Evidence of effective axial U(1) symmetry restoration at high temperature QCD

Akio Tomiya (CCNU)

G. Cossu, S. Aoki, H. Fukaya, T. Kaneko, J. Noaki for JLQCD collaboration

Based on: arXiv:1612.01908 (now submitting to PRD)

PRD 93, no. 3, 034507 (2016)

and related proceedings
(This is not related to this talk but...)  

<Advertisement>  
My paper about Kibble-Zurek physics (in 1+1 dim.) will be available on the arXiv tonight...

Quantum Quench and Scaling of Entanglement Entropy

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   Central China Normal University, Wuhan 430079, CHINA

Global quantum quench with a finite rate which crosses critical points is known to lead to universal scaling of correlation functions as functions of the quench rate. We explore scaling properties of the entanglement entropy of a subsystem in a harmonic chain during a mass quench which asymptotes to finite constant values at early and late times and for which the dynamics is exactly solvable. Both for fast and slow quenches we find that the entanglement entropy has a constant term plus a term proportional to the subsystem size. For slow quenches, the constant piece is consistent with Kibble-Zurek predictions. Furthermore, the quench rate dependence of the extensive piece enters solely through the instantaneous correlation length at the Kibble-Zurek time, suggesting a scaling hypothesis similar to that for correlation functions.

Cf. Deep inelastic scattering as a probe of entanglement  
Dmitri E. Kharzeev, and Eugene M. Levin (arXiv 1702.03489)
Evidence of effective axial U(1) symmetry restoration at high temperature QCD

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QCD phase transition for various mass?

What happens when $N_f=2$ at massless limit?

Not directly related to the real physics but useful for model building.
Our Question:

Does the massless two flavor QCD have $U(1)_A$ symmetry above $T_c$?

Tool: Lattice QCD

Our Conclusion:

The massless two flavor QCD has $U(1)_A$ symmetry above $T_c$, if the action has EXACT chiral symmetry.

Key word: Chiral symmetry on the lattice
Contents

1. Introduction: \( U(1)_A \) sym. in QCD
2. Our observables: Dirac spectrum
3. overlap & domain-wall fermion
4. Setup & Results
5. “Ginsparg-Wilson violation” for Domain-wall fermion in low-laying modes
6. Summary
1. Introduction for $U(1)_A$ sym. in QCD
SU(2) chiral symmetry is broken spontaneously, $U(1)$ is by the anomaly

$T = 0$

QCD Lagrangian

$S U(2)_L \times S U(2)_R \times U(1)_V \times U(1)_A$

SSB

$\rightarrow S U(2)_V \times U(1)_V$ : Symmetry of theory

What is the anomaly?
1. Introduction for $U(1)_A$ sym. in QCD

SU(2) chiral symmetry is broken spontaneously, $U(1)$ is by the anomaly

$$T = 0$$

QCD Lagrangian

$$\frac{SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A}{SSB}$$

$$\rightarrow SU(2)_V \times U(1)_V$$

: Symmetry of theory

What is the anomaly?

$$\psi = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$S = \int d^4 x \bar{\psi} \gamma^5 \gamma_5 \psi$$

is invariant under

$$\begin{cases} 
\psi \rightarrow e^{i\theta \gamma_5} \psi \\
\bar{\psi} \rightarrow \bar{\psi} e^{i\theta \gamma_5} 
\end{cases}$$

Namely sym.

Because:

$$\gamma_5 \gamma_5 + \gamma_5 \gamma_5 = 0$$
1. Introduction for $U(1)_A$ sym. in QCD

SU(2) chiral symmetry is broken spontaneously, $U(1)$ is by the anomaly

$$T = 0$$

QCD Lagrangian

$$\frac{SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A}{SSB} \rightarrow SU(2)_V \times U(1)_V$$

: Symmetry of theory

What is the anomaly?

$$\psi = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$S = \int d^4x \bar{\psi} \mathcal{D}\psi$$ is invariant under

$$\psi \rightarrow e^{i\theta\gamma_5} \psi$$

$$\bar{\psi} \rightarrow \bar{\psi} e^{i\theta\gamma_5}$$

Namely sym.

