Stony Brook University

Dileptons and Photons at Collider Energies

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BNL17

Collaborators

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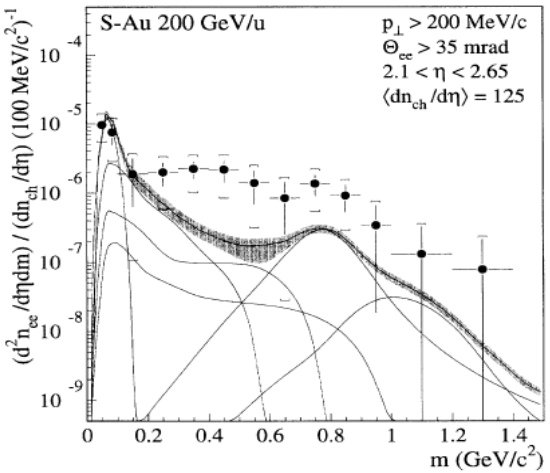
- Y. Kim
- D. Teaney
- C.H. Lee

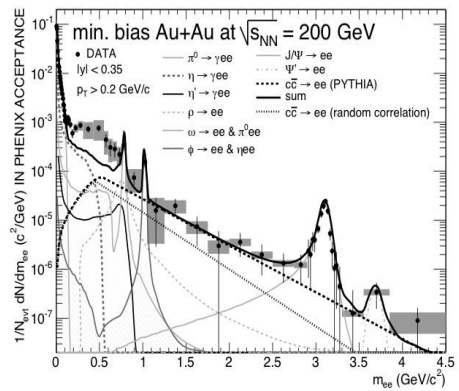
Outline

- Motivation
- Rates, σ_E, χ
- Evolution
- Experiments

Na45: 95'

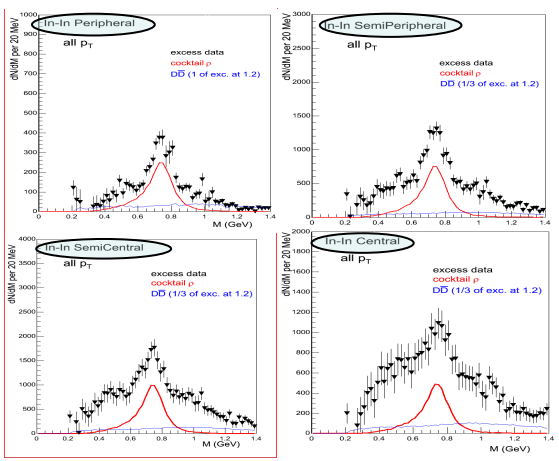


Phenix: 07'

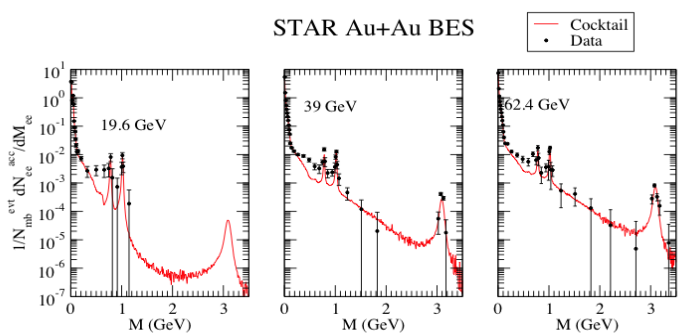


Phenix-v2: 12'-14' Alice-v2: 12'

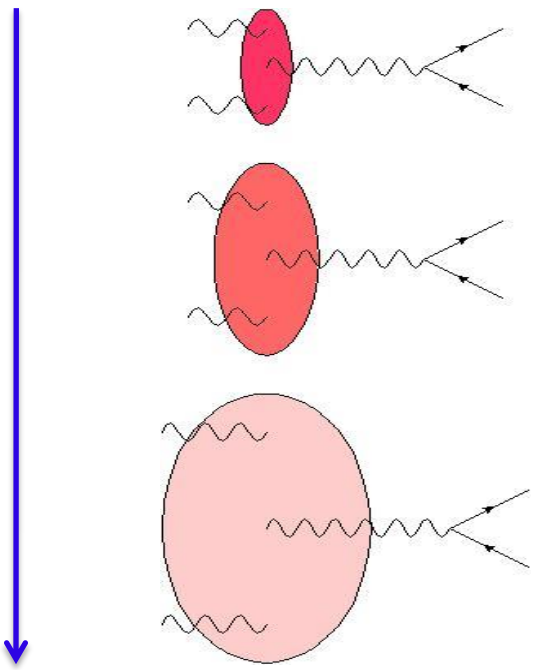
Na60: 03'



Star: 13'



Formally



time

acceptance

$$\frac{dN}{dM} = 2\pi M \int dy \int dq_\perp \cdot q_\perp \times Acc(M, q_\perp, y)$$

$$\times \int_{\tau_0}^{\tau_{f.o}} \tau d\tau \int_{-\infty}^{\infty} d\eta \int_0^{r_{max}} r dr \int_0^{2\pi} d\theta \frac{d^4R}{d^4q d^4x}(M, |\mathbf{q}|, T, \mu_B, x)$$

evolution

rate

Rates: Hadrons

Equilibrium averaging

$$\frac{dR}{d^4q} = \frac{2\alpha}{3\pi M^2}\left(1+\frac{2m_l^2}{M^2}\right)\left(1-\frac{4m_l^2}{M^2}\right)^{1/2}\left(\frac{q^0 dN^*}{d^3q}\right).$$

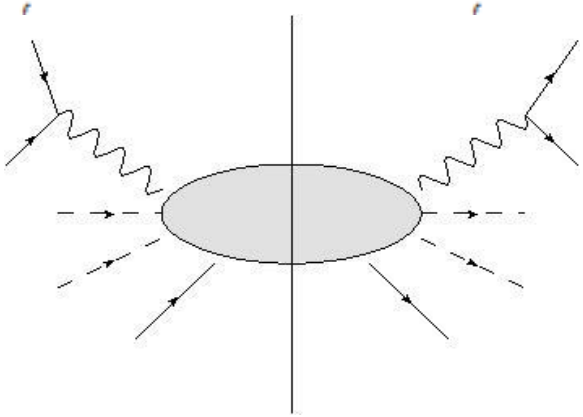
$$\frac{q^0 dN}{d^3q} = -\frac{\alpha}{4\pi^2}\mathbf{W}(q)$$

$$\mathbf{W}(q) = \int d^4x e^{-iq\cdot x}\mathrm{Tr}\left[e^{-(\mathbf{H}-\mathbf{F})/T}\,\mathbf{J}^\mu(x)\mathbf{J}_\mu(0)\right].$$

$$\mathbf{W}(q) = \frac{2}{e^{q^0/T}+1}\,\mathrm{Im}\mathbf{W}^F(q)$$

$$\mathbf{J}_\mu(x) = \sum_f \tilde{e}_f\,\overline{\mathbf{q}}_f\gamma_\mu\mathbf{q}_f(x)$$

$$\mathbf{J}_\mu(x) = \sum_f \tilde{e}_f\,\overline{\mathbf{q}}_f\gamma_\mu\mathbf{q}_f(x)$$



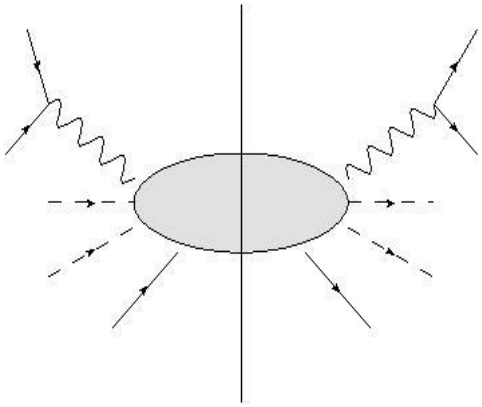
Hadronic Expansion

$$\mathbf{W}^F(q) = \mathbf{W}_0^F(q) + \frac{1}{f_\pi^2}\int d\pi \mathbf{W}_\pi^F(q,k) + \frac{1}{2!}\frac{1}{f_\pi^4}\int d\pi_1 d\pi_2 \mathbf{W}_{\pi\pi}^F(q,k_1,k_2) + \cdots$$

$$\mathbf{W}_0^F(q) = i\int d^4x e^{iq\cdot x}\langle 0|T^*\mathbf{J}^\mu(x)\mathbf{J}_\mu(0)|0\rangle$$

$$\mathbf{W}_\pi^F(q,k) = if_\pi^2\int d^4x e^{iq\cdot x}\langle \pi^a(k)|T^*\mathbf{J}^\mu(x)\mathbf{J}_\mu(0)|\pi^a(k)\rangle$$

$$\mathbf{W}_{\pi\pi}^F(q,k_1,k_2) = if_\pi^4\int d^4x e^{iq\cdot x}\langle \pi^a(k_1)\pi^b(k_2)|T^*\mathbf{J}^\mu(x)\mathbf{J}_\mu(0)|\pi^a(k_1)\pi^b(k_2)\rangle$$

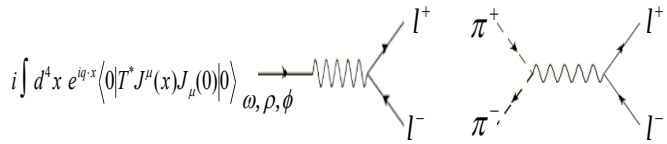


$$\int d\pi = \int \frac{d^3k}{(2\pi)^3}\frac{n(E_\pi - \mu_\pi)}{2E_\pi}$$

In Diagrams

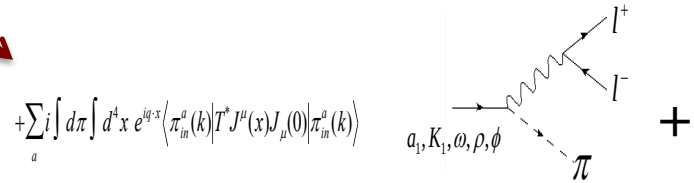
Resonance Gas

$$= i\int d^4x\, e^{iq\cdot x}\left\langle 0\left|T^* J^\mu(x) J_\mu(0)\right|0\right\rangle$$



1,2,3,
...-Pi,

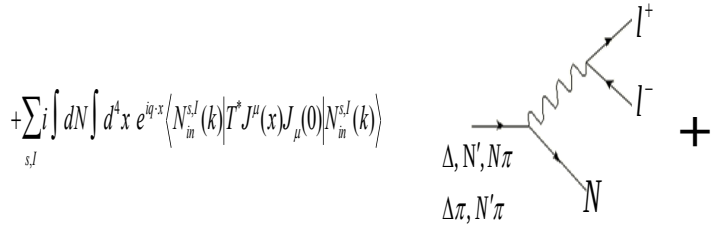
$$+\sum_a i\int d\pi \int d^4x\, e^{iq\cdot x}\left\langle \pi^a_{in}(k)\left|T^* J^\mu(x) J_\mu(0)\right|\pi^a_{in}(k)\right\rangle + \cdots$$



...

1,2,3
-N

$$+\sum_{s,I} i\int dN \int d^4x\, e^{iq\cdot x}\left\langle N^{s,I}_{in}(k)\left|T^* J^\mu(x) J_\mu(0)\right|N^{s,I}_{in}(k)\right\rangle + \cdots$$



...

