Phenomenological QCD equation of state for *massive* neutron stars

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arXiv : 1412.1108 [hep-ph]

Plan of this talk

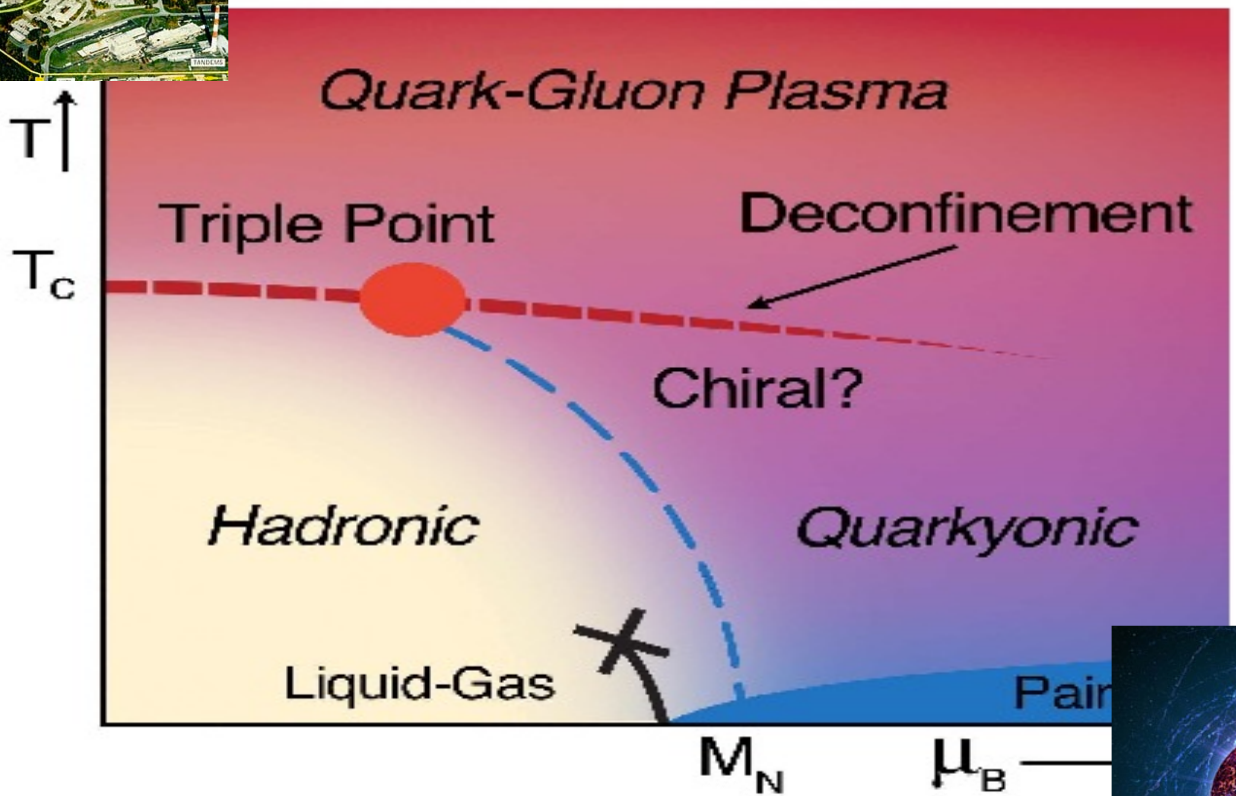
1, Introduction : Quick reminder

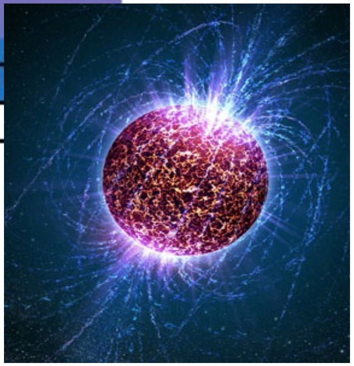
2, Phenomenological QCD EoS

3, Summary & Outlook

QCD phase diagram

RHIC, LHC : high-energy, temperature QCD





Neutron star :
" *Cosmic laboratory* "
for cold, dense QCD

Neutron stars (NSs)

Mass : $M \sim$ (1-2) M_{sun}

Radius : $R \sim$ 10 km

Temperature : $T \sim 10^6 - 10^9$ K $\sim$KeV

Small but heavy : $GM/R \sim 1/10$

→ General relativity (GR) is important

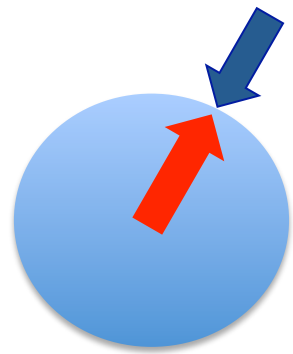
Energy density : $M/(4\pi R^3/3) \sim O(0.1\text{-}1)$ GeV fm^{-3}

(very dense)

→ Physics at the QCD scale

From QCD to NSs : TOV-eq.

$P(\varepsilon)$ ***EoS*** **: from QCD**



Einstein eq.

Tolman-Oppenheimer-Volkov (TOV) eq.

$$\frac{\mathrm{d}P(r)}{\mathrm{d}r} = -\underbrace{\frac{GM(r)\varepsilon(r)}{r^2}}_{\text{Newtonian}} \underbrace{\left(1+\frac{P}{\varepsilon}\right)\left(1+\frac{4\pi r^3 P}{M}\right)\left(1-\frac{GM}{r}\right)^{-1}}_{\text{GR effects (>1)}}$$

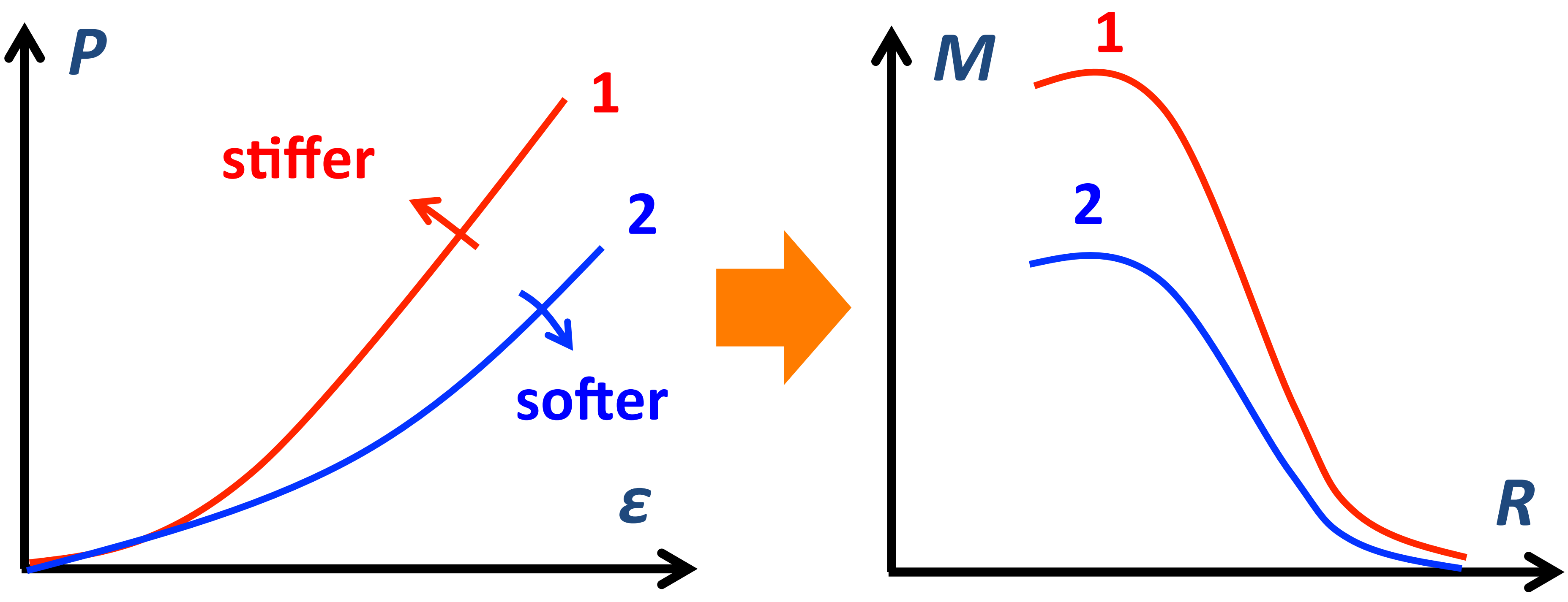
$$M(r) = \int_0^r \mathrm{d}r'\, 4\pi r'^2 \varepsilon(r')$$

mass inside of radius r

" Stiff " EoS & Maximum mass

" Stiff " EoS : P is large at given ε

(NOT at given μ !)



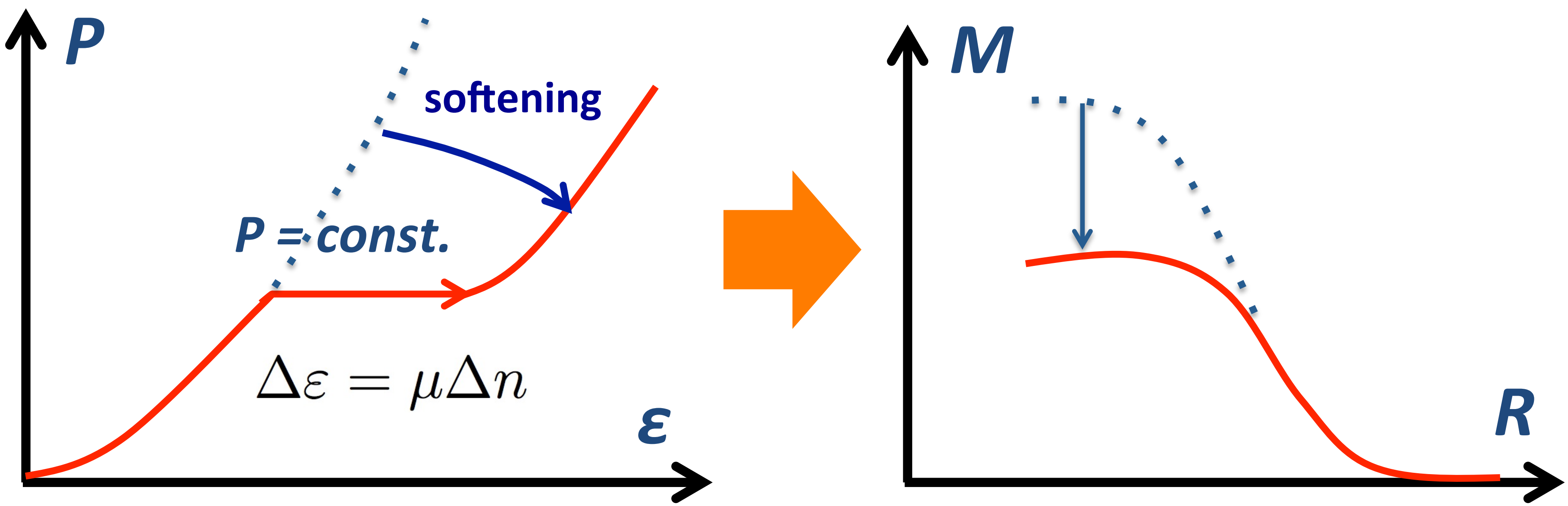
" Stiff " EoS & Phase transitions

$$P = \mu n - \varepsilon$$: thermodynamic relation

P & μ must be continuous everywhere

e.g.) At 1st order phase transitions : $\Delta P = \mu \Delta n - \Delta\varepsilon = 0$

