Forward scattering in SCET: Glauber exchange and the BFKL equation

Sean Fleming, University of Arizona

BNL, December 2014

Outline

- Soft Collinear Effective Theory
- The Glauber region
- Glauber interactions in SCET and the BFKL equation
- Outstanding Issues

Motivation for SCET

Reproduce the factorization proofs of QCD in an EFT framework

- Derive factorization at the operator level rather than diagrammatically
- Large logarithms summed through renormalization group methods
- Systematically incorporate corrections:

  perturbative corrections

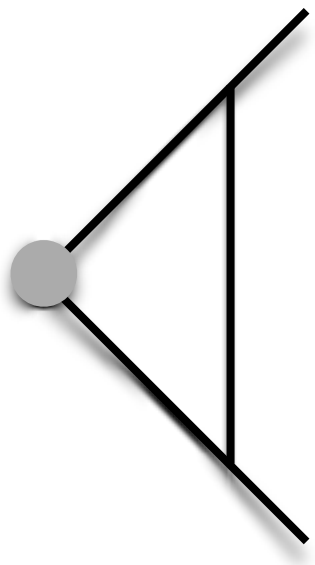
  power corrections

Degrees of Freedom?

What degrees of freedom are needed for an EFT?

- Those that reproduce the infra-red physics of QCD

Scalar Vertex as Example (Dim. Reg.)



$$\propto \frac{1}{(-\epsilon)^2} - \frac{3}{2(-\epsilon)}$$

Double and Single (InfraRed) Poles!

Origin of IR Poles in the Scalar Vertex

- All components of the loop momentum vanish together

  $k^\mu \to 0$ "Soft" region

- One light-cone component remains finite, all other components vanish

  $k^- \sim \text{fixed} \quad k^i_\perp \to 0 \quad k^+ \to 0$ "Collinear" region

All Order Analysis: Landau Equations

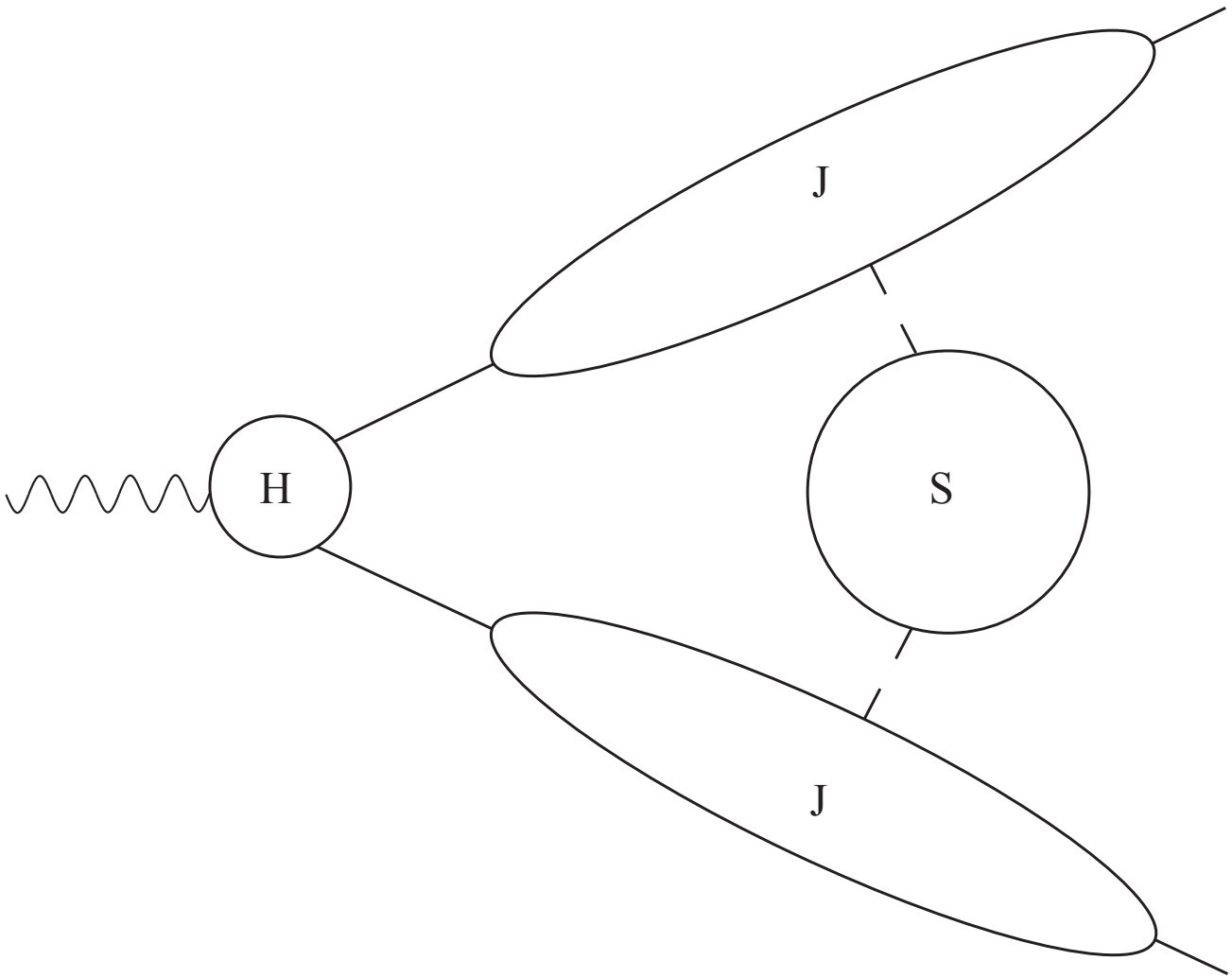
- All order behavior of diagrams is analyzed using the Landau Equations

  One loop results hold at all orders: IR singularities arise from "Soft" and "Collinear" regions

All Order Analysis: Landau Equations

- Diagrams of IR regions: Reduced Diagrams

  E.g. Electromagnetic Form Factor



H: $k^\mu \sim \sqrt{q^2}$

J: "Collinear"

S: "Soft"

Soft Collinear Effective Theory

- Effective Field Theory of "long-distance" modes

SCET Degrees of Freedom

modes	$p^\mu = (+, -, \perp)$	p^2	fields
collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$	$\xi_n,\ A_n^\mu$
soft	$Q(\lambda, \lambda, \lambda)$	$Q^2\lambda^2$	$q_s,\ A_s^\mu$
usoft	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$	$q_{us},\ A_{us}^\mu$

