QCD thermodynamics: beyond perturbation theory

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Description of the strongly interacting matter

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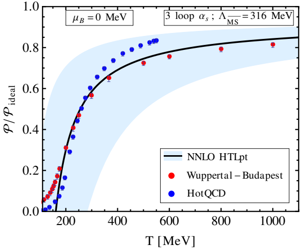

(J.O. Andersen *et.al.* 2014)

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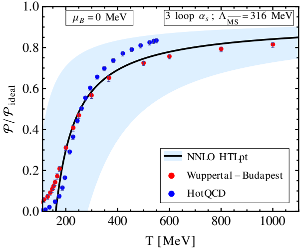

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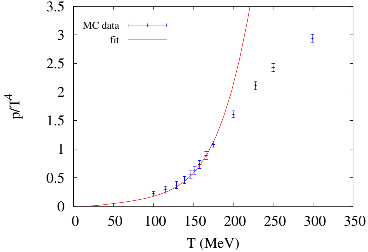
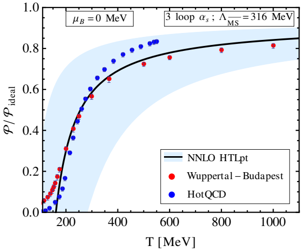


(T.S. Biró, A.J. 2014)

- at high energy scales (high temperature): asymptotic freedom $\Rightarrow$ perturbative QCD; from $T \gtrsim 200 - 250$ MeV
- at low energy scales (low temperature): bound states are formed (hadrons) which interact "weakly" $\Rightarrow$ perturbative hadron gas (HRG) description; up to $T \lesssim 170$ MeV

What drives the phase transition?



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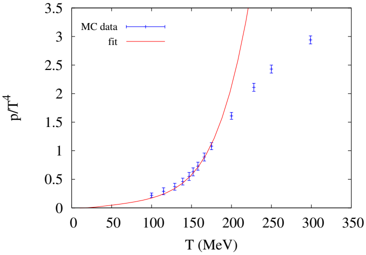

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Problem

- p_{HRG} overshoots the real pressure
- $p_{HRG} \gtrsim p_{pert\ QCD} \quad \Rightarrow \quad F_{HRG} \lesssim F_{pert\ QCD}$, hadronic phase is always more stable

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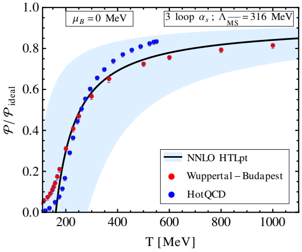

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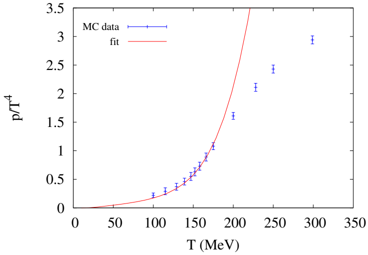

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$$\Downarrow$$

hadronic degrees of freedom must disappear from the system!

Is it possible without an abrupt change of ground state?

Phase transition regime: quasiparticles ideas

Possible explanation:

- hadrons/quarks exist, but have large self-energies

$$m_h \stackrel{T>T_c}{\longrightarrow} 0, \quad m_{q,g} \stackrel{T<T_c}{\longrightarrow} \infty$$

- leads to small thermal weights $\sim e^{-\beta m} \ll 1$
- BUT: MC data do not show drastic variation in particle masses

  direct mass, and correlation measurments

- hadrons do not disappear at T_c

  (J. Liao, E.V. Shuryak PRD73 (2006) 014509 [hep-ph/0510110])

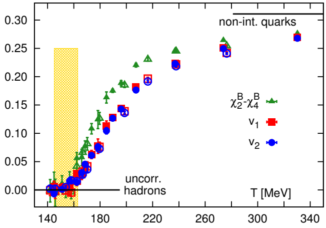
  (AJ., P. Petreczky, K. Petrov, A. Velytsky, PRD75 (2007) 014506)

