Light-Front Methods and Non-Perturbative QCD

Stan Brodsky

SLAC

Hadron Structure from Lattice QCD.

BNL - Riken

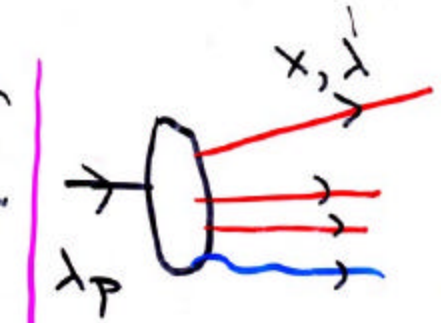
March 18, 2002

Schmidt
W.-M. Zhang

Light-Cone Wavefunctions

encode all helicity, transversity distributions

$$q_{\lambda/\lambda_p} = \sum \left| \right|^2$$



$q_{\lambda/\lambda_p}(x, \Lambda)$ { transversity: density matrix; light-cone helicity }

$$= \sum_{n,\xi} \int \left| \psi^{(\Lambda)}_{n,\lambda_p}(x_i, \vec{k}_{\perp i}, \lambda_i) \right|^2 \prod_{j=1} dx_j \prod_{j=1} d^2 k_{\perp j}$$

$$\delta\left(\sum_i x_i - 1\right) \delta\left(\sum_i \vec{k}_{\perp i}\right)$$

$$\delta(x - x_q)\, \delta_{\lambda,\lambda_q}$$

$$\Theta(\Lambda^2 - m_n^2)$$

Factorization

Light-cone Schema

$\tau = t + z/c$

Dirac
Bjorken, Kogut, Soper
Lepage + SJB
Pauli + SJB

Equation of motion

$$i\frac{\partial}{\partial\tau}|\Psi_H\rangle = P^-|\Psi_H\rangle = \frac{M_H^2 + \vec{P}_\perp^{\,2}}{P^+}|\Psi_H\rangle$$

$$H_{LC} = P^- P^+ - \vec{P}_\perp^{\,2}$$

$P^+, \vec{P}_\perp$ kinematical

$$\boxed{H_{LC}|\Psi_H\rangle = M_H^2|\Psi_H\rangle}$$

⇒ eigenvalue problem for LC Hamiltonian

Insert complete set of H_{LC}^0 color-singlet eigenstates

$$\sum_n |n\rangle\langle n| = I$$

$$\sum_n \langle m|H_{LC}|n\rangle\langle n|\Psi_H\rangle = M_H^2\langle m|\Psi_H\rangle$$

⇒ Heisenberg matrix form of eigenvalue problem — DLCQ

$$|\Psi_H\rangle = \sum_n |n\rangle\langle n|\Psi_H\rangle = \sum_n |n\rangle \psi_{n/H}(x_i, \vec{k}_{\perp i}, \lambda_i)$$

⇒ LC Fock expansion of eigenstate $|\Psi_H\rangle$

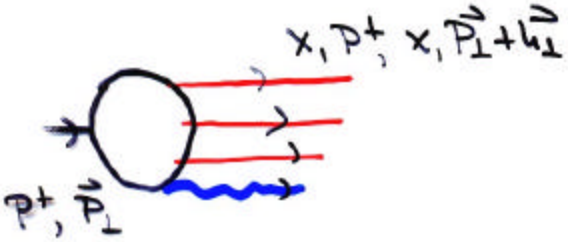
* Given $\{\psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)\}$

wavefunction known for all P^μ !

relative coordinates

$$|P^+, \vec{P}_\perp\rangle = \sum_n \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \prod_j \frac{1}{\sqrt{x_j}} |x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i\rangle$$

absolute coordinates



In equal time theory (instant form)

boost's mix with interactions

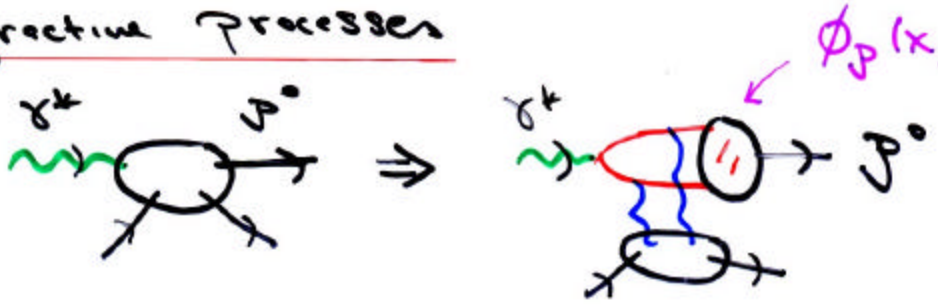
changing $\vec{P} \to \vec{P}'$ as complicated

