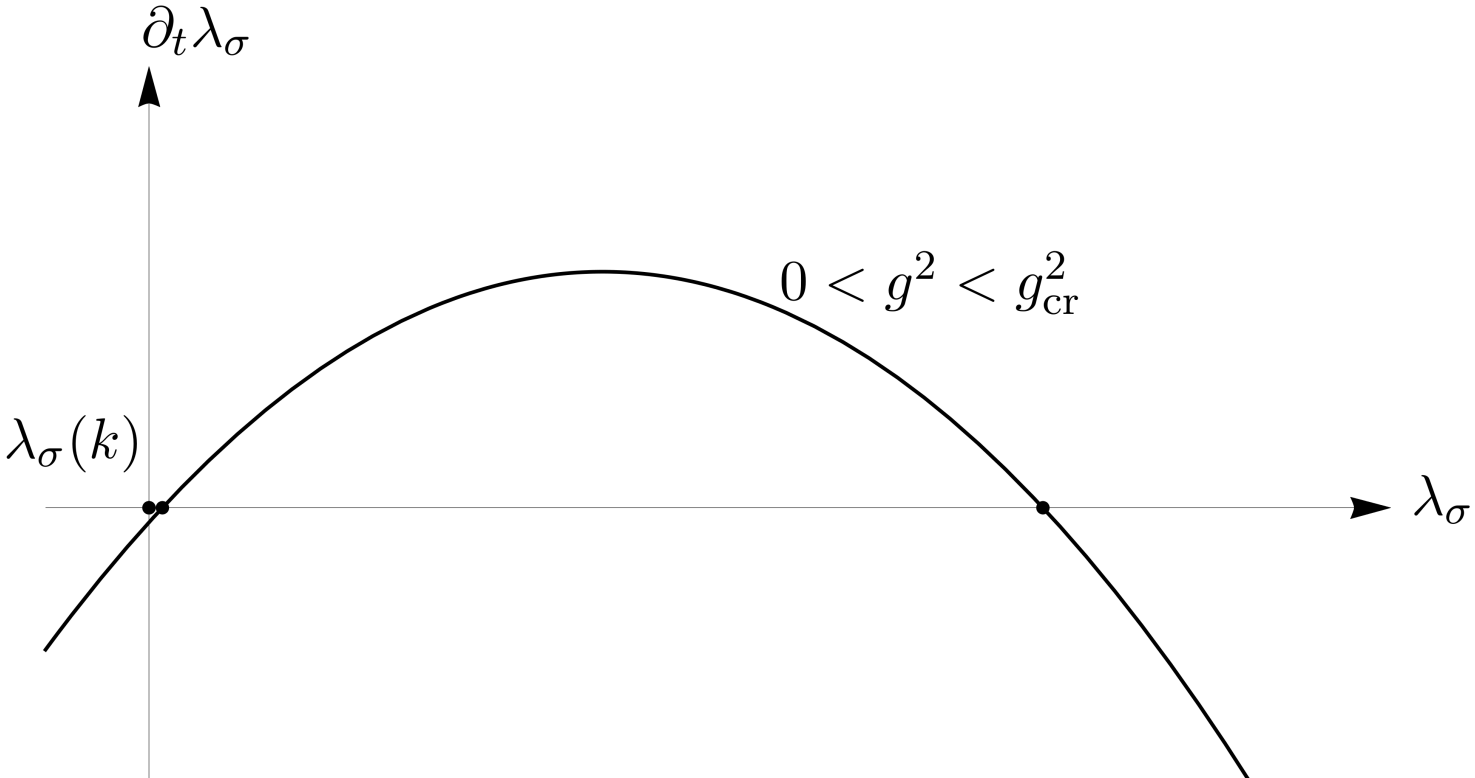
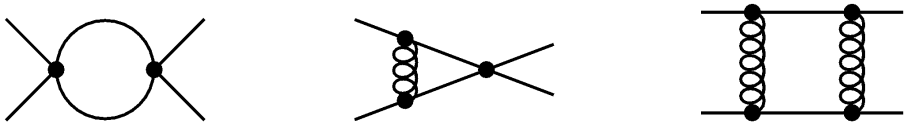


