The LPM Effect in Jet Energy Loss and Sequential Bremsstrahlung

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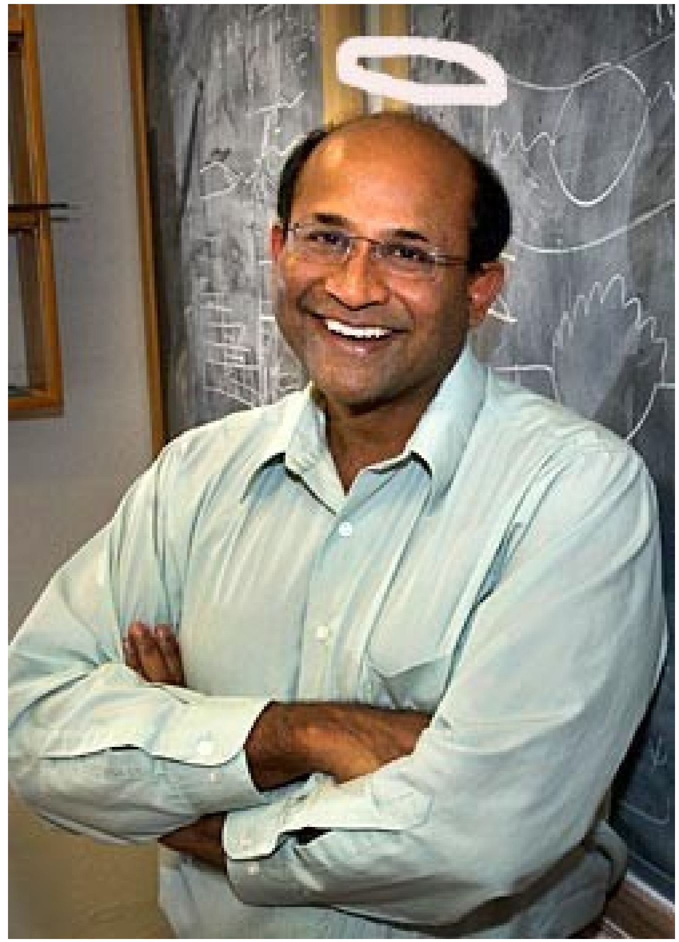
Reporting on work with Shahin Iqbal and Han-Chih Chang:

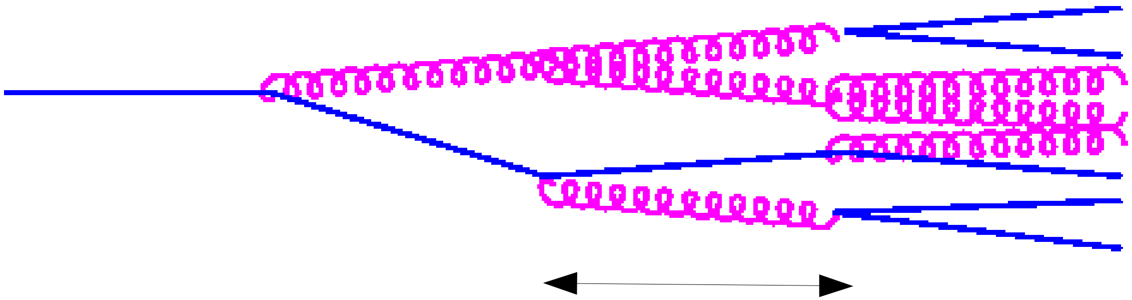
JHEP 04 (2015) 070, arXiV:1501.04964

+ work in progress

This talk dedicated to the memory of



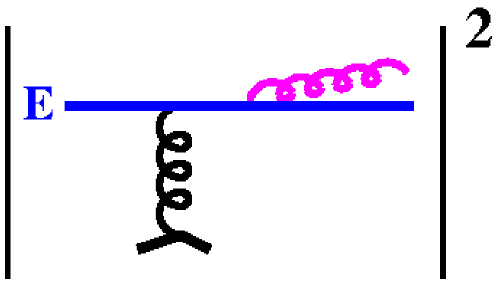
At high energy, energy loss is dominated by nearly-collinear bremsstrahlung and pair production.



Think

- cosmic ray shower
- shower chamber

Naively

brem rate ~ $n\sigma v$ ~ (density of scatterers) ×  × 1

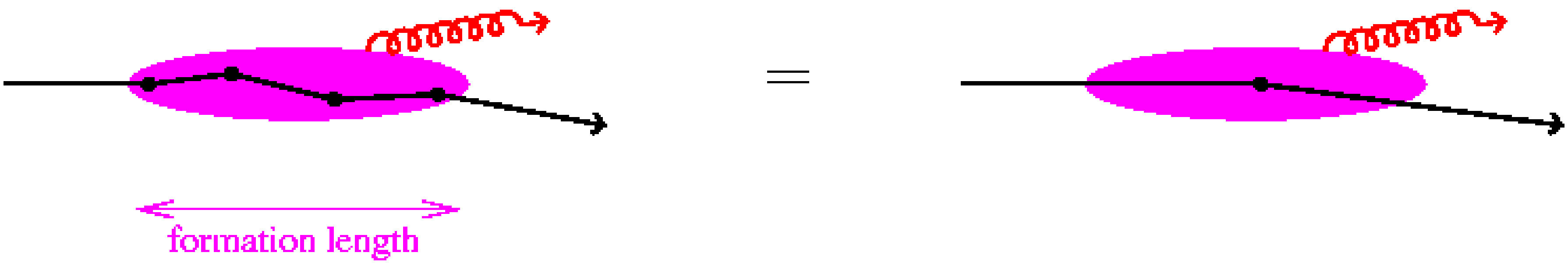
$$\Gamma_{\mathbf{brem}} \propto E^0$$ (up to logs)

Complication: The Landau-Pomeranchuk-Migdal (LPM) Effect

At very high energy,

probabilities of brem from successive scatterings no longer independent;

brem from several successive (small angle) collisions not very different from brem from one collision.



