Upsilon Suppression at RHIC and LHC

Michael Strickland
Gettysburg College, Gettysburg, PA

RBRC Quarkonium Workshop
June 6, 2011

References:
1101.4651, 1011.3056, 1007.0889, 0903.4703, and forthcoming…

Motivation and Goals I

- Screening → quarkonium suppression in QGP
- Decrease in the real part of the binding energy (E_{bind}) as a function of temperature
- Imaginary part of E_{bind} → thermal width which increases as a function of temperature
- Real and imaginary parts of the heavy quark potential, V, are known to leading order in an isotropic and anisotropic plasma

Isotropic Potential: Laine, Philipsen, Romatschke, and Tassler, hep-ph/0611300;
Anisotropic Potential: Burnier, Laine, Vepsalainen, 0903.3467
Dumitru, Guo, and Strickland, 0903.4703
Philipsen and Tassler, 0908.1746

Motivation and Goals II

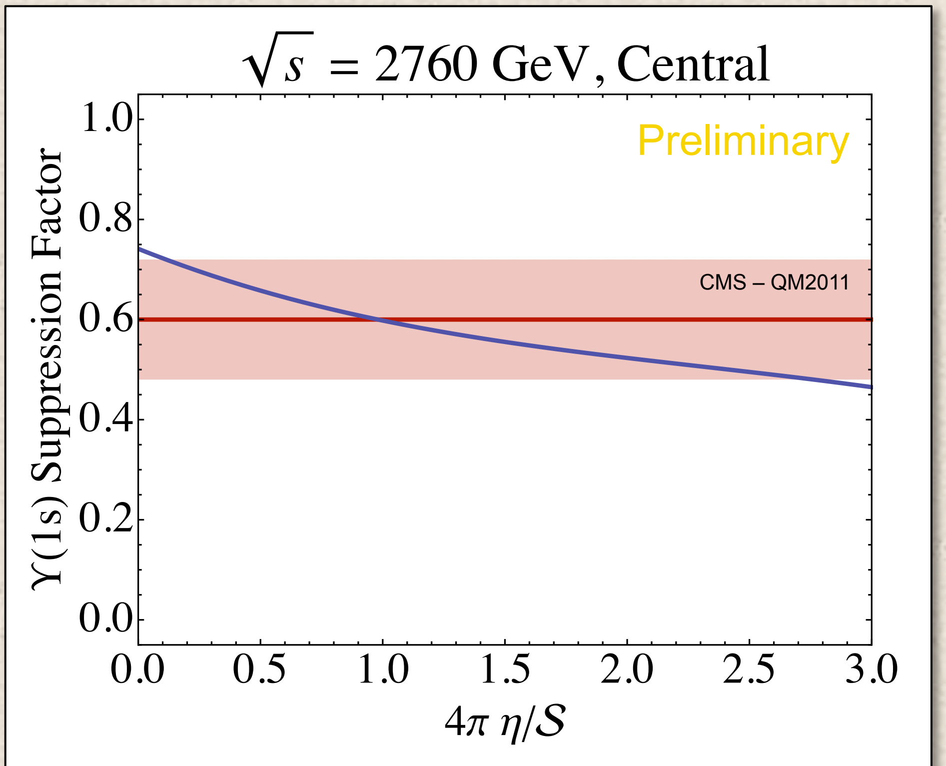
- Can solve Schrodinger equation including both Re[V] and Im[V]
- Gives Re[E_{bind}] and Im[E_{bind}] $\equiv \Gamma$ as function of typical momentum and anisotropy in momentum space
- Evolve system as a function of proper time, rapidity, and transverse coordinates for different QGP viscosities
- Use "Anisotropic Dynamics" method which can describe systems which are highly anisotropic but reduces to 2^{nd} order viscous hydro from small anisotropy

Schrodinger EQ solution: Margotta, McCarty, McGahan, Strickland, and Yager-Elorriaga, 1101.4651
Anisotropic Dynamics: Martinez and Strickland, 1007.0089, 1011.3056

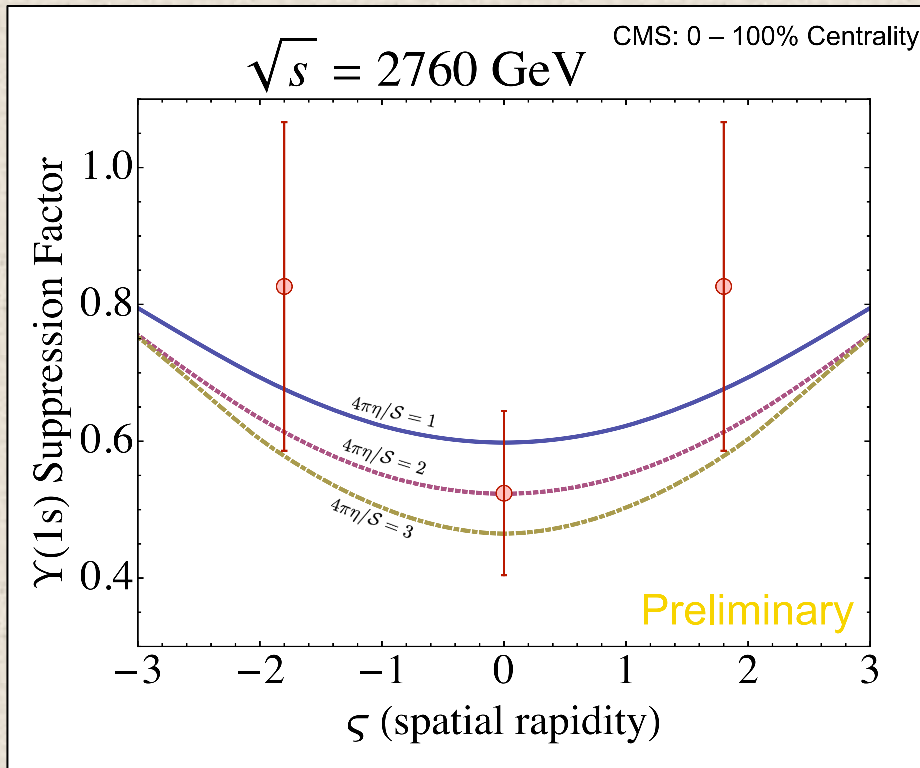
Preview of Results

CMS says:

$\Upsilon(1S)$ R_{AA} in the most central 20%
– 0.60 ± 0.12(stat.) ± 0.10(syst.)

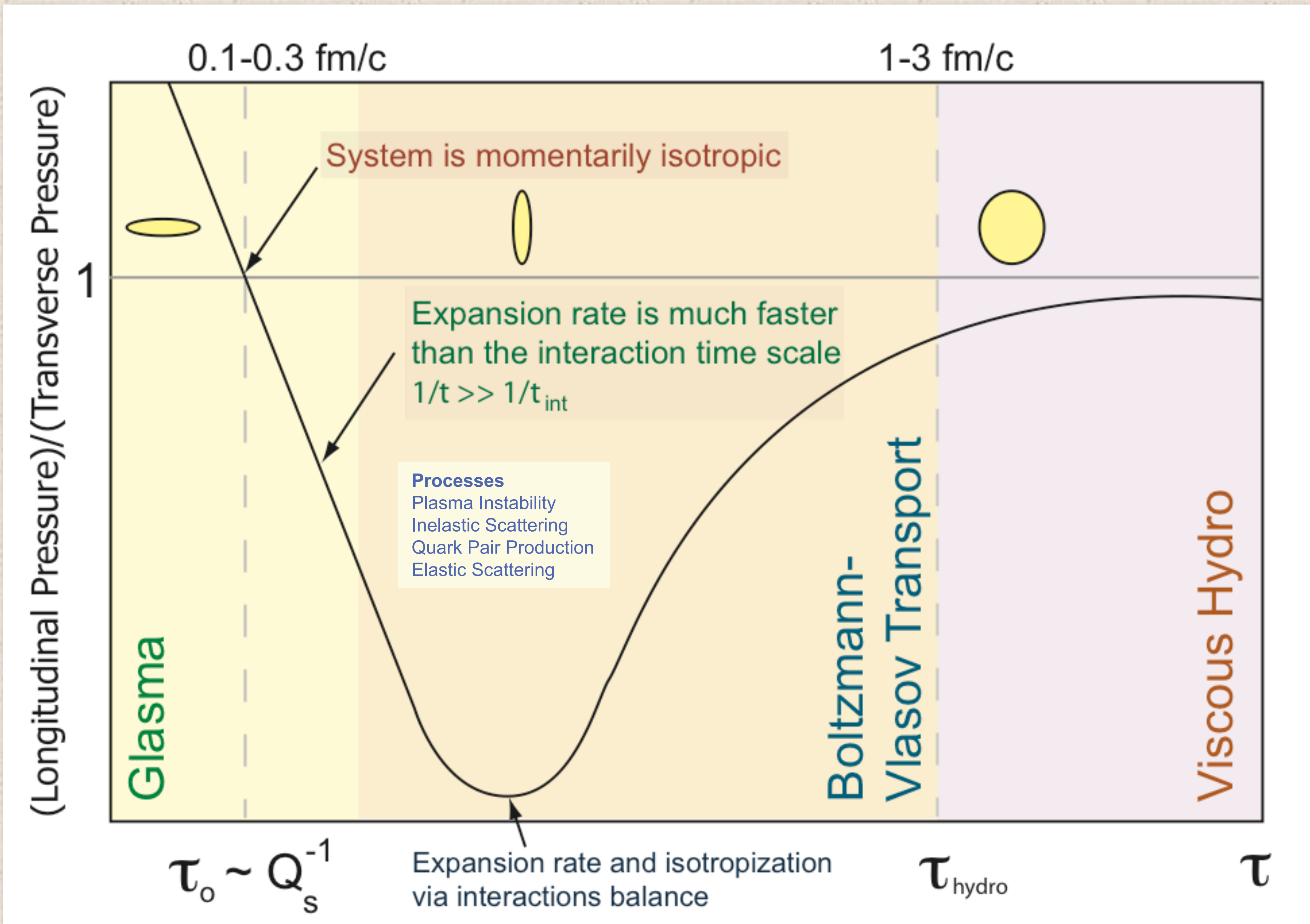


M. Strickland, forthcoming.



M. Strickland, forthcoming.

QGP momentum anisotropy



Anisotropic Plasma

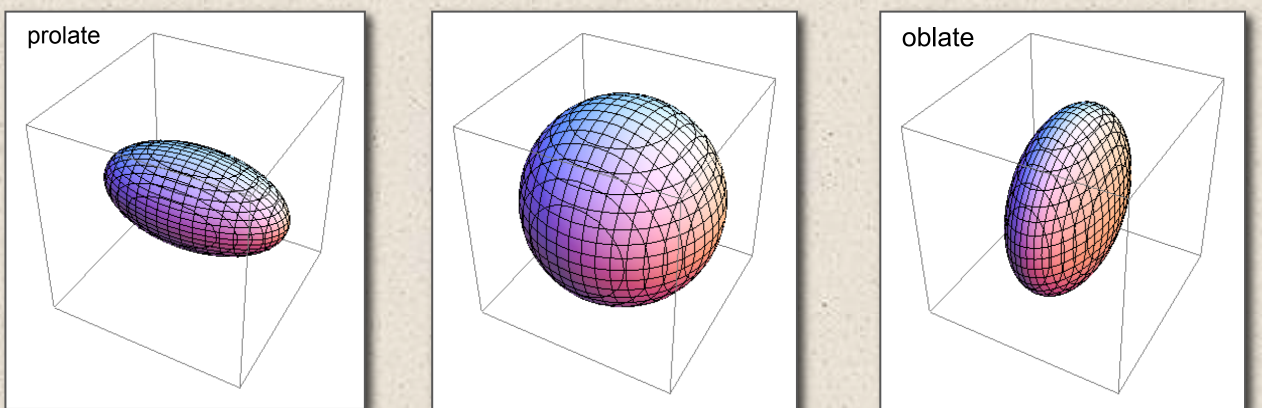
$$f(\tau, \mathbf{x}, \mathbf{p}) = f_{RS}(\mathbf{p}, \xi(\tau), p_{\rm hard}(\tau))$$
$$= f_{\rm iso}([\mathbf{p^2} + \xi(\tau) p_z^2]/p_{\rm hard}^2(\tau))$$

$$\xi = \frac{\langle p_T^2 \rangle}{2\langle p_L^2 \rangle} - 1$$

Small Anisotropy Limit (Thermal f_{iso})

$$f \approx f_{\rm iso}(p)\left[1 - \xi \frac{p_z^2}{2\, p_{\rm hard}\, p}\left(1 \pm f_{\rm iso}(p)\right)\right]$$

Anisotropy parameter, ξ, is related to pressure anisotropy of the system.



$-1 < \xi < 0$ $\quad$ $\xi = 0$ $\quad$ $\xi > 0$

Navier-Stokes Limit

$$\xi \to \frac{10}{T\tau}\frac{\eta}{S}$$

Perturbative Anisotropic Potential

Using real-time formalism can express potential in terms of average of static advanced and retarded propagators

$$V(\mathbf{r},\xi) = -g^2 C_F \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left(e^{i\mathbf{p}\cdot\mathbf{r}} - 1\right) \frac{1}{2} \left(D^{*L}_{R} + D^{*L}_{A} \right)$$

Real part can be written

$$\mathrm{Re}[V(\mathbf{r},\xi)] = -g^2 C_F \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left(e^{i\mathbf{p}\cdot\mathbf{r}} - 1\right) \frac{\mathbf{p}^2 + m_\alpha^2 + m_\gamma^2}{(\mathbf{p}^2 + m_\alpha^2 + m_\gamma^2)(\mathbf{p}^2 + m_\beta^2) - m_\delta^4}$$

With direction-dependent masses, e.g.

$$m_\alpha^2 = -\frac{m_D^2}{2p_\perp^2\sqrt{\xi}} \left(p_z^2 \arctan\sqrt{\xi} - \frac{p_z \mathbf{p}^2}{\sqrt{\mathbf{p}^2 + \xi p_\perp^2}} \arctan \frac{\sqrt{\xi} p_z}{\sqrt{\mathbf{p}^2 + \xi p_\perp^2}} \right)$$

Potential calculation: Dumitru, Guo, and Strickland, 0903.4703
Gluon propagator in anisotropic plasma: Romatschke and Strickland, hep-ph/0304092

Perturbative Parameterization

Efficient parameterization of the potential can be constructed by introducing a direction-dependent Debye mass such that:

M. Strickland, forthcoming.

$$V(r,\theta,\xi,p_{\rm hard}) = -C_F \alpha_s \frac{e^{-\mu(\theta,\xi,p_{\rm hard})r}}{r}$$

$$\mu(\theta,\xi,p_{\rm hard}) = m_D \left[1+\xi\left(1+\frac{\sqrt{2}(1+\xi)^2\,(\cos(2\theta)-1)}{(2+\xi)^{5/2}}\right)\right]^{-1/4}$$

$$m_D^2 = (1.4)^2(1+N_f/6)g_s^2\,p_{\rm hard}^2$$

Factor of $(1.4)^2$ from lattice fit of Kaczmarek et al, 2004

This form reproduces (to acceptable accuracy) the small and large ξ limits of the perturbative anisotropic potential

But … pQCD is not enough

- Need to supplement the perturbative (short range) potential with an appropriate long range potential
- We use a generalization of the Karsch-Mehr-Satz (KMS) potential which has had the entropy contribution removed

$$V_{\rm R}(\mathbf{r}) = -\frac{\alpha}{r}\,(1+\mu\,r)\exp\left(-\mu\,r\right) + \frac{2\sigma}{\mu}\left[1-\exp\left(-\mu\,r\right)\right] - \sigma\,r\,\exp(-\mu\,r) - \frac{0.8\,\sigma}{m_Q^2\,r}$$

- With $\alpha = 0.385$ and $\sigma = 0.223$ GeV

KMS potential: Karsch, Mehr, and Satz, Z. Phys. C37, 617 (1988)
Isotropic case: Mocsy and Petreczky, 0705.2558
Anisotropic case: Dumitru, Guo, Mocsy, and Strickland, 0901.1998

Imaginary Part of the Potential

- So far only able to evaluate the imaginary part of the potential in the small anisotropy limit:

$$\boxed{V_{\mathrm{I}}(\mathbf{r}) = -C_F \alpha_s p_{\mathrm{hard}} \left[\phi(\hat{r}) - \xi \left(\psi_1(\hat{r},\theta) + \psi_2(\hat{r},\theta) \right) \right]}$$

$$\phi(\hat{r}) = 2\int_0^\infty dz \frac{z}{(z^2+1)^2}\left[1 - \frac{\sin(z\,\hat{r})}{z\,\hat{r}}\right] ,$$

$$\psi_1(\hat{r},\theta) = \int_0^\infty dz \frac{z}{(z^2+1)^2}\left(1 - \frac{3}{2}\left[\sin^2\theta \frac{\sin(z\,\hat{r})}{z\,\hat{r}} + (1-3\cos^2\theta) G(\hat{r},z)\right]\right) ,$$

$$\psi_2(\hat{r},\theta) = -\int_0^\infty dz \frac{\frac{4}{3}z}{(z^2+1)^3}\left(1 - 3\left[\left(\frac{2}{3} - \cos^2\theta\right)\frac{\sin(z\,\hat{r})}{z\,\hat{r}} + (1-3\cos^2\theta) G(\hat{r},z)\right]\right)$$

- With $\hat{r} \equiv m_D r$ and $G(\hat{r},z) = \dfrac{\hat{r}z\cos(\hat{r}z) - \sin(\hat{r}z)}{(\hat{r}z)^3}$

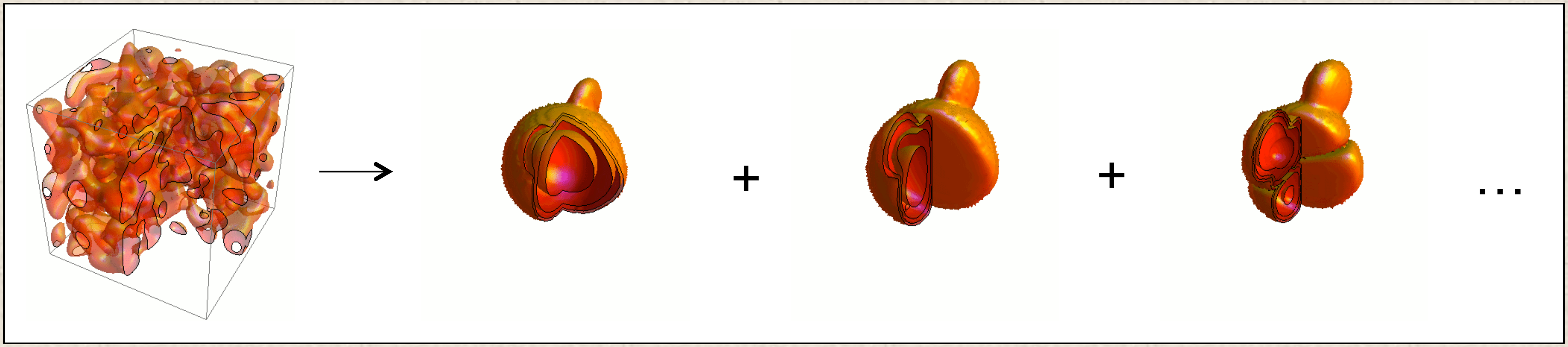
- Analytically → linear combination of hypergeometric functions

Solving the Schrodinger EQ

- Need to solve the 3d Schrodinger EQ
- Transform EQ to imaginary time, start with random wave function, and evolve to large imaginary time

$$\frac{\partial}{\partial\tau}\Psi(\vec{r},\tau) = \frac{1}{2}\nabla^2\Psi(\vec{r},\tau) - V(\vec{r})\Psi(\vec{r},\tau)$$