Vector-Axial Mixing: κ

$$\sum_a i\int d\pi \int d^4x\; e^{iq\cdot x}\left\langle \pi^a_{in}(k)\middle| T^* J^\mu(x) J_\mu(0)\middle|\pi^a_{in}(k)\right\rangle$$

$$\int \frac{d^3k}{(2\pi^3)}\frac{n(k_0)}{2k_0 f_\pi^2}\times\left\{W_1^F(q,k)\right\}$$

K

$$\begin{aligned} W_1^F = &-12q^2\,\mathrm{Im}\Pi_V(q^2) \\ &+6(k+q)^2\,\mathrm{Im}\Pi_A[(k+q)^2] \\ &-8((k\cdot q)^2 - m_\pi^2 q^2)\,\mathrm{Im}\Pi_V(q^2)\times \mathrm{Re}[\Delta_R(k+q)] \\ &+(q\to -q) \end{aligned}$$

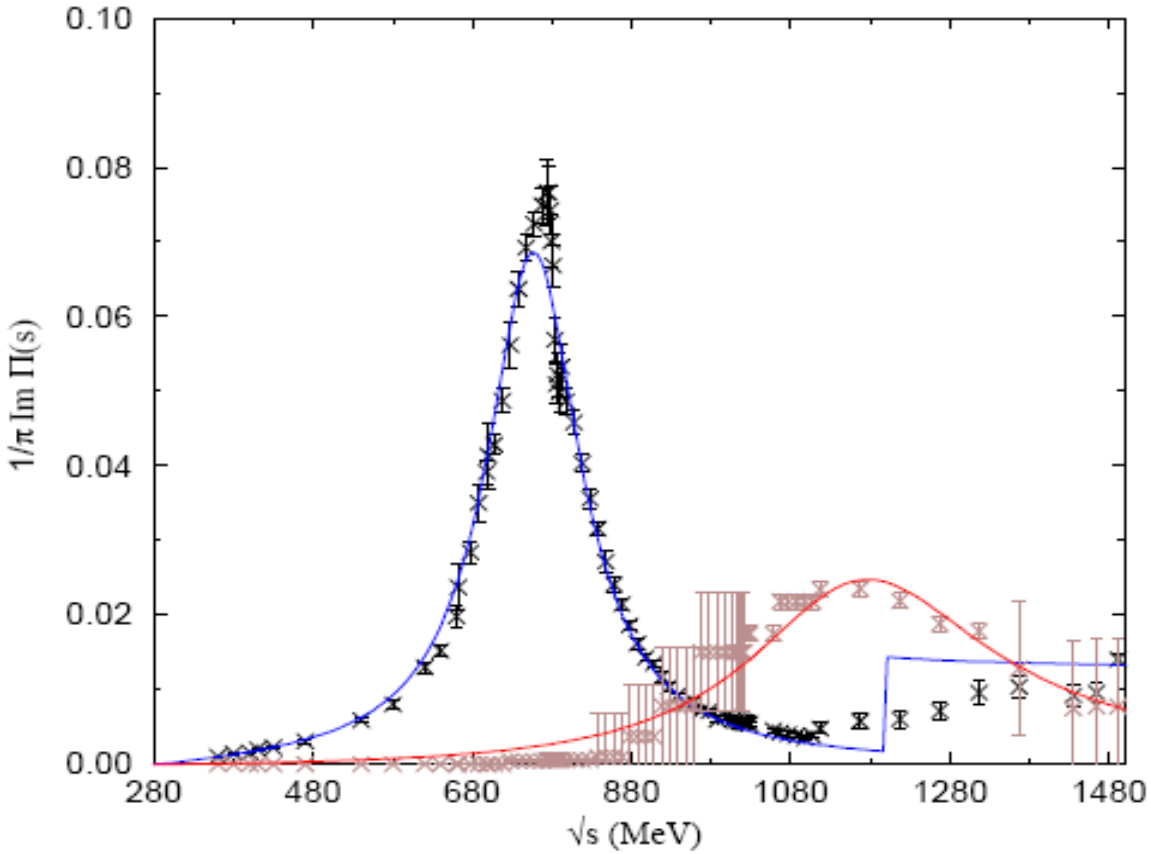
Chiral Limit and zero-mtm pion:

$$\mathrm{Im}\mathbf{W}^F(q) \approx -3q^2\left[(1-4\kappa)\,\mathrm{Im}\mathbf{\Pi}_V(q^2) + 4\kappa\,\mathrm{Im}\mathbf{\Pi}_A(q^2)\right]$$

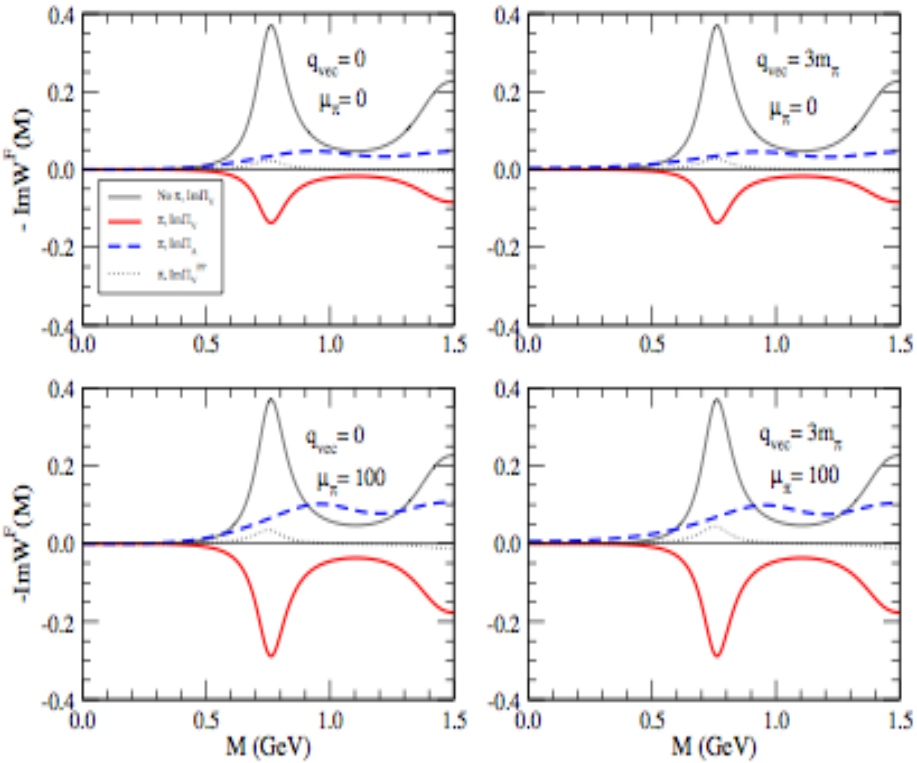
Data :

$$\Pi_V(q^2) = \langle 0 | T^* VV | 0 \rangle \quad \leftarrow \text{e+e-}$$

$$\Pi_A(q^2) = \langle 0 | T^* \, \mathrm{j_A j_A} | 0 \rangle \quad \leftarrow \tau \text{ Decay}$$



VA-Mixing explicitly



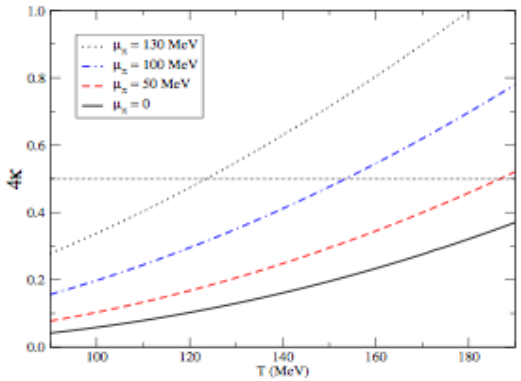


FIG. 1: Pion Density parameter κ vs temperature for different μ_π

$$\Pi_V(T) = (1-\epsilon)\,\Pi_V(0) + \epsilon\,\Pi_A(0) \to (1-\frac{1}{2})\,\Pi_V(0) + \frac{1}{2}\,\Pi_A(0)$$

$$\Pi_A(T) = (1-\epsilon)\,\Pi_A(0) + \epsilon\,\Pi_V(0) \to (1-\frac{1}{2})\,\Pi_A(0) + \frac{1}{2}\,\Pi_V(0)$$

$$\epsilon = \frac{T^2}{6 f_\pi^2} \Rightarrow 4\kappa$$

Eletski-Ioffe $m_\pi = 0$

Vector Spectral Function

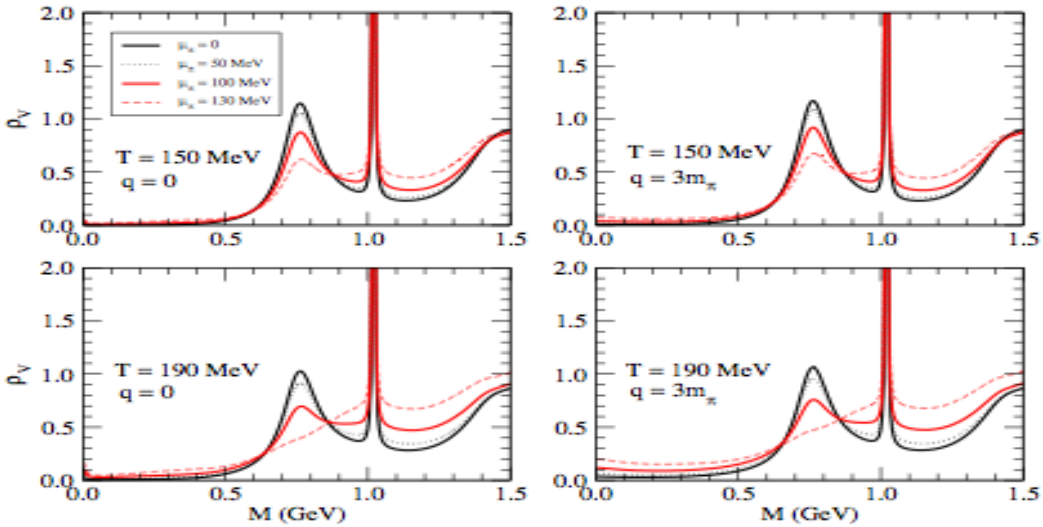


FIG. 4: Spectral function ρ_V of the hadronic gas for $T = 150$ & 190 MeV with various $|\vec{q}|$ and μ_π

$$\rho_H(M,\vec{0}) = \rho_V(M,\vec{0})$$

$$\rho_V(M,\vec{q}) = -\frac{2}{\vec{q}^2}\,\mathrm{Im}\mathbf{W}^R(M,\vec{q})$$

2-pion Final state: κ^2

$$
\begin{aligned}
\frac{1}{f_\pi^4} W^F_{\pi\pi}(q,k_1,k_2) \;=\; & \frac{2}{f_\pi^2}\left[g_{\mu\nu} - (2k_1+q)_\mu k_{1\nu} \mathrm{Re}\Delta_R(k_1+q)\right] \mathrm{Im}\mathcal{T}^{\mu\nu}_{\pi\gamma}(q,k_2) \\
& + (q \to -q) + (k_1 \to -k_1) + (q, k_1 \to -q, -k_1) \\
& + \frac{1}{f_\pi^2} k_1^\mu (2k_1+q)^\nu \mathrm{Re}\Delta_R(k_1+q) \epsilon^{a3e}\epsilon^{e3g} \mathrm{Im}\mathcal{B}^{ag}_{\mu\nu}(k_1,k_2) \\
& - \frac{1}{f_\pi^2}\left[g^{\mu\nu} - (k_1+q)^\mu (2k_1+q)^\nu \mathrm{Re}\Delta_R(k_1+q)\right] \\
& \times \epsilon^{a3e}\epsilon^{a3f} \mathrm{Im}\mathcal{B}^{ef}_{\mu\nu}(k_1+q,k_2) \\
& + \frac{1}{f_\pi^2}(k_1+q)^\mu (k_1+q)^\nu (2k_1+q)^2 \left[\mathrm{Re}\Delta_R(k_1+q)\right]^2 \\
& \times \epsilon^{a3e}\epsilon^{a3f} \mathrm{Im}\mathcal{B}^{ef}_{\mu\nu}(k_1+q,k_2) + (k_1 \to -k_1)
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Im}\mathcal{B}^{ef}_{\mu\nu}(k_1,k_2) \;=\; & \frac{2}{f_\pi^2}\delta^{ef}\left[g_{\mu\nu}(k_1+k_2)^2 - (k_1+k_2)_\mu (k_1+k_2)_\nu\right] \mathrm{Im}\Pi_V\left((k_1+k_2)^2\right) \\
& + (k_2 \to -k_2) - \frac{4}{f_\pi^2}\delta^{ef}\left[g_{\mu\nu}k_1^2 - k_{1\mu}k_{1\nu}\right] \mathrm{Im}\Pi_A\left(k_1^2\right)
\end{aligned}
$$

