Quarkonium at finite temperature: A QCD sum rule approach

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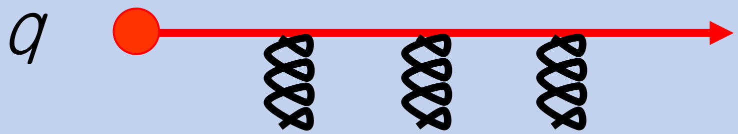
1. Heavy quark system and sum rule approach
2. Heavy quark system at finite temperature
3. Dimension 6 contribution
4. Summary

Acknowledgement:
K. Morita, T. Song, H. Kim, Che-Ming Ko + … (2008) +

I: Heavy Quark System and sum rule approach

Quark propagation



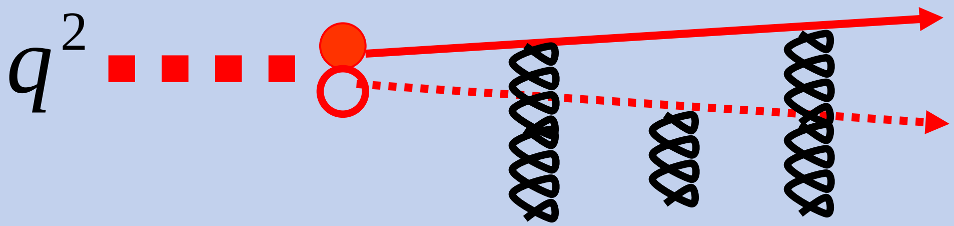
$$S_A(q) = \frac{1}{\not q - m} + \frac{1}{\not q - m} g \not A \frac{1}{\not q - m} + \ldots\ldots\ldots$$

Perturbative expansion is possible when

$$\not q - m \gg g\Lambda$$

1) When momentum $q >> g\ \Lambda$
2) For q=0, the mass is large

System with two heavy quarks



$$\Pi(q) = \int dx\, e^{iqx} \langle \bar{c}\gamma c(x), \bar{c}\gamma c(0) \rangle$$

$$= \ldots + \int_0^1 dz \frac{F(q^2, z)}{\left(4m^2 - q^2 + (2z-1)^2 q^2\right)^n} \cdot \langle G^n \rangle + \ldots$$

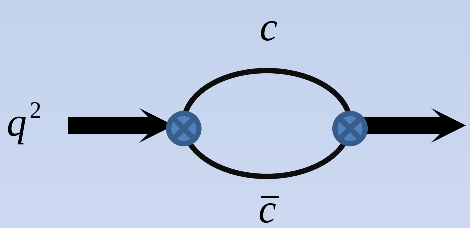
Perturbative expansion is possible when $4m^2 - q^2 \gg \Lambda_{QCD}^2$

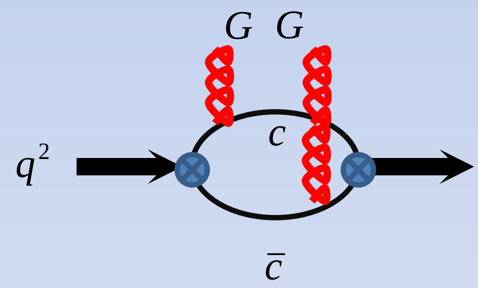
- $q^2=0$ → photo production of open charm
- $q^2=m^2_{J/\psi}$ → bound state problem (pNRQCD)
- $-q^2>0$ → QCD sum rule; OPE in this work

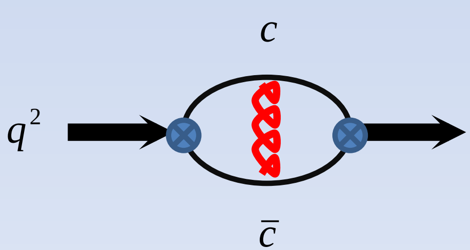
$$\Pi_{\mu\nu}(q) = i\int d^4x e^{ikx} \left\langle \bar{c}(x)\gamma_\mu c(x), \bar{c}(0)\gamma_\nu q(0) \right\rangle$$

i) OPE of the vector correlation function: $4m^2 + Q^2 \gg \Lambda^2_{QCD}$

$$\Pi(q) = F\left(Q^2, m^2, g\right) \quad + \quad \frac{C_G}{\left(4m^2+Q^2\right)^2} \cdot < \frac{\alpha}{\pi} G^2 >$$





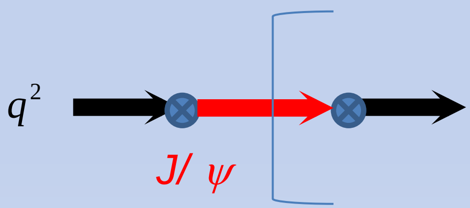


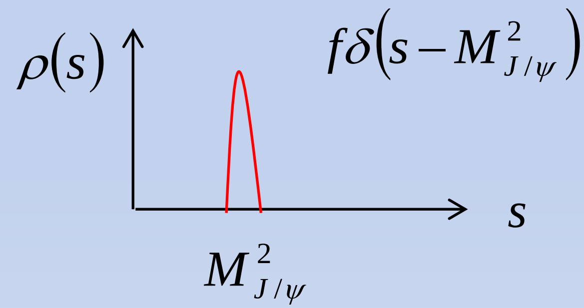
ii) Imaginary part of the vector correlator

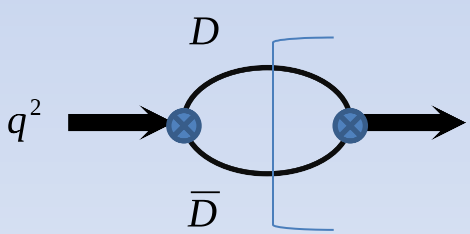
$$\left|\langle \bar{c}\gamma_\mu c \mid J/\psi\rangle\right|^2 \propto \left|\Psi(0)\right|^2$$

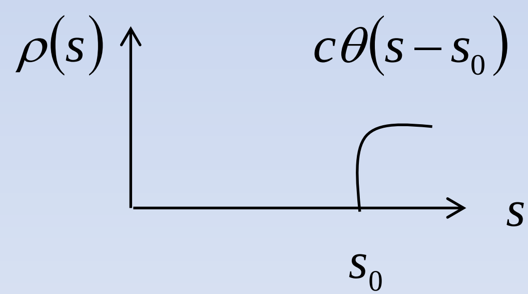
$$\Pi_{\mu\nu}(q) = i\int d^4x e^{ikx}\langle \bar{c}(x)\gamma_\mu c(x), \bar{c}(0)\gamma_\nu q(0)\rangle \quad = \quad -\frac{f}{q^2 - M_{J/\psi}^2} + \ldots\ldots$$

