Chiral magnetic effect & anomalous transport from real-time lattice simulations

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Based on:
M. Mace, N. Mueller, S. Schlichting and S. Sharma, 1612.02477

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“QCD in Finite Temperature and Heavy-Ion Collisions”
CME in Heavy-Ion Collisions

Quantitative theoretical understanding of anomaly induced transport phenomena (CME, CMW, …) in heavy-ion collisions important experimental searches for these effects

Challenges:

Since life time of magnetic field is presumably very short (~0.1-1 fm/c) system is out-of-equilibrium during the time scales relevant for CME & Co.

Need to understand non-equilibrium dynamics of axial and vector charges during the early-time pre-equilibrium phase

Existing theoretical approaches such as anomalous hydro or chiral kinetic theory effectively treat axial charge as a conserved quantity

In order to correctly describe generation of axial charge imbalance (e.g. due to sphalerons) field theoretical description is required

-> Develop field theoretical approach to describe early time dynamics and possibly devise improved macroscopic description of anomalous transport
Early-time dynamics of HIC

Early time dynamics described in terms of classical field dynamics amenable to non-perturbative real-time lattice simulations

-> Include dynamical fermions to study anomalous transport
Simulation technique

Classical-statistical lattice simulation with dynamical fermions

(Aarts, Smit; Berges, Hebenstreit, Kasper, Mueller; Tranberg, Saffin; …)

- Discretize theory on 3D spatial lattice using the Hamiltonian lattice formalism

- Solve operator Dirac equation in the presence of SU(N) and U(1) gauge fields

\[ i \gamma^0 \partial_t \hat{\psi} = ( -i \hat{D}_W^s + m ) \hat{\psi} \]

- Compute expectation values of vector and axial currents to study anomalous transport processes

\[ j^\mu_v (x) = \langle \hat{\psi}(x) \gamma^\mu \hat{\psi}(x) \rangle \quad j^\mu_a (x) = \langle \hat{\psi}(x) \gamma^\mu \gamma^5 \hat{\psi}(x) \rangle \]
Dynamical fermions

Solving the operator Dirac equation can be achieved by expanding the fermion field in operator basis at initial time

\[ \hat{\psi}(x, t) = \sum_{p, \lambda} \hat{b}_{p, \lambda}(t = 0) \phi^{p, \lambda}_u(x, t) + \hat{d}^\dagger_{p, \lambda}(t = 0) \phi^{p, \lambda}_v(x, t) \]

and solving the Dirac equation for evolution of $4N_cN^3$ wave-functions

Not clear to what extent stochastic estimators are useful to reduce problem size
Computationally extremely demanding (~TB memory, ~M CPU hours)

So far first results on small lattices 24 x 24 x 64 in a clean theoretical setup

SU(N): Single sphaleron transition
U(1): constant magnetic field
Back-reaction of fermions on gauge field evolution not considered
Axial anomaly in real-time

Definition of chiral properties (axial charge) of fermions on the lattice generally a tricky issue

Naive fermion discretization: Cancellation of axial anomaly due to Fermion doublers

\[ \partial_{\mu} j^\mu_5(x) = 2m \langle \bar{\psi}(x)i\gamma_5\psi(x) \rangle \]

Exploit knowledge from Euclidean lattice simulations

Wilson fermions: Explicit symmetry breaking term added to the Hamiltonian to decouple doublers (c.f. Aarts,Smit)

\[ \partial_{\mu} j^\mu_5(x) = 2m \langle \bar{\psi}(x)i\gamma_5\psi(x) \rangle + r_W \langle W(x) \rangle \rightarrow -\frac{g^2}{8\pi^2} \text{Tr} F_{\mu\nu} F^{\mu\nu} \]

Overlap fermions: Non-local derivative operator with chiral properties on the lattice
Axial anomaly in real-time

Non-trivial cross check of axial charge production (B=0)

Over the course of the sphaleron transition Chern-Simons number

$$\Delta N_{CS} = \frac{g^2}{8\pi^2} \int d^4x \; \vec{E}_a \vec{B}_a$$

changes by an integer amount leading to an imbalance of axial charge

$$\Delta J_5^0 = -2\Delta N_{CS} + 2m_f \int d^4x \langle \bar{\psi} i\gamma_5 \psi \rangle$$

Excellent agreement for (almost) massless fermions from simulations with improved Wilson fermions and Overlap fermions

(Mace,Mueller,SS, Sharma, 1612.02477)
CME Dynamics

Axial charge $j_5^0$

Vector current $j^z_V$

Vector charge $j_1^0$

Sphaleron transition induces local imbalance of axial charge density

Non-zero magnetic field $B_z$ leads to separation of electric charges $j_1^0$ along the z-direction

Vector current $j^z_V$ leads to separation of electric charges $j_1^0$ along the z-direction

(N.Mueller, SS, S. Sharma PRL 117 (2016) no.14, 142301)
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CMW Dynamics

Vector charge imbalance $j_0^V$ generates an axial current $j_5^\tilde{V}$ so that axial charge also flows along the B-field direction.

