Gauge invariant off-shell amplitudes and their applications

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PLAN

- Introduction
  - Off-shell amplitudes
  - Gauge (non-)invariance
- Gauge invariant off-shell amplitudes using Wilson lines
- Applications
  - Gauge invariant decompositions
  - Recursion relations and connection to light-front amplitudes
  - High Energy (or k_T) factorization and Generalized TMD factorization
- Future plans

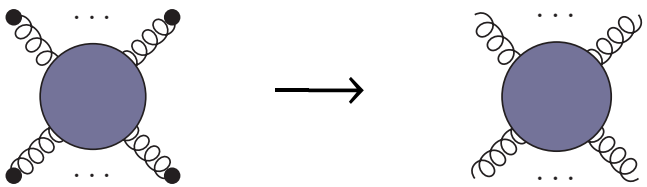
Off-shell gluon amplitudes

Recollection: on-shell amplitudes

Reduction formula (color is suppressed):

$$\mathcal{M}(\varepsilon_1, \ldots, \varepsilon_N) = \lim_{k_1^2 \to 0} \left(\varepsilon_1^{\mu_1} k_1^2\right) \ldots \lim_{k_N^2 \to 0} \left(\varepsilon_N^{\mu_N} k_N^2\right) \; \tilde{G}_{\mu_1 \ldots \mu_N}(k_1, \ldots, k_N)$$

where polarization vectors satisfy $\varepsilon_i \cdot k_i = 0$.



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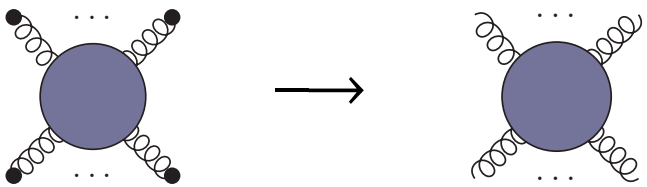
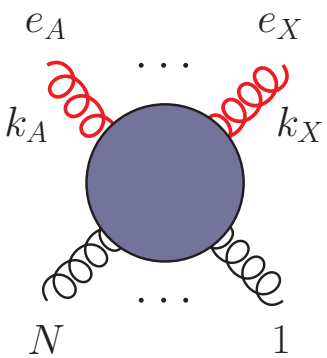


Off-shell amplitudes (non-gauge-invariant)

$$\mathcal{M}_{e_A e_B \ldots e_X}(\varepsilon_1, \ldots, \varepsilon_N) =$$



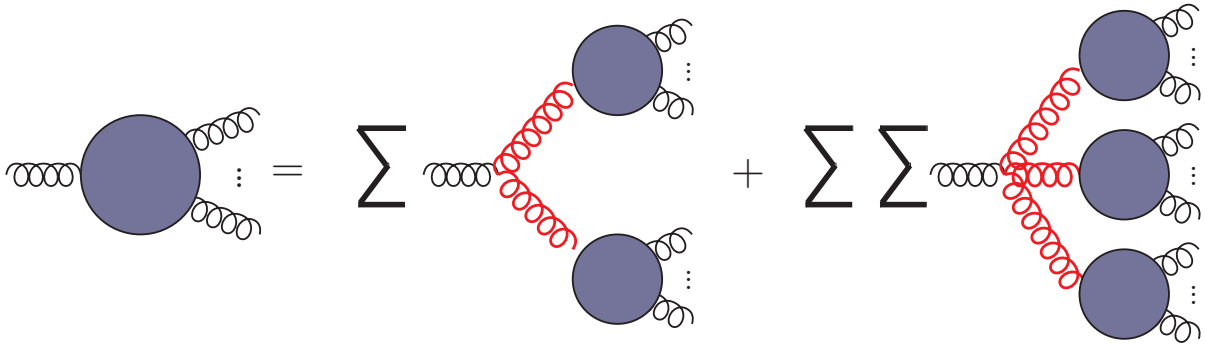
$$k_A^2 \neq 0, \ldots, k_X^2 \neq 0$$

$$k_A \cdot e_A = 0, \ldots, k_X \cdot e_X = 0$$

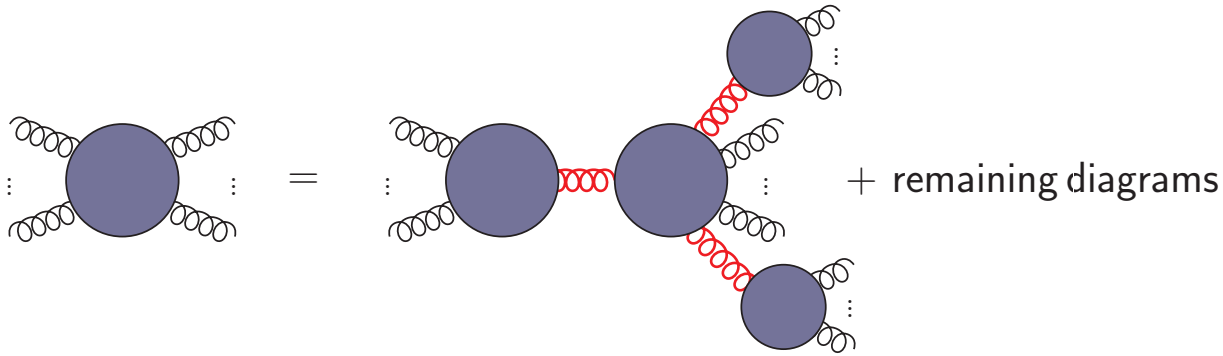
The legs with momenta $k_A, k_B, \ldots, k_X$ are not reduced. They are contracted with four-vectors $e_A, e_B, \ldots, e_X$ playing a role of "polarization" vectors for off-shell legs.

Examples

Berends-Giele recursion



Internal part of a subset of diagrams



'Polarization' vectors for off-shell gluons:

$$g^{\mu\nu} - k^{\mu}k^{\nu}/k^2 = \sum_{i=0}^{2} e^{(i)\nu}(k)\, e^{(i)\mu}(k)\, d_i(k)$$

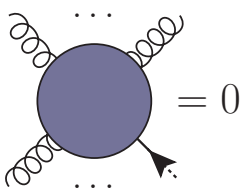
where $k \cdot e^{(i)}(k) = 0$, $e^{(i)}(k) \cdot e^{(j)}(k) = d_i(k)\,\delta_{ij}$, $d_0(k) = \pm 1$, $d_1(k) = d_2(k) = -1$.

Gauge invariance

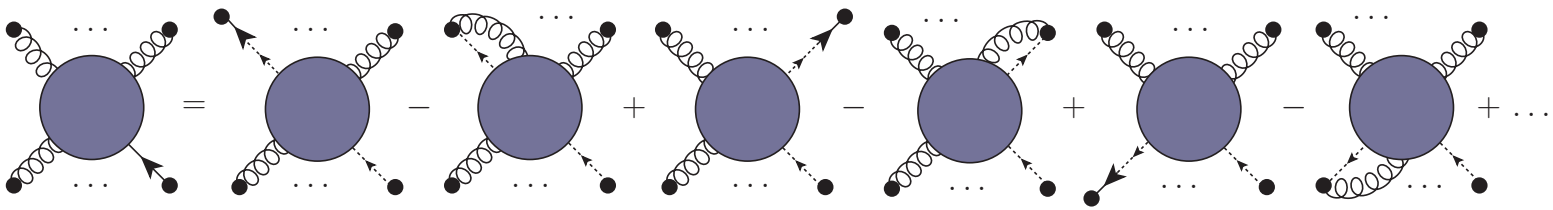
On-shell amplitudes

Gauge invariance implies the Ward identities

$$\mathcal{M}(\varepsilon_1, \ldots, k_i, \ldots, \varepsilon_N) = 0$$



It follows from the Slavnov-Taylor identity (and the reduction):



