Quasinormal modes of charged magnetic black branes & chiral magnetic transport

QCD at finite temperature and heavy ion collisions, BNL

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Magnetic black branes are solutions to Einstein-Maxwell-Chern-Simons (EMCS) theory

- magnetic analog of (charged) Reissner-Nordstrom black brane
- asymptotically AdS

\[
S_{\text{grav}} = \frac{1}{2\kappa^2} \left[ \int_{\mathcal{M}} d^5x \sqrt{-g} \left( R + \frac{12}{L^2} - \frac{1}{4} F_{mn}F^{mn} \right) - \frac{\gamma}{6} \int_{\mathcal{M}} A \wedge F \wedge F \right]
\]

Ansatz

\[
ds^2 = -r_H^2 \tilde{U}(u) dt^2 + \frac{du^2}{4u^2 \tilde{U}(u)} + e^{2V(u)} (dx^2 + dy^2) + e^{2W(u)} dz^2 ,
\]

\[
F = b dx \wedge dy .
\]
Magnetic black branes are solutions to Einstein-Maxwell-Chern-Simons (EMCS) theory

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Ansatz

\[ ds^2 = -r_H^2 \tilde{U}(u) dt^2 + \frac{du^2}{4u^2 \tilde{U}(u)} + e^{2V(u)}(dx^2 + dy^2) + e^{2W(u)} dz^2, \]

\[ F = bdx \wedge dy. \]

EOMs at vanishing charge density already complicated:

\begin{align*}
0 &= 2b^2 + 4e^{4V(u)} (u^3 \tilde{U}'(u)(2V'(u) + W'(u)) + u^2 \tilde{U}(u)(2(u(2V''(u) + W''(u) + W'(u)^2) + 3uV'(u)^2) + 3W'(u)) - 3), \\
0 &= 2u^2 e^{4V(u)} (2u \tilde{U}''(u) + \tilde{U}'(u)(4u(V'(u) + W'(u)) + 3) + \tilde{U}(u)(4uV''(u) + W''(u) + W'(u)^2 + 3W'(u)) - 3), \\
0 &= b^2 e^{-4V(u)} + u^2 (2(2u \tilde{U}''(u) + \tilde{U}(u)(4uV''(u) + 6V'(u)(uV'(u) + 1)) + \tilde{U}'(u)(8uV'(u) + 3)) - 6, \\
0 &= b^2 e^{-4V(u)} + 2u^3 (\tilde{U}'(u)(2V'(u) + W'(u)) + 2\tilde{U}(u)V'(u)(V'(u) + 2W'(u))) - 6. \quad (24)
\end{align*}
Magnetic black branes are solutions to Einstein-Maxwell-Chern-Simons (EMCS) theory

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Ansatz EOMs at vanishing charge density already complicated:

Uncharged magnetic black branes

Rhett Butler to Scarlett O’Hara
- Gone with the wind (1939)
Magnetic black branes are solutions to Einstein-Maxwell-Chern-Simons (EMCS) theory

- **magnetic analog of (charged) Reissner-Nordstrom black brane**
- **symptotically AdS**

**Ansatz**

EOMs at vanishing charge density already complicated:

**Uncharged magnetic black branes**

**Rhett Butler to Scarlett O’Hara**

- Gone with the wind (1939)

**Holography**
Turn holography into a controlled approximation

- quantify deviation from reality (QCD, experiment)
- compute corrections where needed (e.g. large coupling)
  
  e.g. [Steineder, Stricker, Vuorinen: PRL (2013); Waeber, Schaefer, Vuorinen, Yaffe; JHEP (2015); Grozdanov, v.d. Schee (2016)]

- holographic model has to be consistent (existence)
Example: Chiral transport effects in charged anisotropic plasma within a magnetic field at strong coupling

Front and side view of collision between gold ions at Brookhaven National Lab's Relativistic Heavy Ion Collider, captured by the Solenoidal Tracker at RHIC (STAR detector).
Example: Chiral transport effects in charged anisotropic plasma within a magnetic field at strong coupling

Front and side view of collision between gold ions at Brookhaven National Lab's Relativistic Heavy Ion Collider, captured by the Solenoidal Tracker at RHIC (STAR detector).

Challenge: blind spot between collision & detection
Methods: holography & hydrodynamics

Assume we have a hard problem that is difficult to solve in a given theory, for example QCD

gravity dual to QCD or standard model?

not known yet

(Hard) problem in “similar” model theory

Simple problem in a particular gravitational theory

model

holography (gauge/gravity correspondence)
Methods: holography & hydrodynamics

Assume we have a hard problem that is difficult to solve in a given theory, for example QCD.