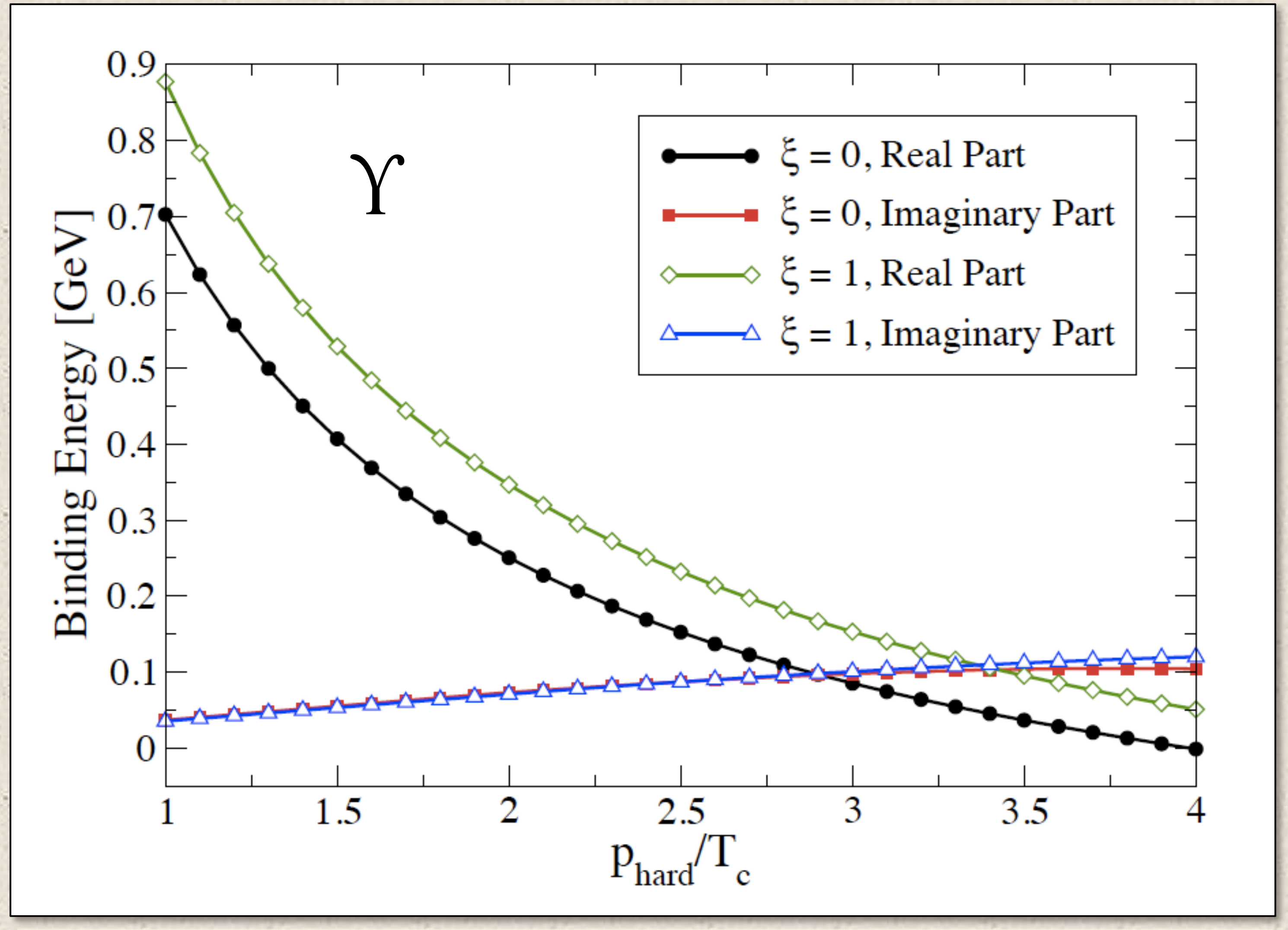
$$\Psi(\vec{r},\tau) = \sum_{n=0}^{\infty} a_n \psi_n(\vec{r}) e^{-E_n \tau}$$



For details see Strickland and Yager-Elorriaga, A Parallel Algorithm for Solving the 3d Schrodinger Equation, Journal of Computational Physics 229, 6015 (2010).

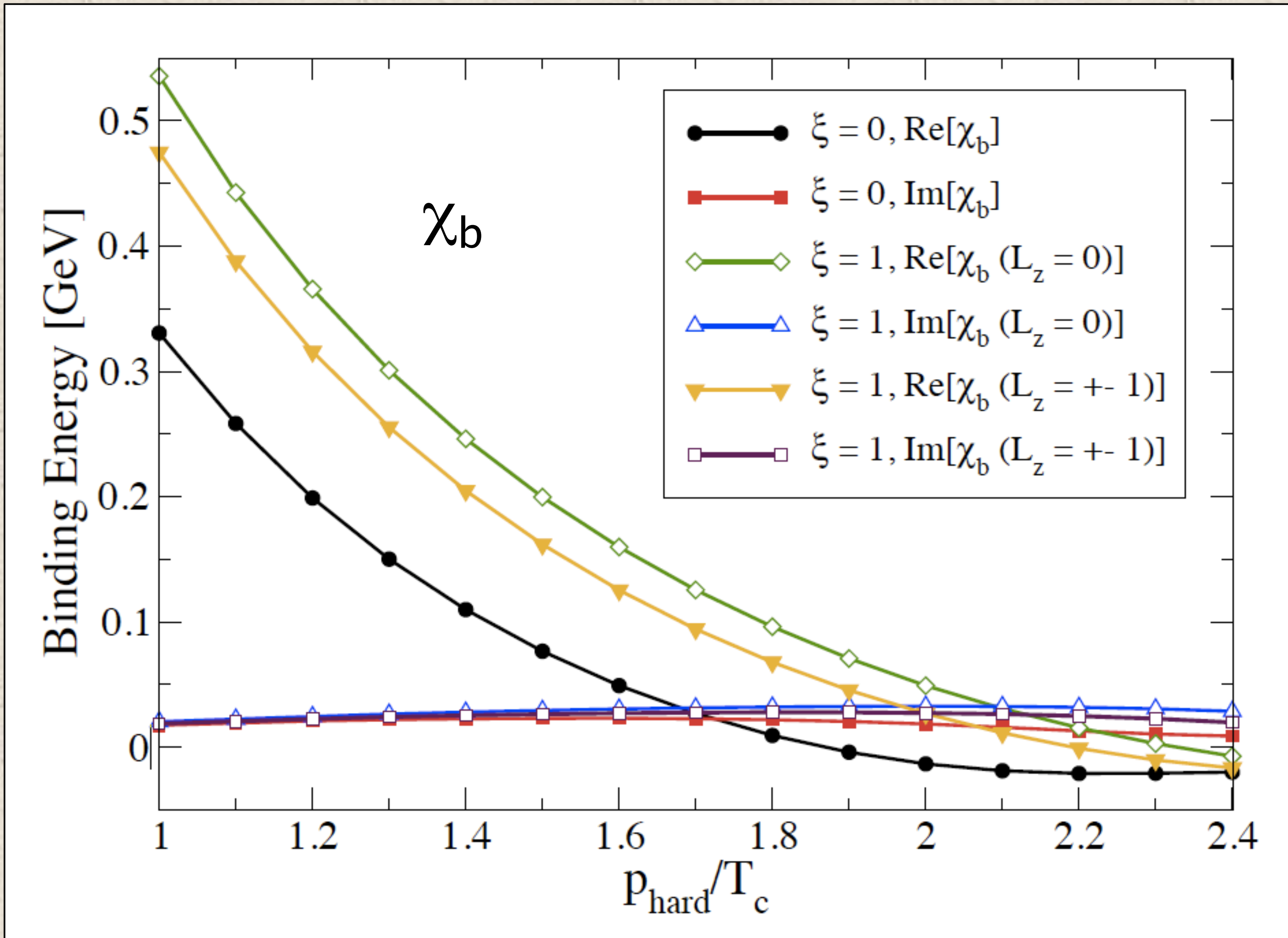
Results for the Υ binding energy

First consider the case that we hold p_{hard} fixed as we vary ξ



Margotta, McCarty, McGahan, Strickland, and Yager-Elorriaga, 1101.4651

Results for the χ_b binding energy



Margotta, McCarty, McGahan, Strickland, and Yager-Elorriaga, 1101.4651

Near Equilibrium QGP Evolution

- If the system is close to equilibrium and has pressures in the local rest frame which are approximately isotropic ($P_T \cong P_L$) then we might try to use relativistic viscous hydrodynamics

$$T^{\mu\nu} = T^{\mu\nu}_{ideal} + \Pi^{\mu\nu}$$

- The ideal stress tensor is thermal and isotropic
- Large amplitudes of the shear tensor compared to the ideal stress tensor indicate a problem with the hydrodynamic expansion itself

Relativistic Hydro from Transport

- Describe evolution of the system using the Boltzmann equation

$$p^\alpha \partial_\alpha f = -C[f]$$

C[f] = Collisional Kernel

- Can extract hydro equations from the Boltzmann equation by taking "moments" of the equation using an integral operator

$$\hat{I}^{\mu\nu\cdots\sigma} \equiv \int \frac{d^3p}{2E}\, p^\mu p^\nu \cdots p^\sigma$$

eg.

$$\hat{I} \equiv \int \frac{d^3p}{2E}$$

0th moment operator

$$\hat{I}^\mu \equiv \int \frac{d^3p}{2E}\, p^\mu$$

1st moment operator

0^{th} Moment

N^α : Particle Number and Current

$$\boxed{\partial_\alpha N^\alpha = -\int \frac{d^3p}{E} C[f]}$$

if number conserving $\longrightarrow$ collisional kernel

$$\boxed{\partial_\alpha N^\alpha = 0}$$

Number conservation

If particle number changing processes in kernel, eg 2 → 3, RHS is nonzero

1^{st} Moment

$T^{\alpha\beta}$: Energy-Momentum Tensor

$$\boxed{\partial_\alpha T^{\alpha\beta} = -\int \frac{d^3p}{E} p^\beta C[f]}$$

if energy conserving $\longrightarrow$ collisional kernel

$$\boxed{\partial_\alpha T^{\alpha\beta} = 0}$$

Energy-momentum conservation!

2^{nd} Moment and generalities

• The first two moments are enough to generate equations of motion for ideal hydrodynamics.

• Canonically the second moment gives the first non-trivial (dissipative) equation of motion and can be used to derive 2nd-order viscous hydro using transport theory.

• If the system is homogeneous in the transverse directions, the energy-momentum tensor in the local rest frame has the following form

$$T^{\alpha\beta} = \begin{pmatrix} \mathcal{E} & 0 & 0 & 0 \\ 0 & \mathcal{P}_T & 0 & 0 \\ 0 & 0 & \mathcal{P}_T & 0 \\ 0 & 0 & 0 & \mathcal{P}_L \end{pmatrix}$$

$\mathcal{E}$: Energy Density

$\mathcal{P}_T$: Transverse Pressure

$\mathcal{P}_L$: Longitudinal Pressure

Boost Invariant 1d Hydro

- Consider a boost invariant system that is homogeneous in the transverse directions.
- Expand the energy momentum tensor to first order around an isotropic state → 1d viscous hydro.
- The 1d second order viscous hydro equations can be written in terms of the isotropic energy density/pressure and the rapidity-rapidity (ς-ς) component of the shear tensor $\Pi = \Pi^{\varsigma}{}_{\varsigma}$.
- Can also include bulk viscosity; In QCD comes from breaking of conformality due to running coupling; maximal effect near T_c

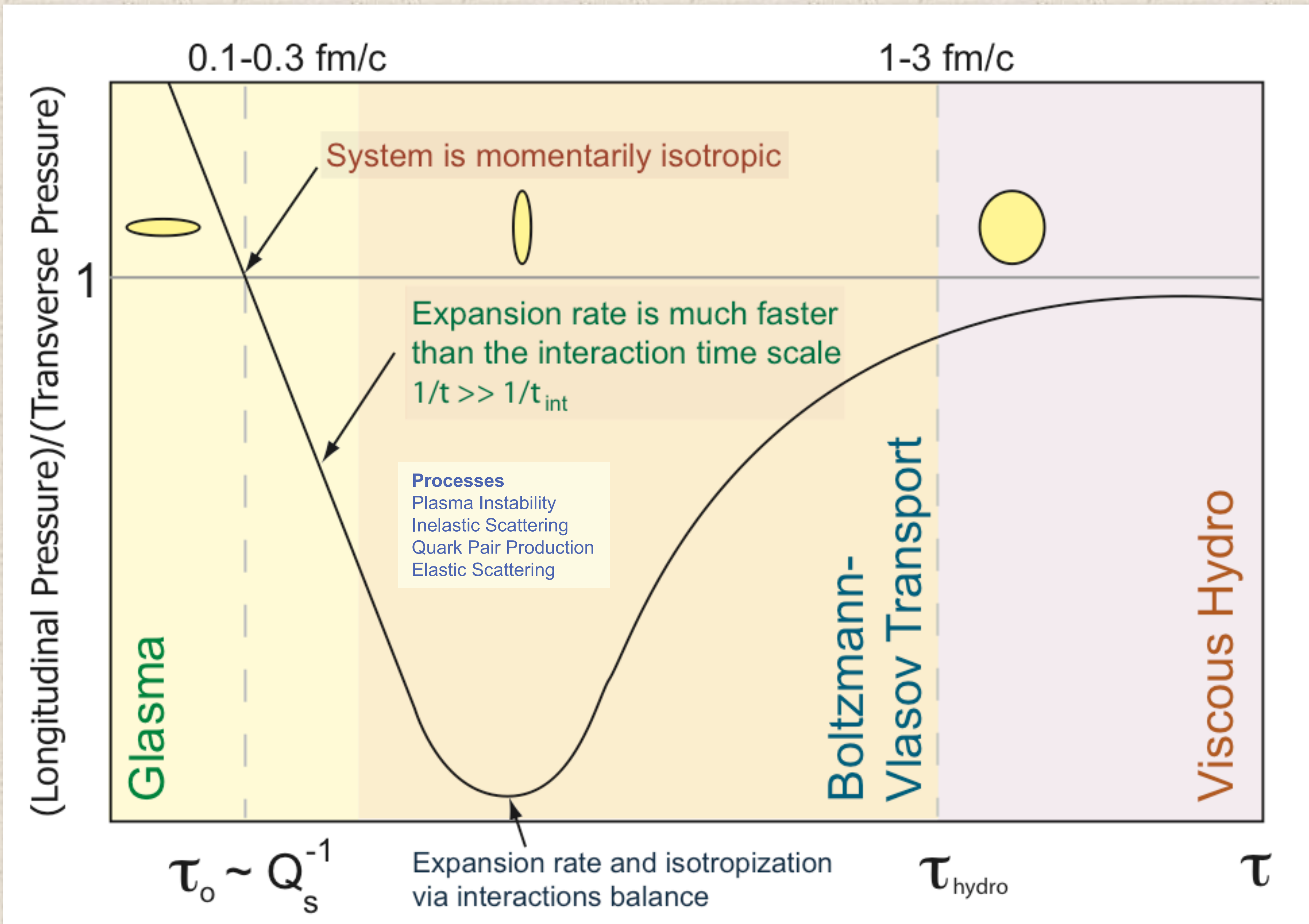
$$\partial_\tau \mathcal{E} = -\frac{\mathcal{E}+\mathcal{P}}{\tau} + \frac{\Pi}{\tau}$$

$$\partial_\tau \Pi = -\frac{\Pi}{\tau_\pi} + \frac{4}{3}\frac{\eta}{\tau_\pi \tau} - \frac{4}{3}\frac{\Pi}{\tau}$$