Soft Collinear Effective Theory

$\mathrm{SCET_I}$ $\longrightarrow$ Energetic jets

usoft $\quad p^\mu \sim \Lambda$

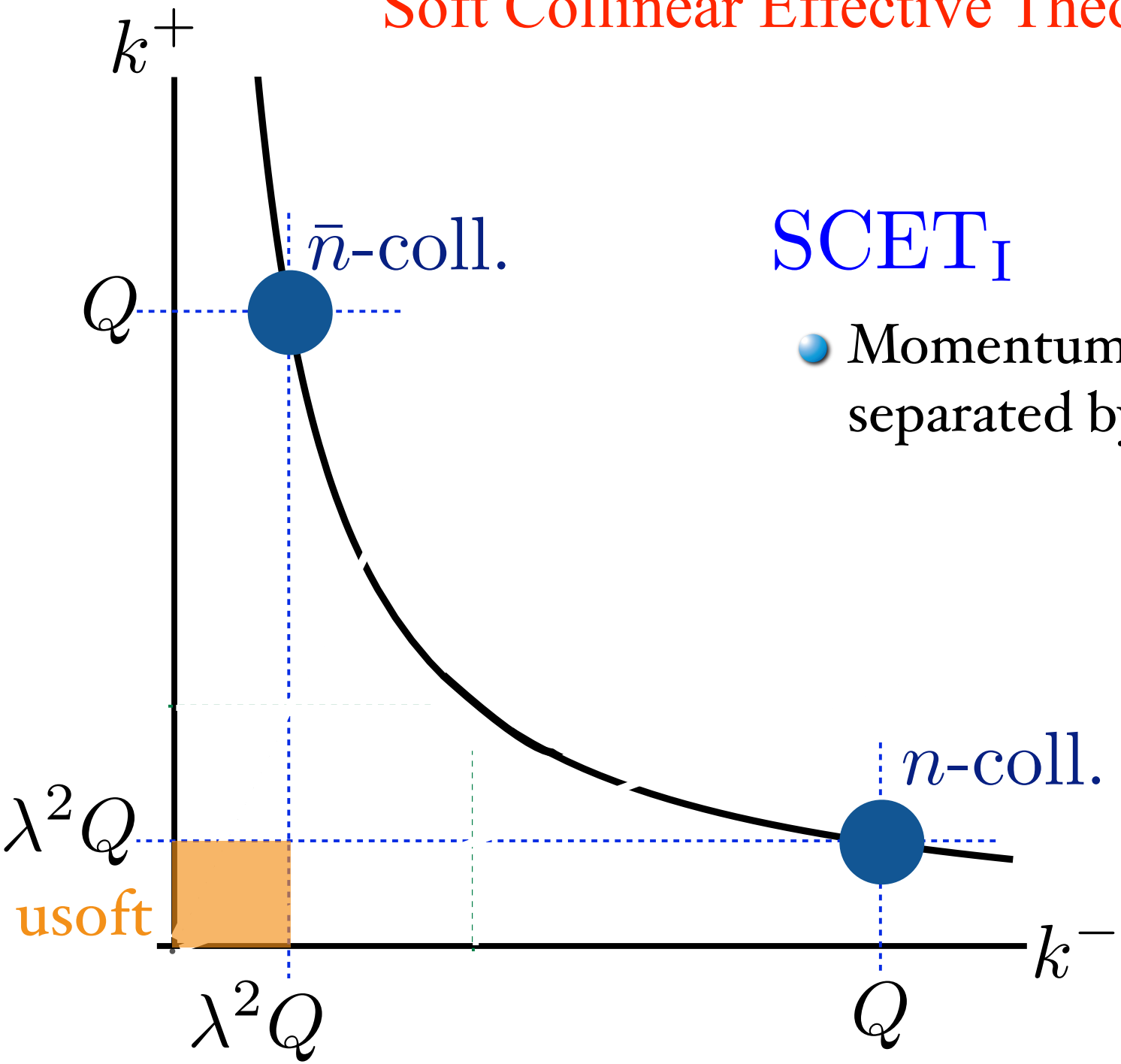
collinear $\quad p_c^2 \sim Q\Lambda, \quad \lambda = \sqrt{\Lambda/Q}$