- *Dispersion relation* $\quad \Pi(q) = \int ds \frac{\rho(s)}{s - q^2}$





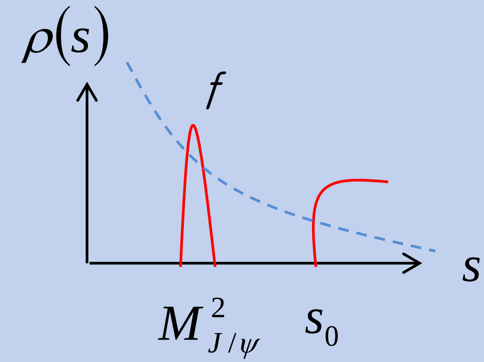




iii) Matching: QCD sum rule for J/ψ and η_c

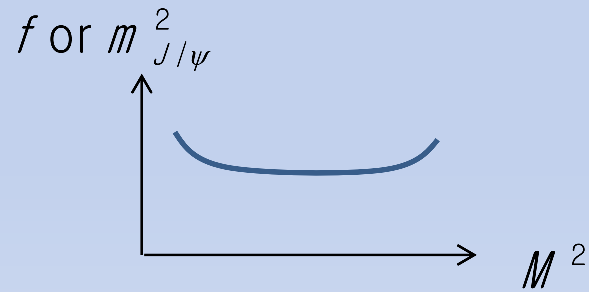
- *Borel transformed Dispersion relation*

$$B.T[\Pi(q)] = \mathrm{M}^{OPE}\left(M^2\right) = \sum_n \frac{C_n(m,M)}{n!\left(M^2\right)^n}\left\langle G^n \right\rangle = \int ds e^{-s/M^2}\rho(s)$$



$$f = e^{M^2_{J/\psi}/M^2}\left[\mathrm{M}^{OPE}\left(M^2\right) - \mathrm{M}^{cont}\left(M^2; s_0\right)\right]$$

$$m^2_{J/\psi} = -\frac{\partial / \partial\left(1/M^2\right)\left[\mathrm{M}^{OPE}\left(M^2\right) - \mathrm{M}^{cont}\left(M^2; s_0\right)\right]}{\left[\mathrm{M}^{OPE}\left(M^2\right) - \mathrm{M}^{cont}\left(M^2; s_0\right)\right]}$$

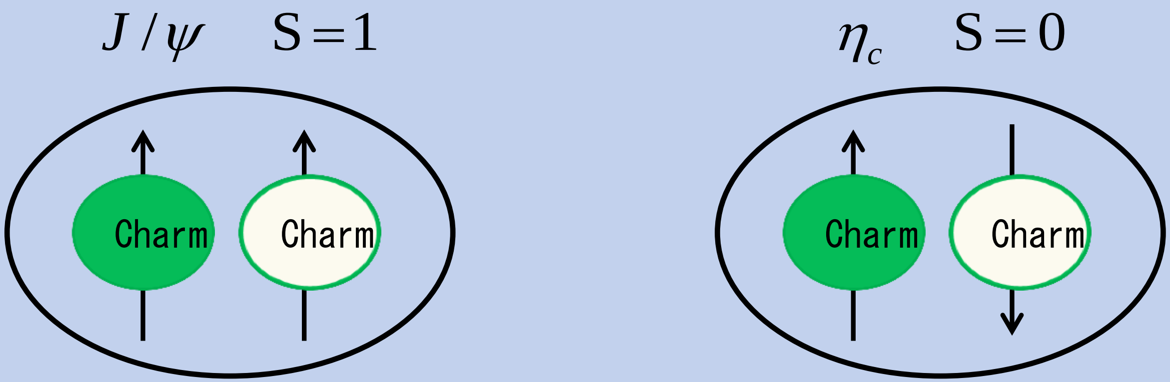


- *Works well for* $f \propto \left|\Psi(0)\right|^2 \propto \Gamma^{e^+e^-}_{J/\psi}$ *then* $m^2_{J/\psi}$

- *Predicted* $\Delta = m_{J/\psi} - m_{\eta_c} \approx 100$ *MeV before experiment* → *non trivial result*

Mass difference between J/ψ and η_c

- *Naïve constituent quark model*



$$V_{ij}^{SS} = \frac{\kappa}{m_i m_j}\sigma_i \cdot \sigma_j = \frac{\kappa}{m_i m_j}\frac{1}{2}\left[\left(\sigma_i + \sigma_j\right)^2 - \sigma_i^2 - \sigma_j^2\right]$$

- *Extrapolation from light meson system does not work*

$$\Delta = M_\rho - M_\pi \approx 640 \ \ \text{MeV} \qquad \Delta = M_{J/\psi} - M_{\eta_c} \approx 100$$

$$\Delta = M_{J/\psi} - M_{\eta_c} \approx \left(M_\rho - M_\pi\right)\text{MeV} \times \frac{m_u m_{\bar{u}}}{m_c m_{\bar{c}}} = 25 \ \ \text{MeV} << \text{experiment} = 100\,\text{MeV}$$

Mass difference between J/ψ and η_c

- *Naïve constituent quark model*

Mass diff	$M_\rho - M_\pi$	M_{K^*}-M_K	M_{D^*}-M_D	M_{B^*}-M_B
Formula	635 MeV	381 MeV	127 MeV	41 MeV
Experiment	635 MeV	397 MeV	137 MeV	46 MeV

Mass diff	$M_\Delta - M_N$	M_Σ-M_Λ	$M_{\Sigma c}$-$M_{\Lambda c}$	$M_{\Sigma b}$-$M_{\Lambda b}$
Formula	290 MeV	77 MeV	154 MeV	180 MeV
Experiment	290 MeV	75 MeV	170 MeV	192 MeV

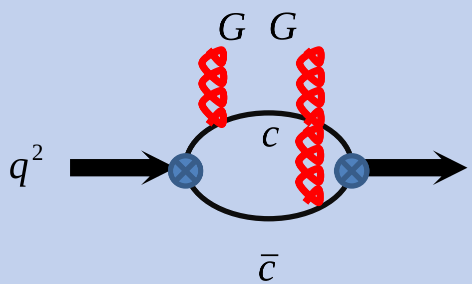
$$V_{ij}^{SS} = \frac{\kappa}{m_i m_j} \sigma_i \cdot \sigma_j = \frac{\kappa}{m_i m_j} \frac{1}{2} \left[\left(\sigma_i + \sigma_j \right)^2 - \sigma_i^2 - \sigma_j^2 \right]$$