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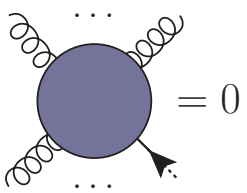

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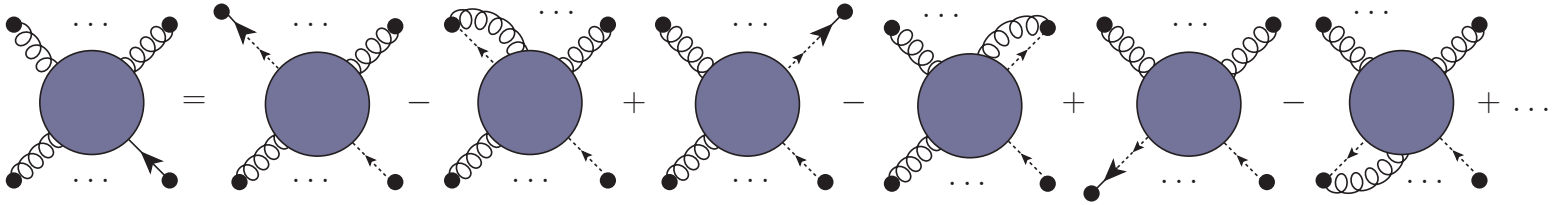
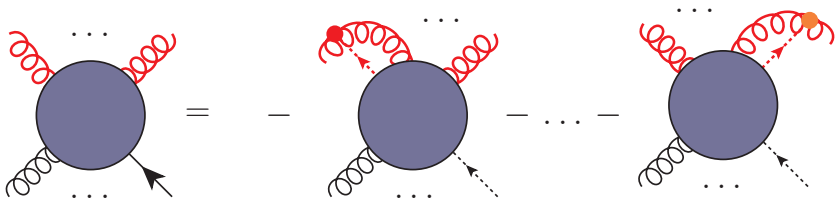
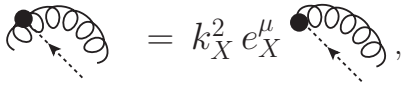


Off-shell amplitudes

Reduction of the Green's function gives



, where  $= k_X^2 \, e_X^\mu$, $X = A, B, \ldots$

$$\Longrightarrow \quad \mathcal{M}_{e_A e_B \ldots}(\varepsilon_1, \ldots, k_i, \ldots, \varepsilon_N) \neq 0$$

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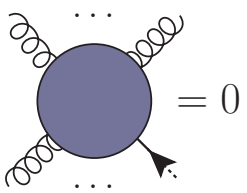

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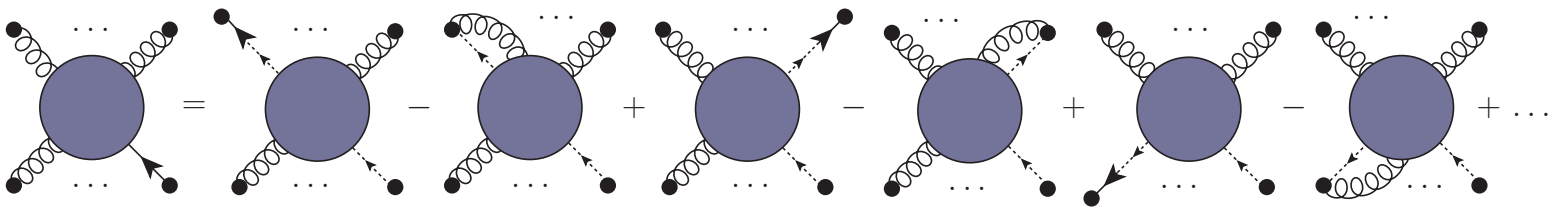

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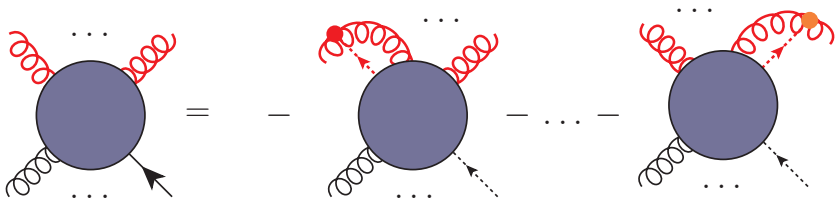

, where

$$\Longrightarrow \quad \mathcal{M}_{e_A e_B \ldots}(\varepsilon_1, \ldots, k_i, \ldots, \varepsilon_N) \neq 0$$

Off-shell *gauge invariant* amplitudes

$\tilde{\mathcal{M}}_{e_A \ldots e_X}(\varepsilon_1, \ldots, \varepsilon_N)$ such that $\tilde{\mathcal{M}}_{e_A \ldots e_X}(\varepsilon_1, \ldots, k_i, \ldots, \varepsilon_N) = 0$.

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Gauge invariant off-shell amplitudes

Methods of constructing $\tilde{\mathcal{M}}$

- One or two off-shell legs (suitable for k_T factorization)
  - Lipatov's effective action
    [E. Antonov, L. Lipatov, E. Kuraev, I. Cherednikov, Nucl.Phys. B721 (2005) 111-135]
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  - Off-shell quarks
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- Any number of off-shell legs
  - Using matrix elements of Wilson lines [PK, JHEP 1407 (2014) 128]
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Gauge invariant off-shell amplitudes

Wilson line operators

$$\Re_n^{(\varepsilon)c}(k) = \int d^4y\, e^{iy\cdot k}\, \mathrm{Tr}\left\{\frac{1}{\pi g}\, t^c\, [y]_n^{(\varepsilon)}\right\}$$

where

$$[y]_n^{(\epsilon)} \equiv [y - \frac{2}{\epsilon}n, y + \frac{2}{\epsilon}n]$$

$$[x, y] = \mathcal{P}\exp\left\{ig \int_C dz_\mu A_b^\mu(z)\, t^b\right\}$$

with the path defined as

$$z_\epsilon^\mu(s) = y^\mu + \frac{2}{\epsilon}\tanh\left(\frac{\epsilon s}{2}\right) n^\mu, \quad s \in (-\infty, \infty)$$

For small ϵ the path looks like a straight line

$$z_\epsilon^\mu(s) = x^\mu + sn^\mu + O\left(\epsilon^2\right)$$

and it becomes infinite.

Gauge invariant off-shell amplitudes

Gauge invariance

Transformation of Wilson line under local gauge transformation $U(x)$:

$$[x, y]_C \to U(x)\,[x, y]_C\,U^\dagger(y)$$

For n having any spatial component non-zero the operator $\lim_{\varepsilon\to 0}[y]_n^{(\epsilon)}$ is invariant under small gauge tranformations, $U(x) \to \mathbf{1}$ if $\left|\vec{x}\right| \to \infty$.