as solving $H|\Psi\rangle = E|\Psi\rangle$

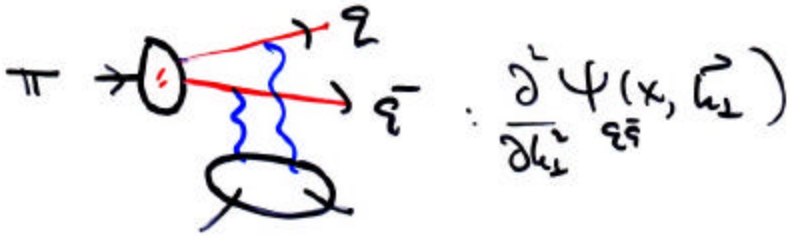
L.C. Wfns — { rest frame
P+≠0
Frame-independent!

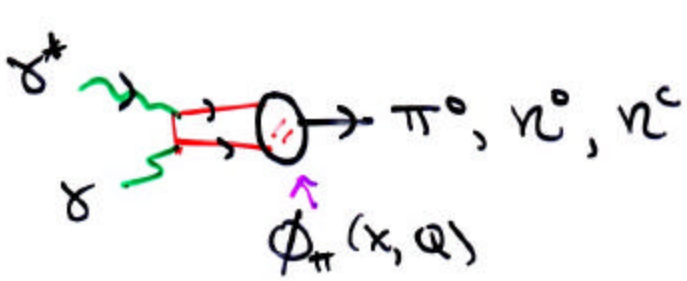
Recent Applications of ψ_{LC}

Diffractive Processes

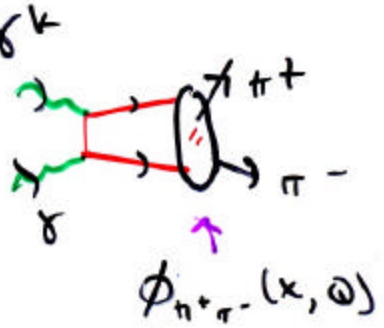


Gunion Frankfurt Strikman Mueller SJB Collins



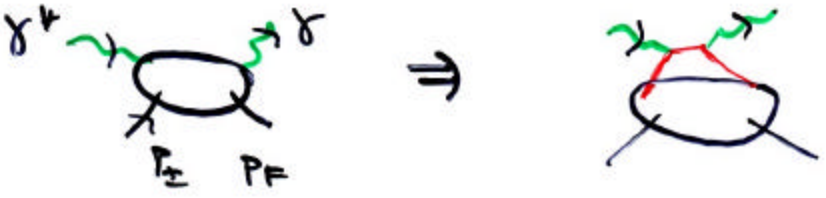


Lepage, SJB



Diehl, et al

Deeply virtual Compton scatt



Einhorn Burkardt QCD $(1+1)_{N_c\to\infty}$

X. Ji A. Radyushkin Close, Gunion, SJB

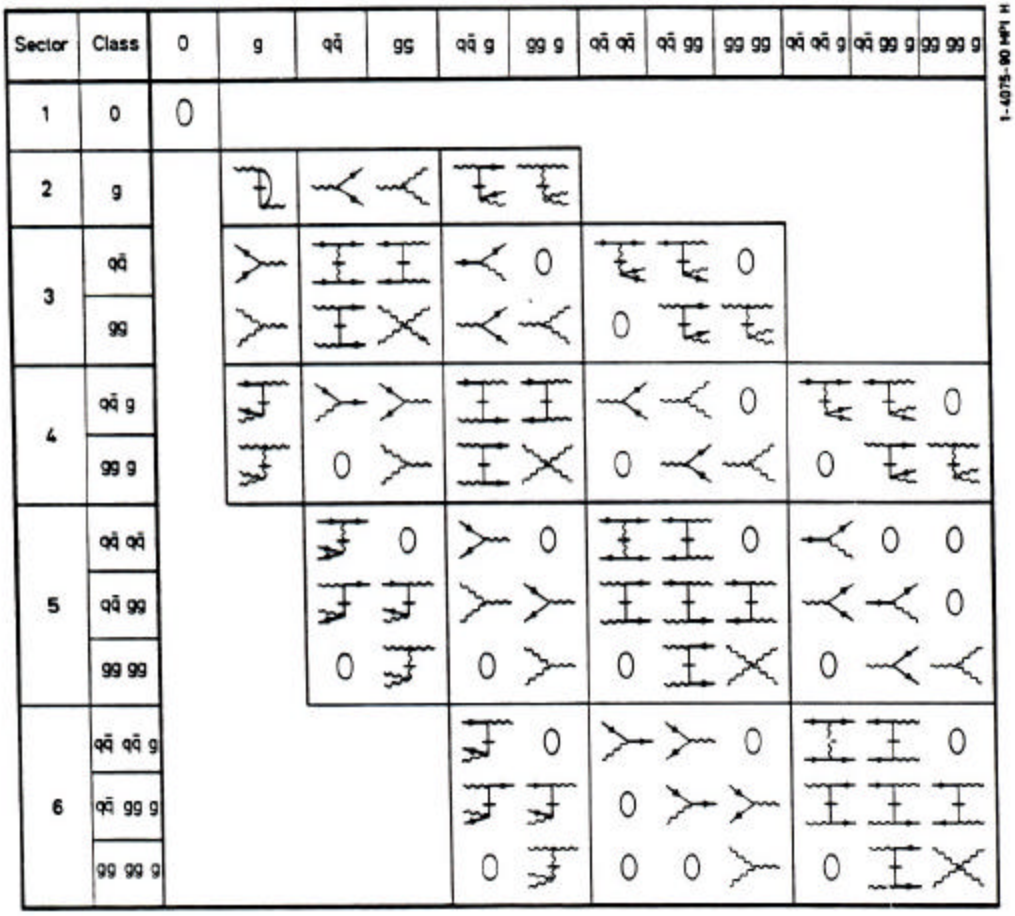
Hwang * Non-forward parton distributions and ψ_{LC}

Schmidt * Sum rules, * J_z cons.

Ma * Relations to EM, Gravity sum rules

SJB * Tests on quantum fluctuations of electron

QCD (3+1)