Electric Conductivity: σ_E

$$\sigma_E = \lim_{M\to 0} \frac{\vec{e}^2 \rho_{ii}(M,\vec{0})}{6M} = \lim_{M\to 0} \frac{-\mathrm{Im}\mathbf{W}^R(M,\vec{0})}{3M} = \lim_{M\to 0} \frac{-\mathrm{Im}\mathbf{W}^F(M,\vec{0})}{6T}.$$

$$\frac{\sigma_E}{T} \approx \frac{(N_f^2-1)}{2T^2} \sum_{s=\pm} \int \frac{d\pi_1}{f_\pi^2} \frac{d\pi_2}{f_\pi^2} (k_1+sk_2)^2 \,\mathrm{Im}\Pi_V\left((k_1+sk_2)^2\right).$$

1loop

$$\frac{\sigma_E}{T} \approx \frac{(N_f^2-1)}{96\pi T^2} \sum_{s=\pm} \int \frac{d\pi_1}{f_\pi^2} \frac{d\pi_2}{f_\pi^2} \Theta\left((k_1+sk_2)^2 - 4m_\pi^2\right) (k_1+sk_2)^2 \left(1 - \frac{4m_\pi^2}{(k_1+sk_2)^2}\right)^{3/2} ($$

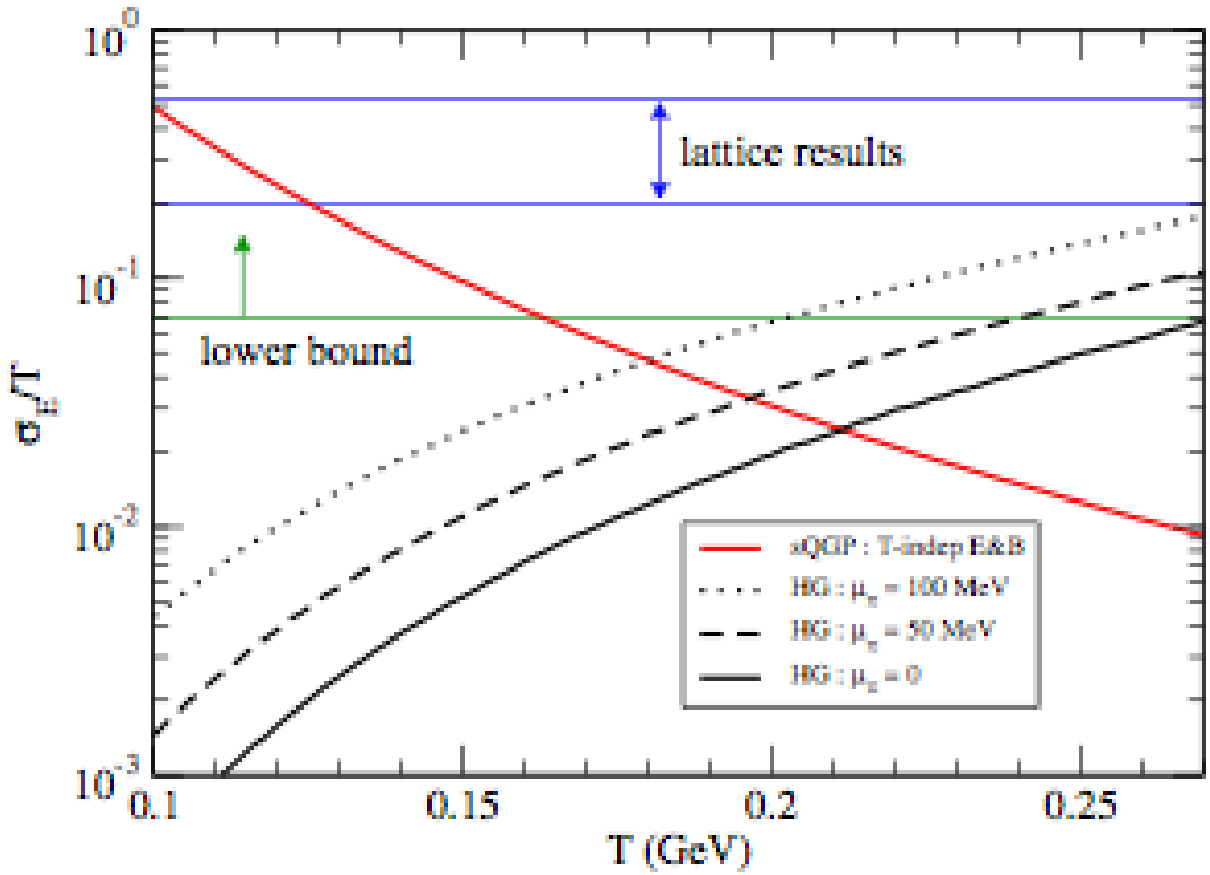
$m_\pi \to 0$

$$\frac{(N_f^2-1)}{24\pi} \frac{\kappa^2 m_\pi^2}{T^2} + \mathcal{O}\left(\frac{m_\pi^3}{T^3}\right)$$

$$\frac{(N_f^2-1) m_\pi^6}{96\pi T^2 f_\pi^4} e^{-2m_\pi/T}$$

$T \to 0$

Hadronic vs Lattice: σ_E



$$\frac{\sigma_E}{T} \approx \frac{(N_f^2-1)}{2T^2} \sum_{s=\pm} \int \frac{d\pi_1}{f_\pi^2} \frac{d\pi_2}{f_\pi^2} (k_1+sk_2)^2 \, \mathrm{Im}\Pi_V \left((k_1+sk_2)^2\right).$$

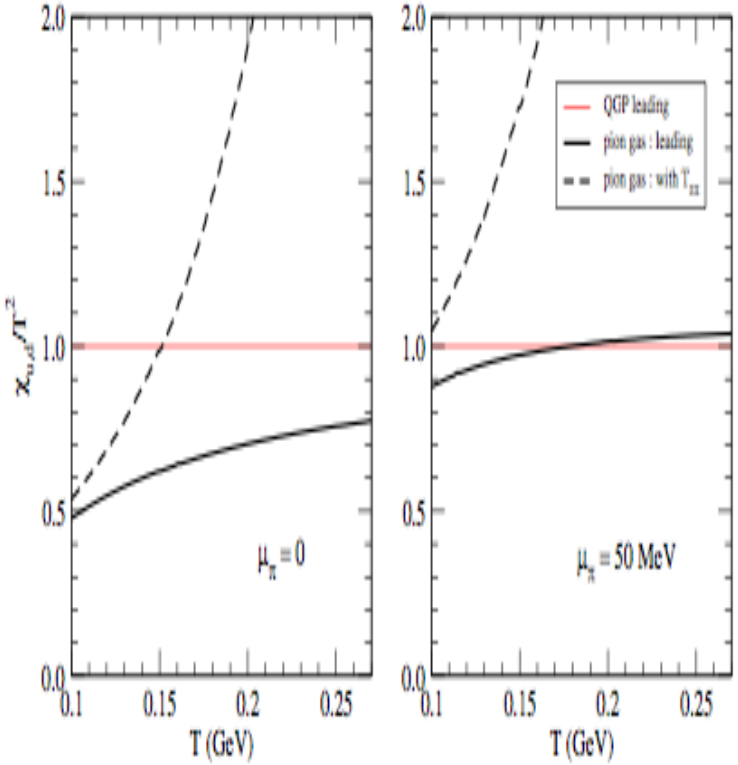
Flavour Susceptibility: $\chi_{u,d}$

$$\chi_{u,d} = \frac{1}{TV_3}\langle(\mathbf{Q}^{\mathrm{I}})^2\rangle$$
$$\approx \mathbf{I}_\pi^2\left[\frac{N_\pi}{T}\int\frac{d^3k}{(2\pi)^3}n\,(1+n)+\frac{1}{T^2}\int\frac{d^3k_1}{(2\pi)^3}\frac{n_1}{2E_1}\frac{d^3k_2}{(2\pi)^3}\frac{n_2}{2E_2}\,\mathrm{Re}\mathcal{T}_{\pi\pi}(s,t,u)\right]$$

$$\mathcal{T}_{\pi\pi}(s,t,u) \equiv \sum_{a=d,b=c}\mathcal{T}_{\pi\pi}(p_2d,k_2b\leftarrow k_1a,p_1c)\big|_{p_2=k_1,p_1=k_2}$$
$$= \mathcal{T}_{\mathrm{tree}}(s,t,u)+\mathcal{T}_{\mathrm{vector}}(s,t,u)+\mathcal{T}_{\mathrm{scalar}}(s,t,u)+\mathcal{T}_{\mathrm{rest}}(s,t,u)$$

$$\boxed{\mathbf{Q}^{\mathrm{I}} = \int d\vec{x}\,q^\dagger\frac{\lambda^3}{2}q = \int d\vec{x}\,\frac{1}{2}\left(u^\dagger u - d^\dagger d\right)}$$

