Spectral Densities in a Leading-Log Approximation

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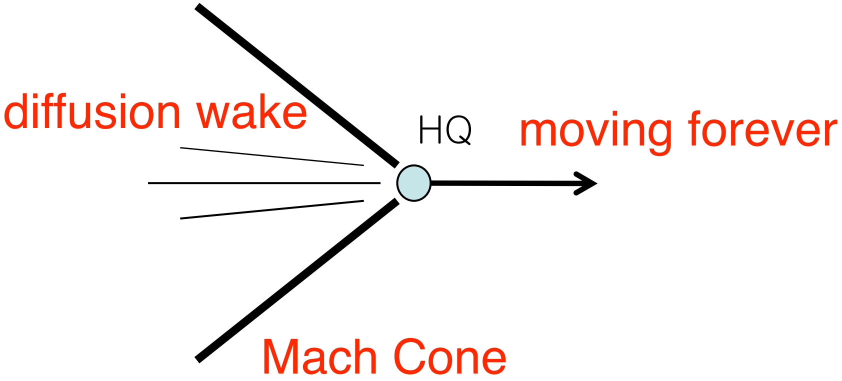
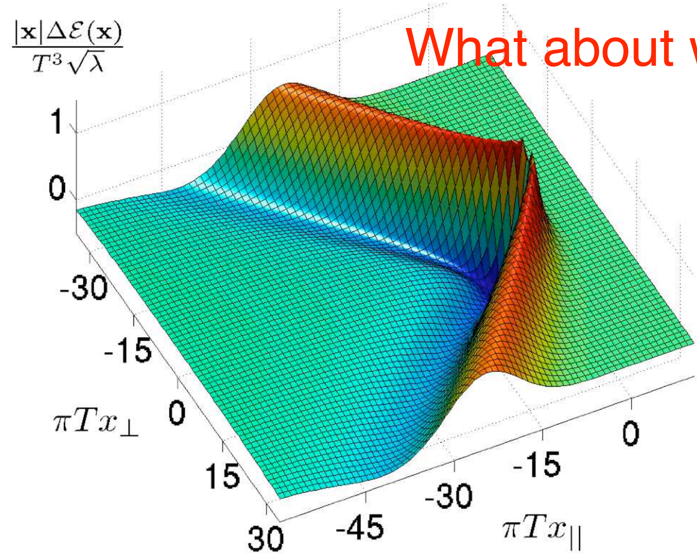
- Base on collaboration with Juhee Hong – arXiv:1003.0699, PRC
- P.Chesler, J. Hong, D. Teaney – arXiv:1110.5292, PRC submitted

Motivation

- Simulate the Mach Cone at Weak coupling



Strong coupling result:



What about weak?

- Solve the linearized Boltzmann equation

$$(\partial_t + v_{\boldsymbol{p}} \cdot \partial_{\boldsymbol{x}}) f(\boldsymbol{p}, \boldsymbol{x}) = \mathrm{C}[f, \boldsymbol{p}] \, ,$$

We got stuck – find out why!

Spectral Densities with Boltzmann Equation – LOTS OF PHYSICS

- Lattice QCD

$$\int \mathrm{d}^3\boldsymbol{x}\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \, \langle J(\tau,\boldsymbol{x}) J(0,\boldsymbol{0})\rangle = \int \frac{\mathrm{d}\omega}{2\pi}\, \rho^{JJ}(\omega,\boldsymbol{k})\, \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}\,.$$

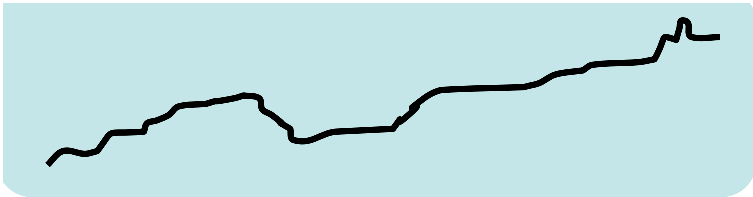
- Hydrodynamics
  - How small do ω and k need to be actually
  - Extract transport coefficients $\Rightarrow$ new ones $\tau_\Pi, \xi_5, \xi_6, \kappa^*$
- Compare Spectral Densities with AdS/CFT

The linearized Boltzmann Equation

1. Collinear Bremmms
2. Hard collisions between particles
3. Random Walk of Hard Particles due to soft background field
   - All of the screening is in here.
   - Want to separate this from hard collisions.

We did it

Fokker Planck Equation



- Random Walk of Hard Particles

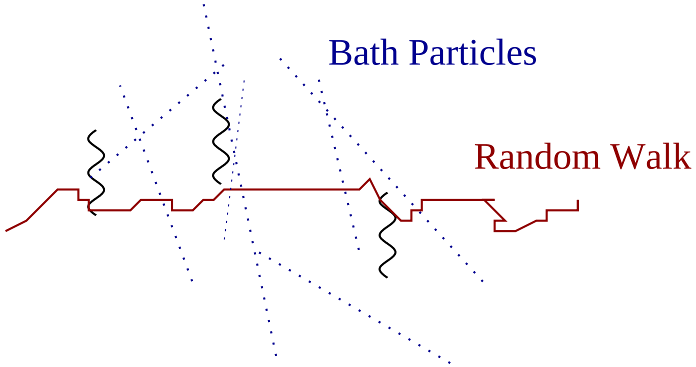
$$\partial_t \delta f \quad = \quad T\mu_A \frac{\partial}{\partial \boldsymbol{p}^i} \left(n_p(1+n_p) \frac{\partial}{\partial \boldsymbol{p}^i} \left[\frac{\delta f}{n_p(1+n_p)} \right] \right) + \text{gain terms}$$

- Could re-interpret this as a Langevin equation

$$\begin{aligned} \frac{d\boldsymbol{p}}{dt} &= -\mu_A (1+2n_p)\hat{\boldsymbol{p}} + \xi(t) \\ \left\langle \xi^i(t)\xi^j(t') \right\rangle &= 2T\mu_A \delta^{ij} \end{aligned}$$

- General form of Langevin equation will hold at leading order and NLO(?)
  - But need hard (unscreened) collisions, and collinear emission

Gain terms:



- We have random walk

$$\partial_t \delta f(\boldsymbol{p}) = \frac{\partial}{\partial \boldsymbol{p}^i} \left(T\mu_A n_p(1+n_p) \frac{\partial \chi(\boldsymbol{p})}{\partial \boldsymbol{p}^i} \right)$$

- Work done on the excess

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \int_{\boldsymbol{p}} p\, \partial_t \left[\delta f\right] = -\int_{\boldsymbol{p}} \hat{p}^i \cdot T\mu_A n_p(1+n_p) \frac{\partial \chi(\boldsymbol{p})}{\partial p^i}$$

- Gain terms

$$\text{gain terms} = \frac{1}{\xi_B}\left[\frac{1}{p^2}\frac{\partial}{\partial p} p^2 n_p(1+n_p)\right]\frac{\mathrm{d}E}{\mathrm{d}t} + \frac{1}{\xi_B}\left[\frac{\partial}{\partial \boldsymbol{p}} n_p(1+n_p)\right] \cdot \frac{\mathrm{d}\boldsymbol{P}}{\mathrm{d}t}\ ,$$

$$\xi_B = \int_p n_p(1+n_p) = \frac{T^3}{6}$$

Summary

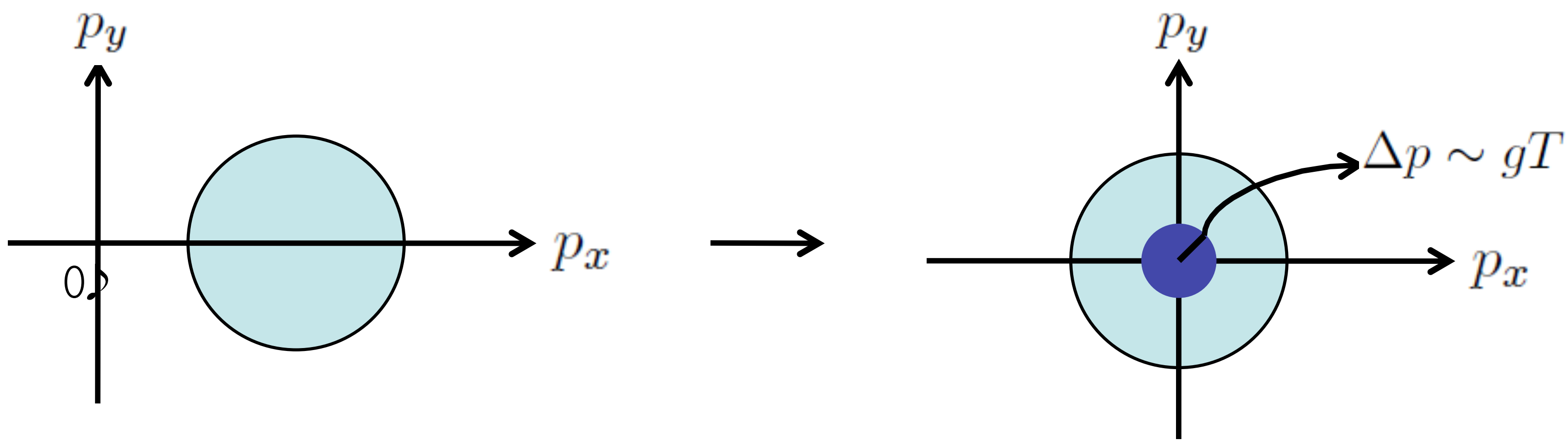
- The Boltzmann Equation

$$
\begin{aligned}
(\partial_t + v_{\boldsymbol{p}} \cdot \partial_{\mathbf{x}})\delta f \quad &= \quad T\mu_A \frac{\partial}{\partial \boldsymbol{p}^i}\left(n_p(1+n_p)\frac{\partial}{\partial \boldsymbol{p}^i}\left[\frac{\delta f}{n_p(1+n_p)}\right]\right) \\
&\quad + \frac{1}{\xi_B}\left[\frac{1}{p^2}\frac{\partial}{\partial p}p^2 n_p(1+n_p)\right]\frac{\mathrm{d}E}{\mathrm{d}t} + \frac{1}{\xi_B}\left[\frac{\partial}{\partial \boldsymbol{p}} n_p(1+n_p)\right]\cdot\frac{\mathrm{d}\boldsymbol{P}}{\mathrm{d}t}\ ,
\end{aligned}
$$

