Transverse Structure of Nucleon Parton Distributions

Dru B. Renner

SciDAC All Hands Meeting
Brookhaven National Laboratory
March 26, 2004

Generalized parton distributions determine the angular momentum decomposition of the nucleon and the transverse distribution of partons in the nucleon. I will review generalized parton distributions and present our full QCD lattice calculations of the quark helicity and orbital angular momentum in the nucleon and the transverse distribution of quarks in the nucleon.

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Lattice Hadron Physics Collaboration

http://talks.drubryantrenner.org/scidac_3-8-04.pdf

Lattice Hadron Physics Collaboration
Physics Program

Spectroscopy

N^*
Exotic Mesons
Pentaquarks

Hadronic Structure

Elastic Form Factors
Parton Distributions
Generalized Parton Distributions
Transition Form Factors
Hadron Polarizability

Hadron-Hadron Interactions

Heavy-Light Potential

Fundamental Aspects of QCD

Chiral Lagrangian
Large N_c

White Paper: http://www-ctp.mit.edu/∼negele/WhitePaper.pdf

In Short

Generalized Parton Distributions determine

- decomposition of nucleon spin into quark helicity, quark orbital, and gluon contributions

- 3D distribution of quarks in a mixed representation - 2 transverse coordinates and 1 longitudinal momentum

Continuum Physics

I. Form Factors

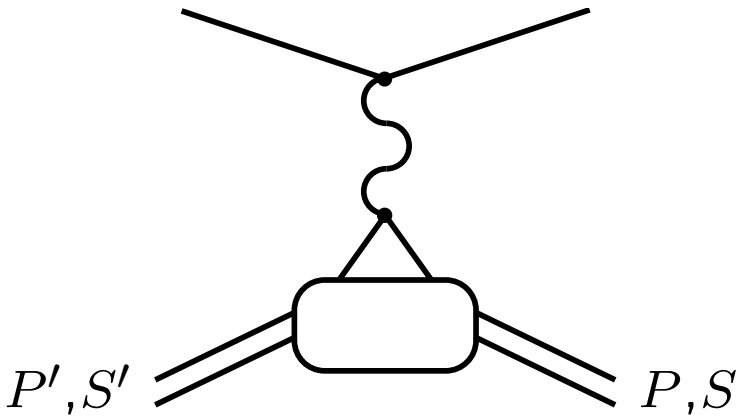
II. Parton Distributions

III. Generalized Parton Distributions

IV. Generalized Form Factors

Form Factors

- lepton-nucleon scattering, $lN \to lN$, exclusive



$$\Delta = P' - P$$
$$t = \Delta^2$$

- off-forward matrix element of the electromagnetic current

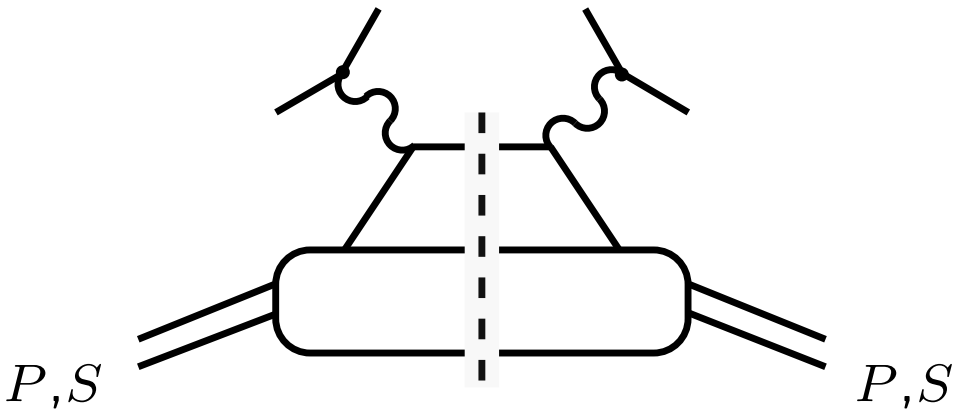
$$< P', S' | J^\mu | P, S >= \overline{U}(P', S') \left(\gamma^\mu F_1(t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m} F_2(t) \right) U(P, S)$$

- interpretation as Fourier transform of charge and current densities *in certain cases*

- magnetic moment, charge & current radii

Parton Distributions

- deep inelastic scattering, $lN \to lX$, inclusive



$$P^+ = \frac{1}{2}\left(P^t + P^z\right)$$
$$y^- = \frac{1}{2}\left(y^t - y^z\right)$$

- forward matrix element of light-cone quark correlator

$$\mathcal{O}_q(x, \vec{b}_\perp) = \int \frac{dy^-}{4\pi}\, e^{ixP^+y^-}\, \overline{q}\left(-\frac{y^-}{2}, \vec{b}_\perp\right) \gamma^+ q\left(\frac{y^-}{2}, \vec{b}_\perp\right)$$

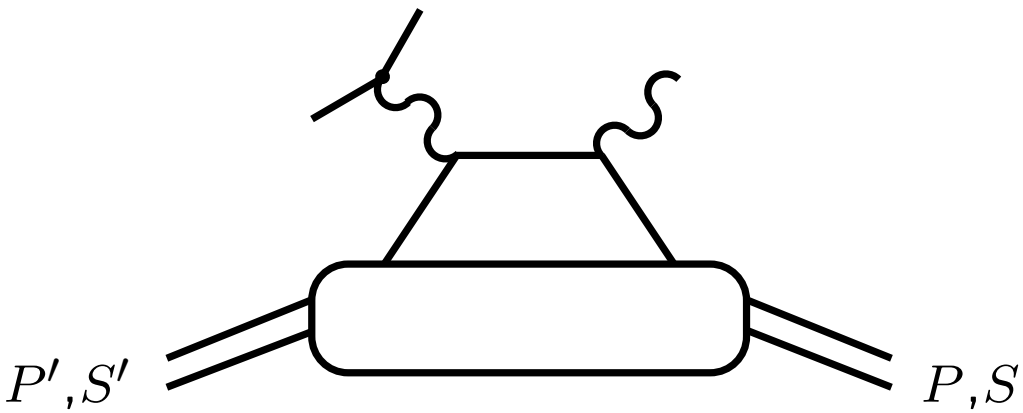
$$\left\langle P, S \middle| \mathcal{O}_q(x, \vec{0}_\perp) \middle| P, S \right\rangle = q(x)$$