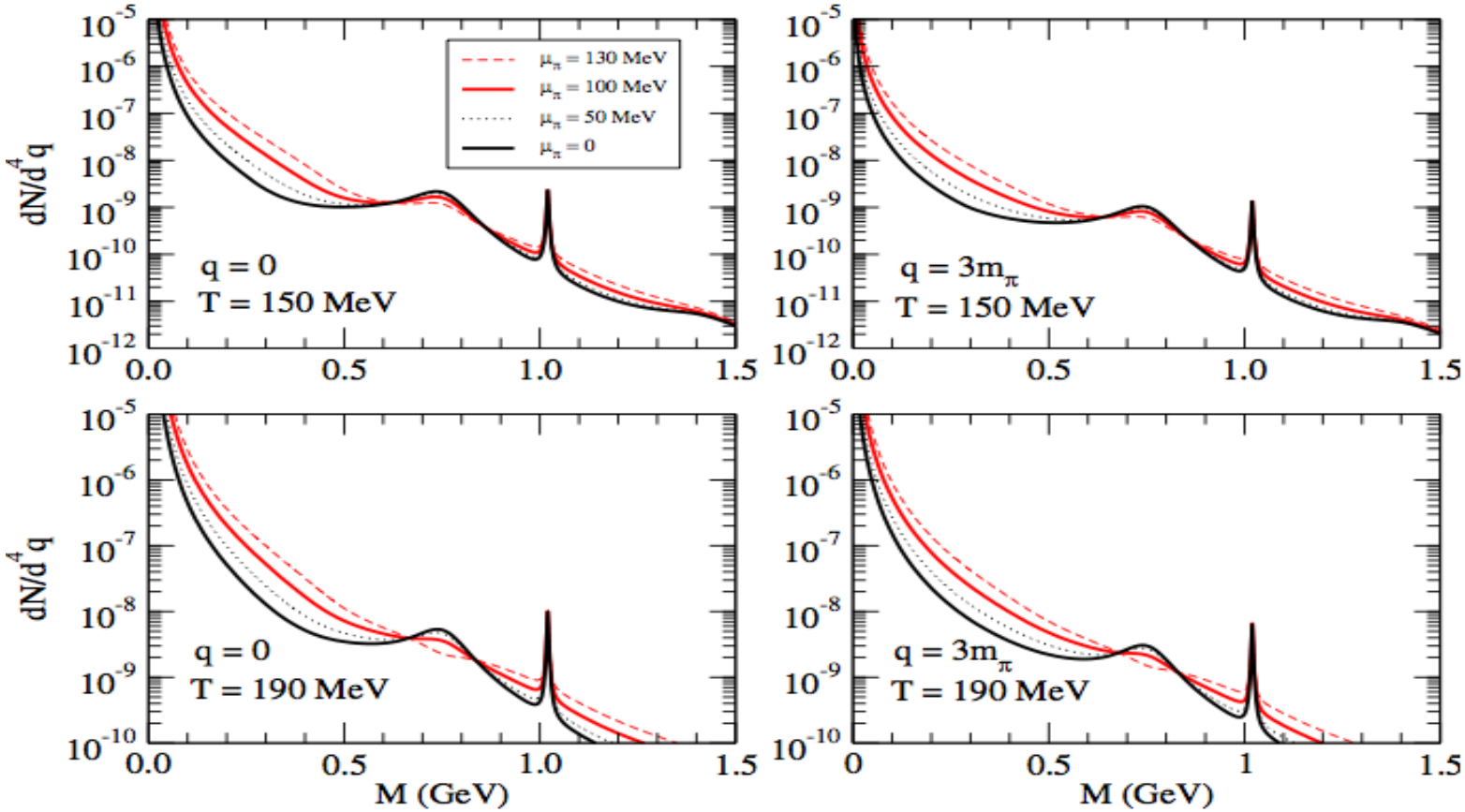
Flavour Susceptibility: $\chi_{u,d}$



$$\chi_{u,d} = \frac{1}{TV_3}\langle(\mathbf{Q}^{\mathbf{I}})^2\rangle$$
$$\approx \mathbf{I}_\pi^2\left[\frac{N_\pi}{T}\int\frac{d^3k}{(2\pi)^3}n(1+n)+\frac{1}{T^2}\int\frac{d^3k_1}{(2\pi)^3}\frac{n_1}{2E_1}\frac{d^3k_2}{(2\pi)^3}\frac{n_2}{2E_2}\,\mathrm{Re}\mathcal{T}_{\pi\pi}(s,t,u)\right]$$

$$\boxed{\chi_{u,d} \approx \mathbf{I}_\pi^2\left[\frac{N_\pi}{T}\int\frac{d^3k}{(2\pi)^3}n(1+n)-\kappa^2 N_\pi(N_\pi-2)\frac{m_\pi^2 f_\pi^2}{T^2}\right]}$$

Rates


μ_π = 130 MeV
μ_π = 100 MeV
μ_π = 50 MeV
μ_π = 0
q = 0
T = 150 MeV
q = $3m_\pi$
T = 150 MeV
q = 0
T = 190 MeV
q = $3m_\pi$
T = 190 MeV
dN/d^4q
M (GeV)

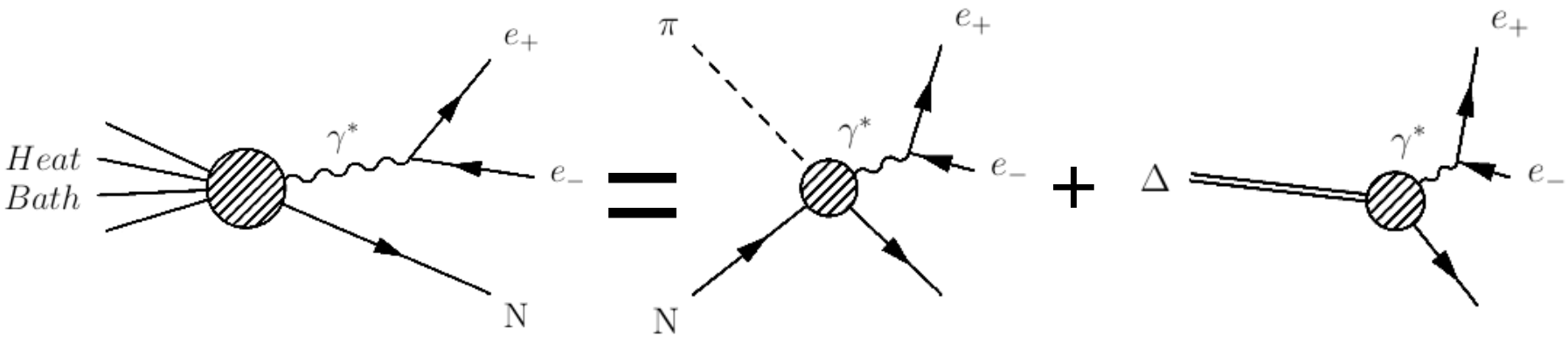
Rates: 1 Nucleon

$$W_N^F(q) = i\sum_{s,I}\int d^4x\; e^{iq\cdot x}\left\langle N_{in}^{s,I}(k)\middle|T^* J^\mu(x) J_\mu(0)\middle|N_{in}^{s,I}(k)\right\rangle$$

γ :

$$e^2 W_N^F(q,p) = -4(s-m_N^2)\sum_I \sigma_{\text{tot}}^{\gamma N}(s)$$

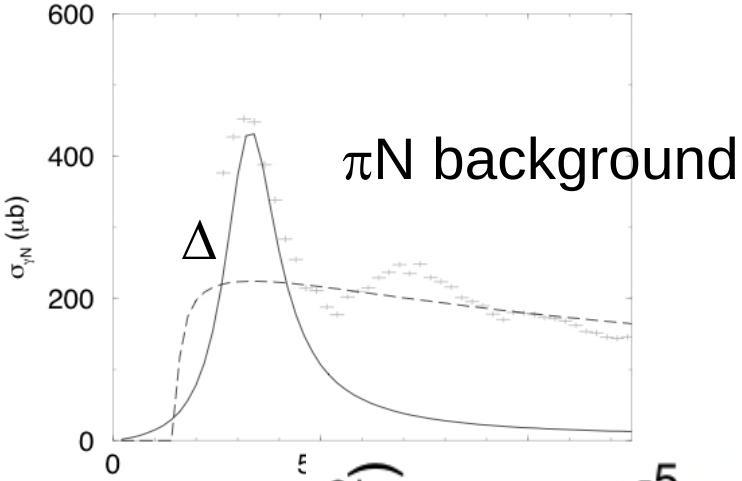
γ^* :



