Heavy quarkonium in plasma: comparison between perturbation theory and lattice

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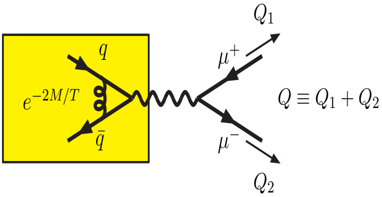
Outline

Heavy quarkonium as probe for QGP

Heavy quarkonium is an important probe of the properties of a quark-gluon plasma [T. Matsui, H. Satz (1986)].

- In heavy ion collisions $\longrightarrow$ short lived quark-gluon plasma.
- In the primary collisions heavy quarkonium is created.
- Depending on the plasma temperature it decays (to muons for instance).
- Muon escape $\leftrightarrow$ carry information out of the interior of the plasma.
- The frequency of the emitted muons is measured.



Different methods

Despite asymptotic freedom at the temperature of interest, the theoretical determination of the properties of heavy quarkonium is not more tractable than at $T = 0$.

Many different approaches:

1. Potential models
2. Perturbation theory
3. Lattice QCD
4. AdS/QCD

In this talk: What can we get from first principles?
$\rightarrow$ Perturbation theory and comparison to lattice results.

Effective field theory: $T = 0$ case first

Starting form the QCD Lagrangian, we separate the light quarks (u,d,s) from the heavy quark (c):

$$\begin{aligned}
\mathcal{L}_{\mathrm{QCD}} &= \mathcal{L}_{\mathrm{gluons}} + \mathcal{L}_{\mathrm{light\ quarks}} + \mathcal{L}_{\mathrm{heavy\ quark}}, \\
&\quad \mathcal{L}_{\mathrm{gluons}} = \frac{1}{4} F^{\mu\nu\ a} F^{a}_{\mu\nu}, \\
&\quad \mathcal{L}_{\mathrm{light\ quarks}} = \bar{\psi}_i (i\gamma^\mu D_\mu)\psi_i, \\
&\quad \mathcal{L}_{\mathrm{heavy\ quark}} = \bar{\Psi}(i\gamma^\mu D_\mu - M)\Psi.
\end{aligned}$$

We want to built an effective description for the bound state of two heavy quarks.

- Heavy quarks have a small binding energy $E_b \ll M$.

$\Rightarrow$ We have the following hierarchy of scales:

$$M \gg p \sim Mv \sim 1/r_b \gg E_b \sim Mv^2$$

$\Rightarrow$ The velocities v of the heavy quarks are small.

$\Rightarrow$ Use the Non-Relativistic QCD for the heavy quarks.

NRQCD

NRQCD is an effective low energy $E \sim Mv$ description for the heavy quark

- Relativistic spinors are decomposed in non-relativistic components $\Psi = \binom{\phi}{\chi}$.
- The Lagrangian of NRQCD reads

$$\mathcal{L}^{\mathrm{NRQCD}}_{\mathrm{heavy\ quark}} = \phi^\dagger (iD_0 + \frac{\mathbf{D}^2}{2M})\phi + \chi^\dagger (iD_0 - \frac{\mathbf{D}^2}{2M})\chi + \mathcal{O}\left(\frac{1}{M^2}\right)$$

- In NRQCD terms are arranged in inverse powers of M.
- It can be obtained from QCD by a Foldy-Wouthuysen transformation.

With NRQCD we integrated the hard scale M, but we can do better:

$\Rightarrow$ Integrate the soft scale $p \sim Mv \sim 1/r_b$. $\Rightarrow$ New effective field theory: potential NRQCD.

pNRQCD, Definition

pNRQCD is an effective ultra low energy $E \sim Mv^2$ description for a heavy quark-antiquark bound state.

- The bound state is represented by a wave function $\psi(\mathbf{x_1}, \mathbf{x_2}, \mathbf{t})$.
- Coordinates $\rightarrow$ relative and CM coordinate $\mathbf{x} = \mathbf{x_1} - \mathbf{x_2}$ and $\mathbf{X} = \frac{1}{2}(\mathbf{x_1} + \mathbf{x_2})$.
- The electric and magnetic fields are multipole expanded around the CM coordinate.
- The color structure of ψ is best split in singlet S and octet O part.
- The heavy quark Lagrangian is now ($r = |\mathbf{x}|$):

$$\mathcal{L}^{\text{pNRQCD}}_{\text{heavy quark}} = S^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{M} - V_S(r) \right) S + \mathcal{O}(M^{-2}) + O^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{M} - V_O(r) \right) O + gV_A(r)\mathbf{x}O\mathbf{E}(\mathbf{X}, t)S^\dagger + \ldots$$

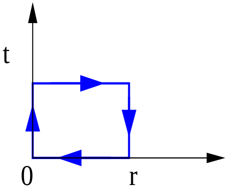
pNRQCD, Matching

- The potential for the octet part is repulsive.
- The potential for the singlet is attractive, at LO:

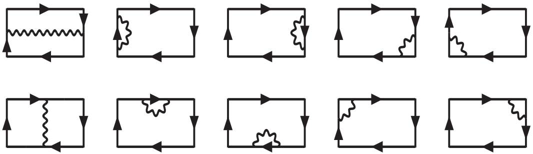
$$V_S^{LO}(r) = -\frac{4}{3}\frac{\alpha_s}{r}.$$