- longitudinal momentum distribution in the *infinite momentum frame*

- momentum fraction & spin fraction

Generalized Parton Distributions

- deeply virtual Compton scattering, $lN \to lN\gamma$, exclusive



$$\overline{P} = \frac{1}{2}\left(P' + P\right)$$

$$\xi = -\frac{\Delta^+}{2\overline{P}^+}$$

- off-forward matrix element of light-cone quark correlator

$$\left\langle P', S' \middle| \mathcal{O}_q(x, \vec{0}_\perp) \middle| P, S \right\rangle =$$

$$\frac{1}{2\overline{P}^+}\overline{U}(P', S')\left(\gamma^+ H(x,\xi,t) + i\frac{\sigma^{+\nu}\Delta_\nu}{2M}E(x,\xi,t)\right)U(P,S)$$

- meson production, Compton scatering

GPD reviews: Ji hep-ph/9807358, Diehl hep-ph/0307382

Generalized Parton Distributions

- familiar limits: parton distributions & form factors

$$H_q(x,0,0) = q(x)$$

$$\sum_q e_q \int dx\ H_q(x,\xi,t) = F_1(t)$$

$$\sum_q e_q \int dx\ E_q(x,\xi,t) = F_2(t)$$

- quark angular momentum [1]

$$J_q = \frac{1}{2}\int dx\ x\,(H_q(x,0,0) + E_q(x,0,0)) = \frac{1}{2}\Sigma_q + L_q$$

- transverse quark distribution in *infinite momentum frame* [2]

$$q(x,\vec{b}_\perp) = \int d^2\Delta_\perp\ e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} H_q(x,0,-\vec{\Delta}_\perp^2)$$

- both results require transverse momentum transfer, $\xi = 0$

[1] Ji hep-ph/9603249
[2] Burkardt hep-ph/0005108

Moments of Generalized Parton Distributions

- generalized parton distribtuions

$$\left\langle P', S' \left| \mathcal{O}_q(x, \vec{0}_\perp) \right| P, S \right\rangle \;=$$

$$\frac{1}{2\overline{P}^+} \overline{U}(P', S') \left(\gamma^+ H_q(x, \xi, t) + i \frac{\sigma^{+\nu} \Delta_\nu}{2M} E_q(x, \xi, t) \right) U(P, S)$$

$$\mathcal{O}_q(x, \vec{b}_\perp) = \int \frac{dy^-}{4\pi} \, e^{ixP^+ y^-} \, \overline{q} \left(-\frac{y^-}{2}, \vec{b}_\perp \right) \gamma^+ \, q \left(\frac{y^-}{2}, \vec{b}_\perp \right)$$

- light-cone expansion & tower of twist 2 operators

$$\mathcal{O}_q^{\mu_1 \mu_2 \ldots \mu_n} = \overline{q} \gamma^{\{\mu_1} i D^{\mu_2} \ldots i D^{\mu_n\}} q$$

- moments of generalized parton distribtuions

$$\langle P | \mathcal{O}^{\mu_1 \mu_2 \ldots \mu_n} | P \rangle = 2 \left\langle x^{n-1} \right\rangle P^{\{\mu_1} P^{\mu 2} \ldots P^{\mu_n\}}$$

$$\left\langle P' | \mathcal{O}^{\mu_1 \mu_2 \ldots \mu_n} | P \right\rangle \sim \int dx \, x^{n-1} H_q(x, \xi, t) \quad \& \quad \int dx \, x^{n-1} E_q(x, \xi, t)$$

Generalized Form Factors

- $n+1$ generalized form factors [1]

$$\left\langle P' | \mathcal{O}_q^{\mu_1 \mu_2 \cdots \mu_n} | P \right\rangle \sim A_{ni}^q(t),\ B_{ni}^q(t),\ C_n^q(t)$$

- moments of parton distributions

$$\left\langle x^{n-1} \right\rangle_q = A_{n0}^q(0)$$

- form factors

$$F_1(t) = \sum_q e_q A_{10}^q(t) \qquad F_2(t) = \sum_q e_q B_{10}^q(t)$$

[1] Ji, Lebed hep-ph/0012160

Generalized Form Factors

- moments of generalized parton distributions

$$\int_{-1}^{1} dx\ x^{n-1} H_q(x,\xi,t) = \sum_{i=0}^{[(n-1)/2]} A^q_{n,2i}(t)(-2\xi)^{2i} + \mathrm{mod}(n+1,2) C^q_n(t)(-2\xi)^n$$

$$\int_{-1}^{1} dx\ x^{n-1} E_q(x,\xi,t) = \sum_{i=0}^{[(n-1)/2]} B^q_{n,2i}(t)(-2\xi)^{2i} - \mathrm{mod}(n+1,2) C^q_n(t)(-2\xi)^n$$

- transverse momentum transfer, $\xi \to 0$

$$\int_{-1}^{1} dx\ x^{n-1} H_q(x,0,t) = A^q_{n0}(t)$$

$$\int_{-1}^{1} dx\ x^{n-1} E_q(x,0,t) = B^q_{n0}(t)$$

Quark Angular Momentum

- angular momentum operator

$$J_q^i = \frac{1}{2}\epsilon^{ijk}\int d^3x \left(T_q^{0k}x^j - T_q^{0j}x^k\right)$$

- energy momentum tensor

$$T_q^{\mu\nu} = \overline{q}\gamma^{\{\mu}iD^{\nu\}}q = O_q^{\mu\nu}$$

- angular momentum

$$J_q = \langle P, 1/2|J_q^z|P, 1/2\rangle \quad \text{involves} \quad \langle P, 1/2|O_q^{\mu\nu}|P, 1/2\rangle$$

- matrix elements of $n = 2$ twist 2 operator, $O_q^{\mu\nu}$: $A_{20}(t)$, $B_{20}(t)$, $C_2(t)$

$$J_q = \frac{1}{2}\left(A_{20}^q(0) + B_{20}^q(0)\right)$$

Transverse Quark Distribution

- wave packet in transverse coordinates

$$|\psi\rangle = \int \frac{d^2 p_\perp}{(2\pi)} \frac{\psi(\vec{p}_\perp)}{\sqrt{2E_{\vec{p}}}} |\vec{p}_\perp, P_z\rangle \qquad \vec{p} = (\vec{p}_\perp, P_z)$$

- transverse quark distribution

$$q_\psi(x, \vec{b}_\perp) = \langle\psi|O_q(x, \vec{b}_\perp)|\psi\rangle \quad \text{involves} \quad \langle\vec{k}_\perp, P_z|O_q(x, \vec{b}_\perp)|\vec{p}_\perp, P_z\rangle$$