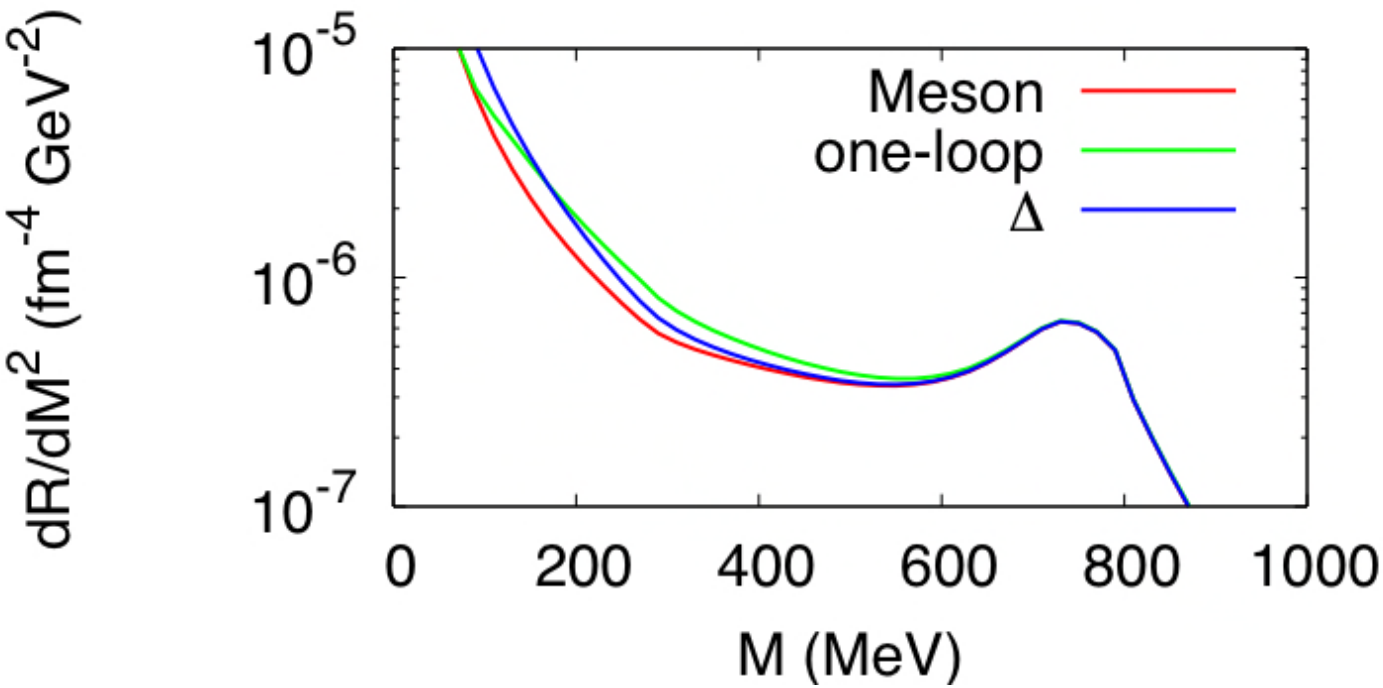
1 Loop ChPT

N-Δ Transition

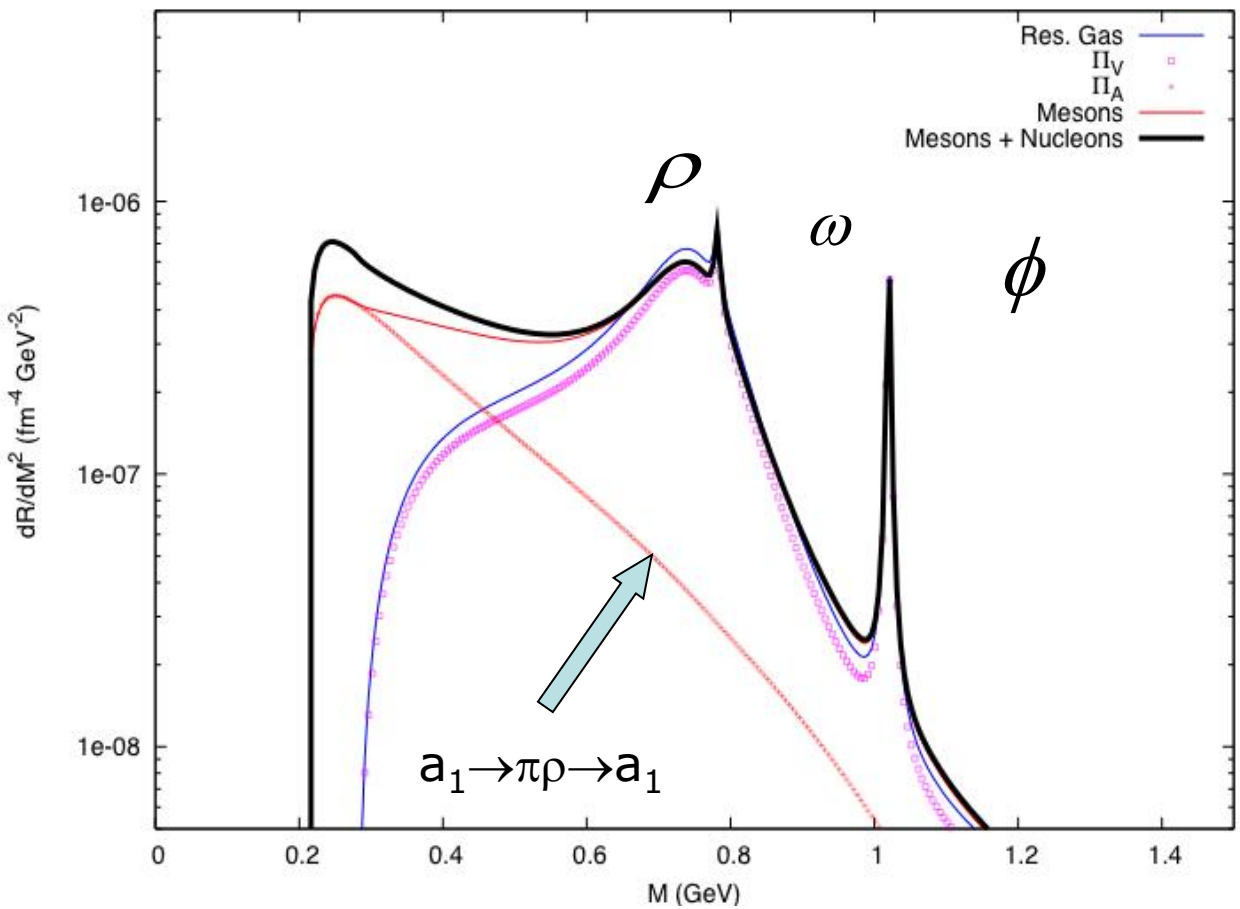
Input at the Photon Point



$$q^2 = 0 \rightarrow q^2 < 0$$



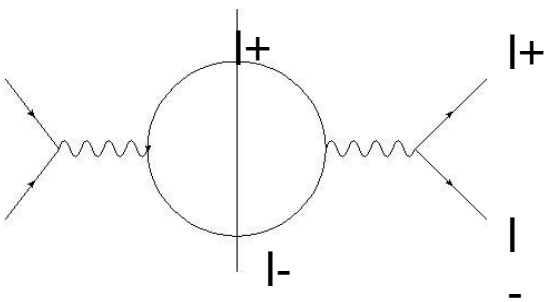
Typical Mixing in $\mu^+\mu^-$

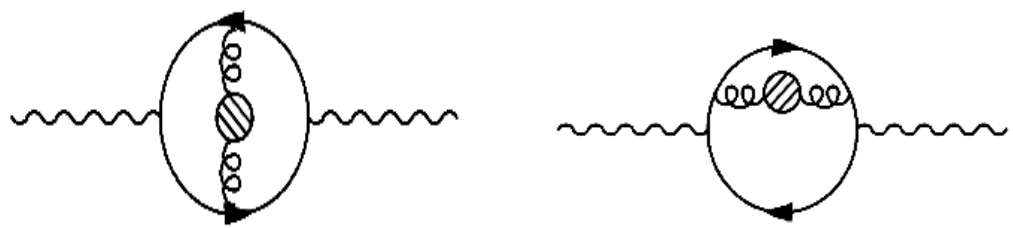


T=150 MeV μb=225 MeV

Rates: sQGP

sQGP: l^+l^-





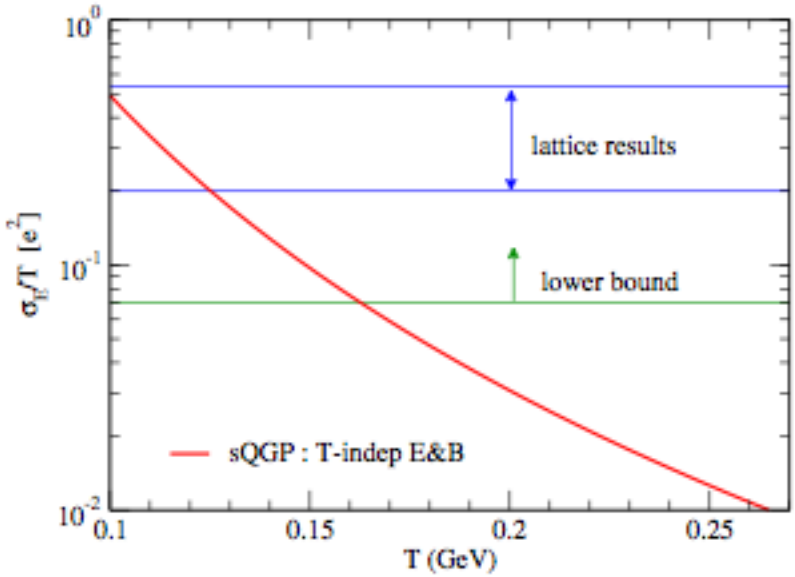
$$\mathrm{Im}\,W_{11}^{R} = \frac{1}{4\pi}\left(N_C \sum_q e_q^2\right) q^2 \left[1 + \frac{2T}{|\vec{q}|}\ln\left(\frac{n_+}{n_-}\right)\right]$$

$$\mathrm{Im}\,W_{11}^{R} = \frac{1}{4\pi}\left(N_C \sum_q e_q^2\right) q^2 \left[\left\langle \frac{\alpha_s}{\pi} A_4^2 \right\rangle \left(\frac{4\pi^2}{T|\vec{q}|}\right)\left(n_+(1-n_+) - n_-(1-n_-)\right)\right]$$

Hansson-Zahed 91'

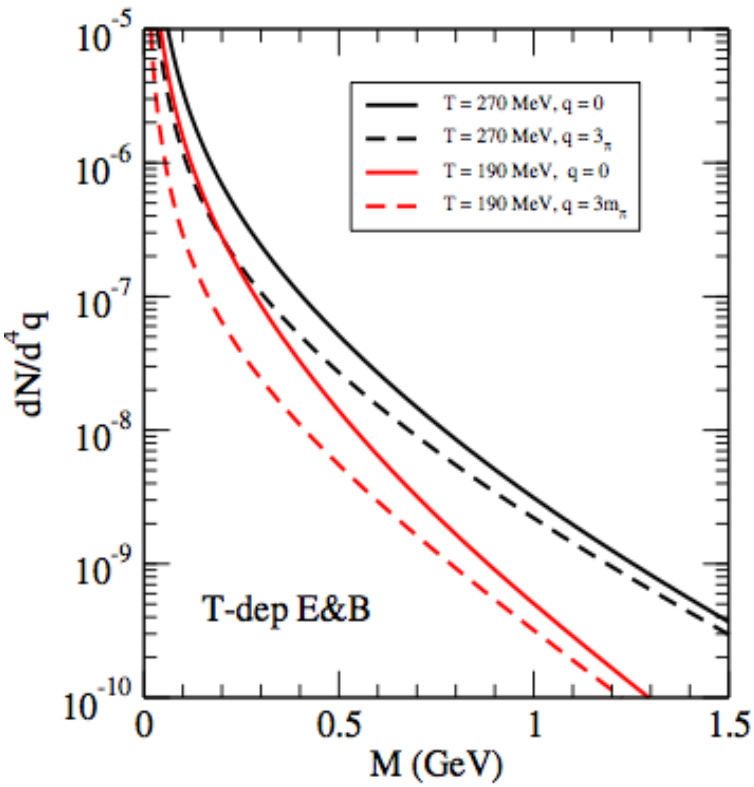
Pisarski et al semi-qgp

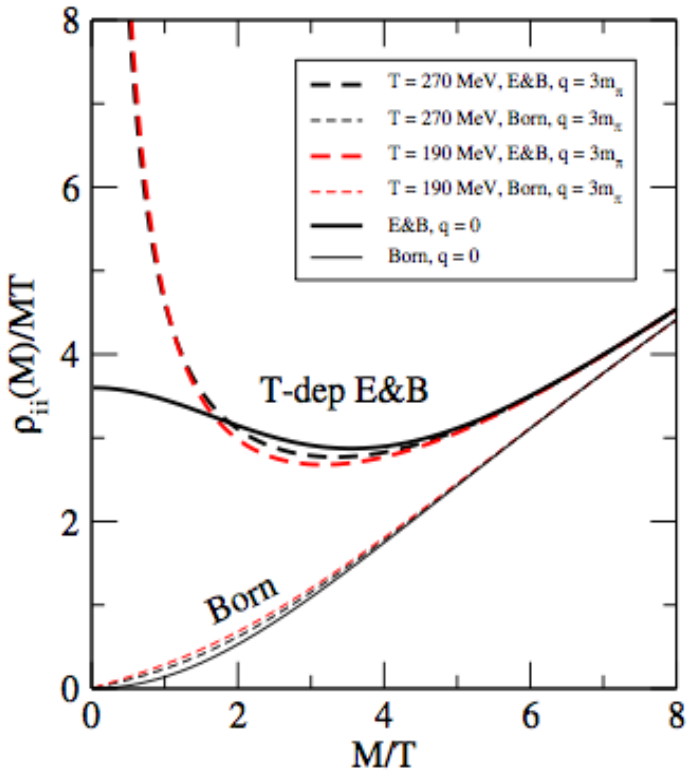
Electric Conductivity



$$\sigma_E \approx \frac{\pi N_c \bar{e}^2}{48T^3}\left(-\frac{1}{6}\left\langle\frac{\alpha_s}{\pi}E^2\right\rangle + \frac{1}{3}\left\langle\frac{\alpha_s}{\pi}B^2\right\rangle\right).$$