Because:

$$\gamma_5 \mathcal{D} + \mathcal{D} \gamma_5 = 0$$

but the path integral measure is not invariant! **non-trivial Jacobian.**

$$\mathcal{D}\bar{\psi} \mathcal{D}\psi \rightarrow \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{i\Gamma}$$

Anomaly(Fujikawa 1972)

This effect must exist for explanation of heavy $\eta'$
1. Introduction for U(1)$_A$ sym. in QCD

SU(2) chiral symmetry is broken spontaneously, U(1) is by the anomaly

\[ T = 0 \quad \text{QCD Lagrangian} \]

\[ SU(2)_L \times SU(2)_R \times U(1)_V \times \frac{U(1)_A}{\text{Anomaly}} \]

\[ \rightarrow SU(2)_V \times U(1)_V : \text{Symmetry of theory} \]

On the other hand,
1. Introduction for $U(1)_A$ sym. in QCD
SU(2) chiral symmetry is broken spontaneously, $U(1)$ is by the anomaly

\[
T = 0 \quad \text{QCD Lagrangian}
\]

\[
\underline{SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A} \quad \text{SSB}
\]

\[\rightarrow SU(2)_V \times U(1)_V : \text{Symmetry of theory}\]

On the other hand,

\[
T > T_c
\]

\[
\underline{SU(2)_V \rightarrow SU(2)_L \times SU(2)_R} \quad \text{Restored}
\]

\[
U(1)_A \rightarrow ??
\]
1. Introduction for $U(1)_A$ sym. in QCD

SU(2) chiral symmetry is broken spontaneously, $U(1)$ is by the anomaly

\[ T = 0 \]

\[
\frac{SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A}{SSB} \rightarrow SU(2)_V \times U(1)_V \quad \text{: Symmetry of theory}
\]

On the other hand,

\[ T > T_c \]

\[
SU(2)_V \rightarrow SU(2)_L \times SU(2)_R \quad \text{Restored}
\]

$U(1)_A \rightarrow ??$

What happens to the anomaly above $T_c$?
1. Introduction for $U(1)_A$ sym. in QCD

Symmetry leads degeneracy between mesons

$\langle \pi(x)\pi(0) \rangle$ \hspace{1cm} $SU(2)_L \times SU(2)_R$

$\langle \sigma(x)\sigma(0) \rangle$

$U(1)_A$

$\langle \delta(x)\delta(0) \rangle$ \hspace{1cm} $SU(2)_L \times SU(2)_R$

$\langle \eta(x)\eta(0) \rangle$

$U(1)_A$
Evidence of effective axial U(1) symmetry restoration at high temperature QCD

1. Introduction for U(1)\(_A\) sym. in QCD

Symmetry leads degeneracy between mesons

\[
\begin{align*}
\langle \pi(x)\pi(0) \rangle & \quad \overset{SU(2)_L \times SU(2)_R}{\leftrightarrow} \quad \langle \sigma(x)\sigma(0) \rangle \\
\langle \delta(x)\delta(0) \rangle & \quad \overset{SU(2)_L \times SU(2)_R}{\leftrightarrow} \quad \langle \eta(x)\eta(0) \rangle \\
\end{align*}
\]

\[
\chi_{U(1)_A} \equiv \int d^4x \left[ \langle \pi(x)\pi(0) \rangle - \langle \delta(x)\delta(0) \rangle \right] 
\]

“Order parameter” of U(1)\(_A\)

If this quantity(susceptibility) is 0 at V \(\rightarrow\infty\), m \(\rightarrow\) 0, U(1)\(_A\) symmetry is effectively “restored” (in other wards, invisible)
2. Our observables: Dirac spectrum

$\rho(\lambda)$ is a spectrum of the Dirac operator with QCD background

Our observable

$$\left( \gamma_5 \partial \right) \psi_j = \lambda_j \psi_j$$

(Covariant derivative has information of the gauge field)

Eigenvalue equation can be solved for a given gauge configuration
2. Our observables: Dirac spectrum

$\rho(\lambda)$ is a spectrum of the Dirac operator with QCD background

Our observable

$$(\gamma_5 D)\psi_j = \lambda_j \psi_j$$

(Covariant derivative has information of the gauge field)

Eigenvalue equation can be solved for a given gauge configuration

One can repeat for all configurations

$\Rightarrow$ $\lambda$ s are distributed in a certain way,

$\Rightarrow$ the Dirac spectrum $\rho(\lambda)$
2. Our observables: Dirac spectrum

\( \rho(\lambda) \) is a spectrum of the Dirac operator with QCD background

Our observable

\[
(\gamma_5 D) \psi_j = \lambda_j \psi_j
\]

(Covariant derivative has information of the gauge field)

Eigenvalue equation can be solved for a given gauge configuration

One can repeat for all configurations

\( \rightarrow \) \( \lambda \)'s are distributed in a certain way, \( = \) the Dirac spectrum \( \rho(\lambda) \)

The Dirac spectrum \( \rho(\lambda) \) has information of symmetry of quarks
2. Our observables: Dirac spectrum

If \( \rho \) has a (volume insensitive) gap, U(1) is effectively restored.