DLCQ: $\langle n | H_{LC}^{I} | m \rangle$

$$\left(M^2 - \sum_n \frac{k_\perp^2 + m^2}{x}\right) \psi_n = \sum_m \langle n | H_{LC}^{I} | m \rangle \psi_m$$

HCP

PBC: $k^+ = \frac{2\pi}{L} n$, $\vec{k}_\perp = \frac{2\pi}{L_\perp} \vec{n}_\perp$

H.C. Pauli
SJB

DLCQ (3+1)

Periodic boundary conditions

$$-L < x^- < L \quad , \quad -L_\perp < x, y < L_\perp$$

$\Rightarrow$ Discrete momenta:

$$p_i^+ = \frac{\pi}{L} n_i \quad , \quad \vec{p}_{\perp i} = \frac{\pi}{L_\perp}(n_{xi}, n_{yi})$$

$$P^+ = \frac{\pi}{L} K \quad , \quad \sum_i n_i = K$$

$$x_i = \frac{p_i^+}{P^+} = \frac{n_i}{K} > 0$$

* Fock state number limited by K

$$\sum_i \frac{m_i^2 + p_{\perp i}^2}{x_i} < \Lambda^2$$

limits n_x, n_y

* Continuum limit $K \to \infty$

Applications to Yukawa theory (3+1)