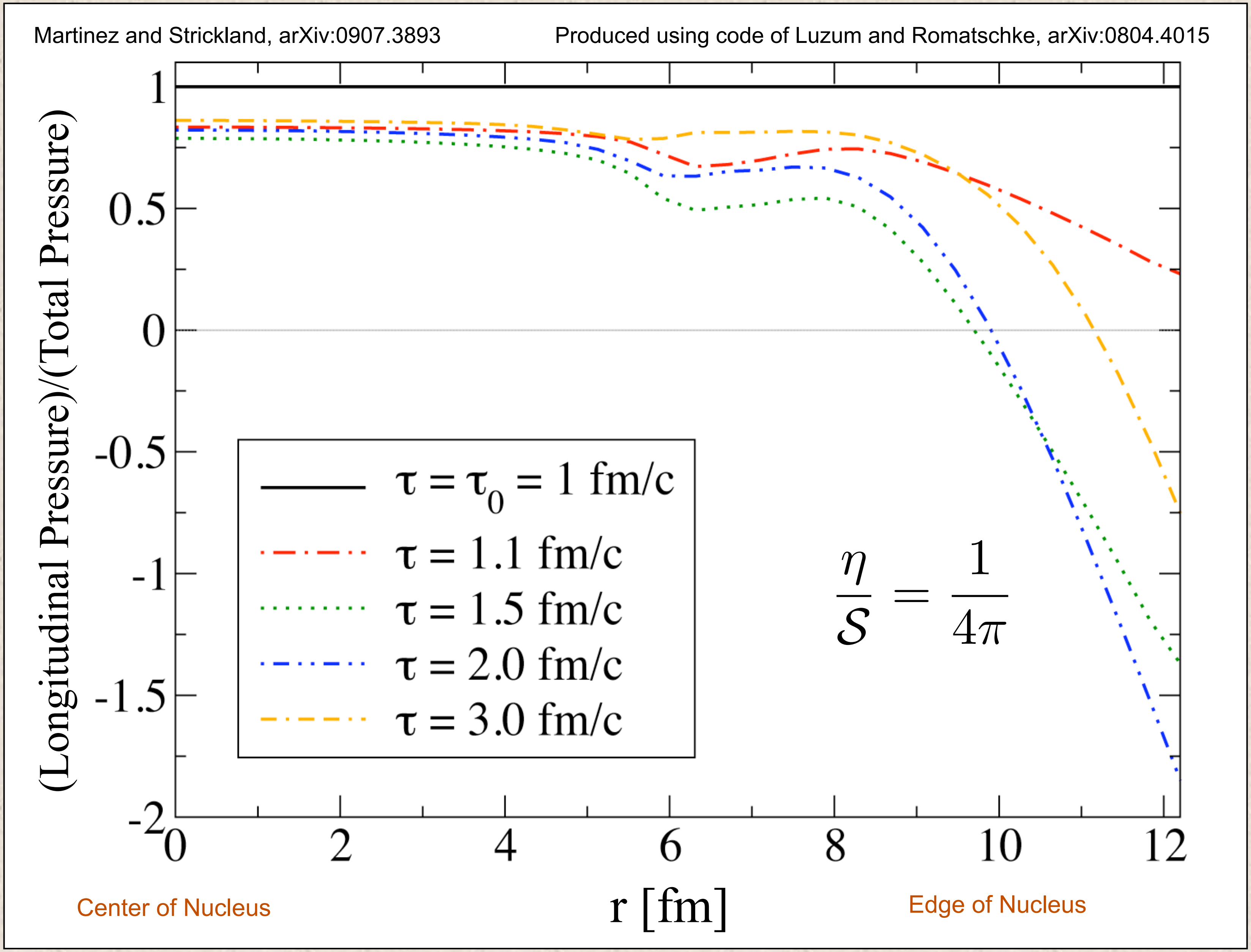
$\eta =$ Shear viscosity

$\tau_\pi =$ Shear relaxation time

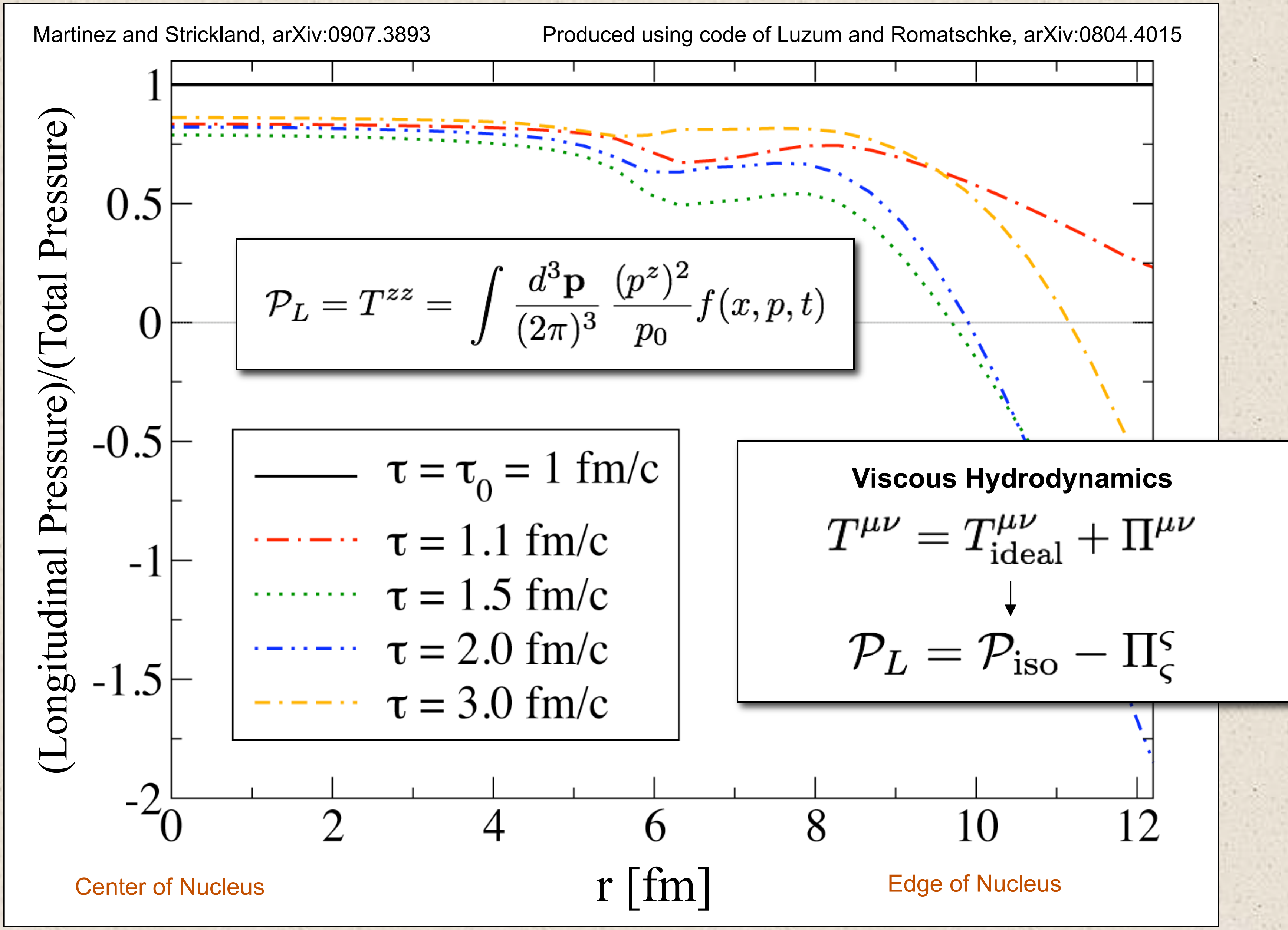
QGP momentum anisotropy



Hydro Results - Strong Coupling



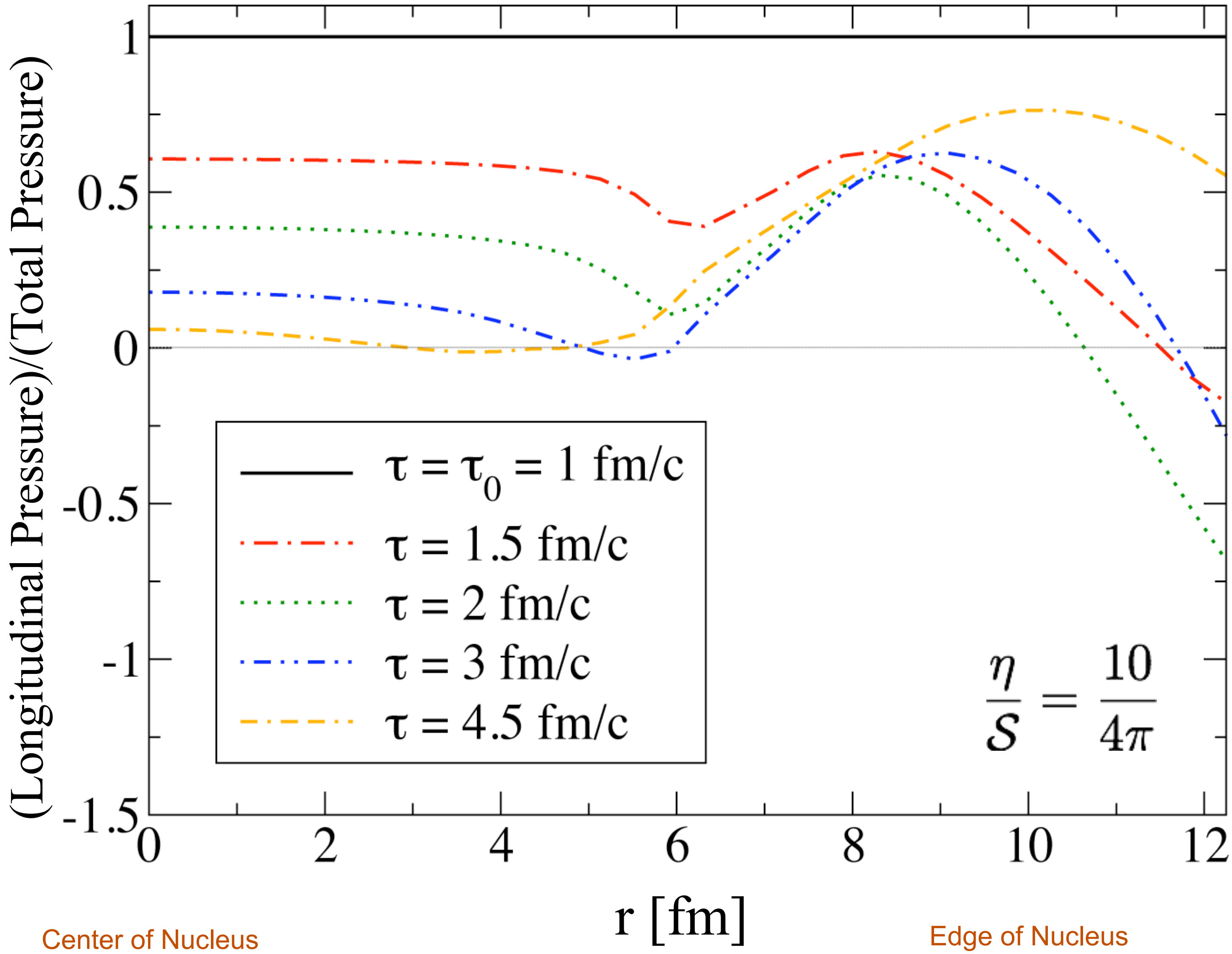
Hydro Results - Strong Coupling



Hydro Results - Weak Coupling

Martinez and Strickland, arXiv:0907.3893

Produced using code of Luzum and Romatschke, arXiv:0804.4015



Center of Nucleus

Edge of Nucleus

Start over from scratch

Viscous Hydrodynamics Expansion

$$\boxed{f(\mathbf{x},\mathbf{p},\tau) = \underline{f_{\rm eq}(|\mathbf{p}|,T(\tau))} + \delta f_1 + \delta f_2 + \cdots}$$

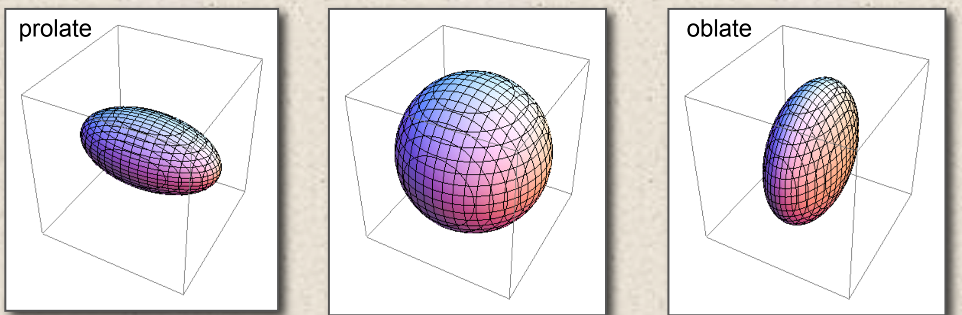
Isotropic in momentum space

Anisotropic Dynamics (**AD**) Expansion

$$\boxed{f(\mathbf{x},\mathbf{p},\tau) = f_{\rm aniso}(\mathbf{p},p_{\rm hard}(\tau),\xi(\tau)) + \delta f_1' + \delta f_2' + \cdots}$$

$$\xi = \frac{\langle p_T^2\rangle}{2\langle p_L^2\rangle} - 1$$

$$\begin{aligned} f(\tau,\mathbf{x},\mathbf{p}) &= f_{RS}(\mathbf{p},\xi(\tau),p_{\rm hard}(\tau)) \\ &= f_{\rm iso}([\mathbf{p^2}+\xi(\tau)p_z^2]/p_{\rm hard}^2(\tau)) \end{aligned}$$



$-1 < \xi < 0$ $\qquad$ $\xi = 0$ $\qquad$ $\xi > 0$

Collisional Kernel

$$p^\alpha \partial_\alpha f = -C[f]$$

- Relaxation time approximation

$$\mathcal{C}[f(t,z,\mathbf{p})] = p_\mu u^\mu \, \Gamma \, [f(t,z,\mathbf{p}) - f_{\rm eq}(t,z,|\mathbf{p}|,T(\tau))]$$

- Where Γ is the relaxation rate
- Γ is fixed by matching to 2nd order viscous hydro in the weak anisotropy limit
- $T(\tau)$ is the self-consistent isotropic temperature which we fix by requiring energy conservation at all proper times

Using relaxation-time approximation scattering kernel gives

0th Moment of Boltzmann EQ

$$\frac{1}{1+\xi}\partial_\tau \xi - \frac{2}{\tau} - \frac{6}{p_{\rm hard}}\partial_\tau p_{\rm hard} = 2\Gamma\left[1 - \mathcal{R}^{3/4}(\xi)\sqrt{1+\xi}\right]$$

1st Moment of Boltzmann EQ

$$\frac{\mathcal{R}'(\xi)}{\mathcal{R}(\xi)}\partial_\tau \xi + \frac{4}{p_{\rm hard}}\partial_\tau p_{\rm hard} = \frac{1}{\tau}\left[\frac{1}{\xi(1+\xi)\mathcal{R}(\xi)} - \frac{1}{\xi} - 1\right]$$

where

$$\mathcal{R}(\xi) \equiv \frac{1}{2}\left(\frac{1}{1+\xi} + \frac{\arctan\sqrt{\xi}}{\sqrt{\xi}}\right)$$

Linearized Equations

If we linearize everything by keeping the lowest non-vanishing order in the anisotropy parameter we find the following relation

$$\frac{\Pi}{\mathcal{E}_{\rm eq}} = \frac{8}{45}\xi + \mathcal{O}(\xi^2)$$

and the coupled nonlinear differential equations reduce to

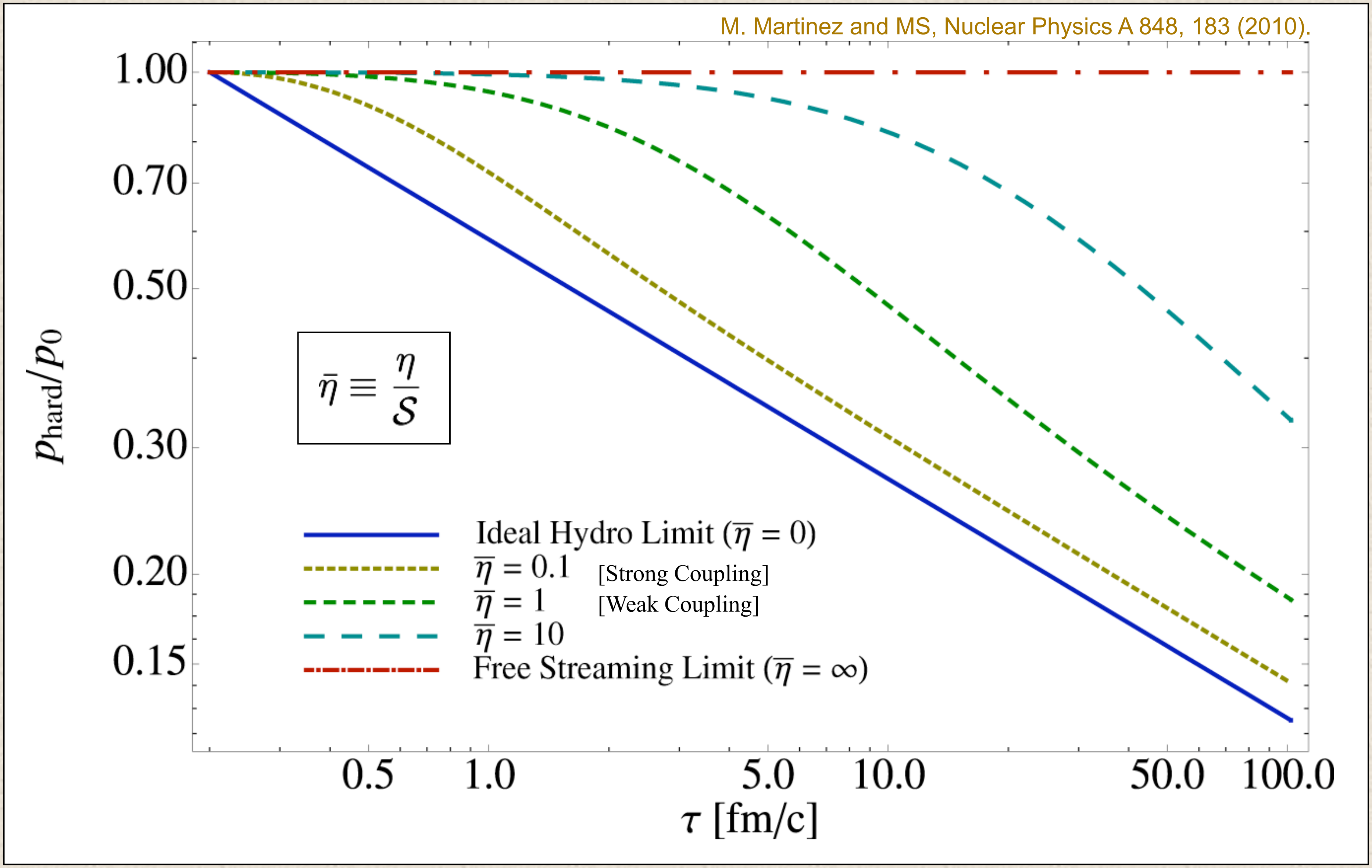
$$\boxed{\begin{aligned} \partial_\tau \mathcal{E} &= -\frac{\mathcal{E}+\mathcal{P}}{\tau} + \frac{\Pi}{\tau} \\ \partial_\tau \Pi &= -\frac{\Pi}{\tau_\pi} + \frac{4}{3}\frac{\eta}{\tau_\pi \tau} - \frac{4}{3}\frac{\Pi}{\tau} \end{aligned}} \qquad \boxed{\begin{aligned} \Gamma &= \frac{2}{\tau_\pi} \\ \tau_\pi &= \frac{5}{4}\frac{\eta}{\mathcal{P}} \end{aligned}}$$

Reproduces 2nd order viscous hydro in small anisotropy limit!

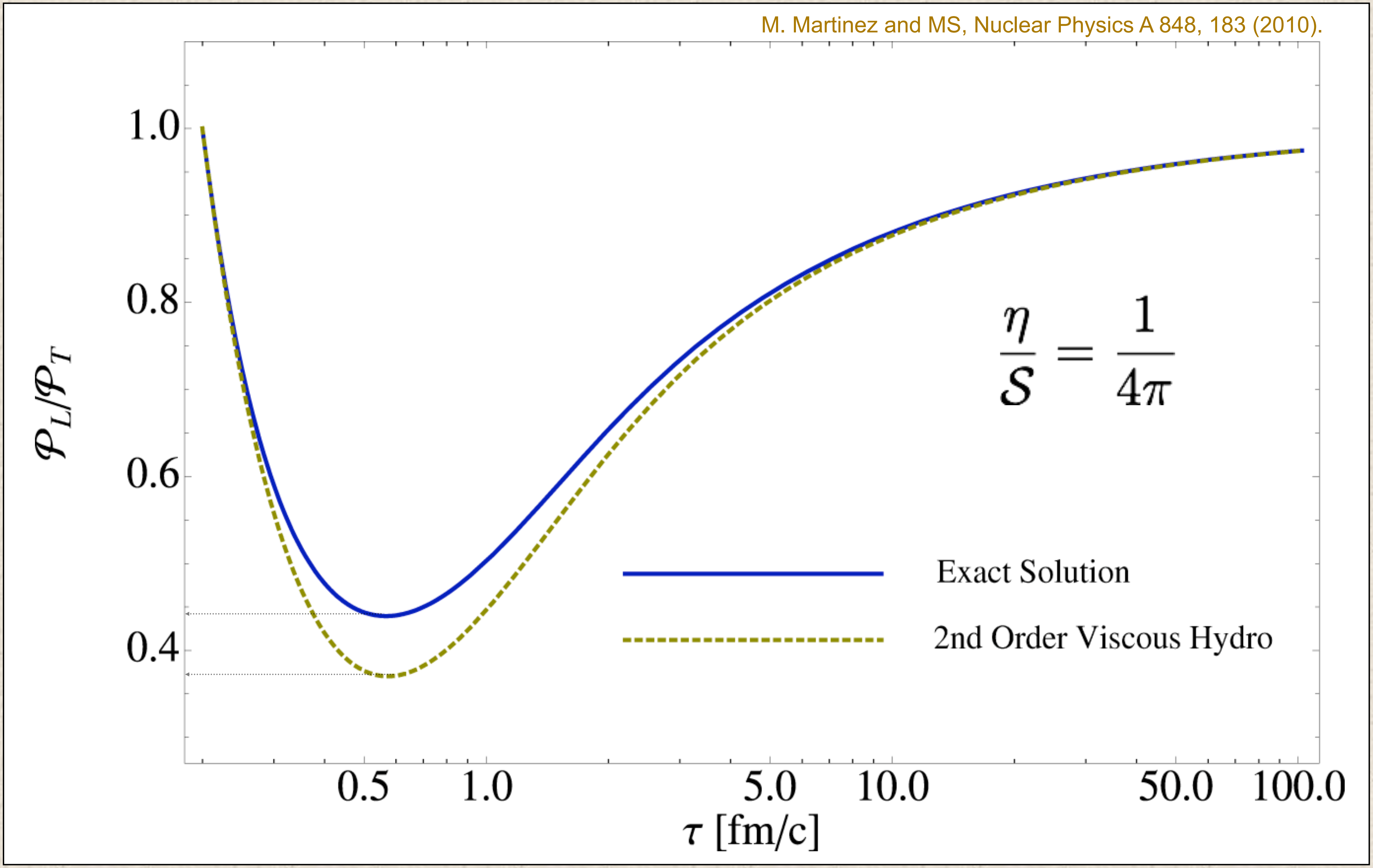
Hard Momentum vs Time

$$\frac{1}{1+\xi}\partial_\tau \xi - \frac{2}{\tau} - \frac{6}{p_{\rm hard}}\partial_\tau p_{\rm hard} = 2\Gamma\left[1 - \mathcal{R}^{3/4}(\xi)\sqrt{1+\xi}\right]$$

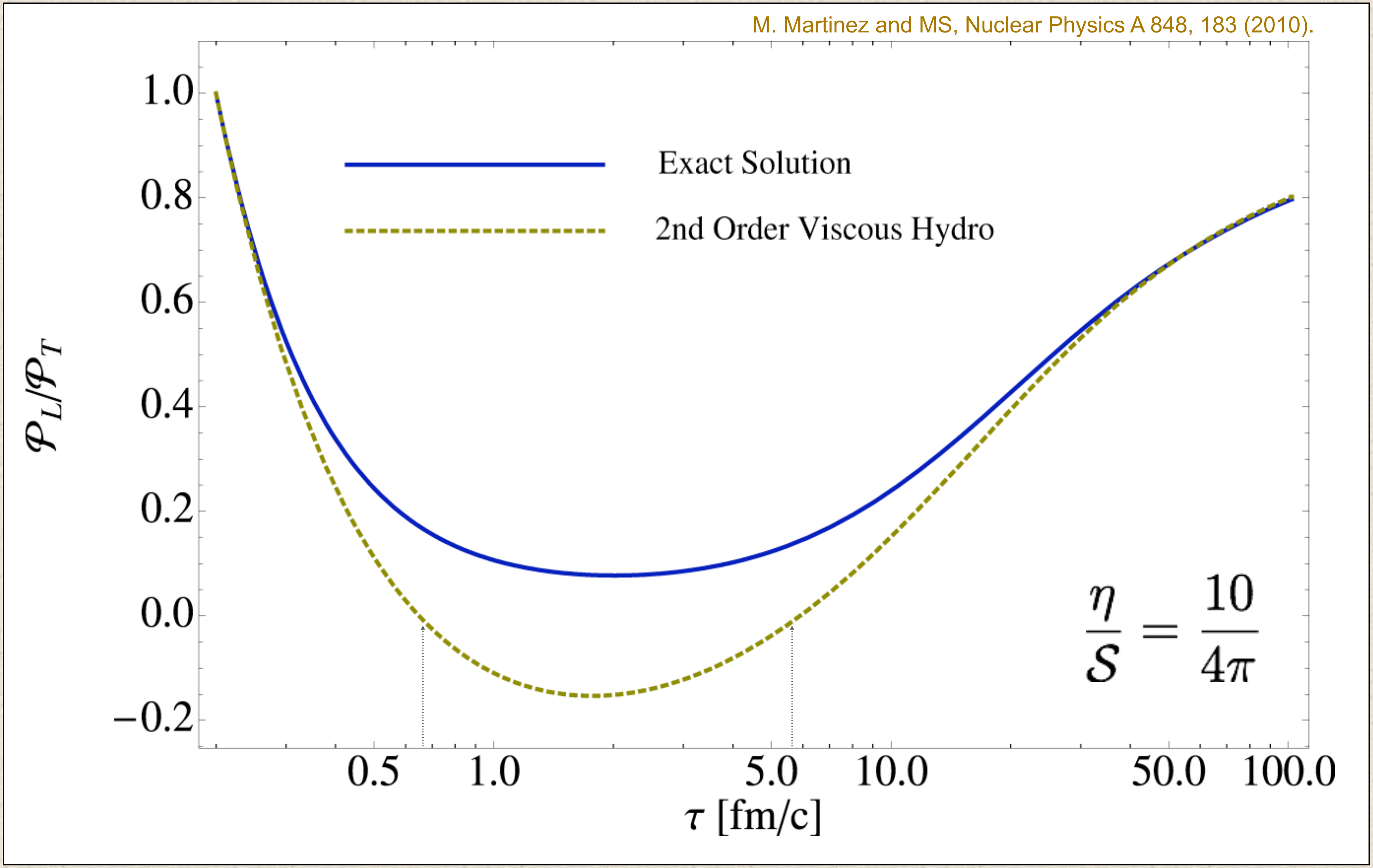
$$\frac{\mathcal{R}'(\xi)}{\mathcal{R}(\xi)}\partial_\tau \xi + \frac{4}{p_{\rm hard}}\partial_\tau p_{\rm hard} = \frac{1}{\tau}\left[\frac{1}{\xi(1+\xi)\mathcal{R}(\xi)} - \frac{1}{\xi} - 1\right]$$



Hydro vs AD : Strong Coupling



Hydro vs AD : Weak Coupling



Breaking Boost Invariance

M. Martinez and MS, Nuclear Physics A 856, 68 (2011).

$$\xi(\tau) \to \xi(\tau,\varsigma)$$
$$p_{\rm hard}(\tau) \to p_{\rm hard}(\tau,\varsigma)$$
$$u^\mu = (\cosh(\varsigma),\mathbf{0},\sinh(\varsigma)) \to (\cosh(\theta(\tau,\varsigma)),\mathbf{0},\sinh(\theta(\tau,\varsigma)))$$

Need three dynamical equations. ξ, $p_{\rm hard,}$ and θ are functions of proper time and spatial rapidity, ς.