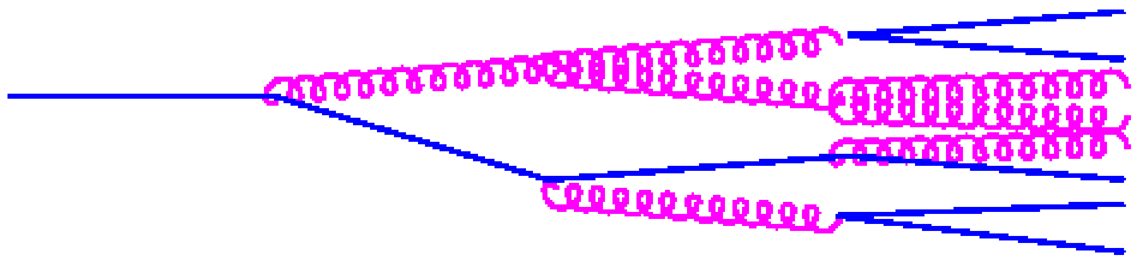
formation length $\propto \sqrt{E}$

Result: a reduction of the naive brem rate.

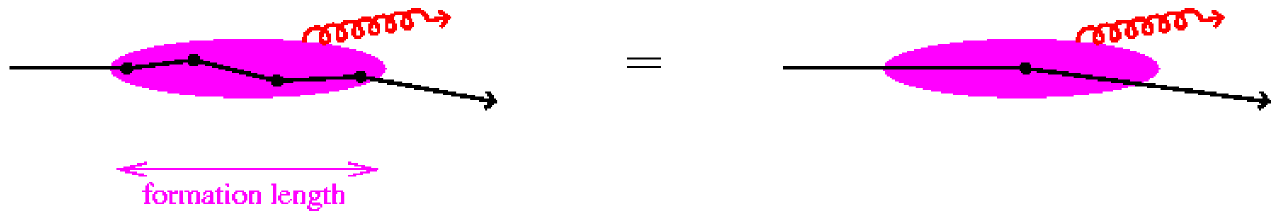
$$\boxed{\Gamma_{\mathbf{brem}} \propto E^{-1/2}}$$

Are splittings independent?

Can I just put LPM result for Γ_{brem} etc. into a Monte Carlo to get



?

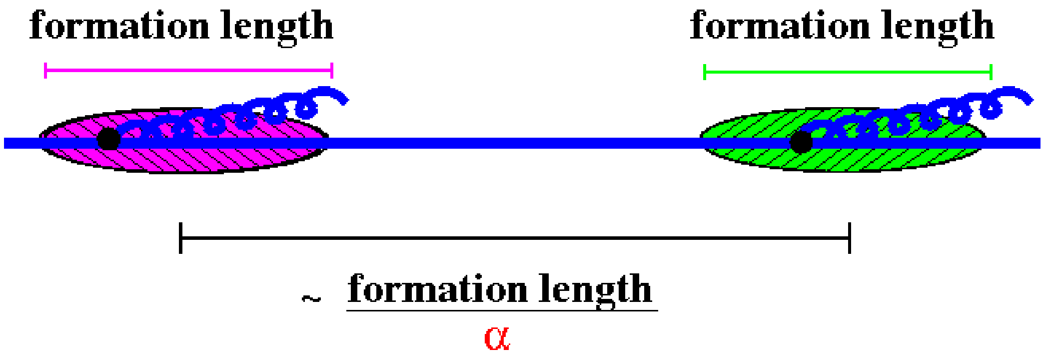


implies roughly α probability of brem per formation time.

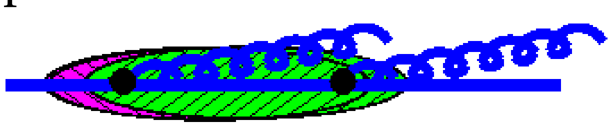
Situation #1: Democratic Splitting

Democratic $\equiv$ no daughter has energy << E.

So two consecutive splittings will typically look like



Chance of overlap



$\propto \alpha$

How big is that α?

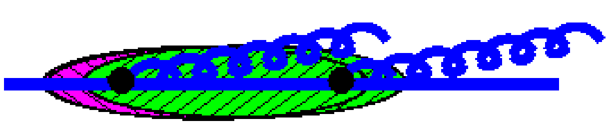
costs roughly $\alpha_s(Q_\perp)$.

$Q_\perp$ grows with formation length, which grows with E, but *slowly:*

$$Q_\perp \sim (\hat{q}E)^{1/4} \qquad [\hat{q} \sim \mathrm{GeV}^3]$$

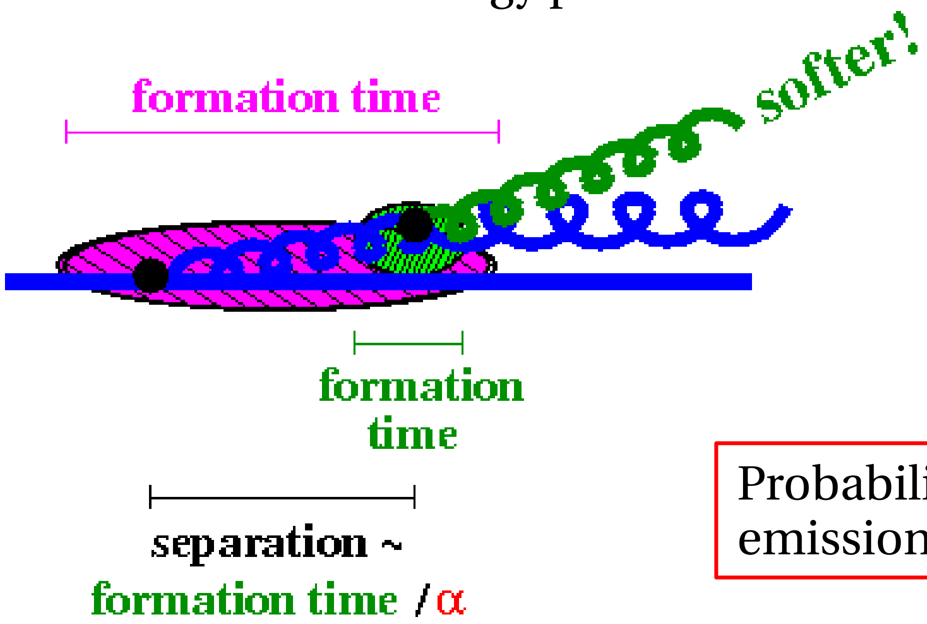
So $\alpha_s(Q_\perp)$ may not be large, but it won't be tiny for energies of interest.