(1) gravity dual to QCD or standard model?
(2) not known yet

Solve problems in effective field theory (EFT), e.g.:
- hydrodynamic approximation of original theory
- hydrodynamic approximation of model theory
Methods: holography & hydrodynamics

Assume we have a hard problem that is difficult to solve in a given theory, for example QCD

- gravity dual to QCD or standard model?
- not known yet

Holography is good at predictions that are \textbf{qualitative} or \textbf{universal}.

\textbf{Compare} holographic result to hydrodynamics of model theory.

\textbf{Compare} hydrodynamics of original theory to hydrodynamics of model.

\textbf{Understand} holography as an effective description.
Methods: holography & hydrodynamics

Assume we have a hard problem that is difficult to solve in a given theory, for example QCD.

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- Compare hydrodynamics of original theory to hydrodynamics of model.
- Understand holography as an effective description.
Chiral transport effects in charged anisotropic plasma within a magnetic field at strong coupling — Model

Theoretical plasma resulting from a collision.
Chiral transport effects in charged anisotropic plasma within a magnetic field at strong coupling — Model

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Chiral anomaly - a classically conserved current is not conserved after quantization

\[ \partial_{\mu} J_{A}^{\mu} = C \epsilon^{\mu \nu \rho \sigma} F_{\mu \nu} F_{\rho \sigma} \]

simplification: no vector current

Theoretical plasma resulting from a collision.
EFT calculation I: strong B thermodynamics

For any theory with chiral anomaly
\[ \partial_{\mu} J_{A}^{\mu} = C \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \]

Axial current with strong external B field:
\[ \langle J_{EFT}^{\mu} \rangle = n_{0} u^{\mu} + \xi_{B} B^{\mu} + \mathcal{O}(\partial) \]

Energy momentum tensor with strong external B field:
\[ \langle T_{EFT}^{\mu\nu} \rangle = \varepsilon_{0} u^{\mu} u^{\nu} + P_{0} \Delta^{\mu\nu} + q^{\mu} u^{\nu} + q^{\nu} u^{\mu} \]
\[ + M^{\mu\alpha} g_{\alpha\beta} F_{\beta\nu} + u^{\mu} u^{\alpha} (M_{\alpha\beta} F^{\beta\nu} - F_{\alpha\beta} M^{\beta\nu}) + \mathcal{O}(\partial) \]

\[ q^{\mu} = \xi_{V} B^{\mu}, \quad M^{\mu\nu} = \chi_{BB} \varepsilon^{\mu\nu\alpha\beta} B_{\alpha} u_{\beta} \]

based on previous work: [Kovtun; JHEP (2016)]
[Jensen, Loganayagam, Yarom; JHEP (2014)]
[Israel; Gen.Rel.Grav. (1978)]
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Energy momentum tensor with strong external \( B \) field:
\[ \langle T_{EFT}^{\mu\nu} \rangle = \epsilon_0 u^\mu u^\nu + P_0 \Delta^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu \\
+ M^{\mu\alpha} g_{\alpha\beta} F_{\beta\nu} + u^\mu u^\alpha (M_{\alpha\beta} F_{\beta\nu} - F_{\alpha\beta} M_{\beta\nu}) + \mathcal{O}(\partial) \]

in thermodynamic frame:
\[ q^\mu = \xi_V B^\mu, \quad M^{\mu\nu} = \chi_{BB} \epsilon^{\mu\nu\alpha\beta} B_{\alpha} u_\beta \]
\[ \xi_V = -3C \mu^2 + \tilde{C} T^2, \quad \xi_B = -6C \mu \]

based on previous work: [Kovtun; JHEP (2016)]
[Jensen, Loganayagam, Yarom; JHEP (2014)]
[Israel; Gen.Rel.Grav. (1978)]
EFT result I: strong B thermodynamics

[Ammon, Kaminski et al. (2017)]
[Ammon, Leiber, Macedo JHEP (2016)]

Strong B thermodynamics with anomaly in **thermodynamic frame:**

Energy momentum tensor:

\[
\langle T^{\mu \nu}_{\text{EFT}} \rangle = \begin{pmatrix}
\epsilon_0 & 0 & 0 & \xi^{(0)}_V B \\
0 & P_0 - \chi_{BB} B^2 & 0 & 0 \\
0 & 0 & P_0 - \chi_{BB} B^2 & 0 \\
\xi^{(0)}_V B & 0 & 0 & P_0
\end{pmatrix} + \mathcal{O}(\partial)
\]