0th moment

$$\frac{1}{1+\xi}\left(\partial_\tau\xi - \frac{2(1+\xi)}{\tau}\partial_\varsigma\vartheta\right) - \frac{6}{p_{\rm hard}}\partial_\tau p_{\rm hard} = 2\Gamma\left[1-\mathcal{R}^{3/4}(\xi)\sqrt{1+\xi}\right]$$

1st moment

Contract with u^μ

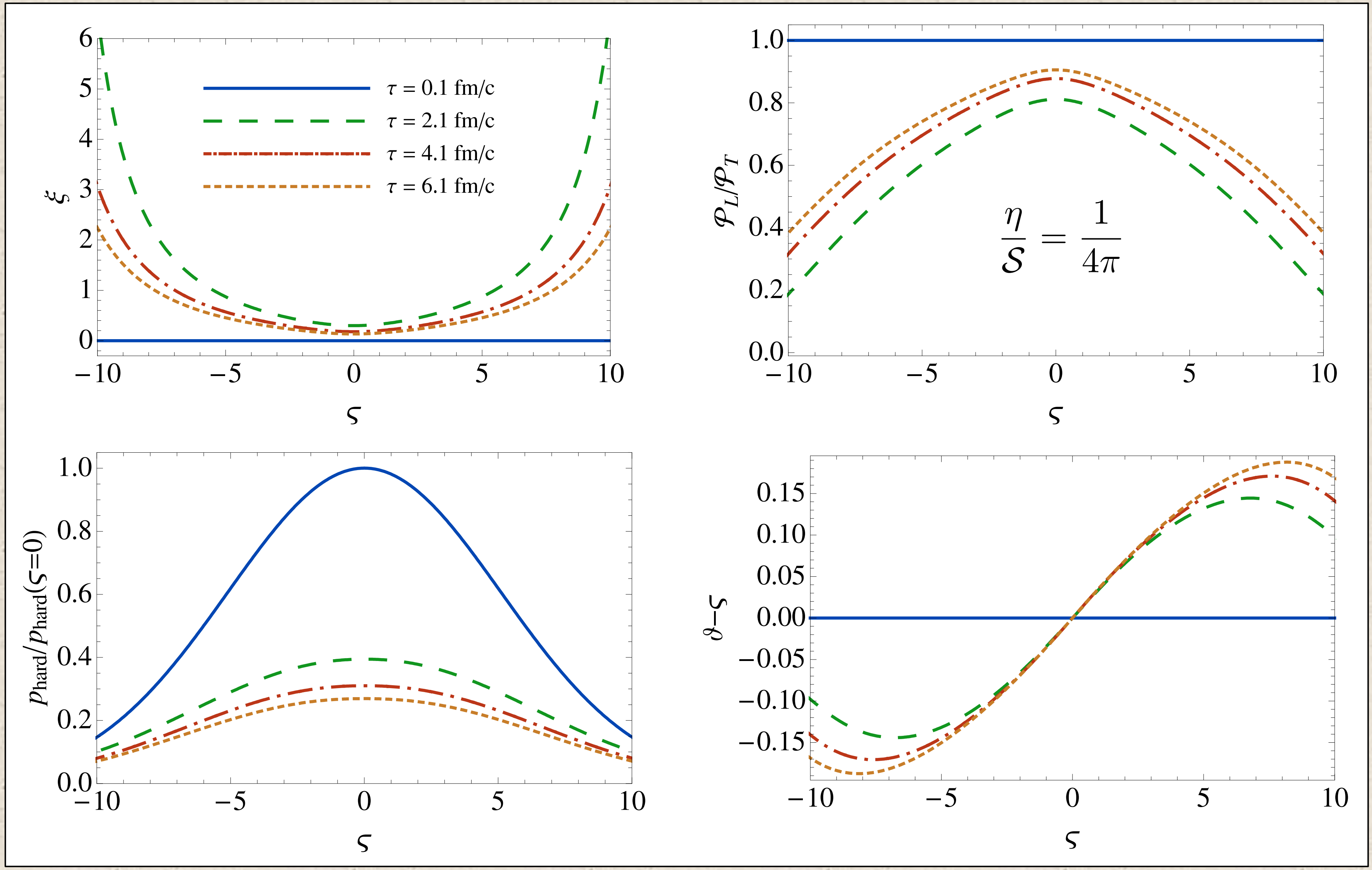
$$\frac{\mathcal{R}'(\xi)}{\mathcal{R}(\xi)}\partial_\tau\xi + 4\frac{\partial_\tau p_{\rm hard}}{p_{\rm hard}} + \frac{\tanh(\vartheta-\varsigma)}{\tau}\left(\frac{\mathcal{R}'(\xi)}{\mathcal{R}(\xi)}\partial_\varsigma\xi + 4\frac{\partial_\varsigma p_{\rm hard}}{p_{\rm hard}}\right) = -\left(1+\frac{1}{3}\frac{\mathcal{R}_L(\xi)}{\mathcal{R}(\xi)}\right)\left(\tanh(\vartheta-\varsigma)\,\partial_\tau + \frac{\partial_\varsigma}{\tau}\right)\vartheta\,,$$

Contract with vector orthogonal to u^μ

$$\tanh(\vartheta-\varsigma)\left(\frac{\mathcal{R}_L'(\xi)}{\mathcal{R}_L(\xi)}\partial_\tau\xi + 4\frac{\partial_\tau p_{\rm hard}}{p_{\rm hard}}\right) + \frac{1}{\tau}\left(\frac{\mathcal{R}_L'(\xi)}{\mathcal{R}_L(\xi)}\partial_\varsigma\xi + 4\frac{\partial_\varsigma p_{\rm hard}}{p_{\rm hard}}\right) = -\left(3\frac{\mathcal{R}(\xi)}{\mathcal{R}_L(\xi)}+1\right)\left(\partial_\tau + \frac{\tanh(\vartheta-\varsigma)}{\tau}\partial_\varsigma\right)\vartheta\,,$$

Non-Boost Invariant Evolution

M. Martinez and MS, Nuclear Physics A 856, 68 (2011).



Putting the pieces together…

- We have the imaginary part of the binding energy energy as a function of p_{hard} and ξ
- And now we know p_{hard} and ξ as a function of proper time and spatial rapidity
- **Missing piece**: transverse coordinate dependence!
- We will model this by considering central collisions in which the temperature profile is a Gaussian given by

$$\boxed{T(r,\varsigma) = T_{\rm central}(\varsigma)\, e^{-\frac{1}{6}(r/a)^2} \qquad a = 3 \text{ fm}}$$

- Radial dependence is important since the thermal width approaches zero at the edges of the plasma

The suppression factor

- Resulting width is a function of τ, r, and ς. First we need to integrate of proper time

$$\overline{\Gamma}(r,\varsigma) = \int_{\tau_0}^{\tau_f} d\tau\, \Gamma(\tau,r,\varsigma)$$

- Next we should geometrically average over r

$$\langle \overline{\Gamma}(\varsigma) \rangle \equiv \frac{\int_0^{r_c} r dr\, \overline{\Gamma}(r,\varsigma)}{\frac{1}{2} r_c^2}$$

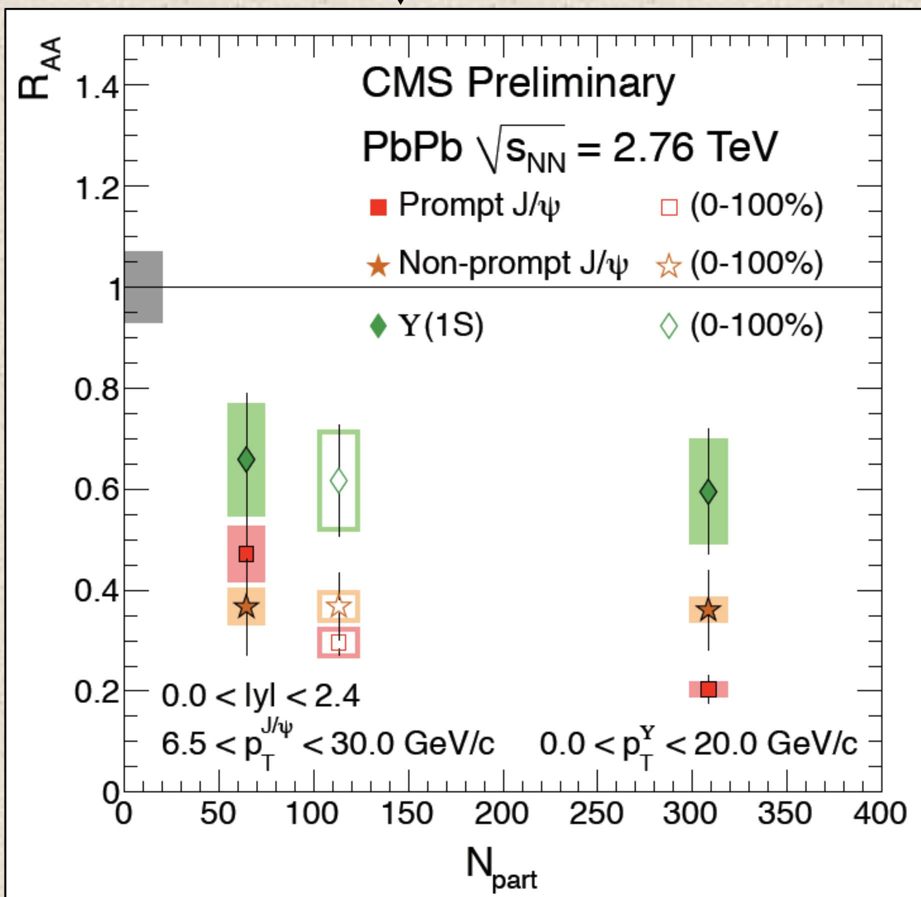
- Suppression factor is then

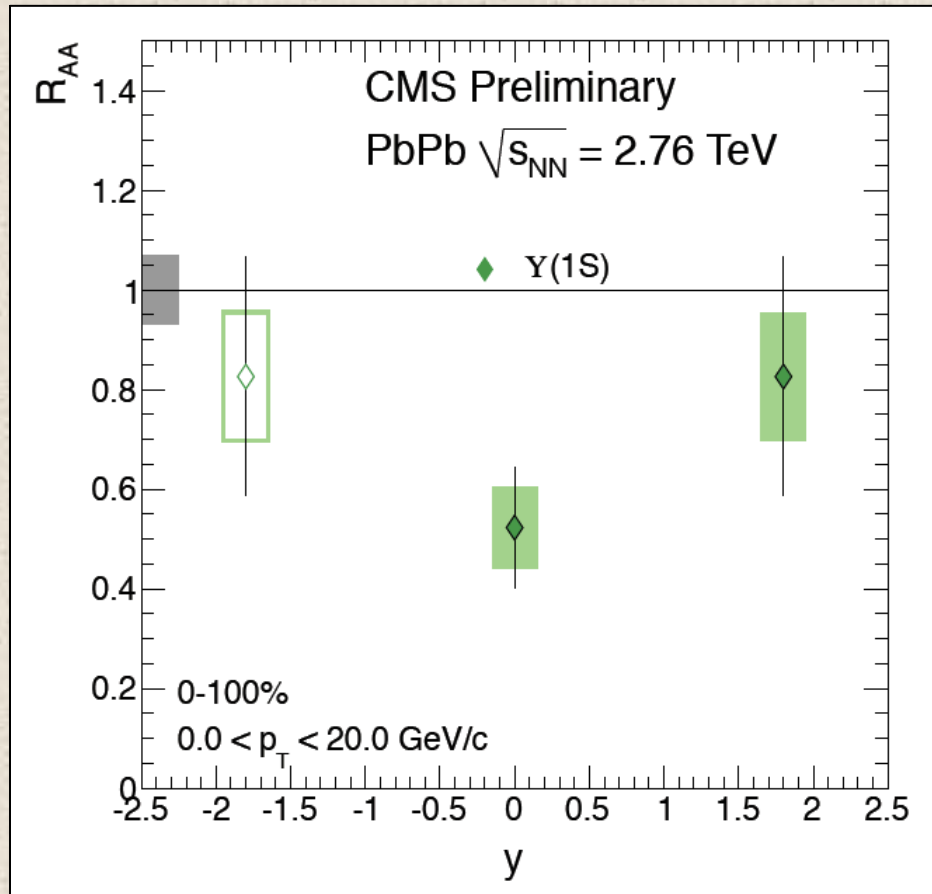
$$f \equiv e^{-\langle \overline{\Gamma}(\varsigma) \rangle}$$

QM 2011 - CMS Results

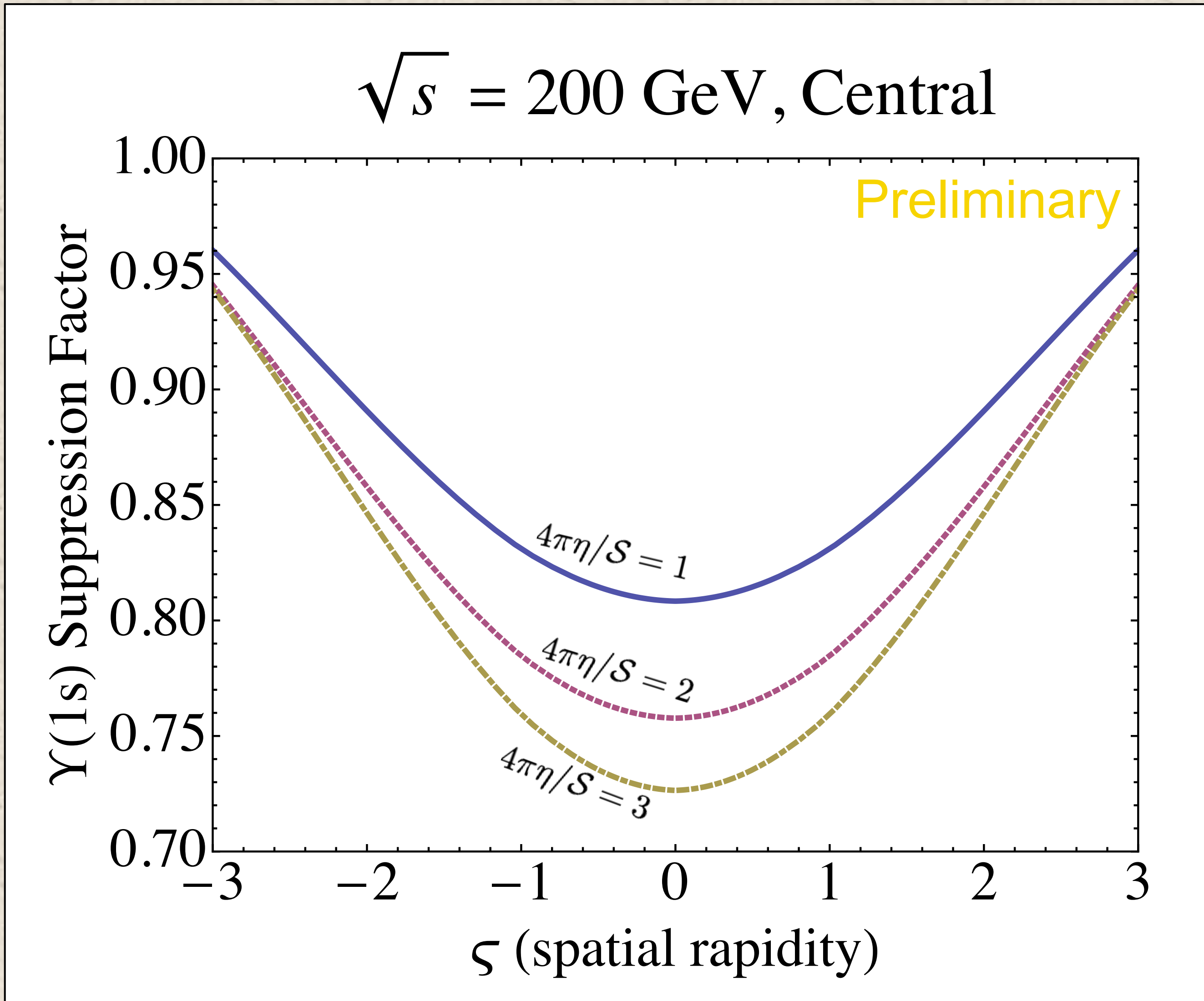
$\Upsilon(1S)$ R_{AA} in the most central 20%