Moral: it's interesting to figure out how to calculate

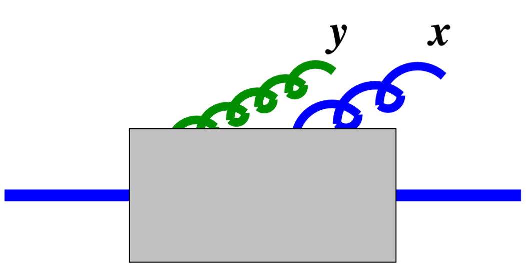


Situation #2: Soft Secondary Emission

2nd emission involves much lower energy particle:



Probability of 2nd (and so 3rd, etc.) emission can be ~ 1.



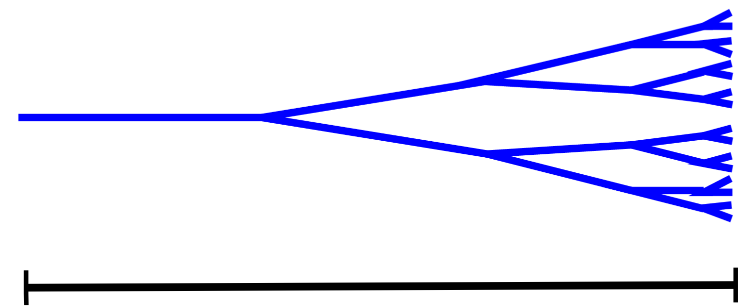
analyzed in case $x \ll y \ll 1$ by

Blaizot & Mehtar-Tani; Iancu; Wu (2014)

Interesting consequence for energy loss …

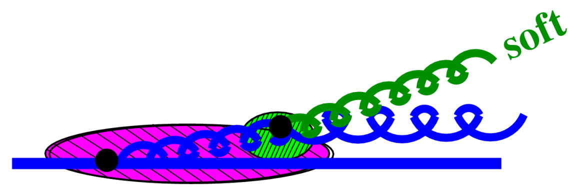
Independent splitting:

$\Gamma_{\mathbf{brem}} \propto \boldsymbol{E}^{-1/2}$ implies



$\mathbf{stopping\ distance} \propto \boldsymbol{E}^{1/2}$

Corrections due to



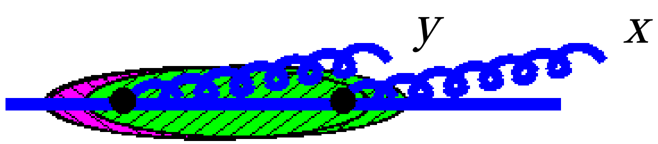
modify this to

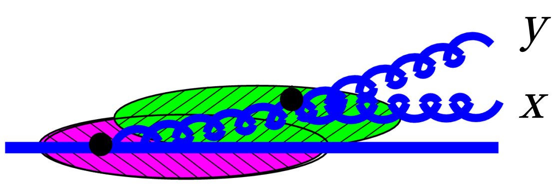
$$\boxed{\mathbf{stopping\ distance} \propto \boldsymbol{E}^{\frac{1}{2} - \#\sqrt{\alpha}}}$$

[Blaizot & Mehtar-Tani (2014)]

Our Primary Goal

Compute effects of overlapping formation times

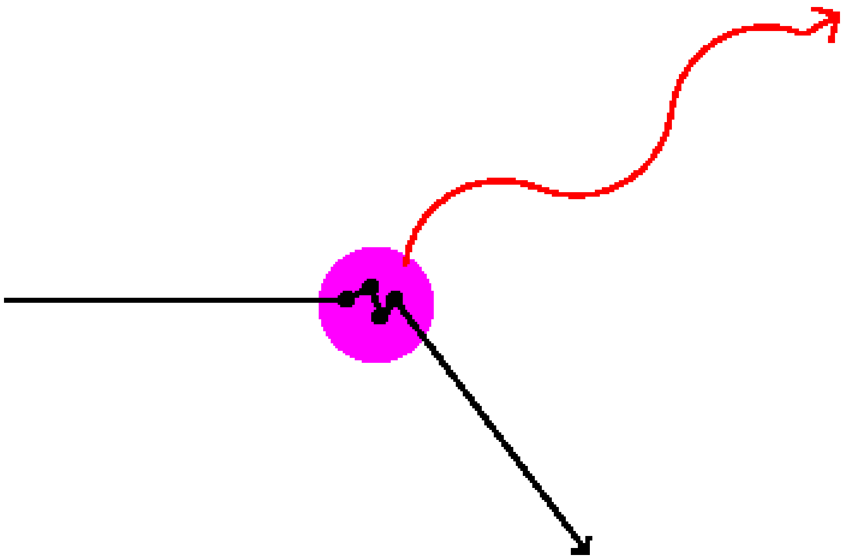




for any x and y, not just $y \ll x \ll 1$.

The LPM Effect (QED)

Warm-up: Recall that light cannot resolve details smaller than its wavelength.



