The extended Linear Sigma Model within the FRG approach

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QCD in Finite Temperature and Heavy-Ion Collisions

BNL | February 14, 2017

Overview

1. Introductory part

2. Methods

   The extended Linear Sigma Model (eLSM)

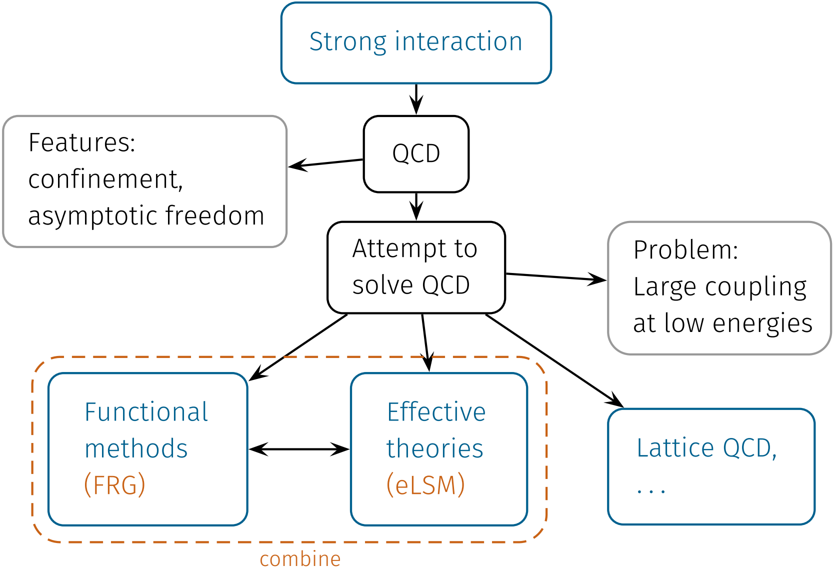
   Functional renormalization group (FRG)

3. The chiral phase transition in the presence of (axial-)vector mesons

4. Summary and outlook

Introductory part

Quantum chromodynamics



Phase diagram of QCD

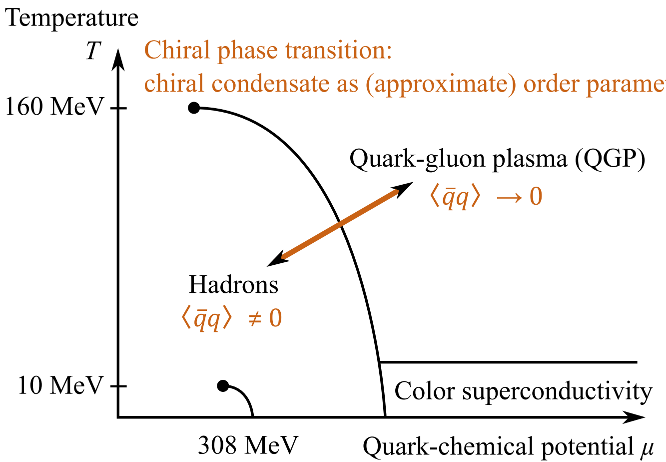


Figure 1: Schematic QCD phase diagram and the chiral phase transition; following [Rischke 2004; Alford et al. 2008; Fukushima et al. 2011; Petreczky 2012].

Chiral symmetry

- Chiral symmetry of QCD for N_f quark flavors:
  $U(N_f)_L \times U(N_f)_R \cong SU(N_f)_V \times SU(N_f)_A \times U(1)_V \times U(1)_A$
- Symmetry breaking:
  1. Axial anomaly breaks $U(1)_A$
  2. Explicitly broken for nonvanishing quark masses
  3. Spontaneously broken by the chiral condensate $\langle \bar{q}q \rangle \neq 0$
- Effective theories of the strong interaction:
  1. eLSM
  2. eLSM with quarks (quark-meson model)

Why is this study relevant?/What is new?

- Why do we focus on the chiral phase transition in the presence of vector mesons?