$\mathrm{SCET_{II}}$ $\longrightarrow$ Energetic hadrons

soft $\quad p^\mu \sim \Lambda$

collinear $\quad p_c^2 \sim \Lambda^2, \quad \lambda = \Lambda/Q$

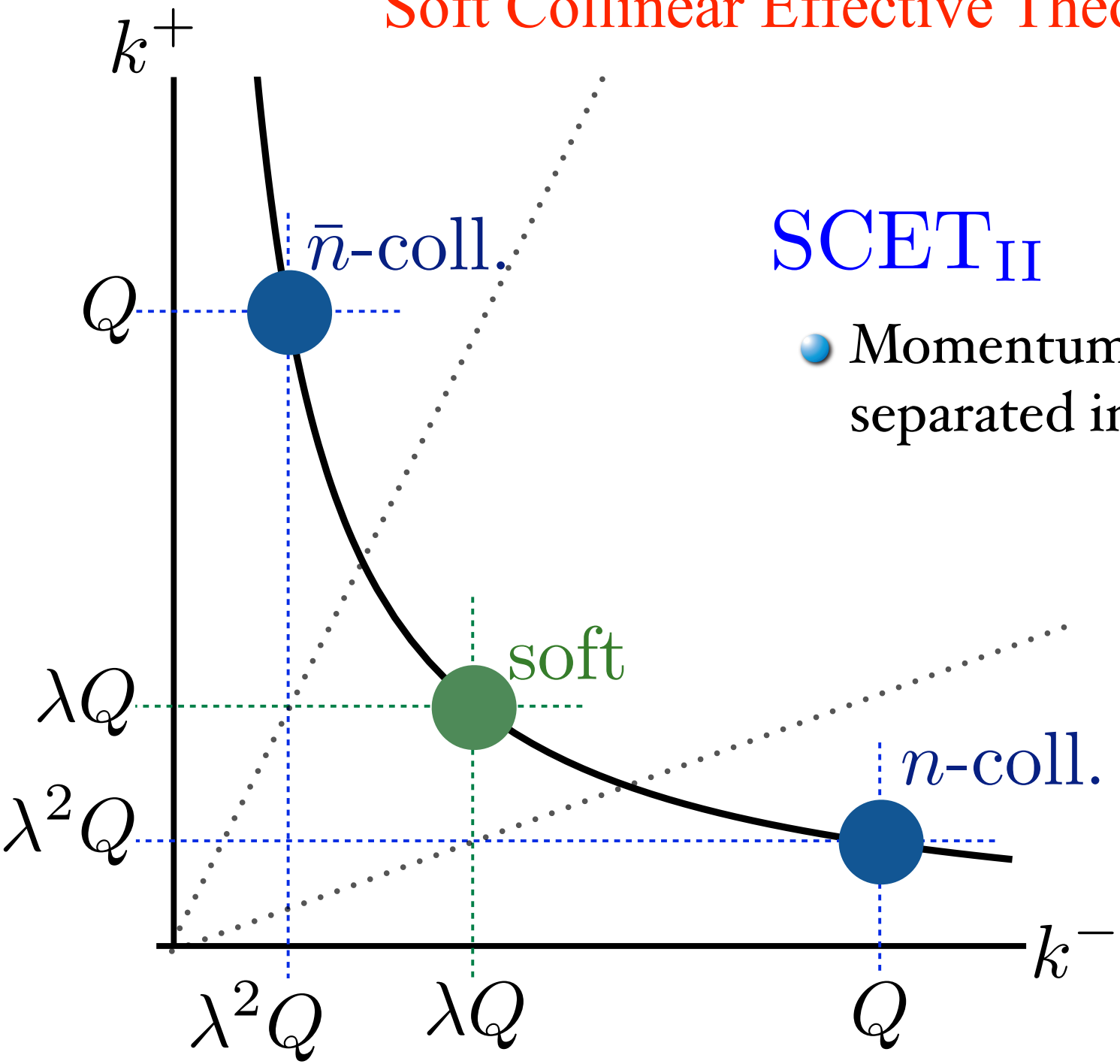
Soft Collinear Effective Theory



$\mathrm{SCET_I}$

- Momentum modes are separated by invariant mass

Soft Collinear Effective Theory

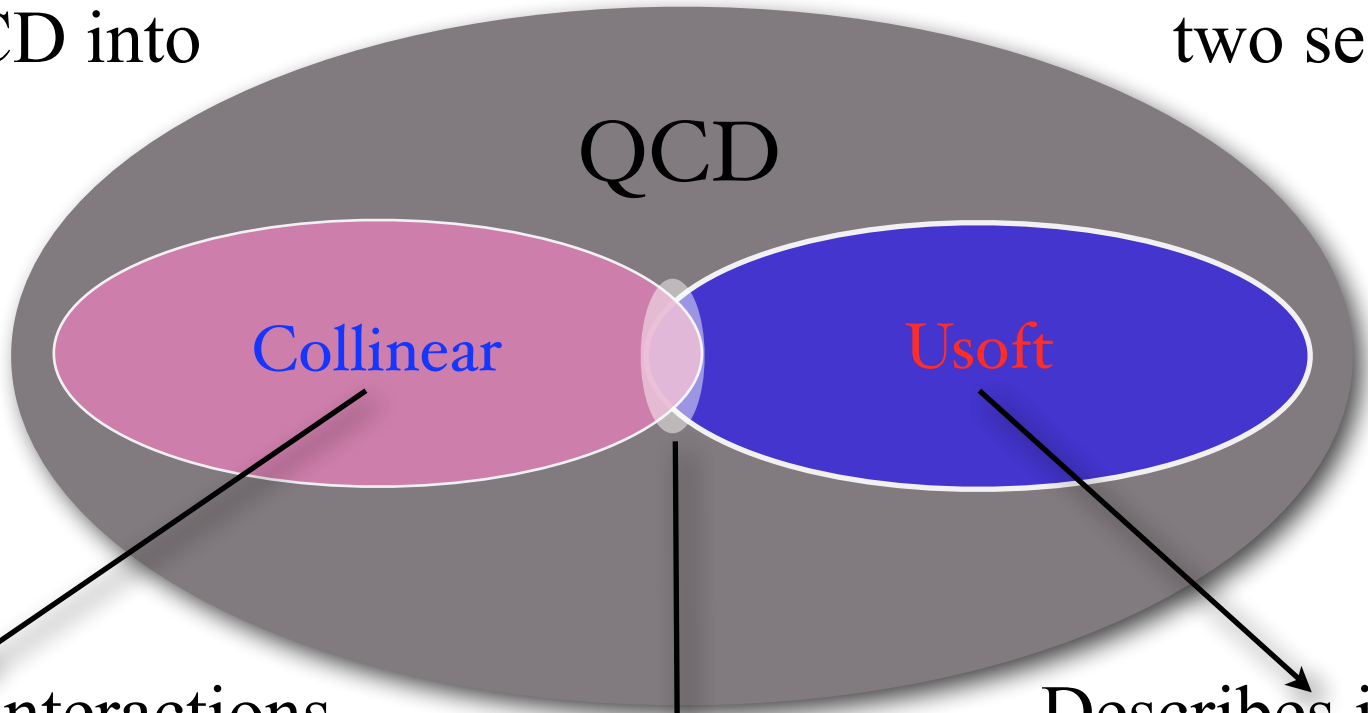


$\mathrm{SCET_{II}}$

- Momentum modes are separated in rapidity

$\mathrm{SCET_I}$

- Split QCD into two sectors:



Describes interactions among the energetic particles

Interactions between usoft and collinear

Describes interactions amongst usoft particles

$$\mathcal{L}_{us} = \bar{\psi}_{us} i \not{D}_{us} \psi_{us}$$

$$\mathcal{L}_c = \bar{\xi}_n \left\{ in \cdot D_c + i\not{D}_c^{\perp} \frac{1}{i\bar{n} \cdot D_c} i\not{D}_c^{\perp} + gn \cdot A_{us} \right\} \frac{\not{\bar{n}}}{2} \xi_n$$

$\mathrm{SCET_I}$

Before:

$$\mathcal{L}_c = \bar{\xi}_n \left\{ in \cdot D_c + i\not{D}_c^\perp \frac{1}{i\bar{n} \cdot D_c} i\not{D}_c^\perp + gn \cdot A_{us} \right\} \frac{\not{\bar{n}}}{2} \xi_n$$

- Field redefinition via usoft Wilson line:

$$\xi_{n,p} \to Y_n \xi_{n,p} \,, \qquad A^\mu_{n,p} \to Y_n A^\mu_{n,p} Y_n^\dagger$$

$$Y_n(x) = \overline{\mathrm{P}} \, \exp\left(-ig \int_0^\infty ds \, n \cdot A_{us}(ns + x) \right)$$

After:

$$\mathcal{L}_c^{(0)} = \bar{\xi}_n \left\{ in \cdot D_c + i\not{D}_c^\perp \frac{1}{in \cdot D_c} i\not{D}_c^\perp \right\} \frac{\not{\bar{n}}}{2} \xi_n$$

$\mathrm{SCET_I}$

- Two collinear sectors in opposite direction n^μ & $\bar{n}^\mu$:

$$n^2 = \bar{n}^2 = 0 \qquad n \cdot \bar{n} = 2$$

$$\mathcal{L}_{\mathrm{SCET}} = \mathcal{L}_n[\xi_n, A_n] + \mathcal{L}_{\bar{n}}[\xi_{\bar{n}}, A_{\bar{n}}] + \mathcal{L}_{us}[\psi_{us}, A_{us}]$$

- Operators before usoft decoupling

$$\bar{\psi}(x)\Gamma\psi(x) \longrightarrow \cancel{O_{n\bar{n}} = \bar{\xi}_n \Gamma \xi_{\bar{n}}}$$

$$\longrightarrow O_{n\bar{n}} = \bar{\xi}_n W_n \Gamma C(\bar{n}\cdot\mathcal{P}^\dagger, n\cdot\mathcal{P}) W_{\bar{n}}^\dagger \xi_{\bar{n}}$$

Collinear Wilson line $\quad W_n(y, -\infty) = \mathrm{P}\,\exp\left[ig\int_{-\infty}^{y} \mathrm{d}s\, \bar{n}\cdot A_n(s\bar{n})\right]$

SCET Factorization

- Operators after usoft decoupling

$$O_{n\bar{n}} = \bar{\xi}_n W_n Y_{\bar{n}}^\dagger \Gamma C(\bar{n}\cdot\mathcal{P}^\dagger, n\cdot\mathcal{P}) Y_n W_{\bar{n}}^\dagger \xi_{\bar{n}}$$

- Factorization of Hard from "long-distance" modes comes from Matching

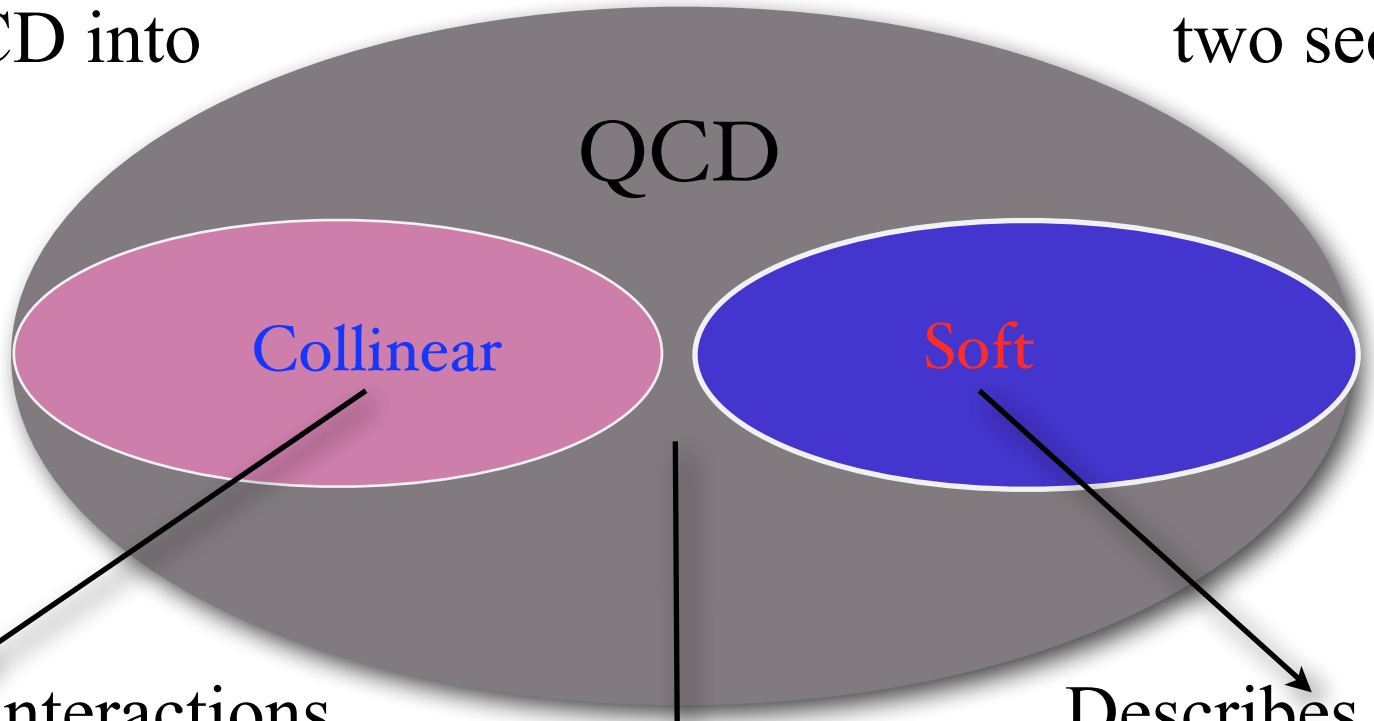
$$W = \int d\omega_n d\omega_{\bar{n}} H(\omega_n, \omega_{\bar{n}}, \mu) W_{\mathrm{SCET}}(\omega_n, \omega_{\bar{n}}, \mu)$$

