Quarkonium Production at Hadron-Hadron Colliders

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Factorization of the Inclusive Quarkonium Production Cross Section

- In heavy-quarkonium hard-scattering production, high-momentum scales appear: $m$ and $p_T$.
- We would like to use NRQCD to separate the perturbative physics at these high-momentum scales from the low-momentum, nonperturbative effects in the heavy-quarkonium dynamics.
- The probability for a $Q\bar{Q}$ pair to evolve into a heavy quarkonium can be calculated as a vacuum-matrix element in NRQCD:

$$\mathcal{O}^H_n(\Lambda) = \langle 0 | \chi \kappa_n \psi \left( \sum_X |H + X\rangle \langle H + X| \right) \psi^\dagger \kappa'_n \chi |0 \rangle.$$ 

- This is the matrix element of a four-fermion operator, but with a projection onto an intermediate state of the quarkonium $H$ plus anything.
  - $\kappa_n$ and $\kappa'_n$ are combinations of Pauli and Color matrices.
Conjecture (GTB, Braaten, Lepage (1995)):
The inclusive cross section for producing a quarkonium at large momentum transfer ($p_T$) can be written as a sum of “short-distance” coefficients times NRQCD matrix elements.

$$\sigma(H) = \sum_n F_n(\Lambda) \langle 0 | \mathcal{O}_n^H(\Lambda) | 0 \rangle.$$ 

- The part of the diagram inside the box corresponds to an NRQCD matrix element.

- The points $A(C)$ and $B(D)$ are within $\sim 1/m$ of each other.
  - Kinematics implies that the virtual $Q$ is off shell by order $m$.
- The points $A(B)$ and $C(D)$ are within $1/p_T$ of each other.
  - The part of the diagram outside the box is insensitive to changes of momentum flow from $A(B)$ to $C(D)$ of order $p_T$. 

• The “short-distance” coefficients $F_n(\Lambda)$ are essentially the process-dependent partonic cross sections to make a $Q\bar{Q}$ pair convolved with the parton distributions.
  – They have an expansion in powers of $\alpha_s$.

• The operator matrix elements are universal (process independent).
  – Only the color-singlet production and decay matrix elements are simply related.

• The matrix elements have a known scaling with $v$.

• The NRQCD factorization formula is a double expansion in powers of $\alpha_s$ and $v$.

• A key feature of NRQCD factorization: Quarkonium production can occur through color-octet, as well as color-singlet, $Q\bar{Q}$ states.

• If we drop all of the color-octet contributions and retain only the leading color-singlet contribution, then we have the color-singlet model (CSM).
  – Inconsistent for $P$-wave production: IR divergent.
Status of a Proof of Factorization

- A proof is complicated because gluons can dress the basic production process in ways that apparently violate factorization.

- A proof of factorization would involve a demonstration that diagrams in each order in $\alpha_s$ can be re-organized so that
  - All soft singularities cancel or can be absorbed into NRQCD matrix elements,
  - All collinear singularities and spectator interactions can be absorbed into parton distributions.

- Nayak, Qiu, Sterman (2005, 2006): The color-octet NRQCD matrix elements must be modified by the inclusion of eikonal lines to make them gauge invariant.
  - The eikonal lines are path integrals of the gauge field running from the creation and annihilation points to infinity.
  - Essential at two-loop order to allow certain soft contributions to be absorbed into the matrix elements.
  - Does not affect existing phenomenology, which is at tree order or one-loop order in the color-octet contributions.
• Nayak, Qiu, Sterman (2005, 2006): A key difficulty in proving factorization to all orders is the treatment of gluons with momenta of order $m$ in the quarkonium rest frame.

• If the orange gluon has momentum of order $m$, it can’t be absorbed into the NRQCD matrix element as a quarkonium constituent.

• But the orange gluon can have non-vanishing soft exchanges with the quarkonium constituents.

• The orange gluon can be treated as the eikonal-line part of the NRQCD matrix element, provided that the answer does not depend on the direction of the eikonal line (universality of the matrix elements).