  Keywords: In-medium spectral function, dilepton spectrum

- What is the benefit of using the eLSM for this study?

  Keywords: Linear realization of chiral symmetry, chiral partners

- Why do we use the FRG?

  Keywords: Extend study of (axial-)vector mesons within the FRG, alternative/complementary to lattice QCD

- Connection to experiments:

  “Compressed baryonic matter” (CBM) experiment at FAIR, Darmstadt, Germany

Methods

Degrees of freedom

- (Pseudo-)scalar meson matrix $\Sigma = \Sigma_a t_a$, generators t_a of $U(N_f)$
- Vector and axial-vector mesons $V_\mu = V_{\mu,a} t_a$ and $A_\mu = A_{\mu,a} t_a$
- Left-/right-handed vector fields $L_\mu = L_{\mu a} t_a$, $R_\mu = R_{\mu a} t_a$:

$$L_\mu = V_\mu + A_\mu, \quad R_\mu = V_\mu - A_\mu.$$

- Transformations under $U(N_f)_L \times U(N_f)_R$:

$$\Sigma \to U_L \Sigma U_R^\dagger, \quad L_\mu \to U_L L_\mu U_L^\dagger, \quad R_\mu \to U_R R_\mu U_R^\dagger.$$

- “Covariant derivative”:

$$D_\mu \Sigma = \partial_\mu \Sigma - i g_1 \left(L_\mu \Sigma - \Sigma R_\mu \right).$$

Mesonic Lagrangian of the eLSM

$$
\begin{aligned}
\mathcal{L}_{\text{mesons}} = {} & \operatorname{tr}\left[(D_\mu \Sigma)^\dagger D_\mu \Sigma\right] + m_0^2 \operatorname{tr}\left(\Sigma^\dagger \Sigma\right) \\
& + \lambda_1 \left[\operatorname{tr}\left(\Sigma^\dagger \Sigma\right)\right]^2 + \lambda_2 \operatorname{tr}\left[\left(\Sigma^\dagger \Sigma\right)^2\right] \\
& + \frac{1}{4} \operatorname{tr}\left(L_{\mu\nu}^2 + R_{\mu\nu}^2\right) + \operatorname{tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)\left(L_\mu^2 + R_\mu^2\right)\right] \\
& - \operatorname{tr}\left[H\left(\Sigma + \Sigma^\dagger\right)\right] - c_A \left(\det\, \Sigma + \det\, \Sigma^\dagger\right) \\
& - i\frac{g_2}{2}\left(\operatorname{tr}\left\{L_{\mu\nu}\left[L_\mu, L_\nu\right]\right\} + \operatorname{tr}\left\{R_{\mu\nu}\left[R_\mu, R_\nu\right]\right\}\right) \\
& + \frac{h_1}{2} \operatorname{tr}\left(\Sigma^\dagger \Sigma\right) \operatorname{tr}\left(L_\mu^2 + R_\mu^2\right) + h_2 \operatorname{tr}\left(\left|L_\mu \Sigma\right|^2 + \left|\Sigma R_\mu\right|^2\right) \\
& + 2h_3 \operatorname{tr}\left(\Sigma R_\mu \Sigma^\dagger L_\mu\right) - g_3\left[\operatorname{tr}\left(L_\mu L_\nu L_\mu L_\nu\right) + \operatorname{tr}\left(R_\mu R_\nu R_\mu R_\nu\right)\right] \\
& - g_4\left[\operatorname{tr}\left(L_\mu L_\mu L_\nu L_\nu\right) + \operatorname{tr}\left(R_\mu R_\mu R_\nu R_\nu\right)\right] - g_5 \operatorname{tr}\left(L_\mu L_\mu\right) \operatorname{tr}\left(R_\nu R_\nu\right) \\
& - g_6\left[\operatorname{tr}\left(L_\mu L_\mu\right) \operatorname{tr}\left(L_\nu L_\nu\right) + \operatorname{tr}\left(R_\mu R_\mu\right) \operatorname{tr}\left(R_\nu R_\nu\right)\right].
\end{aligned}
$$