- infinite momentum limit & transverse localization

$$\vec{\Delta}_\perp \ll P_z \qquad \psi(\vec{k}_\perp) \approx \psi(\vec{p}_\perp)$$

- matrix elements of $O_q(x, \vec{b}_\perp)$: $H_q(x, \xi, t)$, $E_q(x, \xi, t)$

$$q(x, \vec{b}_\perp) = \int d^2\Delta_\perp \; e^{-i\vec{b}_\perp \cdot \vec{\Delta}_\perp} H_q(x, 0, -\vec{\Delta}_\perp^2)$$

- relativistic corrections controlled by

$$\frac{1}{\sqrt{M^2 + P_z^2}} \quad \text{not} \quad \frac{1}{M}$$

Our Results

I. Quark Angular Momenta

II. Transverse Quark Distributions

Lattice Parameters

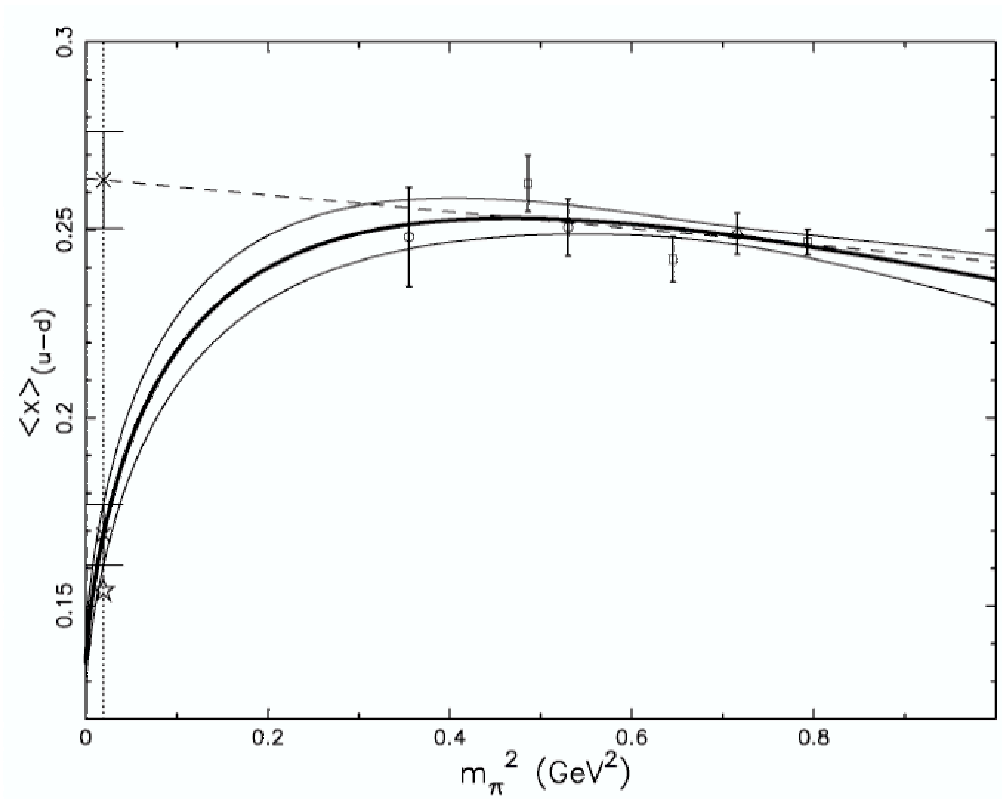
- SESAM full QCD gauge fields
- $N_F = 2$ Wilson fermions
- $a = 0.092$ fm, $16^3 \times 32$, $L = 1.48$ fm
- O(200) gluon configurations
- $M_\pi = 744$, 831, 897 MeV

Chiral Extrapolations or Heavy Pion World

- leading order χPT & heavy quark limit [1]

$$\langle x^n \rangle_{u-d} = a_n \left(1 - \frac{(3g_A^2 + 1) m_\pi^2}{(4\pi f_\pi)^2} \ln \left(\frac{m_\pi^2}{m_\pi^2 + \mu^2} \right) \right) + b_n m_\pi^2$$

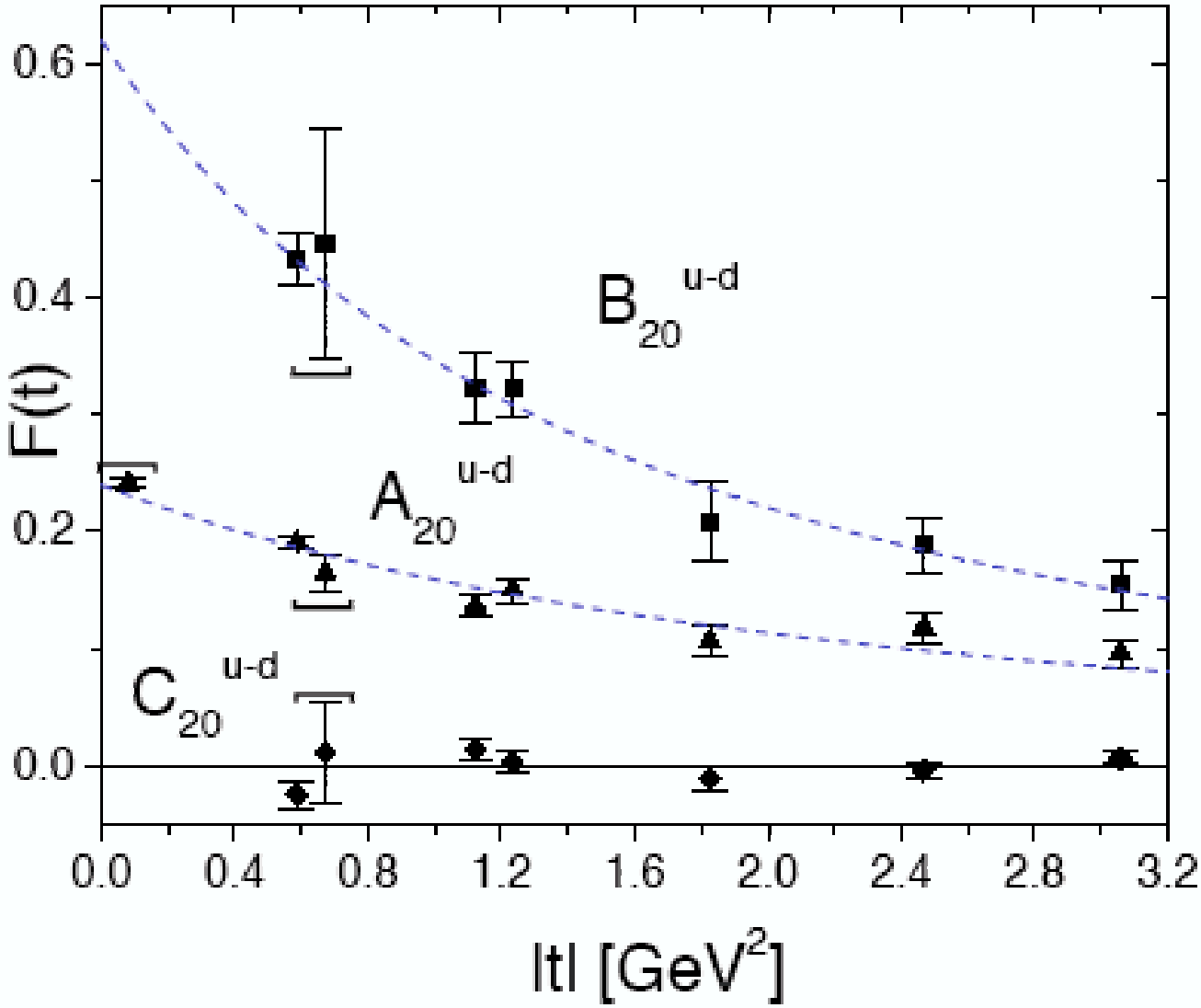
- $\langle x \rangle$ for SESAM full QCD & MIT quenched [2]