- Conserves energy and momentum
  - Is a symmetric pos-definite matrix

Could have guessed the form based on general symmetry arguments

Boundary conditions



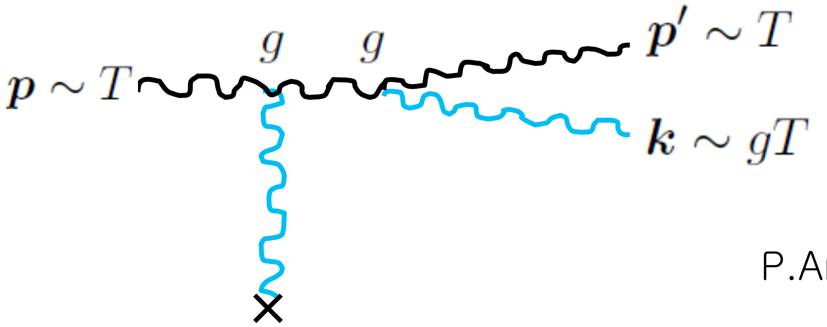
- The excess of gluons in a ball of $\Delta p \sim gT$

$$\int_0^{\Delta p} \frac{d^3\boldsymbol{p}}{(2\pi)^3} n_p(1+n_p)\chi(\boldsymbol{p}) \propto \overbrace{T^2}^{p^2 n_p(1+n_p)} \chi(0)\Delta p$$

- Its easy to create or destroy a soft gluon:

$$\lim_{\boldsymbol{p}\to 0} \chi(\boldsymbol{p}) = 0$$

Emission of Soft Glue:



P.Arnold, C.Dogan, G.D.Moore, (

- Rate for hard particles to Emit/Absorb a soft gluon with $p \sim gT$

$$\Gamma_{\rm soft} \sim g^2 T \times g^2 \times \int \frac{dk}{k} \times [1 + f(\boldsymbol{k})] \sim g^4 T^2 \int \frac{dk}{k^2}$$

$$\sim \frac{g^4 T^2}{m} \sim g^3 T \quad \Leftarrow \quad \text{Fast for us}$$

- Time scale we are considering $\tau \sim 1/g^4 T \log(1/g))$
- Then that the excess of gluons goes like $\sim 1/[\Gamma_{\rm soft} \tau]$

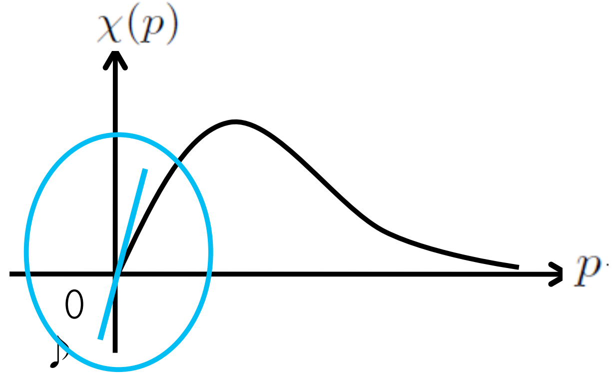
$$\chi(gT) = O(g)$$

The leading order weakly coupled limit $g \to 0$ is found with $\chi(0) = 0$

Number Change♪

- BE :♪ $\frac{\partial}{\partial t}\delta f = T\mu_A \frac{\partial}{\partial \boldsymbol{p}^i}\left[n_p(1+n_p)\frac{\partial \chi}{\partial \boldsymbol{p}^i}\right] + \frac{1}{\xi_B}\left[\frac{1}{p^2}\frac{\partial}{\partial p}p^2 n_p(1+n_p)\right]\frac{\mathrm{d}E}{\mathrm{d}t}$

- Particle Number Change :♪



$$\frac{\partial}{\partial t}\delta N_{FP} = \underbrace{-T\mu_A \lim_{p\to 0}\frac{1}{(2\pi)^3}\int \mathrm{d}\Omega_p \hat{\boldsymbol{p}}\cdot p^2 n_p(1+n_p)\frac{\partial \chi}{\partial \boldsymbol{p}}}_{\text{Diffusion Flux♪}} + \underbrace{\frac{-T^2}{2\pi^2 \xi_B}\frac{\mathrm{d}E}{\mathrm{d}t}}_{\text{Disturbed Gluons♪}}$$

- In Equilibrium : Average♪ $\delta N_{FP} = \mathrm{const}$

Summary

- With gain terms Energy and Momentum are conserved
- With boundary condition particle is not.
- Equation provides a good intial value problem – watch it equilibrate

What you gonna do with it ?

Equilibrium Correlation Functions

- Spectral Functions of $T^{\mu\nu}$

$$\rho^{\mu\nu\alpha\beta}(\omega, \boldsymbol{k}) = -2\,\mathrm{Im} G_R^{\mu\nu\alpha\beta}(\omega, \boldsymbol{k})$$

- Then the retarded Green Function is

$$G_R^{\mu\nu\alpha\beta}(\omega, \boldsymbol{k}) = -i \int_{-\infty}^{\infty} \mathrm{d}t \int_{-\infty}^{\infty} \mathrm{d}\boldsymbol{x}\, e^{+i\omega t - i\boldsymbol{k}\cdot\boldsymbol{x}}\, \theta(t) \left\langle \left[T^{\mu\nu}(t, \boldsymbol{x}), T^{\alpha\beta}(0, \boldsymbol{0}) \right] \right\rangle'$$

- Classify with four modes: $\boldsymbol{k}$ along z-axis
  - $G_R^{zxzx}(\omega, \boldsymbol{k})$ – Shear mode
  - $G_R^{zzzz}(\omega, \boldsymbol{k})$ – Sound mode
  - $G_R^{xyxy}(\omega, \boldsymbol{k})$ – Tensor mode
  - $\eta_{\mu\nu}\eta_{\alpha\beta}G_R^{\mu\nu\alpha\beta}(\omega, \boldsymbol{k})$ – Bulk mode

Linear Response:

- Turn on a small gravitational field – can do this in Kinetics a theory too

$$S(g_{\mu\nu}) \simeq S_o + \frac{1}{2}\int d^4X \, T^{\mu\nu}(X) \, h_{\mu\nu}(X) \,,$$

- This disturbs $T^{\mu\nu}$ from equilibrium

$$\langle T^{\mu\nu}(X) \rangle_{h_{\alpha\beta}} = T^{\mu\nu}_{\rm eq}(X) - \frac{-i}{2}\int d^4Y \, \theta(X^0 - Y^0) \left\langle [T^{\mu\nu}(X), T^{\alpha\beta}(Y)] \right\rangle' h_{\alpha\beta}(Y)$$

- In Fourier space then

$$\underbrace{\langle T^{\mu\nu}(\omega, \boldsymbol{k}) \rangle_{h_{\alpha\beta}}}_{\text{Compute in KT}} = \left.\frac{\partial T^{\mu\nu}_{\rm eq}}{\partial h_{\alpha\beta}}\right|_{h=0} h_{\alpha\beta}(\omega, \boldsymbol{k}) - \frac{1}{2} G_R^{\mu\nu\alpha\beta}(\omega, \boldsymbol{k}) \, h_{\alpha\beta}(\omega, \boldsymbol{k})$$

Can use this to compute spectral densities in kinetic theory

Linearize Response in Kinetic Theory

$$\frac{1}{E_p}\left(P^\mu \frac{\partial}{\partial X^\mu} - \Gamma^\lambda_{\mu\nu} P^\mu P^\nu \frac{\partial}{\partial P^\lambda}\right) f(t,\boldsymbol{x},\boldsymbol{p}) = C[f,\boldsymbol{p}]\,,$$

- Turn on a small $h_{xy}(t,z)$

$$f = n_p^h + \delta f\,, \qquad n_p^h = \frac{1}{\exp(\sqrt{\boldsymbol{p}^i(\eta_{ij}+h_{ij})\boldsymbol{p}^j}/T) \mp 1}\,.$$

- Find an EOM in fourier space

$$(-i\omega + iv_{\boldsymbol{p}}\cdot\boldsymbol{k})\delta f(p,\omega,\boldsymbol{k}) + \underbrace{n_{\boldsymbol{p}}(1+n_{\boldsymbol{p}})\frac{p^i p^j}{2E_{\boldsymbol{p}}T}(-i\omega h_{ij}(\omega,\boldsymbol{k}))}_{\text{Source}}$$

$$= T\mu_A \frac{\partial}{\partial p^i}\left(n_{\boldsymbol{p}}(1+n_{\boldsymbol{p}})\frac{\partial\chi(\boldsymbol{p})}{\partial p^i}\right) + \text{ gain terms}$$