Yukawa coupling and $N_f = 2$

- Left-/right-handed spinors $\psi_{L/R} = \frac{1}{2}(\mathbb{1} \mp \gamma_5)\psi$
- Quark Lagrangian (flavor symmetric quark-chemical potential μ):

$$\begin{aligned}\mathcal{L}_{\text{quarks}} &= \bar{\psi}\left(\gamma_\mu \partial_\mu + \mu\gamma_0\right)\psi + y\left(\bar{\psi}_L \Sigma \psi_R + \bar{\psi}_R \Sigma^\dagger \psi_L\right) \\ &\equiv \bar{\psi}\left(\gamma_\mu \partial_\mu + \mu\gamma_0 + y\Sigma_5\right)\psi.\end{aligned}$$

- Full Lagrangian:

$$\mathcal{L}_{\text{total}} = \mathcal{L}_{\text{mesons}} + \mathcal{L}_{\text{quarks}}.$$

- $N_f = 2$:

$$\begin{aligned}\Sigma &= (\sigma + i\eta)t_0 + (\vec{a}_0 + i\vec{\pi}) \cdot \vec{t}\,, \\ \Sigma_5 &= (\sigma + i\gamma_5\eta)t_0 + (\vec{a}_0 + i\gamma_5\vec{\pi}) \cdot \vec{t}\,, \\ R_\mu &= (\omega_\mu - f_{1\mu})t_0 + (\vec{\rho}_\mu - \vec{a}_{1\mu}) \cdot \vec{t}\,, \\ L_\mu &= (\omega_\mu + f_{1\mu})t_0 + (\vec{\rho}_\mu + \vec{a}_{1\mu}) \cdot \vec{t}.\end{aligned}$$

π-a_1 and η-f_1 mixing

- Nonvanishing vacuum expectation value of the σ field:

$$\sigma \to \sigma_0 + \sigma.$$

- Generates unphysical mixing terms in the full Lagrangian:

$$g_1 \sigma_0 \eta \partial_\mu f_{1\mu}, \quad g_1 \sigma_0 \vec{\pi} \cdot \partial_\mu \vec{a}_{1\mu}.$$

- Elimination of these terms by shifting the axial-vector mesons:

$$f_{1\mu} \to f_{1\mu} + w \partial_\mu \eta, \quad \vec{a}_{1\mu} \to \vec{a}_{1\mu} + w \partial_\mu \vec{\pi}, \quad w = \frac{g_1 \sigma_0}{m_1^2 + (g_1 \sigma_0)^2}.$$

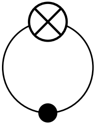
- Results in the canonical renormalization of the pseudoscalars:

$$\pi_a \to \sqrt{Z_\pi} \pi_a, \quad Z_\pi = 1 + \frac{(g_1 \sigma_0)^2}{m_1^2}.$$

FRG

- Implementation of the Wilsonian RG idea
- Renormalization scale(k)-dependent effective action Γ_k
- FRG flow equation [Wetterich 1993]:

$$\partial_k \Gamma_k = \frac{1}{2} \operatorname{str} \left[\partial_k \mathbf{R}_k \left(\mathbf{\Gamma}_k^{(2)} + \mathbf{R}_k \right)^{-1} \right] = \frac{1}{2}$$



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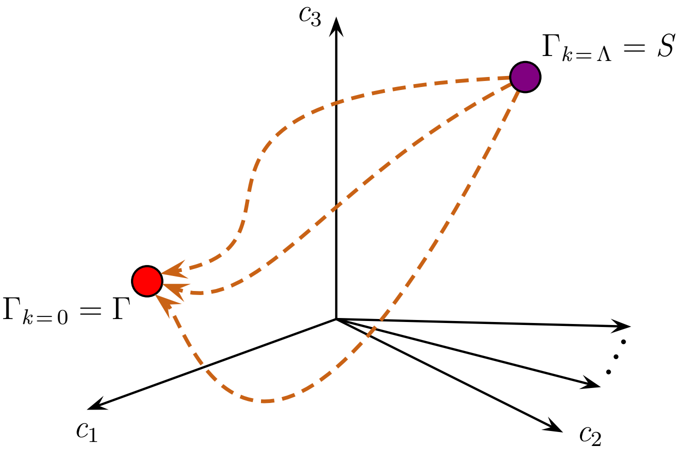
- Regulator function $\mathbf{R}_k$ provides correct integration limits