For SU(2): The Banks-Casher relation

\[
\langle \bar{\psi} \psi \rangle = \int_{0}^{\infty} d\lambda \, \rho(\lambda) \frac{2m}{\lambda^2 + m^2} \quad \rho(\lambda) = \lim_{V \to \infty} \frac{1}{V} \sum_{n} \langle \delta(\lambda_n^A - \lambda) \rangle_A
\]

\[
|\langle \bar{\psi} \psi \rangle| = \pi \rho(0) = 0 \quad \rightarrow \quad \text{SU(2) restoration}
\]
2. Our observables: Dirac spectrum

If $\rho$ has a (volume insensitive) gap, U(1) is effectively restored

For SU(2): The Banks-Casher relation

$$\langle \bar{\psi} \psi \rangle = \int_0^\infty d\lambda \, \rho(\lambda) \frac{2m}{\lambda^2 + m^2}$$

$$\rho(\lambda) = \lim_{V \to \infty} \frac{1}{V} \sum_n \langle \delta(\lambda_n^A - \lambda) \rangle_A$$

$$|\langle \bar{\psi} \psi \rangle| = \pi \rho(0) = 0 \quad \rightarrow \quad \text{SU(2) restoration}$$

For U(1): Cohen’s argument

$$\chi_{U(1)}^A \equiv \int d^4 x [\langle \pi(x)\pi(0) \rangle - \langle \delta(x)\delta(0) \rangle]$$

$$\chi_{U(1)}^A = \int_0^\infty d\lambda \, \rho(\lambda) \frac{4m^2}{(\lambda^2 + m^2)^2}$$

$$|\langle \bar{\psi} \psi \rangle| = 0 \quad \rightarrow \quad \chi_{U(1)}^A = 0$$

SU(2) and U(1) restoration

2. Our observables: Dirac spectrum

If $\rho$ has a (volume insensitive) gap, U(1) is effectively restored

**Argument by Cohen (1996)**

If there is a gap in the Dirac spectrum

$$\int d^4 x [\langle \pi(x)\pi(0) \rangle - \langle \delta(x)\delta(0) \rangle] = \int_0^\infty d\lambda \frac{4m^2 \rho(\lambda)}{(m^2 + \lambda^2)^2}$$

$\to 0 \quad (m \to 0)$

in invisible

Cf: Aoki-Fukaya-Taniguchi (2012):

$\lambda^3$ may be enough for U(1)$_A$ effective restoration.

**low-laying modes are essential for this argument!**

Evidence of effective axial U(1) symmetry restoration at high temperature QCD
## 1. Introduction for $U(1)_A$ sym. in QCD

Symmetry leads degeneracy between mesons

### Previous studies (DW type) are controversial!

<table>
<thead>
<tr>
<th>Group</th>
<th>Fermion</th>
<th>Size</th>
<th>Gap in the spectrum</th>
<th>$U_A(1)$ Correlator</th>
<th>$U(1)_A @T_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>JLQCD (2013)</td>
<td>Overlap (Top. fixed)</td>
<td>2 fm</td>
<td>Gap</td>
<td>Degenerate</td>
<td>Restored</td>
</tr>
<tr>
<td>TWQCD (2013)</td>
<td>Optimal domain-wall</td>
<td>3 fm</td>
<td>No gap</td>
<td>Degenerate</td>
<td>Restored ?</td>
</tr>
<tr>
<td>LLNL/RBC, Hot QCD (2013, 2014)</td>
<td>(Mobius)-Domain-wall (W/ ov)</td>
<td>2, 4, 11 fm</td>
<td>No gap</td>
<td>No degeneracy</td>
<td>Violated</td>
</tr>
</tbody>
</table>

What makes such difference? Fermion (Chiral sym.), Volumes or Topology?
3. Domain-wall and overlap fermion

Chiral symmetry on the lattice = Ginsparg-Wilson relation

SU(2) and U(1) are parts of chiral symmetry in the action:

- Chiral symmetry in continuum theory

\[ \gamma_5 \slashed{D} + \slashed{D} \gamma_5 = 0 \]
3. Domain-wall and overlap fermion

Chiral symmetry on the lattice = Ginsparg-Wilson relation

SU(2) and U(1) are parts of chiral symmetry in the action:

- Chiral symmetry in continuum theory

\[ \gamma_5 \bar{D} + D \gamma_5 = 0 \]

☆ Chiral symmetry on the lattice (Cf. Nielsen-Ninomiya thm)

\[ \gamma_5 \bar{D} + D \gamma_5 = 2aD \gamma_5 \bar{D} \]

(Here “a” is a lattice spacing)

“Ginsparg-Wilson relation”
3. Domain-wall and overlap fermion
Chiral symmetry on the lattice = Ginsparg-Wilson relation

“Ginsparg-Wilson relation”

$$\gamma_5 \slashed{D} + \slashed{D} \gamma_5 = 2a \slashed{D} \gamma_5 \slashed{D}$$

If D satisfies GW relation...
3. Domain-wall and overlap fermion

Chiral symmetry on the lattice = Ginsparg-Wilson relation

“Ginsparg-Wilson relation”

\[ \gamma_5 \mathcal{D} + \mathcal{D} \gamma_5 = 2a \mathcal{D} \gamma_5 \mathcal{D} \]

If D satisfies GW relation...