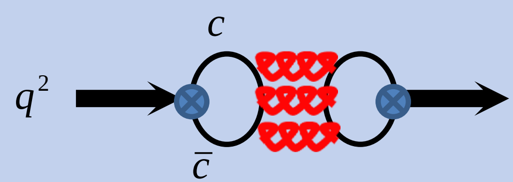
- *Extrapolation from light meson system does not work*

$$\Delta = M_\rho - M_\pi \approx 640 \quad \mathrm{MeV} \qquad \Delta = M_{J/\psi} - M_{\eta_c} \approx 100$$

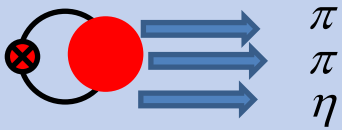
$$\Delta = M_{J/\psi} - M_{\eta_c} \approx \left(M_\rho - M_\pi \right) \mathrm{MeV} \times \frac{m_u m_{\bar{u}}}{m_c m_{\bar{c}}} = 25 \quad \mathrm{MeV} << \text{experiment} = 100 \, \mathrm{MeV}$$

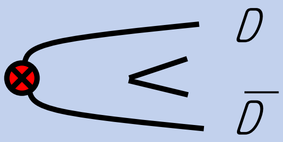
Small contribution from disconnected diagrams for J/ψ





- Look at width of Quarkonium that can not decay into D Dbar type or open charm





- Vector and Axial vector: width is small

$J^{PC}=0^{-+}$	η_c	J/ψ	χ_c 0^{++}	χ_c 1^{++}
Mass	2983 MeV	3097 MeV	3414 MeV	3510 MeV
Width	31.8 MeV	0.093 MeV	10.5 MeV	0.84 MeV

→ J/ψ sum rule should work best if only connected diagrams are included

Application of QCD sum rules for heavy quark system at finite T

- J/ ψ near Tc
  → K. Morita, SHL, PRL 100(2008),+ …

- Potential near Tc
  → SHL, K. Morita, T. Song, C.Ko, PRD89(2014)

- J/Ψ formation time near Tc
  → T. Song, C.M Ko, SHL, PRC 91 (2015)

- External E&M field
  → S. Cho, K. Hattori, SHL, K. Morita, Ozaki, PRL 113 (2014)

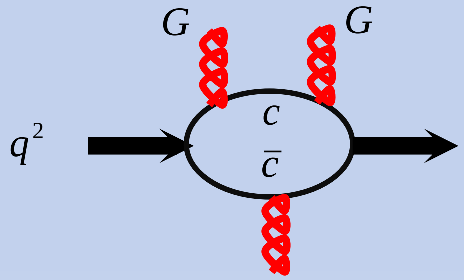
- Application to nuclear matter
  → Jparc, FAIR

II: Heavy Quark system at Finite Temperature

QCD sum rule method (Morita, SHL 08 →)

$$\Pi_{\mu\nu}(q) = i\int d^4x e^{ikx}\langle \bar{c}(x)\gamma_\mu c(x), \bar{c}(0)\gamma_\nu q(0)\rangle = -\frac{\langle \bar{c}\gamma_\mu c \mid J/\psi\rangle\langle J/\psi \mid \bar{c}\gamma_\nu c\rangle}{q^2 - M_{J/\psi}^2} + \ldots\ldots$$

OPE of the vector correlation function at low T



- when $-q^2 = Q^2 \to$ large

$$\Pi(q) = \ldots + \int_0^1 dx \frac{F(q^2, x)}{\left(4m^2 - q^2 + (2x-1)^2 q^2\right)^n} \cdot \langle G^n \rangle + \ldots \to \sum_n \frac{F_n}{\left(4m^2 + Q^2\right)^n}\left(\Lambda_{QCD} + aT\right)^n$$

Convergence ? Borel curve stability and/or $\Delta\langle \alpha G^2\rangle_T \leq \langle \alpha G^2\rangle_{T=0}$

Two independent dim 4 operators at finite Temperature

- Two independent operators

Gluon condensate $\left\langle \frac{\alpha}{\pi} G^2 \right\rangle = G_0$

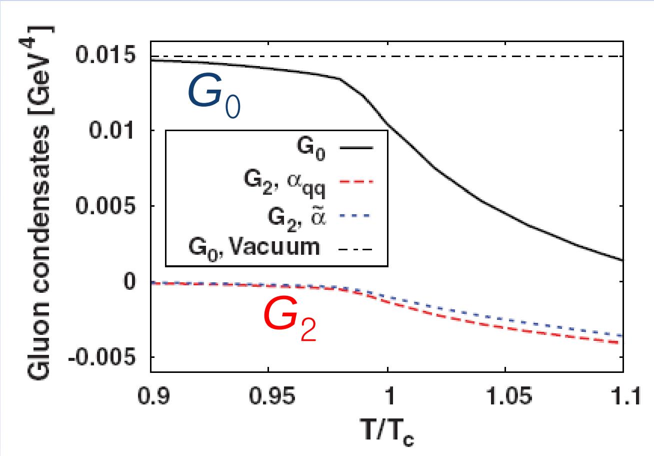
Twist-2 Gluon $\left\langle \frac{\alpha_s}{\pi} G^{\alpha\mu} G^{\beta\mu} \right\rangle = \left(u^\alpha u^\beta - \frac{1}{4} g^{\alpha\beta} \right) G_2$