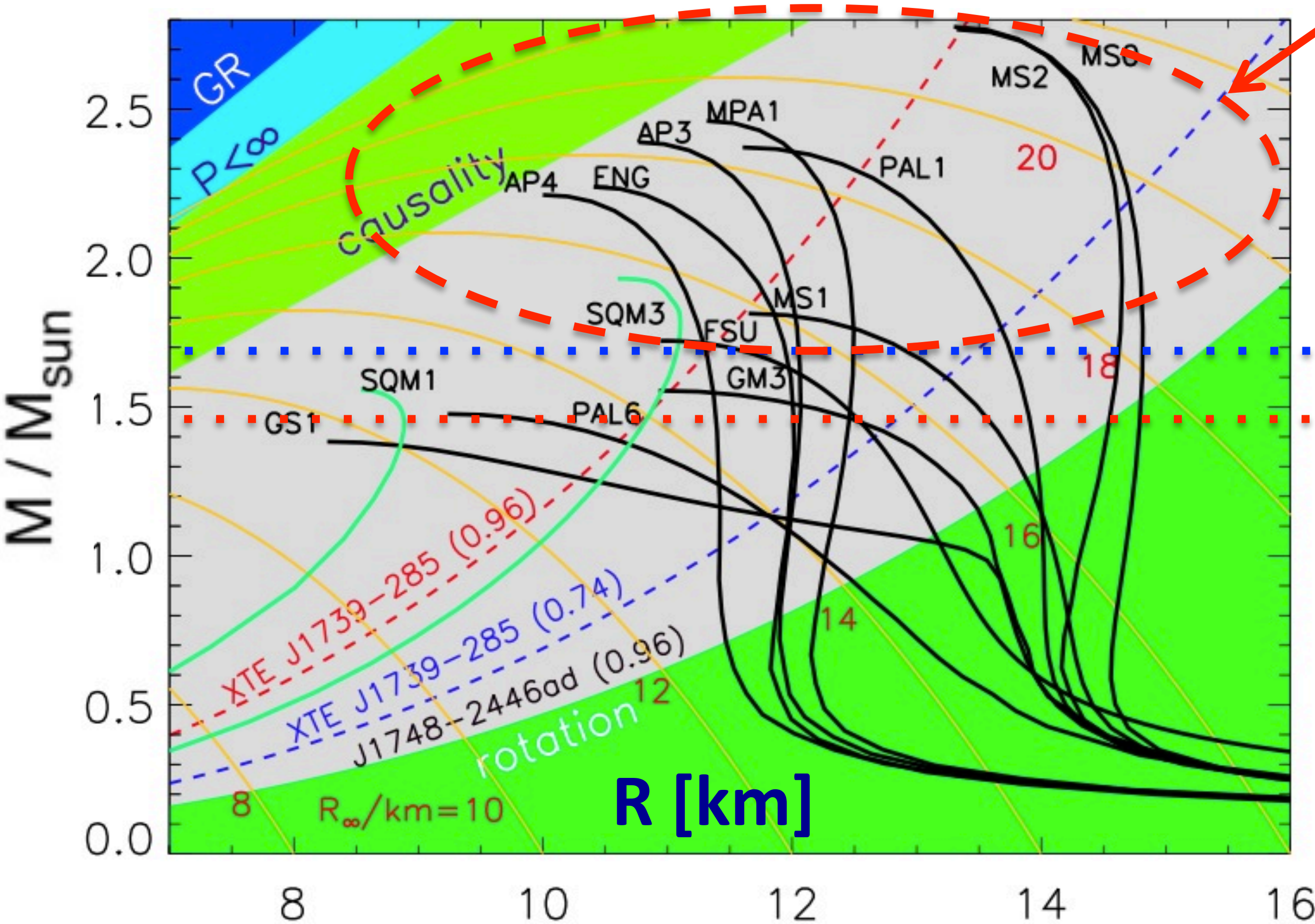
(hadron-quark, meson condensates, ...)



M-R relation for various EoSs

Lattimer-Prakash (2007)

allowed EoSs



J1903-0327

J1909-3744

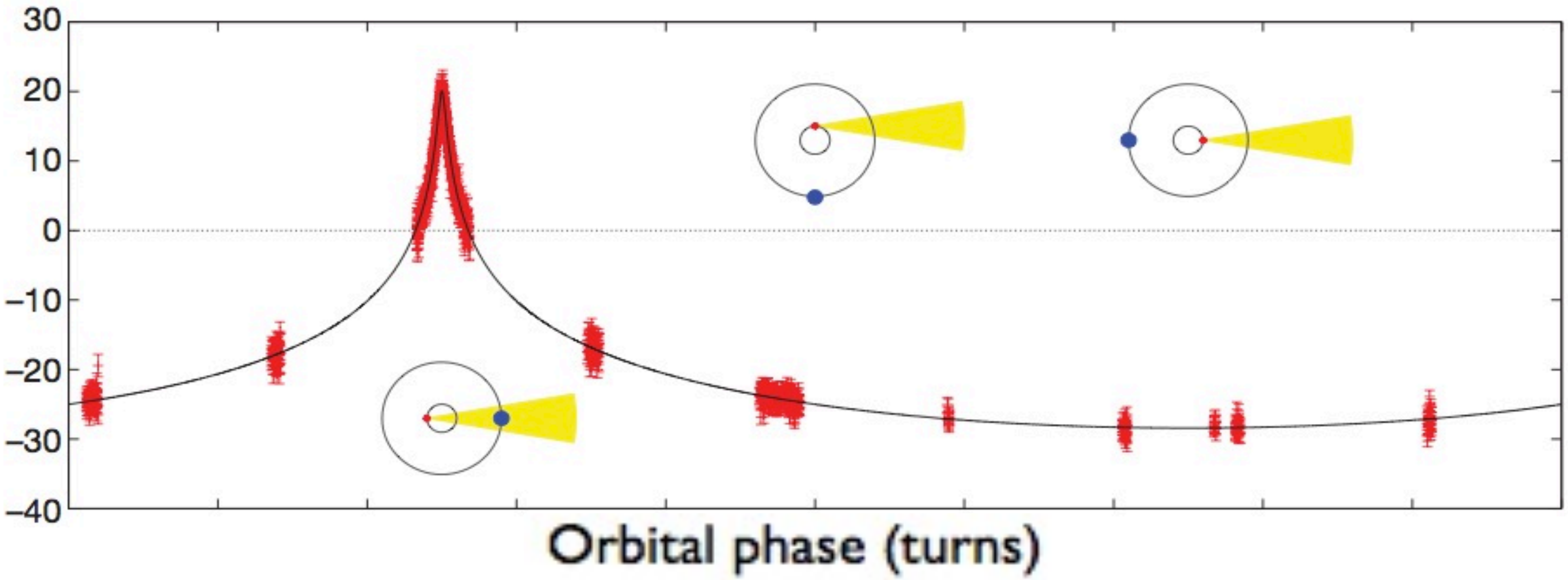
masses precisely measured,
radiuses have uncertainties.

Too soft EoSs are excluded !

Recent discoveries (2010-)

PSR J1614 - 2230 : White Dwarf – Neutron Star binary

“ Shapiro delay ” (change of arrival time of pulse)



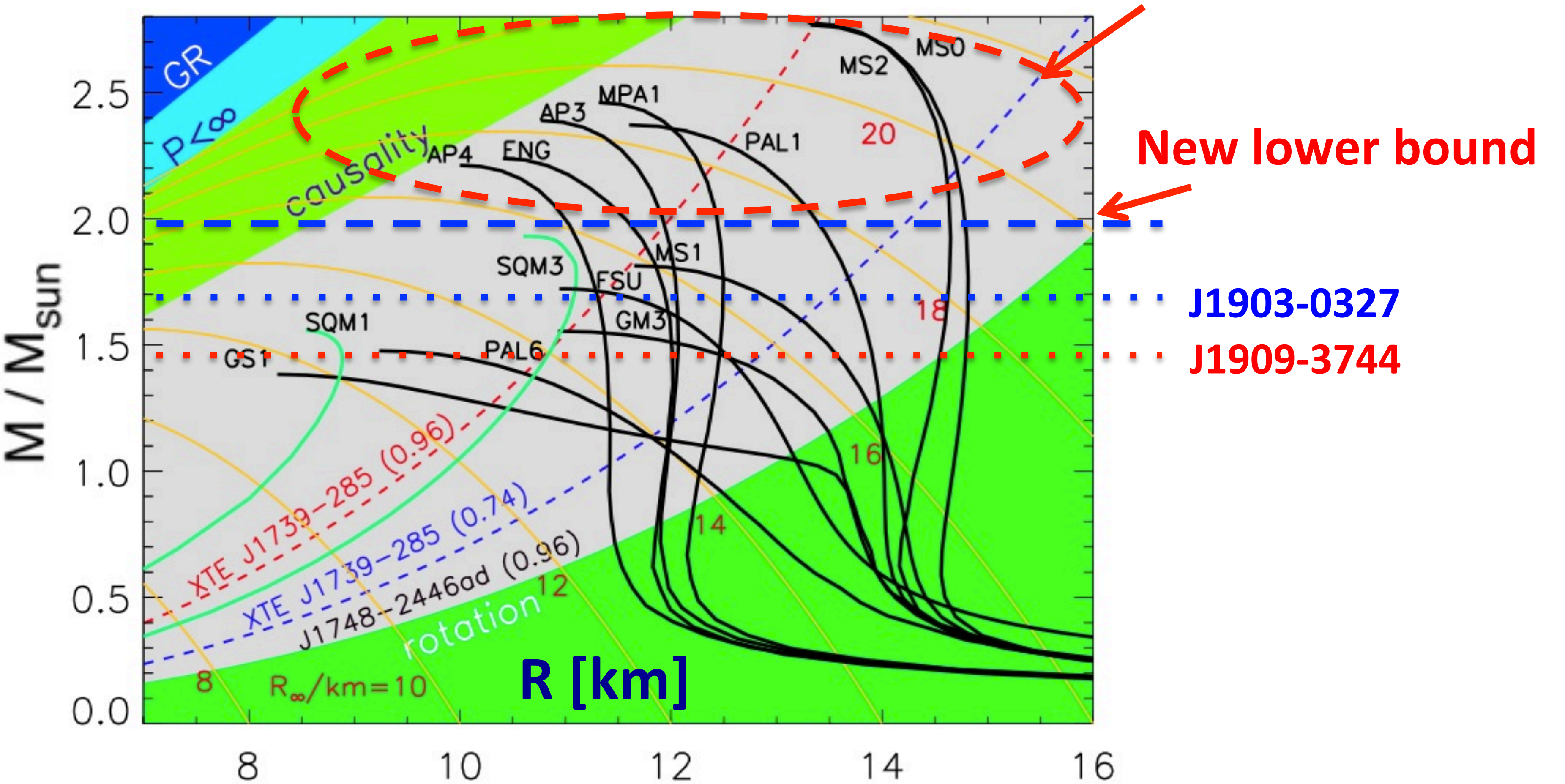
$$M_{\rm WD} = (0.500 \pm 0.006) M_\odot$$

$$M_{\rm NS} = (1.97 \pm 0.04) M_\odot$$

Demorest et al. (2010)

M-R relation for various EoSs

Demorest et al. (2010)



Theoretical challenge !

Hadronic EoSs

e.g.) Akmal-Pandharipande-Ravenhall (APR) 1998

(A18 + δv + UIX version)

based on :

Av-18 two-body potential → fit 2N scattering data very well

UIX : 3-body interactions → important for saturation properties

δv : boost corrections

Good descriptions from saturation properties to spectra of light nuclei

Extrapolation to $n_B \sim 6\ n_0$ ➡ $M_{max} \sim 2.2\ M_{sun}$

But there are important qualifications here **:**

The EoS ignores hyperons which cause considerable softening.

It is not clear how higher-body int. can be truncated.

Convergence ?