  – Source vanishes for time indep grav fields

Solve $2D$ partial diffeq for $\delta f(\omega,k)$ by matrix inversion

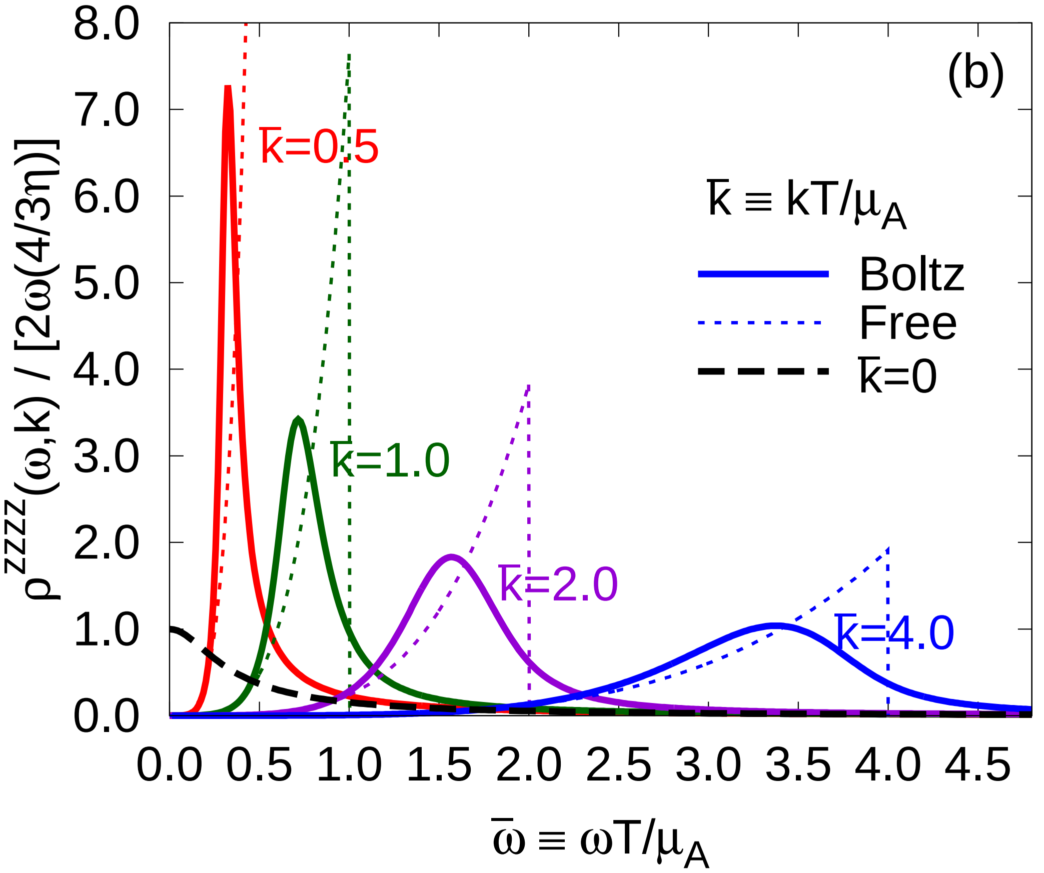
Finally computing G_R^{zxzx}

- The spectral weight is $\rho = -2\,\mathrm{Im} G_R$
- Given delta $f(\omega, k)$ we compute the stress tensor

$$T^{zx}_{eq} + \delta T^{zz} = \nu_g \int \frac{\mathrm{d}^3\boldsymbol{p}\sqrt{-g}}{(2\pi)^3} \frac{p^z p^x}{E_p} \left(n_P^h + \delta f(\omega, k) \right) \,,$$

$$= -\mathcal{P}_o h_{zx}(\omega, k) + \underbrace{\left[\nu_g \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} \frac{p^z p^x}{E_{\boldsymbol{p}}} \frac{\delta f(\omega, \boldsymbol{k})}{h_{zx}(\omega, \boldsymbol{k})} \right]}_{\text{This is } G_R^{zxzx}} h_{zx}(\omega, k)$$

Sound mode – $\langle T^{zz}(x)T^{zz}(0)\rangle$



- Free streaming (easy) – high $\boldsymbol{k}, \omega$
- Hydro – small $\boldsymbol{k}, \omega$

Analysis with Linearized (Conformal) Second Order Hydro

- Stress Tensor:

$$T^{\mu\nu} = T^{\mu\nu} + \pi_1^{\mu\nu} + \pi_2^{\mu\nu} \,,$$

- 2nd Order dissipative part of the stress tensor (Static) :

$$\pi_2^{\mu\nu} = \eta\tau_\pi \left[2u_\alpha R^{\alpha\langle\mu\nu\rangle\beta}u_\beta - 2\left\langle\nabla^\mu\nabla^\nu \ln T\right\rangle\right] + \kappa\left[R^{\langle\mu\nu\rangle} - 2u_\alpha R^{\alpha\langle\mu\nu\rangle\beta}u_\beta\right],$$

  - Baier, Romatschke, Son, Starinets, Stephanov '07

- Using lower order order EOM, they rewrote it as a Dynamic equation

$$\pi^{\mu\nu} = \pi_1^{\mu\nu} - \tau_\pi \left\langle D\pi^{\mu\nu}\right\rangle + \kappa\left[R^{\langle\mu\nu\rangle} - 2u_\alpha R^{\alpha\langle\mu\nu\rangle\beta}u_\beta\right],$$

Which one is better?

Determine the coefficients, τ_π and κ

- Turn on small h_{xy} and solve linearized equations of motion $\nabla_\mu T^{\mu\nu} = 0$

$$e(t, \boldsymbol{x}) \simeq e_o + \epsilon(t, z)\,, \qquad u^\mu \simeq (1, u^i(t, z))\,,$$

- and find (BRSSS)

$$\delta T^{xy} = \underbrace{\left[-i\eta\omega + \eta\tau_\pi\omega^2 - \frac{1}{2}\kappa(\omega^2 + k^2)\right]}_{G_R^{xyxy}} h_{xy}$$

- We expand our numerical results at small ω and k

$$\frac{\tau_\pi}{\eta/sT} = 6.32 \qquad\qquad \kappa = 0$$

An approximate result $\eta\tau_\pi \sim 1/g^8$ while $\kappa = O(1)$. Notice κ is a static: $G_R^{xyxy}(0, k) = -\frac{1}{2}\kappa k^2$.

Now τ_π and κ are fixed

Predictions for sound peak (with fixed coefficients)

- Turn on h_{zz} solve $\nabla_\mu T^{\mu\nu} = 0$
- For the static theory

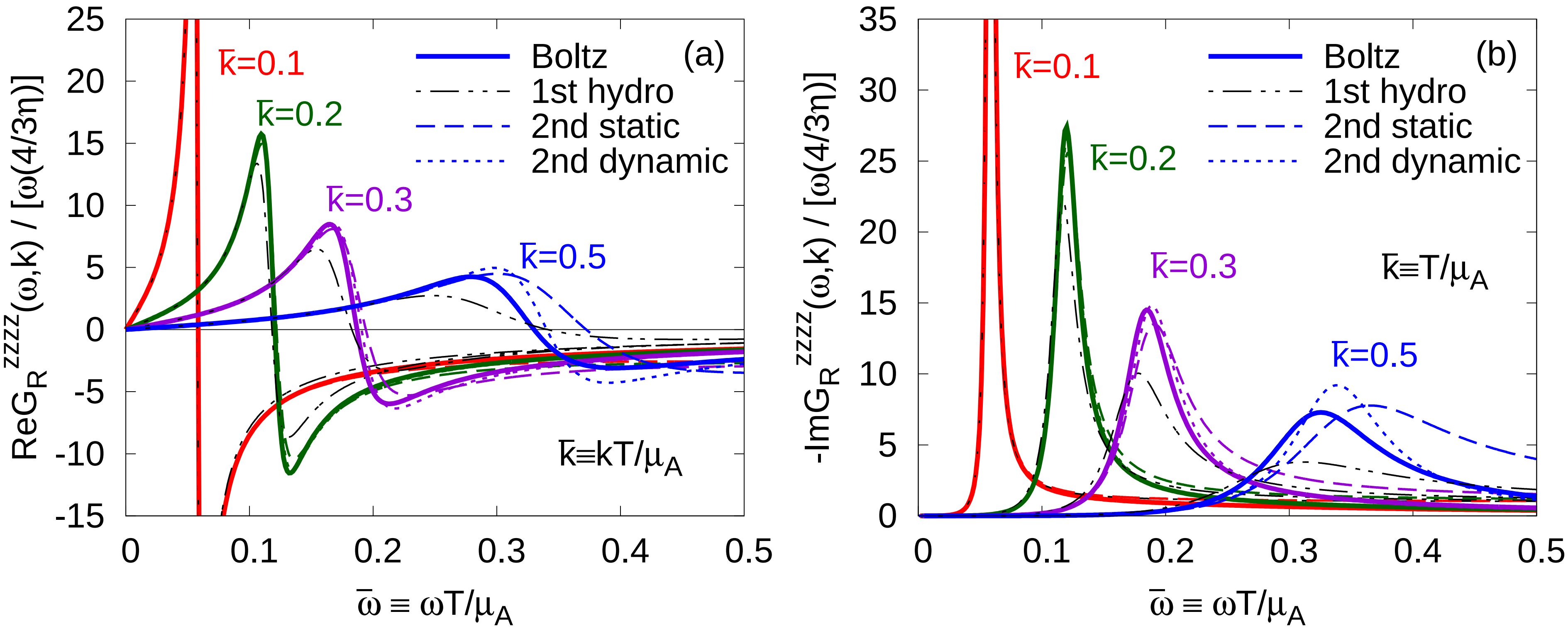
$$T^{zz}(\omega, k) = \underbrace{\left[(e_o + \mathcal{P}_o) \frac{c_s^2 \omega^2 - i\Gamma_s \omega^3 + \tau_\pi \Gamma_s \omega^4 + \tau_\pi \Gamma_s c_s^2 k^2 \omega^2}{\omega^2 - c_s^2 k^2 + i\Gamma_s \omega k^2 - \tau_\pi \Gamma_s c_s^2 k^4} \right]}_{\text{This is } G_R^{zzzz}} h_{zz}$$

- For the dynamic theory

$$G_R^{zzzz}(\omega, k) = (e_o + \mathcal{P}_o) \frac{c_s^2 \omega^2 - i\Gamma_s \omega^3 - i\tau_\pi c_s^2 \omega^3}{\omega^2 - c_s^2 k^2 + i\Gamma_s \omega k^2 + i\tau_\pi c_s^2 \omega k^2 - i\tau_\pi \omega^3}$$