- Factorization Collinear from Usoft by decoupling

$$W_{\mathrm{SCET}}(\omega_n, \omega_{\bar{n}}, \mu) = \int d\ell^+ d\ell^- J_n(\omega_n, \ell^+, \mu) J_{\bar{n}}(\omega_{\bar{n}}, \mu) U(\ell^+, \ell^-, \mu)$$

$\mathrm{SCET_{II}}$

- Split QCD into two sectors:



Describes interactions among the energetic particles

No interactions between soft and collinear

Describes interactions amongst soft particles

$$\mathcal{L}_{\mathrm{SCET}} = \mathcal{L}_n[\xi_n, A_n] + \mathcal{L}_s[\psi_s, A_s]$$

SCET Factorization

- Operators

$$O_{n\bar{n}} = \bar{\xi}_n W_n S_{\bar{n}}^\dagger \Gamma C(\bar{n}\cdot\mathcal{P}^\dagger, n\cdot\mathcal{P}) S_n W_{\bar{n}}^\dagger \xi_{\bar{n}}$$

$$S_n(x) = \overline{\mathrm{P}}\ \exp\Big(-ig\int_0^\infty ds\, n\cdot A_s(ns+x)\Big)$$

- Factorization

$$W = \int d\omega_n d\omega_{\bar{n}} H(\omega_n, \omega_{\bar{n}}, \mu) W_{\mathrm{SCET}}(\omega_n, \omega_{\bar{n}}, \mu)$$

$$W_{\mathrm{SCET}}(\omega_n, \omega_{\bar{n}}, \mu) =$$

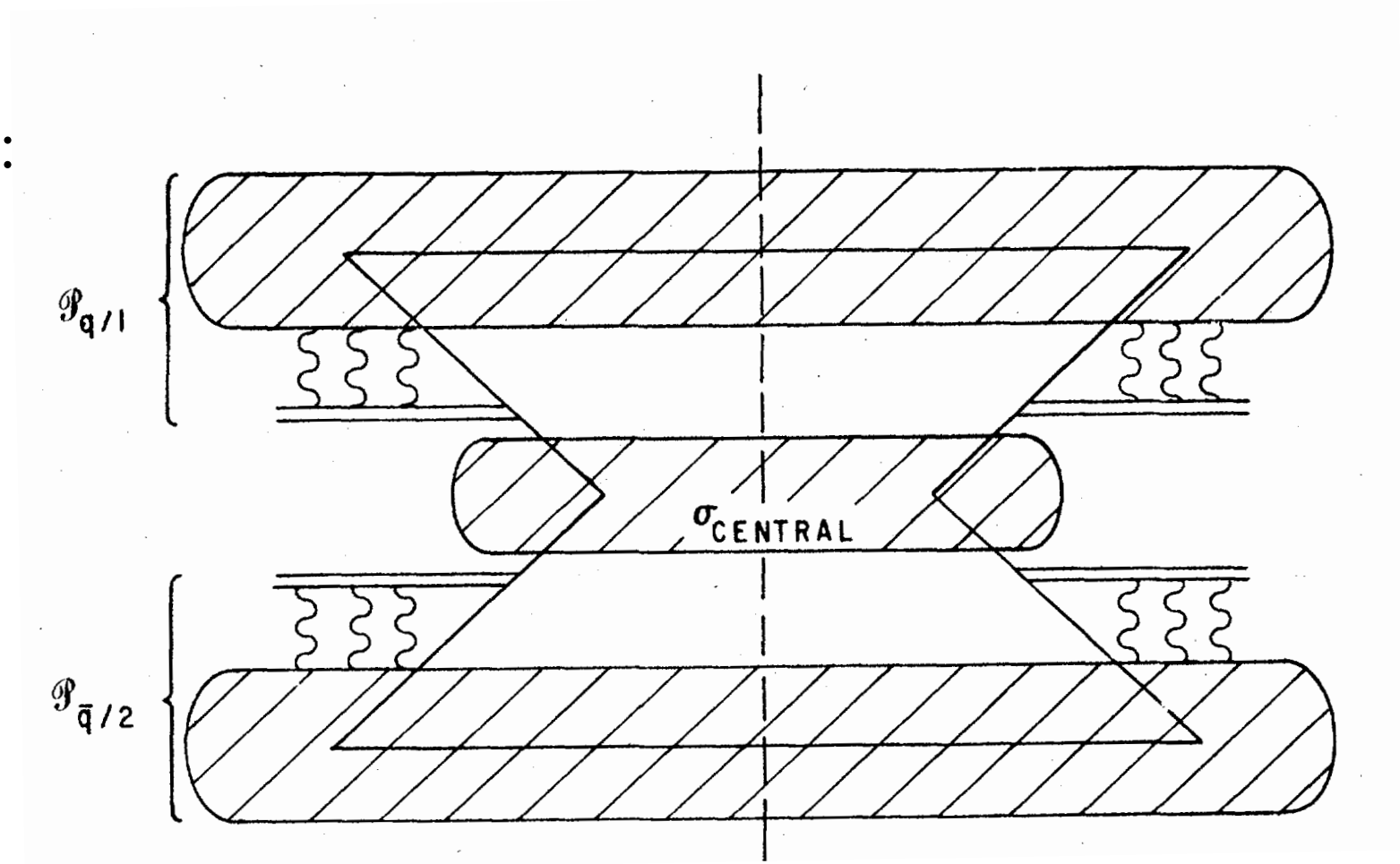
$$\int d\ell^+ d\ell^- J_n(\omega_n, \ell^+, \mu, \nu) J_{\bar{n}}(\omega_{\bar{n}}, \mu, \nu) S(\ell^+, \ell^-, \mu, \nu)$$