- In general it can be calculated from the Wilson loop:

$$\begin{aligned} V_S(r) &= \lim_{t\to\infty} \frac{i}{t} \ln\langle W(t,r)\rangle \\ W(r,t) &= P\exp\left[-ig\oint dz^\mu A_\mu(z)\right]. \end{aligned}$$



At leading order, the diagrams contributing are:



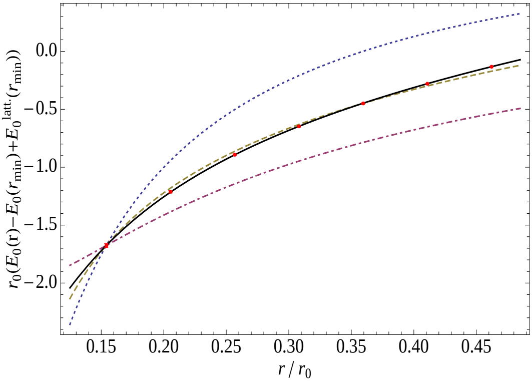
$T = 0$, pNRQCD Results

The singlet potential has been calculated

- Perturbatively to high order in α_s.

$\Rightarrow$ Computed till NNNLL ($\ln \alpha_s$).

- Nonperturbatively on the lattice.



[Brambilla, Vario, Tormo, Soto]

Effective theories for the finite temperature case

We built the same effective theory cascade, with the difference that one has another additional scale:

the Debye mass m_{D}

How does it compare to other scales?

- If $m_{\mathrm{D}} \sim r_b^{-1}$ the bound state is melted.
- If $m_{\mathrm{D}} \ll r_b^{-1}$ the bound state is not affected.

The interesting range is when $M \gg Mv \gtrsim m_{\mathrm{D}} = gT > Mv^2$.

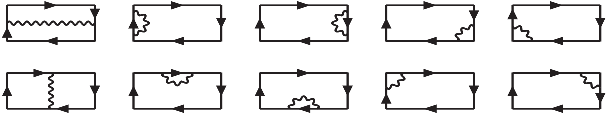
$\Rightarrow$ The reduction QCD$\rightarrow$NRQCD is the same as $M \ll T$.

$\Rightarrow$ NRQCD$\rightarrow$pNRQCD changes, exchanged gluons are screened.

$\Rightarrow$ Potential calculated from the Euclidean Wilson loop with time extend τ and the limit $\tau \rightarrow it \rightarrow i\infty$ is taken.

Potential for heavy quarks

The potential at LO is obtained by summing:



Where the HTML resummed propagator is used for the gluons:

$$V_S(r) = -\frac{g^2 C_F}{4\pi}\left[m_\mathrm{D} + \frac{\exp(-m_\mathrm{D} r)}{r} + iT\,\phi(m_\mathrm{D} r)\right] + \mathcal{O}(g^4)$$

- First term $\rightarrow$ 2$\times$thermal mass correction for heavy quarks.
- Second term $\rightarrow$ standard Debye-screened potential.
  - $C_F = (N_\mathrm{c}^2 - 1)/2N_\mathrm{c}$;
  - $m_\mathrm{D} = gT$ is the Debye mass

[Laine, Philipsen, Romatschke, Tassler (2007); Brambilla, Ghiglieri, Vairo and Petreczky (2008); Beraudo, Blaizot, Ratti (2008)]

Potential for heavy quarks

$$V_S(r) = -\frac{g^2 C_F}{4\pi}\left[m_{\mathrm{D}} + \frac{\exp(-m_{\mathrm{D}} r)}{r} + iT\,\phi(m_{\mathrm{D}} r)\right] + \mathcal{O}(g^4)$$

- Third *imaginary* term → heavy quark damping:

$$\phi(x) \equiv 2\int_0^\infty \frac{\mathrm{d}z\, z}{(z^2+1)^2}\left[1 - \frac{\sin(zx)}{zx}\right]$$

  - $\phi(x)$ is strictly increasing from $\phi(0) = 0$, $\phi(\infty) = 1$.
  - $r \to \infty$ contribution, $2\times$single quark damping (quark absorption in the plasma)
  - Destructive interference between the dampings.

Spectral function from the singlet wave function

From the potential:

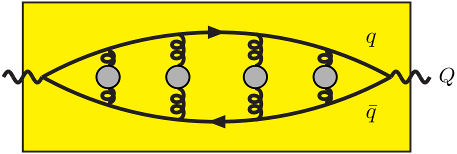
$\longrightarrow$ solve the Schrödinger equation for S

$\longrightarrow$ compute the spectral function:

$$\rho^V(\omega) = \frac{\left(1-e^{-\omega/T}\right)}{2} \int_{-\infty}^{\infty} dt e^{i\omega t} S(t,0,0)$$