• Nayak, Qiu, Sterman (2005, 2006): At two-loop order, the eikonal lines contribute but a “miracle” occurs: The dependence on the direction of the eikonal line cancels.

• In general, factorization of the inclusive cross section beyond two-loop order is still an open question.

• An all-orders proof is essential because the $\alpha_s$ associated with soft gluons is not small.
Nayak, Qiu, Sterman (2007, 2008): If an additional heavy quark is approximately co-moving with the $Q\bar{Q}$ pair that forms the quarkonium, there are soft color exchanges between the heavy quark and the $Q\bar{Q}$ pair.

- This process does not fit into the NRQCD factorization picture.
  It requires production matrix elements that contain additional heavy quarks beyond the $Q\bar{Q}$ pair.
- The process is nonperturbative: It can’t be calculated reliably.
- Can search for the process experimentally:
  The signature is additional heavy-meson production in a narrow cone ($\sim mv/p_T$) around the quarkonium.
- This effect might be eliminated from the measured cross section through the use of an isolation cut.
The Fragmentation Approach
Kang, Qiu, and Sterman (2010)

- Writes the cross section in terms of
  - single-parton production cross sections convolved with the fragmentation functions for a single parton into a quarkonium
    \[ d\hat{\sigma}_{A+B\to i+X} \otimes D_{i\to H} \]
  - \( Q\bar{Q} \) production cross sections convolved with fragmentation functions for a \( Q\bar{Q} \) pair into a quarkonium
    \[ d\hat{\sigma}_{A+B\to Q\bar{Q}+X} \otimes D_{Q\bar{Q}\to H} \]

- Re-organizes the perturbation expansion as an expansion in powers of \( 1/p_T \).
- Believed to hold to all orders in perturbation theory up to corrections of order \( m_Q^4/p_T^4 \).
- If NRQCD factorization holds, then the fragmentation functions can be written as a sum of NRQCD matrix elements times perturbatively calculable short-distance coefficients.
Comparisons of NRQCD Factorization with Experiment
Quarkonium Production and Polarization at the Tevatron

Production Cross Section in LO

- The CDF (1997) data are more than an order of magnitude larger than the LO predictions of the color-singlet model.
- $p_T$ distributions are consistent with NRQCD prediction (Krämer (2001)), but not with the LO color-singlet model.
- Color-octet matrix elements are determined from fits to the data.
- Good fits for $J/\psi$, $\psi(2S)$, $\chi_c$, $\Upsilon(1S)$ production, as well.
- Use color-octet matrix elements from these fits to predict quarkonium production in other processes (test universality).
Polarization in LO

- Transverse quarkonium polarization may be a signature of the color-octet mechanism.
- In LO quarkonium production at large $p_T (p_T \gtrsim 4m_c$ for $J/\psi$), gluon fragmentation via the color-octet $^3S_1 Q\bar{Q}$ state dominates.
- At large $p_T$, the gluon is nearly on mass shell, and, so, is transversely polarized.
- In color-octet gluon fragmentation, most of the gluon’s polarization is transferred to the quarkonium (Cho, Wise (1994)).
  - Spin-flip interactions are suppressed as $v^2$.
  - Verified in a lattice calculation of decay matrix elements (GTB, Lee, Sinclair (2005)).
- Radiative corrections dilute this (Beneke, Rothstein (1995); Beneke, Krämer (1996)).
$J/\psi$ Polarization in LO

Run I:

$d\sigma/d(\cos \theta) \propto 1 + \alpha \cos^2 \theta$.
- $\alpha = 1$ is completely transverse;
- $\alpha = -1$ is completely longitudinal.

NRQCD prediction from Braaten, Kniehl, Lee (1999).
- Feeddown from $\chi_c$ states is about 30% of the $J/\psi$ sample and dilutes the polarization.
- Feeddown from $\psi(2S)$ is about 10% of the $J/\psi$ sample and is largely transversely polarized.