Hille, McCartor, SJB

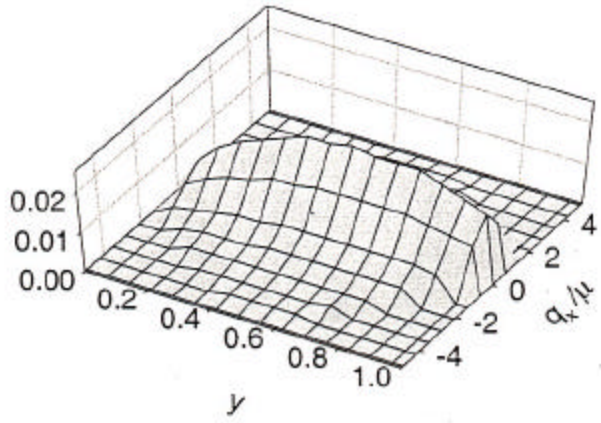


FIG. 13. The parallel-helicity, one-boson amplitude $\phi^{(1,0,0,0)}_{\sigma\sigma}$ as a function of longitudinal momentum fraction y and one transverse momentum component q_x in the $q_y = 0$ plane. The parameter values are $K = 29$, $N_\perp = 7$, $M = \mu$, $\Lambda^2 = 50\mu^2$, $\mu_1^2 = 10\mu^2$, $\mu_2^2 = 20\mu^2$, $\mu_3^2 = 30\mu^2$, and $\langle{:}\phi^2(0){:}\rangle = 0.25$.

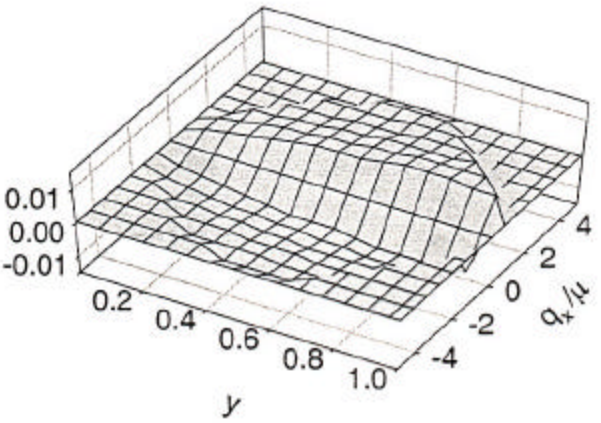


FIG. 14. Same as Fig. 13 but for antiparallel bare helicity.

Anomalous Gravitomagnetic Moment $B(0)$

QED

$$F_2(0) = \frac{\alpha}{2\pi}$$

$$B(0) = \frac{\alpha}{3\pi} - \frac{\alpha}{3\pi} = 0.$$

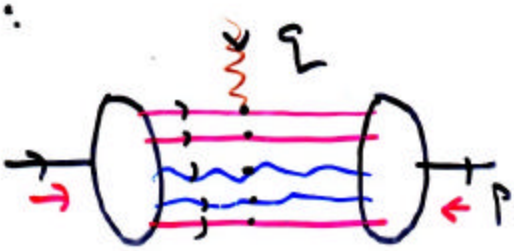
$$B(q^2) \sim \frac{\alpha}{\pi}\sqrt{q^2/m^2}$$

Equivalence Principle : $B(0) = 0$

Okun + Kobzarev (62)
Y. I. , Tergaev

any spin ½ system

LF :



$(\Delta L_z = 1)$

$$\lim_{q^2\to 0} \sum_{i=1}^{n} \qquad \sum_{i=1}^{n} B_i(0) = 0$$

Fock state by Fock state

Result of Lorentz prop of LF wavefunctions.

Hwang, Ma, Schmidt, YB

key Question for LGTh

$$B_p(0) = 0 \ ?$$

Important indicator of lattice errors.

Some advantages of Light-Front-Quantized QCD

- No Fermion doubling
- Multiple fermion flavors
- Minkowski space
- Gauge-Fixed : Physical Degrees of Freedom
- Manifest Frame-Independence : $P^+, \vec{P}_\perp$ arbitrary
  J_z kinematical
- L.F. Vacuum Trivial — Zero modes
- Vanishing anomalous gravitomagnetic moment

$$B(0) \equiv 0 \; ; \quad \lim_{MR \to 0} M_A = 0$$

- DLCQ discretization retains symmetries
  continuum limit : $K \to \infty$
- LF Wavefunctions, continuum solns.,
  Amplitudes, Phases as well as Spectra
- Solns for QCD(1+1), SUSY(1+1)
  QCD(3+1) : Challenging!