  $\Rightarrow$ hadronic states are observable even at $T \sim 1.5\,T_c$

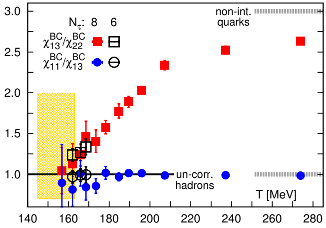
But if hadrons survive T_c why do not they dominate the pressure?

at $T \sim 156$ MeV (crossover) phase transition

Observations vs. quasiparticle predictions



B and BS fluctutations
(A. Bazazov *et.al.* 2013)



BC fluctutations
(A. Bazazov *et.al.* 2014)

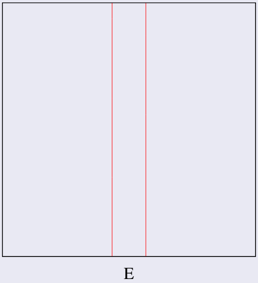
$150 \lesssim T \lesssim 250$ MeV:
non-quasiparticle regime, changing degrees of freedom

nonperturbative methods are needed to describe this regime

same quantum number $\Rightarrow$ only their mass can differ!

What do we observe in a mass spectrometer?

spectrum of two particles



- ideally: 2 thin spectral lines

Two particles with the same quantum numbers

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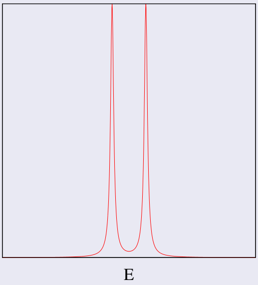

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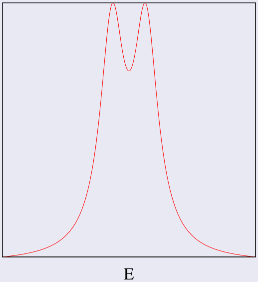

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- widht $\sim$ mass difference:
  no measurements can resolve the peak structure!
  the sates become indistinguishable
  $\Rightarrow$ represent 1 dof

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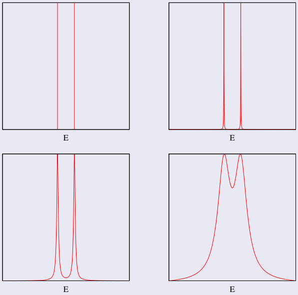

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- **Lesson**:
  changing width (changing spectrum)
  $\Rightarrow$ **changing # of dof.!**

Thermodynamics from spectral function

Assume that we know the spectrum (measurement).

Goal: calculate pressure $P(\varrho)$

(T.S. Biro, A.J. and Zs. Schram 2016; T.S. Biro and A.J. 2014; AJ. 2012,2013)

Strategy

- **represent** ϱ with a (quadratic) effective model
- calculate thermodynamics from this theory
  energy density $\varepsilon = \frac{1}{Z}\,\mathrm{Tr}\, e^{-\beta H} T_{00}$, use KMS relation

Scalar field case

$$S = \int \frac{d^4q}{(2\pi)^4}\,\frac{1}{2}\Phi^*(q)\mathcal{K}(q)\Phi(q)$$

for consistency we need a physical spectrum only!
unitary, causal, Lorentz-invariant, E, $\vec{p}$ conserving

Thermodynamics from spectral function II.

We start from the Lagrangian:

$$\mathcal{L} = \tfrac{1}{2}\Phi^*(q)\mathcal{K}(q)\Phi(q)$$

- In order to reproduce the given ϱ spectral function we need

$$\varrho = \mathrm{Disc}\, i\mathcal{K}^{-1}, \qquad \mathcal{K}^{-1}(q) = \int \frac{d\omega}{2\pi}\,\frac{\varrho(\omega,\mathbf{q})}{q_0-\omega}$$

- Energy momentum tensor (Noether-current):

$$T_{\mu\nu}(x) = \frac{1}{2}\varphi(x)\, D_{\mu\nu}\mathcal{K}(i\partial)\,\varphi(x)$$

where

$$D_{\mu\nu}\mathcal{K}(i\partial) = \left[\frac{\partial\mathcal{K}(p)}{\partial p^\mu}p_\nu - g_{\mu\nu}\mathcal{K}(p)\right]_{p\to i\partial, sym}$$

and the symmetrized derivative is defined as

$$f(x)[(i\partial)^n]_{sym}g(x) = \frac{1}{n+1}\sum_{a=0}^{n}[(-i\partial)^a f(x)][(i\partial)^{n-a}g(x)].$$

- We take its expectation value using KMS relation

$$\langle\varphi\varphi\rangle(q) = n_{BE}(q_0)\varrho(q)$$

$\Rightarrow$ symmetrized derivative becomes normal one.