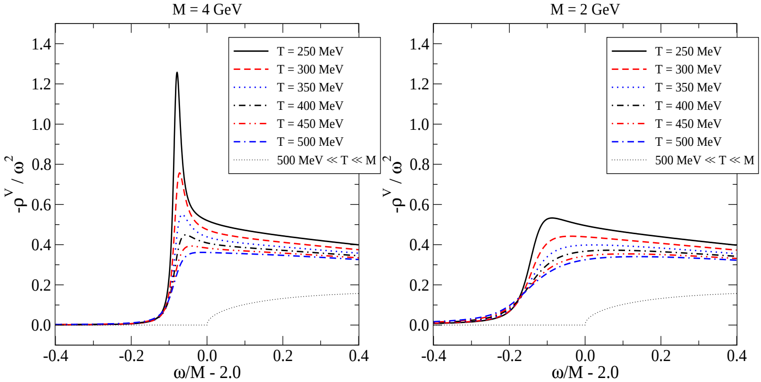
[YB, Laine, Vepsäläinen (2008)]

This is in fact equivalent to resumming the following graphs:



where heavy quarks lines have been replaced by Wilson lines.

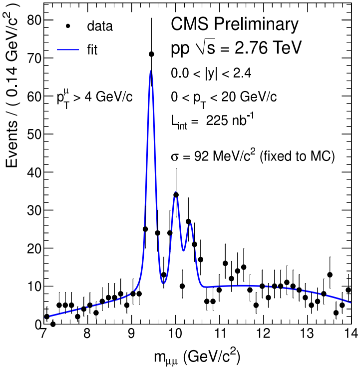
Numerical results for the spectral function

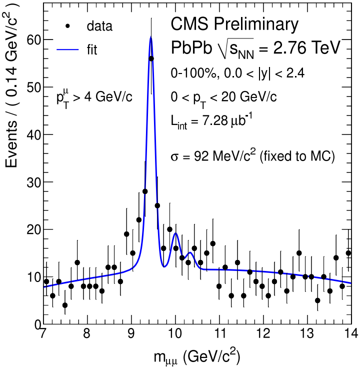


- Charmonium peak almost gone above T_c.
- Bottomonium has only one resonance left.
- No real bound state: Quarkonium is a resonance, which broadens at high temperature.

Hint from experiment?

Our result is certainly not in contradiction with the last results form CMS: [arXiv:1012.5545, CMS-HIN-10-006]





High energy limit $\omega \gg M \gg T$ perturbative expansion

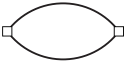
We no longer have $M \gg \delta\omega$ but for large ω quarks will fly apart very fast and do not have time to interact $\rightarrow$ perturbative expansion possible

We define the quark current correlator:

$$C_E^V(\tau) = \int \mathrm{d}^3\mathbf{x} \left\langle \hat{\bar{\psi}}(\tau,\mathbf{x})\,\gamma^\mu \hat{\psi}(\tau,\mathbf{x}) \;\; \hat{\bar{\psi}}(0,0)\,\gamma_\mu \hat{\psi}(0,0) \right\rangle_T .$$

and its Fourier transform $C_E^V(\omega_n^b)$, which is calculated as

At LO:



$$= [Q - \text{indep.}] + 2N_c \SumInt_{\{P\}} \frac{(D-2)Q^2 - 4M^2}{(P^2+M^2)\left((P-Q)^2+M^2\right)}$$

Computation at finite T

NLO:  $= \dots$

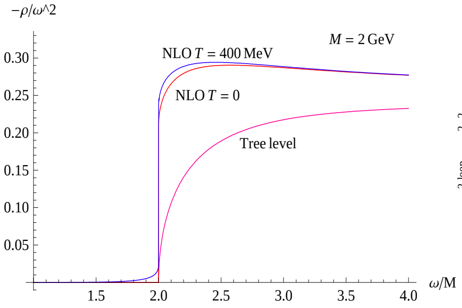
Diagrams for $C_E^V(\omega_n^b)$ are computed at finite T

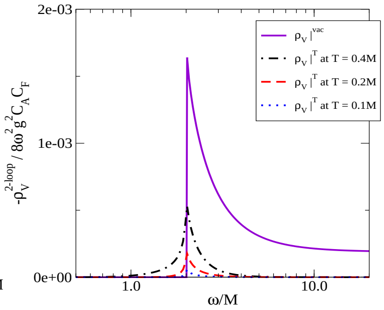
- Gluon propagator in spectral representation
- Carry out the sums
- Check the absence of infrared divergences
- Neglect $e^{-M/T}$ terms
- Carry out the integrals
- Spectral function $\rho^V(\omega) \leftarrow$ imaginary part of $C_E^V(\omega_n^b)$

At LO:

$$\begin{aligned}\rho^V(\omega) &= -\theta(\omega - 2M)\frac{N_c}{4\pi\omega}\sqrt{\omega^2 - 4M^2}(\omega^2 + 2M^2)\tanh\left(\frac{\omega}{4T}\right) \\ &\quad +4\pi N_c \omega \delta(\omega) I_2\end{aligned}$$