Axial current:

\[
\langle J^\mu_{\text{EFT}} \rangle = \begin{pmatrix}
n_0, 0, 0, \xi^{(0)}_B B
\end{pmatrix} + \mathcal{O}(\partial)
\]

based on previous work:

[Kovtun; JHEP (2016)]
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Strong B thermodynamics with anomaly in **thermodynamic frame**:

Energy momentum tensor:

\[
\langle T_{\text{EFT}}^{\mu \nu} \rangle = \begin{pmatrix}
\epsilon_0 & 0 & 0 & \xi_0 B \\
0 & P_0 - \chi BB^2 & 0 & 0 \\
0 & 0 & P_0 - \chi BB^2 & 0 \\
\xi V & 0 & 0 & P_0
\end{pmatrix} + O(\partial)
\]

Axial current:

\[
\langle J_{\text{EFT}}^{\mu} \rangle = \left( n_0, 0, 0, \xi B \right) + O(\partial)
\]

**“vacuum” heat current**

**“magnetic pressure shift”**

**“vacuum” charge current**

**⇒ new contributions to thermodynamic equilibrium observables**

Based on previous work:

[Kovtun; JHEP (2016)]
[Jensen, Loganayagam, Yarom; JHEP (2014)]
[Israel; Gen.Rel.Grav. (1978)]
EFT calc. II: chiral hydrodynamics with magnetic field

For any theory with chiral anomaly
\[ \partial_\mu J_A^{\mu} = C \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \]

Axial current with weak external $B$ field:
\[ \langle J_A^{\mu} \rangle = n u^{\mu} + \sigma E^{\mu} - \sigma T \Delta^{\mu\nu} \nabla_\nu \left( \frac{\mu}{T} \right) + \xi_B B^{\mu} + \xi_V \Omega^{\mu} + \ldots \]

Energy momentum tensor with weak external $B$ field:
\[ \langle T^{\mu\nu} \rangle = \epsilon u^{\mu} u^{\nu} + P \Delta^{\mu\nu} + u^{\mu} q^{\nu} + u^{\nu} q^{\mu} + \tau^{\mu\nu} \]
Axial current with weak external $B$ field:

\[
\langle J_A^\mu \rangle = n u^\mu + \sigma E^\mu - \sigma T \Delta^{\mu\nu} \nabla_\nu \left( \frac{\mu}{T} \right) + \xi_B B^\mu + \xi_V \Omega^\mu + \ldots
\]

charge diffusion
(chiral magnetic conductivity term)
(chiral vortical conductivity term)

Energy momentum tensor with weak external $B$ field:

\[
\langle T^{\mu\nu} \rangle = \epsilon u^\mu u^\nu + P \Delta^{\mu\nu} + u^\mu q^\nu + u^\nu q^\mu + \tau^{\mu\nu}
\]

ideal fluid

\[
q^\mu = \xi_V B^\mu + \xi_3 \omega^\mu
\]

\[
\xi_V = -3C\mu^2 + \tilde{C}T^2, \quad \xi_B = -6C\mu, \quad \xi_3 = -2C\mu^3 + 2\tilde{C}\mu T^2
\]
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\[ \partial_\mu J_A^\mu = C \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \]

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Energy momentum tensor with weak external $B$ field:
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measured in Weyl semi metals
- e.g. [Huang et al; PRX (2015)]
- [Kaminski et al.; (2014)]

neutron stars?
- [Landau, Lifshitz]
- [Kadanoff; Martin]

Now calculate hydrodynamic 1- and 2-point functions and determine their poles!
EFT result II: weak B hydrodynamics

Weak B hydrodynamics - poles of 2-point functions:

spin 1 modes under SO(2) rotations around $B$

$$\omega_\pm = \mp \frac{Bn_0}{\epsilon_0 + P_0} - ik^2 \frac{\eta}{\epsilon_0 + P_0} + k \frac{Bn_0 \xi_3}{(\epsilon_0 + P_0)^2} - \frac{iB^2 \sigma}{\epsilon_0 + P_0}$$

former momentum diffusion modes

spin 0 modes under SO(2) rotations around $B$

$$\omega_0 = v_0 k - iD_0 k^2 + \mathcal{O}(\partial^3)$$

former charge diffusion mode

$$\omega_+ = v_+ k - i\Gamma_+ k^2 + \mathcal{O}(\partial^3)$$

former sound

$$\omega_- = v_- k - i\Gamma_- k^2 + \mathcal{O}(\partial^3)$$

modes

[Ammon, Kaminski et al. (2017)]
[Abbasi et al.; PLB (2016)]
[Kalaydzhyan, Murchikova (2016)]
EFT result II: weak B hydrodynamics