P. Srivastava
S.J.B.

Light-Front Quantization of Standard Model $SU(2) \otimes U(1)$

* $n \cdot A = A^+ = 0$ gauge

  $\Rightarrow$ physical gauge quanta
  Unitary
  renormalizable

* SSB from zero mode of scalar field

$$\phi^{(0)} = \frac{v}{\sqrt{2}}$$

$$\phi(x) = \frac{1}{\sqrt{2}}(v + h(x) + i\eta(x))$$

$$\partial \cdot A = M\eta \quad , \quad M = ev$$

* Goldstone field $\eta(x)$ restores ϵ_L :

$$\epsilon_\mu^{(L)} = \frac{n_\mu M}{n \cdot k} - \frac{k_\mu M}{k^2} \qquad (\epsilon \cdot k = 0)$$

* LC vacuum remains trivial

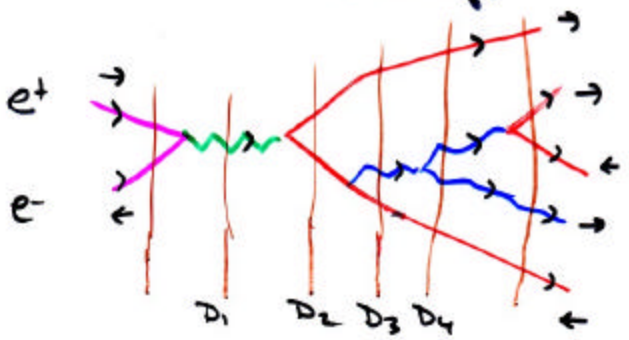
* Amplitude Event Generator

  $\Rightarrow$ Renormalized Amplitudes from LF T-O PTL
  Ghost-Free, $\pi d^2k_\perp dx$, DLCQ discret.

"Event Amplitude Generator"

Generate amplitude from LF TO PTh { Tree + Virtual

$$\mathcal{M} = \sum_{\text{time-ordering}} \mathcal{M}_\alpha \qquad \text{(specific spins } S_z\text{)}$$



$$\mathcal{M}_\alpha = H_I \frac{1}{\not{D}_1} H_I \frac{1}{\not{D}_2} \cdots$$

$\sum k^+$, $\sum k_\perp$, $\sum J_z$ conserved

$k^+ > 0$: few surviving LF time-ordering

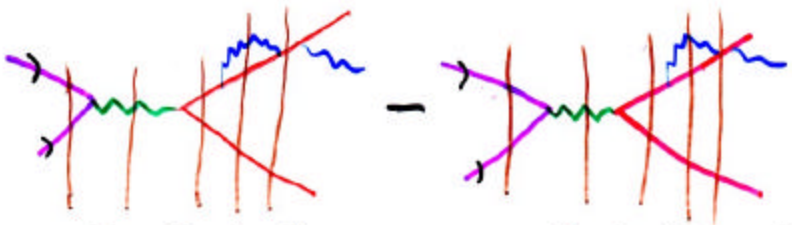
Physical polarization sums: $\sum_{(i)} \epsilon_\mu^{(i)} \epsilon_\nu^{(i)}$

$(i) = 1, 2, 3$ ↑ Standard model W

Compute renormalized amplitude

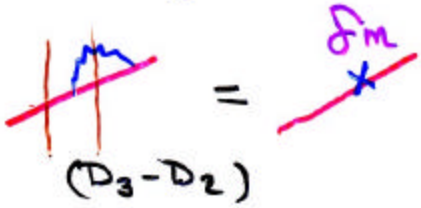
- "alternating denominator" method

Example:



Roskies Suaya SJB

equivalent to subtracting mass counterterm!



$\int d^2k_\perp dx$, Unitary, no ghosts.