The Functional Renormalization Group Analysis



$$\partial_t \lambda_\sigma = 2\lambda_\sigma - a(0,0)\,\lambda_\sigma^2 - b(0,0)\,g^2\lambda_\sigma - c(0,0)\,g^4$$

(H. Gies, J. Jaeckel and C. Wetterich, Phys. Rev. D **69**, 105008 (2004))
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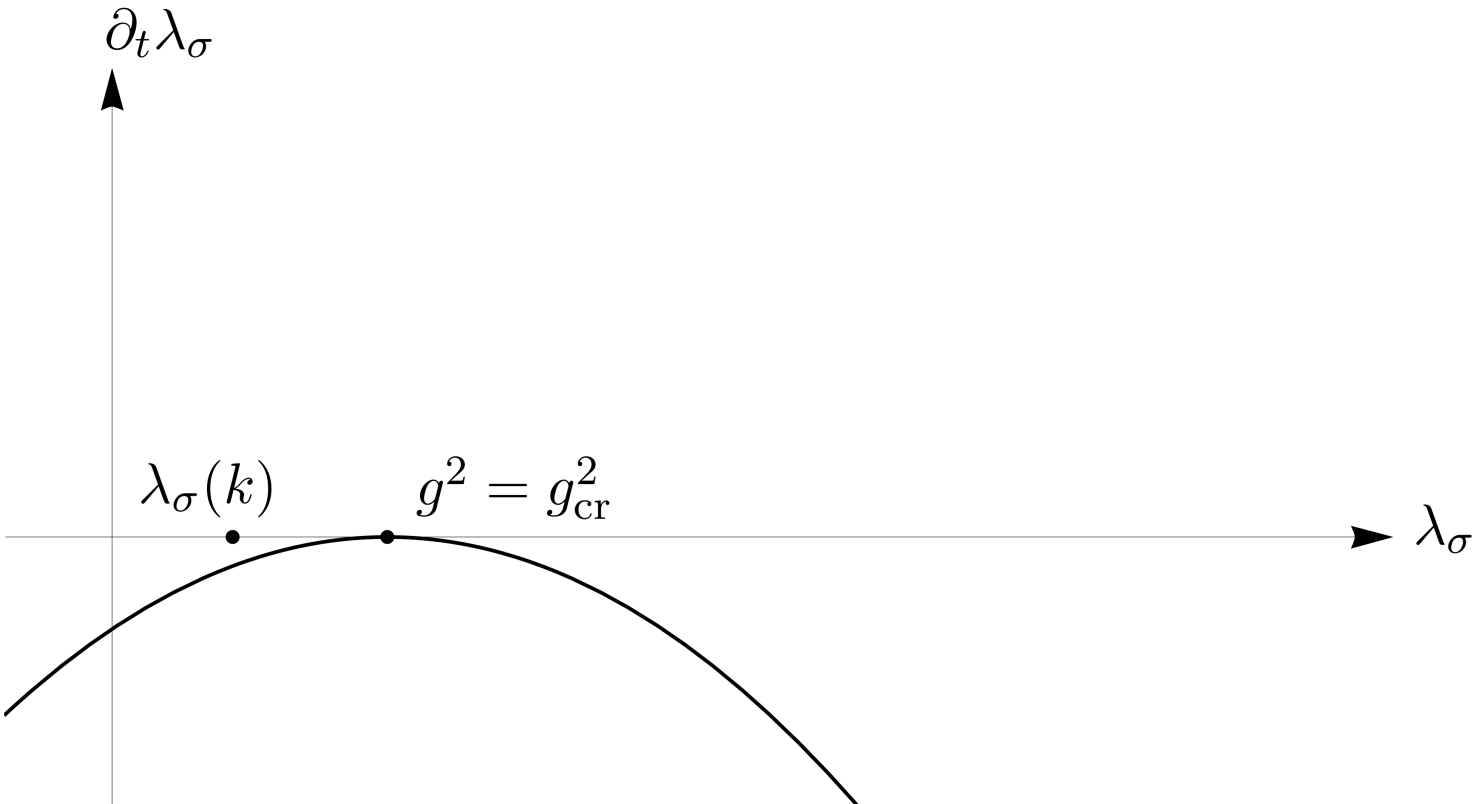
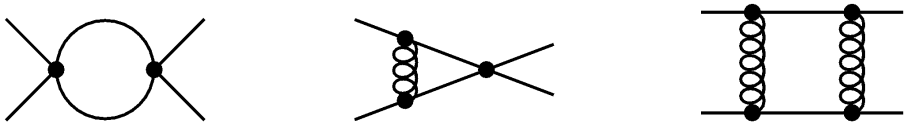