The 800 lb gorilla: the Glauber region

- Drell-Yan Meson-Meson scattering: $MM \to \ell^+\ell^- X$

G. T. Bodwin Phys. Rev. D 31, 2616 (1985)

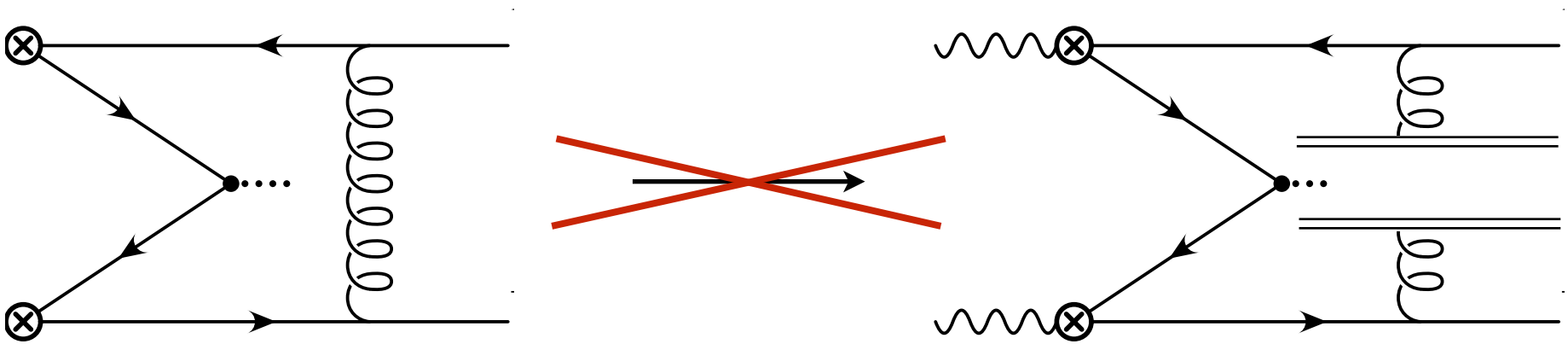
Factored form:



The Glauber region

- Problem at one loop Spectator-Spectator interaction:

C. W. Bauer, B. Lange, G. Ovanesyan, JHEP 07 (2011) 077



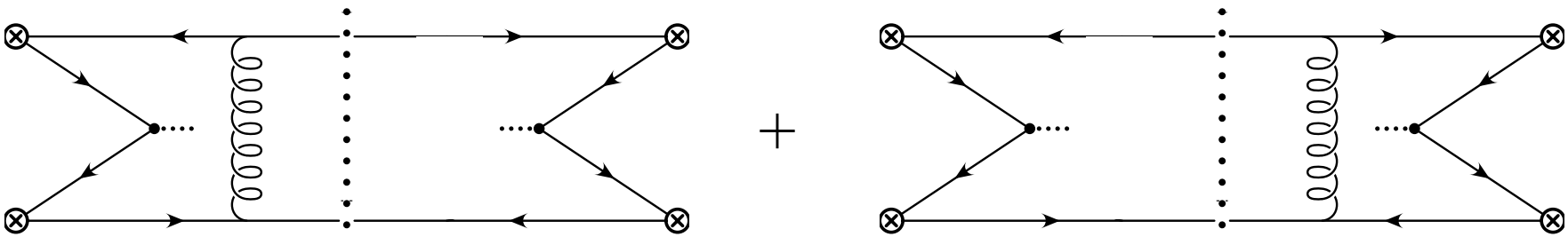
$$I_5 = (-i)\frac{1}{2}\int \frac{d^2 l_\perp}{(2\pi)^2}\int \frac{dl^-}{2\pi} N^-(l^-)\int \frac{dl^+}{2\pi}\prod_{i=0}^{4}\frac{1}{l^+ - z_i(l^-, l_\perp)}$$

- Integral becomes pinched in the Glauber region

$$D(k)_{\text{gluon}} \propto \frac{1}{k_\perp^2}$$

The Glauber region

- For an inclusive observable like $\dfrac{d\sigma}{dq^2 dy}$ Glauber vanishes



sum over all final state cuts is zero

- Holds to all orders in strong coupling

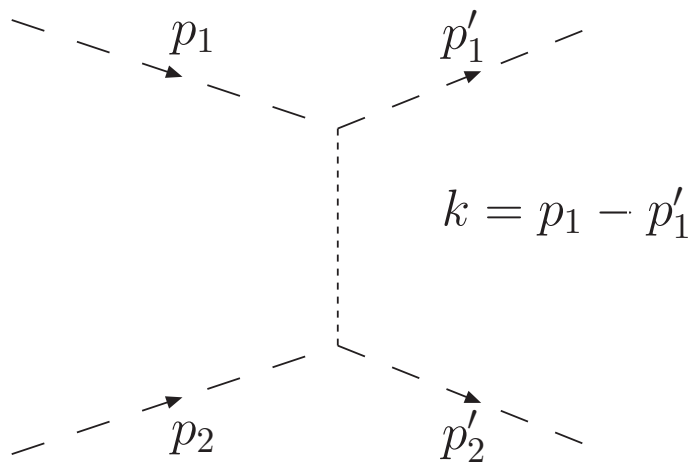
The Glauber interaction in SCET

- Original formulation of SCET did not include Glauber interactions

  SCET is incomplete: validity of factorization from SCET?

- Include Glauber interactions in SCET Lagrangian

F. Liu & J. P. Ma, arXiv:0802.2973; A. Idilbi & A. Majumder, Phys. Rev. D 80 (2009) 054022; S. Fleming Phys.Lett. B735 (2014) 266; I. Z. Rothstein & I. W. Stewart (in progress)



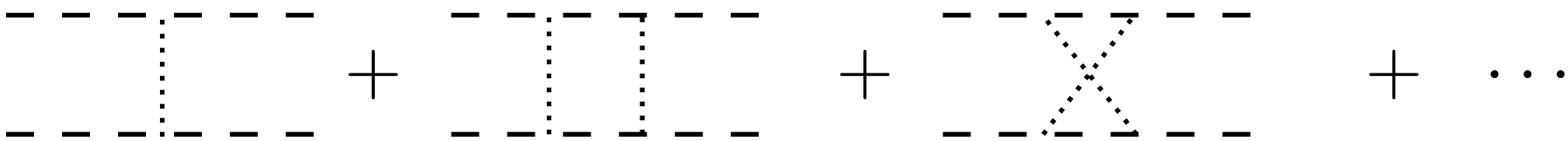
$$\mathcal{L}_G^{n\bar{n}} = -8\pi\,\alpha_s(\mu)\,\bar{\xi}_{p_2',\bar{n}} W_{\bar{n}} S_{\bar{n}}^\dagger T^a \frac{\not{n}}{2} S_{\bar{n}} W_{\bar{n}}^\dagger \xi_{p_2,\bar{n}} \frac{1}{\vec{\mathcal{P}}_\perp^2} \bar{\xi}_{p_1',n} W_n S_n^\dagger T^a \frac{\not{\bar{n}}}{2} S_n W_n^\dagger \xi_{p_1,n}$$

Gauge invariant

The Glauber interaction in SCET

- $\mathcal{L}_G^{n\bar{n}} = \mathcal{O}(1)$ breaks SCET factorization

- Sum the graphs? (abelian)



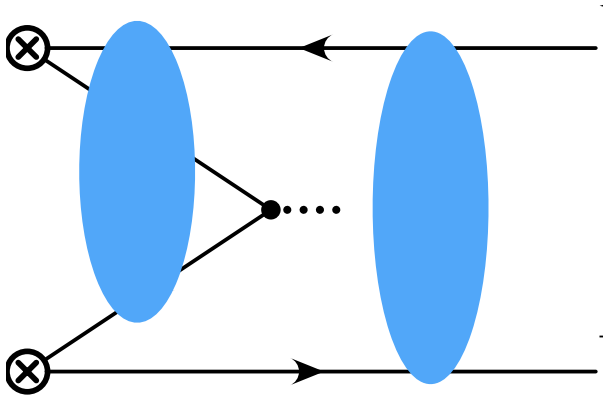
$$= e^{i\tilde{\phi}_G} - 1$$

$$i\tilde{\phi}_G = i\alpha_s e^{\epsilon\gamma_E} 2^{-2\epsilon}\Gamma(-\epsilon)\mu^{2\epsilon}|x_\perp|^{2\epsilon}$$