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Matrix element of Wilson line operators

$$\langle k_1,\varepsilon_1,c_1;\ldots;k_N,\varepsilon_N,c_N|\,\mathfrak{R}^{(\varepsilon)c_A}_{e_A}(k_A)\ldots\mathfrak{R}^{(\varepsilon)c_X}_{e_X}(k_X)\,|0\rangle_c \equiv \mathfrak{M}^{(\varepsilon)}$$

$|k,\varepsilon,c\rangle$ – on-shell gluon states with momentum k, polarization ε and color c.

$$\lim_{\varepsilon\to 0}\mathfrak{M}^{(\varepsilon)} = \delta(k_A\cdot e_A)\ldots\delta(k_X\cdot e_X)$$
$$\delta^4(k_A+\ldots+k_X+k_1+\ldots+k_N)\,\tilde{\mathcal{M}}_{e_A\ldots e_X}(\varepsilon_1,\ldots,\varepsilon_N)$$

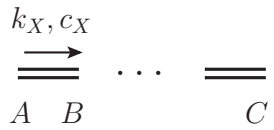
$\mathfrak{M}$ can be calculated using Feynman rules (see next slide) or using the FORM program for automatic analytic calculations

[PK, OGIME, http://annapurna.ifj.edu.pl/~pkotko/OGIME.html]

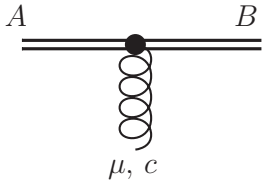
Gauge invariant off-shell amplitudes

The Feynman rules

Besides the standard QCD Feynman rules we introduce the following rules for the gauge link insertion:



$$= \frac{2}{g}\,\delta\left(k_X \cdot e_X\right)\, t^{c_X}_{AB}\,\delta_{AC}$$



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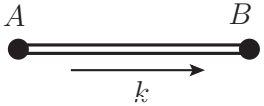

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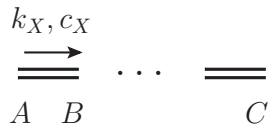

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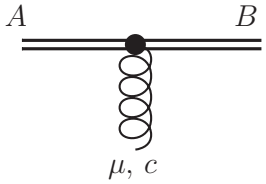

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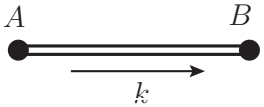
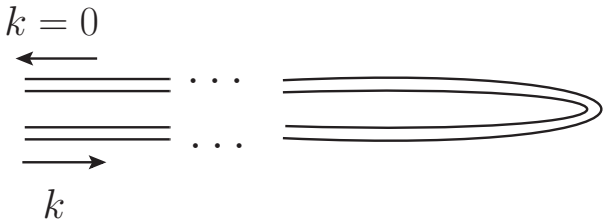


$$= \frac{i\delta_{AB}}{k \cdot e_X + i\epsilon}$$

In order to track the momentum and color flow more easy (and to introduce so-called color-ordered diagrams) it is convenient to use a “bent” gauge link

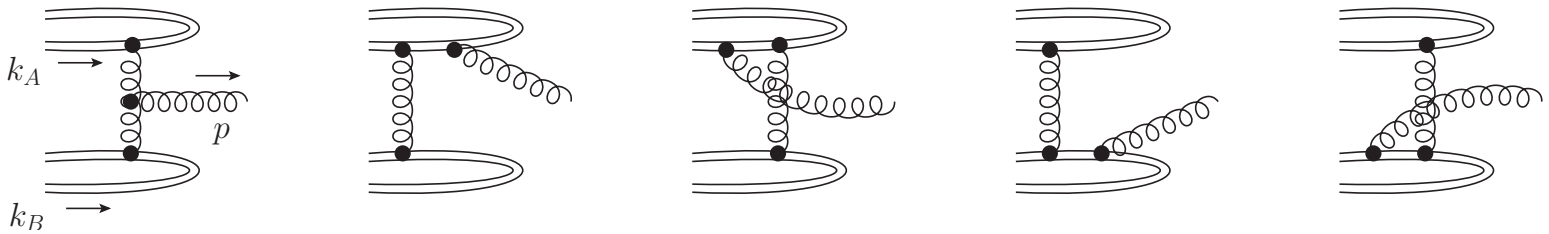


Example I: Lipatov's vertex

We will find the gauge invariant matrix element for the process

$$g^*\left(k_A\right) g^*\left(k_B\right) \to g\left(p\right)$$

where $k_A + k_B = p$ with $p^2 = 0$. We choose the Feynman gauge and the polarization vectors to be e_A, e_B, ε satisfying $e_A \cdot k_A = 0$, $e_B \cdot k_B = 0$, $e \cdot p = 0$. Let the colors of the gluons be c_A, c_B and c respectively.



$$\tilde{\mathcal{M}}_{e_A e_B}\left(\varepsilon\right) = igf^{c_A c_B c}\left\{\frac{-ie_A^\mu e_B^\nu \varepsilon^\gamma V_{\mu\beta\gamma}\left(k_A, k_B, -p\right)}{k_A^2 k_B^2} + \frac{e_A \cdot e_B\, \varepsilon \cdot e_A}{k_B^2\, p \cdot e_A} - \frac{e_A \cdot e_B\, \varepsilon \cdot e_B}{k_A^2\, p \cdot e_B}\right\}$$

This has the same form as the Lipatov's *RRg* vertex. This will no longer be true for more off-shell gluons (see the next slide).

Example II: three off-shell gluons

A more complicated example: $g^*(k_A)\, g^*(k_C) \to g^*(k_B)\, g(p)$

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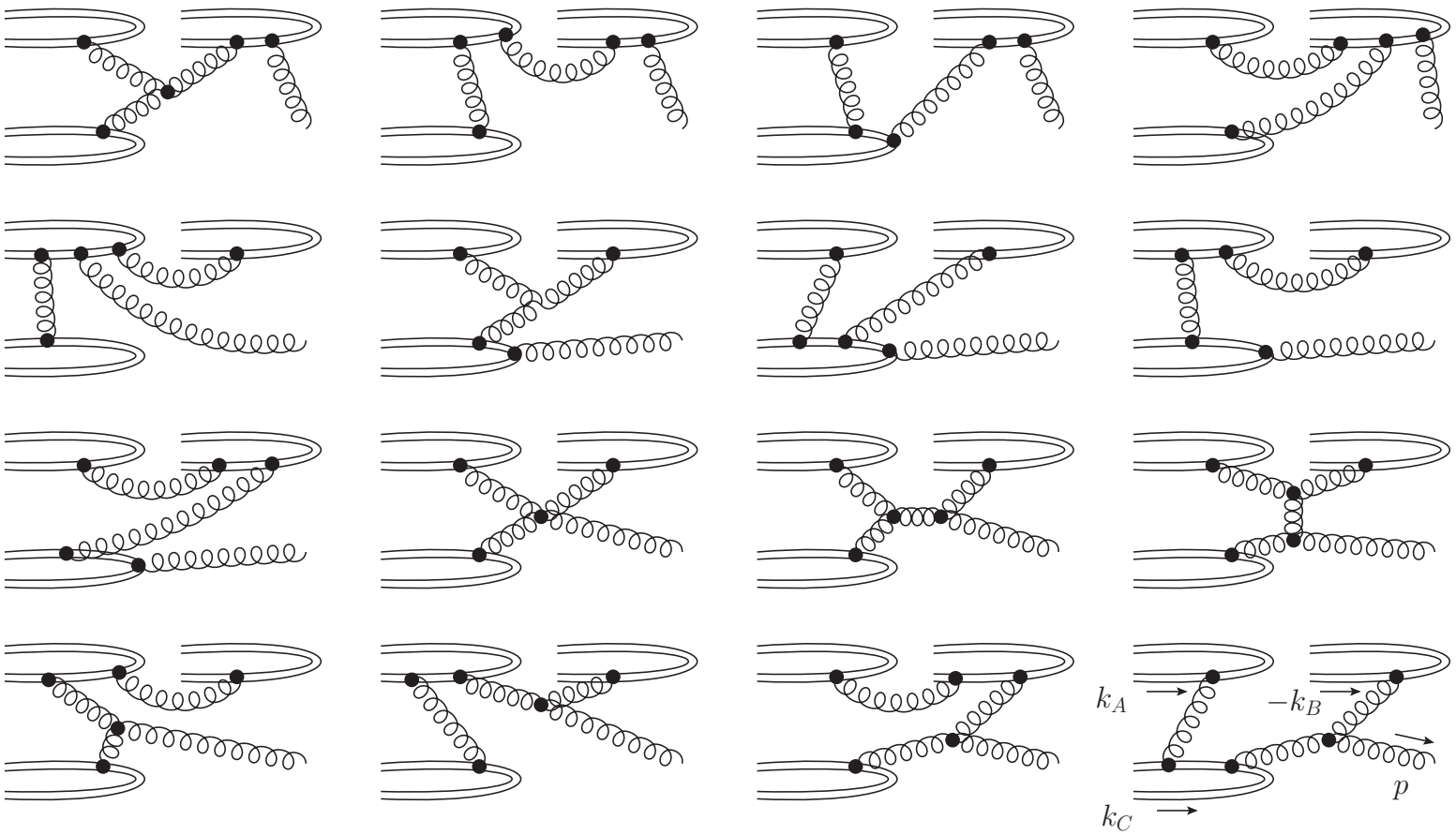
There are 16 color-ordered (for simplicity) diagrams:



For $e_A, e_B, e_C \in \{n_+, n_-\}$ some of the diagrams vanish and the result is consistent with $RRRg$ Lipatov's vertex [M. Braun, M. Vyazovsky, Eur.Phys.J C51, 103 (2007)]

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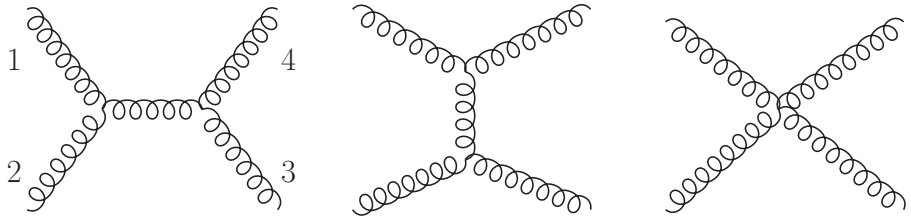
Gauge invariant decompositions

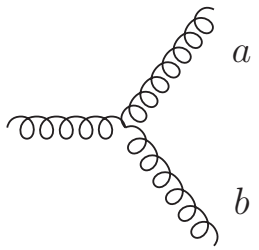
Example: four gluon color-ordered amplitude:



$$\mathcal{M}^{(1234)} = J_\mu^{(12)} \frac{ig^{\mu\nu}}{k_{12}^2} J_\nu^{(34)} + J_\mu^{(41)} \frac{ig^{\mu\nu}}{k_{14}^2} J_\nu^{(23)} + iV_4^{(1234)}$$

with $k_{ab} = k_a + k_b$ and the off-shell color-ordered currents $J_\mu^{(ab)}$

$J_\mu^{(ab)} =$



Gauge invariant decompositions

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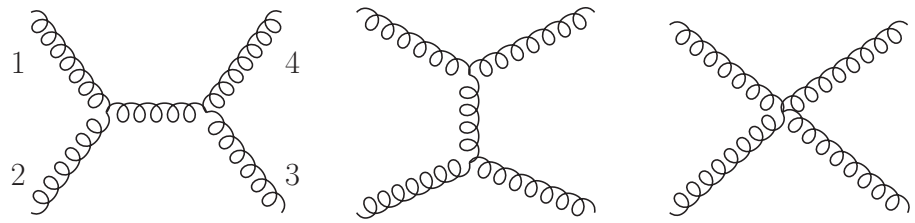

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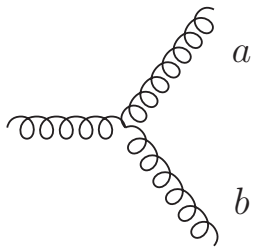

It is possible to write this amplitude in a manifestly gauge invariant way

$$\mathcal{M}^{(1234)} = i\left(k_{12}^2\, \tilde{J}^{(12)} \cdot \tilde{J}^{(34)} + k_{14}^2\, \tilde{J}^{(41)} \cdot \tilde{J}^{(23)} + \tilde{V}_4^{(1234)}\right)$$

$$\tilde{J}^{(ab)} \cdot \tilde{J}^{(cd)} = \sum_{i=0}^{2} \tilde{J}_i^{(ab)} \tilde{J}_i^{(cd)} d_i, \qquad \tilde{J}_i\left(\varepsilon_1, \varepsilon_2; k_{12}\right) \sim \langle k_1, \varepsilon_1; k_2, \varepsilon_2 | \mathcal{R}_{e^{(i)}}\left(k_{12}\right) |0\rangle$$

$k \cdot e^{(i)}(k) = 0$, $e^{(i)}(k) \cdot e^{(j)}(k) = d_i(k)\,\delta_{ij}$, $d_0(k) = \pm 1$, $d_1(k) = d_2(k) = -1$, $\sum_{i=0}^{2} e^{(i)\nu}(k)\, e^{(i)\mu}(k)\, d_i(k) = g^{\mu\nu} - k^\mu k^\nu / k^2$.

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Recursion relations

Fragmentation function within the light front formalism

[C.A. Cruz-Santiago, A. Stasto, Nucl.Phys. B875 (2013) 368-387]

Light-front formalism: all lines on-shell with helicity $\pm$; additional instantenous interactions; energy denominators instead of propagators; no vacuum diagrams.

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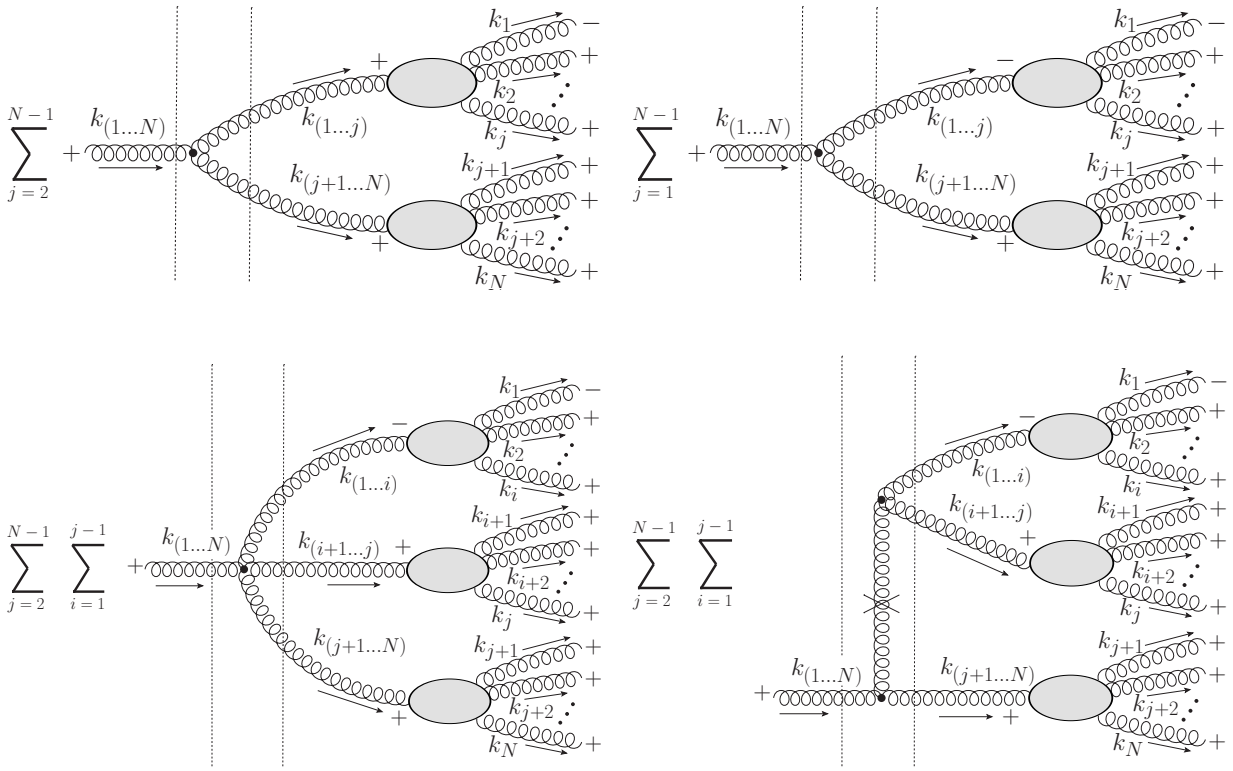
Consider a gluon fragmentation function $1 \to n$ with maximally helicity violating configuration (MHV) $+ \to - + \cdots +$:



Incoming gluon is off-shell in the sense of the corresponding energy denominator being nonzero $D_{1\ldots N} \neq 0$.

Recursion relations

Recursion relation for fragmentation function on the light front

[C.A. Cruz-Santiago, A. Stasto, Nucl.Phys. B875 (2013) 368-387]

$$\mathcal{M}^{(+\to-+\cdots+)}(k_{1\ldots N}) = M^{(+\to-+\cdots+)}(k_{1\ldots N}) -$$

$$z_{1\ldots N}^2 D_{(1,\ldots,N)} \sum_{i=2}^{N-1} \frac{1}{z_{1\ldots i} z_{1\ldots i+1}} \frac{1}{z_{i+1}\ldots z_N} \frac{g^{N-i}}{v_{i+1\ i+2}\ldots v_{N-1\ N}} \frac{M^{(+\to-+\cdots+)}(k_{1\ldots i})}{v_{i+1(1\ldots i+1)} D_{(1,\ldots,i)}}$$

where $M^{(+\to-+\cdots+)}$ is an (off-shell) object which has a simple form

$$M^{(+\to-+\cdots+)}(k_{1\ldots j}) \equiv -2ig^{j-1} \frac{z_{1\ldots j}\ z_1}{z_2 z_3 \ldots z_j} \frac{v^3_{(1\ldots j)1}}{v_{12} v_{23} \ldots v_{j-1\ j} v_{j(1\ldots j)}}$$

Explanation of the variables above:

$$D_{(1,\ldots,i)} = \sum_{j=1}^{i} E_j^- - E_{1\ldots i}^- = \sum_{j=1}^{i} \frac{\vec{k}_{j\perp}^2}{z_j} - \frac{\vec{k}_{1\ldots i\perp}^2}{z_{1\ldots i}}$$

$$v_{ij} = \vec{\varepsilon}_\perp \cdot \left(\frac{\vec{k}_{i\perp}}{k_i^+} - \frac{\vec{k}_{j\perp}}{k_j^+} \right), \quad k_i^+ = z_i P^+, \quad \varepsilon_\perp^\pm = \frac{1}{\sqrt{2}} (0, 1, \pm i, 0)$$

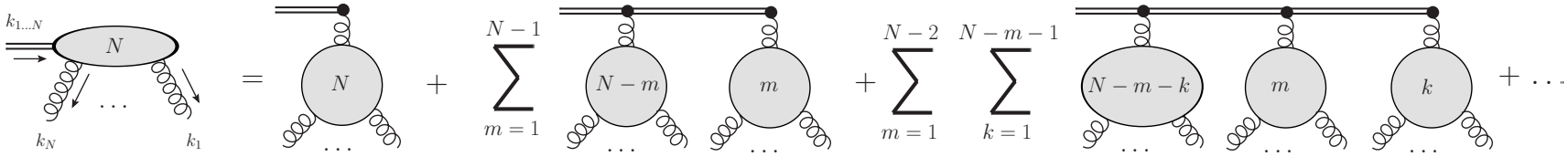
Recursion relations

Recursion relation from Wilson line

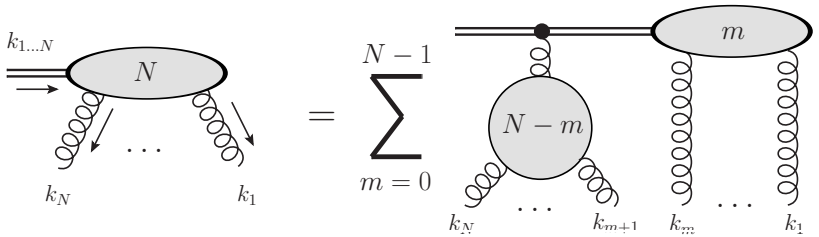
[C.A. Cruz-Santiago, PK, A. Stasto, Nucl.Phys. B895 (2015) 132-160]

It turns out that M corresponds to the gauge invariant off-shell amplitude $\tilde{\mathcal{M}}$.

Expansion of $\tilde{\mathcal{M}}$:



It can be rewritten as a pseudo-recursion relation:



The blob with Wilson line on the rhs is not gauge invariant, because the Wilson line slope is not transverse to the off-shell momentum entering the blob. With the help of a lot of algebra it can be however rewritten in the same form as the recursion from the light-front formalism.

PLAN

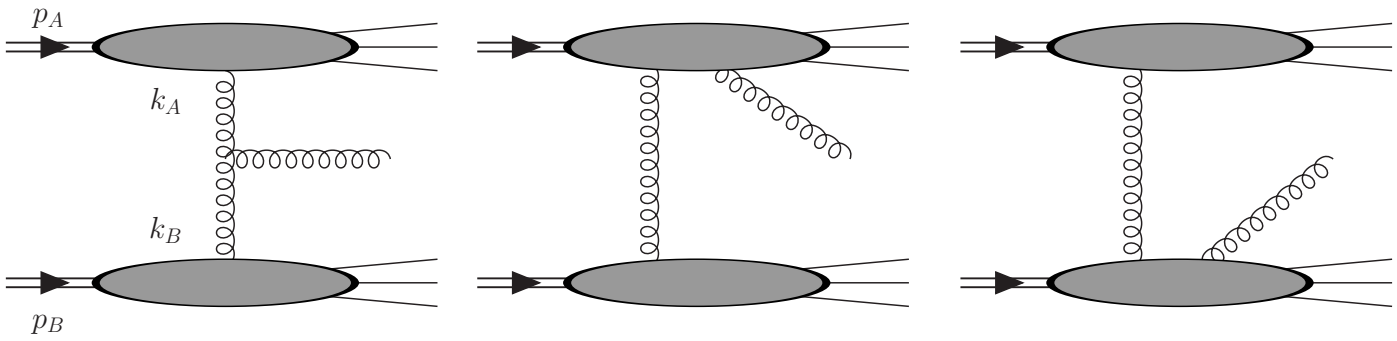
- Introduction
  - Off-shell amplitudes
  - Gauge (non-)invariance
- Gauge invariant off-shell amplitudes using Wilson lines
- Applications
  - Gauge invariant decompositions
  - Recursion relations and connection to light-front amplitudes
  - High Energy (or k_T) factorization and Generalized TMD factorization
- Future plans