The Functional Renormalization Group Analysis



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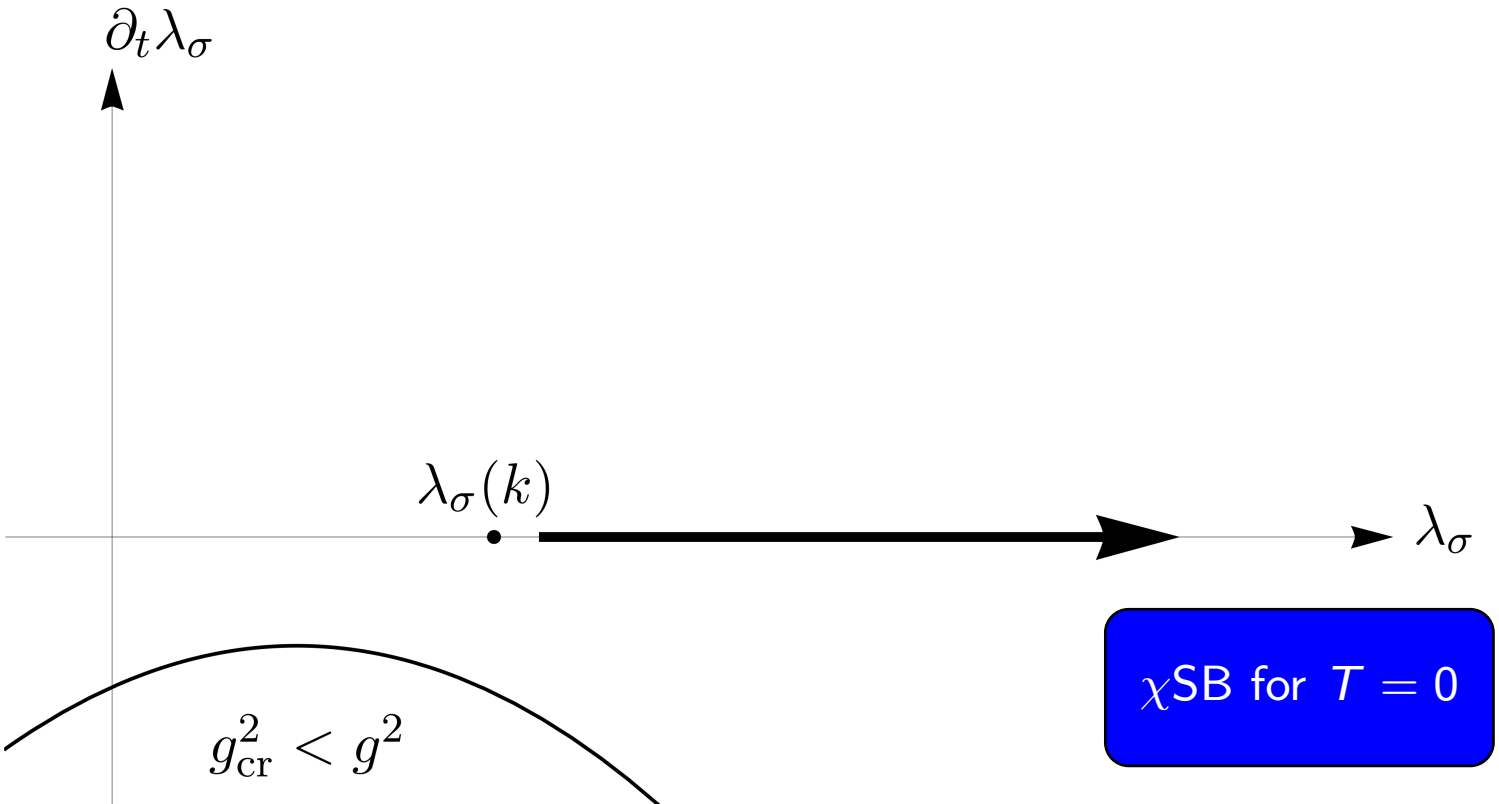
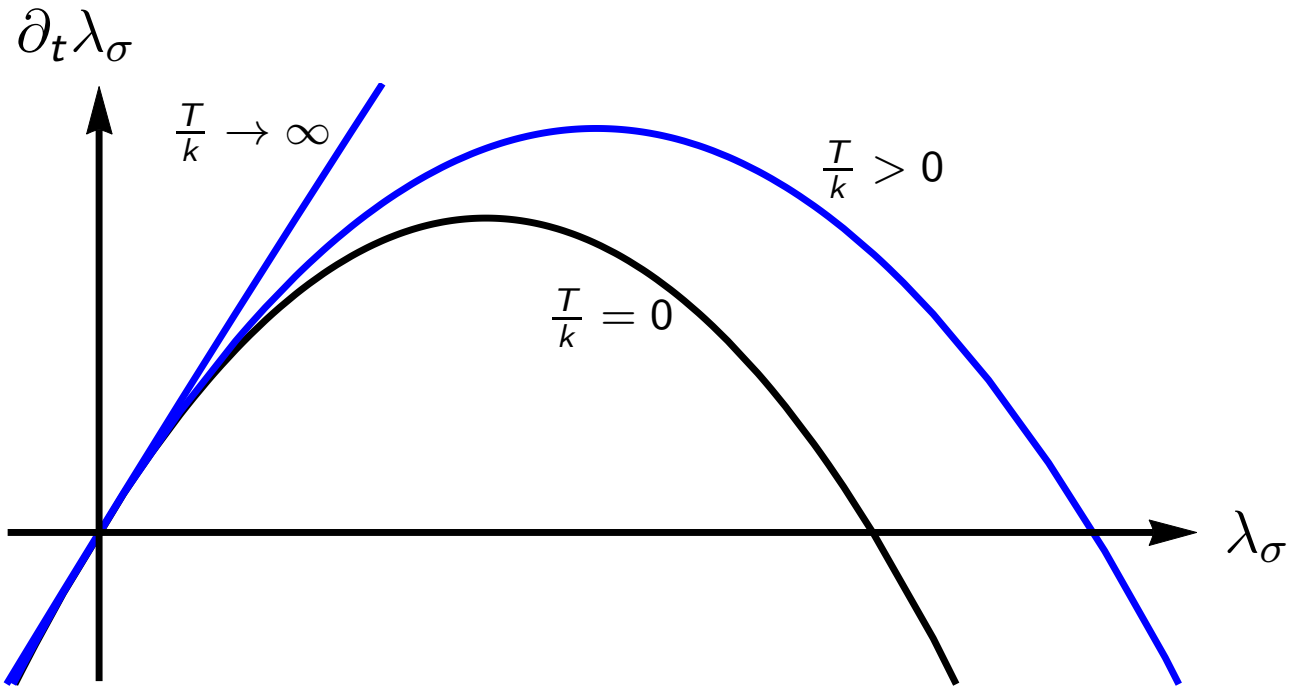


The Functional Renormalization Group Analysis

$$\partial_t \lambda_\sigma = 2\lambda_\sigma - a(\tfrac{T}{k}, \tfrac{B}{k^2})\,\lambda_\sigma^2 - b(\tfrac{T}{k}, \tfrac{B}{k^2})\,g^2\lambda_\sigma - c(\tfrac{T}{k}, \tfrac{B}{k^2})\,g^4$$

The Functional Renormalization Group Analysis

$$\partial_t \lambda_\sigma = 2\lambda_\sigma - a(\tfrac{T}{k}, 0)\, \lambda_\sigma^2$$