Results for the spectral function

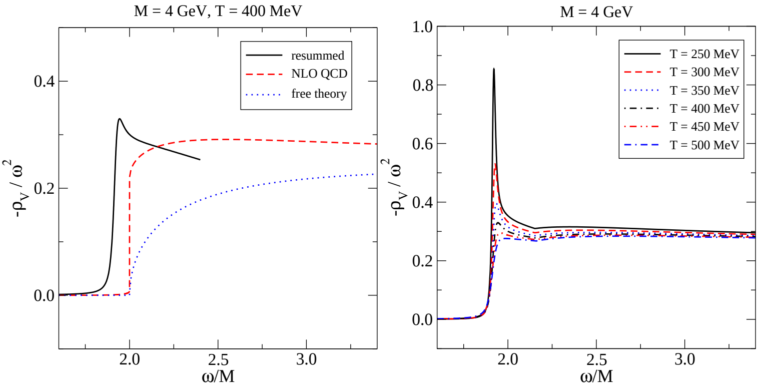




- NLO corrections are large $\xrightarrow{\omega\to\infty}$ const
- Temperature corrections are small $\xrightarrow{\omega\to\infty}$ 0

Gluing small and large energy results

(p)NRQCD/QCD normalization factor...



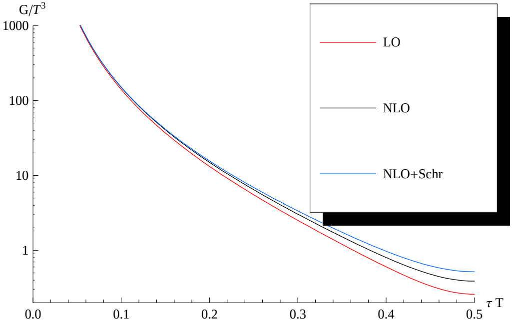
[YB, Laine, Vepsäläinen (2009)]

Results for the Euclidean Correlator

From the spectral function:

$$G(\tau) = \int d\omega\ \rho(\omega) \frac{\cosh(\omega(\tau - \beta/2))}{\pi \sinh(\omega\beta/2)}. \tag{1}$$

Example: Charmonium in pure glue $M = 1.6,\ \ T = 1.5T_c,\ \ N_f = 0$:



For NLO part I use $\alpha(\omega)$.

3. Quarkonium on the lattice

Lattice computations are difficult, Euclidean results have to be analytically continued to Minkowski space-time.

Three kind of computations can be performed on the lattice:

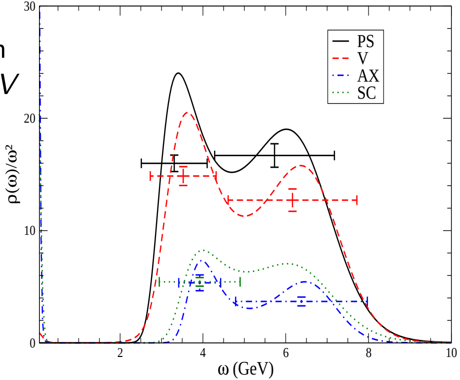
1. The spectral function can be computed directly:
   - More precisely the Euclidean correlator is computed.
   - Has to be analytically continued.

   $\Rightarrow$ Maximal entropy method, ...
2. The heavy quark potential could be computed:
   - Proper potential needs an analytical continuation.
   - Is there an Euclidean definition? At least for the real part?
   - Can perturbation theory help to define that?
3. Reconstruct the potential out of the correlation function.

Spectral function out of the lattice

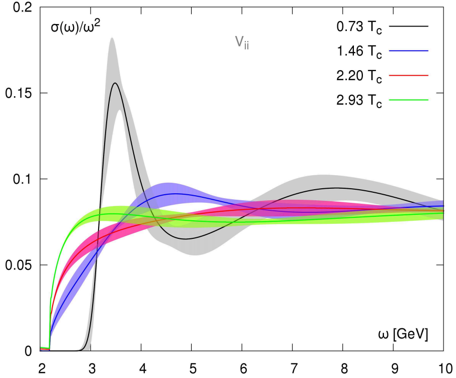
Example of lattice results: Euclidean correlator → Analytical continuation with MEM → Spectral function [Aarts, Allton, Oktay, Peardon, Skullerud (2007)]:

Charmonium
$T = 226 MeV$



Note the second peak, not observed in perturbation theory.

Many progress were done in the quenched case



[Ding, Francis, Kaczmarek, Karsch, Satz, Soeldner (2010)] Note the different threshold behaviour than in perturbation theory.

Comparison at the level of the Euclidean correlator

The analytic continuation is not well defined, we could compare the Euclidean correlator:

Lattice data [Ding, Francis, Kaczmarek, Satz, Karsh, Söldner, 1011.0695]

- Pure glue and charm quark
- Lattice size: $128^3 \times 96,\ 128^3 \times 48,\ 128^3 \times 32,\ 128^3 \times 24$
- Corresponding to $0.73,\ 1.46,\ 2.2,\ 2.93\,T_c$, $(T_c = 270$ MeV$)$
- Small errorbars, $< 10^{-3}$
- Not extrapolated to continuum

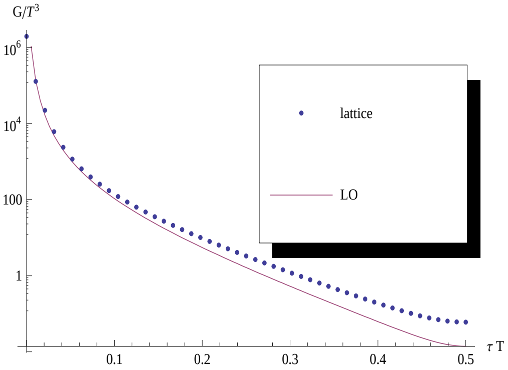
I will still attempt a comparison to continuum "perturbative" results. . .