High energy factorization

Inclusive gluon or heavy quark production

[L.V. Gribov, E.M. Levin, M.G. Ryskin, Phys.Rept. 100 (1983) 1-150]
[S. Catani, M. Ciafaloni, F. Hautmann, Nucl.Phys. B366 (1991) 135-188]
[J.C. Collins, R.K. Ellis, Nucl.Phys. B360 (1991) 3-30]

$$d\sigma_{AB} = \sum_b \int dx_A\, dx_B \int dk_{TA}^2\, dk_{TB}^2\, \mathcal{F}\left(x_A, k_{TA}^2\right) \mathcal{F}\left(x_B, k_{TB}^2\right) d\sigma_{g^* g^*}\left(x_A, x_B, k_{TA}^2, k_{TB}^2\right)$$



- Off-shell momenta: $k_A = x_A p_A + k_{TA}$, $k_B = x_B p_B + k_{TB}$.
- $\mathcal{F}\left(x, k_T^2\right)$ – Unintegrated Gluon Distribution evolving via BFKL (or similar).
- p_A, p_B – the slope of Wilson lines: $\sigma_{g^* g^*} \sim \left|\langle p, \varepsilon| \mathcal{R}_{p_A}\left(k_A\right) \mathcal{R}_{p_B}\left(k_B\right) |0\rangle\right|^2$
- This can be extended to saturation regime.

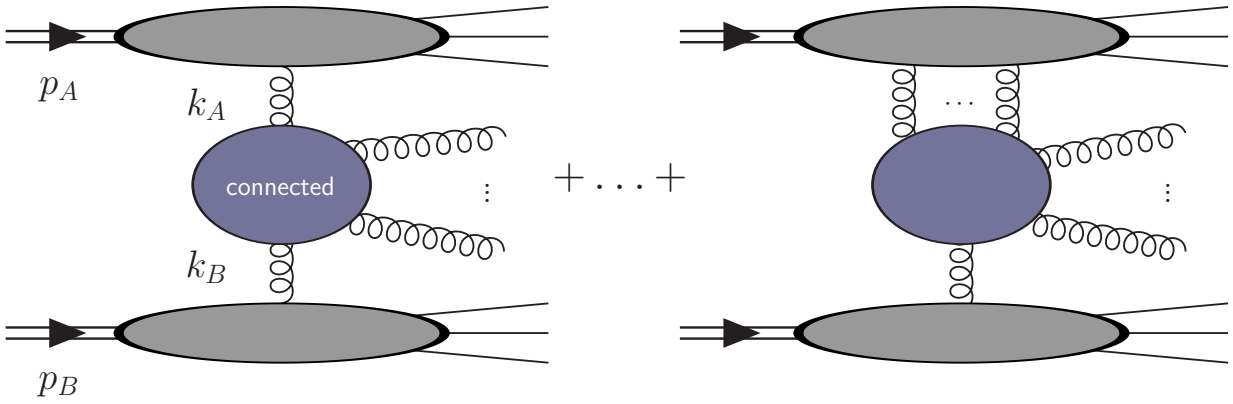
[Y. V. Kovchegov and K. Tuchin, Phys. Rev. D65 (2002) 074026].

High energy factorization

Hybrid approach

[A. Dumitru, A. Hayashigaki and J. Jalilian-Marian, Nucl. Phys. A765 (2006) 464]
[M. Deak, F. Hautmann, H. Jung, K. Kutak, JHEP 0909 (2009) 121]

$$d\sigma_{AB} = \sum_b \int dx_A\, dx_B \int dk_T^2\, \mathcal{F}\left(x_A, k_T^2, \mu^2\right) f_{b/B}\left(x_B, \mu^2\right)\, d\sigma_{g^* b}\left(x_A, x_B, k_T^2, \mu^2\right)$$



- $f_{b/B}\left(x, \mu^2\right)$ – collinear PDF evolving via the DGLAP.
- Off-shell ampltude calculated e.g. from $\langle X| \mathcal{R}_{p_A}\left(k_A\right)|0\rangle$.
- Suitable for forward particle production, where $x_A \ll x_B$.
- For dijet production this formula can derived from CGC in the dilute limit.

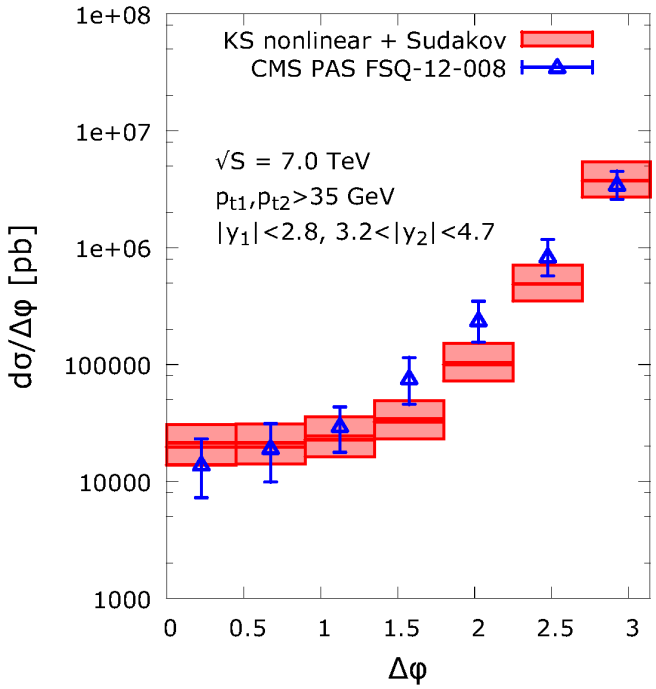
[P.K., K. Kutak, C. Marquet, E. Petreska, S. Sapeta, A. van Hameren, arXiv:150342]

High energy factorization

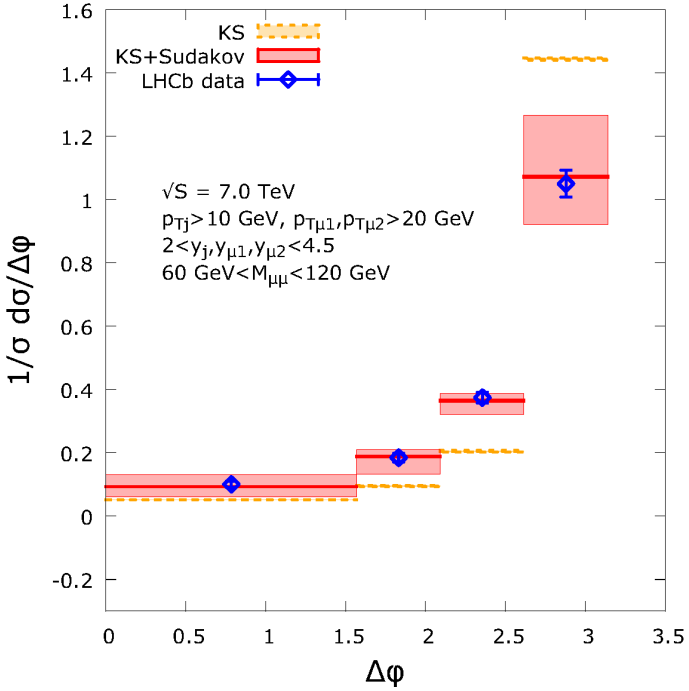
Azimuthal decorrelations at LHC

[A. van Hameren, PK, K. Kutak, S. Sapeta, Phys.Lett. B737 (2014) 335-340]
[A. van Hameren, PK, K. Kutak, arXiv:1505.02763]