The Glauber interaction in SCET

- Drell-Yan Meson-Meson scattering: sum Glauber interactions

I. Z. Rothstein & I. W. Stewart



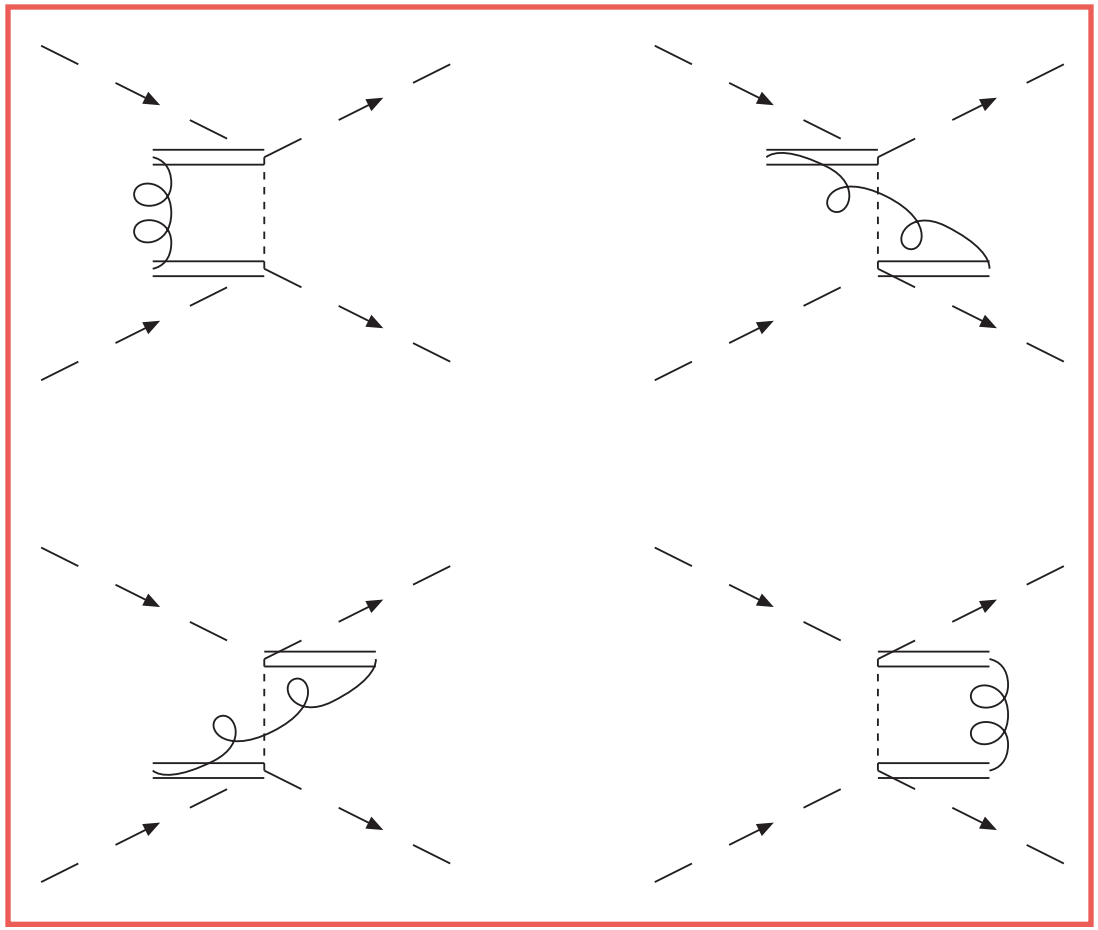
$$\rightarrow J_n(x_{1\perp})J_{\bar{n}}(x_{2\perp})e^{i\tilde{\phi}_G(x_{1\perp}-x_{2\perp})}e^{i\tilde{\phi}_G(x_{1\perp})}$$

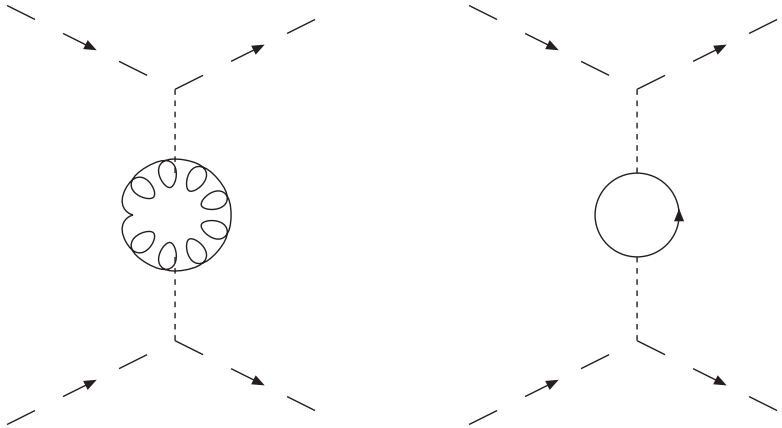
- As long as no measurement on $x_\perp$ is made the Glauber phases cancel in $\dfrac{d\sigma}{dq^2dy}$

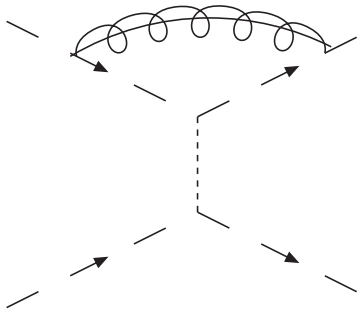
The Glauber interaction in SCET and the BFKL equation

S. Fleming ; J. F. Donoghue, B. K. El-Menoufi, and G. Ovanesyan (2014), arXiv:1405.1731

- Renormalization of the Glauber interaction $\mathcal{L}_G^{n\bar{n}}$







The Glauber interaction in SCET and the BFKL equation

$$\mathcal{A} = -8\pi\alpha_s(\mu)\,\bar{\xi}_n T^a \frac{\not{\bar{n}}}{2}\xi_n \bar{\xi}_{\bar{n}} T^a \frac{\not{n}}{2}\xi_{\bar{n}} \left[i N_c \alpha_s(\mu) \mathcal{I}(\vec{k}_\perp) \right] + \text{counterterms}$$

$$\mathcal{I}(\vec{k}_\perp) = \int \frac{dq^-}{q^-} \int \frac{d^2 q_\perp}{(2\pi)^2} \frac{1}{\vec{q}_\perp^{\,2}} \frac{1}{(\vec{q}+\vec{k})_\perp^2}$$ Gluon Regge Trajectory

- Introduce rapidity regulator J.-Y. Chiu, A. Jain, D. Neill, and I. Z. Rothstein, Phys. Rev. Lett. 108, 151601 (2012) & JHEP 1205, 084 (2012)

$$\mathcal{I}(\vec{k}_\perp) = \frac{-2i}{(4\pi)^2} \frac{w(\nu)^2}{\vec{k}_\perp^2} \left[\frac{1}{\eta} \ln\left(\frac{\vec{k}_\perp^2}{m_g^2}\right) + \ln\left(\frac{\vec{k}_\perp^2}{4\nu}\right) \ln\left(\frac{\vec{k}_\perp^2}{m_g^2}\right) - \frac{1}{4} \ln^2\left(\frac{\vec{k}_\perp^2}{m_g^2}\right) + i\pi \ln\left(\frac{\vec{k}_\perp^2}{m_g^2}\right) \right]$$