Comparison at the level of the Euclidean correlator

First example ($T = 0.7\,T_c$)

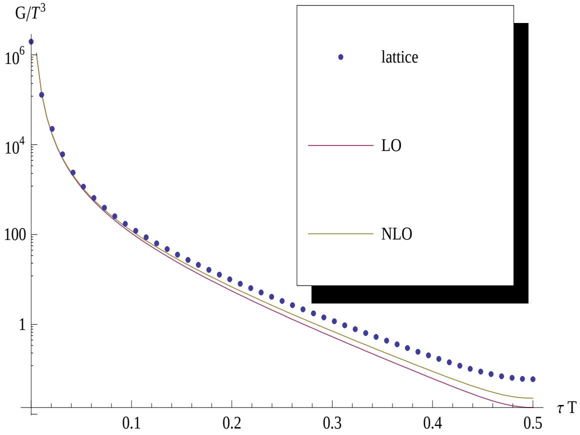
- $n_\tau = 96$ very large
- $T < T_c$, Perturbative approach questionable

Leading Order



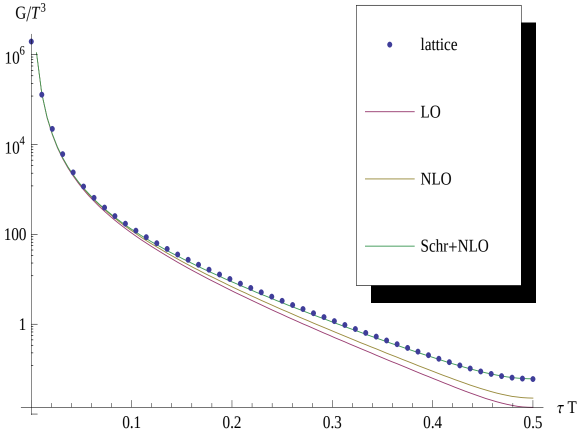
Comparison at the level of the Euclidean correlator

Next to Leading Order ($T = 0.7\,T_c$)



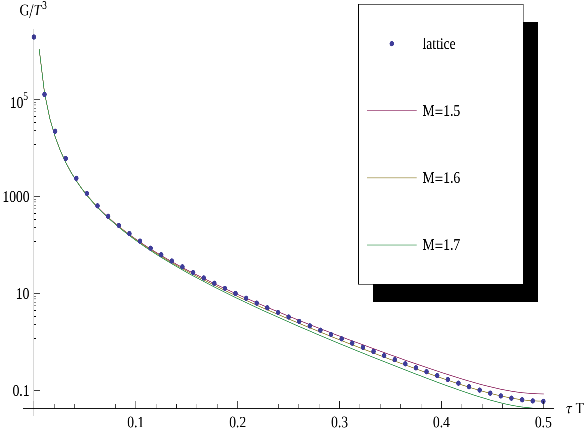
Comparison at the level of the Euclidean correlator

NLO + Schrödinger equation ($T = 0.7 T_c$)



Euclidean correlator, Mass dependence

The good agreement hide that I tuned the mass M to fit the lattice...

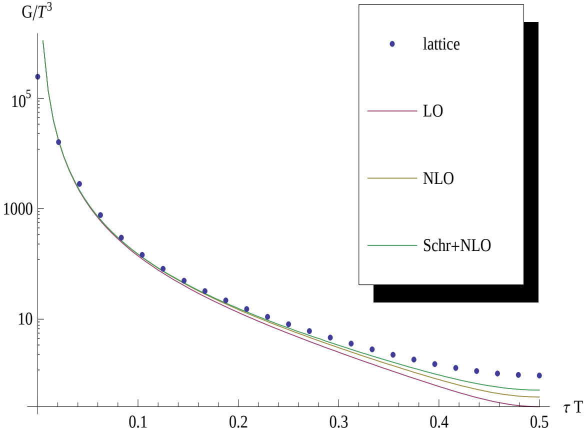


- $M \sim 1.6$ GeV expected as the charmonium mass is $3.1 GeV$

$\Rightarrow$ This gives a way to get M which is an unknown parameter in the EFT framework!