Thermodynamics from spectral function III.

Result:

Pressure as a function of the spectral function

$$P = \mp T \int \frac{d^4q}{(2\pi)^4} \frac{\partial \mathcal{K}}{\partial q_0} \ln\left(1 \mp e^{-\beta q_0}\right) \varrho(q)$$

- generally nonlinear ϱ dependence due to $\mathcal{K} \sim \frac{1}{\varrho}$

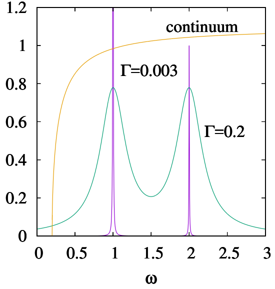
  $\Rightarrow$ P does not depend on the overall normalization of ϱ.
- for free gas mixture $\varrho(p) = \sum_i Z_i \delta(p_0 - E_p)$

  we obtain $P = \sum_i P^{(0)}(m_i)$: sum of partial pressures;

  no dependence on Z_i, while they are nonzero!

Changing degrees of freedom for two particles

How thermodynamics changes when peaks are merged?



- spectrum for two particles with different width, and a typical multiparticle continuum (non-quasiparticle system)

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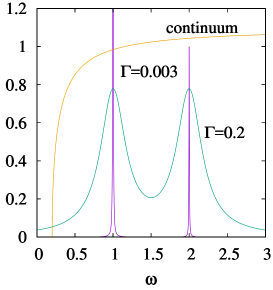

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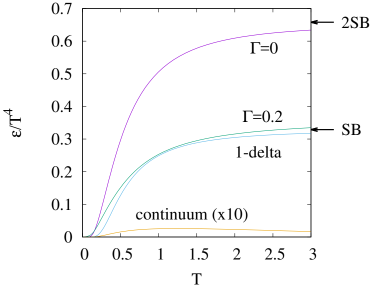

- spectrum for two particles with different width, and a typical multiparticle continuum (non-quasiparticle system)
- at small width ⇒ two-particle energy density
- at large width ⇒ $\sim$ one-particle energy density
- continuum: practically negligible energy density contibution

Gibbs paradox (actualized)

- in mixture of two bosonic gases the SB limit is $P = N_{eff} P_{SB}$, where $N_{eff} = 2$ if the masses are different and $N_{eff} = 1$ if the masses are equal

$\Rightarrow$ discontinuous change for $\Delta m \to 0$!

Gibbs paradox for interacting gases

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  no tool to describe the change in N_{eff}
- With changing spectral functions N_{eff} is dynamical variable

  $\Rightarrow$ **in interacting theories Gibbs paradox is smeared out**

Gibbs paradox for interacting gases

Gibbs paradox (a

- in mixture o ... $N_{eff} P_{SB}$,
  where N_{eff} = ... 1 if the
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  - ⇒ disc
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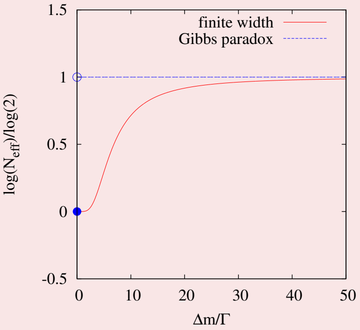
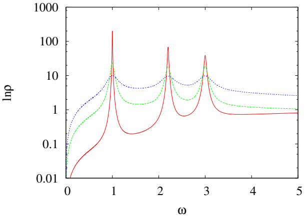
of dof in a gas mixture