[1] Detmold, Melnitchouk, Negele, Renner, Thomas hep-lat/0103006
[2] LHPC, SESAM hep-lat/0201021

Generalized Form Factors

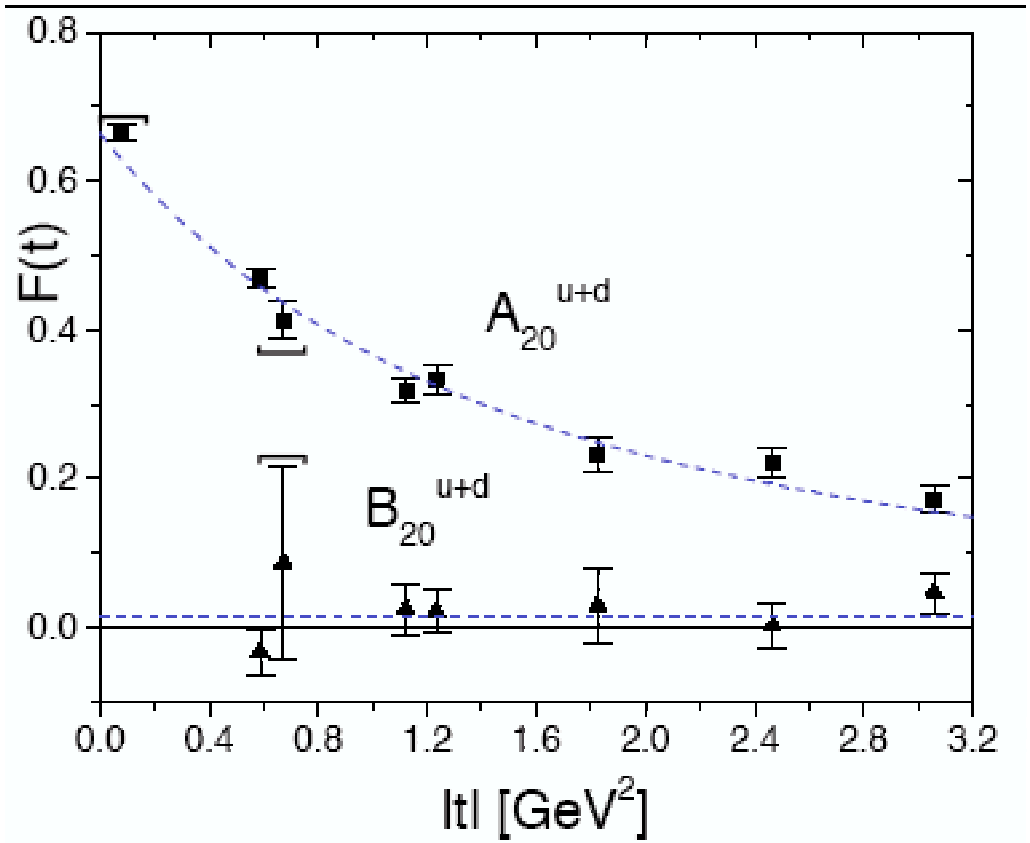
- $\overline{\psi}\gamma^{\{\mu}iD^{\nu\}}\psi$ has 3 GFFs: $A_{20}(t)$, $B_{20}(t)$, $C_2(t)$



Quark Angular Momenta: $m_\pi = 897\text{MeV}$

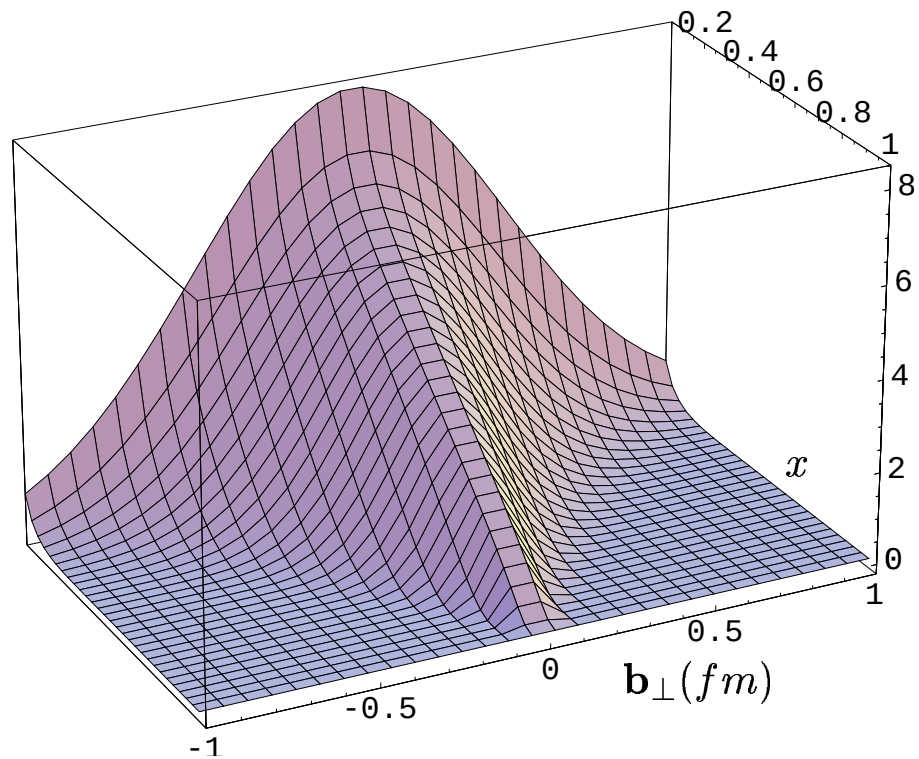
- $\frac{1}{2}\Delta\Sigma_{u\pm d} = \frac{1}{2}\langle 1\rangle_{\Delta u \pm \Delta d}$
- $J_{u\pm d} = \frac{1}{2}\left(A_{20}^{u\pm d}(0) + B_{20}^{u\pm d}(0)\right)$
- $L_{u\pm d} = J_{u\pm d} - \frac{1}{2}\Delta\Sigma_{u\pm d}$
- $J_g = \frac{1}{2} - J_{u+d} = \frac{1}{2}\cdot 0.325(16)$