- The counter term subtracts the $\frac{1}{\eta}$ pole

The Glauber interaction in SCET and the BFKL equation

- Renormalization of rapidity divergence gives rise to a rapidity RGE (R-RGE)

$$\frac{d}{d\ln\nu}\mathcal{L}_G^{n\bar{n}}(\nu) = \gamma(\nu)\mathcal{L}_G^{n\bar{n}}(\nu)$$

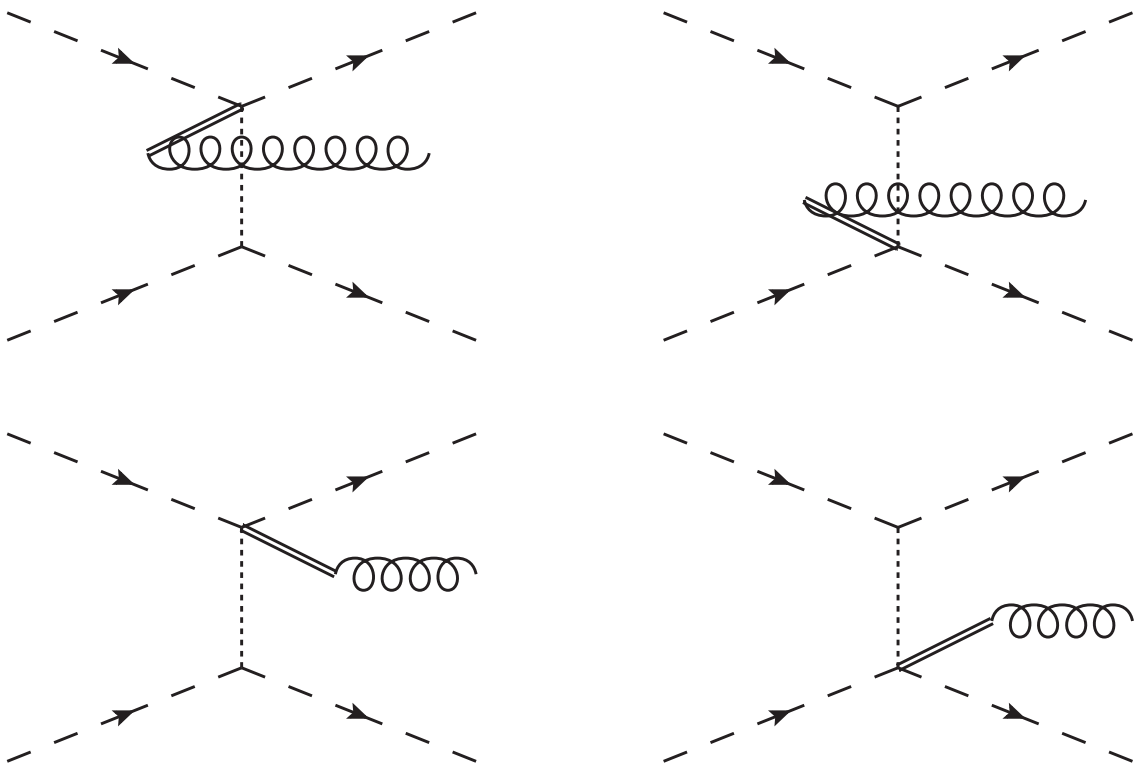
with $$\gamma(\nu) = -\frac{\alpha_s N_c}{4\pi^2}\ln\left(\frac{k_\perp^2}{m_g^2}\right)$$

solving $$\mathcal{L}_G^{n\bar{n}}(\nu) = \mathcal{L}_G^{n\bar{n}}(\nu_0)\left(\frac{\nu}{\nu_0}\right)^{-\frac{\alpha_s N_c}{4\pi^2}\ln\left(\frac{k_\perp^2}{m_g^2}\right)}$$

reggeized gluon

The Glauber interaction in SCET and the BFKL equation

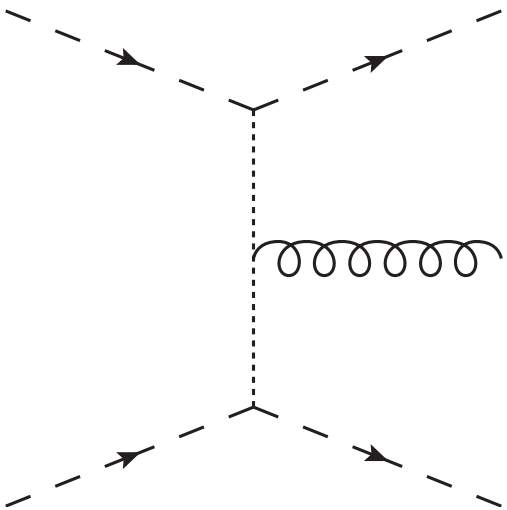
- Real contributions



emission from
soft Wilson line

The Glauber interaction in SCET and the BFKL equation

- Real contributions



new operator in Lagrangian!

$$\mathcal{L}_G^{n\bar{n}S}$$

- Add all real contributions we get the Lipton vertex

The Glauber interaction in SCET and the BFKL equation

- The NLO expression determined from these diagrams is

$$\sigma = \frac{2\alpha_s^2 C_F}{N_c}\int\frac{d^2\vec{k}_\perp}{\vec{k}_\perp^2}\int\frac{d^2\vec{k}_\perp}{\vec{k}_\perp'^2}\left\{\delta^{(2)}(\vec{k}_\perp-\vec{k}_\perp')+\left(\frac{\alpha_s N_c}{\pi^2}\right)\frac{\Gamma(\eta)\Gamma\left(\frac{1}{2}-\eta\right)}{\sqrt{\pi}}\nu^{2\eta}w(\nu)^2\right.$$
$$\left.\times\int\frac{d^2q_\perp}{[(\vec{q}_\perp-\vec{k}_\perp)^2]^{1+\eta}}\left[\delta^{(2)}(\vec{q}_\perp-\vec{k}_\perp')-\frac{\vec{k}_\perp^2}{2\vec{q}_\perp^2}\delta^{(2)}(\vec{k}_\perp-\vec{k}_\perp')\right]\right\}.$$

- Isolating the rapidity divergent term:

$$\sigma = \frac{2\alpha_s^2 C_F}{N_c}\int\frac{d^2\vec{k}_\perp}{\vec{k}_\perp^2}\int\frac{d^2\vec{k}_\perp}{\vec{k}_\perp'^2}\left\{\delta^{(2)}(\vec{k}_\perp-\vec{k}_\perp')\right.$$
$$\left.+\left(\frac{\alpha_s N_c}{\pi^2}\right)\frac{w(\nu)^2}{\eta}\int\frac{d^2q_\perp}{(\vec{q}_\perp-\vec{k}_\perp)^2}\left[\delta^{(2)}(\vec{q}_\perp-\vec{k}_\perp')-\frac{\vec{k}_\perp^2}{2\vec{q}_\perp^2}\delta^{(2)}(\vec{k}_\perp-\vec{k}_\perp')\right]+\ldots\right\}$$

The Glauber interaction in SCET and the BFKL equation

- Assume the cross section factorizes into soft and collinear

$$\sigma = J_n \otimes J_{\bar{n}} \otimes G$$

where $\quad G(\vec{k}_\perp - \vec{k}'_\perp) \equiv \langle O_G^{\rm soft} \rangle$

not known

- Identify the two-dimension Dirac delta function in transverse-momentum space as the leading order value of G:

$$\begin{aligned} G(\vec{k}_\perp - \vec{k}'_\perp, \nu) &= \int d^2\ell_\perp \mathcal{Z}^{-1}(\vec{k}_\perp - \vec{\ell}_\perp; \eta, \nu) G(\vec{\ell}_\perp - \vec{k}'_\perp; \nu)^{(0)} \\ &= \int d^2\ell_\perp \mathcal{Z}^{-1}(\vec{k}_\perp - \vec{\ell}_\perp; \eta, \nu) \delta^{(2)}(\vec{\ell}_\perp - \vec{k}'_\perp) \\ &= \delta^{(2)}(\vec{k}_\perp - \vec{k}'_\perp) + \text{counterterms}\,, \end{aligned}$$