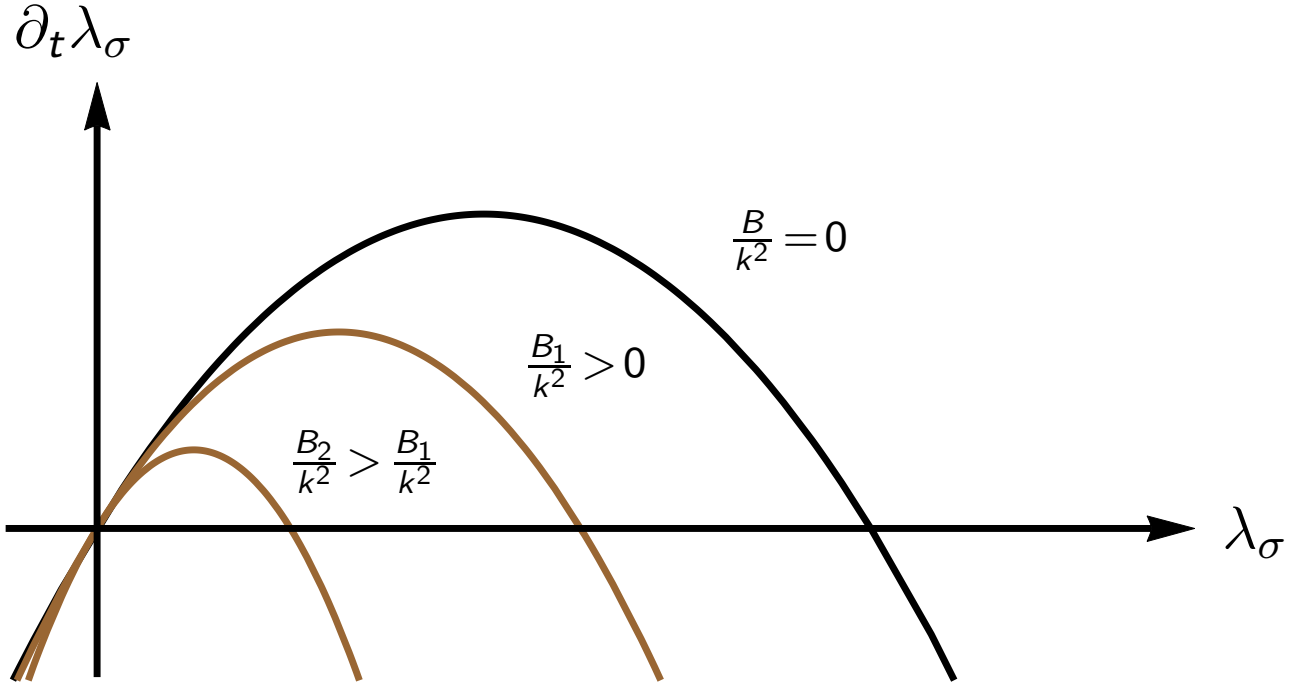


(J. Braun and H. Gies, Phys. Lett. B **645**, 53 (2007), JHEP **0606**, 024 (2006))

symmetry restoration

The Functional Renormalization Group Analysis

$$\partial_t \lambda_\sigma = 2\lambda_\sigma - a(0, \tfrac{B}{k^2})\, \lambda_\sigma^2$$



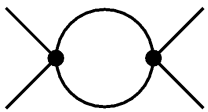
magnetic catalysis

(K. Fukushima and J. M. Pawlowski, Phys. Rev. D **86**, 076013 (2012))
(D. D. Scherer and H. Gies, Phys. Rev. B **85**, 195417 (2012))
(J. Braun, W. A. Mian and S.R., Phys. Lett. B **755**, 265 (2016))

The Functional Renormalization Group Analysis

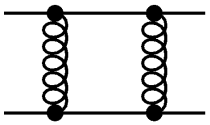
$$\partial_t \lambda_\sigma = 2\lambda_\sigma - a(\tfrac{T}{k}, \tfrac{B}{k^2})\,\lambda_\sigma^2 - b(\tfrac{T}{k}, \tfrac{B}{k^2})\,g^2\lambda_\sigma - c(\tfrac{T}{k}, \tfrac{B}{k^2})\,g^4$$