	$u+d$	$u-d$
$\frac{1}{2}\Delta\Sigma$	$\frac{1}{2}\cdot 0.682(16)$	$0.598(7)$
J	$\frac{1}{2}\cdot 0.675(7)$	$0.432(13)$
L	$-\frac{1}{2}\cdot 0.01(2)$	$-0.163(15)$



Transverse Structure

- 3D quark distribution: $q(x, \vec{b}_\perp)$
- x is longitudinal momentum fraction
- $\vec{b}_\perp$ is transverse displacement of active parton relative to nucleon center of longitudinal momentum
- simple model calculation



plot: Burkardt hep-ph/0207047

Transverse Structure

$$H_q(x, 0, -\vec{\Delta}_\perp^2) = \int d^2 b_\perp \; e^{i\vec{\Delta}_\perp \cdot \vec{b}_\perp} q(x, \vec{b}_\perp)$$

$$A_{n0}^q(t) = \int_{-1}^{1} dx \; x^{n-1} H_q(x, 0, t)$$

- at $x = 1$ a single quark carries all the momentum

$$\lim_{x \to 1} q(x, \vec{b}_\perp) \propto \delta^2(\vec{b}_\perp) \qquad H_q(1, 0, t) = \text{constant}$$

- higher moments A_{n0}^q weight $x \sim 1$ more heavily

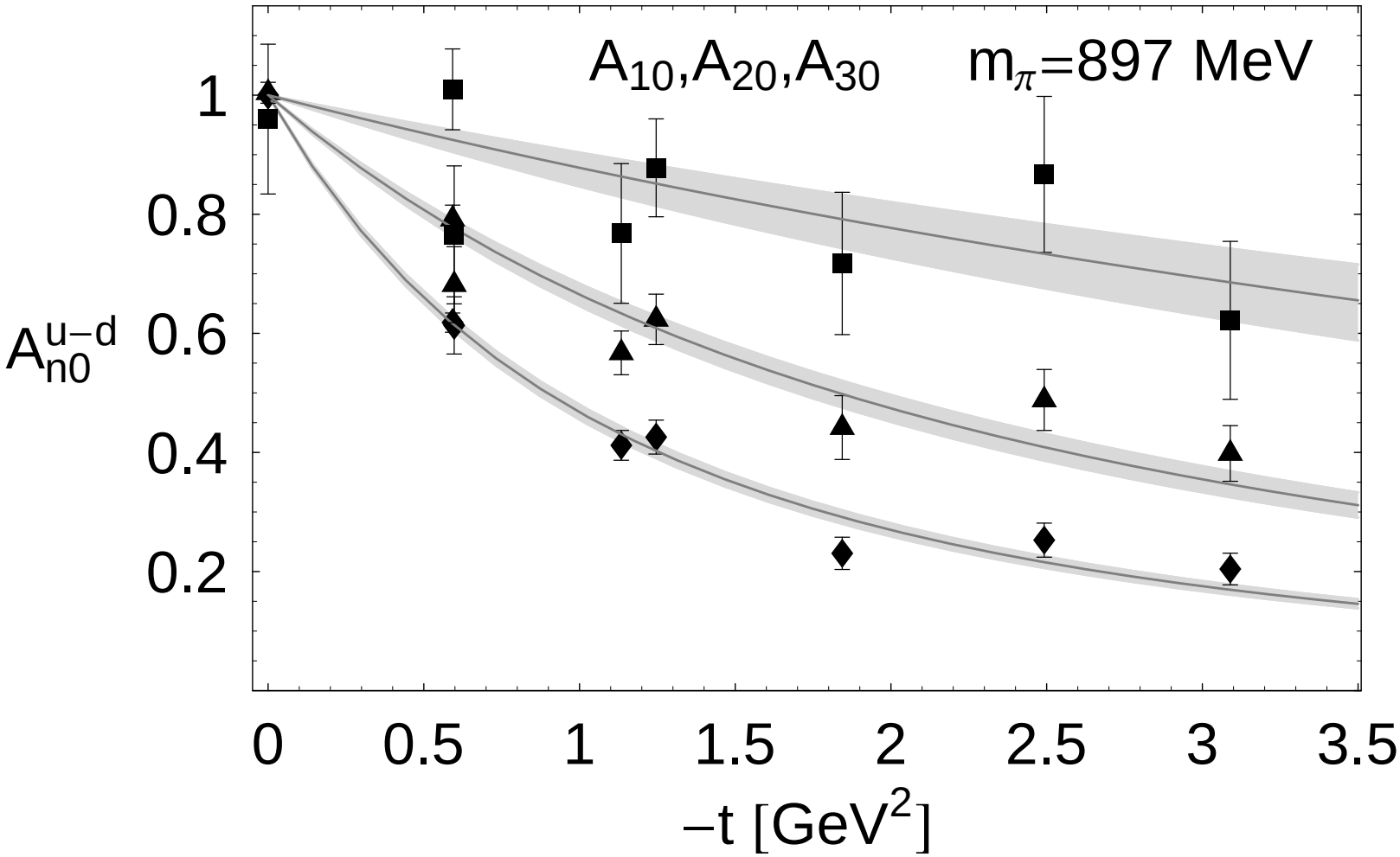
$$\lim_{n \to \infty} A_{n0}^q(t) \propto H_q(1, 0, t)$$