Weak B hydrodynamics - poles of 2-point functions:

**spin 1 modes under SO(2) rotations around \( B \)**

\[
\omega_{\pm} = \mp \frac{B n_0}{\epsilon_0 + P_0} - i k^2 \frac{\eta}{\epsilon_0 + P_0} + k \frac{B n_0 \xi_3}{(\epsilon_0 + P_0)^2} - \frac{i B^2 \sigma}{\epsilon_0 + P_0}
\]

former momentum diffusion modes

**spin 0 modes under SO(2) rotations around \( B \)**

\[
\omega_0 = v_0 k - i D_0 k^2 + \mathcal{O}(\partial^3) \text{former charge diffusion mode}
\]

\[
\omega_+ = v_+ k - i \Gamma_+ k^2 + \mathcal{O}(\partial^3) \text{former sound modes}
\]

\[
\omega_- = v_- k - i \Gamma_- k^2 + \mathcal{O}(\partial^3)
\]

**\( a \) chiral magnetic wave**

\[
v_0 = \frac{2B T_0}{\tilde{c} p n_0} \left( \tilde{C} - 3C s_0^2 \right)
\]

\[
D_0 = \frac{w_0^2 \sigma}{\tilde{c} p n_0^3 T_0}
\]

\[\Gamma_{\pm} = \frac{3\zeta + 4\eta}{6w_0} + c_s^2 \frac{w_0 \sigma}{2n_0^2} \left( 1 - \frac{\alpha p w_0}{\tilde{c} p n_0} \right)^2
\]

\[v_{\pm} = \pm c_s - B \frac{c_s^2}{n_0} \left( 1 - \frac{\alpha p w_0}{\tilde{c} p n_0} \right) \left[ 3CT_0 s_0 + \frac{\alpha p T_0^2}{\tilde{c} p} (\tilde{C} - 3C s_0^2) + \frac{1}{2} \xi_B^{(0)} - \frac{n_0}{w_0} \xi_V^{(0)} \right]
\]

\[+ B \frac{1 - c_s^2}{w_0} \xi_V^{(0)},
\]

\[\rightarrow \text{dispersion relations of hydrodynamic modes are heavily modified by anomaly and } B\]
How to choose a holographic model?

The same way, we chose the hydrodynamic model:

• match symmetries

• include interesting operators

depends on the physical question

dual to $N=4$ Super-Yang-Mills theory coupled to U(1)
How to choose a holographic model?

The same way, we chose the hydrodynamic model:

- match symmetries
- include interesting operators

depends on the physical question

Einstein-Maxwell-Chern-Simons has field theory dual with:

- chiral anomaly, breaking a U(1) axial symmetry
- axial current and energy momentum tensor

\[ S_{grav} = \frac{1}{2\kappa^2} \left[ \int_{\mathcal{M}} d^5x \sqrt{-g} \left( R + \frac{12}{L^2} - \frac{1}{4} F_{mn}F^{mn} \right) - \frac{\gamma}{6} \int_{\mathcal{M}} A \wedge F \wedge F \right] \]

dual to \( N=4 \) Super-Yang-Mills theory coupled to U(1)
Holographic result: thermodynamics

[Ammon, Kaminski et al. (2017)]

Magnetic black branes [D’Hoker, Kraus; JHEP (2009)]

- magnetic analog of RN black brane
- Asymptotically AdS5
- AdS5 near horizon

Breaks rotational invariance from SO(3) in 1-, 2-, 3-directions to SO(2) in 1- and 2-direction

$\langle T^{\mu \nu} \rangle$

Thermodynamics

$\langle J^{\mu} \rangle = (\rho, 0, 0, p_1)$

with near boundary expansion coefficients $u_4, w_4, c_4, p_1$

⇒ agreement with strong B thermodynamics from EFT
Holographic intuition: quasinormal modes (QNMs) are gravitational waves around black holes

Gravitational waves are similar to waves on a pond:

waves on spacetime: solutions to linearized Einstein equation

waves on water: solutions to wave equation

e.g. [Janiszewski, Kaminski; PRD (2015)]
Holographic calculation: QNMs

• start with **gravitational background** (metric, matter content)