Axial charge $j_5^0$  Vector current $j_5^\tilde{V}$  Vector charge $j_0^V$

Emergence of a Chiral Magnetic Shock-wave of vector charge and axial charge propagating along B-field direction

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Non-equilibrium dynamics of vector and axial charges

Clear separation of electric charge $j^0_V$ along the B-field direction

First time anomalous transport phenomena have been confirmed from non-perturbative real-time simulations
Non-equilibrium dynamics of vector and axial charges

Comparison with anomalous hydro (light quarks \( m r_{sph} \ll 1 \))

\[
\partial_\mu j^\mu_a = S(x), \quad \partial_\mu j^\mu_v = 0
\]

\[
j^\mu_v/a = n_v/a u^\mu + \sigma_{v/a}^B B^\mu
\]

Strong field limit ( \( B \gg r_{sph}^{-2}, m^2 \))

\[
\partial_t \begin{pmatrix} j^0_v(t, z) \\ j^0_a(t, z) \end{pmatrix} = -\partial_z \begin{pmatrix} j^0_v(t, z) \\ j^0_a(t, z) \end{pmatrix} + \begin{pmatrix} 0 \\ S(t, z) \end{pmatrix}
\]

Chiral magnetic shock-wave

\[
j^0_v/a(t > t_{sph}, z) = \frac{1}{2} \int_{0}^{t_{sph}} dt' \left[ S(t', z - c(t - t')) + S(t', z + c(t - t')) \right]
\]

\( \Rightarrow \) Evolution for light quarks and strong magnetic fields well described by anomalous hydrodynamics at late times
Validity of constitutive relations

Verify ratios vector/axial currents and axial/vector charge

\[ C_{CME}(t) = \frac{\Delta J^z(t)}{\Delta J^0_a(t)} , \quad C_{CSE}(t) = \frac{\Delta J^z(t)}{\Delta J^0_v(t)} . \]

In the strong field limit related to thermodynamic constitutive relations

\[ C_{CME} = 1 , \quad C_{CSE} = 1 . \]

equal to time independent constants.

Simulation results indicate approach towards constant value with a finite relaxation time

Since lifetime of magnetic field in HIC is short this effect should also be incorporated in phenomenological approaches

(Mace, Mueller, SS, Sharma, 1612.02477)
Quark mass dependence

Explicit violation of axial charge conservation for finite quark mass

\[ \partial_\mu j_\mu^a(x) = 2m\langle \hat{\psi}(x)i\gamma_5\hat{\psi}(x) \rangle + S(x) \]

leads to damping of axial charge

Since chiral magnetic effect current is proportional to axial charge density it will also be reduced

\[ \vec{j}_v \propto j_0^a \vec{B} \]

(N.Mueller, SS, S. Sharma PRL 117 (2016) no.14, 142301)
Quark mass dependence

**Light quarks** \( (m_t^sph \ll 1) \)

Chiral magnetic wave leads to non-dissipative transport of axial and vector charges

**Heavy quarks** \( (m_t^sph \sim 1) \)

Dissipation of axial charge leads to significant reduction of charge separation

(Mace, Mueller, SS, Sharma, 1612.02477)
Quark mass dependence

Significant reduction of the charge separation signal by factor $\sim 5$ already for moderate quark masses

Expect backreaction (not included so far) to suppress the signal even further

Phenomenological consequences

Unlikely that strange quarks participate in CME

Desirable to include dissipative effects in macroscopic description

(Mace, Mueller, SS, Sharma, 1612.02477)
Conclusions & Outlook

Development of first-principle techniques to study dynamics of vector and axial charges out-of-equilibrium

Successful microscopic description of CME & CMW

Observed importance of finite relaxation time and dissipative effects

Should be included in macroscopic descriptions

Next step is to include back-reaction of fermions perform simulations for a realistic heavy-ion environment

- Quark production & electro-magnetic response
- Chiral magnetic effect & anomalous transport in HIC

Several technical developments in progress to reduce numerical complexity / extend lattice size
Backup
Comparison of Wilson & Overlap
Magnetic field dependence

Vector charge separation: $\Delta J_V$

Magnetic field: $qB_{sph}^2$

$\max \frac{t}{t_{sph}} = 1.5$

Asymptotic