Merging with continuum: melting

- one peak dominated regime: $N_{eff} = 1$
- continuum dominated regime: $N_{eff} = 0$
- if peak merges into a continuum $\Rightarrow$ vanishing pressure
- particle ceases to be a thermodynamical dof

thermodynamic definition of # dof: $N_{eff}(T) = \dfrac{P(T)}{P_0(T)}$



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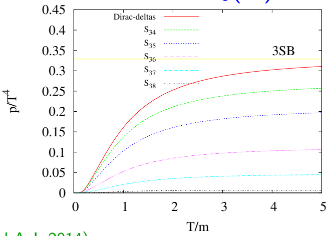

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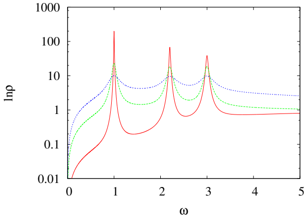

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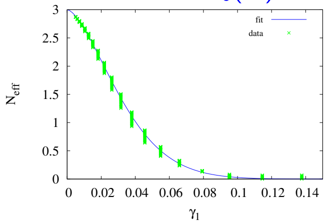


(T.S. Biro, A.J. and Zs. Schram 2016; T.S. Biro and A.J. 2014)

good fitting function: $N_{eff} = N_0 + N_1 e^{-a\gamma^b}$ (typically $b = 1.5 - 2$)

Application to QCD

An oversimplified (statistical) realization of these ideas for QCD

(T.S. Biro, A.J. and Zs. Schram 2016; T.S. Biro and A.J. 2014)

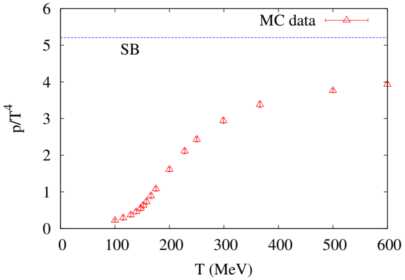
$$P_{hadr}(T) = N_{eff}^{(hadr)} \sum_{n \in \text{hadrons}}^{N} P_0(T, m_n), \qquad \ln N_{eff}^{(hadr)} = -(T/T_0)^b,$$

$$P_{QGP}(T) = N_{eff}^{(part)} \sum_{n \in \text{partons}} P_0(T, m_n), \qquad \ln N_{eff}^{(part)} = G_0 - c(N_{eff}^{(hadr)})^d.$$

$P = P_{hadr} + P_{QGP}$ total pressure, P_0 ideal gas pressure

- hadrons: Hagedorn-sp. up to a certain mass ($m \lesssim 3\,\mathrm{GeV}$)
- partons quark and gluon quasiparticles
- $N_{hadr}(\gamma)$ common suppression factor for all hadrons: stretched exponential, and $\gamma \sim T$
- $N_{part}(N_{hadr})$ partonic suppression factor grows with the # of available hadronic resonances.

Fitting procedure



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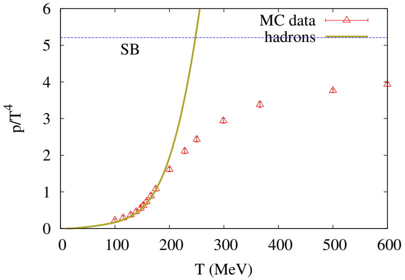

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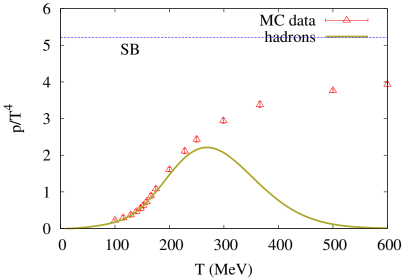

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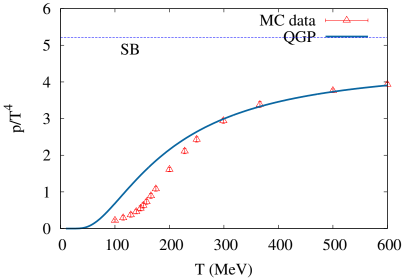

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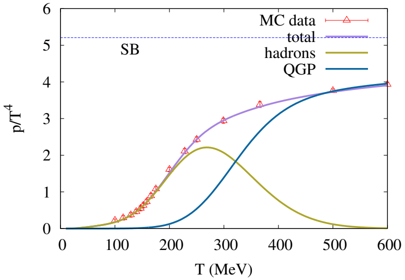