1. It has “exact” chiral symmetry

\[ \psi \rightarrow \psi' = e^{i \gamma_5 (1 - aD) \theta} \psi \]

\[ \bar{\psi} \rightarrow \bar{\psi}' = \bar{\psi} e^{i \gamma_5 \theta} \]

2. \( U(1)_A \) symmetry is broken by the Jacobian as same as the continuum theory

3. It satisfies the Atiyah-Singer index theorem
3. Domain-wall and overlap fermion
Overlap fermion satisfies the Ginsparg-Wilson relation

The overlap Dirac operator satisfies GW relation

\[ D_{ov} = \frac{1 + m}{2} - \frac{1 - m}{2} \gamma_5 \text{sgn}(H_T) \]

However...
numerical cost of the sign function is extremely expensive!

There is an approximate one,
“The domain-wall fermion”
3. Domain-wall and overlap fermion

Overlap fermion satisfies the Ginsparg-Wilson relation

Domain-wall fermion $\sim$ Overlap fermion + $m_{\text{res}}$

$$D_{ov} = \frac{1 + m}{2} - \frac{1 - m}{2} \gamma_5 \text{sgn}(H_T)$$

Domain-wall fermion:

$$\text{tanh} \left[ L_s \text{tanh}^{-1} (2H_T) \right]$$

Qualitative difference can be measured by “residual mass”: $m_{\text{res}}$
Our Setup

1. Sea quarks: Dynamical Möbius domain-wall fermion with small $m_{\text{res}}$.
2. Calculation is done with and without OV/DW reweighting to realize overlap sea-quark effectively.
3. Volume & topology: 3 Volumes (2-4 fm) and frequent topology tunneling.
4. Probes: Domain-wall and overlap valence quarks.
5. Temperature range: 172 MeV to 217 MeV. $T_c \sim 190$ MeV.
4. Setup & Results

Sea quark: Domain-wall(DW) and Reweighted Overlap(OV), Probe: DW and OV

<table>
<thead>
<tr>
<th>$L^3 \times L_t$</th>
<th>$\beta$</th>
<th>$m_a$</th>
<th>$L_s$</th>
<th>$m_{\text{res}}a$</th>
<th>$T$ [MeV]</th>
<th>$N_{\text{conf}}$</th>
<th>$N_{\text{conf}}^{\text{eff}}$</th>
<th>$N_{\text{conf}}^{\text{eff}(2)}$</th>
<th>$\tau_{\text{CG int}}$</th>
<th>$\tau_{\text{int}}^{\text{top}}$</th>
<th>$M_{PSL}$</th>
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<td>$16^3 \times 8$</td>
<td>4.07</td>
<td>0.01</td>
<td>12</td>
<td>0.00166(15)</td>
<td>203(1)</td>
<td>6600</td>
<td>11(13)</td>
<td>45(8)</td>
<td>70</td>
<td>25(6)</td>
<td>5.4(3)</td>
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<td>24</td>
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<td>203(1)</td>
<td>12000</td>
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<td>14(3)</td>
<td>315</td>
<td>23(4)</td>
<td>5.3(4)</td>
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<tr>
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<td>0.01</td>
<td>12</td>
<td>0.00079(5)</td>
<td>217(1)</td>
<td>7000</td>
<td>23(7)</td>
<td>150(17)</td>
<td>134</td>
<td>30(10)</td>
<td>6.9(5)</td>
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<tr>
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<td>24</td>
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<td>121(10)</td>
<td>104</td>
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<td>6.3(9)</td>
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<td>3800</td>
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<td>20(4)*</td>
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<td>84</td>
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<td>7000</td>
<td>(703)</td>
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<td>5600</td>
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<td>16</td>
<td>0.00012(1)</td>
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<td>$32^3 \times 12$</td>
<td>4.24</td>
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<td>195(1)</td>
<td>7600</td>
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<td>0.0025</td>
<td>16</td>
<td>0.00011(2)</td>
<td>195(1)</td>
<td>16000</td>
<td>188</td>
<td>8(10)</td>
<td>7(1)</td>
<td>618</td>
<td>80(20)</td>
</tr>
</tbody>
</table>

(1) $m_{\text{res}}$ is enough small
(2) # of statistics are increased from 2015
(3) We care about finite size effect & “overlapping problem” for reweighting

Calculations done by BG/Q and SR16000 in KEK using Iroiro++
4. Setup & Results
Reweighted Overlap with overlap probe has gap! and volume insensitive!!