sQGP Rates

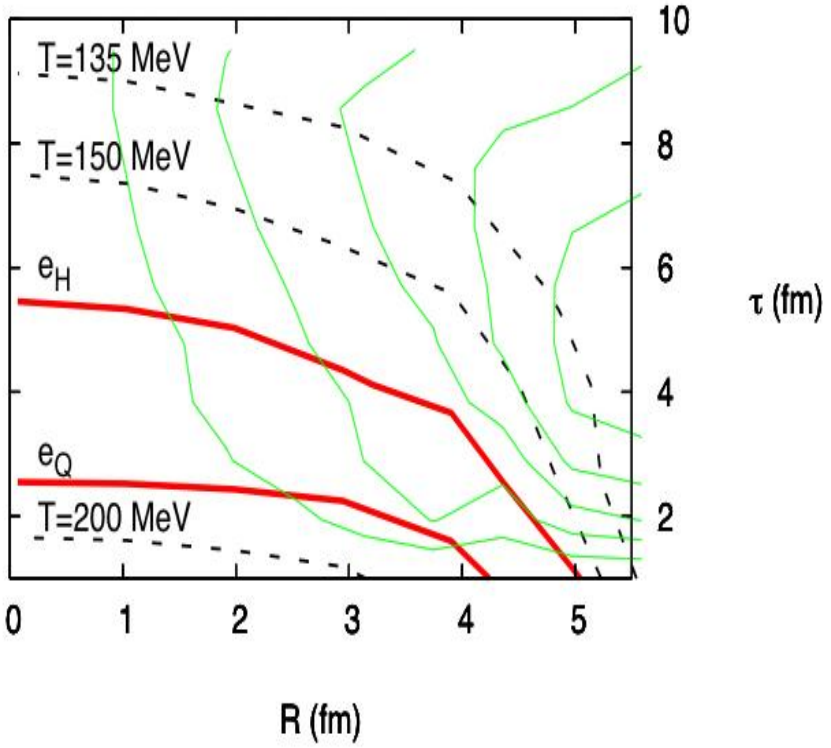


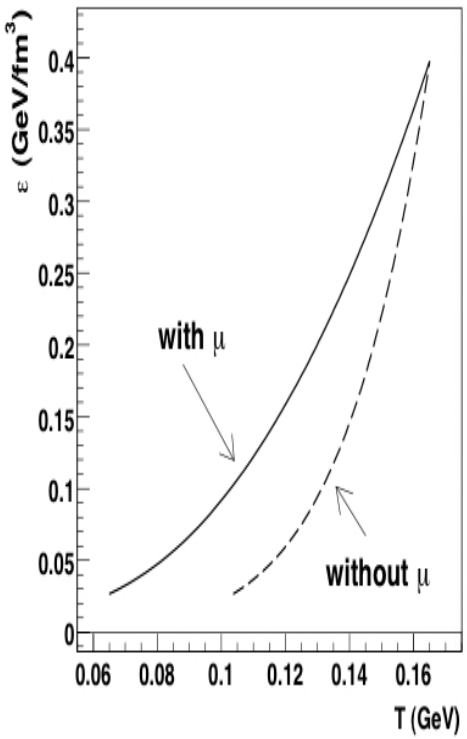


$$\langle \alpha_s E^2 \rangle \approx \langle \alpha_s B^2 \rangle \approx \frac{288}{N_c} \left\langle \frac{\sigma_E}{\bar{e}^2 T} \right\rangle T^4 \approx 48\, T^4$$