Now go compare with full results

Sound Mode



- Translate:

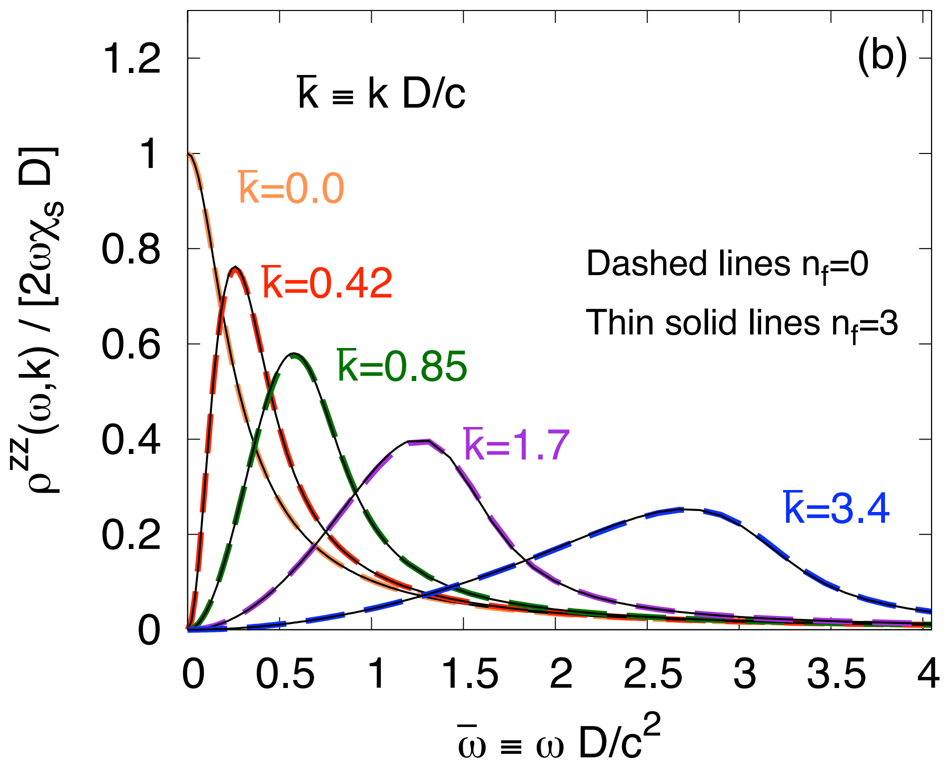
$$0.5\,\bar{k} = 0.7\,\overbrace{\frac{c\eta}{(e+\mathcal{P})c_s^2}}^{\sim\ell_{\rm mfp}}\,k$$

- Dynamic is arguably better than static

2nd order "works until" $\omega, ck \lesssim 0.7\left[\eta/(e_o+\mathcal{P}_o)c_s^2\right]^{-1}$

Current-Current Channel (all other channels studied as well)

$$\rho^{zz}(\omega, \boldsymbol{k}) = \text{Fouier Trans} \left\langle \left[J^{\mu\nu}(t, \boldsymbol{x}), J^{\alpha\beta}(0, \boldsymbol{0}) \right] \right\rangle$$



Expect diffusion structure for $kD/c \lesssim 0.7$. Can use to fit finite $\boldsymbol{k}$ lattice data

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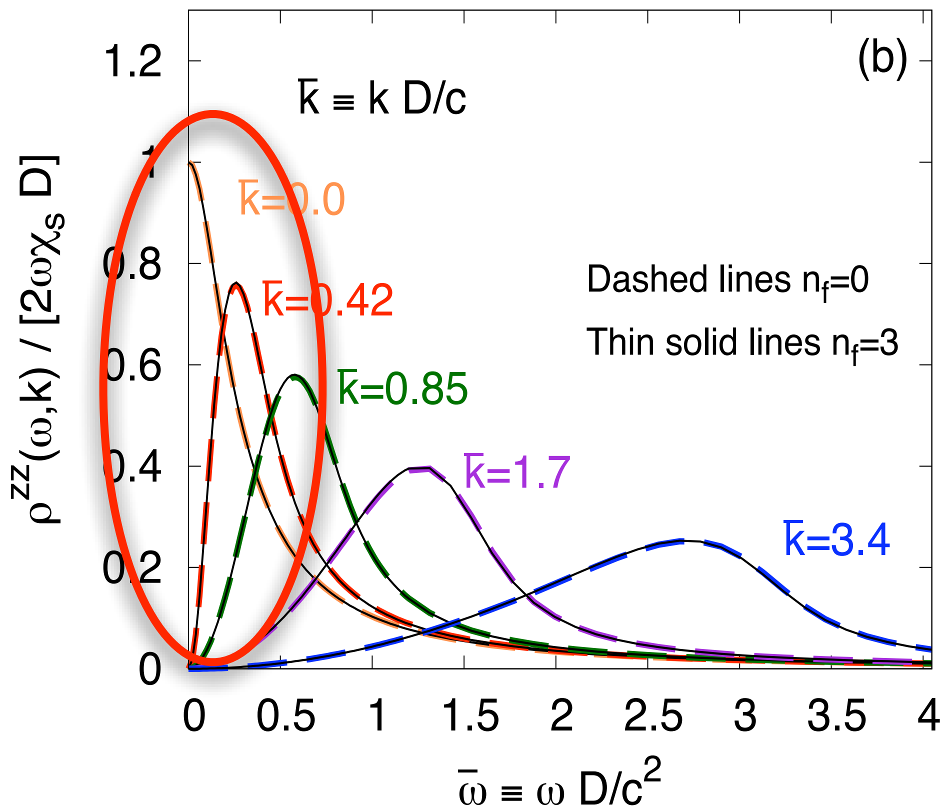

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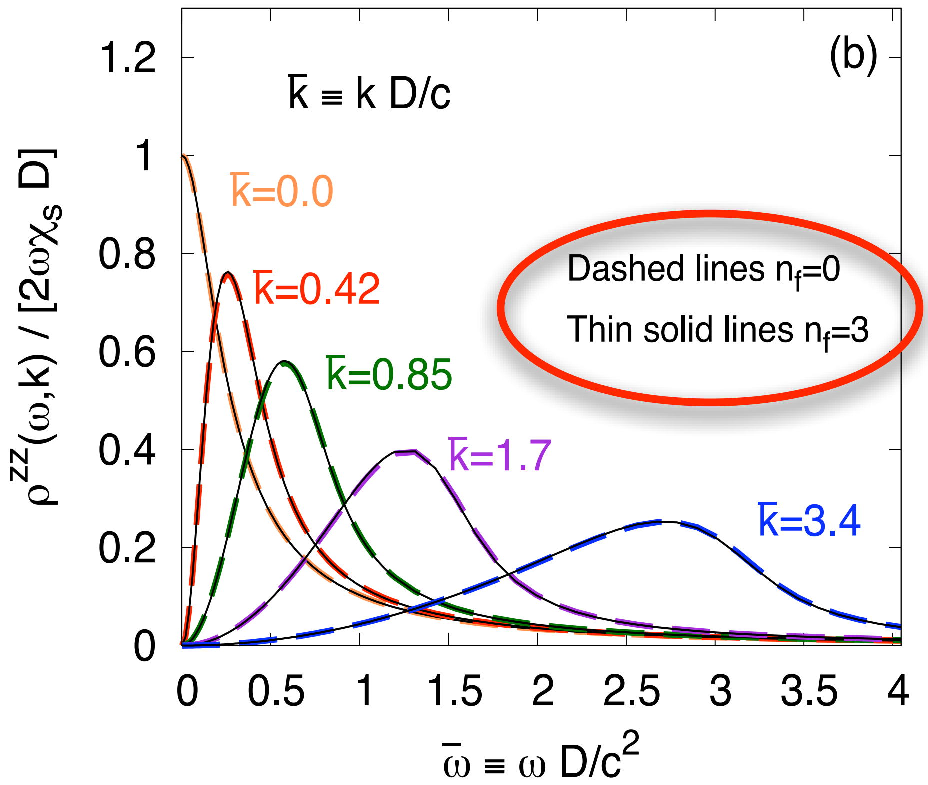

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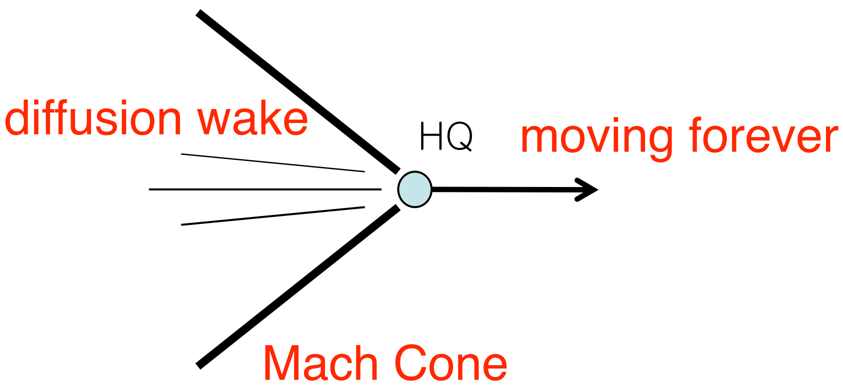
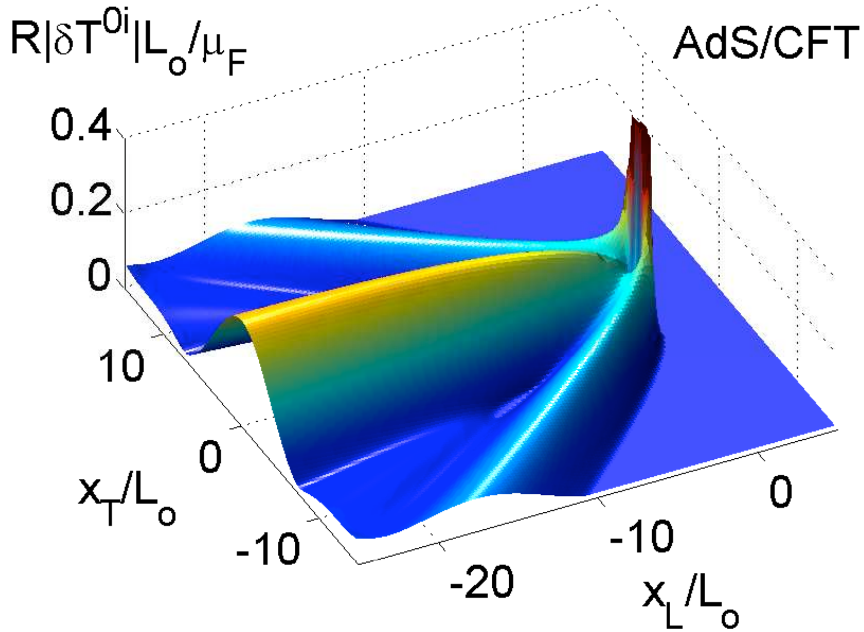
Return to the Mach Cone