– **0.60 ± 0.12(stat.) ± 0.10(syst.)**



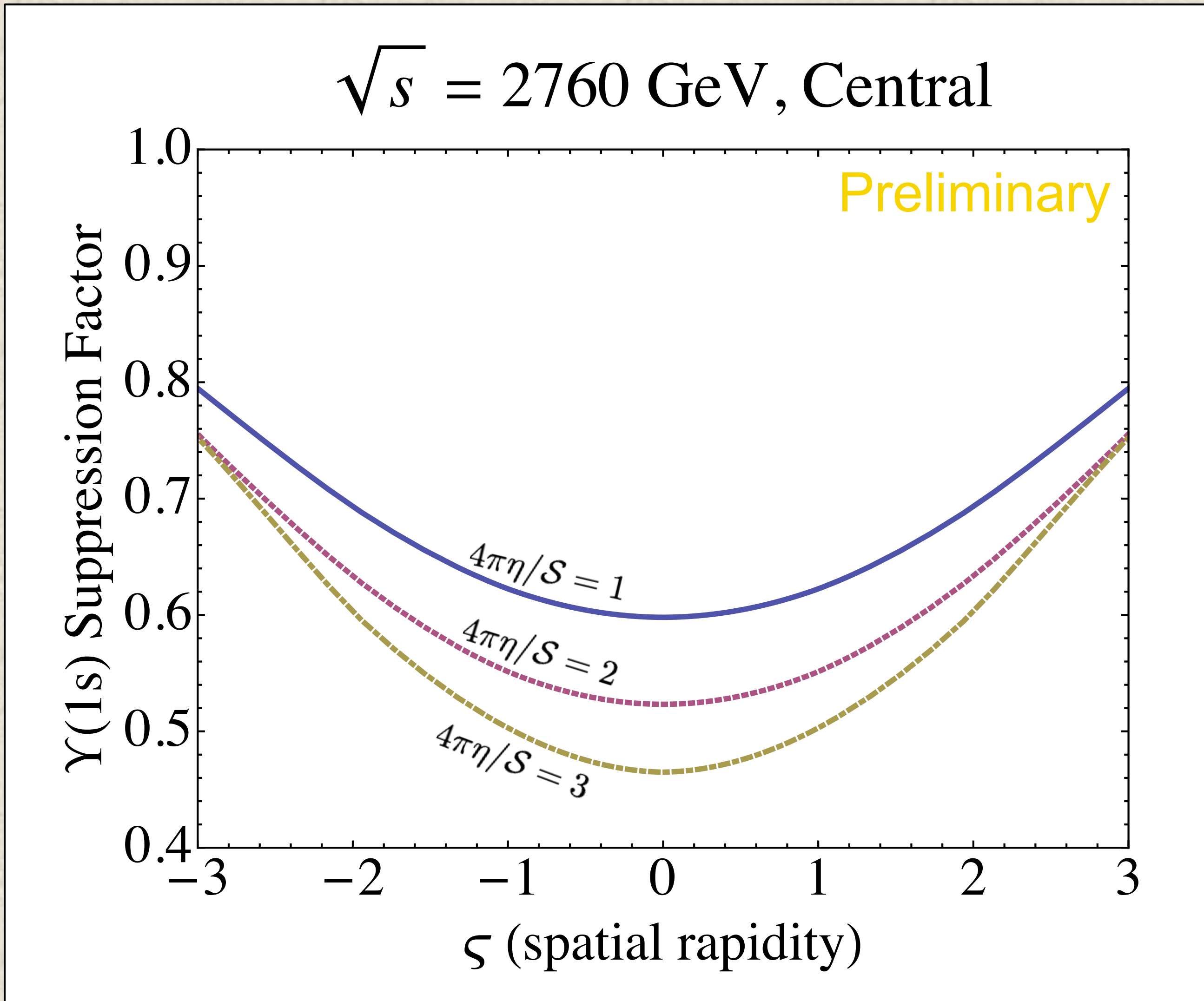


RHIC Suppression



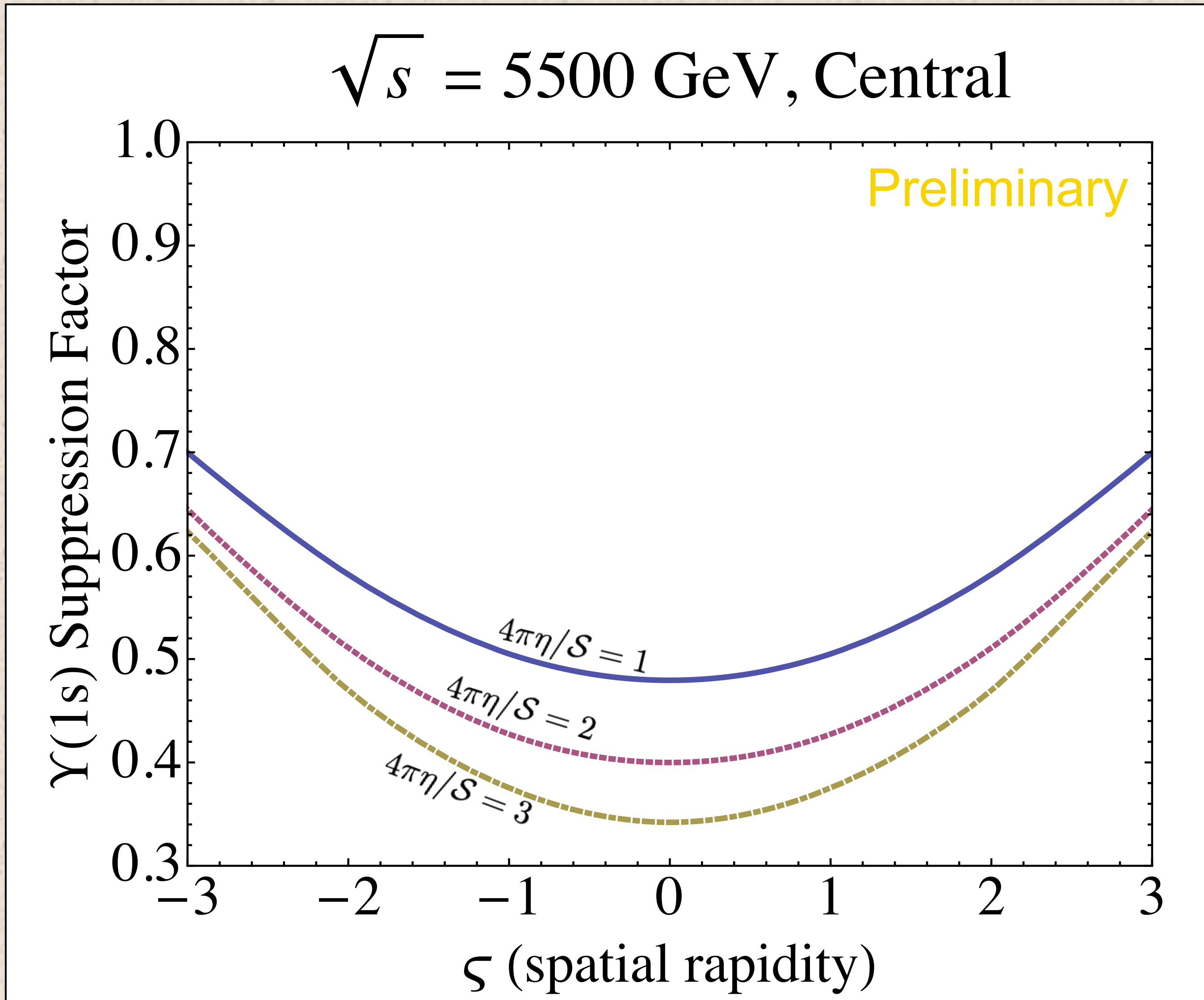
M. Strickland, forthcoming.

LHC $\sqrt{s}$ = 2760 GeV



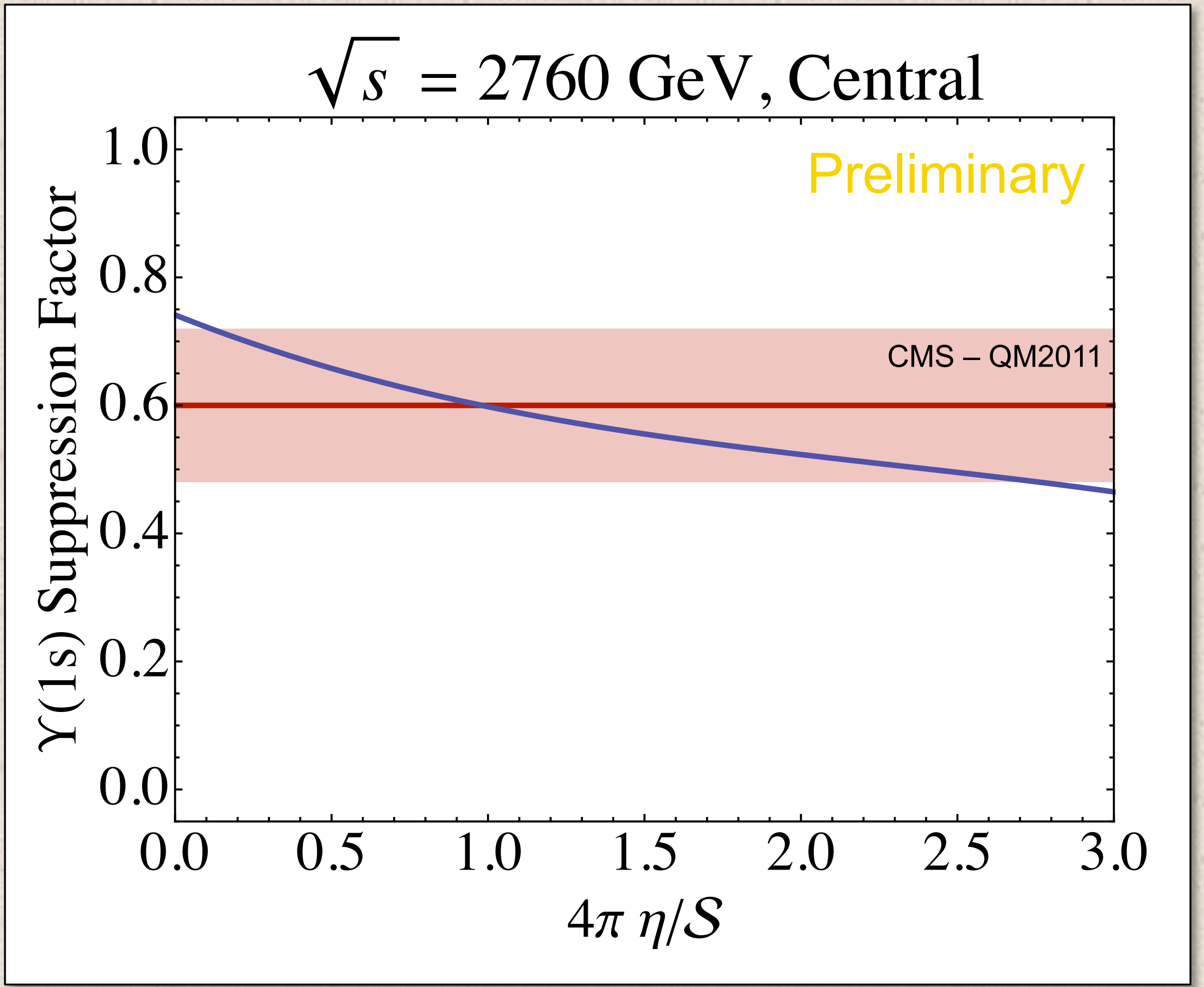
M. Strickland, forthcoming.

LHC $\sqrt{s}$ = 5500 GeV



M. Strickland, forthcoming.

LHC $\sqrt{s}$ = 2760 GeV



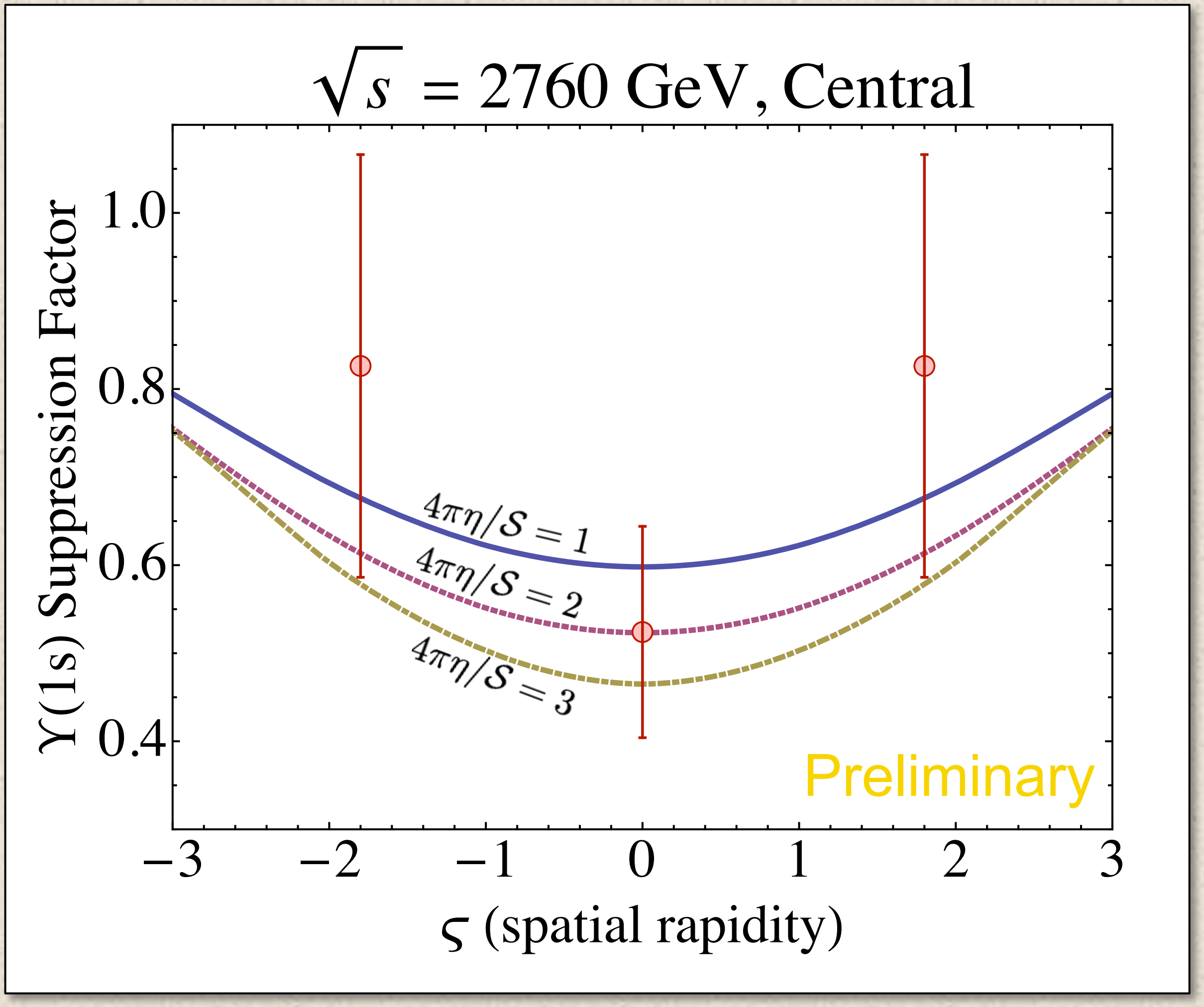
M. Strickland, forthcoming.

Caveats!

(1) No transverse expansion

(2) Temperature independent viscosity

LHC $\sqrt{s}$ = 2760 GeV



M. Strickland, forthcoming.

Caveats!

(1) Here I am comparing CMS 0-100% centrality results with my central curves

(2) I have equated spatial rapidity and "y"

(3) No transverse expansion

(4) Rapidity profile of initial condition fixed by final state ☹

Conclusions and Outlook

• Heavy quark potential → thermal width which determines suppression factor

• Almost all of the phenomenological pieces have been put together!

• Transverse dependence of the temperature is important: Large suppression in center, small at edges. Extracted suppression factor is geometrical average of these

• Today I only showed results for central collisions with no transverse expansion … work in progress …