Evolution

Sample hydro evolution

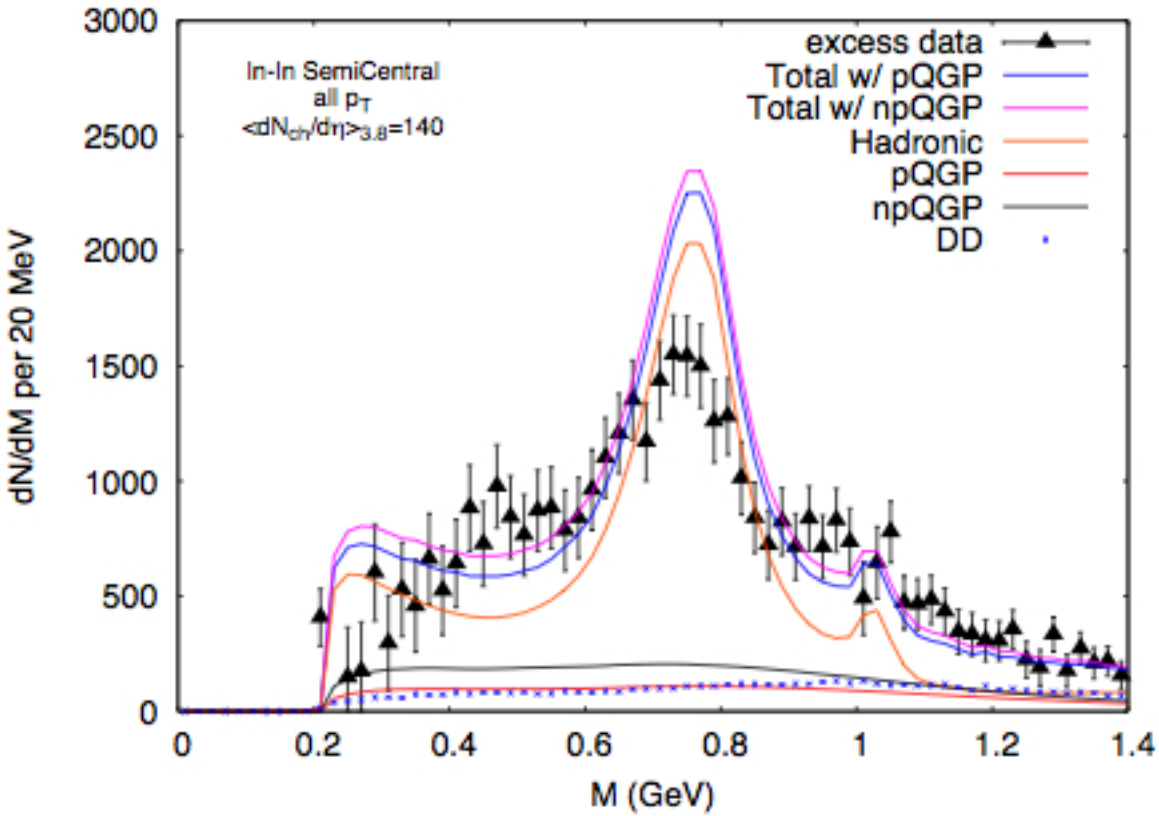




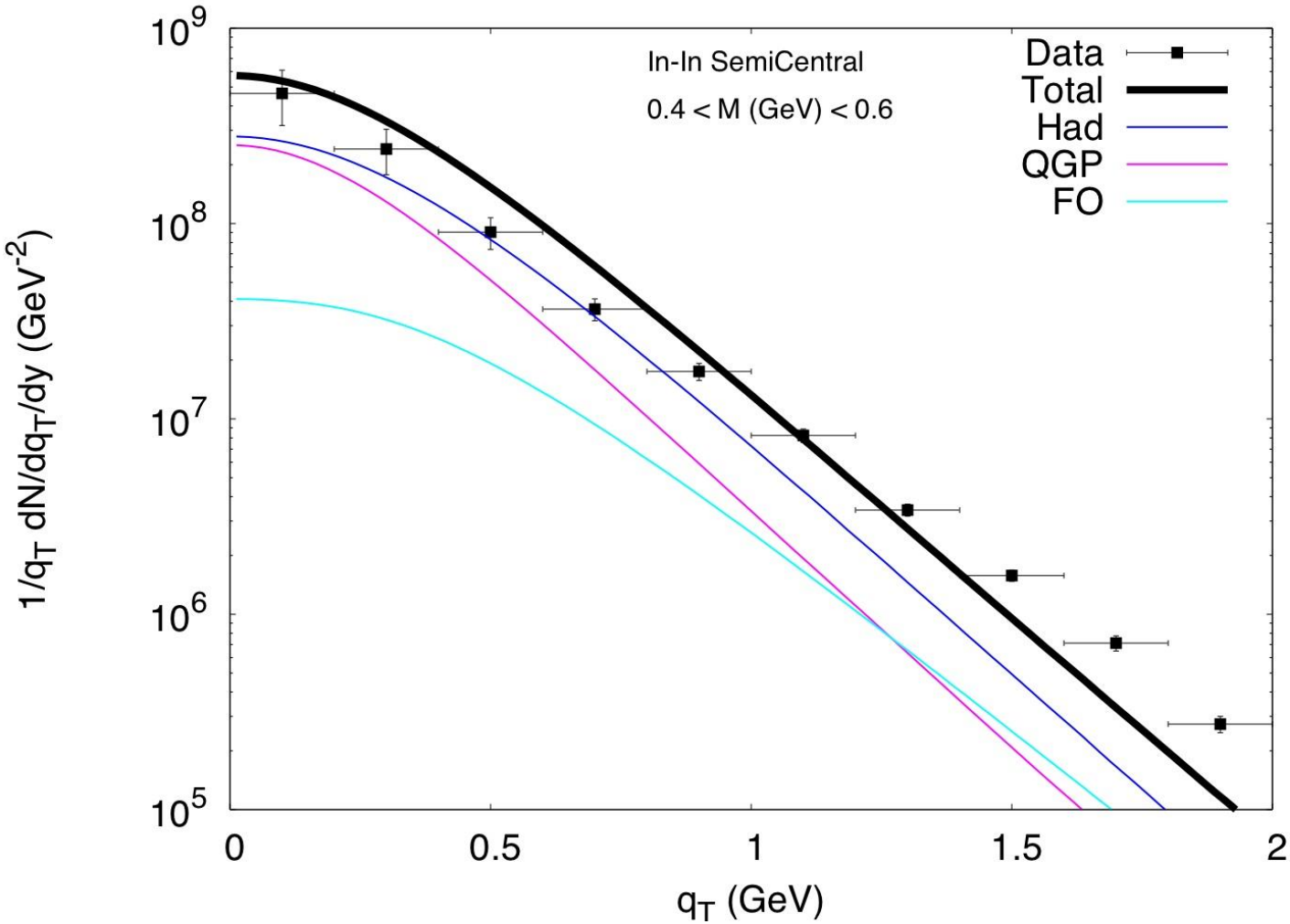
Faster cooling x higher rates: same

Dileptons: NA60, Phenix, Star

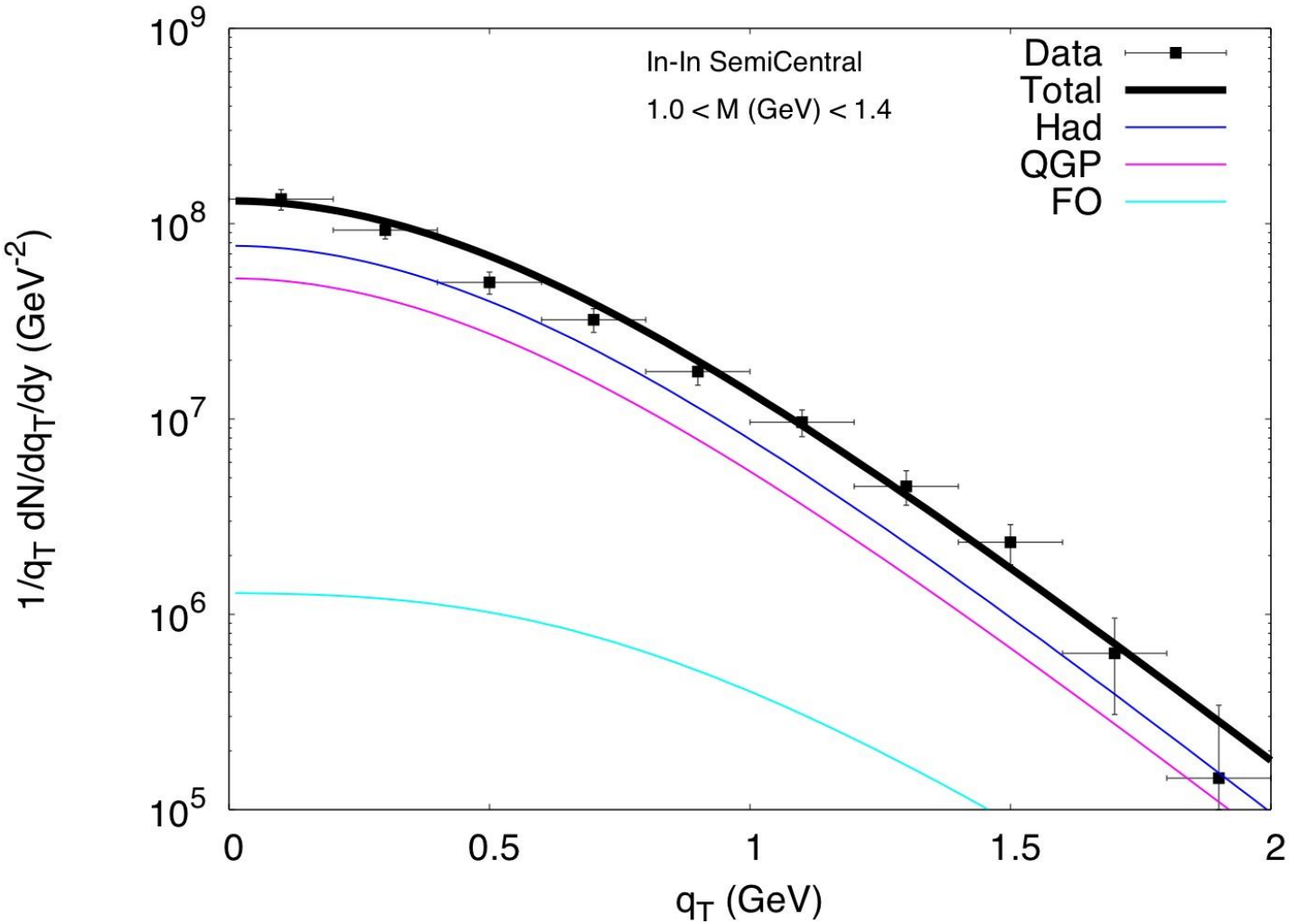
NA60 : dN/dη=140 95'



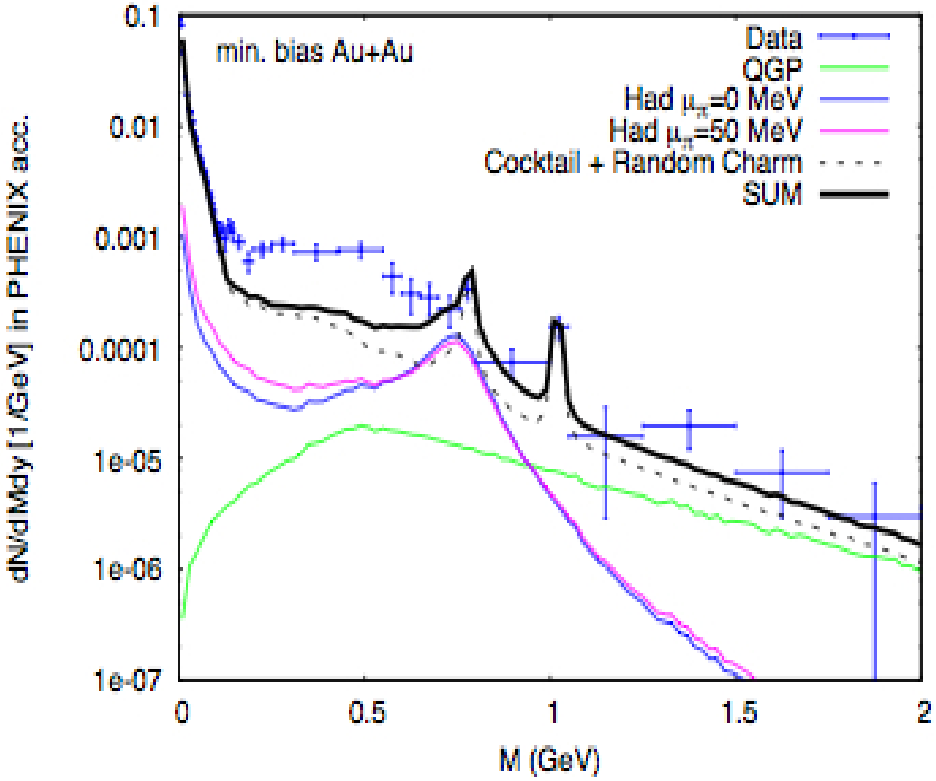
Pt Spectra: low



Pt Spectra: High



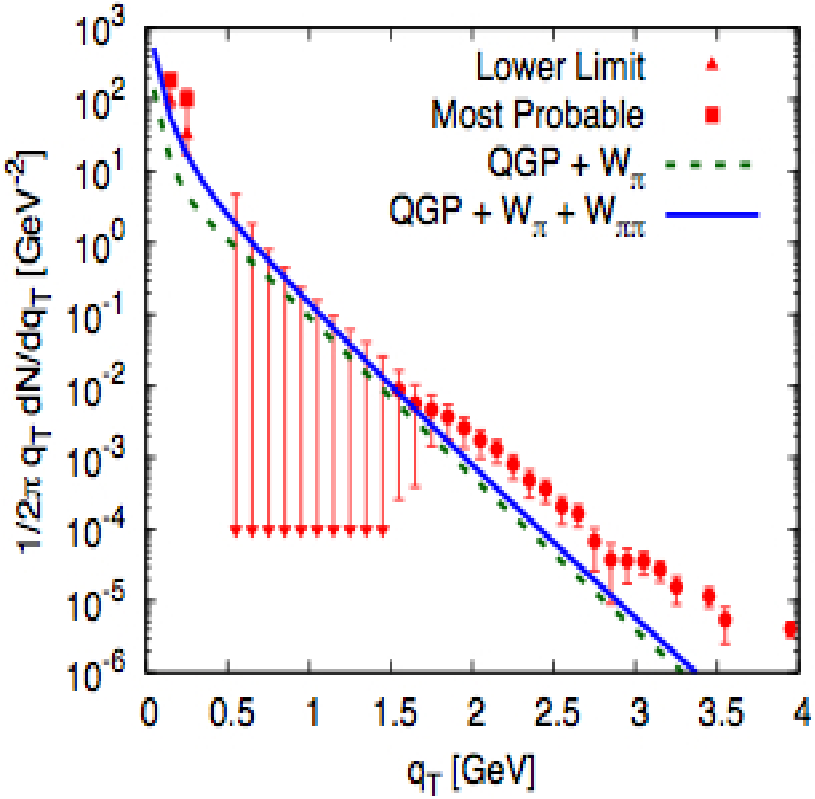
PHENIX



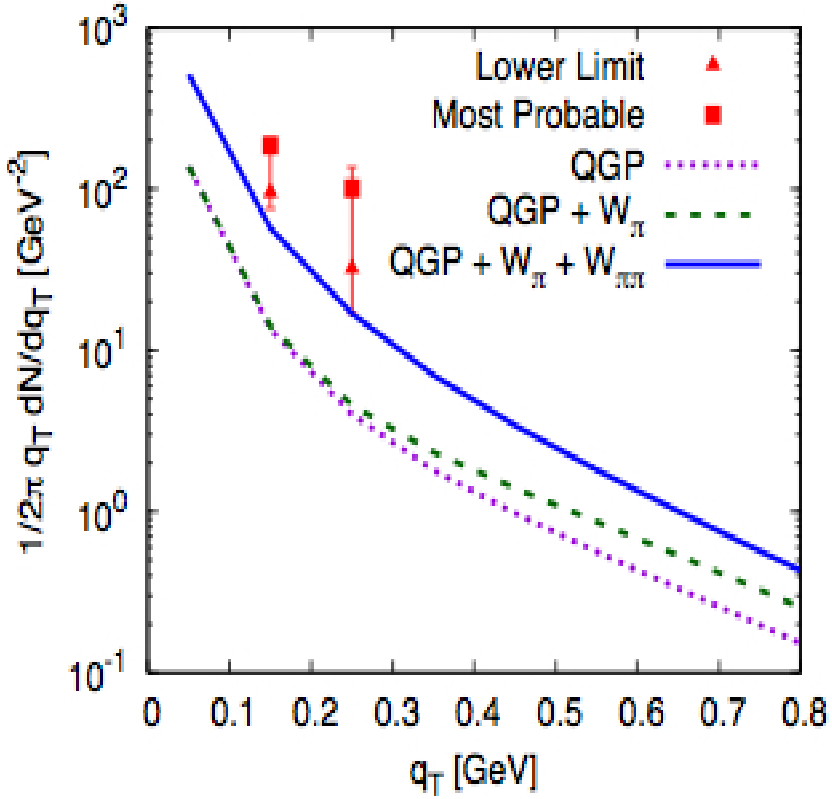
$X2$ only!