- slopes of A_{n0}^q should decrease as n increases

- A_{10}, A_{30}, $\tilde{A}_{20}$ measure $q - \overline{q}$ & $\tilde{A}_{10}$, $\tilde{A}_{30}$, A_{20} measure $q + \overline{q}$

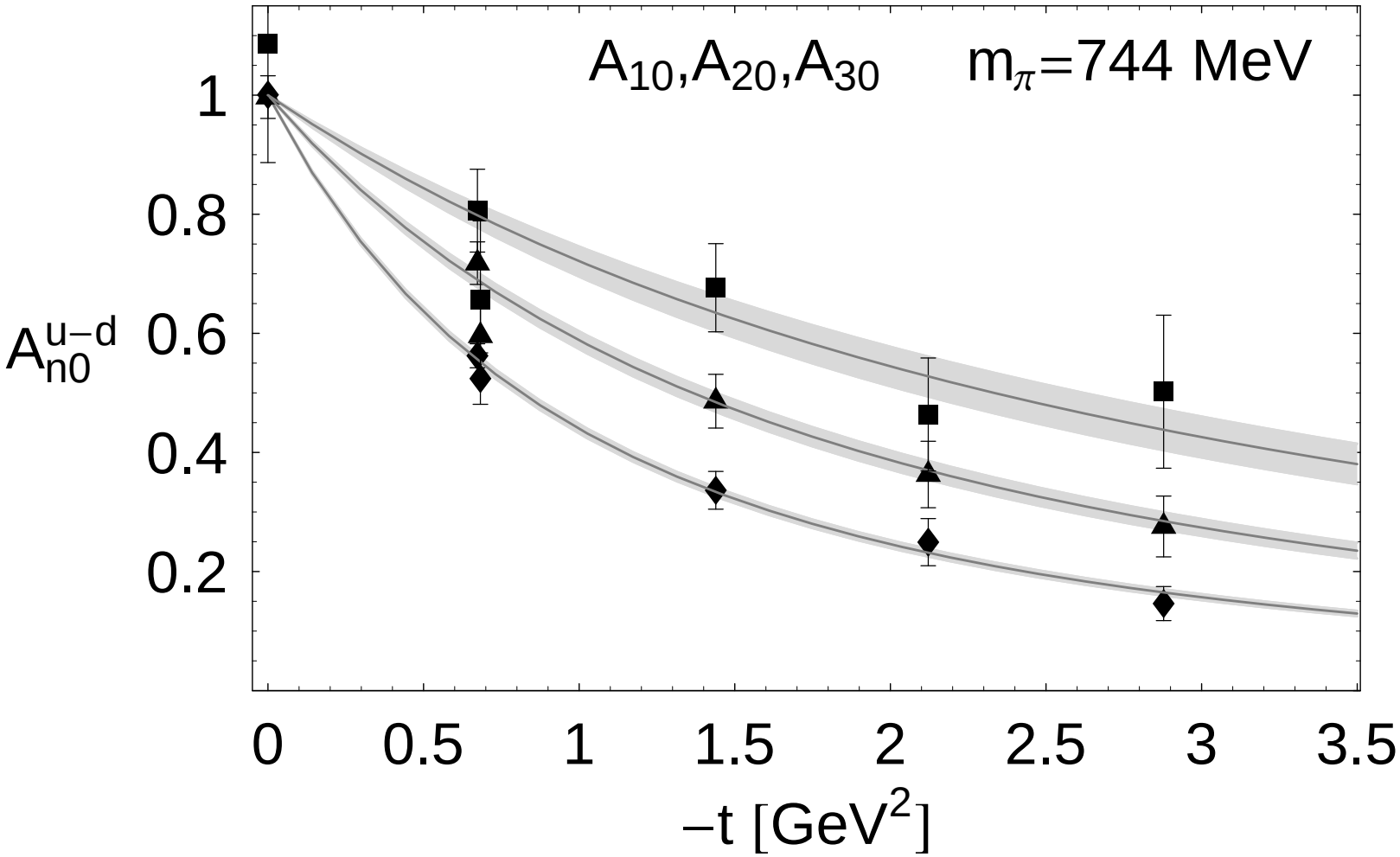
Transverse Structure

- slope of $A_{10}^{u-d} = -0.93 \pm 0.04$ (GeV)$^{-2}$
- slope of $A_{30}^{u-d} = -0.13 \pm 0.03$ (GeV)$^{-2}$ (factor of 7)