• Suppression of excited states and feed down … work in progress …

• Assumed initial spatial rapidity profile is based on final state fits … this needs further consideration …

~~~~ Backup Slides ~~~~

Non-equilibrium ambiguity?

- For finite anisotropy increasing the anisotropy decreases the number and energy densities

$$\boxed{n(\xi, p_{\rm hard}) = \frac{n_{\rm iso}(p_{\rm hard})}{\sqrt{1+\xi}}}$$

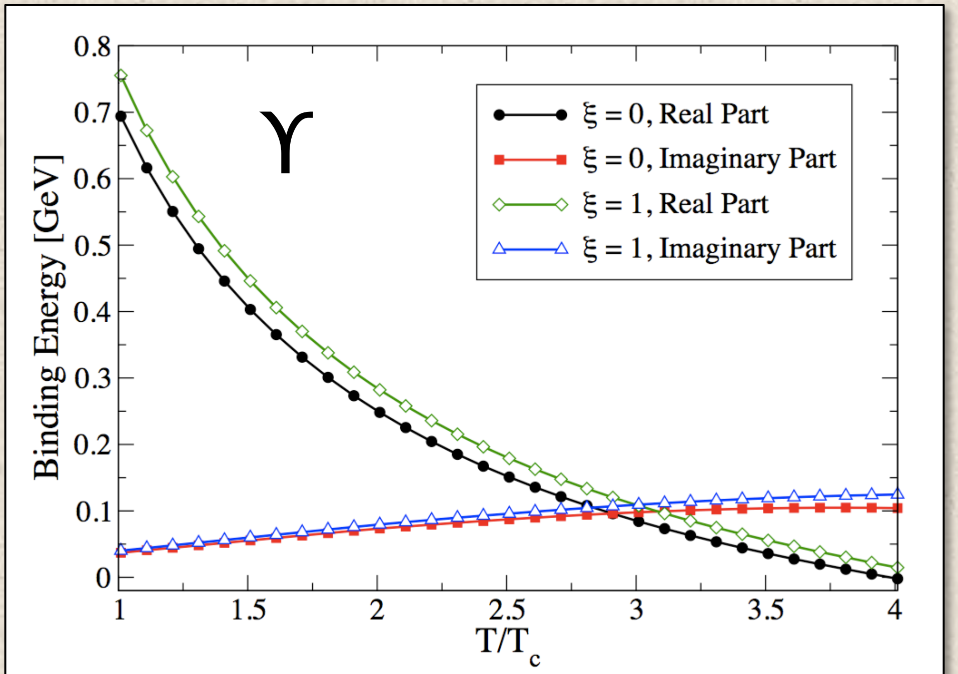
$$\boxed{\mathcal{E}(\xi, p_{\rm hard}) = \mathcal{R}(\xi)\mathcal{E}_{\rm iso}(p_{\rm hard})} \qquad \mathcal{R}(\xi) \equiv \frac{1}{2}\left(\frac{1}{1+\xi} + \frac{\arctan\sqrt{\xi}}{\sqrt{\xi}}\right)$$

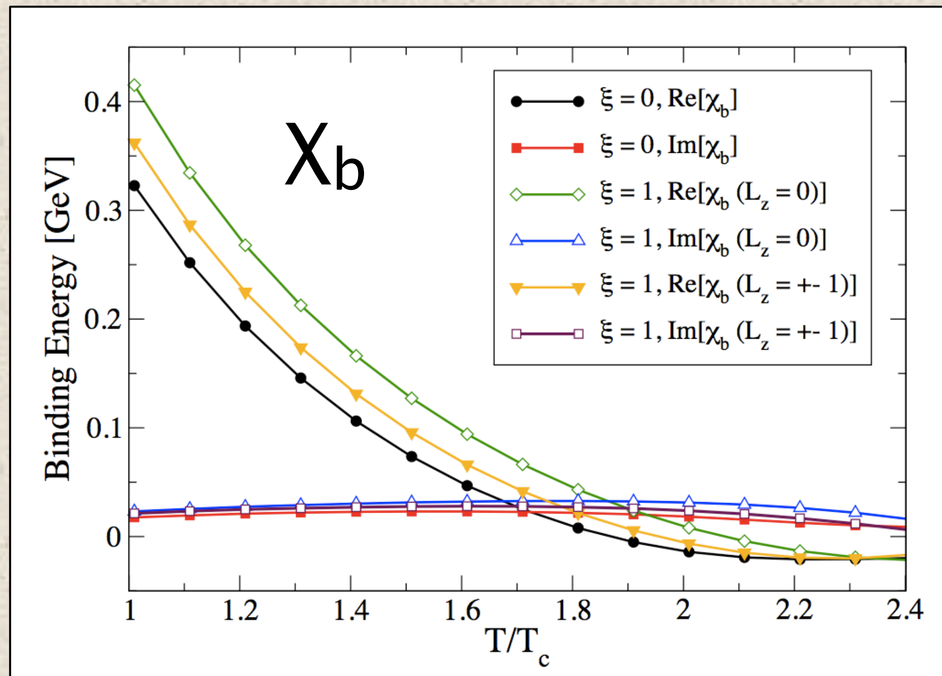
- A large part of the effect seen is due to the number density and hence Debye mass decreasing. Is that all?

- At leading order in ξ the result of holding either one fixed is the same so let's look at the case of holding the number density fixed

Fixed Number Density Results

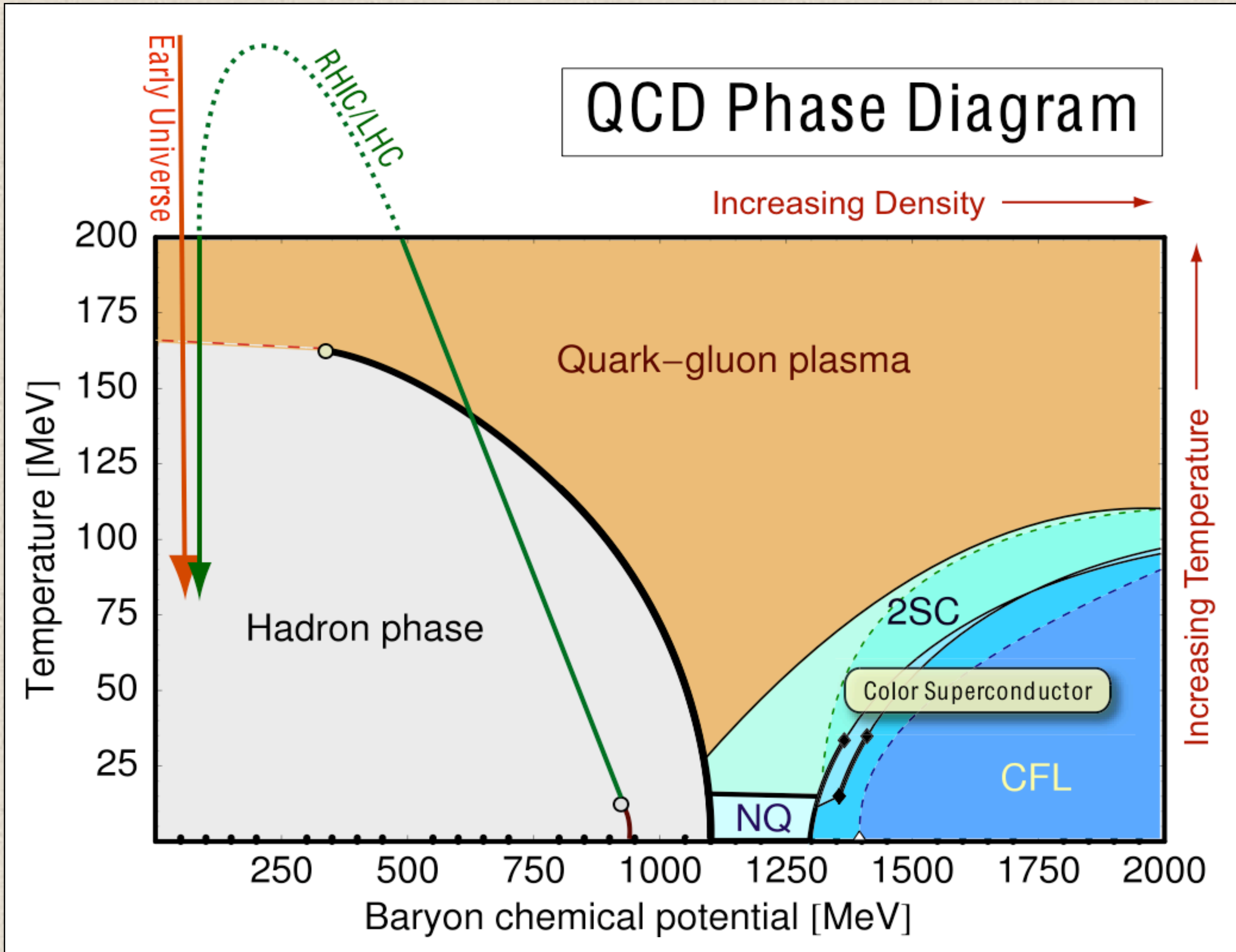
Margotta, McCarty, McGahan, Strickland, and Yager-Elorriaga, 1101.4651





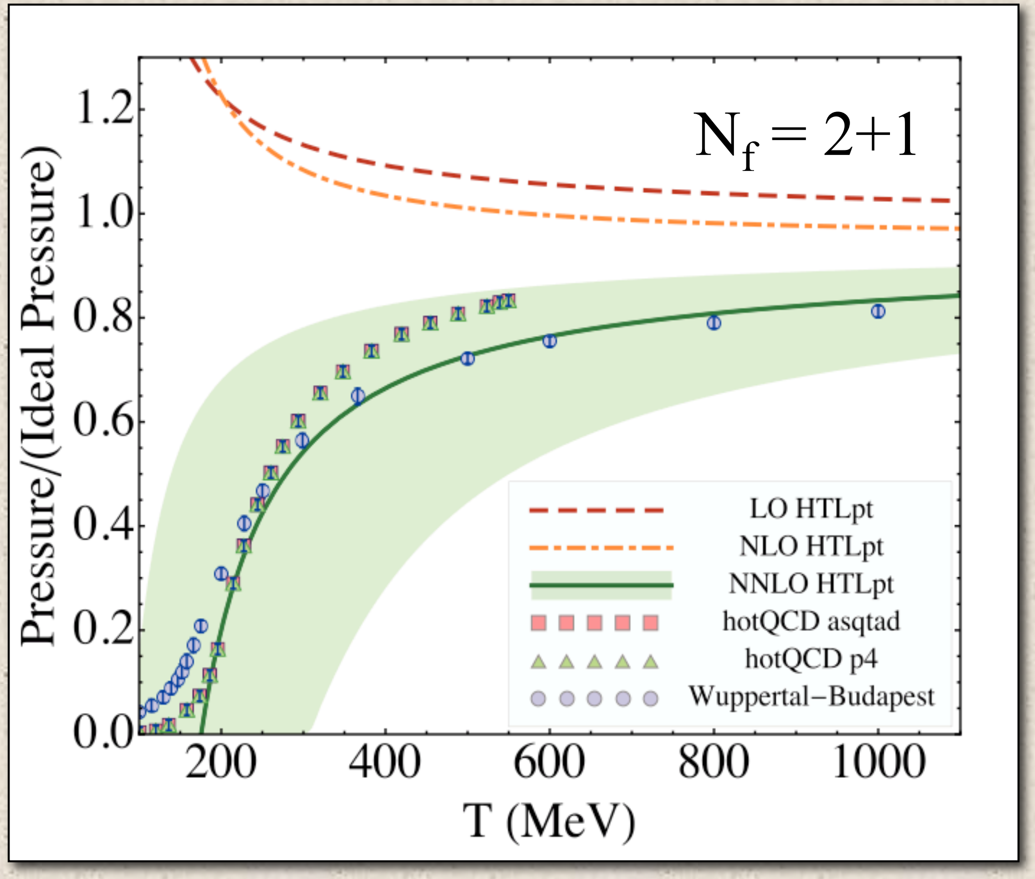
- But .. this discussion is merely academic!
- To really find out what happens we need to evolve p_{hard} and ξ dynamically not holding anything fixed

QGP Thermodynamics


QCD Phase Diagram
Early Universe
RHIC/LHC
Increasing Density
Increasing Temperature
Quark–gluon plasma
Hadron phase
2SC
Color Superconductor
CFL
NQ
Temperature [MeV]
200
175
150
125
100
75
50
25
250
500
750
1000
1250
1500
1750
2000
Baryon chemical potential [MeV]

QGP Thermodynamics

- Lattice data from various groups show smooth crossover near T_c ~ 180 MeV
- Different lattice discretizations give slightly different results
- hotQCD results shown are not continuum extrapolated
- Theory curves are based on Hard Thermal Loop perturbation theory (HTLpt) reorganization of finite temperature QCD



LATTICE

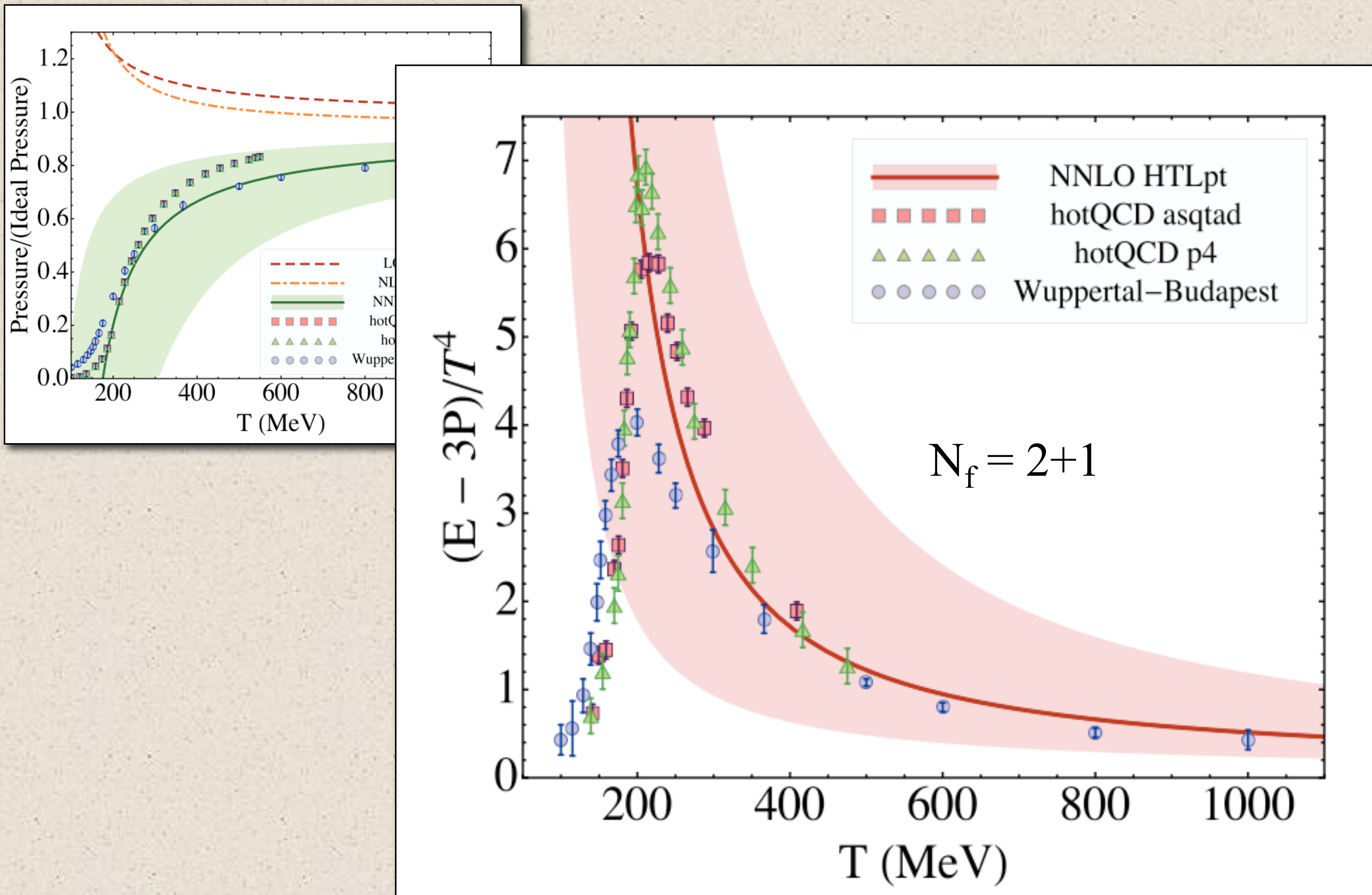
hotQCD - A. Bazavov et. al., Phys. Rev. D 80 (2009) 014504, [arXiv:0903.4379].

Wuppertal-Budapest - S. Borsanyi et. al., The QCD equation of state with dynamical quarks, arXiv:1007.258

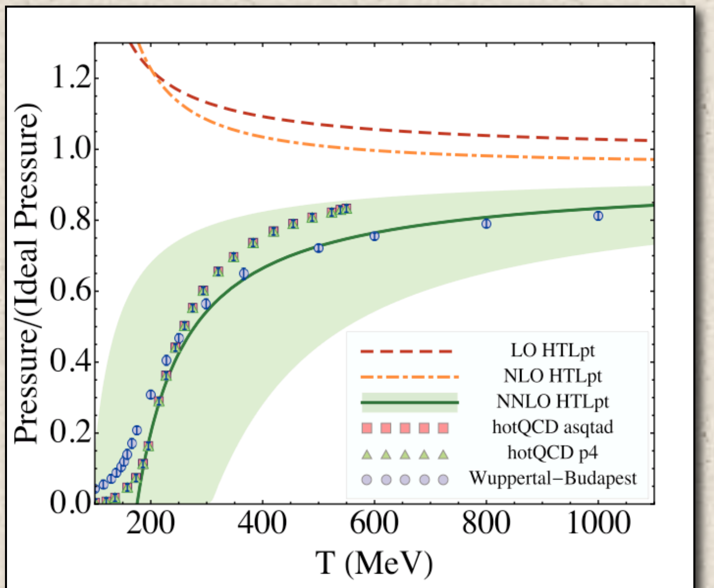
pQCD

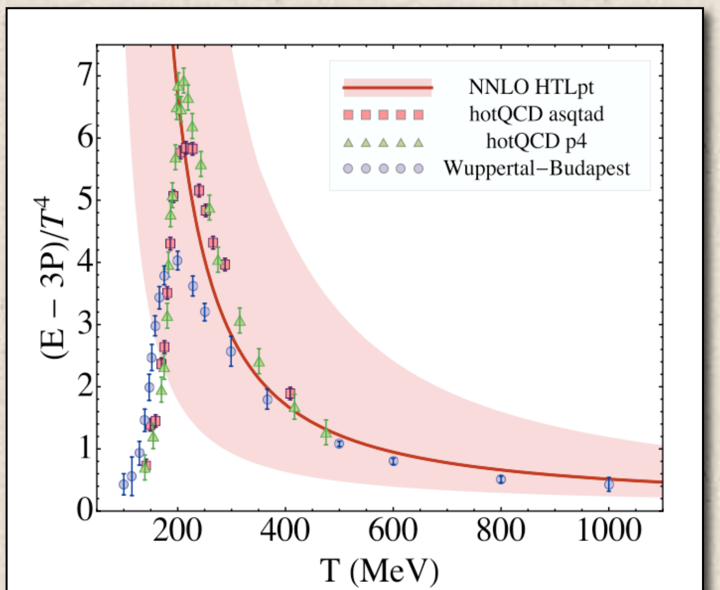
HTLpt - J.O. Andersen, L.E. Leganger, MS, and N. Su, Physics Letters B 696, 5, 468 (2011); arXiv:/1103.2528

The QGP EOS at Finite T



The QGP EOS at Finite T

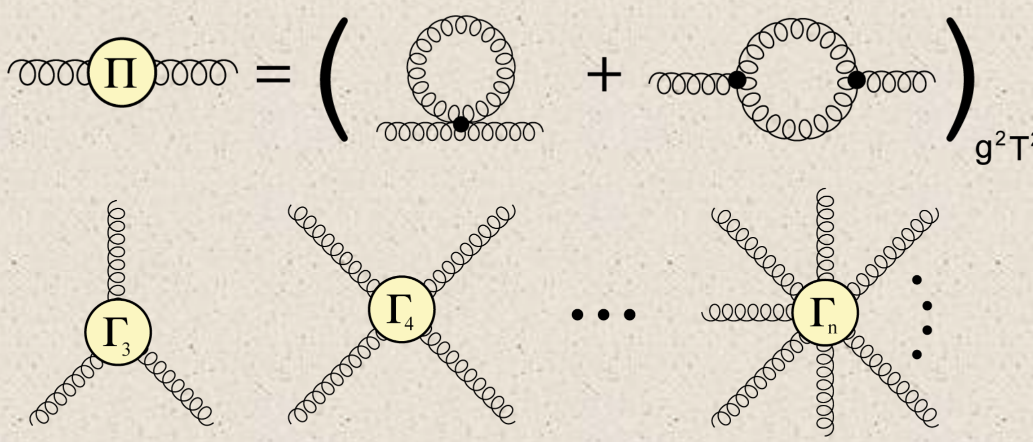




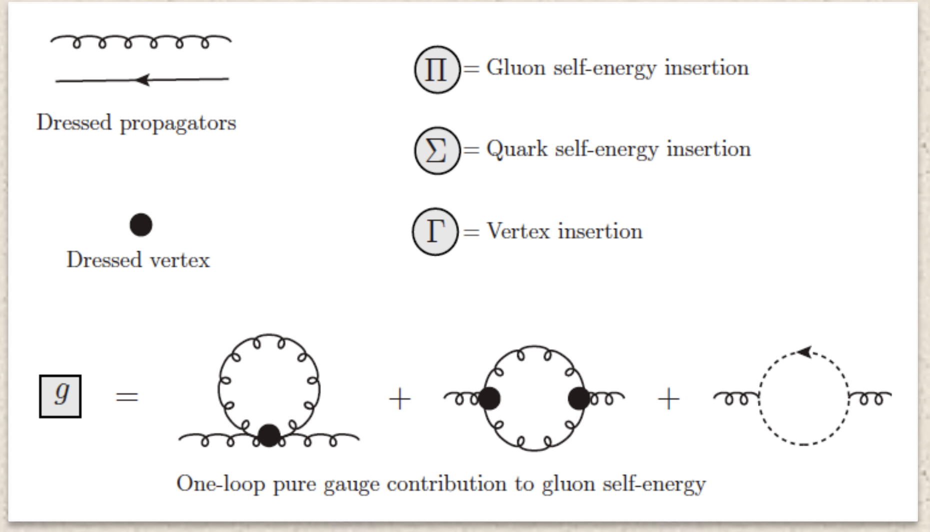
HTLpt : Reorganizes loop expansion around classical state of high temperature QCD which includes "hard-loop" resummed propagators and vertices

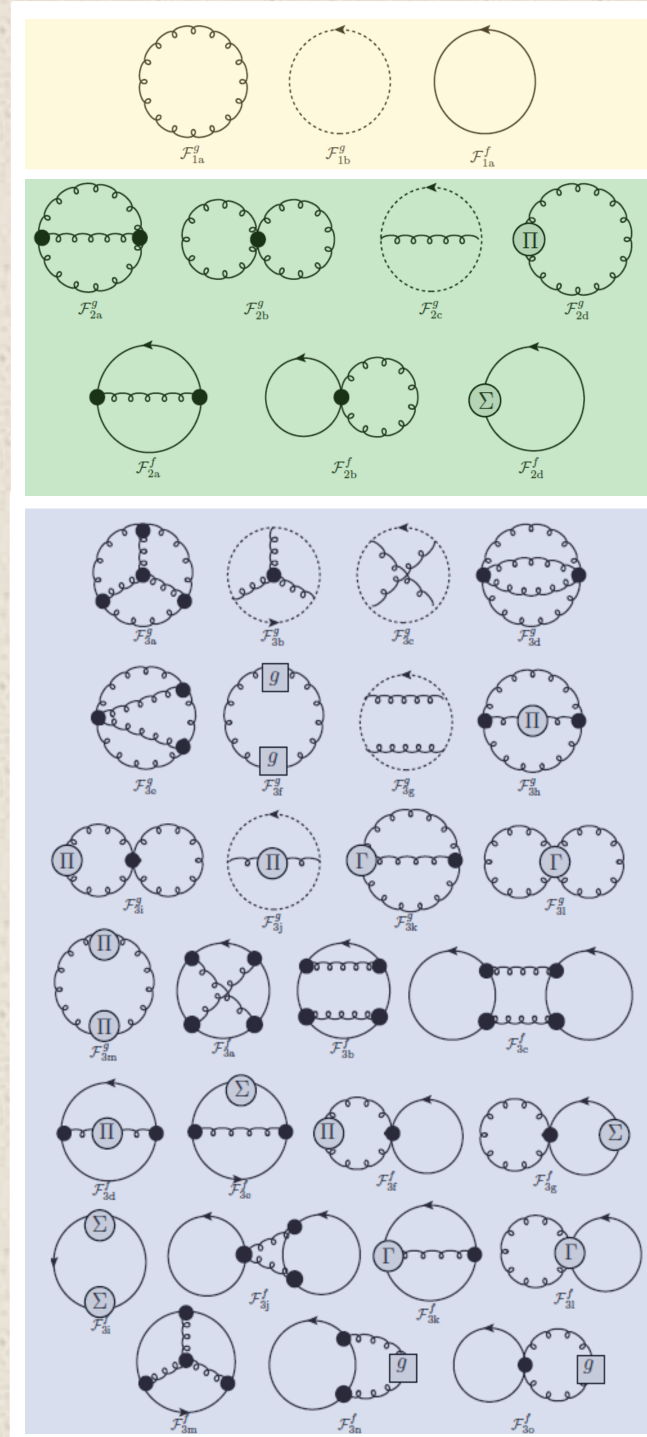
$$\mathcal{L} = (\mathcal{L}_{\rm YM} + \mathcal{L}_{\rm HTL})\Big|_{g_s \to \sqrt{\delta} g_s} + \Delta\mathcal{L}_{\rm HTL}$$

$$\mathcal{L}_{\rm HTL} = -\frac{1}{2}(1-\delta)\, m_D^2\, {\rm Tr}\left(F_{\mu\alpha}\left\langle \frac{y^\alpha y^\beta}{(y\cdot D)^2}\right\rangle_y F^\mu{}_\beta\right)$$



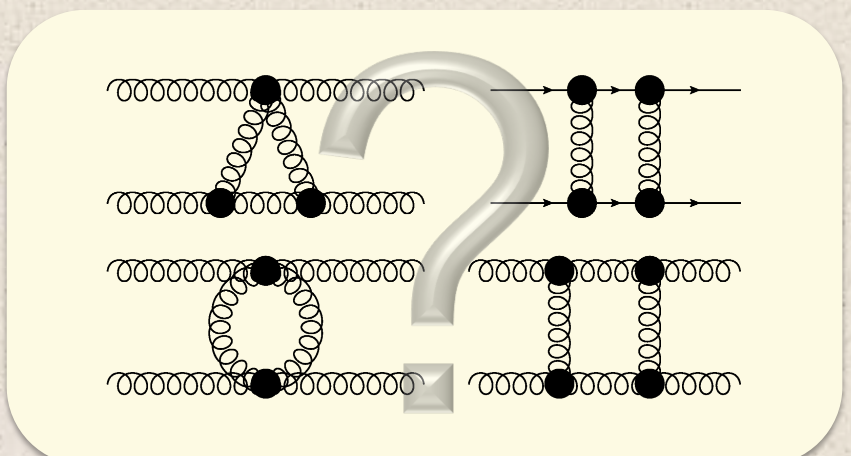
NNLO Diagrams

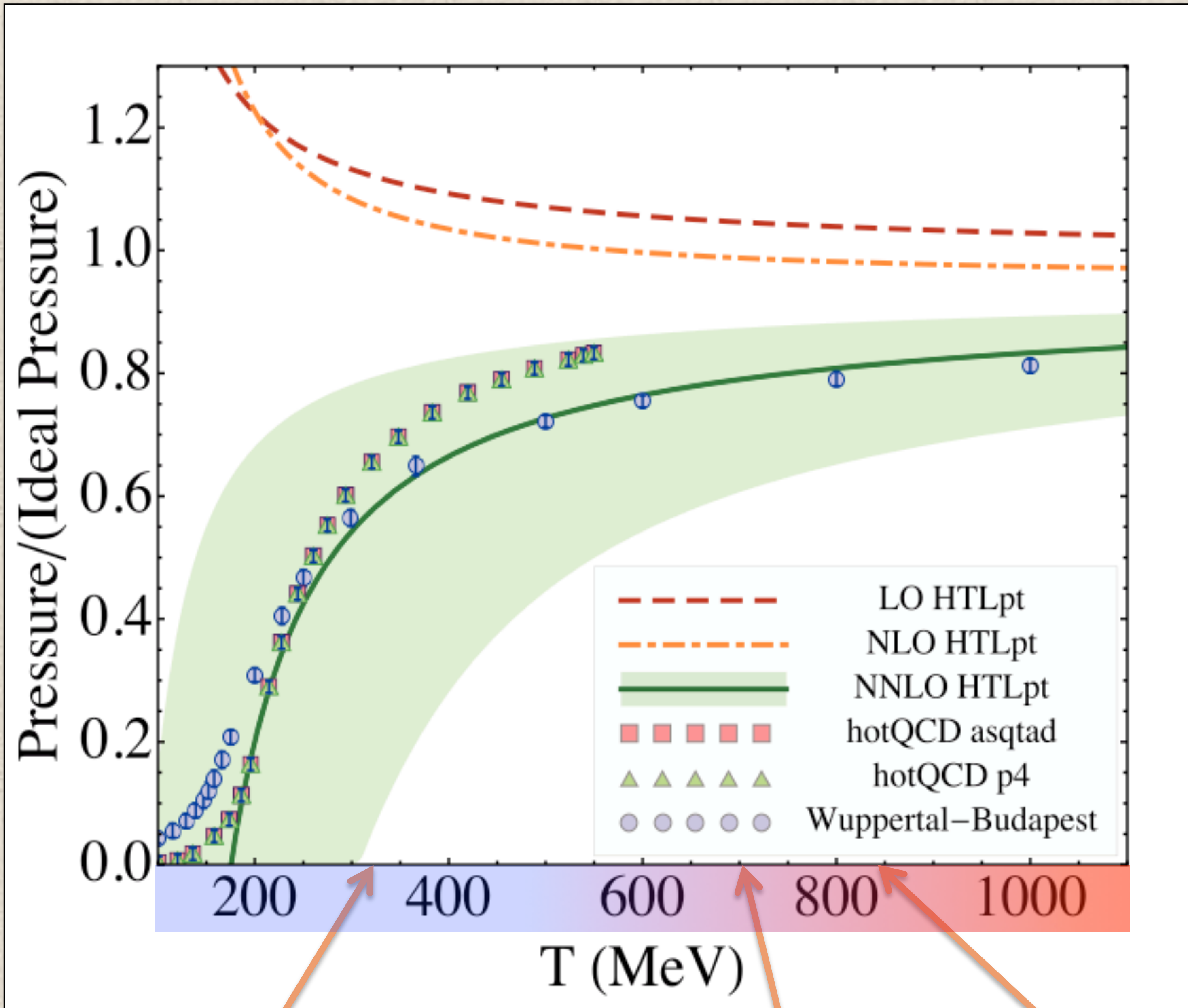




What is special about 3 loops?