Run II:

Run I results are marginally compatible with the NRQCD prediction.
Run II results are inconsistent with the NRQCD prediction.
Also inconsistent with the Run I results.
CDF was unable to track down the source of the Run I-Run II discrepancy.
The Run II data are incompatible with the LO NRQCD prediction.
$\Upsilon(1S)$ Polarization:

- In the $\Upsilon(1S)$ case, the D0 results (red) are incompatible with the CDF results (black).

- Both the CDF and D0 results are incompatible with the LO NRQCD prediction of Braaten and Lee (2000) (green), but in different regions of $p_T$.

$\Upsilon(2S)$ Polarization:

- In the $\Upsilon(2S)$ case, the theoretical and experimental error bars are too large to make a stringent test.
Higher-Order Calculations

- Campbell, Maltoni, Tramontano (2007); Artoisenet, Lansberg, Maltoni (2007): Higher-order corrections to color-singlet quarkonium production at the Tevatron are unexpectedly large.
- At high $p_T$, higher powers of $\alpha_s$ can be offset by a less rapid fall-off with $p_T$.

LO:

\[ \sim \alpha_s^3 \frac{(2m_c)^4}{p_T^8} \]
NLO:

\[ \sim \alpha_s^4 \frac{(2m_c)^2}{p_T^6} \]

\[ \sim \alpha_s^4 \frac{1}{p_T^4} \]

NNLO:

\[ \sim \alpha_s^5 \frac{1}{p_T^4} \]
NLO and NNLO* Color-Singlet $J/\psi$ Production

- Plot from Pierre Artoisenet, based on work by Artoisenet, Campbell, Lansberg, Maltoni, Tramontano.
- The NNLO* calculation is an estimate based on real-emission contributions only.
- The data still seem to require a color-octet contribution.
NLO Color-Octet $S$-Wave $J/\psi$ and $\psi(2S)$ Production

- Gong, Li, and Wang (2008, 2010): NLO corrections to the $S$-wave channels are small.
  - $K$ factors at the Tevatron are about 1.235 for the $^1S_0$ channel and 1.139 for the $^3S_1$ channel.

First Complete NLO Color-Octet Calculations
Ma, Wang, and Chao (2010); Butenschön and Kniehl (2010)

- NLO corrections for all of the color-octet channels through order $\nu^4$.
  Color-octet channels: $^1S_0$, $^3S_1$, $^3P_J$.

- Confirm that the NLO corrections to the $S$-wave channels are small.

- Very large $K$ factor $\sim -10$ for the $^3P_J$ channel.
  A $1/p_T^4$ contribution appears for the first time in NLO.

- The results of Ma, Wang, and Chao and Butenschön and Kniehl for the short-distance cross sections agree.
Ma, Wang, and Chao (2010):

- Matrix elements were fit to the CDF (2005, 2009) Run II data for $p_T > 7$ GeV.
- Feeddown from the $\psi(2s)$ was taken into account by using the CDF (2005, 2009) Run II data.
- Feeddown from the $\chi_{cJ}$ states was taken into account by using the NLO prediction of Ma, Wang, and Chao (2010) for $\chi_{cJ}$ production.
  - Uses a color-octet matrix element that is obtained by fitting to the CDF (2007) measurements of $R_{\chi_c} = \sigma_{\chi_{c2}}/\sigma_{\chi_{c1}}$.
  - The predicted $\chi_{cJ}$ fraction increases with increasing $p_T$, while the $\chi_{cJ}$ fraction measured by CDF (1997) in Run I decreases with increasing $p_T$.
- The fits were used to predict the CMS (2010) data.
• Only the linear combinations

\[ M_{0,r_0} = \langle O^\psi (1S_0^{[8]}) \rangle + (r_0/m_c^2)\langle O^\psi (3P_0^{[8]}) \rangle = (7.4 \pm 1.9) \times 10^{-2} \text{ GeV}^3 \]

\[ M_{1,r_1} = \langle O^\psi (3S_1^{[8]}) \rangle + (r_1/m_c^2)\langle O^\psi (3P_0^{[8]}) \rangle = (0.05 \pm 0.02) \times 10^{-2} \text{ GeV}^3 \]

could be fit unambiguously.