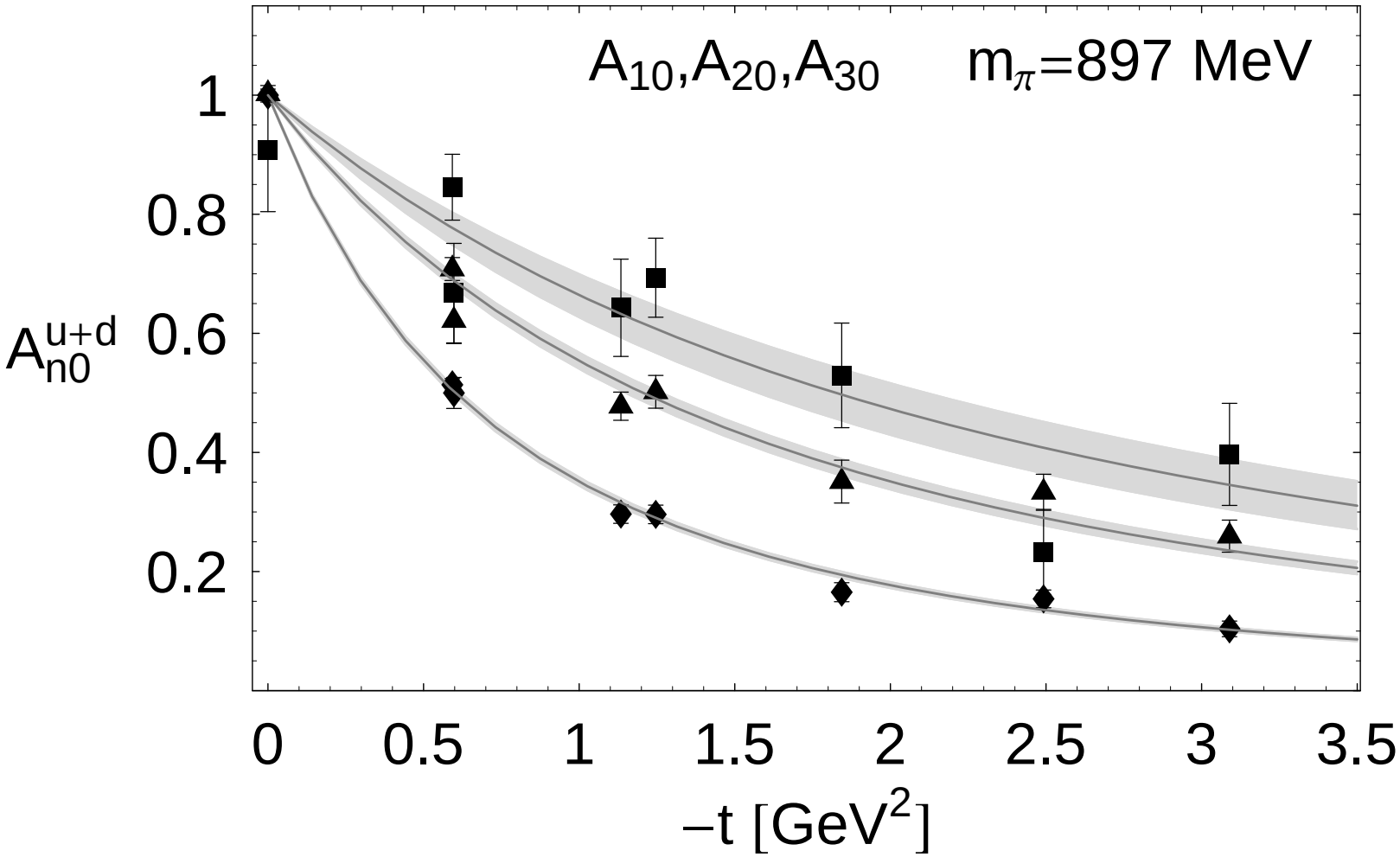
Transverse Structure: Mass Dependence

- slope of $A_{10}^{u-d} = -1.02 \pm 0.03$ (GeV)$^{-2}$
- slope of $A_{30}^{u-d} = -0.36 \pm 0.04$ (GeV)$^{-2}$ (factor of 3)



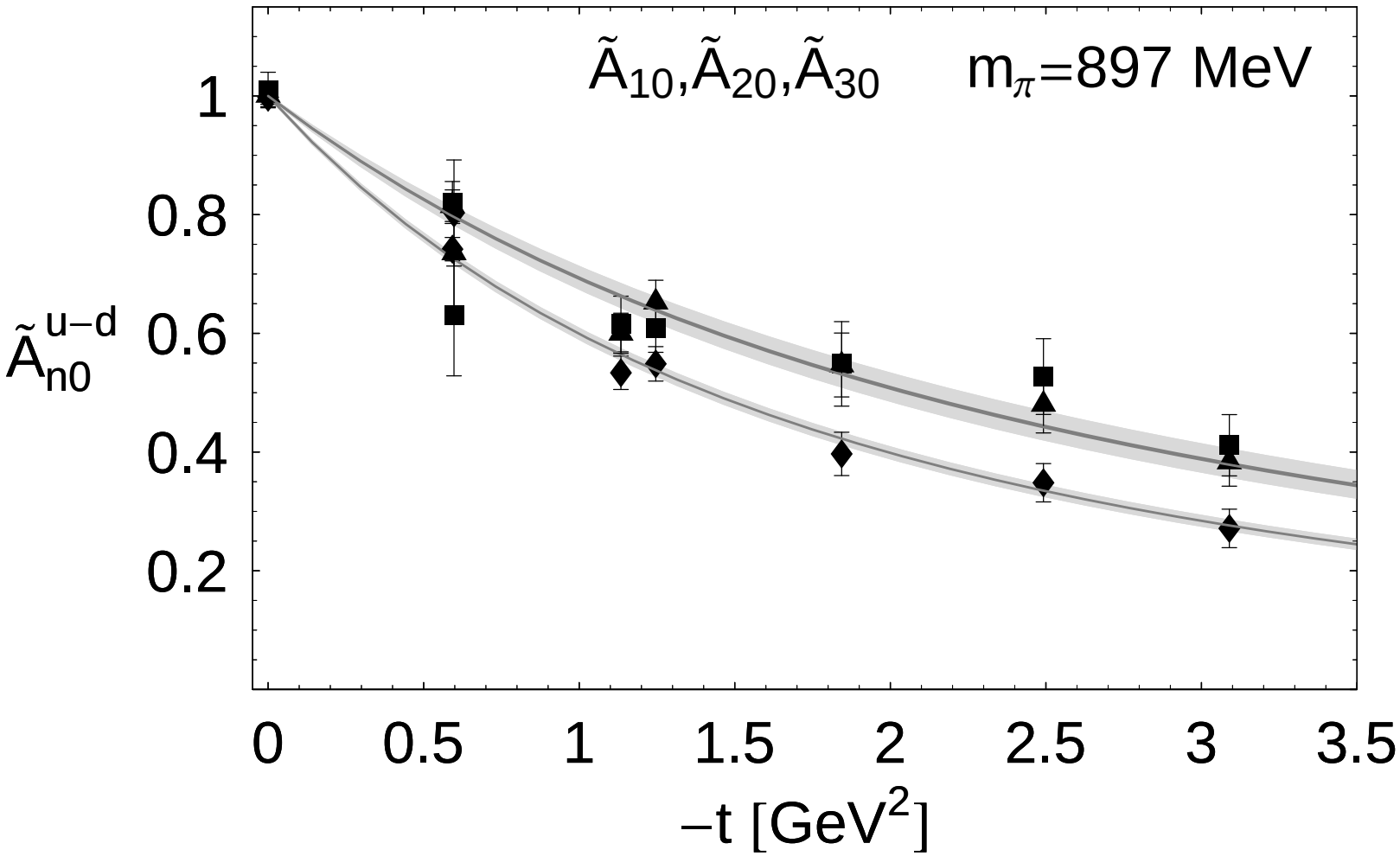
Transverse Structure: Flavor Dependence

- slope of $A_{10}^{u+d} = -1.38 \pm 0.03$ (GeV)$^{-2}$
- slope of $A_{30}^{u+d} = -0.45 \pm 0.07$ (GeV)$^{-2}$ (factor of 3)