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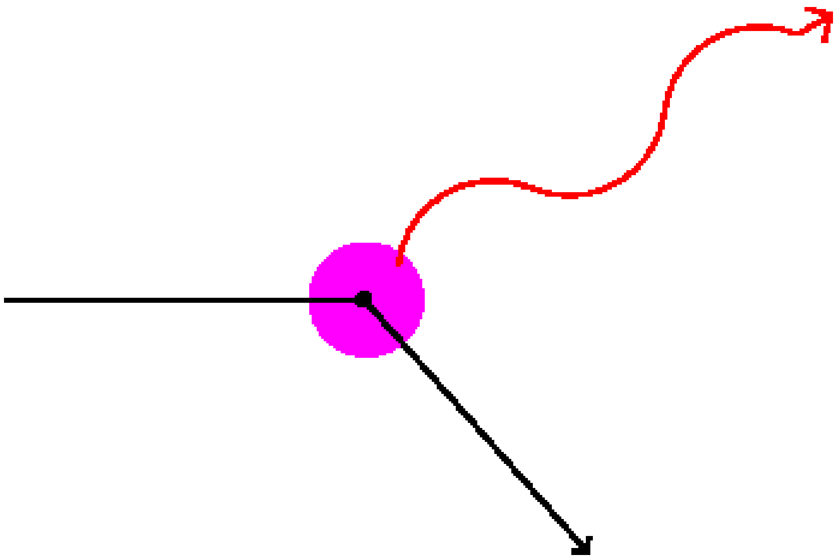
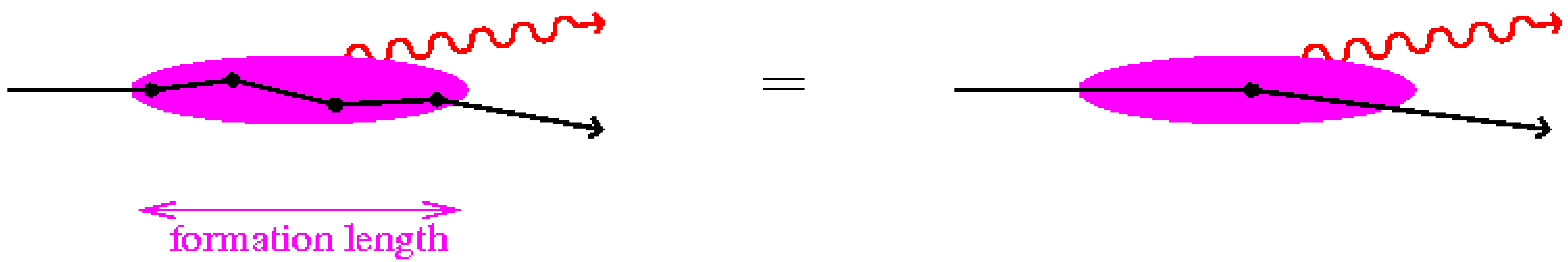


[Photon emission from different scatterings have same phase → coherent.]

Now: Just Lorentz boost above picture by a lot!

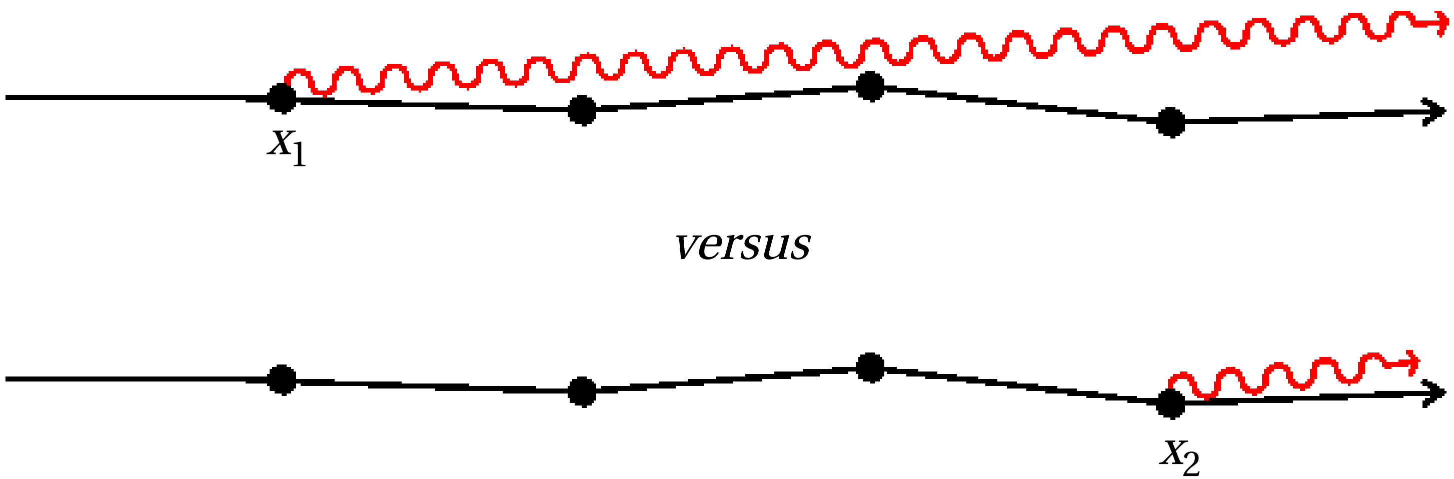
The LPM Effect (QED)



Note: (1) bigger E requires bigger boost → more time dilation → longer formation length

(2) big boost → this process is **very collinear.**

An alternative picture



Are these two possibilities in phase? Or does the interference average to zero?