Many-body interaction (APR-Av18+UIX case)

n_B	2 –body int.		3 –body int.		4 –body int.
	$\langle v^{\pi}_{ij} \rangle$	$\langle v^{R}_{ij} \rangle$	$\langle V^{2\pi}_{ijk} \rangle$	$\langle V^{R}_{ijk} \rangle$	
n_0	-4.1	-29.9	1.2	4.5	
$2\, n_0$	-25.1	-36.4	-17.4	30.6	?
$3\, n_0$	-35.7	-44.7	-34.1	78.0	
$4\, n_0$	-52.2	-41.1	-76.9	160.3	

At $n_B \sim 2n_0$: 2- and 3- body interactions are comparable.

It can be: $V_{2\text{-body}} \sim V_{3\text{-body}} \sim V_{4\text{-body}} \sim \ldots$ at $n_B > 2n_0$

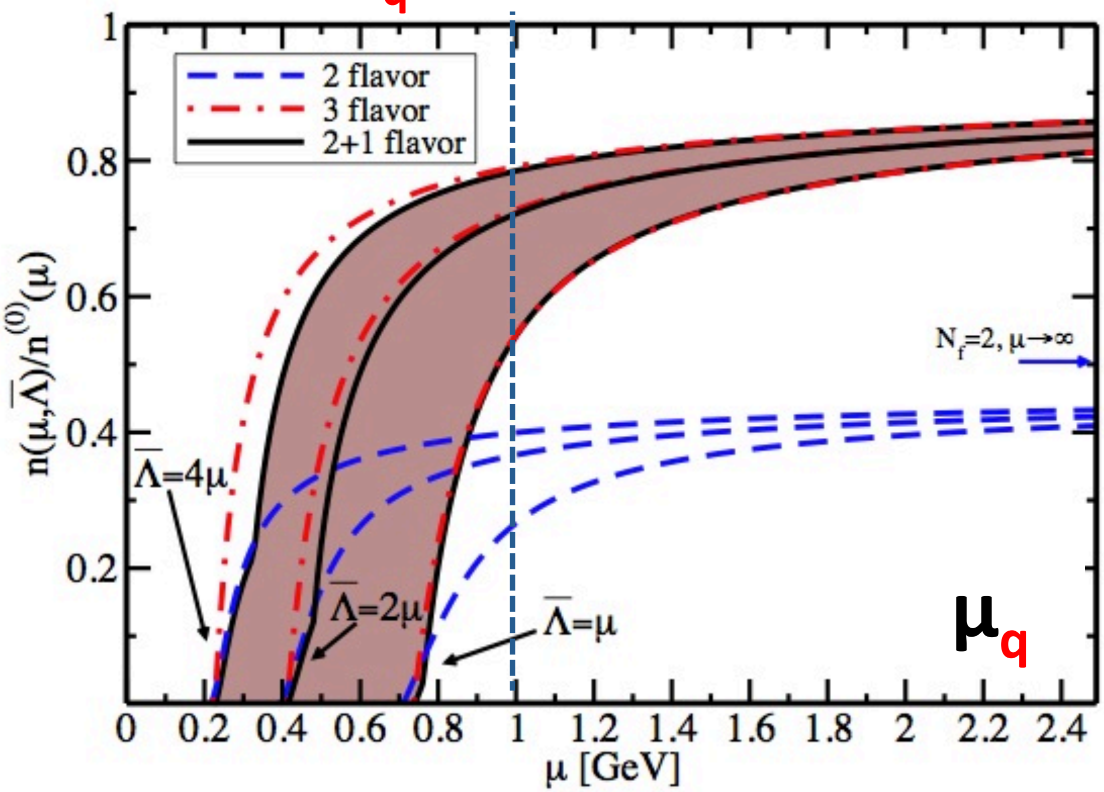
Perturbative QCD EoS

O(α_s^2) & m_s corrections: Freedman-McLerran 78; Baluni 78

Kurkela-Romatschke-Vuorinen 09

e.g.) number density (Kurkela-Romatschke-Vuorinen 09)

$\mu_q \sim 1$ GeV



For $\mu_q < 1$ GeV :

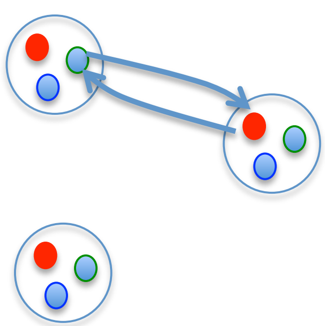
($n_B < 100\ n_0$)

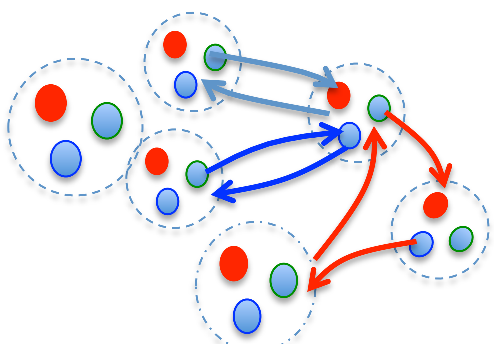
Renormalization scale dependence is large

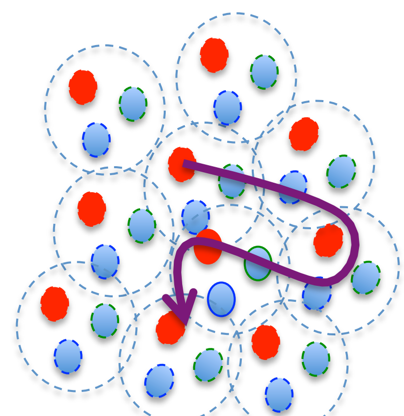
3-window modeling

(Masuda-Hatsuda-Takatsuka 12)

APR →	Interpolated EoS	← NJL + phen. int.	
(confining)	(output)	(non-confining)	
dilute	pre-percolation	percolation	(pQCD)
few meson (quark) exchange	many-meson exchange (many-*quark exchange*)	*Baryon overlap*	
nucleons	excited baryons, Δ, Σ,.. (*quark excitation*)	Quark Fermi sea	







n_B

$\sim 2n_0$ $\sim (4\text{-}7)n_0$ $\sim 100n_0$

Phenomenological QCD equation of states

When do EoSs become stiff ?

simple parameterization at large density :

bag constant.

$$\varepsilon(n) = c_1 n^{4/3} + c_2 n^{2/3} + c_{-2} n^2 + B$$

$\sim p_F^4$ *kinetic* energy

$\sim p_F^2$ *pairing effects*

$\sim p_F^6$ density-density int.

$$P = \frac{\varepsilon}{3} - \frac{2}{3}c_2 n^{2/3} + \frac{2}{3}c_{-2} n^2 - \frac{4}{3}B$$

corrections to the conformal relation

P at given ε becomes large when :

- $c_2 < 0$ → attractive pairing effects (cf. diquark correlation)
- $c_{-2} > 0$ → repulsive density-density interactions (cf. ω-meson exchange or hard-core repuslsion)
- small bag constant

3-flavor quark model (n_B > (4-7) n_0)

(we use the Hatsuda-Kunihiro parameter set & mean field)

$$\mathcal{L} = \bar{q}(\mathrm{i}\partial\!\!\!/ - \hat{m})q + G\sum_{i=0}^{8}\left[(\bar{q}\tau_i q)^2 + (\bar{q}\mathrm{i}\gamma_5\tau_i q)^2\right] + \mathcal{L}_{t'Hooft}$$

Standard NJL : successful for meson phenomenology

(for chiral restoration → generate quark bag constant B_q)

$$-g_V(\bar{q}\gamma^\mu q)^2$$

density-density repulsion ~ ω-meson exchange

$$+H\sum_{A,A'=2,5,7}(\bar{q}\mathrm{i}\gamma_5\lambda_A\tau_{A'}q_c)(\bar{q}_c\mathrm{i}\gamma_5\lambda_A\tau_{A'}q)$$

color-mag. int.
or diquark correlation

+ *effects of the possible gluonic bag constant B_g*

& Constraints : β-equilibrium, charge neutrality (with leptons)
color-neutrality

Impact of the bag constant

For chiral sym. restored, deconfined free quark gas (3-flavor) :

$$P(\mu) = c_0 \mu^4 - B \qquad \varepsilon(\mu) = 3 c_0 \mu^4 + B$$

We can find the scaling :

$$M_{\max} \simeq 1.78\, M_\odot \left(\frac{155\,\mathrm{MeV}}{B^{1/4}} \right)^2 \qquad R \simeq 9.5 \left(\frac{155\,\mathrm{MeV}}{B^{1/4}} \right)^2 \mathrm{km}$$

If B were very small, even free gas could give a very large mass !

NJL model : **"quark" bag const. appears through the chiral restoration**

$$B_q^{\mathrm{NJL}} \simeq 284\,\mathrm{MeV/fm^3} = (219\,\mathrm{MeV})^4 \rightarrow M_{max} \sim 0.9\ M_{sun}$$

Our question is : ***provided*** **this order of magnitude for B,**

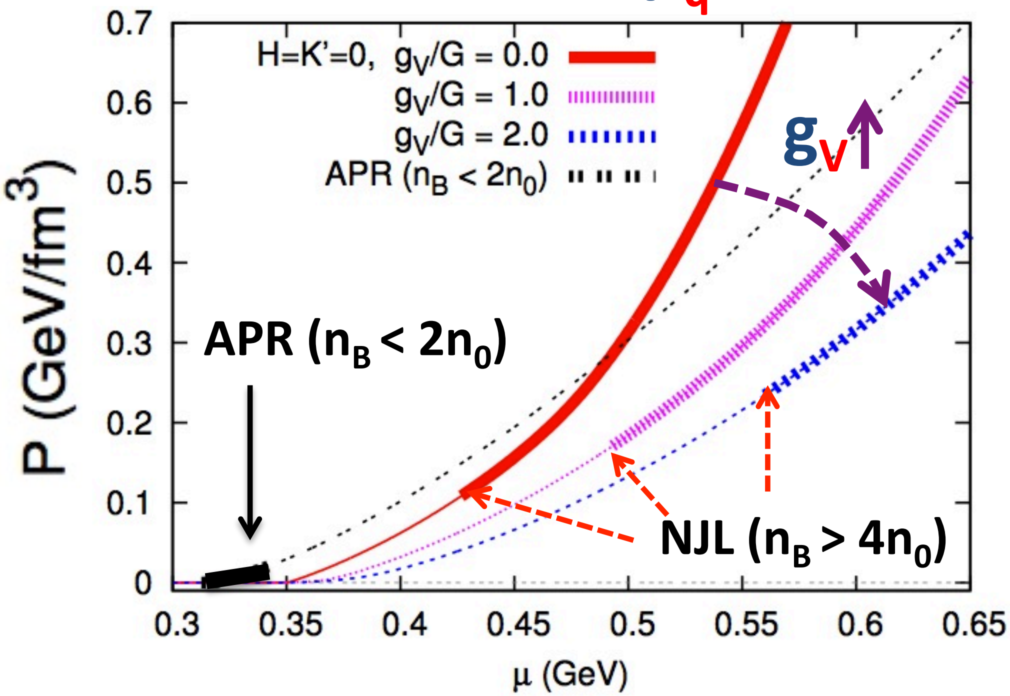
how can we achieve large star masses ?