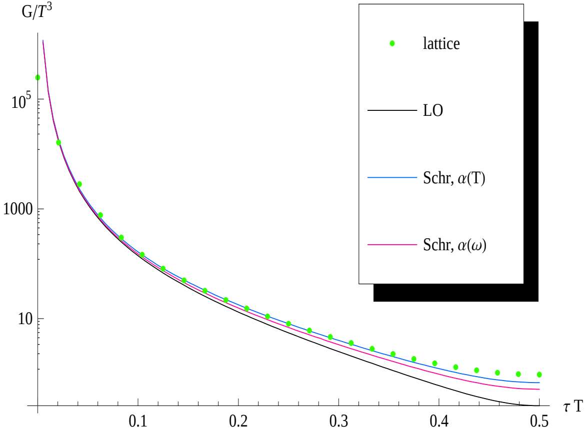
Euclidean correlator, Higher temperatures

At higher temperature, the agreement is not so good ($T = 1.5\,T_c$, $M = 1.6$)



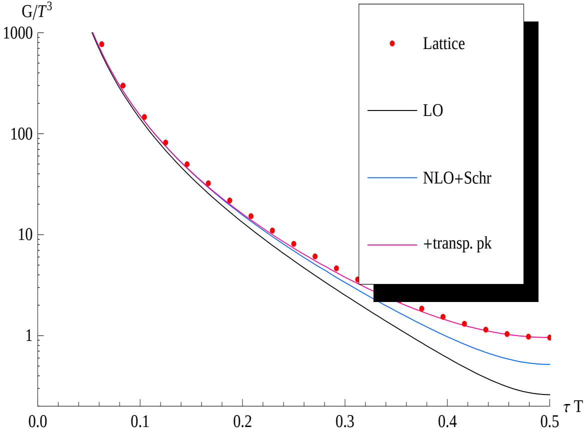
Euclidean correlator, Higher temperatures

- Changing the mass M does not help.
- Changing the running of the coupling helps but not enough ($T = 1.5T_c$, $M = 1.6$)



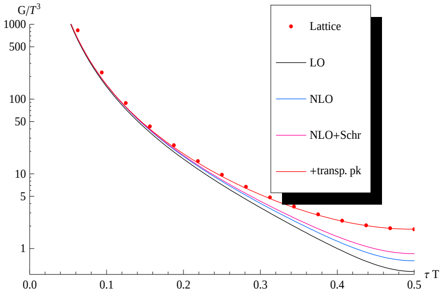
Euclidean correlator, Higher temperatures

- The transport peak is not known at NLO.
- Fitting the data with an arbitrary peak gives good results ($T = 1.5T_c,\ M = 1.6$)

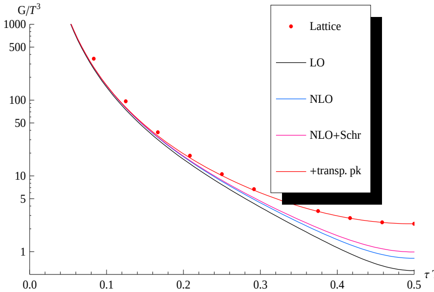


Euclidean correlator, Higher temperatures

$M = 1.6, \;\; T = 2.2\,T_c$



$T = 3\,Tc$



$\Rightarrow$ Next task: Compute the transport peak at NLO!

Euclidean definition of the potential

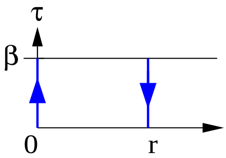
Popular correlators on the lattice:

1. Singlet free energy in Coulomb gauge:

$$\Psi_C = \frac{1}{N_c}\langle \mathrm{Tr}[P_r P_0^\dagger]\rangle_{Coulomb}$$

2. Traced Polyakov loop correlator

$$\Psi_T = \frac{1}{N_c^2}\langle \mathrm{Tr}[P_r]\mathrm{Tr}[P_0^\dagger]\rangle$$

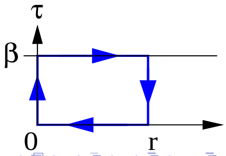


$\Leftarrow$ Gauge invariant.

Other interesting correlators:

3. The singlet free energy in covariant gauge Ψ_ξ.
4. Cyclic Wilson loop:

$$\Psi_W = \frac{1}{N_c}\langle \mathrm{Tr}[P_r W_\beta P_0^\dagger W_0^\dagger]\rangle$$



$\Leftarrow$ Gauge invariant.

Leading order perturbation theory

- The traced Polyakov loop correlator $F_T \sim \alpha^2 \frac{e^{-2m_\mathrm{D} r}}{r^2}$ is not the potential, but the roughly the sum of the singlet and octet potential.
- I will not discuss it more here.

- The free energy is gauge invariant ($\Psi_P = \frac{1}{N_c}\langle \mathrm{Tr}[P]\rangle$),

$$F_C = T \ln\left(\frac{\Psi_C}{|\Psi_P|^2}\right) = F_\xi = -\alpha C_F \frac{e^{-m_\mathrm{D} r}}{r}.$$

- It equates the Wilson loop

$$\Psi_C = \Psi_W.$$

- It is equal to the real part of the potential up to some constant.

Do all these nice properties extend to NLO?

Perturbation theory calculation at NLO

We calculated these different observables using finite T perturbation theory at NLO: [YB, M. Laine and M. Vepsäläinen, 2009]

Difficulties:

- UV divergences:
  - $\frac{1}{N_c}\langle \mathrm{Tr}[P_r P_0^\dagger]\rangle$ depends only on g.

  $\Rightarrow$ Charge renormalization alone should cancel UV divergences.
- IR divergences:
  - Color electric modes at the scale gT.