The Glauber interaction in SCET and the BFKL equation

- Derive R-RGE for G

$$\frac{d}{d\ln\nu}G(\vec{k}_\perp - \vec{k}'_\perp;\nu) = \int d^2\ell_\perp \gamma_\nu(\vec{k}'_\perp - \vec{\ell}_\perp)G(\vec{\ell}_\perp - \vec{k}'_\perp;\nu)$$

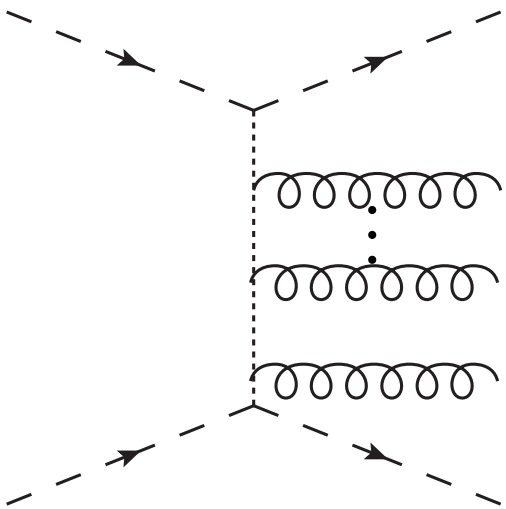
with

$$\gamma_\nu(\vec{k}_\perp - \vec{k}'_\perp) = \left(\frac{\alpha_s N_c}{\pi^2}\right)\left[\frac{1}{(\vec{k}_\perp - \vec{k}'_\perp)^2} - \frac{1}{2}\delta^{(2)}(\vec{k}_\perp - \vec{k}'_\perp)\int \frac{d^2q_\perp}{(\vec{q}_\perp - \vec{k}_\perp)^2}\frac{\vec{k}_\perp^2}{\vec{q}_\perp^2}\right]$$

This is the BFKL equation

Outstanding Issues

- This calculation suggests that Glauber mediated Soft Gluons can be summed



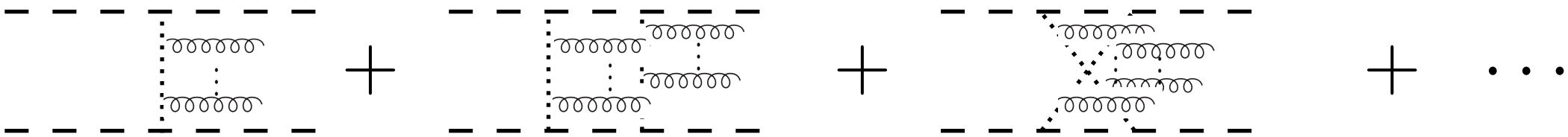
$$\longrightarrow \mathcal{L}_G^{n\bar{n}\,\text{summed}} =$$

$$\bar{\xi}_{p_2',\bar{n}} W_{\bar{n}} S_{\bar{n}}^\dagger T^a \frac{\not{n}}{2} S_{\bar{n}} W_{\bar{n}}^\dagger \xi_{p_2,\bar{n}} \frac{1}{\vec{\mathcal{P}}_\perp^2} E(A_s) \bar{\xi}_{p_1',n} W_n S_n^\dagger T^a \frac{\not{\bar{n}}}{2} S_n W_n^\dagger \xi_{p_1,n}$$

$E(A_s)$ must be gauge invariant

Outstanding Issues

- We also need to sum multiple Glauber exchanges with soft gluon attachments

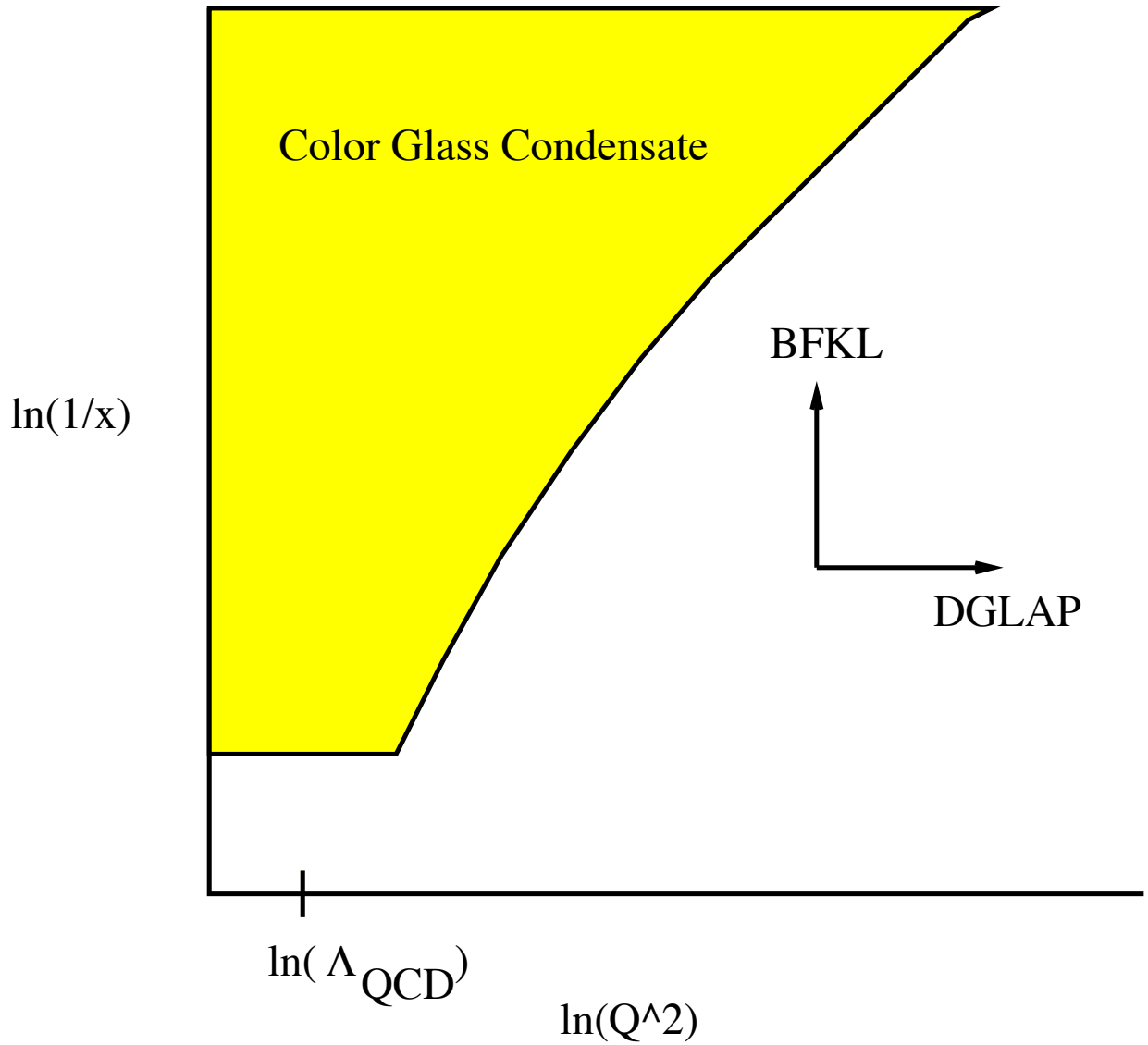


- Can soft still be factored from collinear at the operator level?

If so in which observables? J. R. Gaunt JHEP 07 (2014) 110

Outstanding Issues

- How does SCET connect to the CGC?



?