Repulsive *density-density interaction*

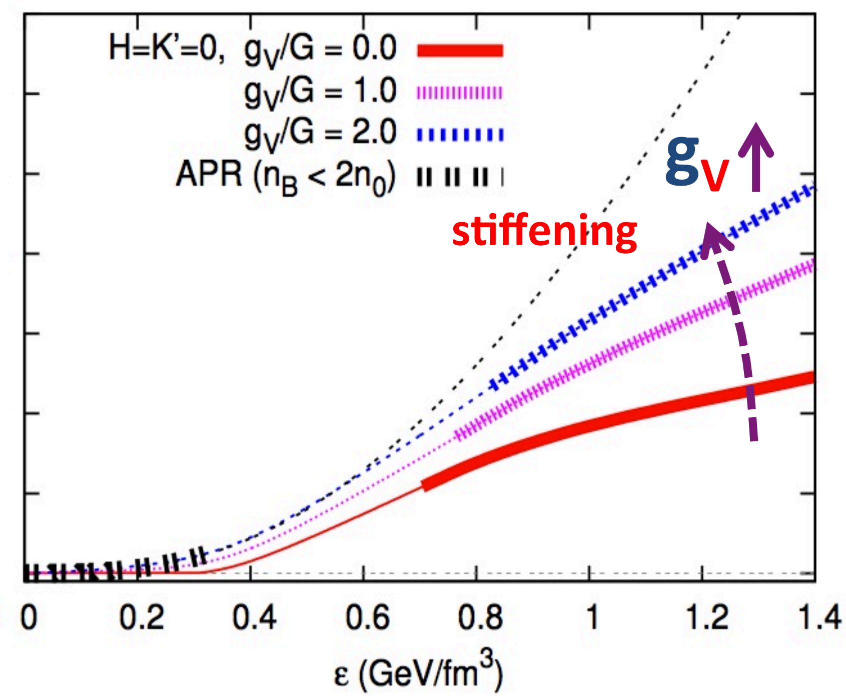
4-Fermi terms : $-g_V(\bar{q}\gamma^\mu q)^2$

$g_V \sim$ 2G (Bratovic et al. 12)

P v.s. μ_q



P v.s. ε



For g_V > 2G : quark EoSs can be stiff enough to allow

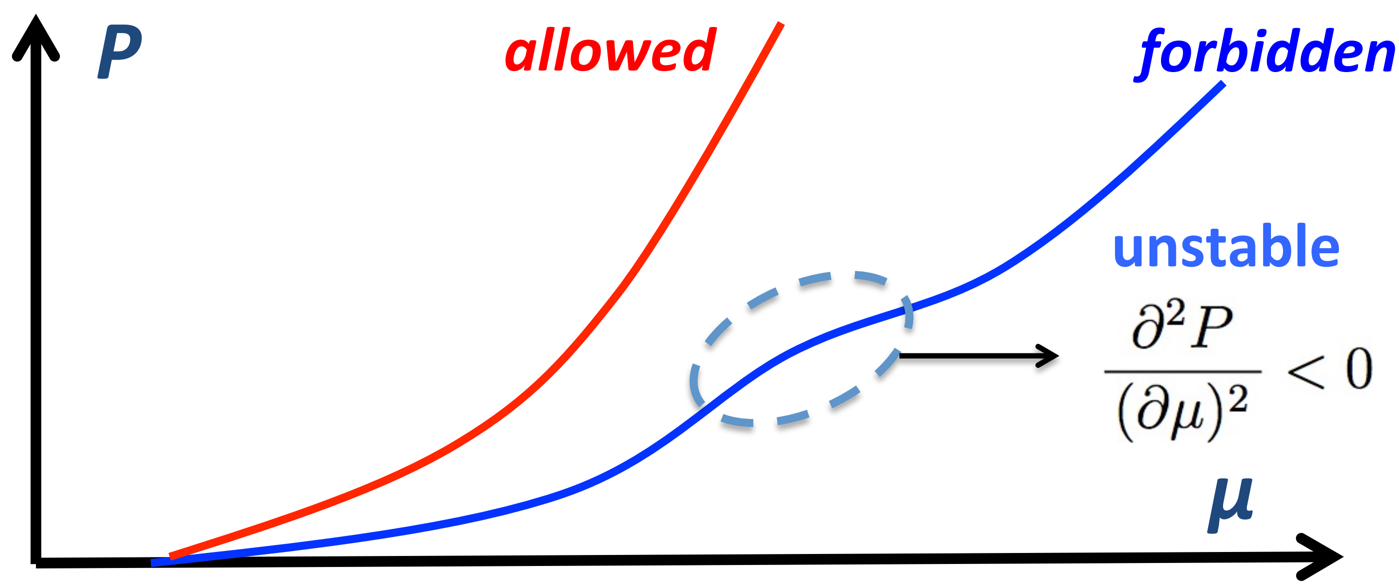
M ∼ 2 M_{sun} (Masuda-Hatsuda-Takatsuka 12)

However, one must check :

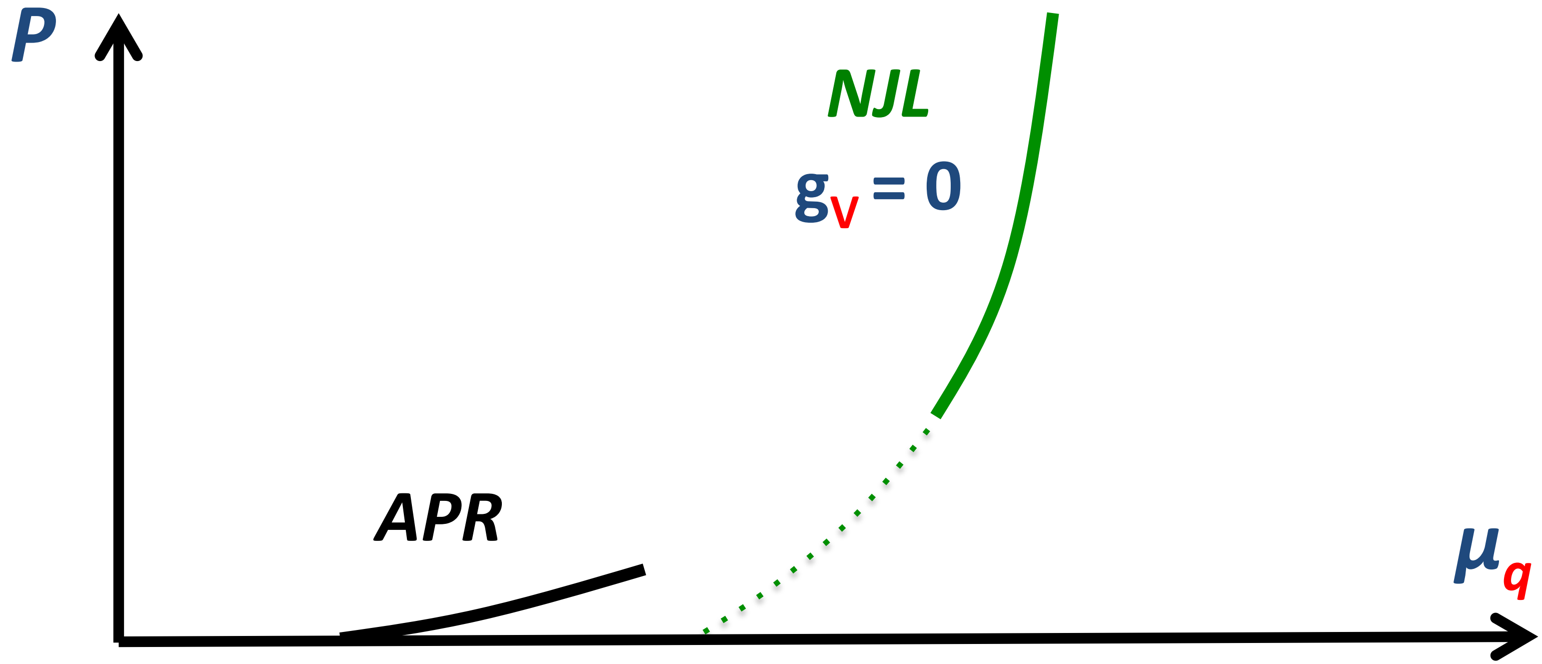
For the *stability* of the matter, we need

$$\frac{\partial^2 P}{(\partial\mu)^2} = \frac{\partial n_q}{\partial\mu} > 0$$ **at any μ**

examples)



What would happen at larger g_V ?



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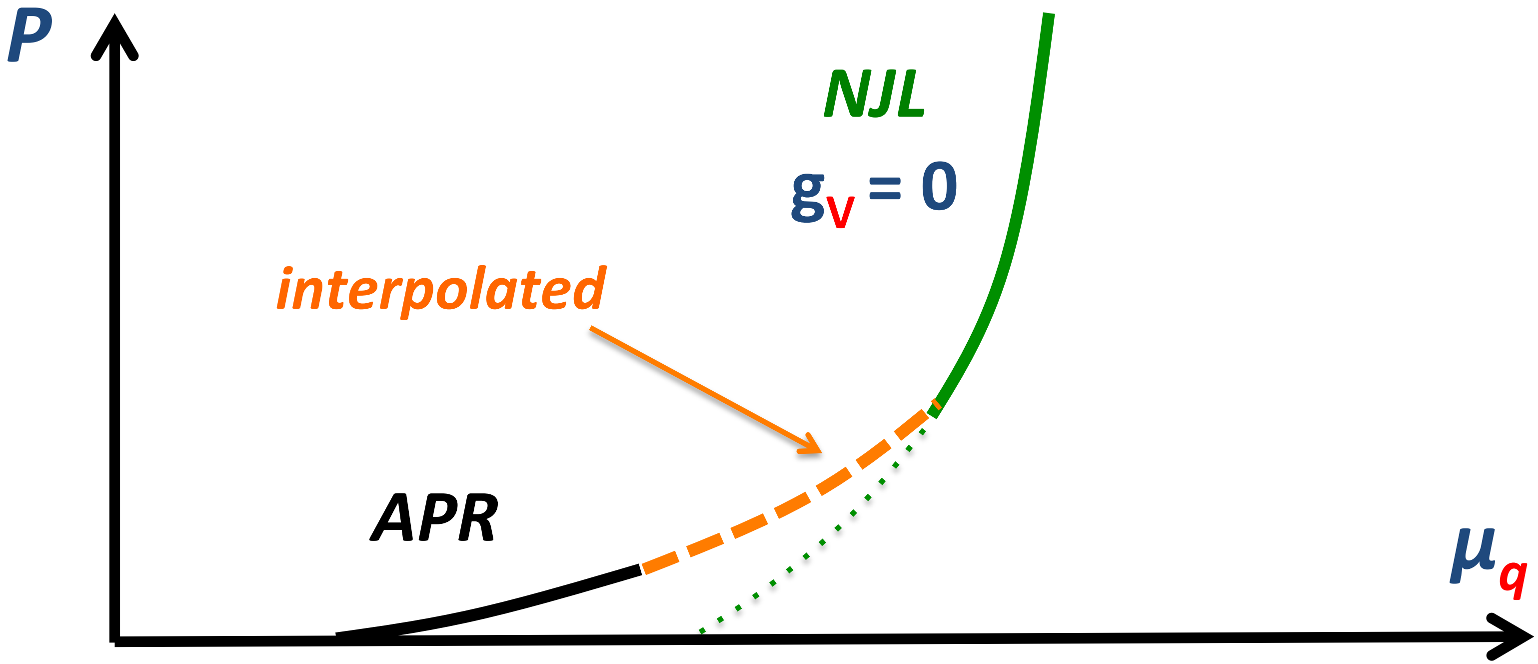