Flow in theory space



The chiral phase transition in the presence of (axial-)vector mesons

Simplifications

- Set couplings $g_2, \dots, g_6$ as well as $h_1, \dots, h_3$ to zero and neglect momentum-dependent vertices
- Assume exact isospin symmetry:

$$\Delta = 0, \quad H = \frac{h_{\mathrm{ESB}}}{2} \mathbb{1}.$$

$$\begin{aligned}
\mathcal{L}_{\mathrm{mesons}} = & \operatorname{tr}\left(\partial_\mu \Sigma^\dagger \partial_\mu \Sigma\right) + m_0^2 \operatorname{tr}\left(\Sigma^\dagger \Sigma\right) \\
& + \lambda_1 \left[\operatorname{tr}\left(\Sigma^\dagger \Sigma\right)\right]^2 + \lambda_2 \operatorname{tr}\left[\left(\Sigma^\dagger \Sigma\right)^2\right] \\
& + \frac{1}{4} \operatorname{tr}\left(L_{\mu\nu}^2 + R_{\mu\nu}^2\right) + \frac{m_1^2}{2} \operatorname{tr}\left(L_\mu^2 + R_\mu^2\right) \\
& - \frac{h_{\mathrm{ESB}}}{2} \operatorname{tr}\left(\Sigma + \Sigma^\dagger\right) - c_A \left(\det \Sigma + \det \Sigma^\dagger\right) \\
& + g_1^2 \operatorname{tr}\left[\left(L_\mu \Sigma - \Sigma R_\mu\right)^\dagger \left(L_\mu \Sigma - \Sigma R_\mu\right)\right].
\end{aligned}$$

Effective action

- Ansatz for the effective action (with quarks):

$$\Gamma_k = \int_x \Bigg[\frac{1}{2}\partial_\mu\phi_i\partial_\mu\phi_i + \frac{1}{4}\mathcal{F}_{i,\mu\nu}\mathcal{F}_{i,\mu\nu} + U_k - h_{\mathrm{ESB}}\sigma$$
$$+\bar{\psi}\left(\gamma_\mu\partial_\mu + \mu\gamma_0 + y\Sigma_5\right)\psi\Bigg],$$

  spin-zero fields ϕ, spin-one fields $\mathcal{A}$, field-strength tensors $\mathcal{F}_{\mu\nu} = \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu$, and spin-one-half fields $\bar{\psi}$, ψ
- Local potential approximation: $\partial_k\Gamma_k \propto \partial_k U_k$
- Grid method:
  1. Discretize effective potential U_k on a grid
  2. Tune the UV potential such that the IR minimum lies at $\sigma_0 = f_\pi m_{a_1}/m_\rho$ or $\sigma_0 = f_\pi$ (pion decay constant f_π)

Effective potential without axial anomaly

- Truncation:

$$U_k(\xi_1, ..., \xi_4) = V_k(\xi_1) + W_k(\xi_1)\xi_2 + X_k(\xi_1)\xi_3 + Y_k(\xi_1)\xi_4.$$