T= 203 MeV for L=2fm, T=1.13 Tc (small lattice)

Domain-wall on DW sea
4. Setup & Results
Reweighted Overlap with overlap probe has gap! and volume insensitive!!

\[ T = 203 \text{ MeV for } L=2 \text{fm, } T=1.13 \text{ Tc (small lattice)} \]

Domain-wall on DW sea

Overlap on domain-wall sea (partially quenched)
4. Setup & Results

Reweighted Overlap with overlap probe has gap! and volume insensitive!!

\[ T = 203 \text{ MeV for } L=2 \text{ fm, } T=1.13 \text{ Tc (small lattice)} \]

Domain-wall on DW sea

Overlap on domain-wall sea (partially quenched)

(reweighted)Overlap sea with overlap probe
4. Setup & Results
Reweighted Overlap with overlap probe has gap! and volume insensitive!!

\[ T = 203 \text{ MeV} \text{ for } L=2\text{fm}, \; T=1.13 \; T_c \text{ (small lattice)} \]

Domain-wall on DW sea

Overlap on domain-wall sea (partially quenched)

(rewighted) Overlap sea with overlap probe

Evidence of effective axial U(1) symmetry restoration at high temperature QCD

A. Tomiya: 15 Feb. 2017 at BNL
4. Setup & Results

Reweighted Overlap with overlap probe has gap! and volume insensitive!!

T = 190 MeV for L=3fm,T=1.05 Tc (middle size, finer lattice)

Domain-wall on DW sea

Overlap on domain-wall sea (partially quenched)

(reweighted) Overlap sea with overlap probe

Unphysical peak
4. Setup & Results

Reweighted Overlap with overlap probe has gap! and volume insensitive!!

**T= 202 MeV for L=4fm, T=1.1 Tc (Large volume)**

- Domain-wall on DW sea

- Overlap on domain-wall sea (partially quenched)
  - Unphysical peak

- (rewighted)Overlap sea with overlap probe
4. Setup & Results

Reweighted Overlap with overlap probe has gap! and volume insensitive!!

\[ T = 217 \text{ MeV for } L=4\text{fm}, \ T=1.2 \ Tc \ (\text{Large volume}) \]

Domain-wall on DW sea

Overlap on domain-wall sea (partially quenched)

(reweighted) Overlap sea with overlap probe

Why they look different??
5. “GW violation” for DW fermion in low-laying modes
Difference coming from violation of Ginsparg-Wilson relation in low-laying modes

To understand difference between spectra, we define Ginsparg-Wilson relation violation for individual eigenmode:

\[ g_i \propto \psi_i^\dagger \gamma_5 [D\gamma_5 + \gamma_5 D - 2aD\gamma_5 D] \psi_i \]

\( \psi \): Eigenmodes of the Dirac operator D

※ This “g” is zero for the overlap Dirac op.
5. “GW violation” for DW fermion in low-laying modes

Difference coming from violation of Ginsparg-Wilson relation in low-laying modes

\[ g_i \propto \psi_i^\dagger \gamma_5 \left[ D \gamma_5 + \gamma_5 D - 2aD \gamma_5 D \right] \psi_i \]

\[ |g_i| = |\text{GW violation}| \]

The lattice artifact can be 100% for the near zero-modes for Domain-wall fermion
Evidence of effective axial $U(1)$ symmetry restoration at high temperature QCD

5. “GW violation” for DW fermion in low-lying modes
Susceptibility is dominated by Ginsparg-Wilson violation

\[ \chi_{U(1)} \] also has GW violation

\[ \chi_{U(1)_A} \equiv \int d^4x [\langle \pi(x)\pi(0)\rangle - \langle \delta(x)\delta(0)\rangle] \]

Ratio of susceptibility:
GW-breaking-modes v.s.
Total
for DW fermion

Even for finer lattice, \(~40\%\) are artifact

Finer lattice
\(1/a \sim 2.2\,\text{GeV}\)
Evidence of effective axial U(1) symmetry restoration at high temperature QCD

5. “GW violation” for DW fermion in low-laying modes
At the massless limit, overlap fermion suggests effective restoration of U(1)

For overlap fermion, after taking of massless limit, physical U(1) violating signal is disappeared
6. Summary

In this work, we examined axial U(1) breaking with Möbius domain-wall (DW), partially quenched overlap (on DW sea), and reweighted overlap fermions. We found,

1. unexpectedly large violation of the Ginsparg-Wilson relation in low-lying modes of DW operator even for small residual mass case

2. precise chiral symmetry both in sea and valence quark is crucial.