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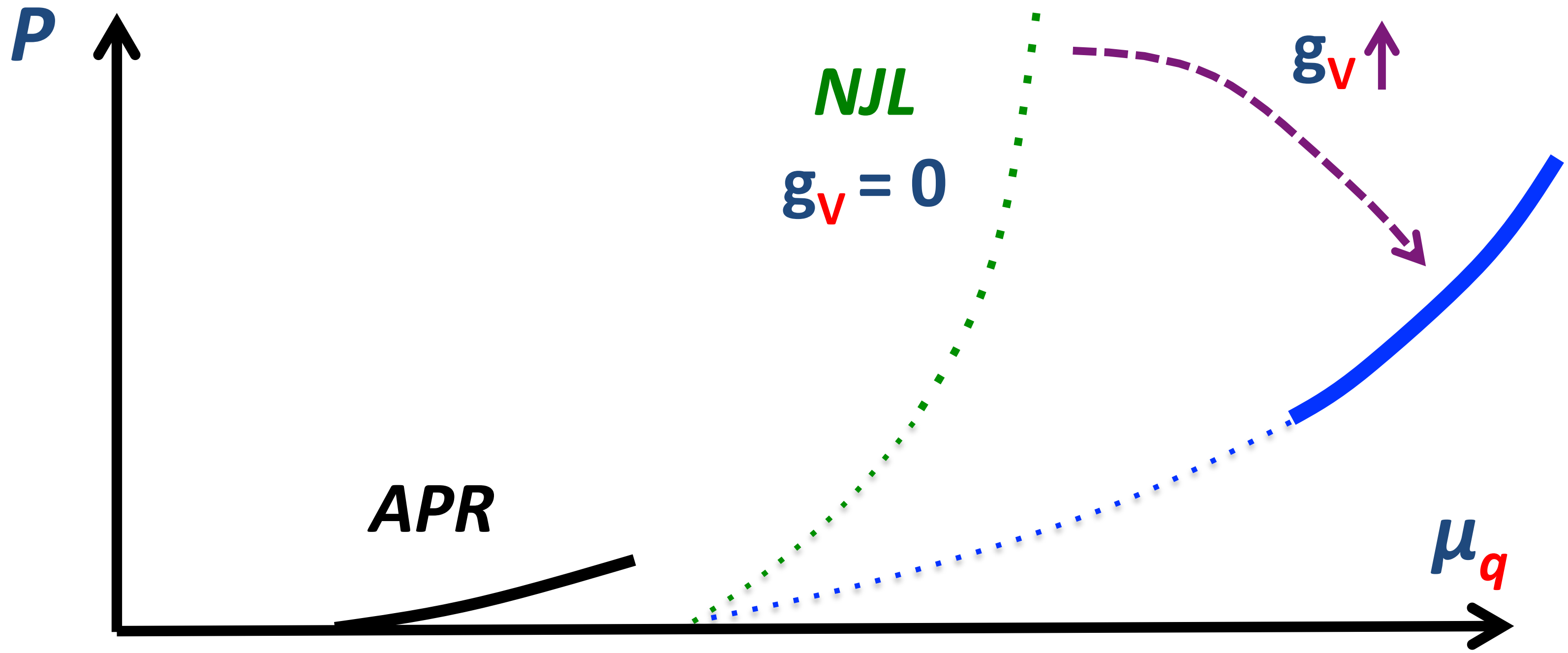

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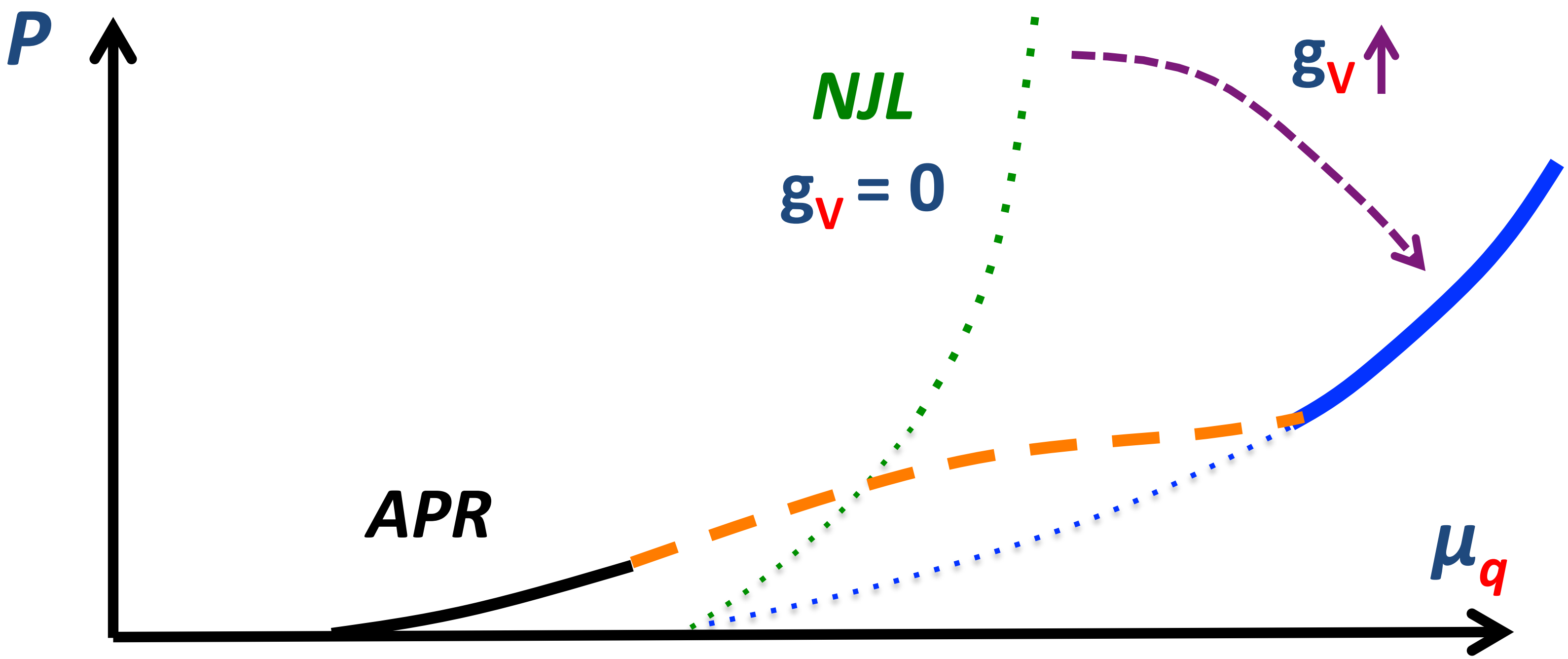
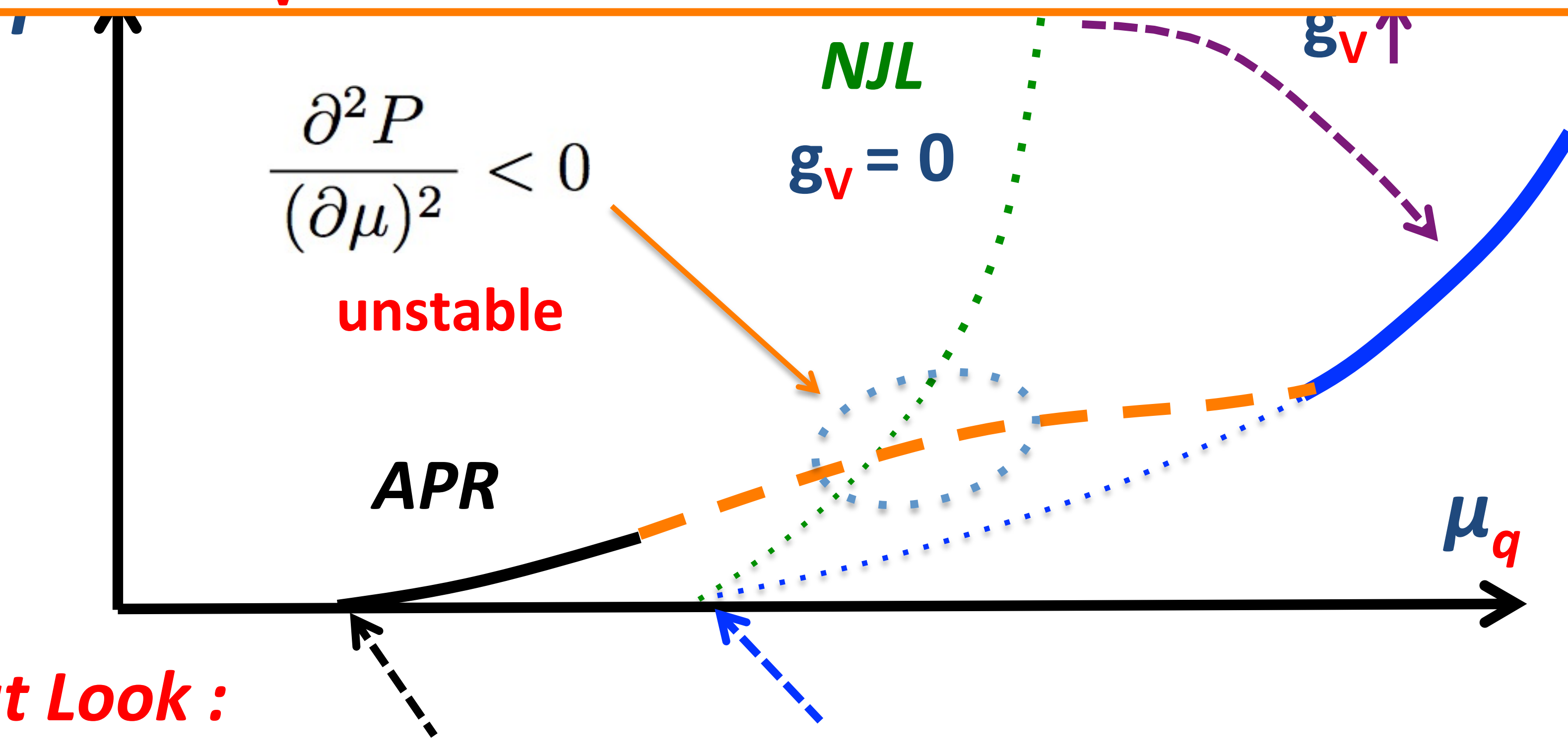


What would happen at larger g_V ?

At larger g_V, more danger to violate stability condition



But Look :

$M_N/3 \sim 313$ MeV

$M_{NJL}^{u,d} \sim 336$ MeV

The single quark energy is larger in the NJL model :

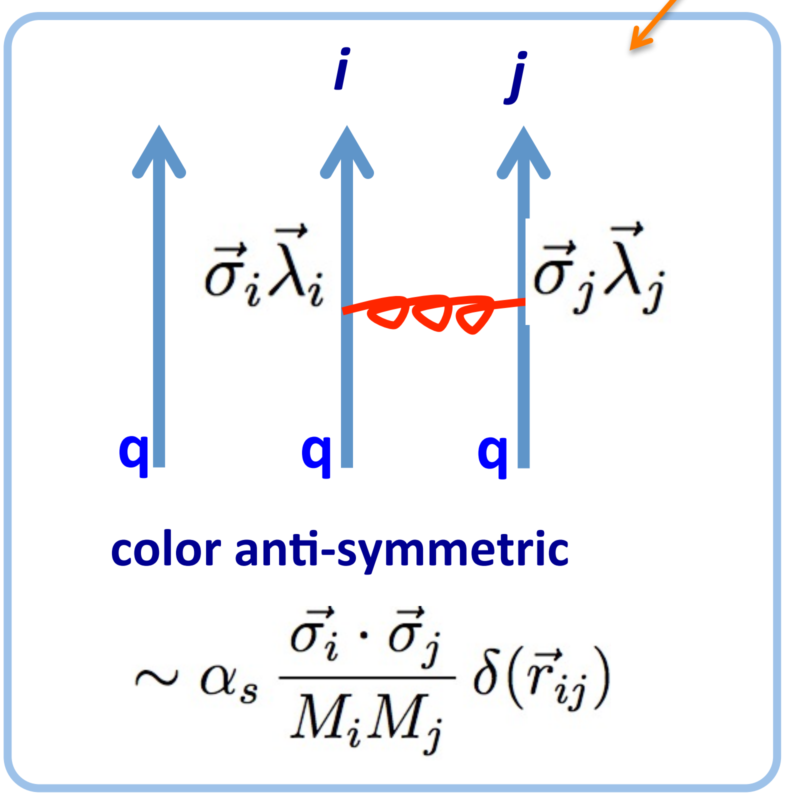
the NJL pressure curve tends to appear at larger μ_q region