Transverse Structure: Spin Dependence

- slope of $\tilde{A}_{10}^{u-d} = -0.58 \pm 0.02$ (GeV)$^{-2}$
- slope of $\tilde{A}_{30}^{u-d} = -0.40 \pm 0.03$ (GeV)$^{-2}$ (factor of 1.5)



Transverse Structure: RMS Radii

- transverse rms radius

$$\left\langle b_\perp^2 \right\rangle_x = \frac{\int d^2b_\perp \, b_\perp^2 \, q(x, \vec{b}_\perp)}{\int d^2b_\perp \, q(x, \vec{b}_\perp)}$$

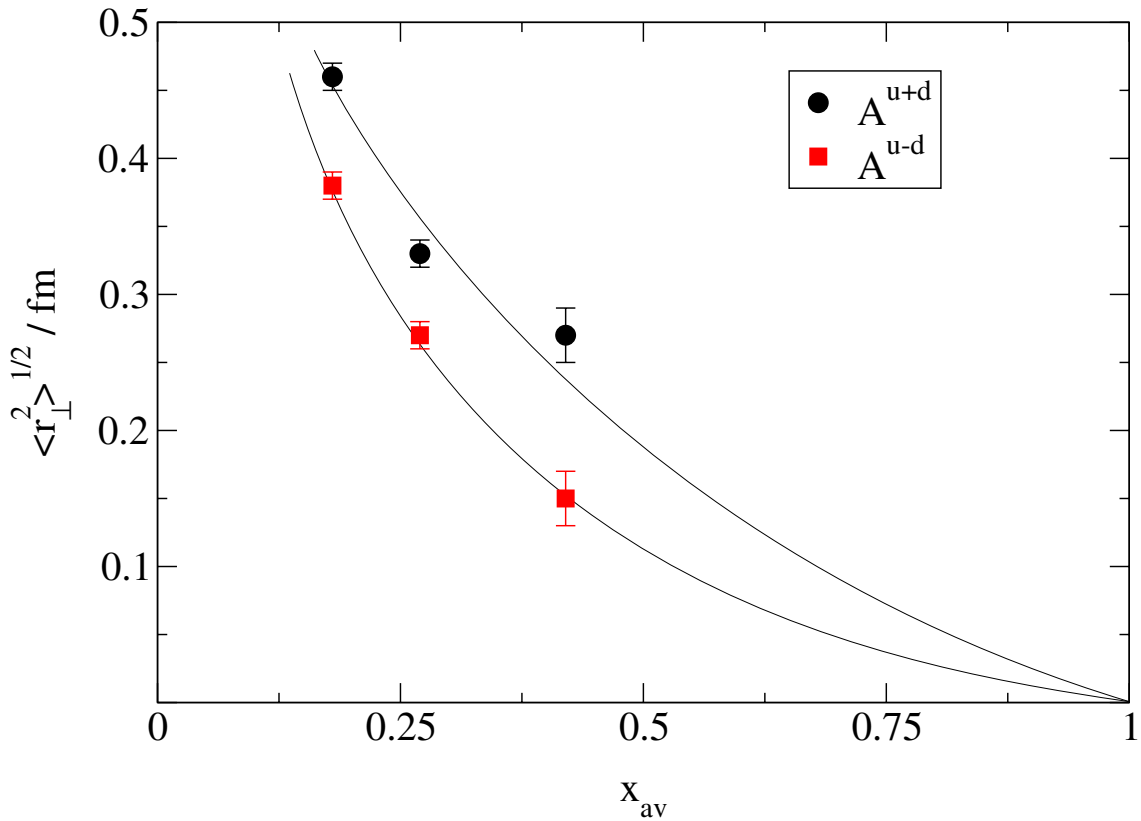
- smeared transverse rms radius

$$\frac{A'_{n0}(0)}{A_{n0}(0)} = -\frac{1}{4} \left\langle b_\perp^2 \right\rangle_{(n)} \qquad \left\langle b_\perp^2 \right\rangle_{(n)} = \frac{\int d^2b_\perp \, b_\perp^2 \, \int_{-1}^{1} dx \, x^{n-1} q(x, \vec{b}_\perp)}{\int d^2b_\perp \, \int_{-1}^{1} dx \, x^{n-1} q(x, \vec{b}_\perp)}$$

- the average x in $\left\langle b_\perp^2 \right\rangle_{(n)}$

$$x_{\text{av}}^{(n)} = \frac{\int d^2b_\perp \, \int_{-1}^{1} dx \, x \cdot x^{n-1} q(x, \vec{b}_\perp)}{\int d^2b_\perp \, \int_{-1}^{1} dx \, x^{n-1} q(x, \vec{b}_\perp)} = \frac{< x^n >}{< x^{n-1} >}$$

Transverse Structure: Smeared RMS Radii



- charge radius: $\sqrt{\langle b_\perp^2 \rangle}_{\text{charge}} = 0.42$ fm (0.72 fm from experiment), no pion cloud
- non-singlet radius: $\sqrt{\langle b_\perp^2 \rangle_{u-d}^{(1)}} = 0.38$ fm & $\sqrt{\langle b_\perp^2 \rangle_{u-d}^{(3)}} = 0.15$ fm, 61% decrease
- singlet radius: $\sqrt{\langle b_\perp^2 \rangle_{u+d}^{(1)}} = 0.46$ fm & $\sqrt{\langle b_\perp^2 \rangle_{u+d}^{(3)}} = 0.27$ fm, 41% decrease

Current Work: Chiral Limit

- hybrid calculation: Asqtad staggered sea quarks [1] & domain wall valence quarks with HYP smearing [2]
- $a = 0.13$ fm, $20^3 \times 32$, $L = 2.6$ fm
- $M_\pi = 338$, 709 MeV
- O(100) gluon configurations
- extended statistics, intermediate masses, volume dependence

[1] MILC hep-lat/0104002
[2] LHPC, SESAM hep-lat/0309065

Summary

- generalized parton distributions:

  (1) contain form factors and ordinary parton distributions

  (2) determine the spin decomposition of the nucleon

  (3) measure Fourier transform of transverse quark distribution

- physics results in a heavy pion world:

  (1) $\frac{1}{2} = \frac{1}{2}(0.68(2) - 0.01(2) + 0.33(2))$ $\qquad L_{u-d} = -0.16(2)$

  (2) observed significant dependence of slopes of generalized form factors A_{n0} on n indicating a significant variation of transverse size with x