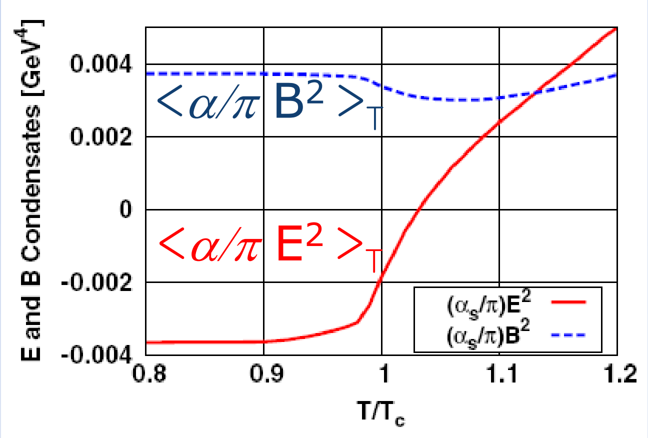
or

$\left\langle \frac{\alpha}{\pi} E^2 \right\rangle$

$\left\langle \frac{\alpha_s}{\pi} B^2 \right\rangle$

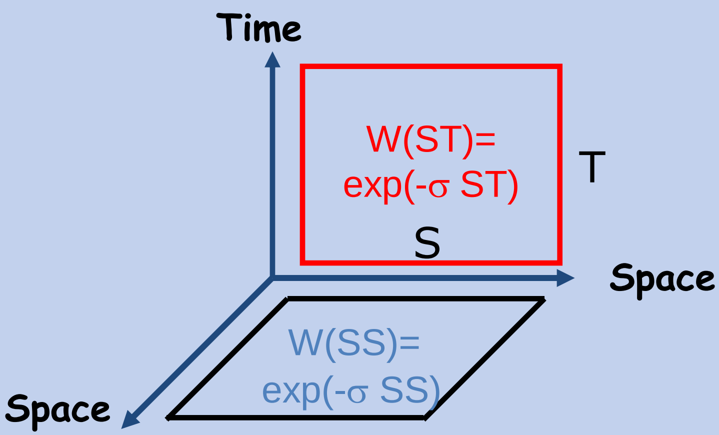
- At finite temperature: using $T_{\mu\nu}$ $\quad G_0 = -\frac{8}{9}(\varepsilon - 3p), \quad G_2 = \frac{\alpha}{\pi}(\varepsilon + p)$





$\langle E^2\rangle$, $\langle B^2\rangle$ vs confinement potential

- Local vs non local behavior



OPE for Wilson lines: Shifman NPB73 (80)

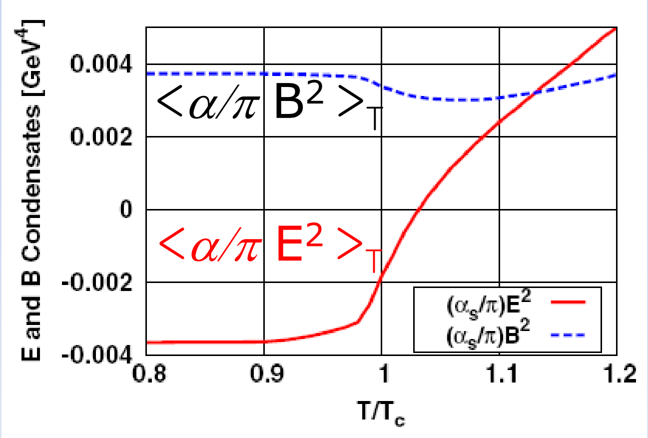
$$W(S\text{-}T) = 1 - \langle \alpha/\pi\, E^2\rangle (ST)^2 + \ldots$$

$$W(S\text{-}S) = 1 - \langle \alpha/\pi\, B^2\rangle (SS)^2 + \ldots$$

- Behavior at T>Tc

$$W(SS) = \exp(-\sigma\, SS)$$

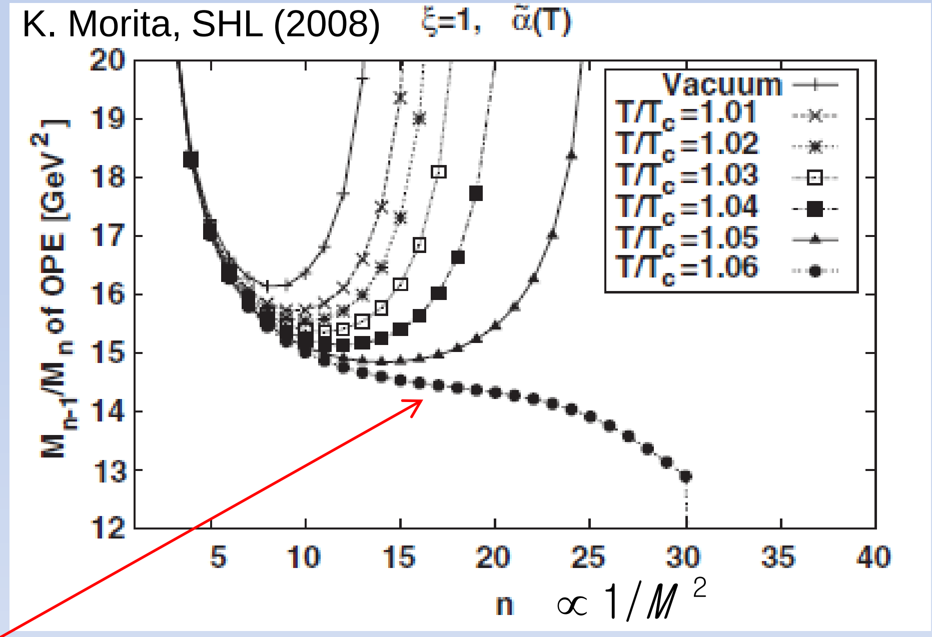
$$W(ST) = \exp(-\, g(1/S)\, S)$$



Result of QCD sum rule analysis -1

1. Mass of J/ψ and $|\Psi(0)|^2$ near Tc using moment sum rule

$$m_{J/\psi}^2 = M_{n-1}/M_n - 4m_c^2 \qquad M_n(Q^2) = \frac{1}{n!}\left(\frac{d}{dq^2}\right)^n \Pi(q^2)$$



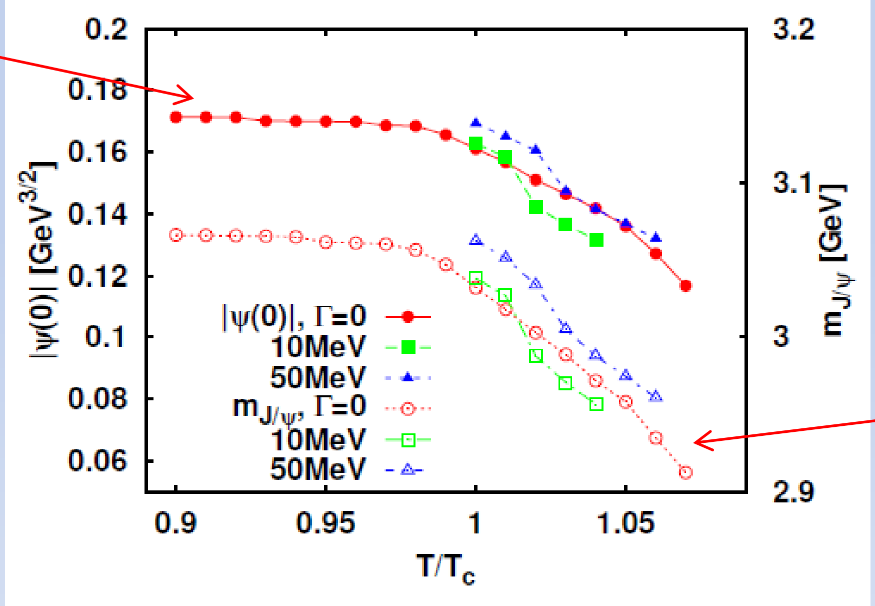
2. For T>1.06 Tc, $\Delta\left\langle \frac{\alpha}{\pi} G^2 \right\rangle > \left\langle \frac{\alpha}{\pi} G^2 \right\rangle_{T=0}$