IN PHASE if (i) everything is nearly collinear ✓
(ii) particle and photon have nearly same velocity ✓ (*speed of light*)

The LPM Effect (QCD)

There is a qualitative difference for ***soft*** bremsstrahlung.:

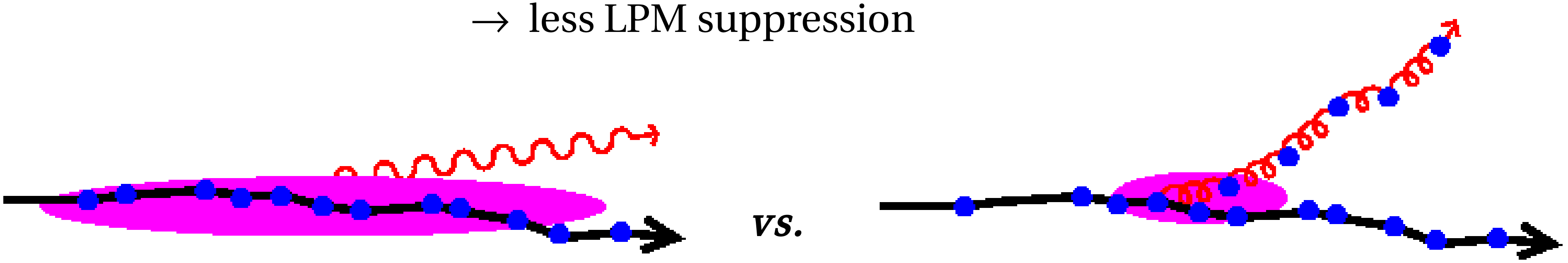
QED

Softer brem photon $\rightarrow$ longer wavelength
$\rightarrow$ less resolution
$\rightarrow$ more LPM suppression

QCD

Unlike a brem photon, a brem gluon can easily scatter from the medium.

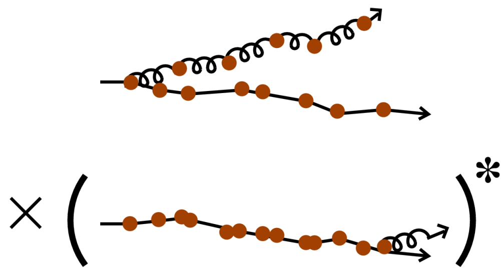
Softer brem gluon $\rightarrow$ easier for brem gluon to scatter
$\rightarrow$ less collinearity
$\rightarrow$ less LPM suppression



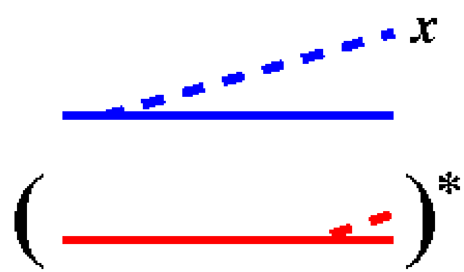
Upshot: In-medium soft brem more important in QCD than in QED.

Formalism for LPM: single brem

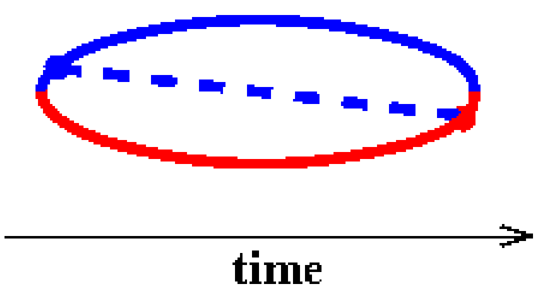
Shorthand henceforth: Draw



as simply

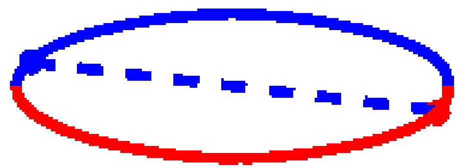


But will be even more convenient to draw as



Can (formally) interpret this as 3 particles moving forward in time [Zakharov 1990's]:

2 particles from the amplitude (evolving with e^{-iHt})
1 particle from the conjugate amplitude (evolving with e^{+iHt})

Will show that evolution in  can be described by

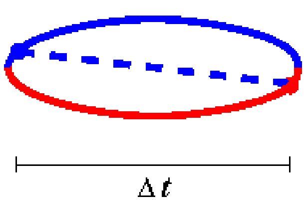
3-particle non-relativistic Quantum Mechanics in 2 dimensions

$$\mathcal{H}_{\text{eff}} = \frac{p_{\perp 1}^2}{2m_1} + \frac{p_{\perp 2}^2}{2m_2} + \frac{p_{\perp 3}^2}{2m_3} + V(b_1, b_2, b_3)$$

with weird properties:

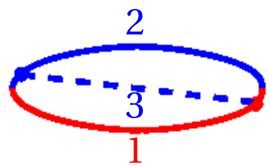
- $m_1 + m_2 + m_3 = 0$
- $V \propto -i$ (i.e. $\mathcal{H}$ is non-Hermitian)

⇨ interference vanishes as $\Delta t \to \infty$, as it must!



Kinetic terms:

Energy of a high-p_z particle: $\quad \epsilon_p = \sqrt{p_z^2 + p_\perp^2} \simeq p_z + \dfrac{p_\perp^2}{2p_z}$

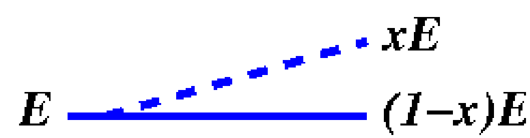
Evolution of 

is $e^{-i\mathcal{H}t}$ with

$$\mathcal{H}_{\text{kin}} = -\epsilon_{p_1} + \epsilon_{p_2} + \epsilon_{p_3} \simeq -\frac{p_{\perp 1}^2}{2p_{z1}} + \frac{p_{\perp 2}^2}{2p_{z2}} + \frac{p_{\perp 3}^2}{2p_{z3}}$$

conjugate evolves with e^{+iHt}

$$\simeq -\frac{p_{\perp 1}^2}{2E} + \frac{p_{\perp 2}^2}{2(1-x)E} + \frac{p_{\perp 3}^2}{2xE}$$



This is 2-dimensional non-relativistic QM with

$$(m_1, m_2, m_3) = (-E, (1-x)E, xE)$$

As promised,

$$m_1 + m_2 + m_3 = 0$$

Potential terms:

Accounts for local interactions with medium.