Note that:



increases with B

magnetic catalysis



decreases with B

anti-catalysis

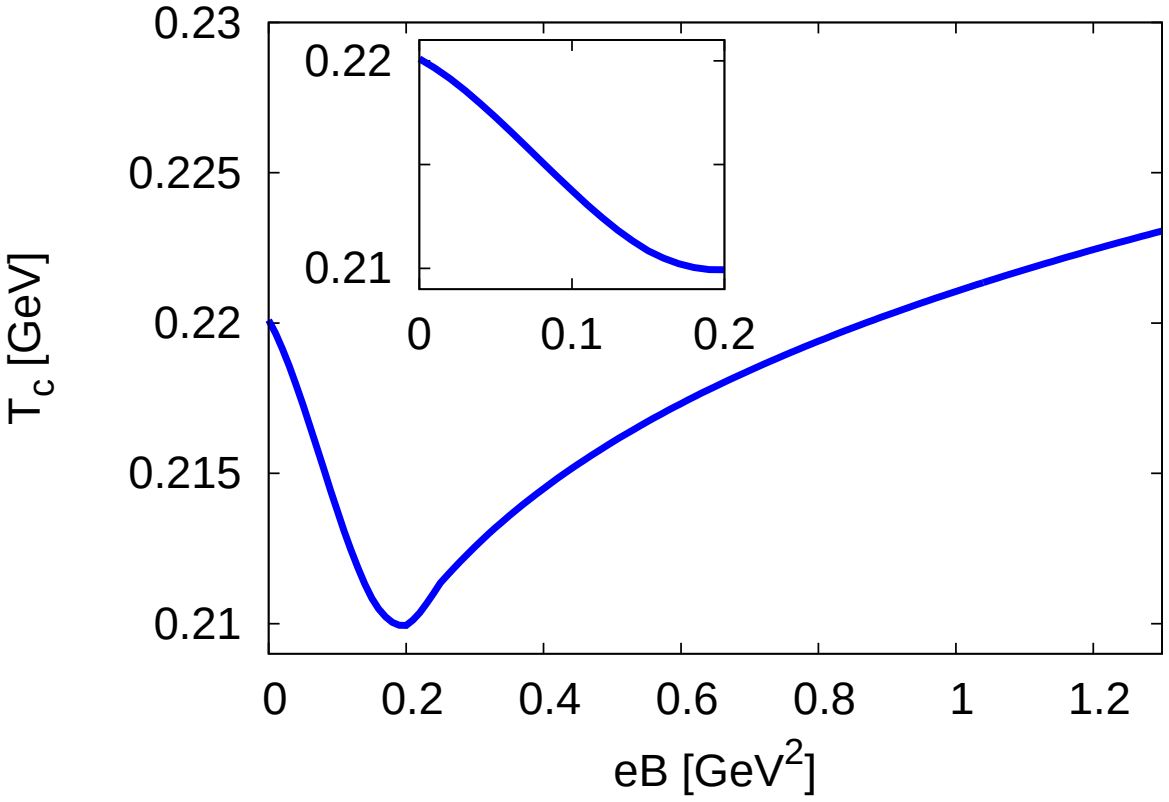
Finally: integrate the coupled system of flow equations

$$k\partial_k \lambda_\sigma = \beta_\lambda(\lambda_\sigma, g; T, B)$$
$$k\partial_k g = \beta_g(\lambda_\sigma, g; T, B)$$

(J. Braun and H. Gies,JHEP **0606**, 024 (2006))

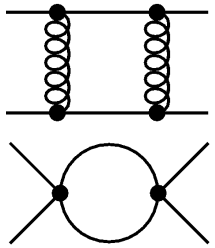
and see whether or not λ_σ diverges.