Result of QCD sum rule analysis - II

1. Mass of J/ψ and $|\Psi(0)|^2$ near Tc

SHL, K. Morita, T. Song, C.M.Ko, PRD89 (2014)094015

$|\Psi(0)|$



Mass of J/ψ

Comparison: Solution of Schrodinger equation

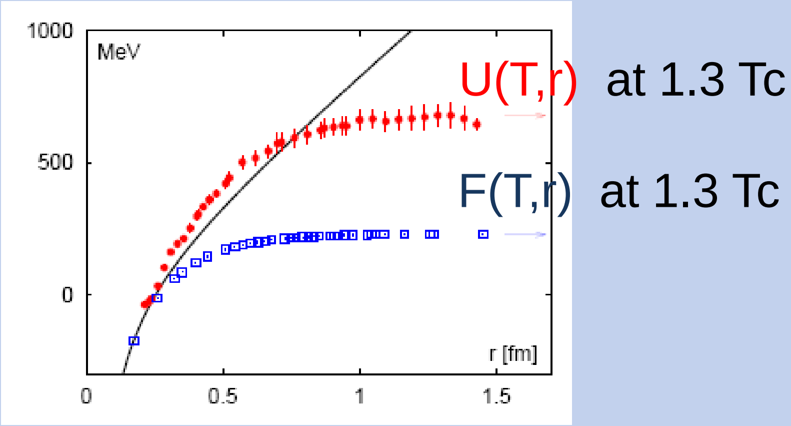
- Solve

$$\left[2m - \frac{1}{m}\nabla^2 + V(r,T)\right]\Psi(r,T) = M_{J/\psi}\Psi(r,T)$$

- By transforming (Karsch, Mehr, Satz (88))

$$\left[-\frac{1}{m}\nabla^2 + \tilde{V}(r,T)\right]\Psi(r,T) = -\varepsilon\,\Psi(r,T)$$

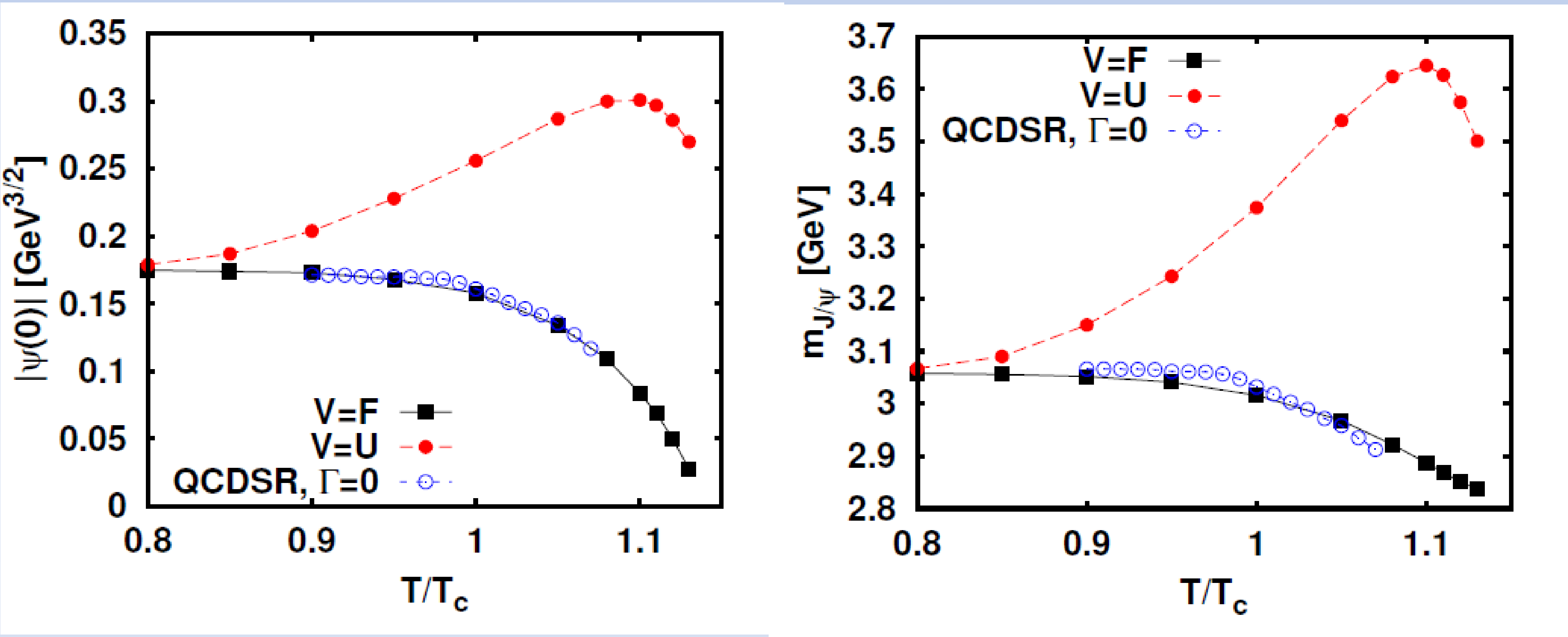
$$\tilde{V}(r,T) \equiv V(r,T) - V(r=\infty,T) \qquad \varepsilon = 2m + V(r=\infty,T) - M_{J/\psi}$$