To motivate form, think of something else...

A classical Boltzman analysis of scattering:

$$\frac{d}{dt} f(p_\perp) = \underbrace{\int_{q_\perp} f(p_\perp - q_\perp) \frac{d\Gamma_{\rm el}}{dq_\perp}}_{\text{gain term}} - \underbrace{f(p_\perp) \int_{q_\perp} \frac{d\Gamma_{\rm el}}{dq_\perp}}_{\text{loss term}}$$

Fourier transform:

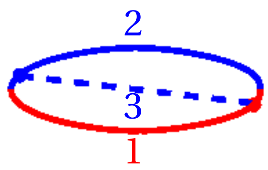
$$\frac{d}{dt} f(b) = f(b) \Big[\Gamma_{\rm el}(b) - \Gamma(0)\Big]$$

$$\text{with} \qquad \Gamma_{\rm el}(b) \equiv \int_{q_\perp} \frac{d\Gamma_{\rm el}}{dq_\perp} e^{-ib\cdot q_\perp}$$

This looks like a Schrodinger-ish equation:

$$i\frac{d}{dt} f = \mathcal{H}_{\rm boltz}\, f \qquad \text{with} \qquad \mathcal{H}_{\rm boltz} = -i\Big[\Gamma_{\rm el}(0) - \Gamma_{\rm el}(b)\Big]$$

In our problem, this physics gives V:



QED: $$V = -ie^2\left[\bar\Gamma(0) - \bar\Gamma_{\rm el}(b_2-b_1)\right]$$

(bar over Γ means charge e^2 factored out)

QCD:

$$V = -ig^2\Big[\tfrac{1}{2}T_1^2\,\bar\Gamma_{\rm el}(0) + \tfrac{1}{2}T_2^2\,\bar\Gamma_{\rm el}(0) + \tfrac{1}{2}T_3^2\,\bar\Gamma_{\rm el}(0)$$

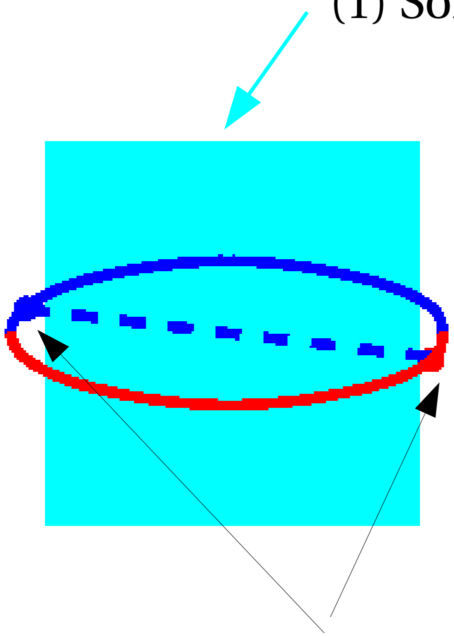
$$+T_2\cdot T_1\,\bar\Gamma_{\rm el}(b_2-b_1) + T_3\cdot T_2\,\bar\Gamma_{\rm el}(b_3-b_2) + T_1\cdot T_3\,\bar\Gamma_{\rm el}(b_1-b_3)\Big]$$

Color factors $T_i \cdot T_j$ are fixed (not dynamical) because $T_1 + T_2 + T_3 = 0$

⇨ e.g. $T_2 \cdot T_1 = -\frac{1}{2}(T_3^2 - T_1^2 - T_2^2) = -\frac{1}{2}(C_1 - C_2 - C_3)$

How to put the calculation together:

(1) Solve for propagation in 3-particle QM in shaded region.



(2) Tie together with QFT matrix elements for vertices

$$\propto \sqrt{\textbf{DGLAP splitting functions}}$$

$$\propto \sqrt{P_{i \to j}(x)}$$

Simplifcation: 3-particle QM → 1-particle QM

$$\mathcal{H}_{\text{eff}} = \frac{p_{\perp 1}^2}{2m_1} + \frac{p_{\perp 2}^2}{2m_2} + \frac{p_{\perp 3}^2}{2m_3} + V(b_1, b_2, b_3)$$

- Translation invariance:

  Factor out COM motion → 2-particle QM

- Results should not depend on exact choice of z axis:

→ can factor out d.o.f. associated with tiny changes of z axis

→ 1-particle QM

[In 2-dim QM language, the last simplification depends on a special property of the case $m_1+m_2+m_3 = 0$.]

Solving 1-particle QM:

$$\mathcal{H} = \frac{P_B^2}{2M} + V(B)$$

[BPMPS-Z (1990's)]

Method 1. Can solve numerically.

[Zakharov (2004+); Caron-Huot & Gale (2010)]

Method 2. High energies → very collinear → b 's small.

So make small b approximation to

$$-i\Big[\Gamma_{\rm el}(0) - \Gamma_{\rm el}(b)\Big] \simeq -i\hat{q}b^2$$

$$\hat{q} \equiv \int_{q_\perp} \frac{d\Gamma_{\rm el}}{dq_\perp}\, q_\perp^2$$

→ a harmonic oscillator problem

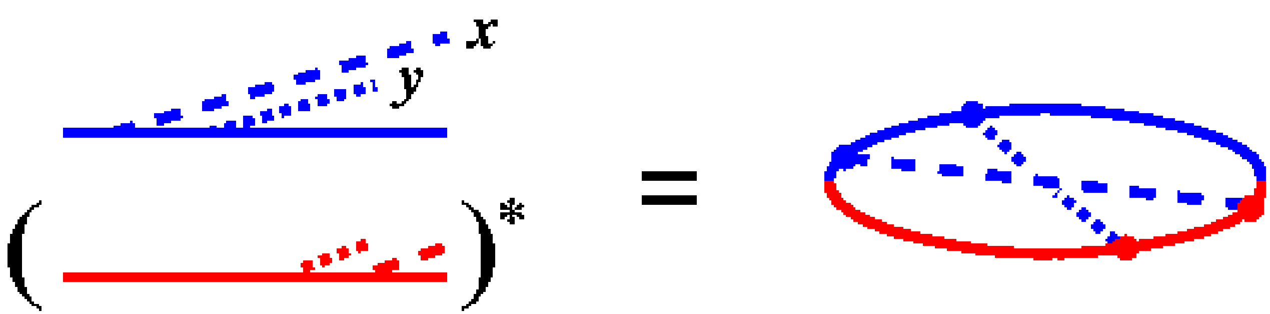
$$\mathcal{H} = \frac{P_B^2}{2M} + \frac{1}{2} M\Omega_0^2 B^2$$