Delayed Magnetic Catalysis

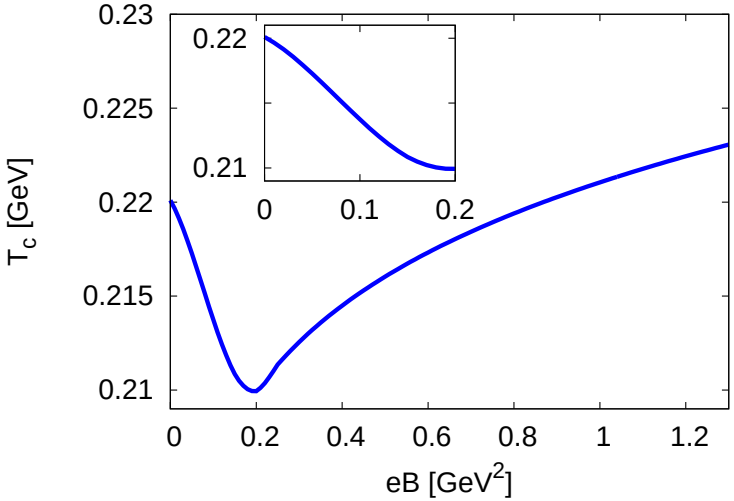


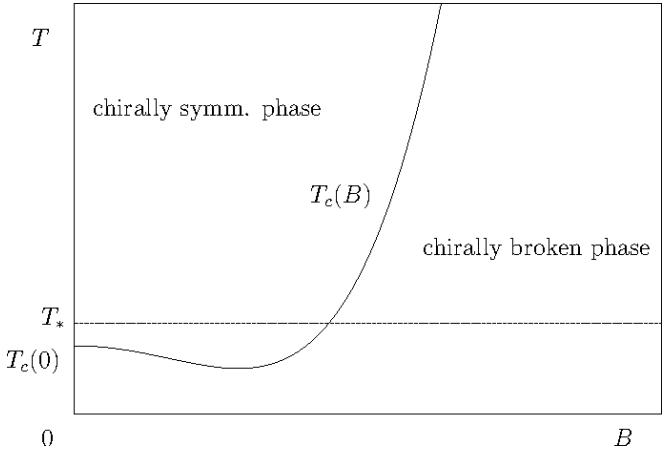
(J. Braun, W. A. Mian and S.R., Phys. Lett. B **755**, 265 (2016))

Conclusion and Outlook



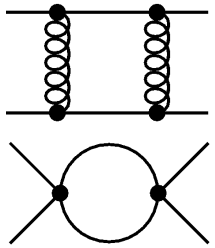
- gluodynamically induced anti-catalysis
- fermionically induced catalysis
- only a single input parameter: strong coupling at UV scale
- anti-catalysis at weak magnetic fields
  prediction of catalysis at strong magnetic fields
- **delayed magnetic catalysis**





(Ilgenfritz et al, Phys. Rev. D **89**, 054512 (2014))

Conclusion and Outlook



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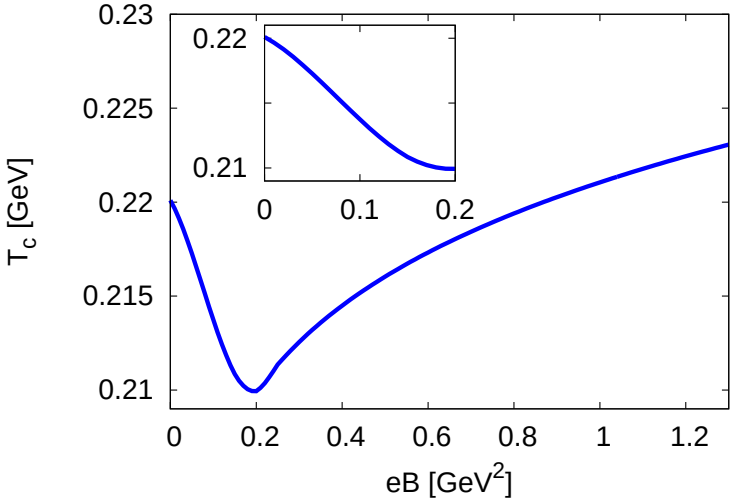

- chiral condensate?
- effect of quark masses?
- quantitative studies