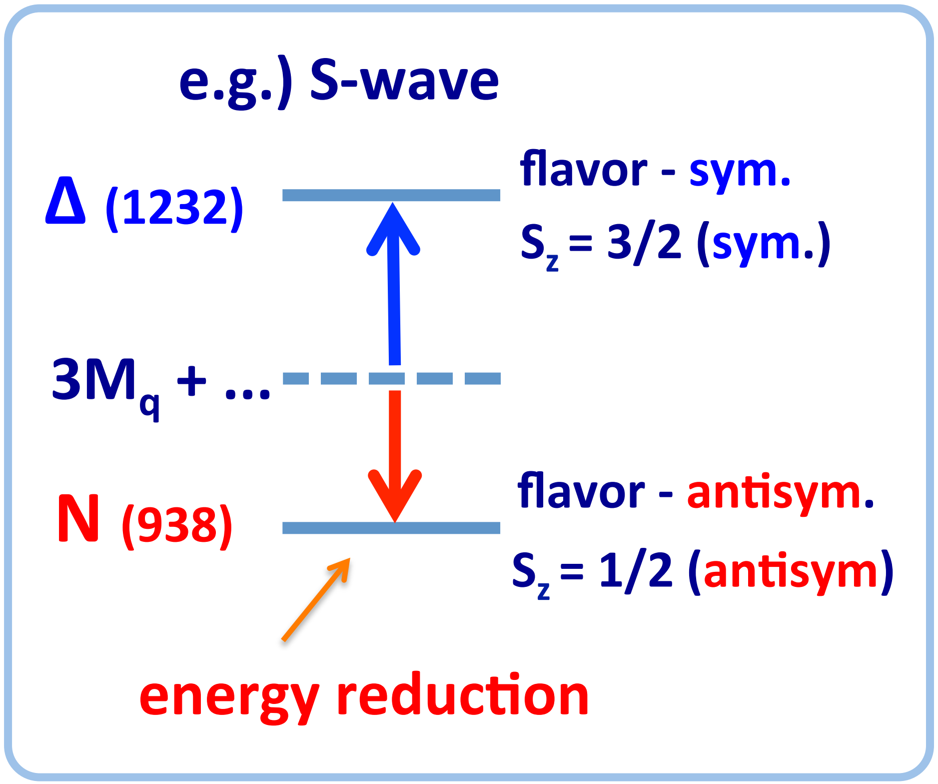
Attractive color-magnetic interaction

$M_q > M_N/3$: This is **NOT** quite unusual in quark models

e.g.) *Non-relativistic constituent quark model*

$$M_B = 3\,M_q + \Delta E_{\text{color-mag}} + \Delta E_{\text{conf}} + \ldots$$



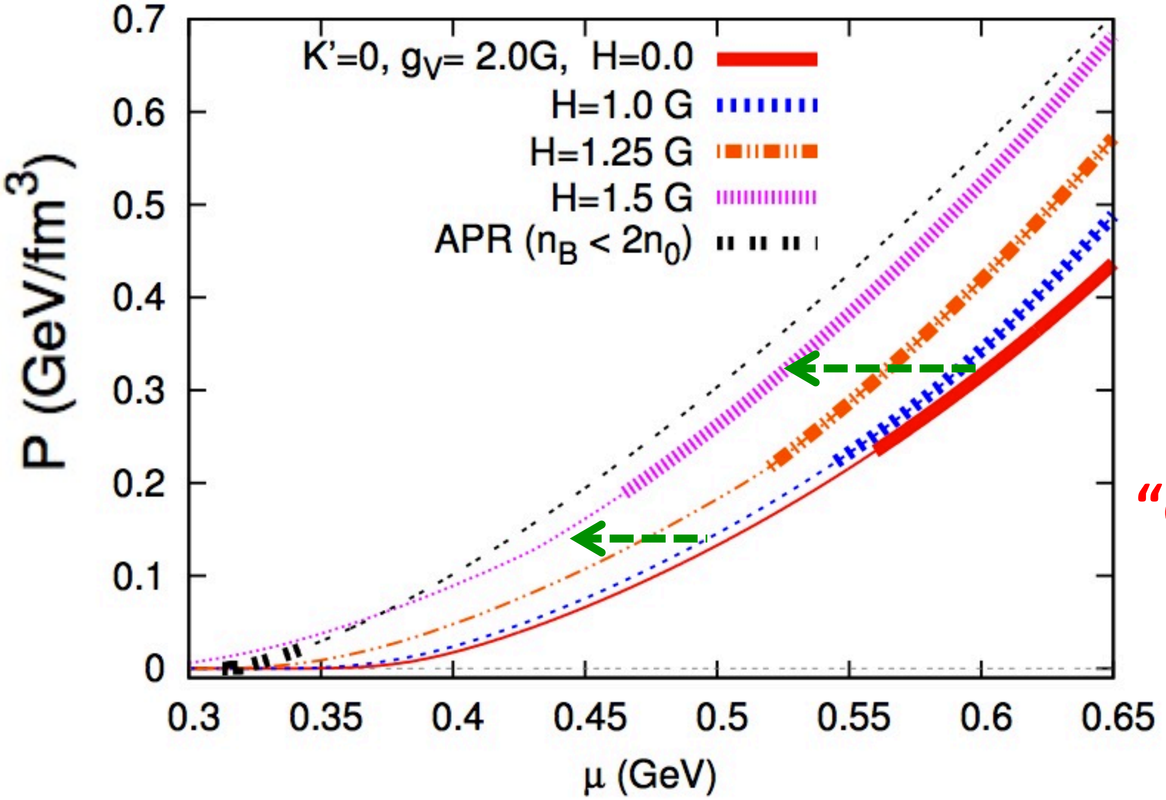


Attractive color-magnetic interaction

Add 4-Fermi terms for color-magnetic interaction:

$$H \sum_{A,A'=2,5,7} (\bar{q}\mathrm{i}\gamma_5\lambda_A\tau_{A'}q_c)\,(\bar{q}_c\mathrm{i}\gamma_5\lambda_A\tau_{A'}q)$$

attractive **for S-wave, color-antisym., flavor-antisym. , spin-singlet**



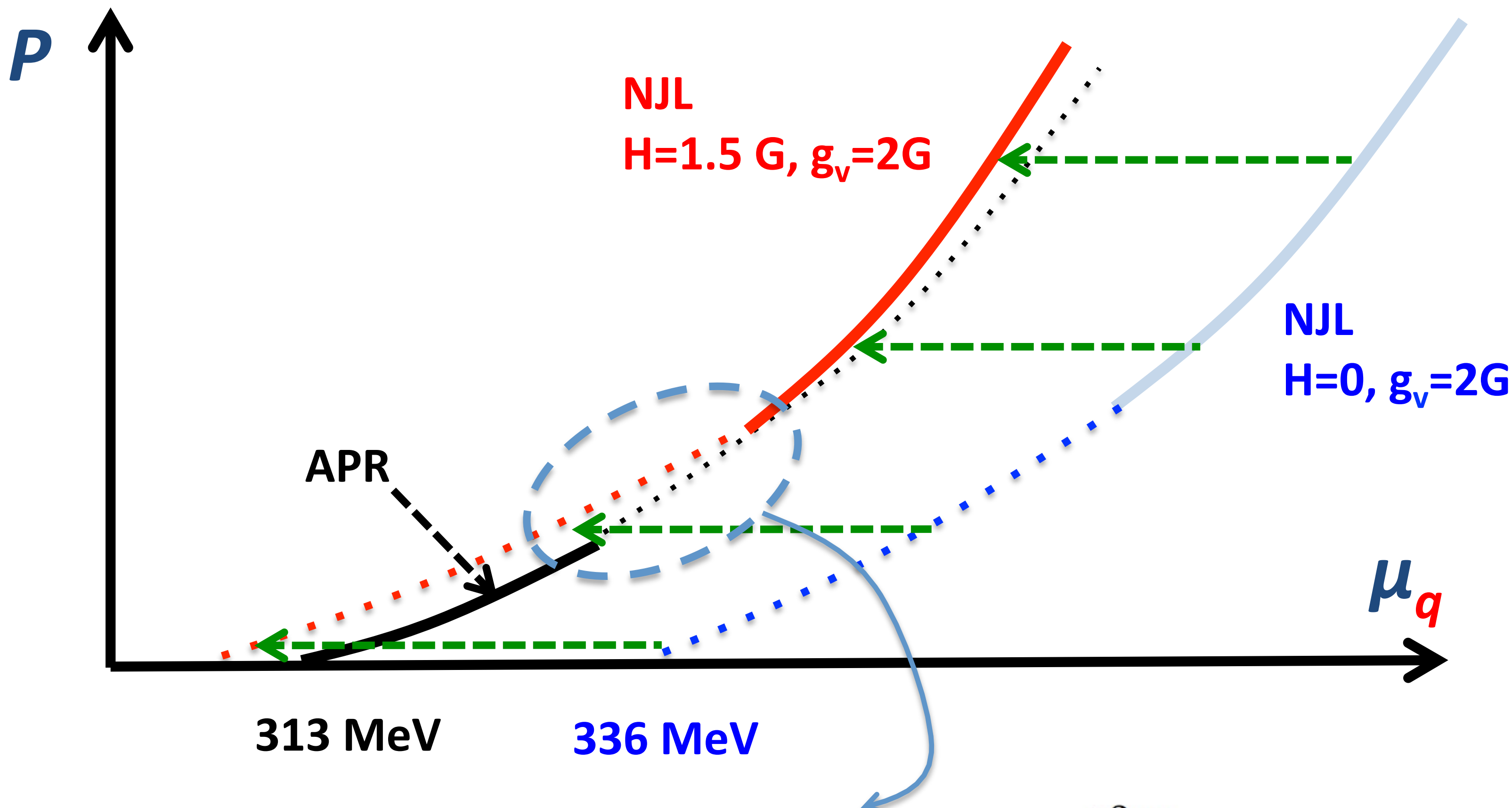
"reduction" of average quark energy

"overall shift" of P(μ) curve toward lower μ- region

Attractive color-magnetic interaction

Overall shift *from low to high densities* :

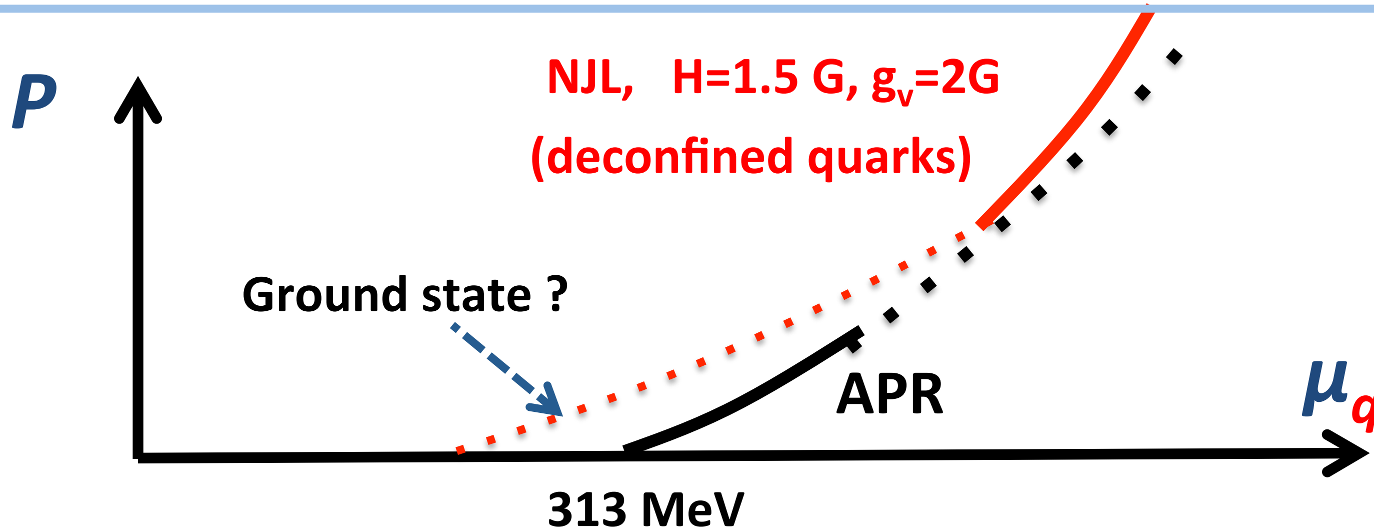
(due to relatively short-range nature)



Easier to interpolate *without* introducing $\frac{\partial^2 P}{(\partial \mu)^2} < 0$ -region

Ground state ?