[Baier *et al*. (1998)]

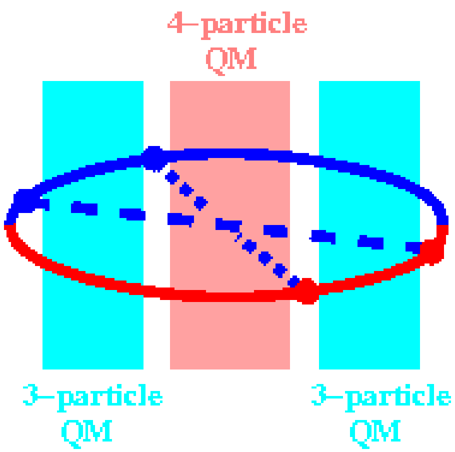
(a non-Hermitian one: $\Omega_0^2 \propto -i$)

Formalism for LPM: double brem

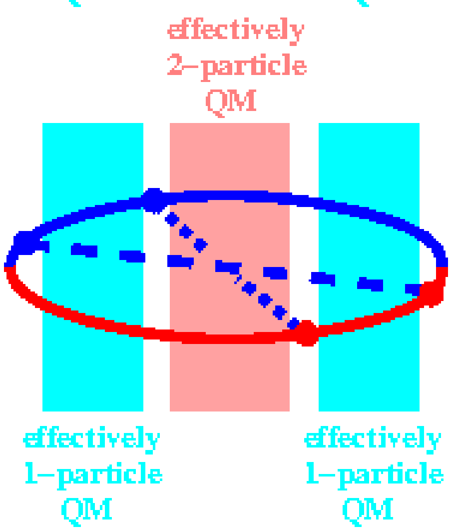
Example of an interference contribution:

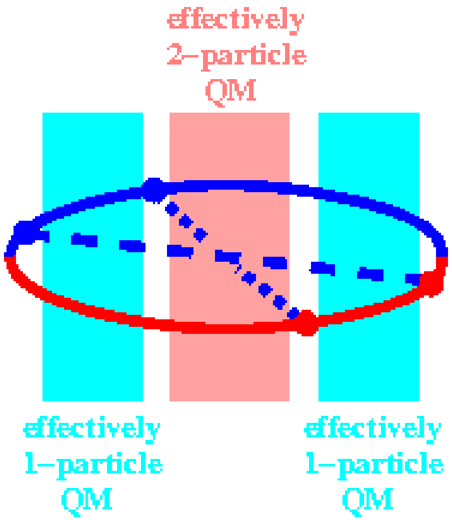


To compute : Sew together QFT matrix element for vertices with QM evolution in between.



Simplify : Using symmetries, as before.





ugliest bit = 2-particle QM evolution

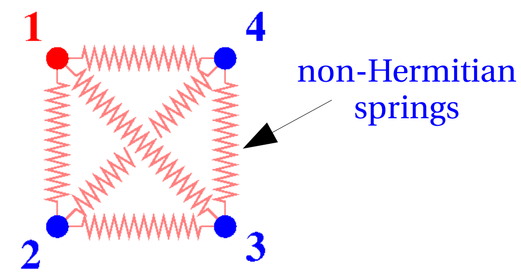
Can imagine

- numerics [have not done]
- harmonic osc. approximation [have done!]

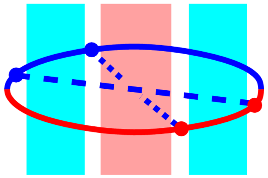
Harmonic osc. ***sounds*** very straight-forward, but in fact quite complicated.

What do we do?

- For 4-particle (effectively 2-particle) evolution, find eigenmodes and frequencies of

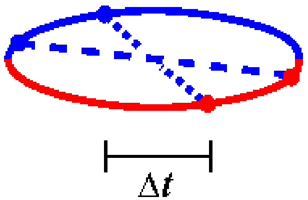


- Construct corresponding propagator for 4-particle (2-particle) evolution.
  [Also do the same for 3-particle (1-particle) evolution.]



- Combine with QFT matrix elements for splitting vertices.

- Analytically integrate over all vertex times except Δt :



- Analytically integrate over all vertex transverse positions.

Result:

$$\boxed{\textbf{answer} = \int_0^\infty d(\Delta t)\ \textbf{complicated formula}}$$

- Final Δt integral easy to do numerically.

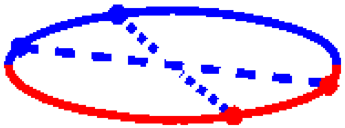
Complications

Formalism: Getting straight the formalism for 4-particles → effectively 2 particles.

Color: During 4-particle evolution, $T_1+T_2+T_3+T_4 = 0$ is not enough to fix color factors $T_i \cdot T_j$.

Color dynamics is non-trivial!

For now : Work in large N_c limit. [Not necessary if the brems are soft.]

Helicities: Helicities of high-energy particles contract non-trivially in 

Must use helicity-dependent DGLAP splitting functions at vertices.