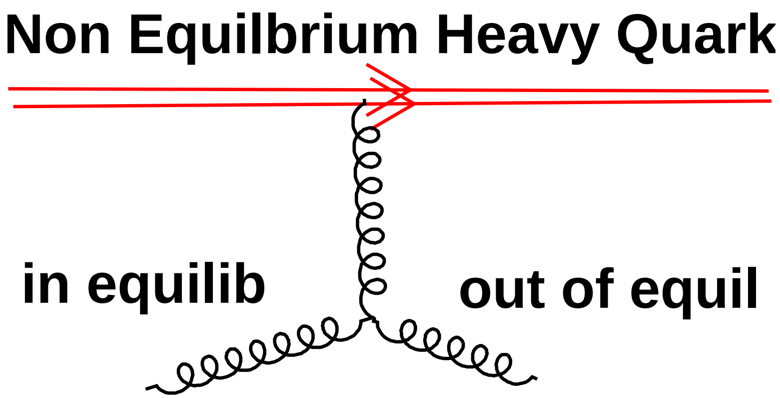


- Most of the structure you see is universal ideal hydro:

$$\partial_t T^{0j} + \partial_i T^{ij}_{\mathrm{hydro}} = \frac{\mathrm{d}p^j}{\mathrm{d}t}\delta^3(\boldsymbol{x} - vt) + \text{corrections}$$

We differences between weak and strong lie at intermediate distances!

The source for kinetic theory:



- The Boltzmann equation becomes

$$(\partial_t + \boldsymbol{v}_p \cdot \partial_{\boldsymbol{x}})\, \delta f(t, \boldsymbol{x}, \boldsymbol{p}) = C[\delta f, \boldsymbol{p}] + S(\boldsymbol{p})\delta^3(\boldsymbol{x} - \boldsymbol{v}t)$$

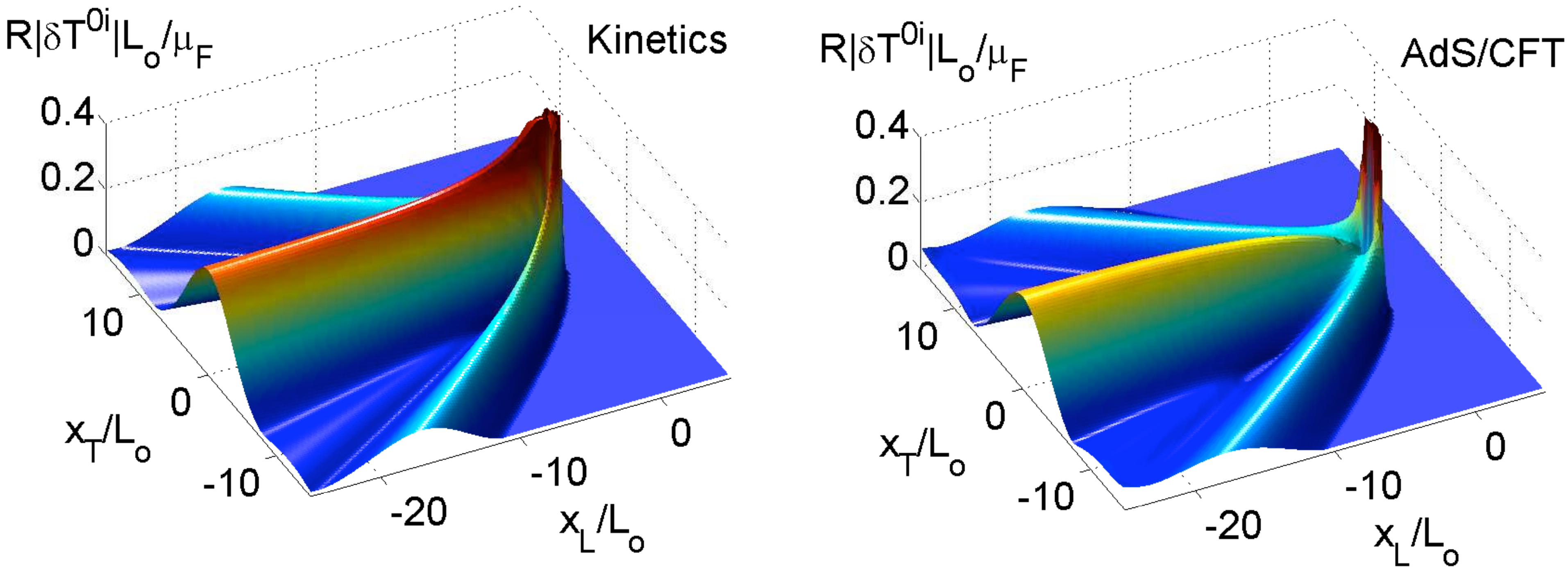
where the kinetic theory source is

$$S(\boldsymbol{p}) \propto \left[\frac{1}{p^2}\frac{\partial}{\partial p} p^2 n_p(1+n_p)\right] \underbrace{\mu_F v^2}_{\text{coll e-loss}} + \left[\frac{\partial}{\partial \boldsymbol{p}} n_p(1+n_p)\right] \cdot \underbrace{\mu_F \boldsymbol{v}}_{\text{coll p-loss}}$$

- E-loss of HQ found by Braaten-Thoma (1995): $\mathrm{d}\boldsymbol{p}/\mathrm{d}t = -\mu_F \boldsymbol{v}$

Final Result

$$L_o = \frac{4}{3}\frac{c\,\eta}{(e+\mathcal{P})c_s^2} = \text{A measure of the mean free path}$$