  $\Rightarrow$ Needs resummation: systematically done from EQCD.

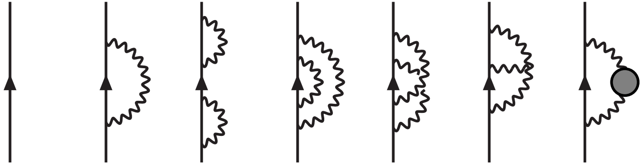
$$\Psi = [\Psi_{QCD} - \Psi_{EQCD}]_{unresummed} + [\Psi_{EQCD}]_{resummed}$$

  - Color magnetic modes at the scale $g^2 T$.

  $\Rightarrow$ No prominent role here.

Calculation: One Polyakov loop as example

$$[\Psi_P]_{QCD} = \left[\frac{1}{N_c}\langle Tr[P_r]\rangle\right]_{QCD} =$$



$$+\,\mathcal{O}(g^6)$$

$$= 1 - \frac{g^2 C_{\mathrm{F}}}{2T}\int_k \frac{1}{\mathbf{k}^2} - \frac{g^4 C_{\mathrm{F}}}{2}\int_{\mathbf{k}} \frac{2}{\mathbf{k}^4} \sum\!\!\!\!\!\!\!\int_Q \cdots + \ldots$$

The IR divergent $1/\mathbf{k}^4$ and further logarithmic divergences in the ... require resummations.

Soft mode resummation

The coefficient of the linear IR divergence is simply the Debye mass:

$$[\Psi_P]_{QCD} = \cdots + \frac{g^4 C_\mathrm{F}}{2}\int_\mathbf{k}\frac{1}{\mathbf{k}^4}\SumInt_Q N_\mathrm{c}(2-D)\left(-\frac{1}{Q^2}+2\frac{q_0^2}{Q^4}\right)$$
$$= \cdots + \frac{g^2 C_\mathrm{F}\beta}{2}\int_\mathbf{k}\frac{1}{k^4}m_\mathrm{D}^2$$

Can be removed by the usual resummation

$$-\frac{g^2 C_\mathrm{F}\beta}{2}\int_\mathbf{k}\frac{1}{\mathbf{k}^2}+\frac{g^2 C_\mathrm{F}\beta}{2}\int_\mathbf{k}\frac{1}{k^4}m_\mathrm{D}^2 = -\frac{g^2 C_\mathrm{F}\beta}{2}\int_\mathbf{k}\frac{1}{\mathbf{k}^2+m_\mathrm{D}^2} = \frac{g^2 C_\mathrm{F} m_\mathrm{D}\beta}{8\pi}$$

Logarithmic divergences remain, a more systematic treatment of the soft mode is needed

EQCD resummation

The Lagrangian of EQCD reads

$$\mathcal{L}_{\mathrm{E}} = \frac{1}{2}\mathrm{Tr}\,[\tilde{F}_{ij}^2] + \mathrm{Tr}\,[\tilde{D}_i, \tilde{A}_0]^2 + m_{\mathrm{D}}^2 \mathrm{Tr}\,[\tilde{A}_0^2] + \ldots \ .$$

The Polyakov loop operator is represented as

$$P_{\mathbf{r}} = [\mathbb{1}\, \mathcal{Z}_0] + ig\tilde{A}_0\beta\, \mathcal{Z}_1 + \frac{1}{2}(ig\tilde{A}_0\beta)^2\, \mathcal{Z}_2 + \ldots + (g^2\tilde{F}_{ij}\beta^2)^2 \mathcal{X}_4 + \ldots$$

g^3, g^4 corrections to the Polyakov loop in EQCD:

$$[\Psi_P]_{EQCD} = -\frac{g^2 C_{\mathrm{F}}}{2T}\int_k \frac{1}{\mathbf{k}^2 + m_D^2} - \frac{g^4 C_{\mathrm{F}}}{2}\int_{\mathbf{k}} \frac{2}{(\mathbf{k}^2 + m_D^2)^2}\int_{\mathbf{q}} \cdots$$

The divergences are regularized and reappear in the $m_D \to 0$ limit. The expression $\Psi_P = [[\Psi_P]_{QCD} - [\Psi_P]_{EQCD}]_{m_D\to 0} + [[\Psi_p]_{EQCD}]$ is finite and contains the correct color electric physics.

NLO results for the free energy in the Coulomb gauge

We obtain a well defined result for Ψ_C :