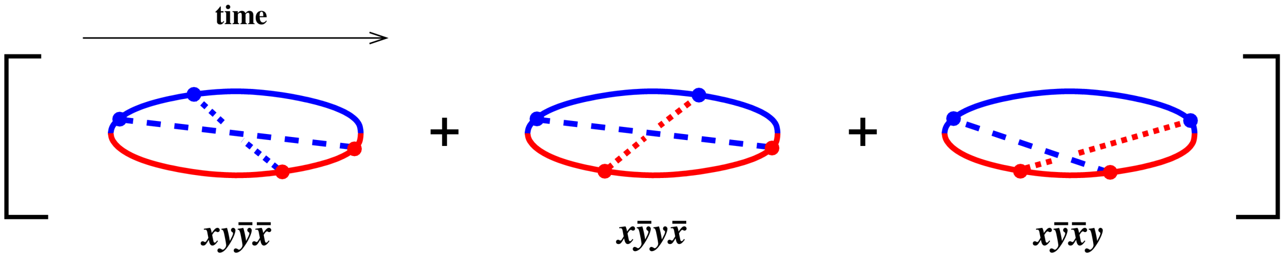
Divergences: Each time-ordered diagram diverges as $\Delta t \to 0$.

Must handle carefully (and non-trivially) with $i\varepsilon$ prescriptions.

Published Work

[all for $g \to gg \to ggg$]

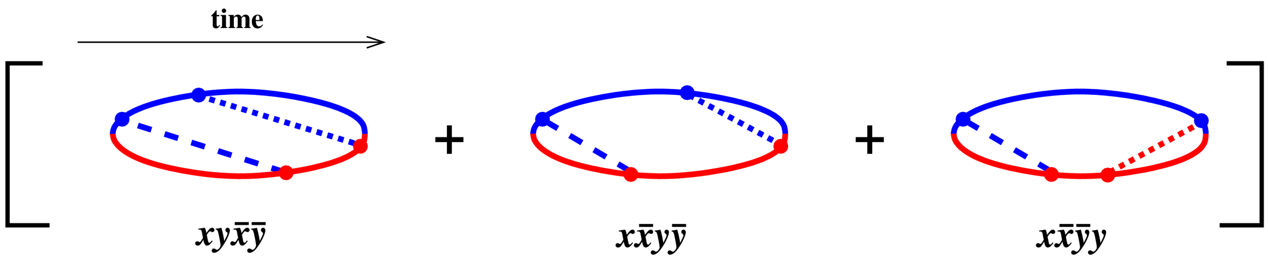
crossed diagrams:

2 Re 

+ permutations of $(x, y, 1-x-y)$

Forthcoming

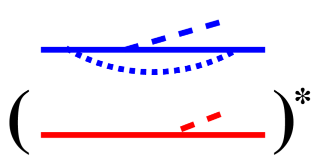
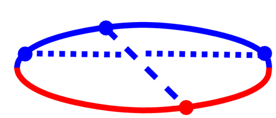
sequential diagrams:

2 Re 

+ permutations of $(x, y, 1-x-y)$

Still in progress

4-gluon vertices, e.g. 

virtual corrections, e.g.  =  correct single brem rate

[parts of which included in $y \ll x \ll 1$ work of earlier refs.]

Preview of one result for g → gg → ggg

The difference

(double brem) − $\left(\begin{array}{c}\text{what you'd get treating it as} \\ \text{independent single brems}\end{array}\right)$

is negative.

(Reminiscent of heuristic model in JEWEL Monte Carlo [Zapp *et al.* (2011+)].)

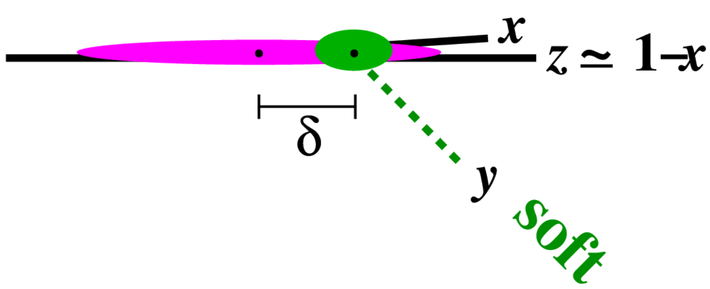
Example: a simple formula for our results in limiting case $y \ll x \ll 1$

$$\frac{\Delta\, d\Gamma}{dx\, dy} \simeq -\frac{C_A^2 \alpha_s^2}{\pi^2 x y^{3/2}} \ln\frac{x}{y}$$

[not analyzed in $y \ll x \ll 1$ work of earlier refs.]

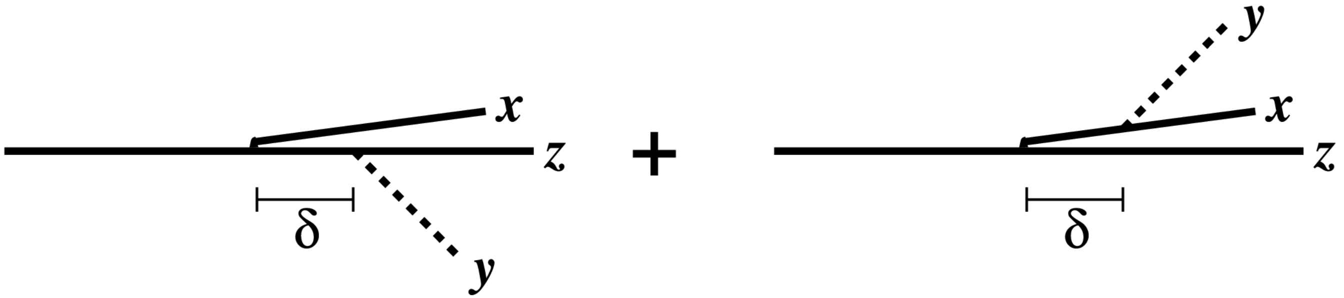
Why is the correction from overlapping formation lengths negative? ...

Actual



x and z gluons are so close during formation time, the soft y sees them as a single adjoint-color particle.

Monte Carlo



Simple Monte Carlo always treats y emission from x and z gluons independently and so double counts the chance of emission during laswt half of x emission formation time.