Chiral Shock Waves

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ArXiv: 1609.07030v2

Outline

- Chiral transport phenomenon.
- What is a shock wave ?
- Shock wave in nonchiral matter.
- Shockwave in chiral matter.
- Example and implications.

Ideal and dissipative hydrodynamics

- Energy momentum conservation and particle number conservation:

$$\partial_\mu T^{\mu\nu} = 0,$$
$$\partial_\mu j^\mu = 0.$$

Where $T^{\mu\nu} = hu^\mu u^\nu - pg^{\mu\nu}$ $\quad j^\mu = nu^\mu$

$$h = \epsilon + p \qquad u^\mu = \gamma(1, \boldsymbol{v})$$

- Dissipative processes require additional terms in the conserved quantities – to be constrained by the second law of thermodynamics.

Background field and anomaly

- Consider massless fermions of single chirality.

- In the presence of a background electromagnetic field

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda \, , \quad \partial_\mu j^\mu = C E^\mu B_\mu$$

- External fields add further dissipative components to conserved currents where as anomaly adds nondissipative terms.

Chiral Transport

- Ignoring dissipation the constitutive relations become in Landau frame:

$$j^\mu = n u^\mu + \xi \omega^\mu + \xi_B B^\mu$$

chiral vortical effect chiral magnetic effect

$$\xi = \frac{C}{2}\mu^2\left(1 - \frac{2}{3}\frac{n\mu}{\epsilon + P}\right) + \frac{D}{2}T^2\left(1 - \frac{2n\mu}{\epsilon + P}\right)$$

$$\xi_B = C\mu\left(1 - \frac{1}{2}\frac{n\mu}{\epsilon + P}\right) - \frac{D}{2}\frac{nT^2}{\epsilon + P}$$

$$\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} u_\nu \partial_\lambda u_\rho \qquad B^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} u_\nu F_{\alpha\beta}$$

D. T. Son and P. Surowka, Phys. Rev. Lett. 103, 191601 (2009)

Fluid flow (sonic)

- Effects of compressibility for high velocity fluid flow important.
- Two types: subsonic and supersonic flow.
- Supersonic flow of two types:
  - Steady continuous flow.
  - Surface of discontinuity → Shock waves.

Surface of discontinuity

- Velocity, pressure, density and temperature can be discontinuous across a surface perpendicular to the flow.
- Certain boundary conditions must be satisfied at this surface.
- Quantities like mass flux and energy flux should remain continuous across the surface for non-relativistic and relativistic systems respectively.

Nonchiral relativistic Shock Waves

- Shockwave traveling along 'x' axis.
- The region behind and ahead of the shock wave front denoted by 1 and 2.
- Impose continuity in particle number flux, energy flux etc.

$$j_1^x = j_2^x \quad T_1^{xx} = T_2^{xx} \quad T_1^{0x} = T_2^{0x}$$

$$T_1^{yx} = T_2^{yx} \quad T_1^{zx} = T_2^{zx}$$

Pressure-volume relation

$$v_1 = \sqrt{\frac{(p_2 - p_1)(\epsilon_2 + p_1)}{(\epsilon_2 - \epsilon_1)(\epsilon_1 + p_2)}}$$

$$v_2 = \sqrt{\frac{(p_2 - p_1)(\epsilon_1 + p_2)}{(\epsilon_2 - \epsilon_1)(\epsilon_2 + p_1)}}$$

The adiabatic: $$\frac{v_1}{V_1\sqrt{1 - v_1^2}} - \frac{v_2}{V_2\sqrt{1 - v_2^2}} = 0$$

$$\Rightarrow h_1^2 V_1^2 - h_2^2 V_2^2 + (p_2 - p_1)(h_1 V_1^2 + h_2 V_2^2) = 0$$

Weak shock waves

$\epsilon_2 \to \epsilon_1 \quad p_2 \to p_1 \quad \Delta\epsilon \equiv \epsilon_2 - \epsilon_1 \quad \Delta p \equiv p_2 - p_1$

$$v_1^2 = (c_{s1})^2 \left(1 + \frac{\Delta\epsilon}{h_1}(1 - c_{s1}^2) + ..\right)$$

$$v_2^2 = (c_{s2})^2 \left(1 - \frac{\Delta\epsilon}{h_1}(1 - c_{s2}^2) + ..\right)$$

$$c_s^2 = \frac{dp}{d\epsilon}$$

Compression and rarefaction shock waves

$\epsilon_2 > \epsilon_1$ ➡ $(v_1)^2 > (v_2)^2$

$p_2 > p_1$ follows from: $c_s^2 = \lim_{2 \to 1} \frac{p_2 - p_1}{\epsilon_2 - \epsilon_1} > 0$

$\longrightarrow$ compression

$\epsilon_2 < \epsilon_1$ ➡ $(v_1)^2 < (v_2)^2$

$p_1 > p_2$ follows from: $c_s^2 = \lim_{2 \to 1} \frac{p_2 - p_1}{\epsilon_2 - \epsilon_1} > 0$

$\longrightarrow$ rarefaction

Entropy discontinuity

Define
$$\Delta H = H_2 - H_1 \quad \Delta S = S_2 - S_1$$
$$\Delta V = V_2 - V_1$$