$$
\begin{aligned}
F_C(r) = &-\frac{\alpha(\bar{\mu}) C_F \exp(-m_\mathrm{D} r)}{r}\left\{1+\alpha(\bar{\mu})\left[\frac{11N_\mathrm{c}}{3}\left(2\ln\frac{\bar{\mu}e^{\gamma_\mathrm{E}}}{4\pi T}+1\right)\right.\right. \\
&\left.\left.-\frac{2N_\mathrm{f}}{3}\left(2\ln\frac{\bar{\mu}e^{\gamma_\mathrm{E}}}{\pi T}-1\right)\right]\right\} - \alpha(\bar{\mu})^2 C_F N_\mathrm{c}\left\{-\frac{\exp(-2m_\mathrm{D} r)}{8Tr^2}\right. \\
&\frac{1}{12Tr^2}+\frac{\mathrm{Li}_2(e^{-4\pi Tr})}{(2\pi r)^2 T}+T\exp(-m_\mathrm{D} r)\Big[2-\ln(2m_\mathrm{D} r)-\gamma_\mathrm{E} \\
&\left.+e^{2m_\mathrm{D} r}E_1(2m_\mathrm{D} r)\Big]+\frac{1}{\pi r}\int_1^\infty \mathrm{d}x\left(\frac{1}{x^2}-\frac{1}{2x^4}\right)\ln\left(1-e^{-4\pi Trx}\right)\right\} \\
&-\alpha(\bar{\mu})^2 C_F N_\mathrm{f}\left[\frac{1}{2\pi r}\int_1^\infty \mathrm{d}x\left(\frac{1}{x^2}-\frac{1}{x^4}\right)\ln\frac{1+e^{-2\pi Trx}}{1-e^{-2\pi Trx}}\right]+\mathcal{O}(g^5)\,.
\end{aligned}
$$

NLO results

For $rT \ll 1$, Ψ_C reproduces the $T=0$ potential

$$V(r) = -\frac{g^2 C_F}{4\pi r} + \frac{g^4 C_F}{(4\pi)^2}\int_{\mathbf{k}} \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{k^2}\left[\frac{2N_{\mathrm{f}}}{3}\left(\ln\frac{\bar{\mu}^2}{k^2}+\frac{5}{3}\right)-\frac{11N_{\mathrm{c}}}{3}\left(\ln\frac{\bar{\mu}^2}{k^2}+\frac{31}{33}\right)\right].$$

However there are some "problems":

- Polyakov loop correlator is not gauge invariant at $\mathcal{O}(g^4)$.

$\Rightarrow$ The choice of the coulomb gauge seems arbitrary!

- Ψ_C is finite after charge renormalization but not Ψ_ξ nor Ψ_W.

$\Rightarrow$ Ideas to solve that problem welcome!

- Ψ_C, Ψ_ξ have a power law tail $\propto \frac{\alpha^2}{T^2 r^2}$.

$\rightarrow$ Gauge artefact since there is a finite screening length in QGP.

- The gauge invariant Ψ_W decreases like $e^{-m_{\mathrm{D}} r}$.

Free Energy as real part of the potential

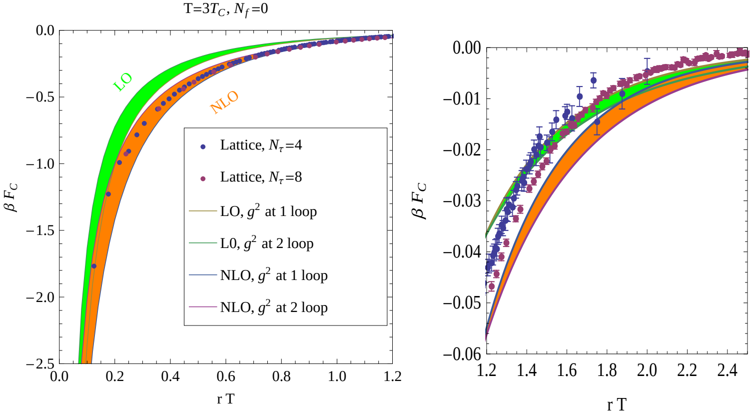
The proper potential hasn't been fully computed to NLO yet

- We cannot make a decisive statement but.
- The nice LO properties do not extend to NLO:
- The free energy is gauge variant.
- Contains artefacts, probably not present in the proper potential.
- If the Free energy matches the potential at $T = 0$ this do not seems to extend to $T \neq 0$.

Perturbation theory breaks down at large r:
$[\Psi_C]^{NLO} > [\Psi_C]^{LO}$ at $r \gg \frac{\pi}{g^2 T}$.

- Probably not so important as large r behaviour is Debye screened.

Comparison with lattice



Perturbative calculation shows a good convergence and fits lattice data very well. [Lattice data from Kaczmarek, Karsch, Petreczky, Zantow, 2002]

4. Conclusion: Free Energy

- The singlet free energy in Coulomb gauge reproduce the correct $Tr \to 0$ behavior.
- This observable might be quite close to the real part of the potential.
- However shows a non physical $1/r^2$ behavior at large distance.

$\Rightarrow$ Using the free energy probably overestimates the binding energy.

- Perturbation theory seems to converge well.

$\Rightarrow$ Computations for the quarkonium decay from perturbative potential should be reliable.

- Motivation to calculate the perturbative potential to $\mathcal{O}(g^4)$.
- Or to get the spectral function by analytical continuation from the lattice euclidean correlator.

Conclusion: Euclidean correlator

- Preliminary work indicates a good match between lattice and perturabtion theory.
- Needed from pertrubation theory:
  - (N)NNLO T=0.
  - Transport peak at NLO.
- Comparison to lattice allows to fix the unknown parameter M.
- Might help for the analytical continuation: The divergent $\tau = 0$ part could be subtracted.

Dimuon spectrum from quarkonium decay now observable in experiment. . .
We should make phenomenological pre(/post)dictions!