forward-central dijets

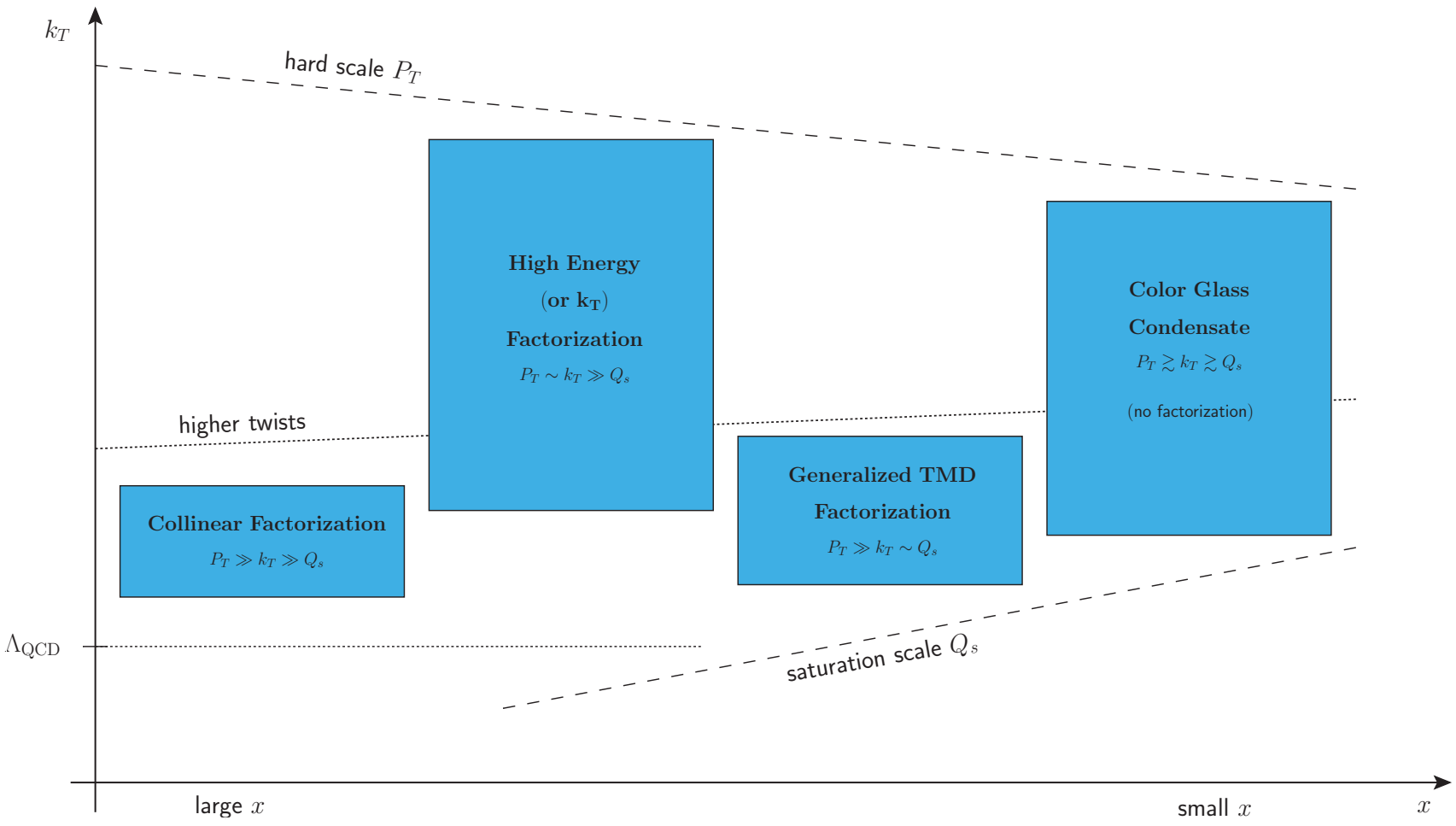


forward Z_0 + jet



Evolution of unintegrated gluon distribution is the KMS approach [J. Kwiecinski, A.D. Martin, A.M. Stasto, Phys.Rev. D56 (1997) 3991-4006] (BFKL+kinematic constraint+DGLAP term) fitted to HERA data [K. Kutak, S. Sapeta, Phys.Rev. D86, 094043 (2012)] and supplemented with the Sudakov 'resummation'.

High energy factorization



Three scales involved:

- hard scale P_T (of the order of the average transverse momentum of jets)
- transverse momentum imbalance k_T
- saturation scale Q_s (increasing with energy)

High energy factorization

Effective generalized TMD factorization

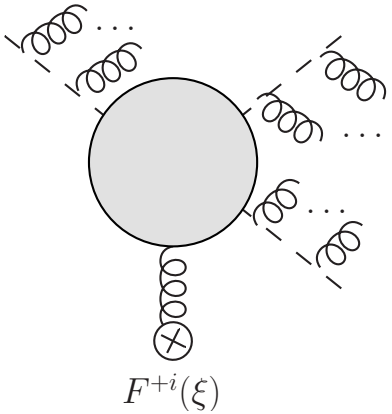
[F. Dominguez, C. Marquet, B-W. Xiao, F. Yuan Phys.Rev. D 83 (2011) 105005]

$$d\sigma_{AB} = \sum_b \int dx_A\, dx_B\, f_{b/B}\left(x_B, \mu^2\right) \sum_i \int dk_T^2 \phi_{bg}^{(i)}\left(x_A, k_T^2, \mu^2\right) d\sigma_{gb}^{(i)}\left(x_A, x_B, \mu^2\right)$$

The hard partonic cross sections $\sigma_{gb}^{(i)}$ are computed with partons a, b on-shell.
$\phi_{bg}^{(i)}\left(x_A, k_T^2, \mu^2\right)$ – TMD Gluon Distributions depending on the process.

$$\phi_{bg}\left(x, k_T\right) = 2 \int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_A p_A^- \xi^+ - i\vec{k}_T \cdot \vec{\xi}_T} \langle p_A | \operatorname{Tr}\left\{F^{+i}\left(\xi\right)[\xi, 0]_{C_1} F^{+i}\left(0\right)[0, \xi]_{C_2}\right\} | p_A \rangle$$

The Wilson lines $[\xi, 0]_{C_i}$ depend on the process-dependent paths.

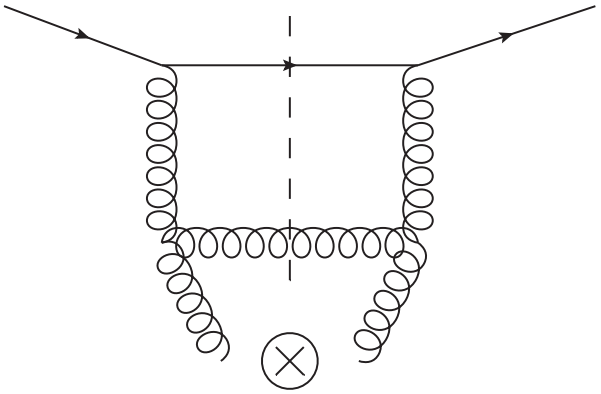


Wilson lines come from resummation of collinear gluons attached to the external lines. Then they are 'glued' by the color structure of the hard process to give $[\xi, 0]_{C_i}$.