• choose one or more **fields to fluctuate**
  (obeying linearized Einstein equations; Fourier transformed $\phi(t) \propto e^{-i\omega t}\phi(\omega)$)

• impose **boundary conditions** that are
  (i) **in-falling** at horizon:

  (ii) **vanishing** at AdS-boundary:
Holographic calculation: QNMs

- start with **gravitational background** (metric, matter content)

  *Example*: (charged) Reissner-Nordstrom black brane in 5-dim AdS

  [Janiszewski, Kaminski; PRD (2015)]

- choose one or more **fields to fluctuate**
  (obeying linearized Einstein equations; Fourier transformed \( \phi(t) \propto e^{-i\omega t} \phi(\omega) \))

  *Example*: metric tensor fluctuation \( \phi := h_{x}^{y} \)

- impose **boundary conditions** that are
  (i) **in-falling** at horizon:
  (ii) **vanishing** at AdS-boundary:
Holographic intuition: QNM frequencies

QNMs of $\phi := h_x^y$ are poles of $\langle T_{xy} T_{xy} \rangle$

Fourier transformation of gravity field:

$$h_{xy}(t) \propto e^{-i\omega t} h_{xy}(\omega)$$

$$e^{-i\omega t} = e^{-i(\text{Re}\omega) t} e^{i(\text{Im}\omega) t}$$

hydrodynamics valid

holoography: poles correspond to quasi-normal mode (QNM) frequencies

[Starinets; JHEP (2002)]
Holographic result: hydrodynamics

Fluctuations around charged magnetic black branes [Ammon, Kaminski et al. (2017)]

- Weak $B$: **holographic results are in full agreement with hydrodynamics.**
- Strong $B$: holographic in agreement with thermodynamics, and numerical result indicates that **chiral waves propagate at ...** the speed of light and without attenuation

confirming conjectures and results in probe brane approach [Kharzeev, Yee; PRD (2011)]

⇒ **hydrodynamic modes have Landau level scaling at large $B$**
Reality check example: comparison to lattice data

Trace anomaly in lattice QCD with $B$

\[ \langle T_{\mu}^{\mu} \rangle_{\text{lattice}} \sim -\frac{1}{2} B^2 + \Delta I \]

\[ \langle T_{\mu}^{\mu} \rangle_{\text{magnetic brane}} \sim -\frac{1}{2} B^2 \]

[Bali, Bruckmann, Endrodi, Katz, Schafer; JHEP (2014)]

see also updated data [Endrodi; JHEP (2015)]

\[ \Rightarrow \text{restrict to} \]

(1) parameter range where discrepancy is negligible
(2) observables which are unaffected by discrepancy
or discard model

[work in progress]
All this was in/near equilibrium
BUT heavy ion collisions are a time-dependent problem

➡ Perform these holographic calculations in time-dependent metric backgrounds: “holographic thermalization”

Investigate:
• evolution of electromagnetic fields
• transport far from equilibrium
• initial excentricities versus flow harmonics
• dynamical evolution of “the ridge”

[Chesler, Yaffe; PRL (2011)]
[Janik; PRD (2006)]
[Fuini, Yaffe; (JHEP) 2015]

[Kaminski; work in progress]
Summary

- holography in parallel with hydrodynamics (effective field theory) is a successful program

- transport properties of plasma change qualitatively with $B$, charge, and anomaly coefficient

- strong $B$ results (fully backreacted) at any $\mu, T, \omega, k$

**Outlook:**
- construct holographic & effective description far from equilibrium (excentricities/flow, transport, ridge, ...)
- compare to QCD (e.g. lattice) and experiments

➞ “love triangle”: EFT + QCD + holography
(Happy Valentine’s Day!)
Collaborators

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**APPENDIX: chiral effects in vector/axial currents**

see e.g. [Jensen, Kovtun, Ritz; JHEP (2013)]

Vector current (e.g. QCD $U(1)$)

\[
J^\mu_V = \cdots + \xi_V \omega^\mu + \xi_{VV} B^\mu + \xi_{VA} B_A^\mu
\]

- chiral magnetic effect

Axial current (e.g. QCD axial $U(1)$)

\[
J^\mu_A = \cdots + \xi \omega^\mu + \xi_B B^\mu + \xi_{AA} B_A^\mu
\]

- chiral vortical separation effect
APPENDIX: chiral effects in vector/axial currents

see e.g. [Jensen, Kovtun, Ritz; JHEP (2013)]