\( r_0 = 3.9 \) and \( r_1 = -0.56 \) chosen on the basis of approximate relations between the short-distance coefficients.

• The small size of \( M_{1,r_1} \) suggests that \( \langle O^\psi (3S_1^{[8]}) \rangle \) is small.

  – Assumes that there is not an accidental cancellation between the \( \langle O^\psi (3S_1^{[8]}) \rangle \) and \( \langle O^\psi (3P_0^{[8]}) \rangle \).

  – Might explain the absence of transverse \( J/\psi \) polarization in the Tevatron data.
Butenschön and Kniehl (2010):

NRQCD matrix elements were extracted in a fit that made use of both the CDF (2005) Run II data and the H1 (2002, 2005) HERA I and HERA II data.

- All three color-octet NRQCD matrix elements were determined in the fit.
- A cut $p_T > 3$ was applied to the CDF data.
- No corrections were made for feed-down.
- This fit describes shape of the CDF data less well than the fit of Ma, Wang, and Chao.
  - May be caused by tension between the theory and the combined CDF and H1 data.
- The results were used to predict cross sections at PHENIX and CMS.
There is a slight discrepancy in shape between the NLO prediction and the H1 data.
Discussion

- The Butenschön and Kniehl matrix elements are not very different from those from LO extractions.

- In comparison to the values in the Ma, Wang, and Chao fit
  - $M_{0,r_0}$ is about a factor 4 smaller,
  - $M_{1,r_1}$ is about a factor 11 larger.

- Since the short-distance cross section agree, the differences between the matrix elements must arise from the differences in the fitting procedures.

- The differences in the matrix-element extractions seem to arise mainly from
  - The use of the HERA data in the fit of Butenschön and Kniehl.
    Note that most of it is at rather low values of $p_T$.
  - The use of approximate relations between the short-distance coefficients to select the linear combinations that are used in the fit of Ma, Wang, and Chao.
  - The inclusion of feeddown from the $\psi(2S)$ and $\chi_{cJ}$ states in the fit of Ma, Wang, and Chao.
    The calculated $\chi_{cJ}$ feeddown may fall less rapidly with $p_T$ than the CDF data.

- The relative size of the $\langle O^{\psi(3S_1)} \rangle$ contribution and the expected $J/\psi$ polarization depend on the resolution of these discrepancies.
NLO and NNLO* Color-Singlet $\Upsilon$ Production

- The data could be explained by color-singlet production alone.

- There is still room for a substantial amount of color-octet production.

- Color-octet production is suppressed as $v^4$.
  Should be smaller for $\Upsilon$ ($v^2 \approx 0.1$) than for $J/\psi$ ($v^2 \approx 0.3$).

Plot from Pierre Artoisenet, based on work by Artoisenet, Campbell, Lansberg, Maltoni, Tramontano (2008)

NLO results confirmed by Gong and Wang (2007).
NLO Color-Octet $S$-Wave $\Upsilon$ Production

- (Gong, Wang, Zhang (2008, 2010)): NLO corrections to the $S$-wave channels are small.
- $K$ factors at the Tevatron are about $1.313$ for the $^1S_0$ channel and $1.379$ for the $^3S_1$ channel.
NLO and NNLO* Color-Singlet Polarization

- Gong and Wang (2008): color-singlet $J/\psi$ polarization at the Tevatron changes from transverse to longitudinal when NLO corrections are included.

- NLO$^-$ excludes $gg \rightarrow J/\psi c \bar{c}$.
- Unlabeled line is contribution of $gg \rightarrow J/\psi c \bar{c}$. 
- Artoisenet, Campbell, Lansberg, Maltoni, Tramontano (2008): color-singlet $\gamma$ polarization at the Tevatron changes from transverse to longitudinal when NLO and NNLO* corrections are included.

NLO Color-Octet $S$-Wave Polarization

- Gong, Li, and Wang (2008): The prediction for the $J/\psi$ polarization is little affected by NLO corrections to the color-octet $^1S_0$ and $^3S_1$ channels.