3. reweighted overlap Dirac spectrum and susceptibility suggest U(1)$_A$ effective restoration at the chiral limit.
No more slides
Backups
Sym. of QCD $\iff$ Degeneracy

\[
\begin{align*}
\langle \pi(x) \pi(0) \rangle & \overset{SU(2)_L \times SU(2)_R}{\longleftrightarrow} \langle \sigma(x) \sigma(0) \rangle \\
\langle \delta(x) \delta(0) \rangle & \overset{SU(2)_L \times SU(2)_R}{\longleftrightarrow} \langle \eta(x) \eta(0) \rangle
\end{align*}
\]

\[
\begin{align*}
\pi(x) &= i \bar{\psi}(x) \gamma_5 \tau \psi(x) \\
\sigma(x) &= \bar{\psi}(x) \psi(x) \\
\delta(x) &= \bar{\psi}(x) \tau \psi(x) \\
\eta(x) &= i \bar{\psi}(x) \gamma_5 \psi(x)
\end{align*}
\]

Degeneracy of these channels
$\iff$ There are symmetries
<table>
<thead>
<tr>
<th>$L^3 \times L_t$</th>
<th>$\beta$</th>
<th>$m$</th>
<th>$\rho_{ov}(0-8\text{MeV})$</th>
<th>$\Delta^\text{direct}_{\pi-\delta} a^2$</th>
<th>$\Delta^\text{ev}_{\pi-\delta} a^2$</th>
<th>$\Delta^G_W_{\pi-\delta}/\Delta^\text{ev}_{\pi-\delta}$</th>
<th>$\Delta^\text{ov}_{\pi-\delta} a^2$</th>
<th>$\bar{\Delta}^\text{ov}_{\pi-\delta} a^2$</th>
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<tbody>
<tr>
<td>$16^3 \times 8$</td>
<td>4.07</td>
<td>0.01</td>
<td>$0.0071(18)$</td>
<td>$0.132(14)$</td>
<td>$0.139(12)$</td>
<td>$0.37(2)$</td>
<td>$0.19(5)$</td>
<td>$0.032(13)$</td>
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<td>4.07</td>
<td>0.001</td>
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<td>0.005</td>
<td>$0.0008(3)$</td>
<td>$0.009(2)$</td>
<td>–</td>
<td>–</td>
<td>$0.0003(1)$</td>
<td>$0.003(1)$</td>
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<tr>
<td>$16^3 \times 8$</td>
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<td>$1.5(1.5) \times 10^{-8}$</td>
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<td>$0.0232(13)$</td>
<td>$0.983(4)$</td>
<td>$6(3) \times 10^{-5}$</td>
<td>$6(3) \times 10^{-5}$</td>
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<td>0.001</td>
<td>$0.00002(1)$</td>
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<td>$32^3 \times 8$</td>
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<td>$1.5(1.3) \times 10^{-5}$</td>
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<td>$0.011(1)$</td>
<td>$0.112(10)$</td>
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<td>$32^3 \times 12$</td>
<td>4.23</td>
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<td>$0.00444 (96)$</td>
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<td>$0.216(41)$</td>
<td>$0.162(22)$</td>
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<td>$0.078(52)$</td>
<td>$0.0030(6)$</td>
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Re-weighting tech. enables us to change another fermion determinant
( = quark loop effect exchange)

\[
\langle \mathcal{O} \rangle_{\text{Overlap}} \propto \int D\bar{\psi}D\psi DA_\mu \mathcal{O} e^{-S_{\text{gauge}}} e^{-\bar{\psi}[D_{\text{OV}}]\psi} \\
= \int DA_\mu \mathcal{O} e^{-S_{\text{gauge}}} \text{Det}[D_{\text{OV}}^2] \\
= \int DA_\mu \mathcal{O} e^{-S_{\text{gauge}}} \text{Det}[D_{\text{OV}}^2] \frac{\text{Det}[D_{\text{DW}}^2]}{\text{Det}[D_{\text{DW}}^2]} \\
= \int D\bar{\psi}D\psi DA_\mu \mathcal{O}_R e^{-S_{\text{gauge}}} e^{-\bar{\psi}[D_{\text{DW}}]\psi} \\
\propto \langle \mathcal{O} R \rangle_{\text{Domain Wall}}
\]

\[
R = \frac{\text{Det}[D_{\text{OV}}^2]}{\text{Det}[D_{\text{DW}}^2]} 
\]