QCD sum rule vs. Schrodinger equation with U or F

SHL, K. Morita, T. Song, C.M.Ko, PRD89 (2014)094015

$|\Psi(0)|$ $M_{J/\psi}$



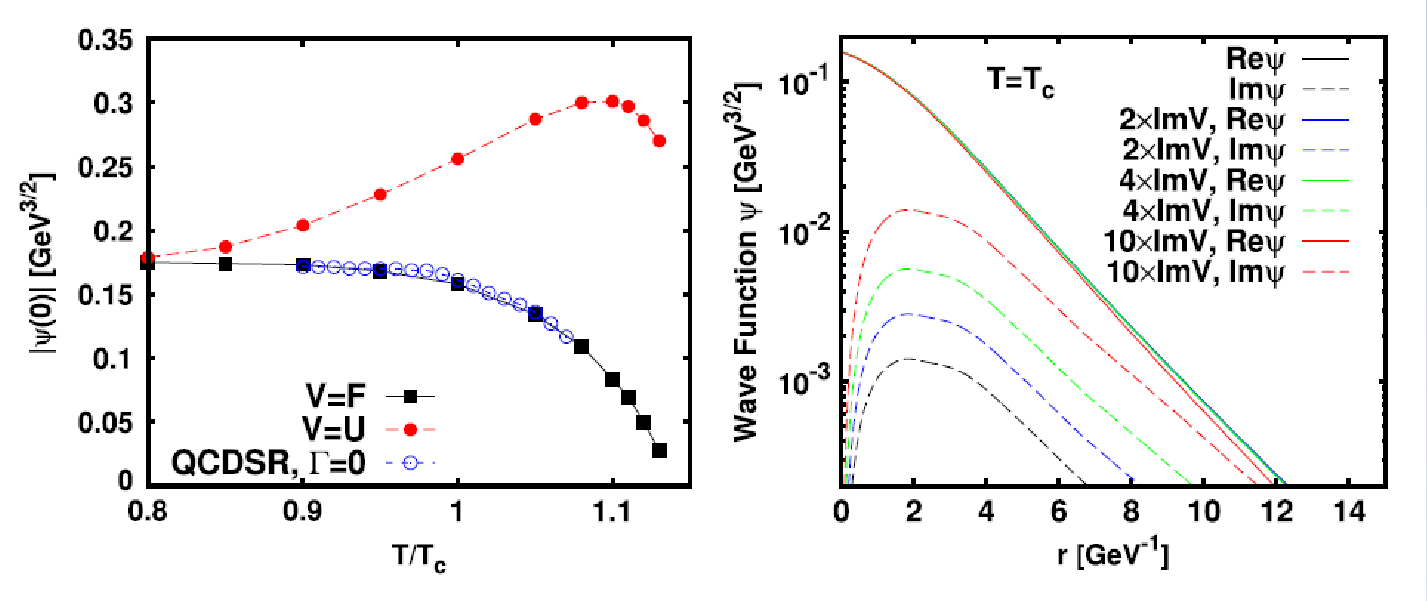
● V= U ■ V= F ○ Sum rule result

Effects of Imaginary potential

- Physical observable is thermal width : obtained separately.
  Peskin, Kharzeev, Satz, SHL (LO QCD), Song, SHL (NLO),
  Kharzeev and McLerran (non perturbative)

- Imaginary potential: resumed QCD:
  Laine, Pilipsen, Tassler, Romatschke, (2007), $\mathrm{Im}\, V^{HTL} \equiv -\frac{i g^2 T C_F}{4\pi} \phi(m_D r)$

T. Song, SHL, K. Morita, C.M.Ko, NPA 931 (2014) 607



- H. Satz, “Quarkonium binding and entropic force” arXiv:1501.03940 (EPJC)

Will an imaginary part in V change F to U ?

- Adding a constant C to V → No change in wave function

$$\left[-\frac{1}{m}\nabla^2+\tilde{V}(r,T)\right]\Psi(r,T)=-\varepsilon\,\Psi(r,T)$$

$$\left[-\frac{1}{m}\nabla^2+\tilde{V}(r,T)+C\right]\Psi(r,T)=-(\varepsilon+C)\Psi(r,T)$$

- Imaginary Coulomb potential → No change in $|\Psi(0)|$

$$\left[-\frac{1}{m}\nabla^2+V(r,T)(1+i\beta)\right]\Psi_C(r,T)=-\varepsilon\,\Psi_C(r,T)$$

$$\left[-\frac{1}{m(1+i\beta)}\nabla^2+V(r,T)\right]\Psi_C(r,T)=-\frac{\varepsilon}{(1+i\beta)}\Psi_C(r,T)$$

$$\rightarrow\quad \Psi_C(r,T)=A\exp\left(-m(1+i\beta)g^2r\right)$$

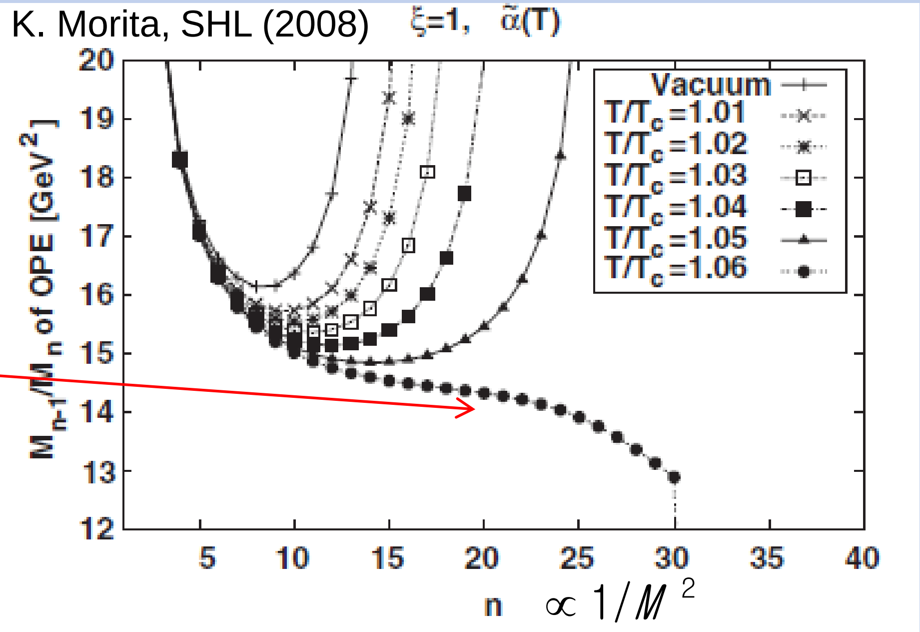
- In general mass dependence is not so simple and $|\Psi(0)|$ will change but expected to be mild

Can we go beyond?