Expand the pressure-volume relation using:

$$\Delta H = T\Delta S + V_1 \Delta p + \frac{1}{2}\frac{\partial V}{\partial p}\bigg|_1 (\Delta p)^2 + \frac{1}{6}\frac{\partial^2 V}{\partial p^2}\bigg|_1 (\Delta p)^3 + \cdots$$

$$\Delta V = \frac{\partial V}{\partial p}\bigg|_1 \Delta p + \frac{1}{2}\frac{\partial^2 V}{\partial p^2}\bigg|_1 (\Delta p)^2 + \frac{1}{6}\frac{\partial^3 V}{\partial p^3}\bigg|_1 (\Delta p)^3 + \frac{\partial V}{\partial S}\bigg|_1 \Delta S + \cdots$$

Entropy discontinuity and shockwaves

$$\Delta S = \frac{1}{12 H_1 T} \frac{\partial^2 (HV)}{\partial p^2}\bigg|_1 (\Delta p)^3 + O\big((\Delta p)^4\big)$$

For any realistic equation of state: $\partial^2 (HV)/\partial p^2 > 0$

Second law of thermodyncamics: $S_2 > S_1 \rightarrow$

$p_2 > p_1$ ✓

$p_1 > p_2$ ✗

Hence, only compression shock waves allowed !

Chiral shock waves

- Does the pressure entropy discontinuity depend on chiral transport ? How ?

- To answer choose a particular limit.

- Set B^μ to *0* such that, but ω^μ is finite

$$j^\mu = n u^\mu + \xi \omega^\mu$$

- Fermions with chemical potential *μ* and temperature *T* such that $T/\mu \ll 1$

Chiral shock waves

- Shock wave front traveling along *'x'*.
- Vorticity chosen to be along *'x'*,

$$\omega_x = \omega, \omega_y = \omega_z = 0$$

- Hydrodynamics makes sense when $\omega \ll \mu$
- Back of the wave-front denoted by '1', Front denoted by '2'.

Chiral shock waves

- Due to this vorticity, we cannot go to a frame with $v_1^y = v_2^y = v_1^z = v_2^z = 0$ everywhere.
- Consider the regime $|\omega|\rho \ll 1$ where the distance from axis: $\rho = \sqrt{y^2 + z^2}$
- In that case the speed perpendicular to the direction of vorticity

$$v_\perp = \omega\rho(1 - v_x^2) + O\big((\omega\rho)^2\big)$$

Continuity equation

- From the continuity equation we have to have
  $v_1^y = v_2^y$ and $v_1^z = v_2^z$ or $v_1^\perp = v_2^\perp$

- This implies

$$\omega_1 \left(1 - (v_1^x)^2\right) = \omega_2 \left(1 - (v_2^x)^2\right)$$

- In the limit $|\omega|\rho \ll 1$ the expressions for v_1^2 and v_2^2 are given by their nonchiral version.

Pressure-volume relation

$$\frac{v_1}{V_1\sqrt{1-v_1^2}} - \frac{v_2}{V_2\sqrt{1-v_2^2}} = -(\xi_1\omega_1 - \xi_2\omega_2)$$

- We know the expansion of the LHS in terms of ΔS and Δp.
- The RHS is a function of μ_1, μ_2, T_1, T_2 as well.
- We do not know the expansion of $\Delta\mu \equiv \mu_2 - \mu_1$ and $\Delta T \equiv T_2 - T_1$ in terms of Δp and ΔS.

A change of variables

- At this point we need to express μ and T as a function of p and S.

- Assume noninteracting Fermi gas to do so:

$$n = \frac{\mu^3}{6\pi^2} + \frac{\mu T^2}{6},$$

$$p = \frac{\epsilon}{3} = \frac{\mu^4}{24\pi^2} + \frac{\mu^2 T^2}{12},$$

$$S = \frac{\pi^2 T}{\mu}.$$

Entropy discontinuity

- The pressure volume relation expanded:

$$\Delta S \approx \frac{216\pi^6}{\mu_1^{11} T_1}(\Delta p)^3 - \frac{\omega_1 \lambda}{T_1}\frac{36\sqrt{2}\pi^4}{\mu_1^8}(\Delta p)^2 + ..$$

Dominates for $\omega_1 < \frac{\Delta p}{\mu_1^3} 3\sqrt{2}\pi^2$

And we are back to nonchiral shockwaves..

Entropy discontinuity

For $\omega_1 > \frac{\Delta p}{\mu_1^3} 3\sqrt{2}\pi^2$

dominates

$$\Delta S \approx \frac{216\pi^6}{\mu_1^{11} T_1} (\Delta p)^3 - \frac{\omega_1 \lambda}{T_1} \frac{36\sqrt{2}\pi^4}{\mu_1^8} (\Delta p)^2 + ..$$

And the entropy discontinuity :

$$\Delta S \approx -\frac{\omega_1 \lambda}{T_1} \frac{36\sqrt{2}\pi^4}{\mu_1^8} (\Delta p)^2 + ..$$

Entropy discontinuity

- ΔS is quadratic in (Δp) in chiral matter instead of being cubic as in nonchiral matter.

- Both rarefaction and compression shockwaves are allowed in chiral matter provided chiral transport dominates!

- Depending on the chirality of fermion, the wave can only propagate either along the vorticity or opposite to the vorticity, but not both.

Conclusion

- We find that rarefaction shockwaves are allowed by the second law of thermodynamics in chiral matter.
- Our result is exemplified in a limit $T/\mu \ll 1$ in a vorticity.
- We expect the qualitative form to hold in other regimes such as that of high temperature and nonzero magnetic field as well.