Photons: WA90, PHENIX, ALICE

WA98

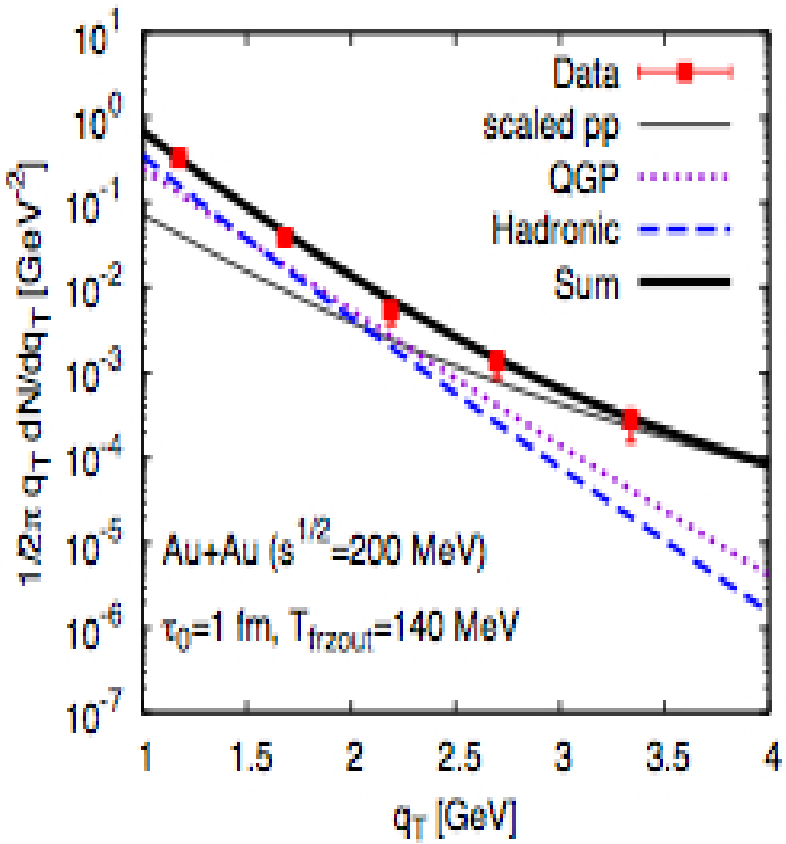


0<qT (GeV) <4

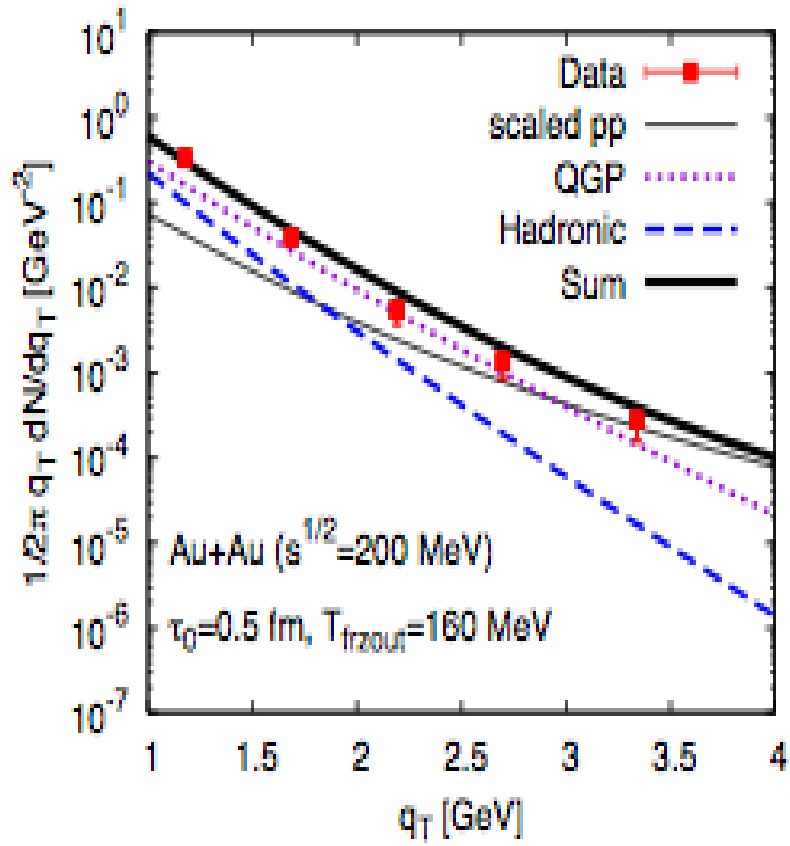


0<qT (GeV)<0.8

PHENIX



RHIC1



RHIC2

PHENIX extrapolation: γ

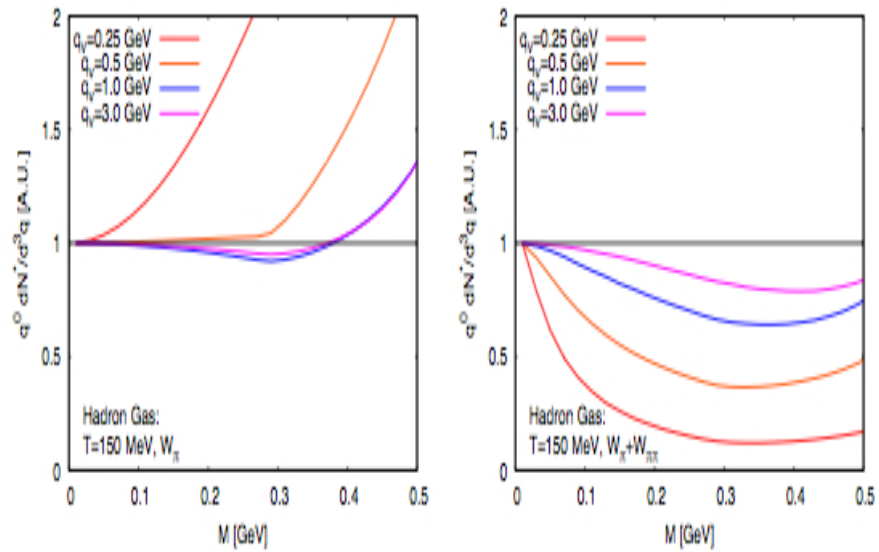
$$\frac{dR}{d^4q} = \frac{-\alpha^2}{3\pi^3 q^2}\left(1+\frac{2m_l^2}{q^2}\right)\left(1-\frac{4m_l^2}{q^2}\right)^{1/2}\frac{1}{1+e^{q^0/T}}\mathrm{Im}\mathbf{W}^F(q)$$

$$\frac{dR}{d^4q} = \frac{2\alpha}{3\pi\, q^2}\left(1+\frac{2m_l^2}{q^2}\right)\left(1-\frac{4m_l^2}{q^2}\right)^{1/2}\left(q^0\frac{dN^*}{d^3q}\right)$$

PHENIX: M<<qv

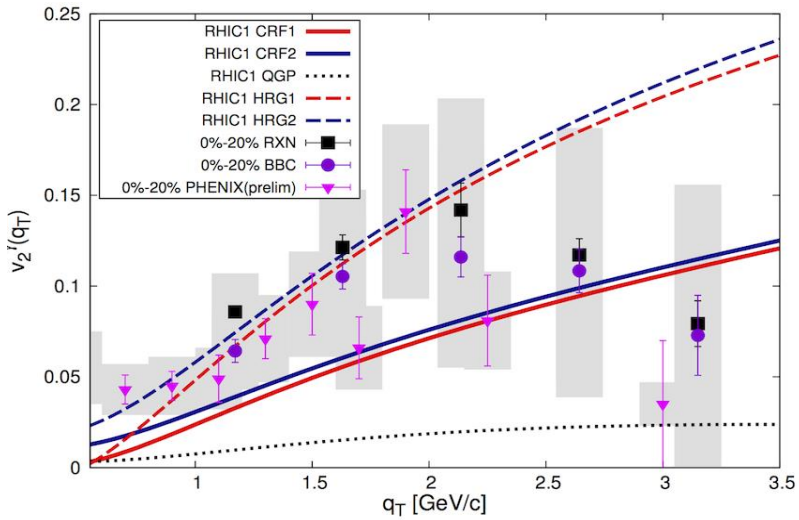
$$\frac{dR}{d^4q} = \frac{2\alpha}{3\pi\, q^2}\left(1+\frac{2m_l^2}{q^2}\right)\left(1-\frac{4m_l^2}{q^2}\right)^{1/2}\mathcal{S}\left(q^0\frac{dN}{d^3q}\right)$$

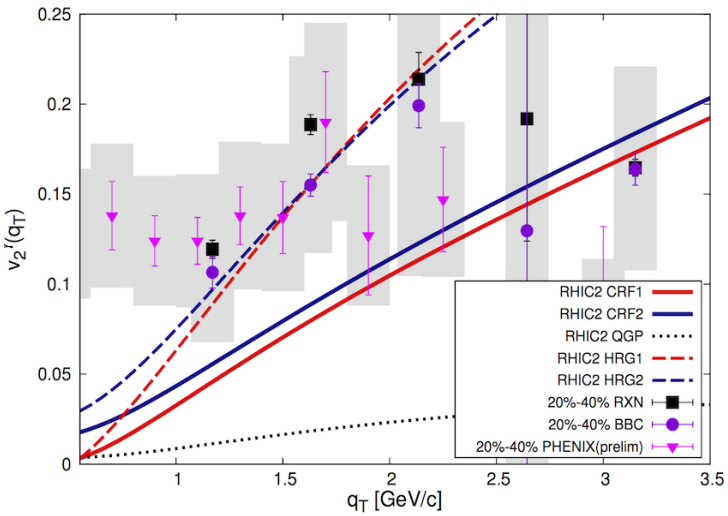
1



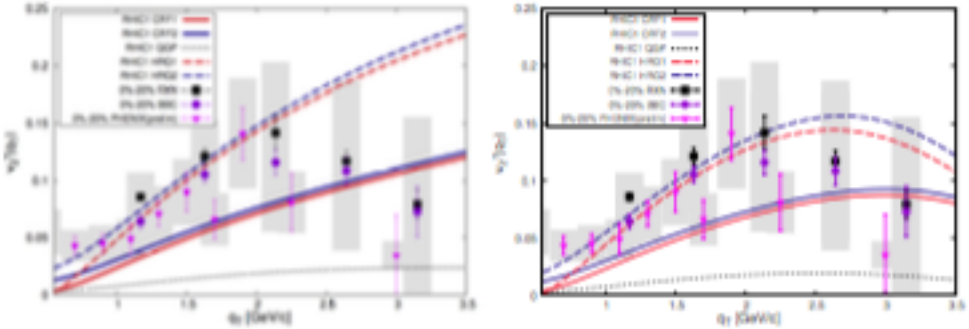
15% Systematics!

Phenix: 0-20%, 20-40%

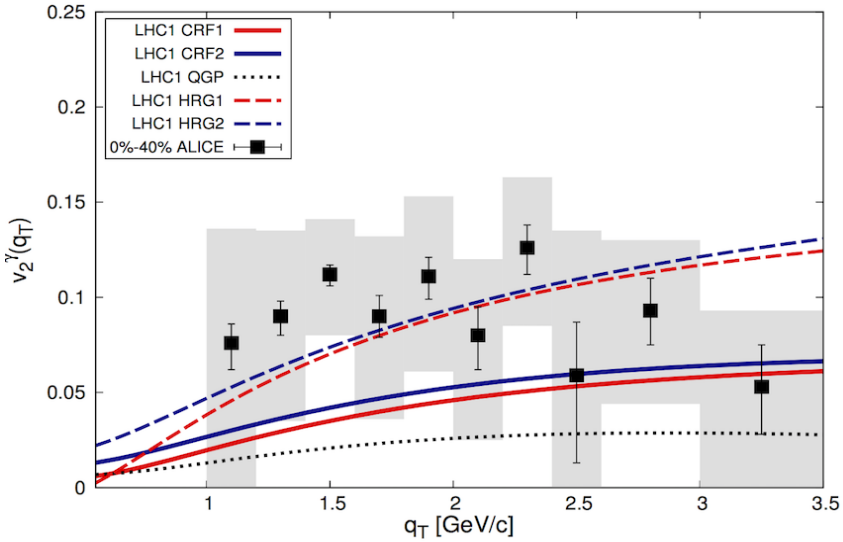


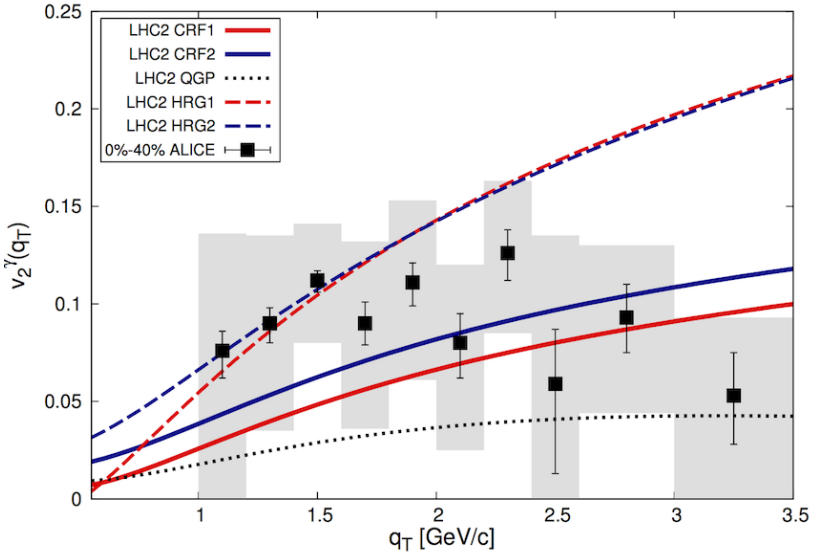


Direct-Phenix



Alice: 0-10%, 10-20%





Achievements

1. EM "OK"*
2. LxR "restored"
3. QGP "visible"
4. V2γ "low"

(* π < e^+e^- < ρ)