- Invariants:

$$\begin{aligned}
\xi_1 &= \sigma^2 + \vec{\pi}^2 + \eta^2 + \vec{a}_0^2, \\
\xi_2 &= \left(\sigma^2 + \vec{\pi}^2\right)\left(\eta^2 + \vec{a}_0^2\right) - \left(\sigma\eta - \vec{\pi}\cdot\vec{a}_0\right)^2, \\
\xi_3 &= \left(\vec{\pi}\cdot\vec{a}_{1\mu} + \eta f_{1\mu}\right)^2 + \left(\vec{a}_0\cdot\vec{a}_{1\mu} + \sigma f_{1\mu}\right)^2 \\
&\quad + \left(\vec{\rho}_\mu \times \vec{a}_0 + \eta\vec{a}_{1\mu} + \vec{\pi} f_{1\mu}\right)^2 + \left(\vec{\pi}\times\vec{\rho}_\mu + \sigma\vec{a}_{1\mu} + \vec{a}_0 f_{1\mu}\right)^2, \\
\xi_4 &= f_{1\mu}^2 + \vec{a}_{1\mu}^2 + \omega_\mu^2 + \vec{\rho}_\mu^2.
\end{aligned}$$

Effective potential with axial anomaly

- Truncation:

$$U_k(\bar{\xi}_1, ..., \xi_4) = \bar{V}_k(\bar{\xi}_1) + \bar{W}_k(\bar{\xi}_1)\bar{\xi}_2 + \bar{X}_k(\bar{\xi}_1)\bar{\xi}_3 + \bar{Y}_k(\bar{\xi}_1)\xi_3 + \bar{Z}_k(\bar{\xi}_1)\xi_4.$$

- Invariants:

$$\begin{aligned}
\bar{\xi}_1 &= \sigma^2 + \vec{\pi}^2, \\
\bar{\xi}_2 &= \eta^2 + \vec{a}_0^2, \\
\bar{\xi}_3 &= \left(\sigma\eta - \vec{\pi} \cdot \vec{a}_0\right)^2, \\
\xi_3 &= \left(\vec{\pi} \cdot \vec{a}_{1\mu} + \eta f_{1\mu}\right)^2 + \left(\vec{a}_0 \cdot \vec{a}_{1\mu} + \sigma f_{1\mu}\right)^2 \\
&\quad + \left(\vec{\rho}_\mu \times \vec{a}_0 + \eta \vec{a}_{1\mu} + \vec{\pi} f_{1\mu}\right)^2 + \left(\vec{\pi} \times \vec{\rho}_\mu + \sigma \vec{a}_{1\mu} + \vec{a}_0 f_{1\mu}\right)^2, \\
\xi_4 &= f_{1\mu}^2 + \vec{a}_{1\mu}^2 + \omega_\mu^2 + \vec{\rho}_\mu^2.
\end{aligned}$$

Local potential approximation

$$\partial_k \bar{V}_k = \frac{k^4}{12\pi^2}\left\{ \frac{3\coth\left(\frac{\sqrt{k^2+2\,\bar{V}_k{}'}}{2\,T}\right)}{\sqrt{k^2+2\,\bar{V}_k{}'}} + \frac{\coth\left(\frac{\sqrt{k^2+4\bar{\xi}_1\,\bar{V}_k{}''+2\,\bar{V}_k{}'}}{2\,T}\right)}{\sqrt{k^2+4\bar{\xi}_1\,\bar{V}_k{}''+2\,\bar{V}_k{}'}} \right.$$

$$+\frac{3\coth\left(\frac{\sqrt{k^2+2\,\bar{W}_k}}{2\,T}\right)}{\sqrt{k^2+2\,\bar{W}_k}} + \frac{\coth\left(\frac{\sqrt{k^2+2\bar{\xi}_1\,\bar{X}_k+2\,\bar{W}_k}}{2\,T}\right)}{\sqrt{k^2+2\bar{\xi}_1\,\bar{X}_k+2\,\bar{W}_k}}$$

$$+\frac{12\coth\left(\frac{\sqrt{k^2+2\bar{Z}_k}}{2\,T}\right)}{\sqrt{k^2+2\bar{Z}_k}} + \frac{12\coth\left(\frac{\sqrt{k^2+2\bar{\xi}_1\,\bar{Y}_k+2\bar{Z}_k}}{2\,T}\right)}{\sqrt{k^2+2\bar{\xi}_1\,\bar{Y}_k+2\bar{Z}_k}}$$