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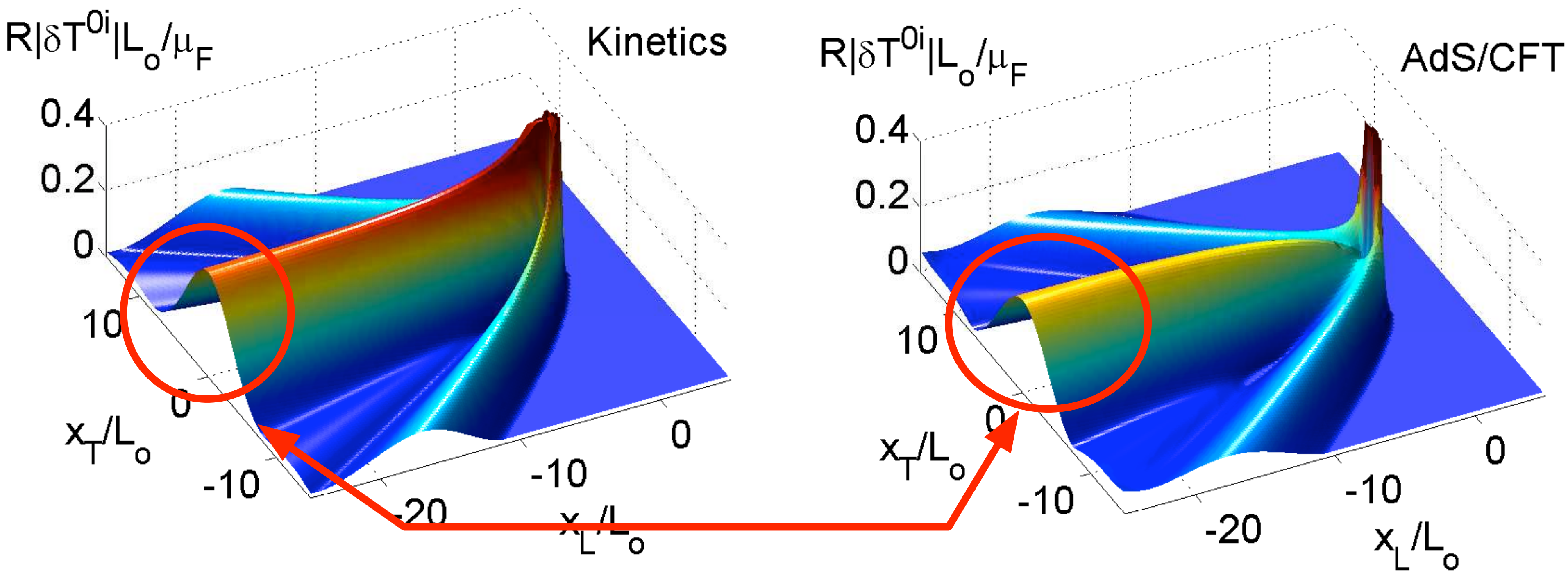
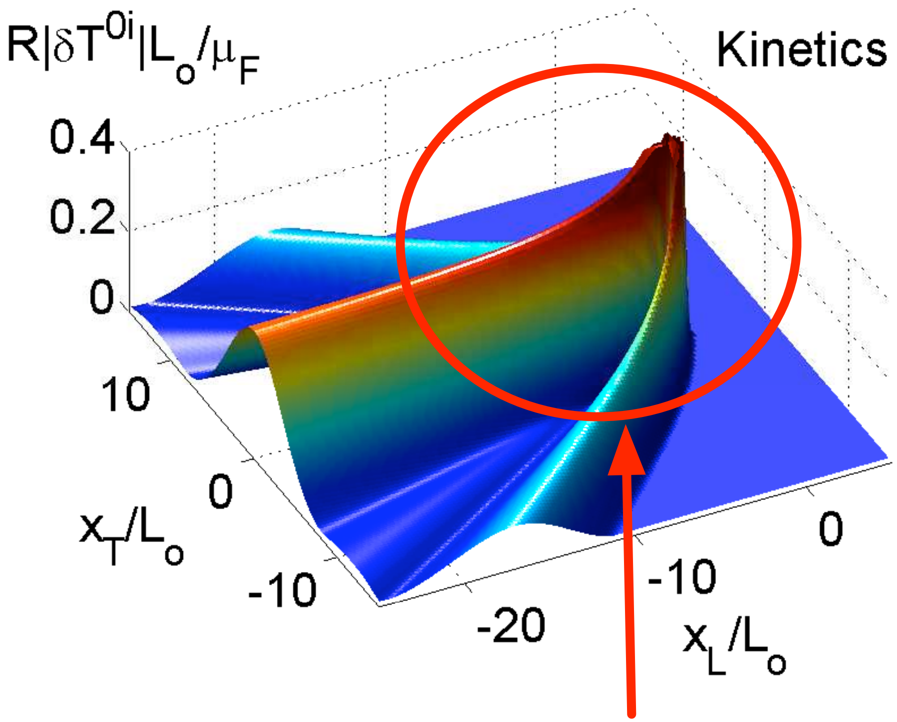
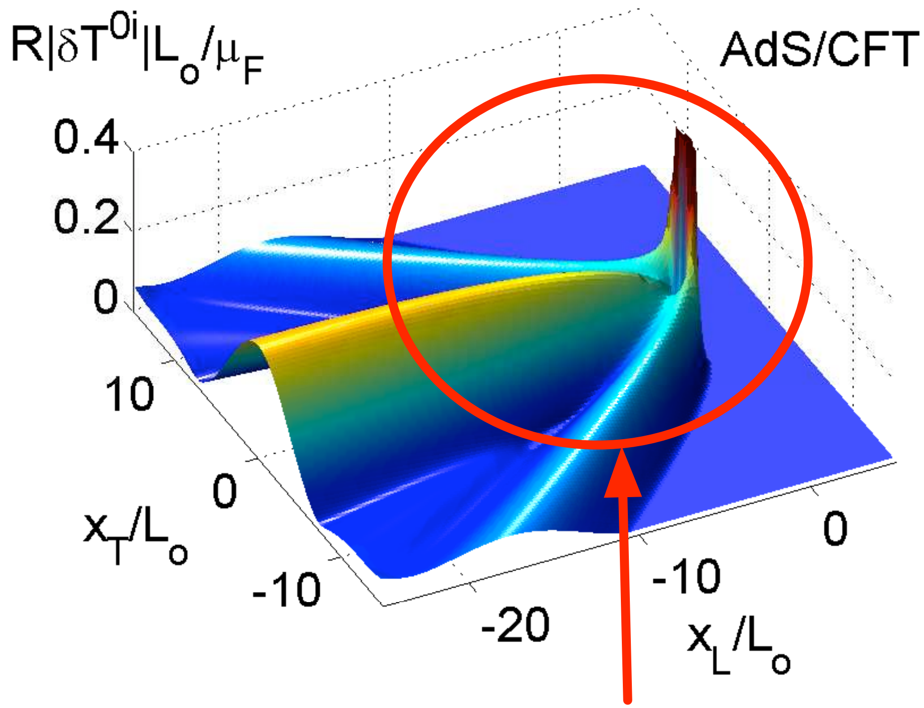


Same at large distances

Final Result

$$L_o \equiv \frac{4}{3}\frac{c\,\eta}{(e+\mathcal{P})c_s^2} = \text{A measure of the mean free path}$$



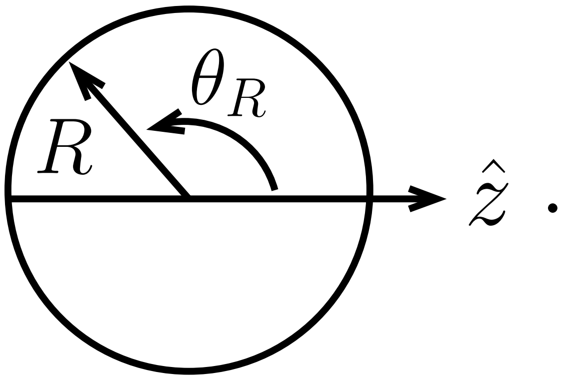


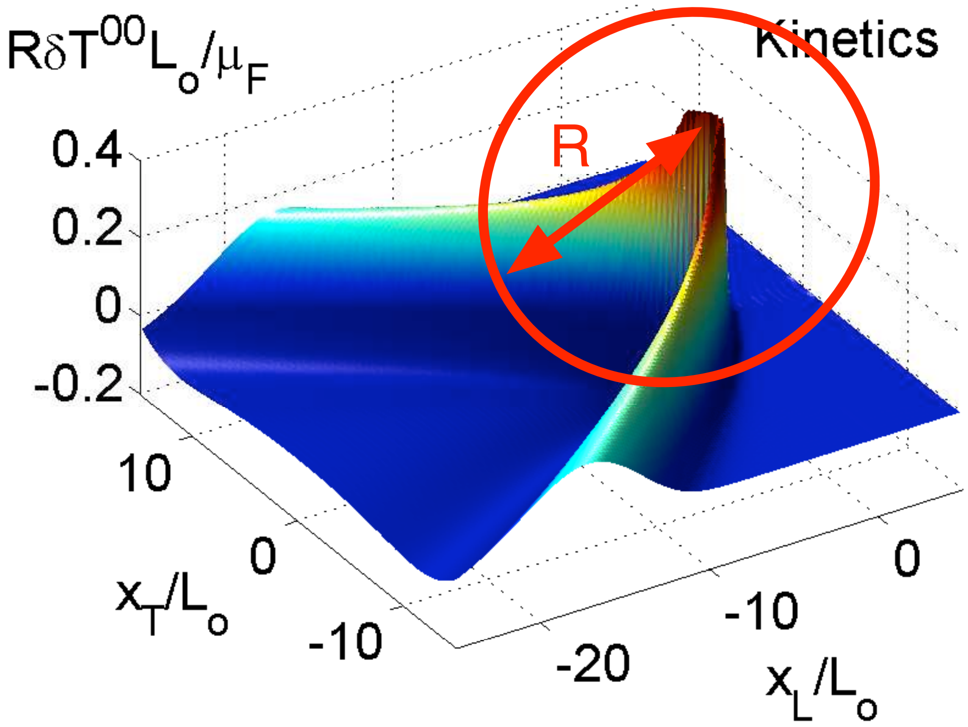
Free streaming, slowly becoming hydro

Dissipate like crazy, hydro emerges

Comparing with hydro quantitatively

$$L_o \equiv \frac{4}{3}\frac{c\,\eta}{(e+\mathcal{P})c_s^2} = \text{“mean free path”}$$