Other Applications of LF Quantization

Light-Front Thermodynamics

Set boundary conditions at fixed $\tau = t + z/c$
not t

$$Z_{LF} = \sum_n \exp - \frac{m_n^2}{T_{LF}}$$

Light-Front Lippmann-Schwinger

$$T = H_I + H_I \frac{1}{m^2 - \sum \frac{m_i^2}{x} + ce} T$$

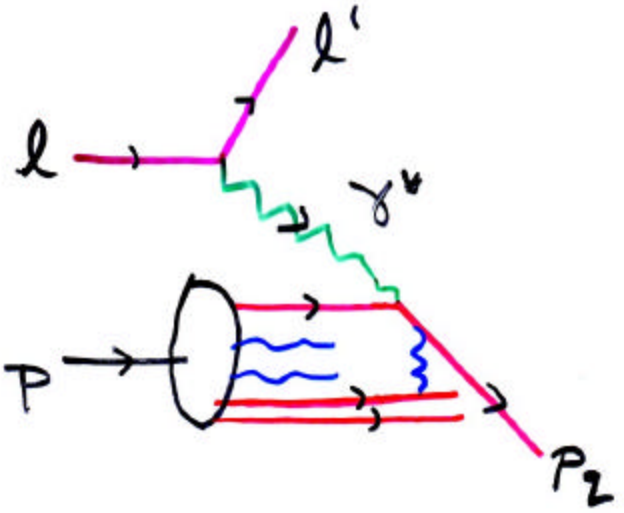
Variational Solutions to Bound-State Problems

minimize $\langle \psi_{trial} | H_{LF} | \psi_{trial} \rangle$

Construct $\langle \psi_{trial} | n \rangle = \psi_n^{trial}(x_i, k_{\perp i}, \lambda_i)$

using numerators from L_z
ladder relations

Unexpected Role of Final State Interactions in Deep Inelastic Scattering



gluon exchange after photon acts

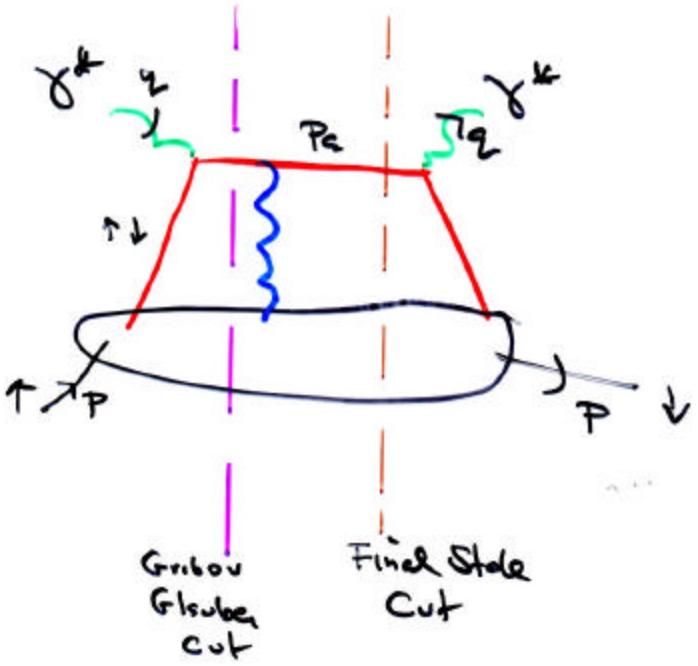
not in LF WF

* Single-spin asymmetry $\vec{S}_p \cdot \vec{q} \times \vec{p}_q$ Bjorken-scaling
* Diffraction at Leading Twist
* Nuclear Shadowing (interference of diff channels)
* Energy Loss , p_T Broadening

Diffraction, Nuclear Shadowing, Pomeron not in nuclear wavefunction!

Explicit calculation of FSI SSA

D. S. Hwang
I. A. Schmidt
SJB
hep/ph/0201296



Overlap of wavefunctions with $\Delta L_z = 1$

$$\left[e^{i(\chi_1 - \chi_2)}\right]$$

$\chi_1 - \chi_2$: IR Finite

$$i\vec{S}_p \cdot \vec{q} \times \vec{p}_q = i\vec{S}_p \cdot \vec{q} \times \vec{r}$$

$$\vec{p}_q = \vec{q} + \vec{r}$$

$$P_y = A_n \simeq \frac{\alpha_s(r_1^2)\, x_{Bj}\, M\, |\vec{r}_1|\, \ln r_1^2}{r_1^2}$$

Bjorken scaling for finite r_1

Same matrix elements as $a_p = F_2(0)$

Conclusions

Important Role of FSI, ISI

- Leading Twist Diffraction in DIS
- Color Dipole Physics
- Nuclear Shadowing : not in nuclear LC w.f. !
- Single. Spin Asymmetries

  - Bjorken scaling !
  - New spin-dep structure functions W_1', W_2', W
  - Nuclear Enhancement

Shadowing / Anti-Shadowing
Energy Loss
P_T Spreading
Diffraction
SSA

} Beginnings of fund. descr. in QCD