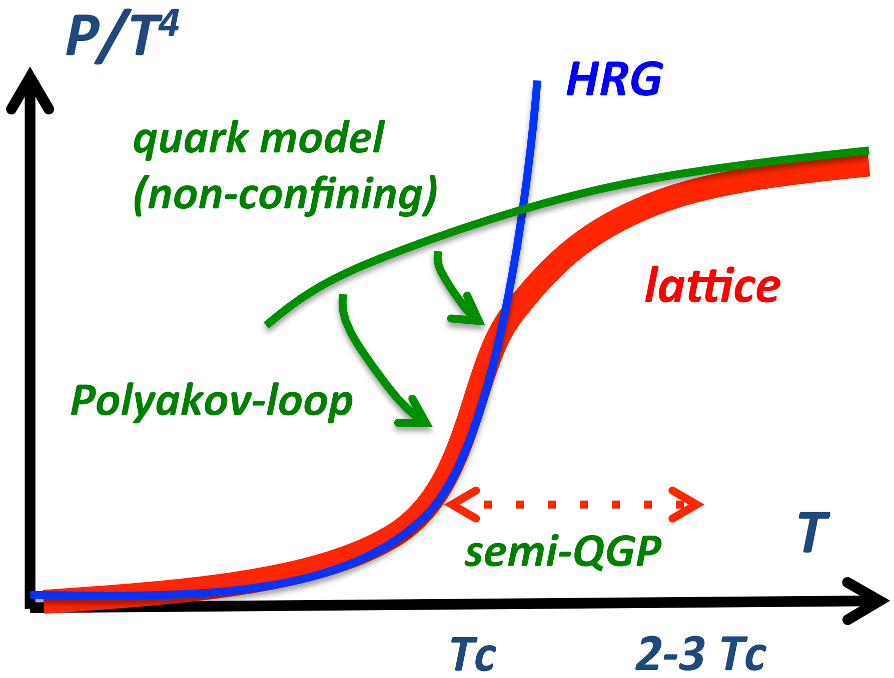
Ground state : a state with largest P at given μ
(or smallest ε at given n)

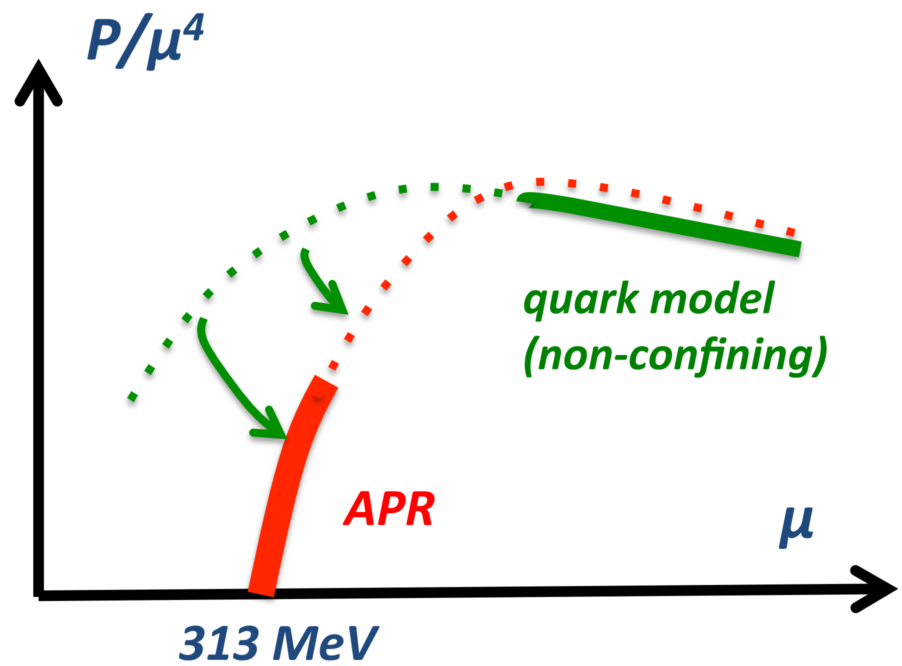


If we insist that "(3-flavor) *deconfined quark* " matter is *more stable* than *nuclear* matter, the picture is similar to " *strange quark star* " –picture.
(Witten 84)

Instead, we shall insist that *at low density*,
the confining effects kill the excess of quark pressure.

Some analogy with *finite-T QCD*





At low *T* :

Polyakov-loop suppresses colored, single quark excitation

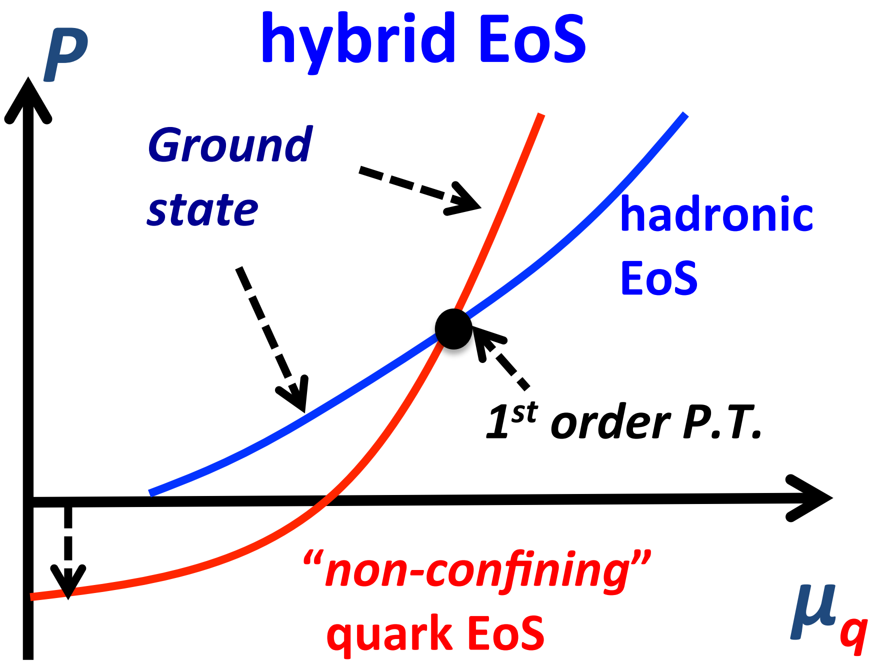
→ *less* single quark pressure

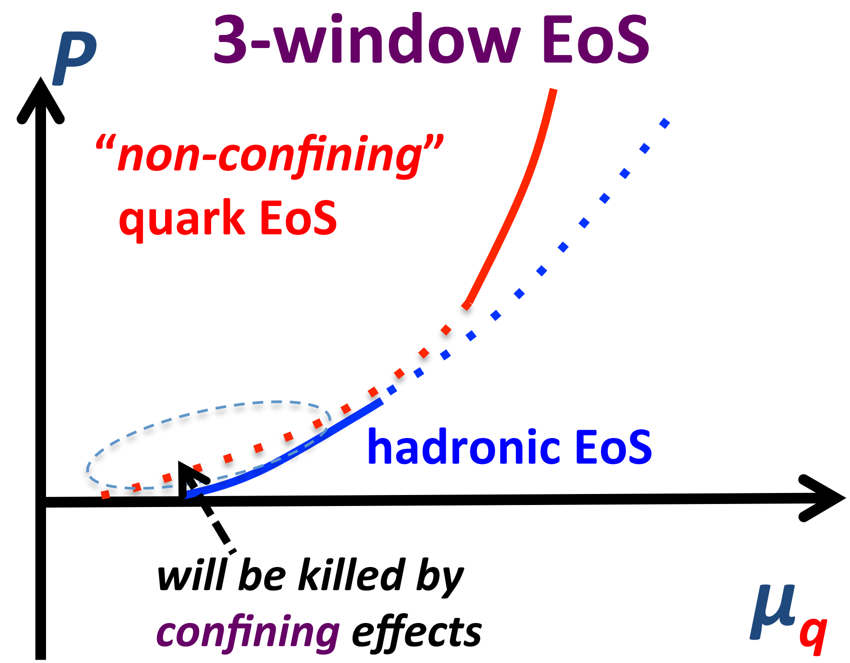
At low *μ* :

Quarks are trapped into baryons

→ *less* single quark pressure

(Conventional) hybrid EoS v.s. 3-window EoS



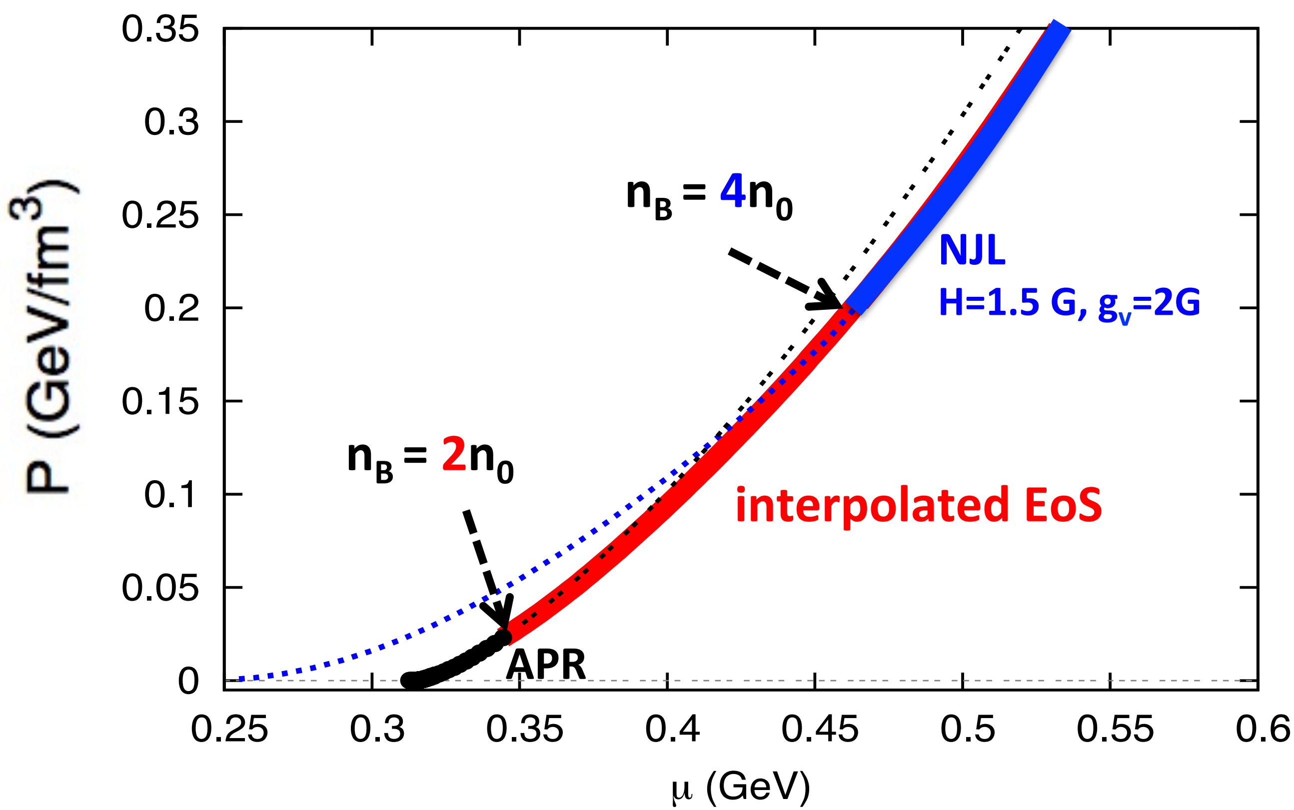


Demands for (non-conf.) quark model parameters

At low μ	At low μ
quark pressure < hadronic pressure	quark pressure > hadronic pressure

The difference affects the high density behaviors of quark EoSs !