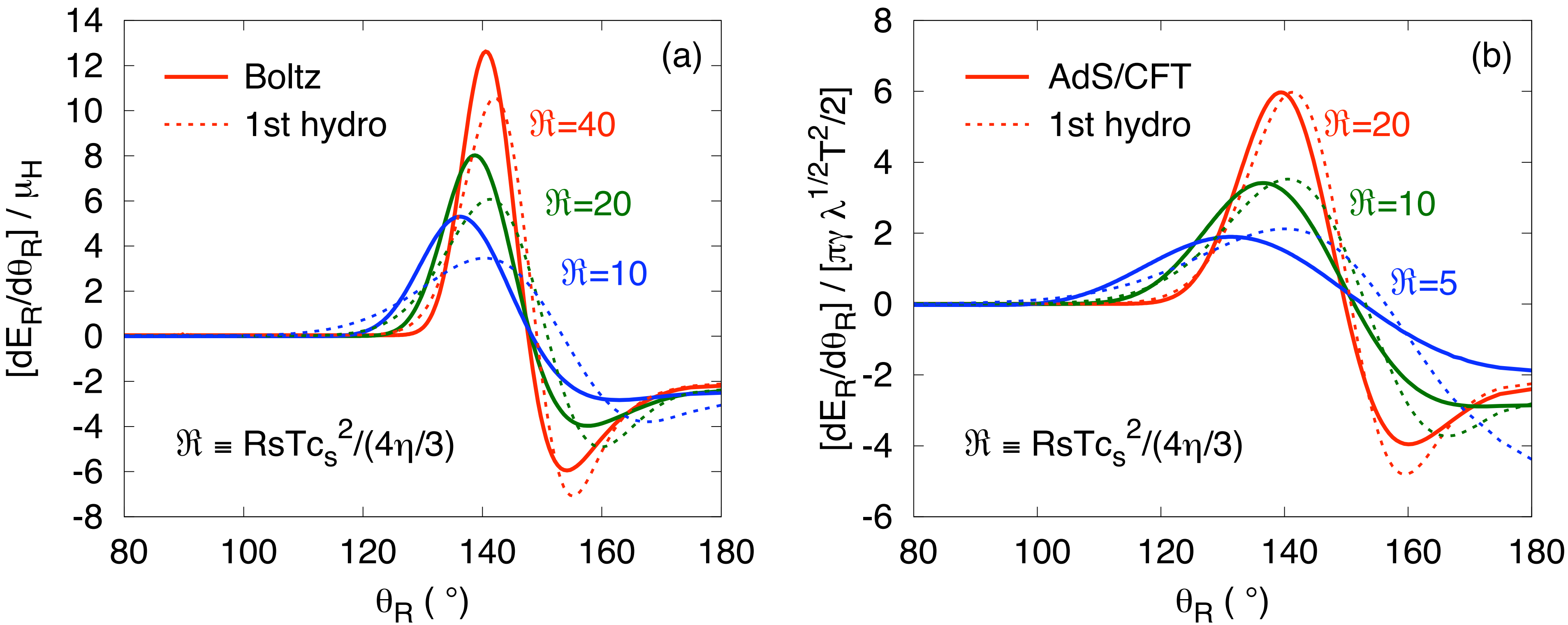




Go around circle and plot the energy density, compare with hydro

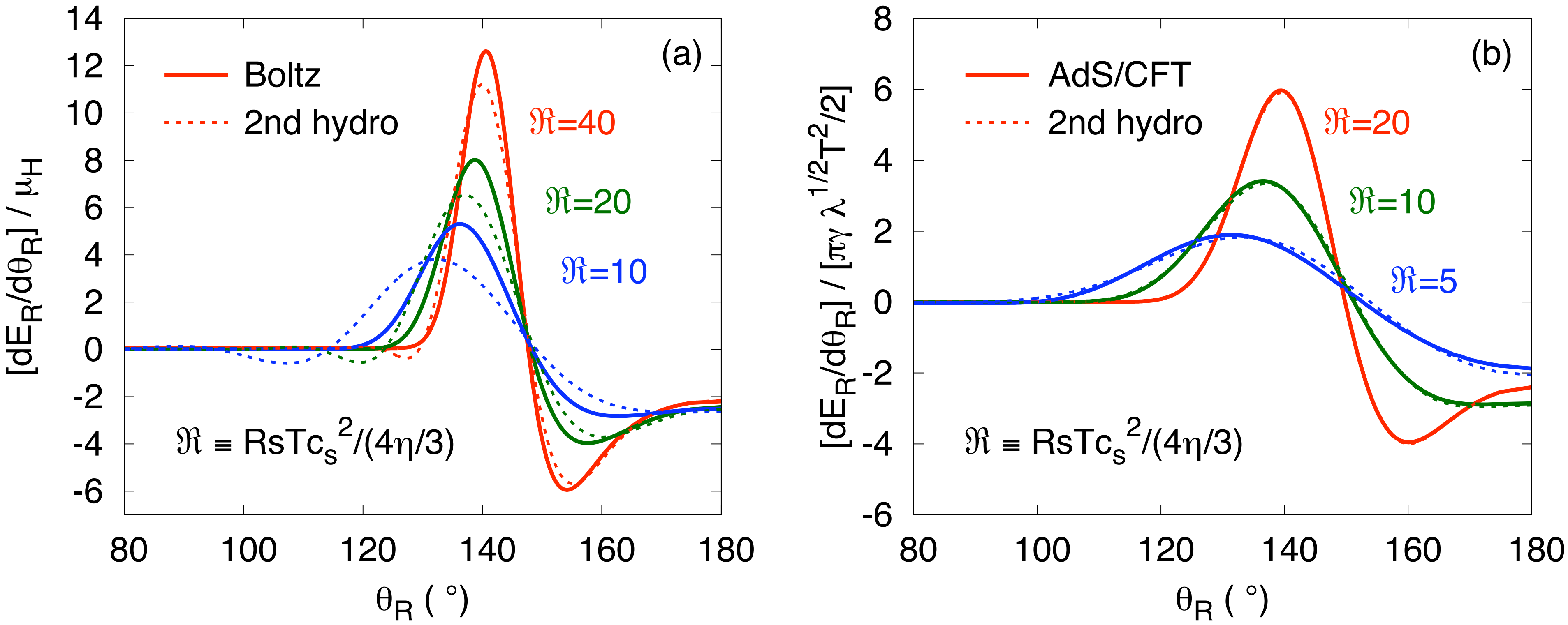
Comparison with hydro

$$\mathfrak{R} = \frac{R}{L_o} \quad \text{where} \quad L_o \equiv \frac{4}{3}\frac{c\,\eta}{(e+\mathcal{P})c_s^2} = \text{“mean free path”}$$



Comparison with hydro

$$\Re = \frac{R}{L_o} \quad \text{where} \quad L_o \equiv \frac{4}{3}\frac{c\,\eta}{(e+\mathcal{P})c_s^2} = \text{“mean free path”}$$



The AdS/CFT works amazingly well once second order corrections are included

Comparison with hydro

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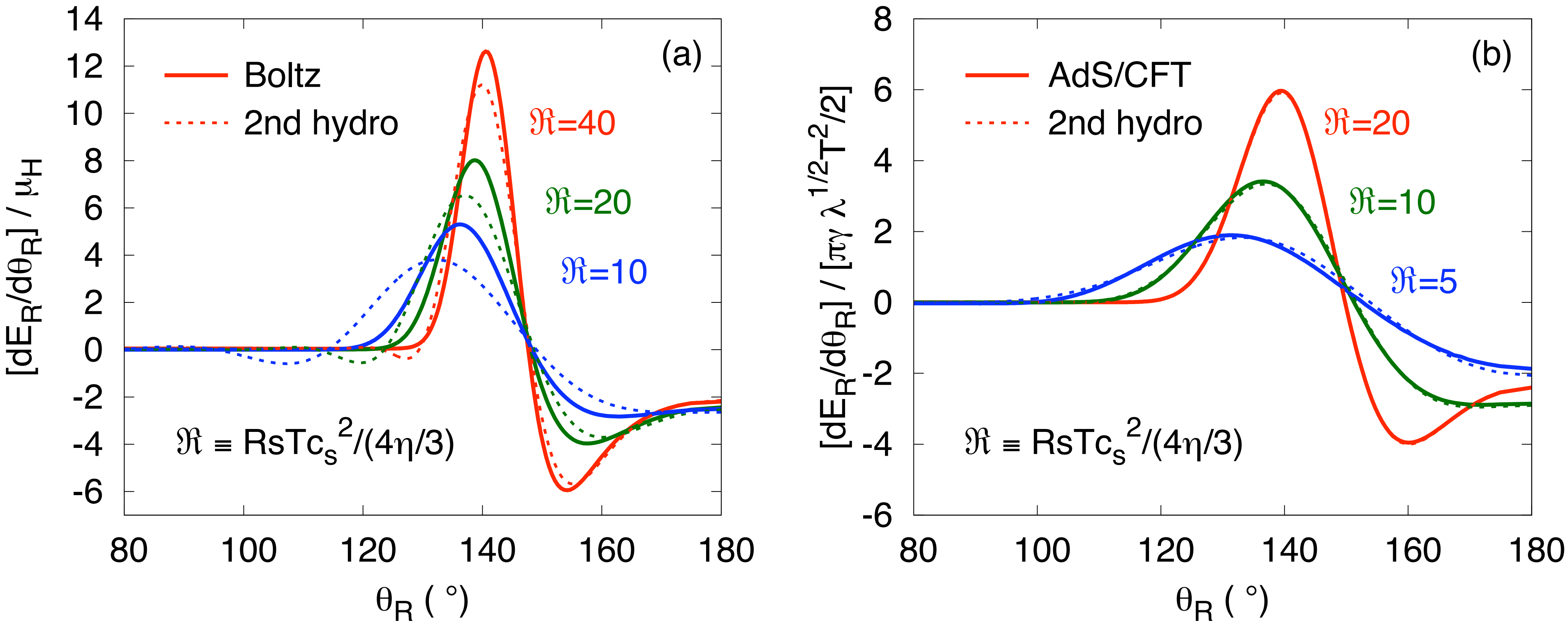


Second order corrections don’t help Boltzmann too much

Why?

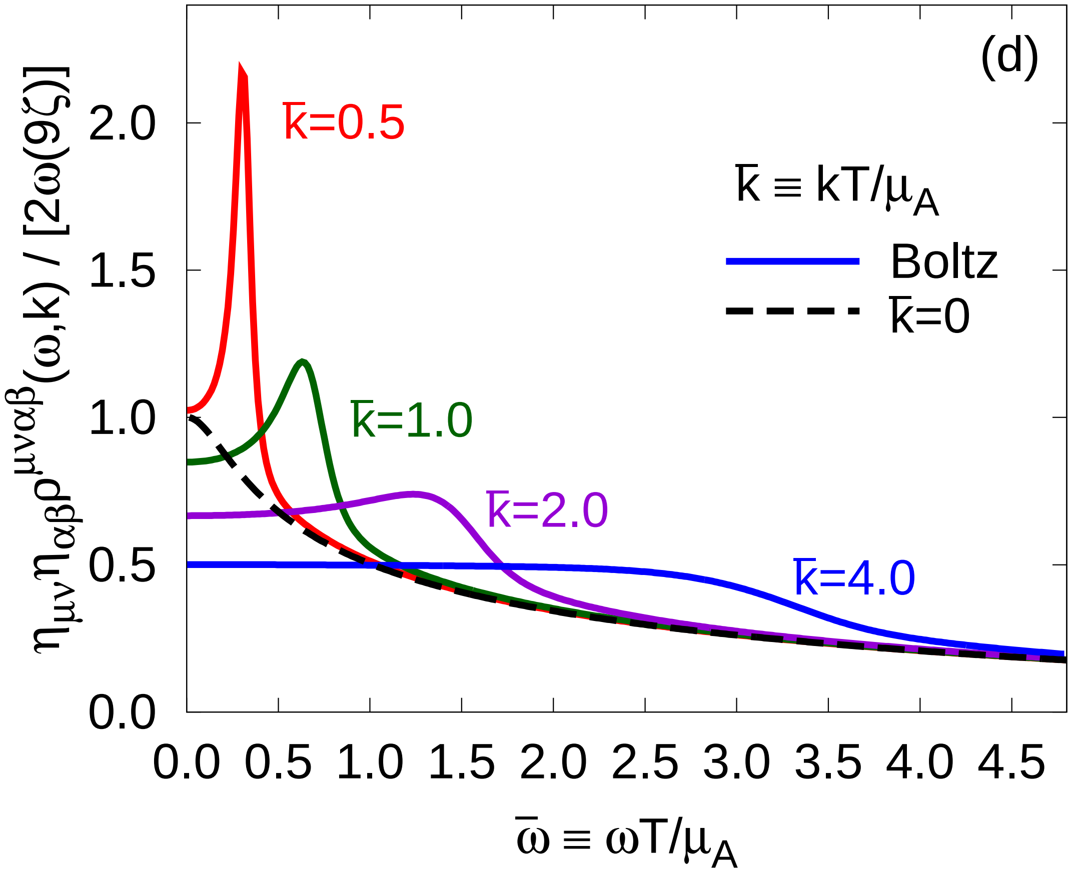
$$\frac{\tau_\pi}{\eta/sT} = 6.32\,, \qquad \text{(Kinetic Theory)}$$

$$\frac{\tau_\pi}{\eta/sT} = 4 - 2\log(2) \simeq 2.61\,. \qquad \text{(AdS/CFT)}$$

Can't have such (relatively) small second order parameters

in any kinetic theory with any collision term!!!