1. For T>1.06 Tc, ?

$$\Delta\left\langle \frac{\alpha}{\pi}G^2\right\rangle > \left\langle \frac{\alpha}{\pi}G^2\right\rangle_{T=0}$$

K. Morita, SHL (2008)



2. OPE for Wilson loop?

W(S-T) = 1- $\langle \alpha/\pi\, E^2\rangle (ST)^2$ +…

W(S-S) = 1- $\langle \alpha/\pi\, B^2\rangle (SS)^2$ +…

III: Dimension 6 contribution

Dim 6 gluonic operators

- Scalar operators: 2 independent operators

$$S_1 = \langle (D_\alpha G_{\mu\nu})(D_\alpha G_{\mu\nu}) \rangle, \qquad S_2 = \langle (D_\alpha G_{\alpha\mu})(D_\beta G_{\beta\mu}) \rangle$$

note $$\langle g f^{abc} G^a_{\alpha\beta} G^b_{\alpha\mu} G^c_{\beta\mu} \rangle = \langle (D_\alpha G_{\alpha\mu})(D_\beta G_{\beta\mu}) \rangle - \frac{1}{2} \langle (D_\alpha G_{\mu\nu})(D_\alpha G_{\mu\nu}) \rangle$$

- Spin-2 operators: 3 independent operators

$$O_1 = \langle (D_\alpha G_{\mu\nu})(D_\beta G_{\mu\nu}) \rangle, \quad O_2 = \langle (D_\mu G_{\mu\alpha})(D_\nu G_{\nu\beta}) \rangle, \quad O_3 = \langle (D_\alpha G_{\beta\mu})(D_\nu G_{\mu\nu}) \rangle$$

external index α β

- In pure gauge, $D_\mu G_{\mu\alpha} = \bar{\psi}\gamma_\alpha \psi \rightarrow 0$

→ *Hence only 1 spin 0 and 1 spin 2 operators survive*

$$S_1 = \langle (D_\alpha G_{\mu\nu})(D_\alpha G_{\mu\nu}) \rangle, \qquad O_1 = \langle (D_\alpha G_{\mu\nu})(D_\beta G_{\mu\nu}) \rangle$$

Dim 6 gluonic operators : Renormalization

- Spin 2 operators (Hyungjoo Kim, SHL, PLB748 (2015)352)

$$O_1 = \langle (D_\alpha G_{\mu\nu})(D_\beta G_{\mu\nu}) \rangle, \quad O_2 = \langle (D_\mu G_{\mu\alpha})(D_\nu G_{\nu\beta}) \rangle, \quad O_3 = \langle (D_\alpha G_{\beta\mu})(D_\nu G_{\mu\nu}) \rangle$$

$$Z = \begin{pmatrix} 1 + \frac{3N\alpha_s}{4\pi\epsilon} & -\frac{N\alpha_s}{12\pi\epsilon} & \frac{2N\alpha_s}{3\pi\epsilon} \\ 0 & 1 + \frac{N\alpha_s}{3\pi\epsilon} & \frac{N\alpha_s}{24\pi\epsilon} \\ 0 & \frac{N\alpha_s}{6\pi\epsilon} & 1 + \frac{7N\alpha_s}{24\pi\epsilon} \end{pmatrix}$$

$$\langle O_{1\text{new}} \rangle = \langle O_1 \rangle$$

$$\langle O_{2\text{new}} \rangle = \left\langle \frac{-653 + 21\sqrt{17}}{424} O_1 + \frac{1 - \sqrt{17}}{8} O_2 + O_3 \right\rangle$$

$$\langle O_{3\text{new}} \rangle = \left\langle \frac{-653 - 21\sqrt{17}}{424} O_1 + \frac{1 + \sqrt{17}}{8} O_2 + O_3 \right\rangle$$

$$\phi_1 = {\alpha_s}^{-\frac{9}{11}} \langle O_{1\text{new}} \rangle$$

$$\phi_2 = {\alpha_s}^{-\frac{15-\sqrt{17}}{44}} \langle O_{2\text{new}} \rangle$$

$$\phi_3 = {\alpha_s}^{-\frac{15+\sqrt{17}}{44}} \langle O_{3\text{new}} \rangle .$$

Dim 6 gluonic operators : Renormalization

- Spin 2 operators (Hyungjoo Kim, SHL, PLB748 (2015)352)

$$O_1 = \langle (D_\alpha G_{\mu\nu})(D_\beta G_{\mu\nu}) \rangle, \quad O_2 = \langle (D_\mu G_{\mu\alpha})(D_\nu G_{\nu\beta}) \rangle, \quad O_3 = \langle (D_\alpha G_{\beta\mu})(D_\nu G_{\mu\nu}) \rangle$$

$$D_\mu G_{\mu\alpha} = \bar{\psi}\gamma_\alpha \psi$$

→ 0 in pure gauge

$$Z = \begin{pmatrix} 1+\frac{3N\alpha_s}{4\pi\epsilon} & -\frac{N\alpha_s}{12\pi\epsilon} & \frac{2N\alpha_s}{3\pi\epsilon} \\ 0 & 1+\frac{N\alpha_s}{3\pi\epsilon} & \frac{N\alpha_s}{24\pi\epsilon} \\ 0 & \frac{N\alpha_s}{6\pi\epsilon} & 1+\frac{7N\alpha_s}{24\pi\epsilon} \end{pmatrix}$$

$$\langle O_{1\text{new}} \rangle = \langle O_1 \rangle$$

$$\langle O_{2\text{new}} \rangle = \left\langle \frac{-653+21\sqrt{17}}{424} O_1 + \frac{1-\sqrt{17}}{8} O_2 + O_3 \right\rangle$$

$$\langle O_{3\text{new}} \rangle = \left\langle \frac{-653-21\sqrt{17}}{424} O_1 + \frac{1+\sqrt{17}}{8} O_2 + O_3 \right\rangle$$

$$\phi_1 = \alpha_s^{-\frac{9}{11}} \langle O_{1\text{new}} \rangle$$

$$\phi_2 = \alpha_s^{-\frac{15-\sqrt{17}}{44}} \langle O_{2\text{new}} \rangle$$

$$\phi_3 = \alpha_s^{-\frac{15+\sqrt{17}}{44}} \langle O_{3\text{new}} \rangle .$$