Interpolated EoS (3-window model)



Check:

speed of sound

$$c_s^2 = \frac{\partial P}{\partial \varepsilon} < 1$$

interpolated EoS :

$$\mathcal{P}(\mu) = \sum_{m=0}^{N} b_m \mu^m$$

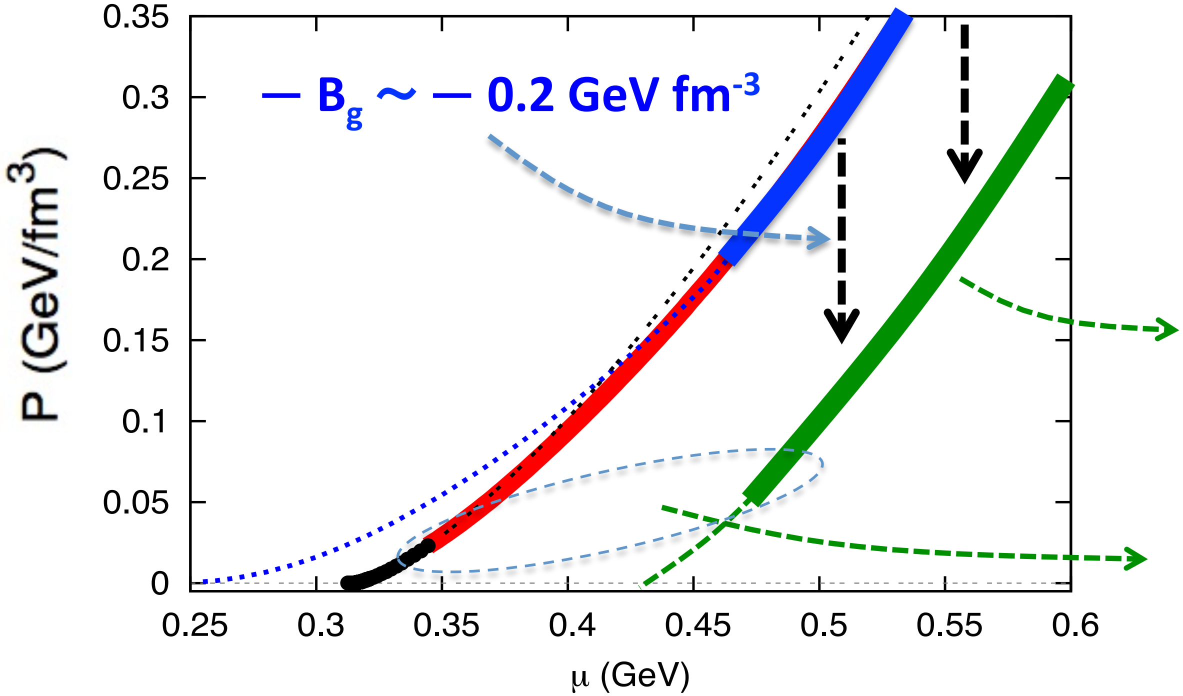
***Matching* : up to 2^{nd} order of derivatives at $n_B = 2n_0$ & $4n_0$**

Gluonic bag constant ?

What would happen if the *gluonic* sector becomes *perturbative* ?

$$\varepsilon \to \varepsilon + B_g \qquad P \to P - B_g$$ **(Softening)**

$$B_g \sim (0.2\,\mathrm{GeV})^4 \sim 0.2\,\mathrm{GeV fm}^{-3}$$



If B_g is such large :

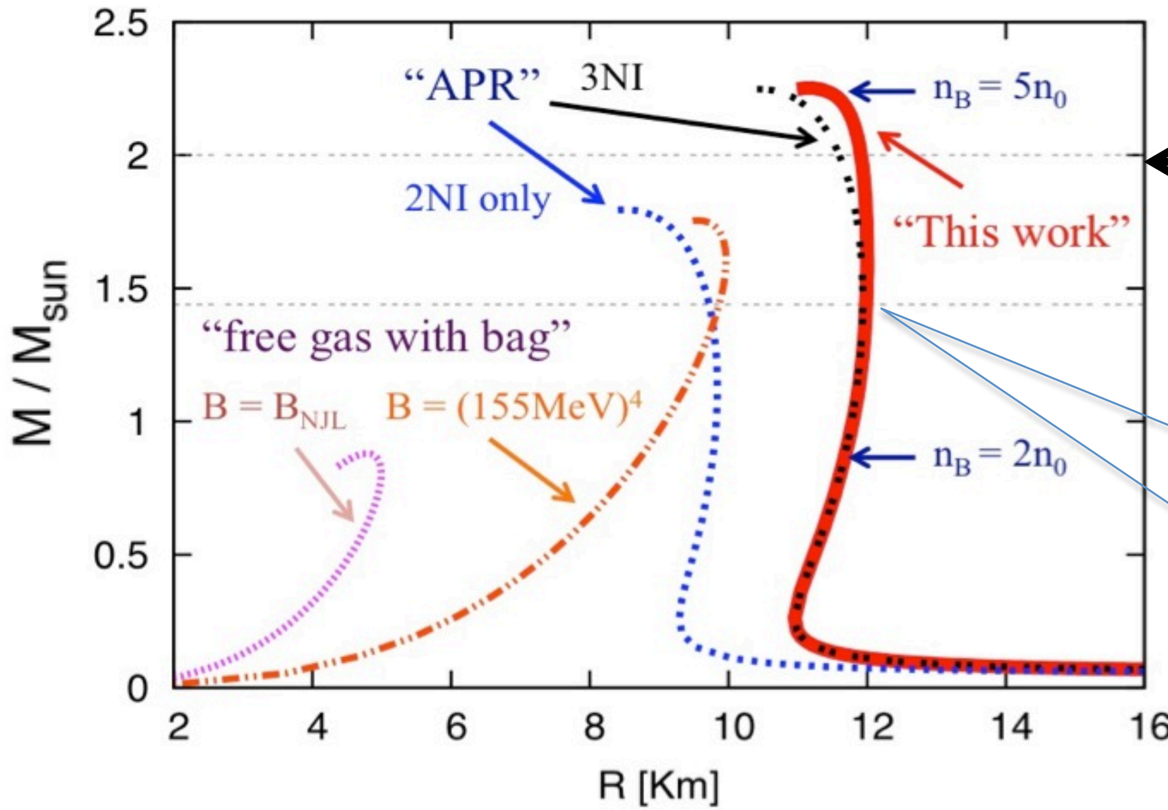
1, Quark EoS becomes *significantly* softer

2, More difficulties for the interpolation

The gluon sector *should remain non-perturbative* to $n_B \sim 10\, n_0$

(The gluonic bag constant should be small)

M – R relation



lower bound

PSR J1614 - 2230

Our result for

g_v = 2G, H = 1.5G

B_g = 0

Our 3-window EoS gives P-ε relation similar to APR:

→ similar M-R relation

But relevant d.o.f in these EoSs are quite different.

Summary

1, For 2 $n_0 < n_B <$ 10 n_0 ; we *should* construct EoSs in which d.o.f. are neither purely hadronic nor weakly int. quarks & gluons.

2, Interactions, *which have been important in the baryon phenomenology* , are carried over to finite n_B , and we clarify the possible impacts.

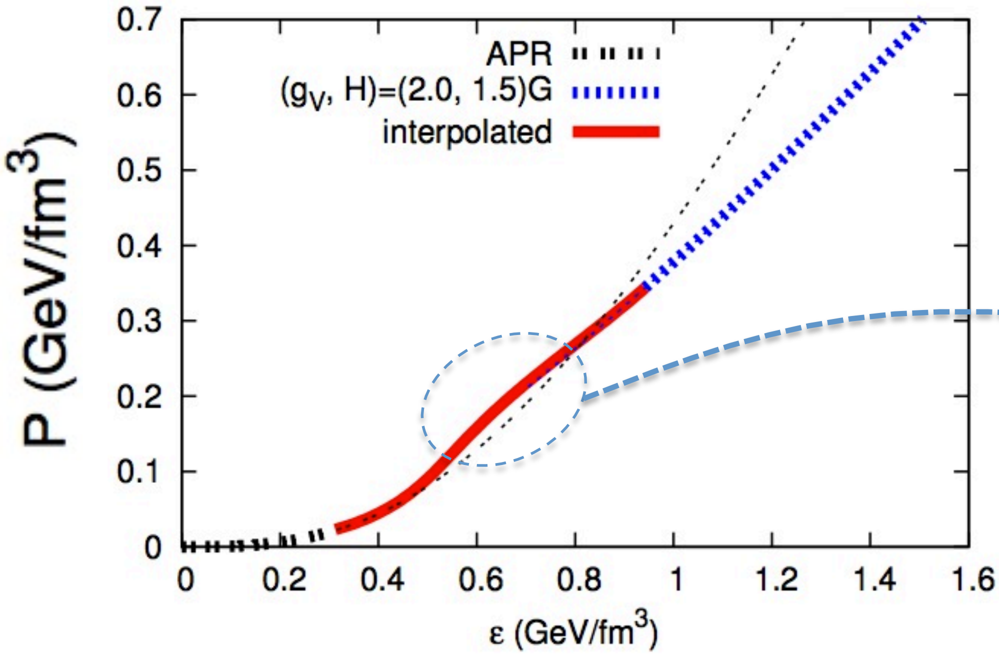
3, Gluons should remain non-perturbative to $n_B \sim 10\, n_0$.

4, Consideration on missing confining effects in quark models require constraints different from conventional hybrid EoSs.

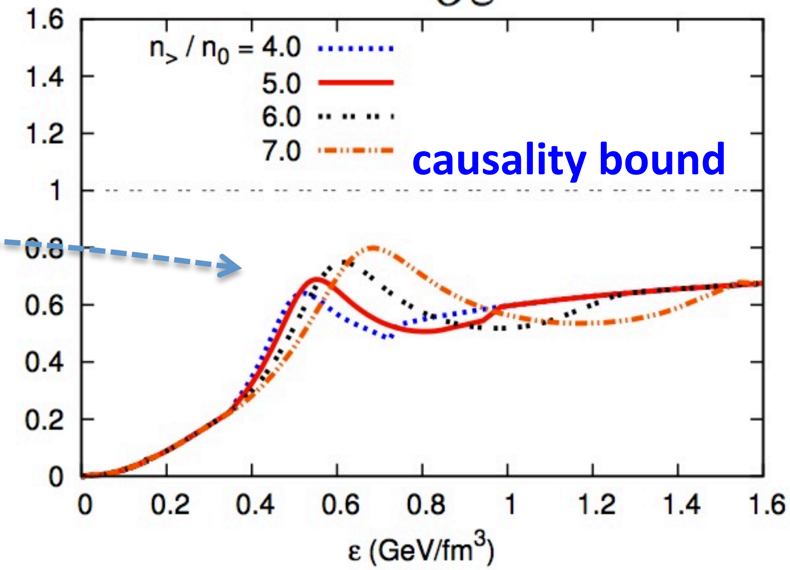
→ This allows us to study quark EoSs unexplored previously.

5, Our challenge is : to replace the sketchy 4-Fermi descriptions with more microscopic calculations.

P v.s. ε & speed of sound



$$c_s^2 = \frac{\partial P}{\partial \varepsilon}$$



speed of sound < the causal bound (speed of light)

Neutron stars