This where attractive double gluon exchange diagrams enter, eg





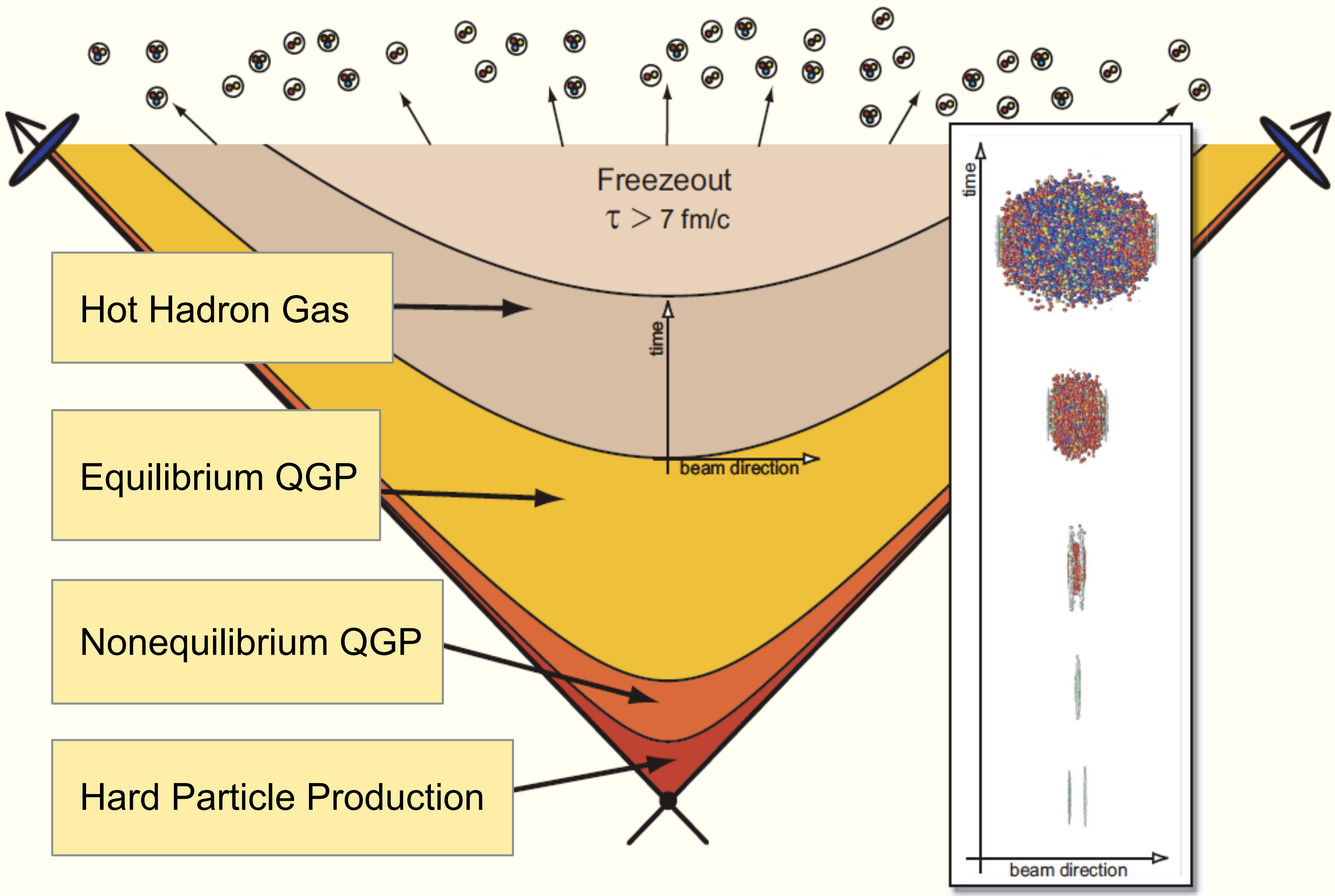
RHIC @ 200 A GeV :
$T_o \sim 360$ MeV $\sim 2\ T_c$

LHC @ 2.76 A TeV :
$T \sim 690$ MeV $\sim 3.9\ T_c$

LHC @ 5.5 A TeV :
$T_o \sim 820$ MeV $\sim 4.6\ T_c$

QGP Dynamics

Heavy-ion collision timescales and “epochs” @ RHIC



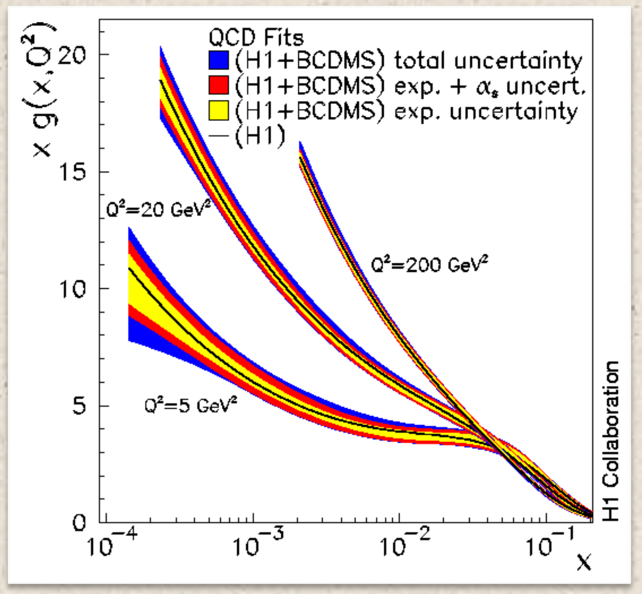
*1 fm/c $\simeq 3 \times 10^{-24}$ seconds

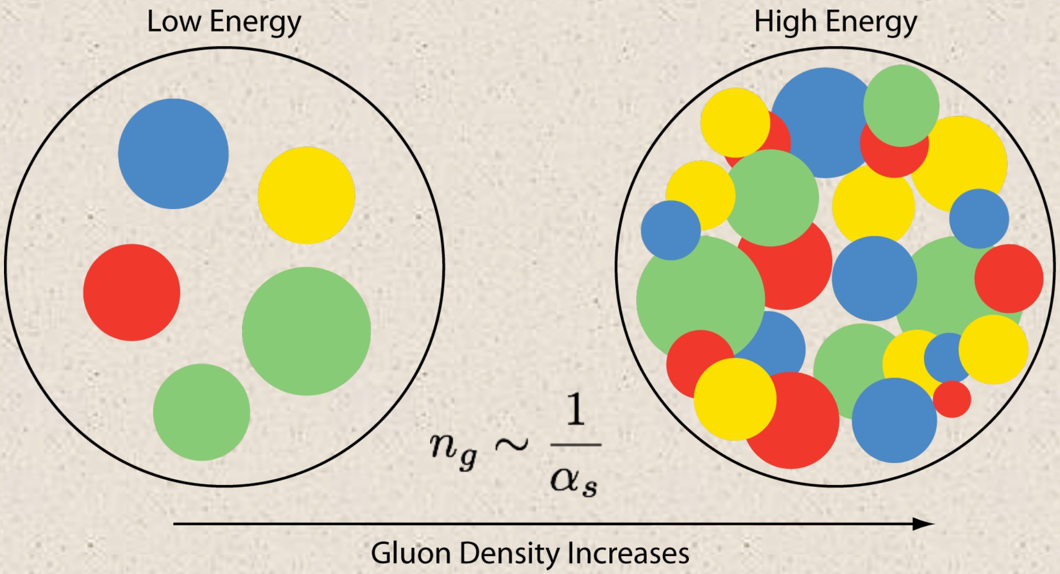
Weak vs Strong Coupling

- Strong coupling constant on the order of $\alpha_s \sim 0.2 – 0.3$
- Hard degrees of freedom can be treated perturbatively [$p_t > 2$ GeV]
- Soft (bulk) degrees of freedom may be strongly coupled [$p_t < 2$ GeV]
- Hadronization : nonperturbative physics – currently treated rather crudely (Cooper-Frye prescription)
- Theoretical studies in both strong and weak coupling limits are needed

QGP Initial State

- Nuclei take 0.1 - 0.2 fm/c to pass through one another
- Most of the quarks (and hence baryon number) continues down the pipe
- Highest energy densities (momenta) occur at early times → perturbative regime → **C**olor **G**lass **C**ondensate / Glasma





See eg

L. McLerran, The CGC and the Glasma: Two Lectures at the Yukawa Insitute, arXiv:1011.3204 (2011).
F. Gelis, Color Glass Condensate and Glasma, Nucl. Phys. A854, 10 (2011).

And references therein

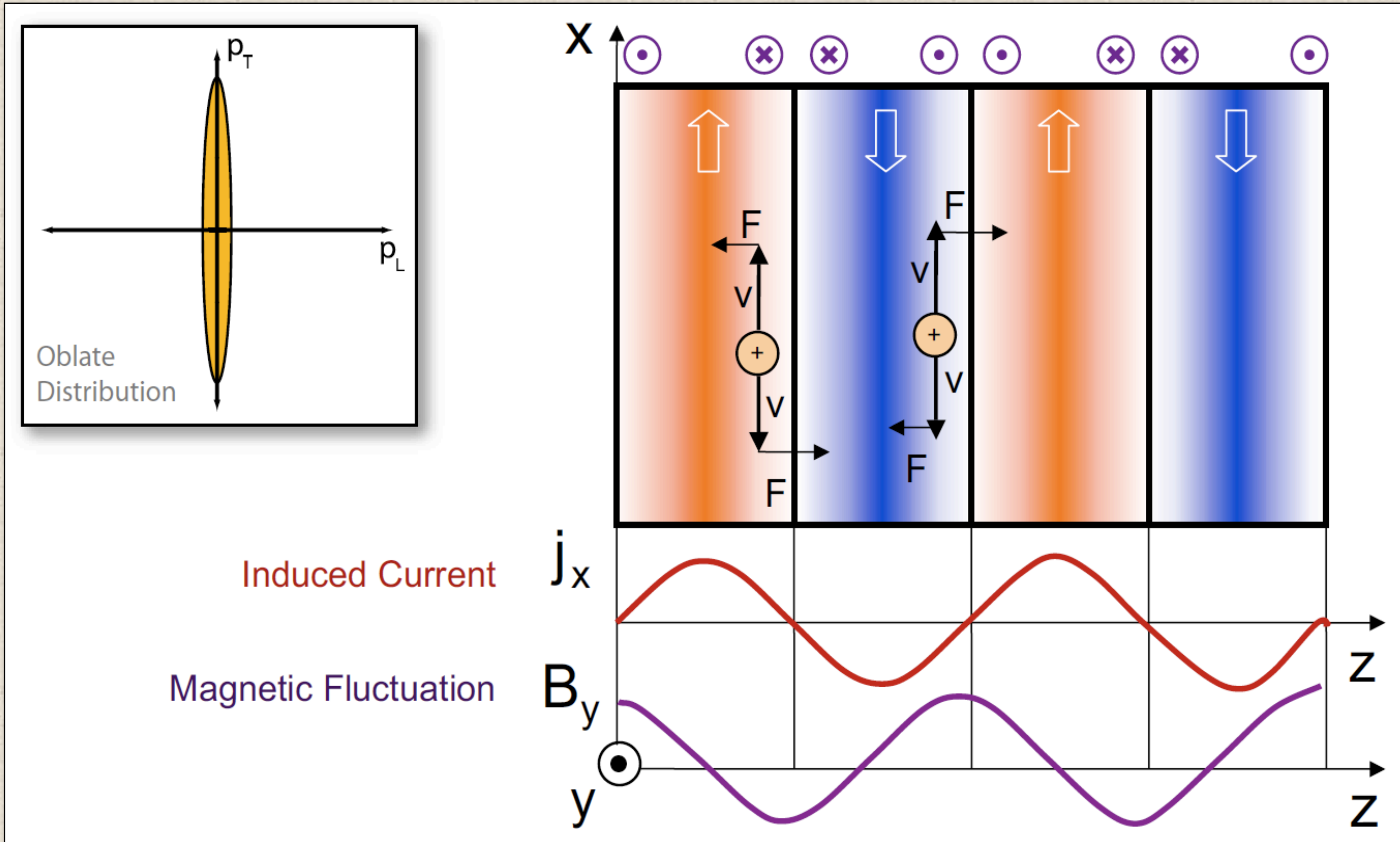
Thermalization

- Binary and multiparticle collisions (number conserving and non-conserving)
- In addition, plasma instabilities are present
- In the weak coupling limit plasma instabilities are the fastest mechanism for plasma isotropization
- Collision time scale : $t_{\text{collisions}} \sim g^{-4}$
- Instability time scale : $t_{\text{instability}} \sim g^{-1}$

α_s	$t_{\text{collisions}}/t_{\text{instability}}$
0.01	1000
0.1	30
0.3	6

S. Mrówczynski and M. Thoma, Phys. Rev. D 62, 036011 (2000).
P. Romatschke and MS, Phys. Rev. D 68, 036004 (2003).
P. Arnold, J. Lenaghan, and G. Moore, JHEP 0308, 002 (2003).
B. Schenke, MS, C. Greiner, and M. Thoma, Phys.Rev. D73, 125004 (2006).
A. Rebhan, MS, and M. Attems, Phys.Rev. D78, 045023 (2008).

Gauge Field Dynamics in Anisotropic Plasmas

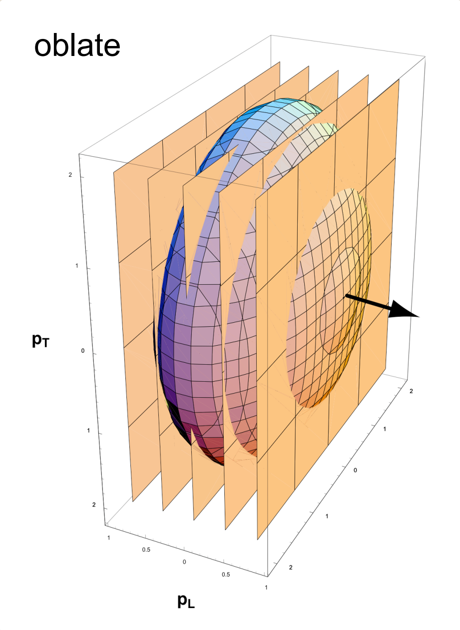


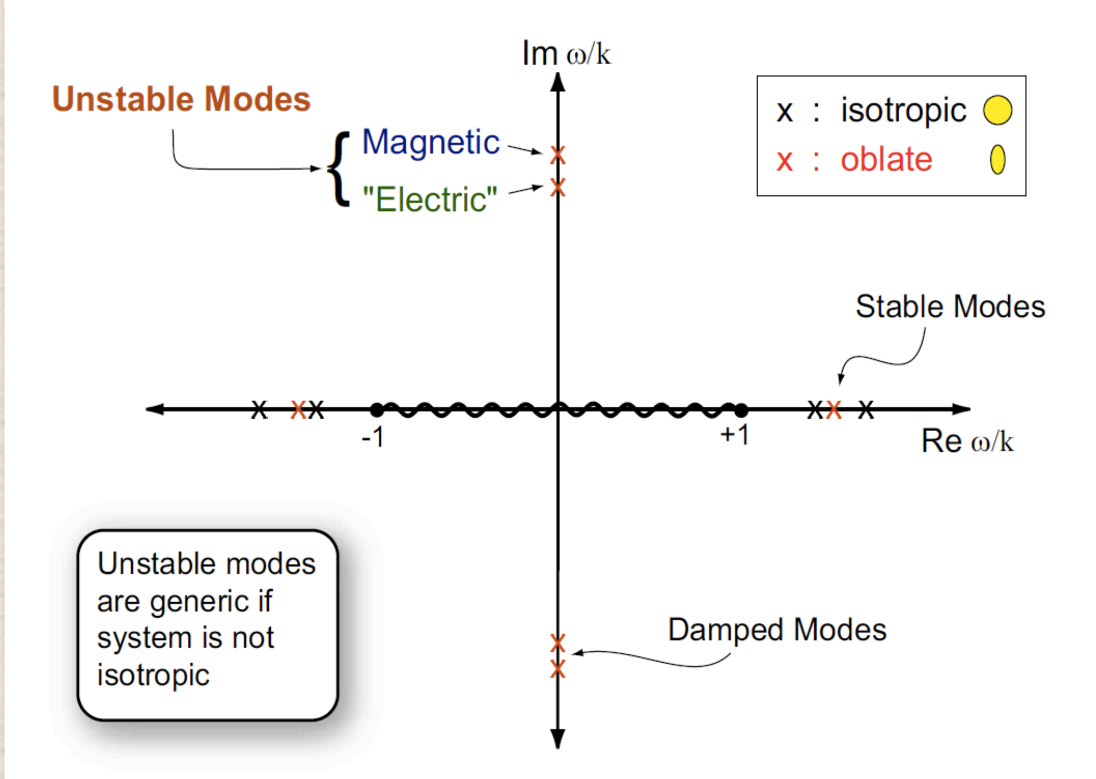
Collective modes

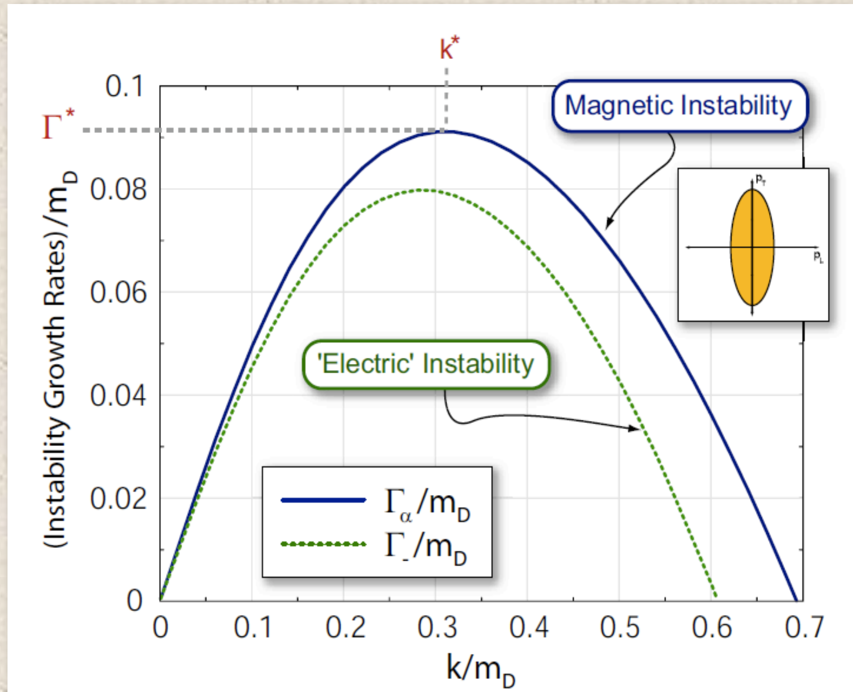
• Solve for the poles in the plasma gluon propagator

• Stable and unstable modes

• Unstable modes work to restore isotropy by generating compensating field pressure





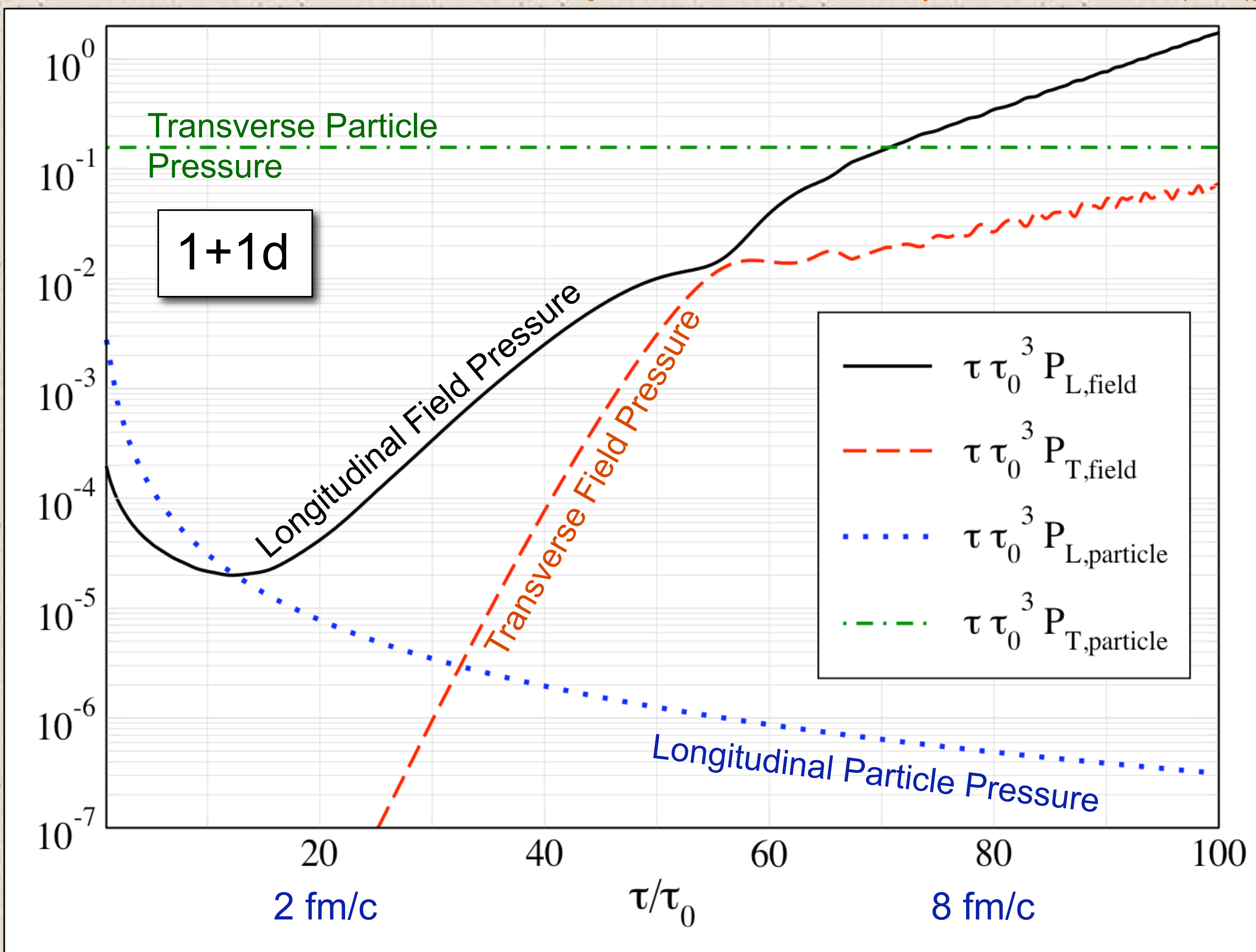


Instability growth rates as a function of momentum for $\langle p_T^2\rangle/2\langle p_L^2\rangle \simeq 10$ and $\theta_{\rm glue} = \pi/8$ with respect to the beamline.

The chromo-Weibel instability

Particle and field pressures → Isotropization

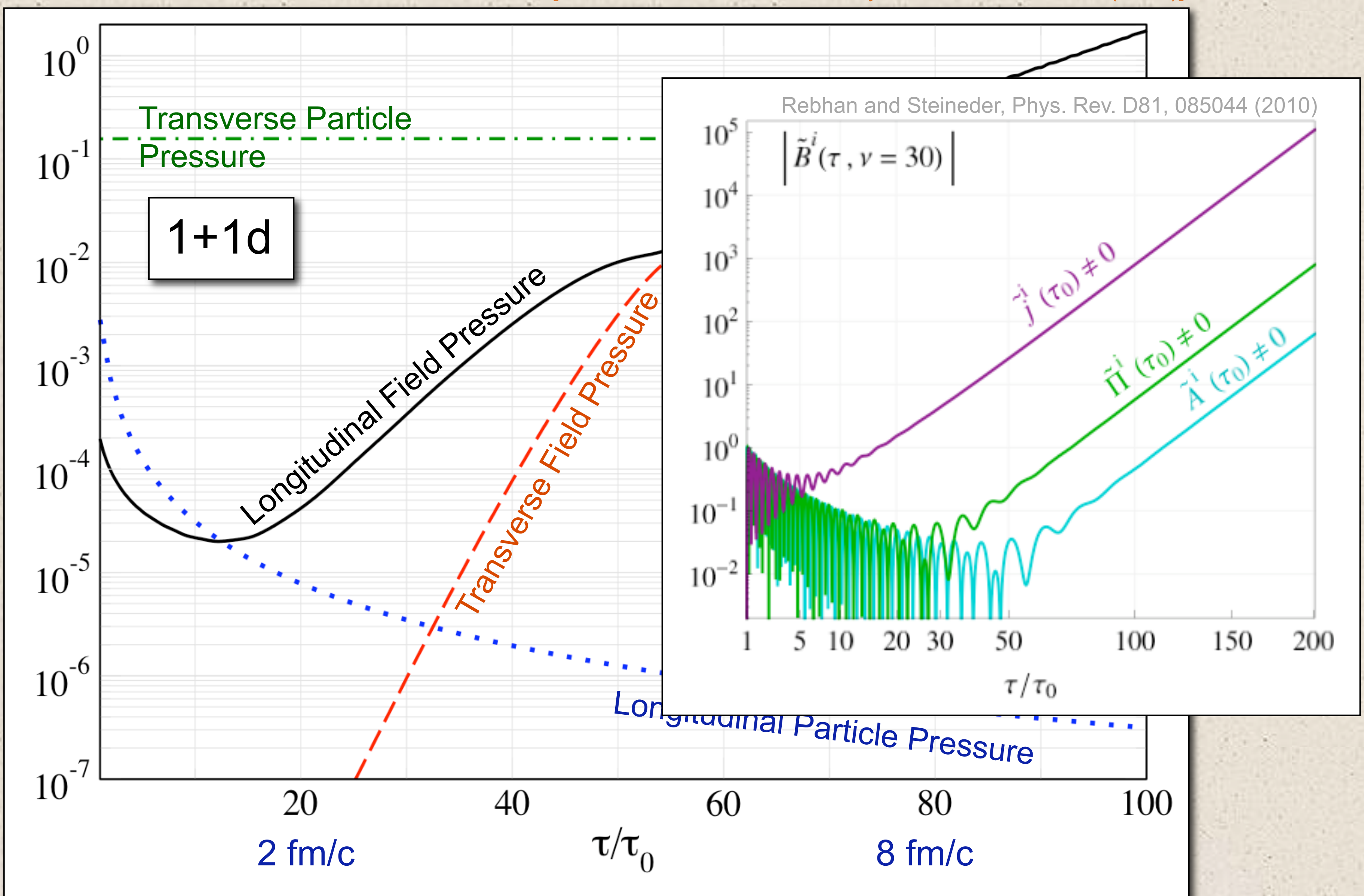
[Rebhan, MS, and Attems, Phys. Rev. D 78, 045023 (2008)]



The chromo-Weibel instability

Particle and field pressures → Isotropization

[Rebhan, MS, and Attems, Phys. Rev. D 78, 045023 (2008)]



Boost Invariant 1d Expansion