$$\left. -12\left[\frac{\tanh\left(\frac{\sqrt{\frac{y^2\bar{\xi}_1}{4}+k^2}-\mu}{2\,T}\right)}{\sqrt{\frac{y^2\bar{\xi}_1}{4}+k^2}} + \frac{\tanh\left(\frac{\sqrt{\frac{y^2\bar{\xi}_1}{4}+k^2}+\mu}{2\,T}\right)}{\sqrt{\frac{y^2\bar{\xi}_1}{4}+k^2}}\right]\right\}.$$

Chiral limit (1)

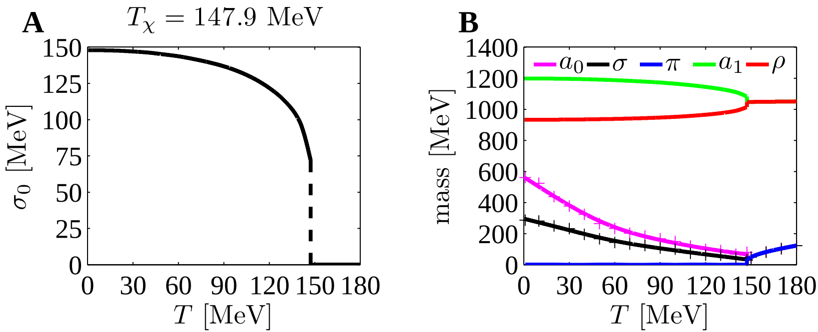


Figure 2: 1st-order phase transition (**A**) and mass degeneracy (**B**) for zero quark masses and in the absence of the axial anomaly. $m_{f_1} = m_{a_1}$, $m_\omega = m_\rho$, $m_\eta = m_\pi$; cf. [Eser et al. 2015].

Chiral limit (2)

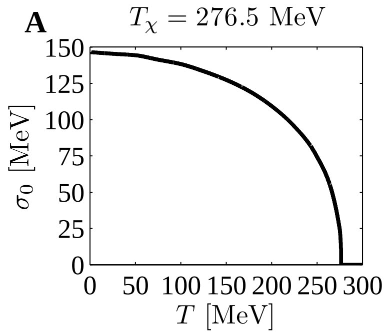


Figure 3: 2nd-order phase transition (**A**) and mass degeneracy (**B**) for zero quark masses and in the presence of the axial anomaly. $m_{f_1} = m_{a_1}$, $m_\omega = m_\rho$; cf. [Eser et al. 2015].

Nonzero quark masses

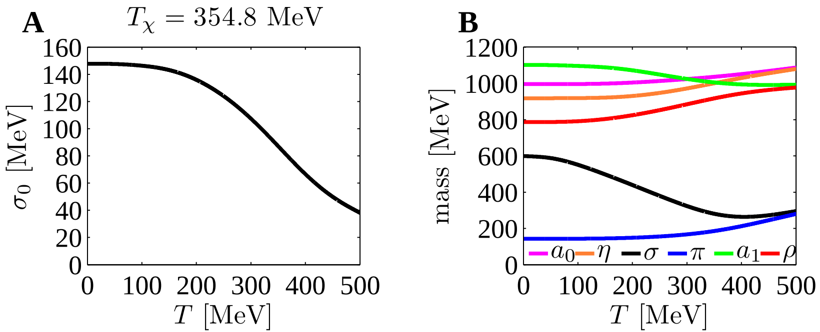


Figure 4: Crossover transition (**A**) and mass degeneracy (**B**) for nonzero quark masses and in the presence of the axial anomaly. $m_{f_1} = m_{a_1}$, $m_\omega = m_\rho$; cf. [Eser et al. 2015].