Vector current (e.g. QCD $U(1)$)

\[ J_\mu^V = \ldots + \xi_V \omega^\mu + \xi_{VV} B_\mu + \xi_{VA} B_\mu^A \]

- chiral magnetic effect

Axial current (e.g. QCD axial $U(1)$)

\[ J_\mu^A = \ldots + \xi_\omega^\mu + \xi_B B_\mu + \xi_{AA} B_\mu^A \]

- chiral vortical effect
- chiral separation effect
Holography far-from equilibrium

examples:
quench, heavy ion collision

thermalization

time

$T=0$ many body system

thermal QFT

plasma at $T>0$

correspondence

horizon formation

$T=0$ QFT

radial AdS coordinate

black hole

boundary of Anti de Sitter space

thermal QFT

examples:
quench, heavy ion collision
Result: tensor QNMs of RN black brane

Equilibrium solution

Reissner-Nordstrom (charged) black branes in 5-dim AdS

Result: tensor QNMs of RN black brane

\[ \text{Re } \Omega = 0 \]

\[ \text{Im } \Omega \]

\[ k=0 \]

\[ \phi := h_x^y \]

SO(3) rotational invariance in \( x, y, z \) (\( xx-zz \) is a spin 2 tensor)

Less stable resonances at larger charges. Equilibration happens faster.

Agreement with far from equilibrium setup at late times, deviation <1%
Result: Imaginary QNMs

[Janiszewski, Kaminski; PRD (2015)]

\[ \phi := h_{x}^{y} \]

two sets of QNMs

imaginary QNMs dominate late-time behavior at large charge densities

increasing charge
**Result: tensor QNMs of magnetic black brane**

[D’Hoker, Kraus; JHEP (2009)]

**Equilibrium solution**

**Magnetic black branes**

- magnetic analog of RN black brane
- Asymptotically AdS5
- AdS5 near horizon

Final state for fluids in magnetic field.

**Quasinormal modes**

\[ \phi := h_{xy} \]

Increasing magnetic field

**Re \( \hat{\omega} \)**

\[ 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \]
Result: scalar QNMs of magnetic black brane

**Equilibrium solution**
[D’Hoker, Kraus; JHEP (2009)]

**Quasinormal modes**
[Janiszewski, Kaminski; PRD(2015)]

Magnetic black branes
- magnetic analog of RN black brane
- Asymptotically AdS5
- AdS5 near horizon

Final state for fluids in magnetic field.

Agreement with far from equilibrium setup at late times: ~10%

\[ \phi = h_x \tilde{z} \]

increasing magnetic field
Holographic calculation: QNMs

- start with **gravitational background** (metric, matter content)

- choose one or more **fields to fluctuate**
  (obeying linearized Einstein equations; Fourier transformed $\phi(t) \propto e^{-i\omega t} \phi(\omega)$)

- impose **boundary conditions** that are
  in-falling at horizon:

  and

  vanishing at AdS-boundary: $\lim_{u \to u_{bdy}} \phi(u) = 0$
Holographic calculation: QNMs

• start with **gravitational background** (metric, matter content)

*Example:* (charged) Reissner-Nordstrom black brane in 5-dim AdS

\[
\begin{align*}
\text{ds}^2 &= \frac{r^2}{L^2} \left( -f dt^2 + dx^2 \right) + \frac{L^2}{r^2} dr^2 \\
A_t &= \mu - \frac{Q}{Lr^2}
\end{align*}
\]

[Janiszewski, Kaminski; PRD (2015)]

• choose one or more **fields to fluctuate**

(obeying linearized Einstein equations; Fourier transformed \( \phi(t) \propto e^{-i\omega t}\phi(\omega) \))

*Example:* metric tensor fluctuation

\[
\phi := h_{xy} \\
0 = \phi'' - \frac{f(u) - uf'(u)}{uf(u)} \phi' + \frac{\omega^2 - f(u)k^2}{4r_H^2uf(u)^2} \phi \\
u = \left( \frac{r_H}{r} \right)^2
\]

• impose **boundary conditions** that are

  _in-falling_ at horizon:

\[
\phi = (1 - u) \pm \frac{i\tilde{\omega}}{2(2-q^2)} \left[ \phi^{(0)} + \phi^{(1)}(1 - u) + \phi^{(2)}(1 - u)^2 + \ldots \right]
\]

and

  _vanishing_ at AdS-boundary:

\[
\lim_{u \rightarrow u_{bdy}} \phi(u) = 0
\]