• Consider boost-invariant one-dimensional expansion and describe the system using the Boltzmann equation

$$p^\mu \partial_\mu f = -C[f]$$

Collisional Kernel

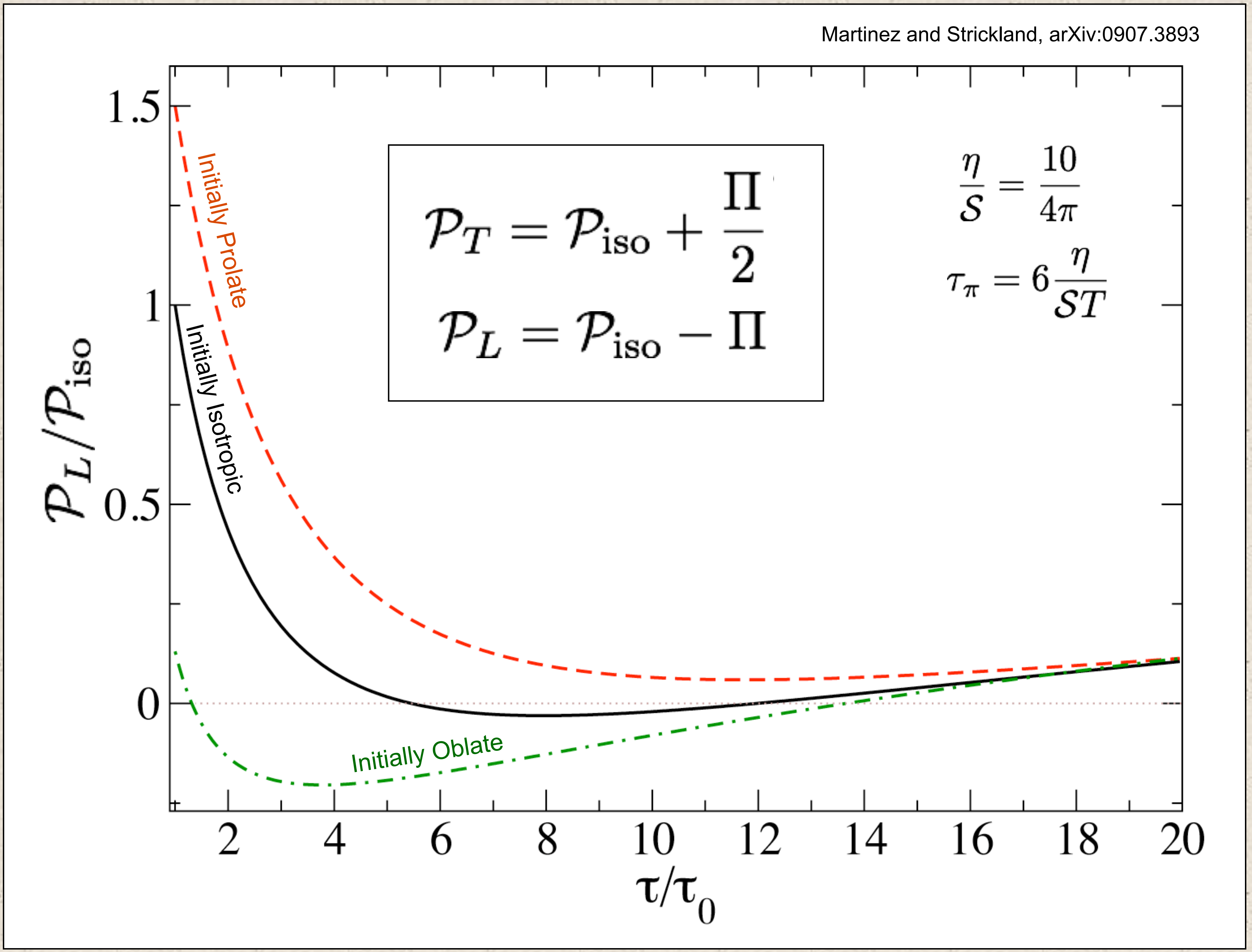
• If system is homogeneous in the transverse direction then the transverse derivatives vanish.

• If system is boost-invariant then the local rest frame has velocity $v_z = z/t$.

• In lab frame the Boltzmann equation becomes

$$p^t \partial_t f(t, z, \mathbf{p}) + p^z \partial_z f(t, z, \mathbf{p}) = -\mathcal{C}[f(t, z, \mathbf{p})]$$

0+1 Boost Invariant Hydro Results



Energy-Momentum Tensor

- Can calculate all components analytically

$$\mathcal{E}(p_{\rm hard},\xi) = T^{\tau\tau} \equiv \mathcal{R}(\xi)\,\mathcal{E}_{\rm iso}(p_{\rm hard})$$

$$\mathcal{P}_T(p_{\rm hard},\xi) = \frac{1}{2}\left(T^{xx}+T^{yy}\right) = \frac{3}{2\xi}\left(\frac{1+(\xi^2-1)\mathcal{R}(\xi)}{\xi+1}\right)\mathcal{P}_{\rm iso}(p_{\rm hard}),$$

$$\equiv \mathcal{R}_{\rm T}(\xi)\mathcal{P}_{\rm iso}(p_{\rm hard})$$

$$\mathcal{P}_L(p_{\rm hard},\xi) = -T^{\varsigma}_{\varsigma} = \frac{3}{\xi}\left(\frac{(\xi+1)\mathcal{R}(\xi)-1}{\xi+1}\right)\mathcal{P}_{\rm iso}(p_{\rm hard})\ ,$$

$$\equiv \mathcal{R}_{\rm L}(\xi)\mathcal{P}_{\rm iso}(p_{\rm hard})$$

where

$$\boxed{\mathcal{R}(\xi) \equiv \frac{1}{2}\left(\frac{1}{1+\xi}+\frac{\arctan\sqrt{\xi}}{\sqrt{\xi}}\right)}$$

Relation between ξ and Π

- Equating the longitudinal pressure with that obtained using linearized hydro we obtain

$$\Pi(\tau) = \frac{\mathcal{E}_{\rm eq}(T(\tau))}{3}\left[1 - \frac{\mathcal{R}_{\rm L}(\xi(\tau))}{\mathcal{R}(\xi(\tau))}\right]$$

- In the limit of small ξ

$$\boxed{\frac{\Pi}{\mathcal{E}_{\rm eq}} = \frac{8}{45}\,\xi + \mathcal{O}(\xi^2)}$$

- For Navier-Stokes viscous hydro one obtains

$$\Pi_{\rm NS} = 4\eta/(3\tau) \longrightarrow \xi_{\rm NS} = \frac{10}{T\tau}\frac{\eta}{\mathcal{S}} + \mathcal{O}(\Pi_{\rm NS}^2)$$

Positivity of Entropy Divergence

$$\partial_\mu \mathcal{S}^\mu \geq 0 \longrightarrow \partial_\tau(\tau\mathcal{S}) \geq 0$$

using

$$\begin{aligned}\mathcal{S} &= -\int \frac{d^3\mathbf{p}}{(2\pi)^3} f(\tau,\mathbf{x},\mathbf{p})\left\{\log\left[f(\tau,\mathbf{x},\mathbf{p})\right]-1\right\} \\ &= \frac{\mathcal{S}_{\rm iso}(p_{\rm hard})}{\sqrt{1+\xi}}\end{aligned}$$

can show

$$\frac{\partial_\tau(\tau\mathcal{S})}{\mathcal{S}} = \tau\Gamma\left[\mathcal{R}^{3/4}(\xi)\sqrt{1+\xi}-1\right] \geq 0$$

Ideal Hydro Limit

If we take the scattering rate Γ to infinity the first and second equations reduce to

$$\xi = 0$$

$$\frac{1}{p_{\rm hard}} \partial_\tau p_{\rm hard} = -\frac{1}{3\tau}$$

Which has the solution

$$\xi = 0$$

$$p_{\rm hard}(\tau) = p_{\rm hard}(\tau_0) \left(\frac{\tau_0}{\tau} \right)^{1/3}$$

Ideal hydro limit is recovered!

Free Streaming Limit

If we take the scattering rate Γ to zero we obtain

$$\partial_\tau \xi = \frac{2}{\tau}(1+\xi)$$

$$\partial_\tau p_{\rm hard} = 0$$

Which has the solution

$$\xi(\tau) = (1+\xi_0)\left(\frac{\tau}{\tau_0}\right)^2 - 1$$

$$p_{\rm hard} = p_0$$

Free streaming limit is recovered!