Multiplying R and taking average, we obtain
the result with the overlap determinant
Some difficulties of the sign function in the overlap-Dirac operator, as we have mentioned in earlier subsection. To overcome these problems, we introduce rational approximation for the sign function i.e. we employ the Möbius domain-wall fermion action [24, 25] for the quarks.

\[ \text{D}^{4DW}(m) = 1 + \frac{m^2}{1 - m^2} \gamma^5 \text{sgn}(HM) \]

Its determinant is equivalent (except for overall constants) to that of a 4-dimensional effective Dirac operator:

\[ D^{4DW}(m) = \frac{1}{1 - H_M} \text{sgn}(HM) = 1 - \left( T(H_M) \right) L_s \]

We introduce three steps of the stout smearing for the gauge links. In order to evaluate explicit chiral symmetry breaking which comes from the approximation of the sign function, we introduce the residual mass, calculated as

\[ m_{\text{res}} = \frac{\langle \text{tr} G^\dagger \Delta_L G \rangle}{\langle \text{tr} G^\dagger G \rangle}, \quad \Delta_L = \frac{1}{2} \gamma_5 (\gamma_5 D_{DW}^{4D} + D_{DW}^{4D} \gamma_5 - 2a D_{DW}^{4D} \gamma_5 D_{DW}^{4D}) \]

\[ G: \text{contact-term-subtracted quark propagator}, \]

\[ \text{D}^{4D}_G(0) = \frac{1}{2} \sum_{\lambda_i < |\lambda_{\text{th}}|} \lambda_i \langle \lambda_i | G \rangle \langle G | \lambda_i \rangle + D^{4D}_{DW}(0)(1 - \sum_{\lambda_i < |\lambda_{\text{th}}|} \lambda_i \langle \lambda_i | G \rangle \langle G | \lambda_i \rangle) \]
R with UV suppression factor
low-mode reweighting

\[ R(A) = \frac{\text{Det } D_{ov}^2(m)}{\text{Det } D_{DW}^2(m)} \frac{\text{Det } D_{DW}^2(1/2a)}{\text{Det } D_{ov}^2(1/2a)}. \quad \text{(for } L = 16^3 \times 8) \]

\[ R(A) \sim \frac{\prod_i^{N_{th}} [(\lambda_{ov}^i m)^2]}{\prod_i^{N_{th}} [(\lambda_{DW}^i m)^2]} = R_{low}(A), \quad \text{(for } L = 16^3 \times 8, 32^3 \times 8) \]
Low-mode reweighting factor does not seem to affect existence of the gap

This is now testing in finer (and larger) lattice...
Massless Dirac spectrum

The Dirac spectrum of the massless fermion can be obtained by subtracting,

\[ \lambda_i a \equiv \frac{\sqrt{a^2(\lambda_i^m)^2 - a^2 m_{ud}^2}}{\sqrt{1 - a^2 m_{ud}^2}}, \]
We measure the violation of the Ginsparg-Wilson relation on each eigenmode of the Hermitian Dirac operator $\gamma_5 D$ through

$$g_i \equiv \frac{\psi_i^\dagger \gamma_5 [D \gamma_5 + \gamma_5 D - 2aD \gamma_5 D] \psi_i}{\lambda_i^m} \left[ \frac{(1 - am_{ud})^2}{2(1 + am_{ud})} \right],$$  \hspace{1cm} (7.2)

where $\lambda_i^m$, $\psi_i$ denotes the $i$–th eigenvalue/eigenvector of massive hermitian Dirac operator respectively. $D$ is the domain-wall or overlap Dirac operator. Last factor in (7.2) comes from the normalization of the Dirac operator. Note that one can obtain the residual mass by an weighted average of $g_i$,

$$m_{res} = \frac{\left\langle \text{tr} \, G^\dagger \Delta_L G \right\rangle}{\left\langle \text{tr} \, G^\dagger G \right\rangle} = \sum_i \frac{\lambda_i^m(1 + am_{ud})}{(1 - am_{ud})^2(a\lambda_i^m)^2} g_i \bigg/ \sum_i \frac{1}{(a\lambda_i^m)^2},$$  \hspace{1cm} (7.3)

where the sum runs over all eigenvalues.
Reweighting factor

FIG. 30:

FIG. 31:
Figure 27: Low-mode reweighting factors for each configuration for $L = 16$, $\beta = 4.10$. Left panel and right panel correspond to $m = 0.01$ and $m = 0.001$ respectively.