- Gong, Wang, and Zhang (2010): The prediction for the $\Upsilon$ polarization is not shifted significantly by NLO corrections to the color-octet $^1S_0$ and $^3S_1$ channels.

- There are large uncertainties because of the feeddown from the $\chi_{bJ}$ states.
Large corrections appear in NLO and NNLO* because new channels that open produce a slower fall-off with increasing $p_T$.

- The new channels spoil the convergence of the perturbation series.
- There are still large renormalization-scale uncertainties in NLO and NNLO*.

The fragmentation approach of Kang, Qiu, and Sterman (2010) potentially brings the higher-order corrections under better control.

Re-organizes the perturbation expansion according to powers of $p_T$.

In the fragmentation functions, an important class of higher-order corrections is resummed by making use of evolution equations for the fragmentation functions.

It may be possible to compute fragmentation contributions to higher orders in $\alpha_s$ than one can compute complete cross sections.
Discussion

- The NNLO* corrections greatly increase the color-singlet contributions to the \( J/\psi \) and \( \Upsilon \) cross sections, but the uncertainties are very large.

- The \( J/\psi \) production data still seem to require a color-octet contribution that dominates at large \( p_T \).

- A color-octet contribution is not required or excluded by the \( \Upsilon \) production data.

- NLO corrections might change our ideas about the relative contributions of the color-octet channels and about the expected quarkonium polarization.

- The fragmentation approach may help to reduce theoretical uncertainties.

- Interpretation of the Tevatron \( J/\psi \) data, both polarized and unpolarized, is complicated by feed-down from the \( \psi(2S) \) and \( \chi_{cJ} \) states.

- High-statistics, high-\( p_T \) measurements of the cross section and polarization for direct production of the \( J/\psi \), \( \chi_{cJ} \), and \( \psi(2S) \) states would be of great help.

- The discrepancies between the CDF and D0 \( \Upsilon \) polarization data must be resolved before any meaningful comparisons can be made with theory.

- NLO calculations of the color-octet \( P \)-wave contributions to quarkonium polarization are needed.
**χ_{cJ} Production**

- Ratio of $P$-wave cross sections:

  \[
  R_{\chi_c} = \frac{d\sigma_{\chi_{c2}}/dp_T}{d\sigma_{\chi_{c1}}/dp_T}.
  \]

- In NRQCD factorization in LO, $R_{\chi_c}$ is dominated by color-octet contributions at large $p_T$. It is predicted at large $p_T$ to be

  \[ R_{\chi_c} = 5/3. \]

- CDF (2007): At large $p_T$

  \[ R_{\chi_c} \approx 0.75. \]

- Ma, Wang, Chao (2010): NLO corrections to $R_{\chi_c}$ are large at large $p_T$.
  - Using the NLO results, they are able to fit the $p_T$ distribution of $R_{\chi_c}$, using plausible values of the color-octet NRQCD matrix elements.
  - The fit predicts that feeddown from the $\chi_{cJ}$ states to the $J/\psi$, may be as large as 30% of the $J/\psi$ rate at $p_T = 20$ GeV.
  - The predicted $\chi_{cJ}$ fraction increases with increasing $p_T$, while the $\chi_{cJ}$ fraction measured by CDF (1997) in Run I decreases with increasing $p_T$. The experimental and theoretical uncertainties are large.
### $J/\psi$ Production at RHIC

#### Production Cross Section

- The STAR collaboration has measured the $J/\psi$ $p_T$ distributions in $p + p$ and $Cu+Cu$ collisions:

  ![Graphs showing $J/\psi$ production in different collision types](image)

- Nayak, Liu, Cooper (2003): An LO NRQCD calculation (color-singlet plus color-octet contributions) fits the data well.
  - Does not include feeddown from $\psi(2S)$, $\chi_c$, or $B$ decays. (Estimated to be a factor 1.5.)

The color-singlet contribution is well below the PHENIX (2009) data.