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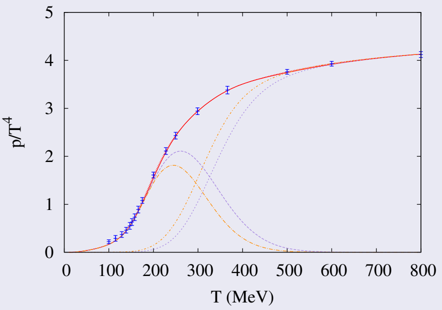
- from pressure at $T > 300\,\mathrm{MeV}$ fit QGP parameters (fixed $m_q = 330\,\mathrm{MeV}$, $m_g = 600\,\mathrm{MeV}$)
- quark and gluon width depends on the number of hadrons $\gamma_{QGP}^2 = \gamma_0^2 + cN_{hadr}^{\alpha}$, $N_{QGP} = e^{-\gamma_{QGP}^2}$.

With this Ansatz: $P = N_{eff}^{(hadr)} P_{hadr} + N_{eff}^{(QGP)} P_{QGP}$

QCD partial pressures



- total pressure is well reproduced
- width of melting interval is tunable
- hadrons do not vanish at T_c: they just start to melt there.
- quarks just start to appear at T_c

Transport coefficients come from correlators of conserved quantities. In particular

$$\eta = \lim_{\omega \to 0} \frac{\langle [T_{12}, T_{12}] \rangle (\omega, \mathbf{k} = 0)}{\omega}$$

In the quadratic nonlocal effective model we know $T_{\mu\nu}$

$\Rightarrow$ η can be calculated (M. Horvath and AJ. 2016)

Pressure as a function of the spectral function

$$\eta = \int \frac{d^4q}{(2\pi)^4} \left(\frac{q_1 q_2}{q_0} \frac{\partial \mathcal{K}}{\partial q_0} \varrho(q) \right)^2 \left(-\frac{dn(q_0)}{dq_0} \right).$$

Given the spectral function (nonperturbative information) we can calculate η/s.

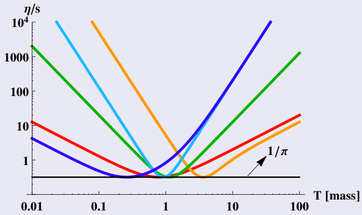
Analytically computable example: Lorentzian peak with large width

$$\rho_L(\omega, p) = \frac{4\gamma\omega}{(\omega^2 - p^2 - \gamma^2)^2 + 4\gamma^2\omega^2}$$

The corresponding ratio

$$\frac{\eta_L}{s_L} = \frac{5}{4\pi^2}\frac{\gamma}{T} + \frac{1}{5}\frac{T}{\gamma}.$$

η/s special example



$\Rightarrow$ reaches minimal value in this case $1/\pi$ at $T \approx 0.8\gamma$

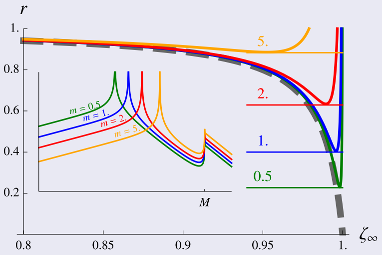
similar to liquid-gas crossover without phase transition

Presence of continuum

The **presence of continuum** can considerably diminish the viscosity. Characterize relative weight of the continuum by ζ_∞. Introduce

$$r = \frac{(\eta/s)_\varrho}{(\eta/s)_{QP}}$$

effect of presence of continuum



Conclusions

- Thermodynamics of strongly interacting matter is perturbative for $T < 150$ MeV (HRG), and $T > 250$ MeV (QCD) (at $\mu = 0$)
- in the critical domain (analytically) changing dof
  $\Rightarrow$ hadron melting
  crucial: correct treatment of spectral properties
- Hadrons start to melt at T_c, but disappear from the system much later (at $\sim 250 - 300$ MeV).
- Transport coefficients can be calculated using the representation of the spectral function.
- QP systems have lower bound for η/s
- Presence of large continuum part diminishes η/s.