Quark-meson model (1)

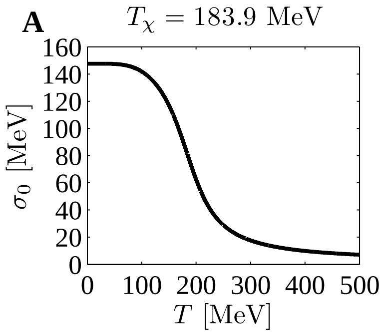


Figure 5: Crossover transition ($\mathbf{A}$) and mass degeneracy ($\mathbf{B}$) for nonzero quark masses and in the presence of the axial anomaly. $\mu = 0$ MeV. $m_{f_1} = m_{a_1}$, $m_\omega = m_\rho$; [preliminary].

Quark-meson model (2)

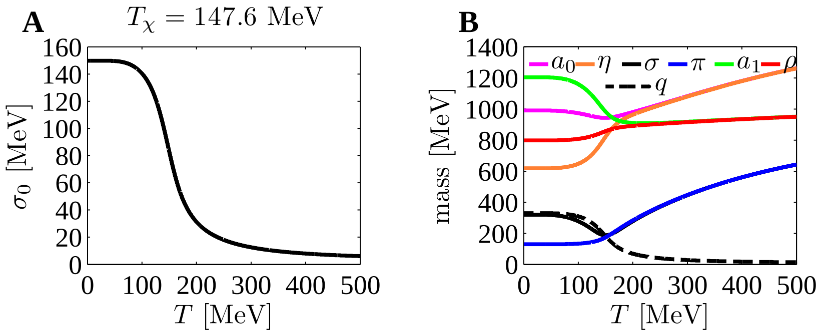


Figure 6: Crossover transition ($\mathbf{A}$) and mass degeneracy ($\mathbf{B}$) for nonzero quark masses and in the presence of the axial anomaly. $\mu = 0$ MeV. $m_{f_1} = m_{a_1}$, $m_\omega = m_\rho$; [preliminary].

Quark-meson model (3)

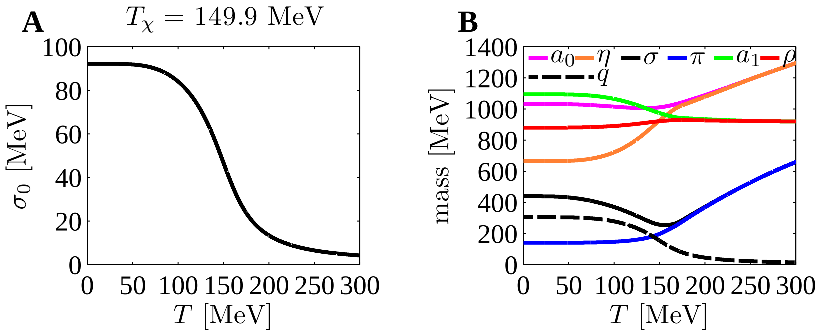


Figure 7: Crossover transition ($\mathbf{A}$) and mass degeneracy ($\mathbf{B}$) for nonzero quark masses and in the presence of the axial anomaly. $\mu = 0$ MeV. $m_{f_1} = m_{a_1}$, $m_\omega = m_\rho$; [preliminary].

Quark-meson model (4)

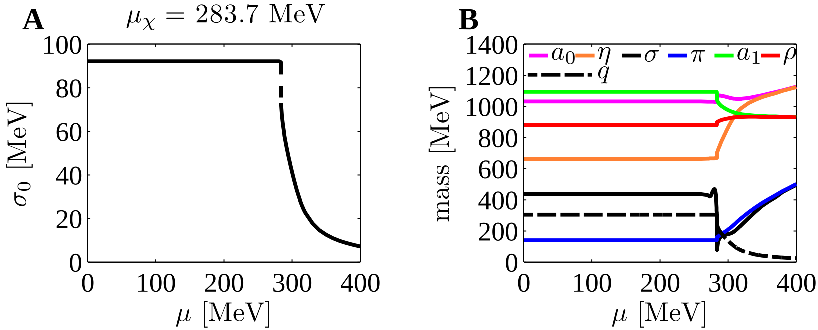


Figure 8: Transition near the critical point (**A**) and mass degeneracy (**B**) for nonzero quark masses and in the presence of the axial anomaly.
$T = 9$ MeV. $m_{f_1} = m_{a_1}$, $m_\omega = m_\rho$; [preliminary].