Figure 28: Low-mode reweighting factors for each configuration for $L = 32$, $\beta = 4.07$. Left panel and right panel correspond to $m = 0.01$ and $m = 0.001$ respectively.

Figure 29: Low-mode reweighting factors for each configuration for $L = 32$, $\beta = 4.10$. Left panel and right panel correspond to $m = 0.01$ and $m = 0.001$ respectively.
Reweighting factors vs configuration
Figure 5: The eigenvalue histograms of the domain wall (left panels) and reweighted overlap (right) Dirac operators. The data for $T \sim 200\,\text{MeV}$ on $L^3 = 16^3$ (top panels) and $L^3 = 32^3$ (bottom) lattices are presented.

Figure 6: The eigenvalue histograms of the domain wall (left panels) and reweighted overlap (right) Dirac operators. The data for $T \sim 190\,\text{MeV}$ on $L^3 \times L^t = 32^3 \times 12$ lattices are presented.

Figure 7: The violation of Ginsparg-Wilson relation ($g_i$). Red symbol corresponds to $g_i$ for domain-wall eigen vector and blue symbol corresponds to $g_i$ for overlap eigen vector.
Topological charge changes along HMC

$L = 16, \beta = 4.10, m = 0.01, L_s = 12$

![Graph showing changes in topological charge along HMC with parameters $L = 16, \beta = 4.10, m = 0.01, L_s = 12$.](image)
Tc Estimation

Polyakov & Chiral condensate

Chiral Condensate

Polyakov loop

Above Tc (T=200MeV)

Around Tc (T=180MeV)

Vol. dependence of Polyakov loop
Decreasing of Chiral condensate
Overlap type=Different “Sign function”

Comparison of sign function and its approximations

- Mobius-Domain-wall(us)
- Domain-wall
- Overlap(JLQCD)
- $\text{sgn}(x)$, Overlap
- $\tanh(ls*\text{atanh}(x))$, Domain-wall/Shamir
- $\tanh(ls*\text{atanh}(2x))$, Mobius-Domain-wall/scaled Shamir

$\sim$ E-val
\[ \text{U}(2)_L \times \text{U}(2)_R \simeq \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_V \times \text{U}(1)_A, \]  
\hspace{5cm} (3.10) \]

where \( \text{SU}(2)_L \times \text{SU}(2)_R \) symmetry corresponds to

\[ \begin{align*}
\psi & \rightarrow e^{i\theta\gamma_5\tau_a}\psi, \\
\bar{\psi} & \rightarrow \bar{\psi}e^{+i\theta\tau_a\gamma_5},
\end{align*} \]  
\hspace{5cm} (3.11, 3.12) \]

(the \( \text{SU}(2) \) chiral symmetry) and

\[ \begin{align*}
\psi & \rightarrow e^{i\theta\tau_a}\psi, \\
\bar{\psi} & \rightarrow \bar{\psi}e^{-i\theta\tau_a}.
\end{align*} \]  
\hspace{5cm} (3.13, 3.14) \]

On the other hand, the \( \text{U}(1)_A \) symmetry, equivalently the \( \text{U}(1) \) chiral symmetry, corresponds to

\[ \begin{align*}
\psi & \rightarrow e^{i\theta\gamma_5}\psi, \\
\bar{\psi} & \rightarrow \bar{\psi}e^{+i\theta\gamma_5}.
\end{align*} \]  
\hspace{5cm} (3.15, 3.16) \]
Cohen’s argument:

\[
\Pi_\sigma(x) - \Pi_\delta(x) = \frac{1}{Z} \int [DA] e^{-S_{YM}} \text{Det} [\mathcal{D} - m] \left[ \text{Tr} [G(x, x)] \text{Tr} [G(0, 0)] \right]
\]

\[
\langle \bar{\psi} \psi \rangle = \frac{1}{Z} \int [DA] e^{-S_{YM}} \text{Det} [\mathcal{D} - m] \text{Tr} [G(x, x)]
\]

\[
\text{Tr} [G(x, x)] = \sum_j \frac{-m \psi_j^\dagger(x) \psi_j(x)}{\lambda_j^2 + m^2}
\]

\[
= \int d\lambda \frac{-m \rho_A(\lambda)}{\lambda^2 + m^2}
\]

\[
e^{-S_{YM}} \text{Det} [\mathcal{D} - m] \text{tr} [G(x, x)] = 0,
\]

Here we ignore zero-modes. This means, the quark propagator is negative semi-definite for any gauge configurations. For all configuration. This leads to 

\[
\text{Tr} \left[ \left( \partial_{\mu} \right)^5 \bar{\psi} \gamma^\mu \gamma^5 \psi \right] = 0
\]