Bulk mode – $\langle T^{\mu}_{\mu}(x) T^{\mu}_{\mu}(0)\rangle$



- Hydro – small $\boldsymbol{k}, \omega$

Bulk perturbations

- Gravitational Perturbation:

$$g_{\mu\nu}(X) = (1 + H(x))\eta_{\mu\nu}$$

- The correct equilibrium is not at constant temperature

$$T_H(X)\sqrt{-g_{00}(X)} = \text{Const}\,, \qquad T_H(x) = T_o\left(1 - \frac{1}{2}H(X)\right)\,,$$

- Expand around this time dependent equilibrium

$$f = n_{\boldsymbol{p}}^H + \delta f \qquad n_{\boldsymbol{p}}^H(t, \boldsymbol{x}, \boldsymbol{p}) = \frac{1}{e^{-P(X)\cdot U(X)/T_H(X)} - 1}$$

Boltzmann Equation for Bulk Perturbation

- Boltzmann equation + mass term

$$\frac{1}{E_p}\Big(P^\mu \frac{\partial}{\partial X^\mu} - \Gamma^\lambda_{\mu\nu} P^\mu P^\nu \frac{\partial}{\partial P^\lambda} - \underbrace{\frac{1}{2}\frac{\partial m^2(X)}{\partial X^\mu}\frac{\partial}{\partial P_\mu}}_{\text{Force term } \frac{\partial E_p}{\partial \boldsymbol{x}}\frac{\partial f}{\partial \boldsymbol{p}}}\Big) f(t,\boldsymbol{x},\boldsymbol{p}) = C[f,\boldsymbol{p}]$$

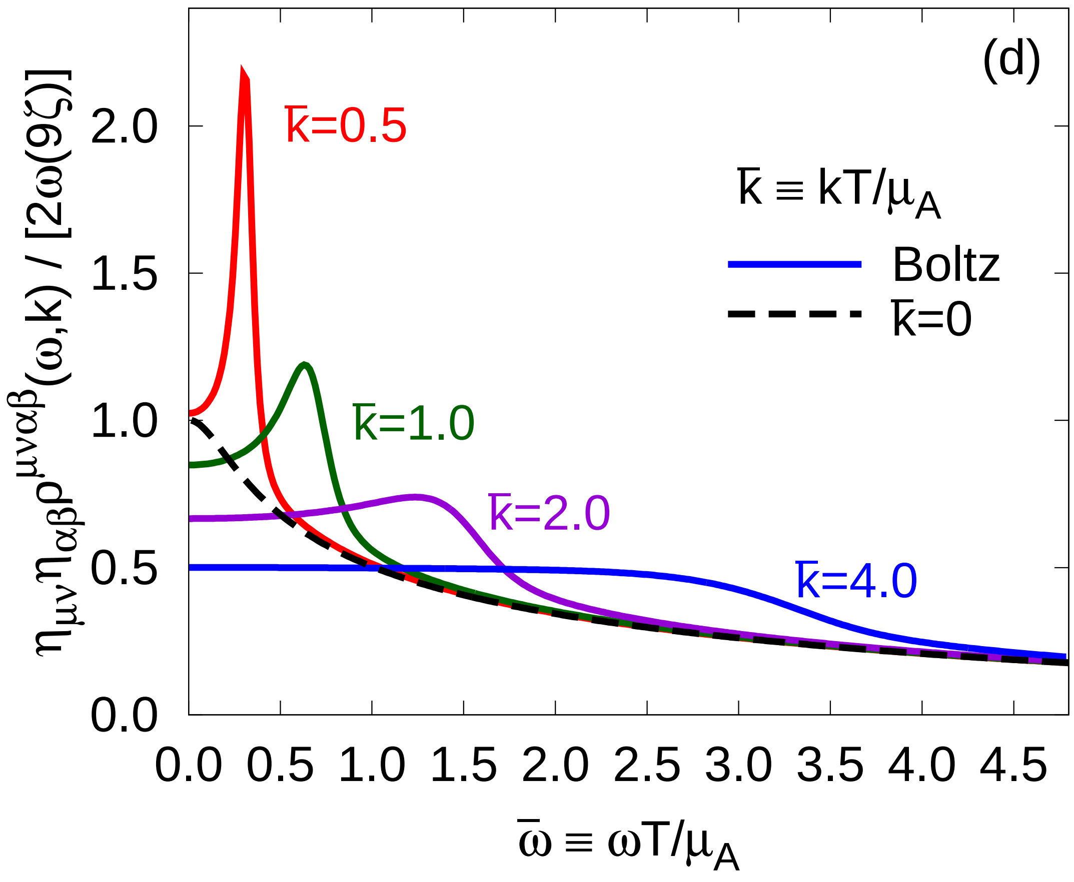
- After Plugging in $f = n_p^H + \delta f$ and careful algebra

$$(\partial_t + v_{\boldsymbol{p}} \cdot \partial_{\boldsymbol{x}})\,\delta f - n_p(1+n_p)\frac{\tilde{m}^2}{2E_{\boldsymbol{p}}T}\partial_t H = \mathcal{C}[\delta f, \boldsymbol{p}]\,,$$

where

$$\tilde{m}^2 \equiv m^2 - T^2 \frac{\partial m^2}{\partial T^2}\bigg|_{T=T_o} = -C_A \beta(g)\frac{T^2}{6}\,.$$

Bulk mode – $\langle T^{\mu}_{\mu}(x) T^{\mu}_{\mu}(0) \rangle$



- Hydro – small $\boldsymbol{k}, \omega$

Should be able to extract second order non-conformal transport coeffs

2nd Order Non-Conformal Hydro (Romatschke)

- At linear order in non-conformal hydro there is the shear tensor

$$\pi^{\mu\nu} = \pi_1^{\mu\nu} + \eta\tau_\pi \langle D\sigma^{\mu\nu}\rangle + \kappa\left[R^{\langle\mu\nu\rangle} - 2u_\alpha R^{\alpha\langle\mu\nu\rangle\beta}u_\beta\right] + \kappa^* 2u_\alpha u_\beta R^{\alpha\langle\mu\nu\rangle\beta}\,,$$

and the bulk tensor

$$\Pi = \Pi_1 + \zeta\tau_\Pi D(\nabla\cdot u) + \xi_5 R + \xi_6 u_\alpha u_\beta R^{\alpha\beta}$$

- Lots of coeffs . . .
  - But $\kappa^* = \xi_5 = \xi_6 = 0$ in kinetic theory for the same reasons as before

2nd Order Non-Conformal Hydro (Romatschke)

- At linear order in non-conformal hydro there is the shear tensor

$$\pi^{\mu\nu} = \pi_1^{\mu\nu} + \eta\tau_\pi \left\langle D\sigma^{\mu\nu}\right\rangle$$

and the bulk tensor

$$\Pi = \Pi_1 + \zeta\tau_\Pi D(\nabla \cdot u)$$

- Turning on a sinusoidal perturbation

$$g_{\mu\nu} = (1 + H(t))\eta_{\mu\nu}$$

- One can, solve the hydro equations of motion and find the hydro prediction

$$\left\langle c_s^2 T_0^0 + T_i^i \right\rangle = -\frac{1}{2}\left[-i\zeta\omega + \zeta\tau_\Pi\omega^2\right] H(\omega, 0)$$

Comparison with kinetic results for $\left\langle c_s^2 T_0^0 + T_i^i \right\rangle$ determines τ_Π

Results for τ_Π

N_f	0	2	3
τ_Π/τ_π	0.510	0.548	0.554

- Bulk viscosity is small because the coupling of T^μ_μ to $H(t)\eta_{\mu\nu}$ small

$$\zeta \sim \beta(g)^2 T^4 \times \frac{1}{g^4 T \log(1/g)}$$

- But the relaxation times of such perturbations is similar to τ_π

$$\frac{\tau_\Pi}{\tau_\pi} \simeq 0.5$$

Conclusions

- Formulated the linearized Boltzmann Equation as a Fokker Planck Equation
- Determined <u>all</u> spectral functions (most not discussed)
  - Found that response is well described by hydrodynamics for

$$\omega, ck \lesssim 0.7 \frac{\eta}{(e + \mathcal{P})c_s^2}$$

  - Completed a second order analysis of bulk response function.
  - Only $\tau_{\Pi} \neq 0$ for linear case.
- Studied the medium response to a heavy quark probe
  - The AdS/CFT converges to hydro extraordinarily quickly (even after measuring units in terms of mean free paths)