Lansberg (2010): NLO corrections increase the size of the color-singlet contribution substantially.

The color-singlet contribution still lies below the PHENIX (2010) and STAR (2009) data at large $p_T$.

Figure courtesy of Hee Sok Chung.
- The NLO NRQCD calculation of Kniehl and Butenschön (2010), with NRQCD matrix elements fit to the CDF (2005) and H1 (2002, 2010) data, agrees well with the PHENIX data:

- Feeddown ($\approx 36\%$) is not included in the theoretical prediction.

- The NLO color-singlet contribution is well below the PHENIX data.

The color-singlet contribution in LO is in poor agreement with the PHENIX data.

Lansberg (2010): The NLO corrections to the color-singlet contribution make it virtually indistinguishable from the color-octet contribution.
The NLO predictions of Ma, Wang and Chao (2010) and Kniehl and Butenschön (2010) agree well with the CMS (2010) data:

Ma, Wang, and Chao (2010) $|y| < 2.4$

Butenschön and Kniehl (2010) $1.4 < |y| < 2.4$

Only the calculation of Ma, Wang, and Chao (2010) includes the effects of feeddown.
• Somewhat surprising that both calculations agree well with the data since the NRQCD matrix elements that are used are so different.
  – The $J/\psi$ cross sections at the Tevatron and the LHC are dominated by $gg$-initiated processes.
  – The fit to the Tevatron $p_T$ distribution produces a mapping of the gluon momentum values into a $p_T$ spectrum.
  – Because the $p_T$ distributions of the three important color-octet channels are not linearly independent, that mapping can be achieved in NRQCD in different ways.
  – Two models that have the same mapping of gluon momenta to $p_T$ distributions will produce the same predictions for $d\sigma/dp_T$ at the Tevatron and the LHC.
  – Explains why both the LO and NLO NRQCD predictions fit the Tevatron data and predict the LHC data accurately.

• The analysis of Kang, Qiu, and Sterman shows that we can predict only the leading and first subleading powers of $p_T^2$ in $d\sigma/dp_T$.

• Clearly, we need additional observables in order to understand the details of the production mechanism.
The LHCb data also agree well with the NLO NRQCD predictions. (There are also Atlas and Alice measurements.)

- The NNLO* color-singlet prediction significantly undershoots the data.
Summary

- The NRQCD factorization approach provides a systematic method for calculating quarkonium decay and production rates as double expansions in powers of $\alpha_s$ and $v$.
- NRQCD factorization for inclusive production rates has not yet been established.
- NRQCD factorization has enjoyed a number of successes:
  - quarkonium production at the Tevatron,
  - $J/\psi$ production at RHIC,
  - $J/\psi$ production at the LHC,
  - $\gamma\gamma \to J/\psi + X$ at LEP,
  - inelastic $J/\psi$ photoproduction at HERA,
  - $J/\psi$ production in DIS at HERA,
  - exclusive double-charmonium production at Belle and BaBar.
The disagreement between theory and experiment for quarkonium polarization at the Tevatron presents a serious challenge.

- The CDF results for the $J/\psi$ and $\psi(2S)$ polarizations are unconfirmed.
- The CDF and D0 results for the $\Upsilon$ polarization do not agree.
- NLO calculations of the $P$-wave contributions to quarkonium polarization are needed in order to draw definite conclusions.

In a number of cases, corrections of higher order in $\alpha_s$ and $\nu$ and resummations near kinematic endpoints have proven to be essential to obtain reliable theoretical predictions.

NNLO* calculations of color-singlet quarkonium production at the Tevatron may reduce the importance of color-octet contributions and could possibly resolve some puzzles.

In many cases, the perturbation expansion converges poorly, and theoretical uncertainties are large.

The fragmentation approach of Kang, Qiu, and Sterman may help to bring theoretical uncertainties under control.

Measurements of direct-production cross sections and polarizations would be of great help in understanding production mechanisms.

We need to make additional measurements beyond $d\sigma/dp_T$ at hadron-hadron colliders in order to pin down the quarkonium production mechanisms.