Phase diagram (1)

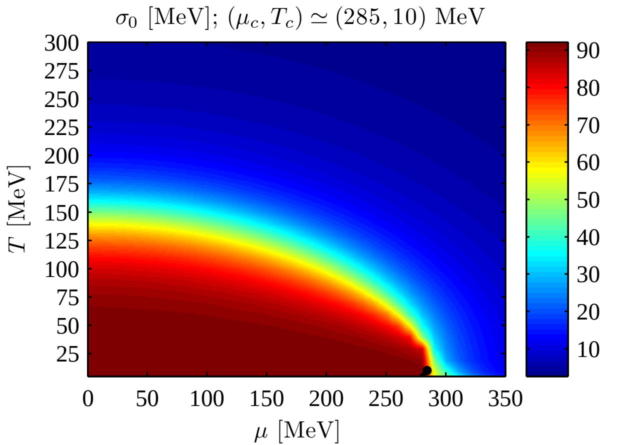


Figure 9: The chiral condensate σ_0 as a function of the temperature T and the quark-chemical potential μ; [preliminary].

Phase diagram (2)

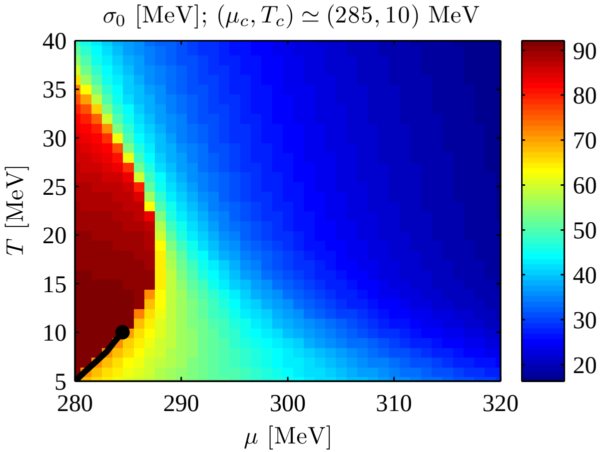


Figure 10: The chiral condensate σ_0 as a function of the temperature T and the quark-chemical potential μ; [preliminary].

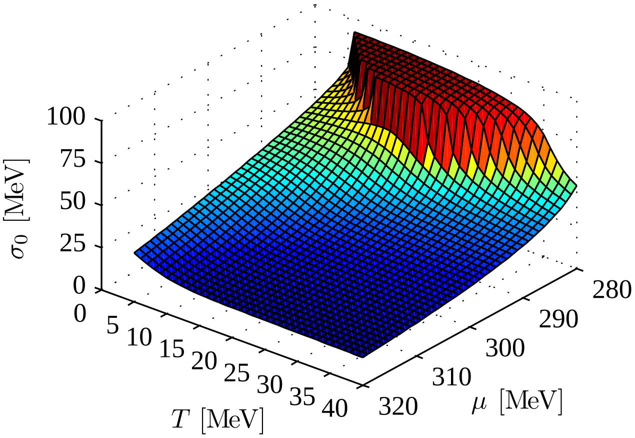


Figure 11: 3D version of the phase diagram; [preliminary].

Summary and outlook

Summary and outlook

Summary:

- Transition order consistent with other studies [Eser et al. 2015]
- T_χ brought into agreement with lattice QCD by including quark fluctuations

Outlook:

- Check quadratic order terms in the effective potential
- Include effective Polyakov loop potential
- $N_f = 3$ (strangeness)
- **Low-energy limit of the eLSM:**
  Nuclear Physics Seminar at Stony Brook
  on Thursday, Feb. 16, 2:30 p.m.