Hence

$$G_0 = \alpha (G_{\mu\nu})(G_{\mu\nu})$$

$$\phi_1 = \alpha_s^{-9/11} (D_\alpha G_{\mu\nu})(D_\beta G_{\mu\nu})$$

Dim 6 operators – pure gauge

- Two independent operators

$$\langle (D_\mu G_{\alpha\beta})(D_\mu G_{\alpha\beta}) \rangle = -2 \langle g f^{abc} G^a_{\mu\nu} G^b_{\mu\alpha} G^c_{\nu\alpha} \rangle$$

$$\langle (D_\mu G_{\alpha\beta})(D_\nu G_{\alpha\beta}) \rangle = -2 \langle g f^{abc} G^a_{\mu\alpha} G^b_{\nu\beta} G^c_{\alpha\beta} \rangle$$

- E, B parametrization of the two operators

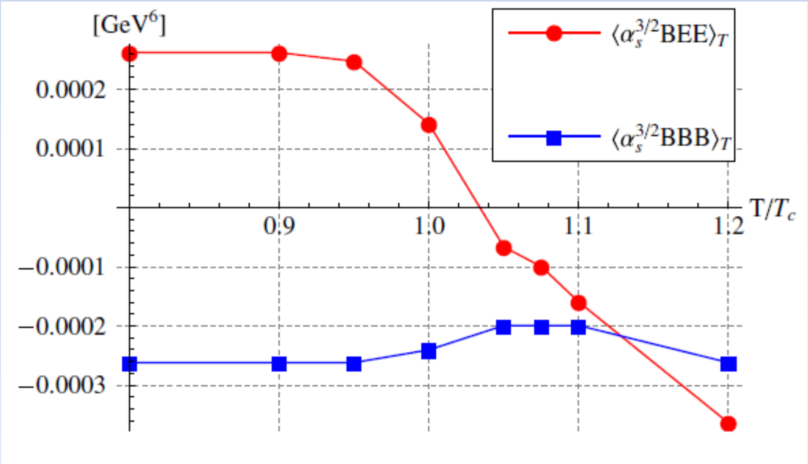
$$\langle f^{abc} (E^a) \cdot (E^b \times B^c) \rangle, \ \langle f^{abc} (B^a) \cdot (B^b \times B^c) \rangle$$

- A naïve parametrization

$$\langle EEB \rangle_T \propto \langle E^2 \rangle_T \langle B^2 \rangle_T^{1/2}$$

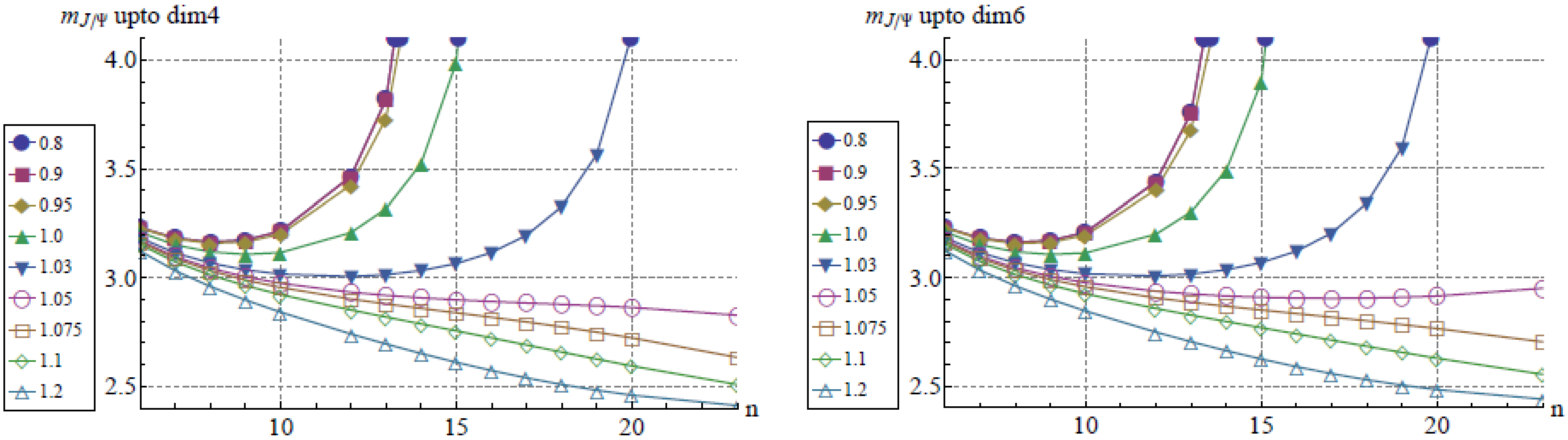
$$\langle BBB \rangle_T \propto \langle B^2 \rangle_T^{3/2}$$

H Kim, K. Morita, SHL, PRD 93(2016)



Dim 6 improves stability at 1.05 Tc

FIG. 3: Contributions to $M_n(\xi)/A_n^V(\xi)$ of dimension 6 condensates at $0.8T_c$(left figure) and at $1.05T_c$(right figure).



Dim 6 operators improves stability at 1.05Tc

But still no convergence at higher T

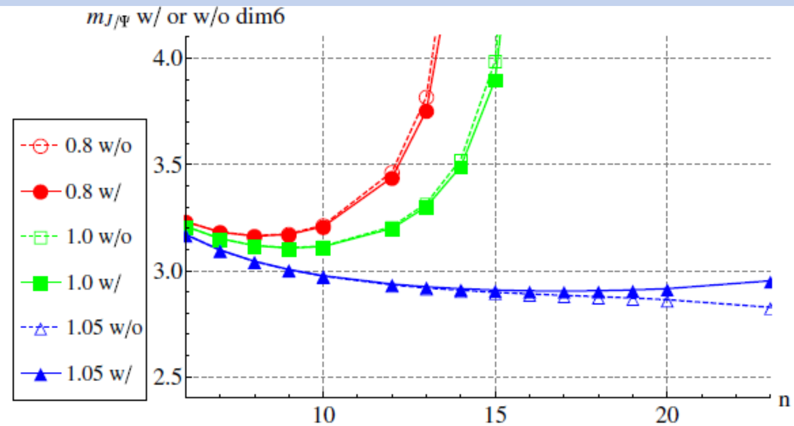


FIG. 5. $m_{J/\psi}$ at $T = 0.8,\ 1.0,\ 1.05T_c$ with or without dimension-6 condensates.

Summary

1. Can use sum rules to investigate properties of J/ψ up to 1.05Tc.
   - Potential near Tc , properties near Tc , +

2. Why does J/ψ become unstable above 1.06Tc ?
   - temperature dependence of dim 6 and dim 8 operators
   - relation to Wislon loops
   - other states

3. QCD sum rules can be applied to J/psi in Strong B field and more
   …..

- Dynamical quark will have little effect on gluon condensate (Morita, Lee 09)