High energy factorization

Example: TMD for a particular diagram

[C.J. Bomhof, P.J. Mulders, F. Pijlman, Eur.Phys.J.C. 47, 147 (2006)]



$$\langle p_A | \,\mathrm{Tr}\{F(\xi)\,\mathcal{U}^{[+]\dagger}\,F(0)\left[\frac{\mathrm{Tr}\mathcal{U}^{[\square]\dagger}}{N_c}\mathcal{U}^{[+]} + \mathcal{U}^{[-]}\right]\}\,|p_A\rangle$$

with the following definitions of Wilson lines and loops:

$$\mathcal{U}^{[\pm]} = U(0,\pm\infty;0_T)U(\pm\infty,\xi^+;\xi_T)\;\; \mathcal{U}^{[\square]} = \mathcal{U}^{[+]}\mathcal{U}^{[-]\dagger} = \mathcal{U}^{[-]}\mathcal{U}^{[+]\dagger}$$

where $U(a,b;x_T) = \mathcal{P}\exp\left[ig\int_a^b dx^+ A_a^-(x^+,x_T)t^a\right]$.

High energy factorization

All TMD structures for dijets

$$\mathcal{F}_{qg}^{(1)} \sim \langle p_A | \mathrm{Tr}\{F(\xi)\,\mathcal{U}^{[-]\dagger} F(0)\,\mathcal{U}^{[+]}\} | p_A \rangle \quad \mathcal{F}_{qg}^{(2)} \sim \langle p_A | \mathrm{Tr}\{F(\xi)\,\frac{\mathrm{Tr}\mathcal{U}^{[\square]}}{N_c}\mathcal{U}^{[+]\dagger} F(0)\,\mathcal{U}^{[+]}\} | p_A \rangle$$

$$\mathcal{F}_{gg}^{(1)} \sim \langle p_A | \mathrm{Tr}\{F(\xi)\,\frac{\mathrm{Tr}\mathcal{U}^{[\square]}}{N_c}\mathcal{U}^{[-]\dagger} F(0)\,\mathcal{U}^{[+]}\} | p_A \rangle \quad \mathcal{F}_{gg}^{(2)} \sim \frac{1}{N_c} \langle p_A | \mathrm{Tr}\left\{F(\xi)\,\mathcal{U}^{[\square]\dagger}\right\} \mathrm{Tr}\left\{F(0)\,\mathcal{U}^{[\square]}\right\} | p_A \rangle$$

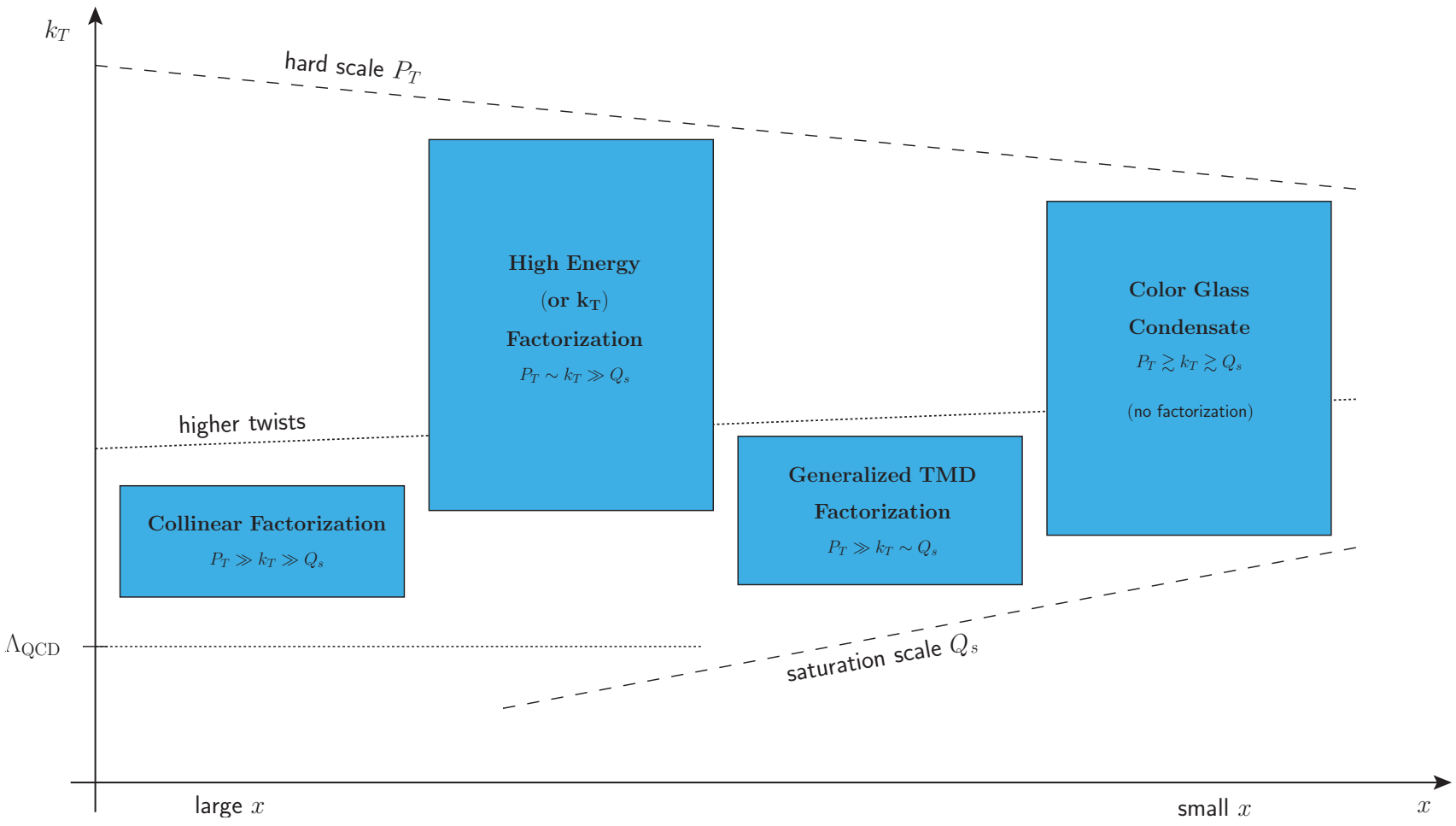
$$\mathcal{F}_{gg}^{(3)} \sim \langle p_A | \mathrm{Tr}\left\{F(\xi)\,\mathcal{U}^{[+]\dagger} F(0)\,\mathcal{U}^{[+]}\right\} | p_A \rangle \quad \mathcal{F}_{gg}^{(4)} \sim \langle p_A | \mathrm{Tr}\left\{F(\xi)\,\mathcal{U}^{[-]\dagger} F(0)\,\mathcal{U}^{[-]}\right\} | p_A \rangle$$

$$\mathcal{F}_{gg}^{(5)} \sim \langle p_A | \mathrm{Tr}\left\{F(\xi)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger} F(0)\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right\} | p_A \rangle \quad \mathcal{F}_{gg}^{(6)} \sim \langle p_A | \mathrm{Tr}\left\{F(\xi)\,\mathcal{U}^{[+]\dagger} F(0)\,\mathcal{U}^{[+]}\right\} \frac{\mathrm{Tr}\mathcal{U}^{[\square]}}{N_c}\frac{\mathrm{Tr}\mathcal{U}^{[\square]}}{N_c} | p_A \rangle$$

In the large N_c limit $\mathcal{F}_{qg}^{(1)}, \mathcal{F}_{gg}^{(3)}$ are the only necessary TMDs:

- $\mathcal{F}_{qg}^{(3)} = xG_1$ is the Weizsacker-Wiliams gluon distribution with parton number interpretation (calculable in CGC but decoupled from most simple processes; appears also in dijet production in DIS)
- $\mathcal{F}_{qg}^{(1)} = xG_2$ is so-called 'dipole' gluon distribution probed in inclusive processes at small x

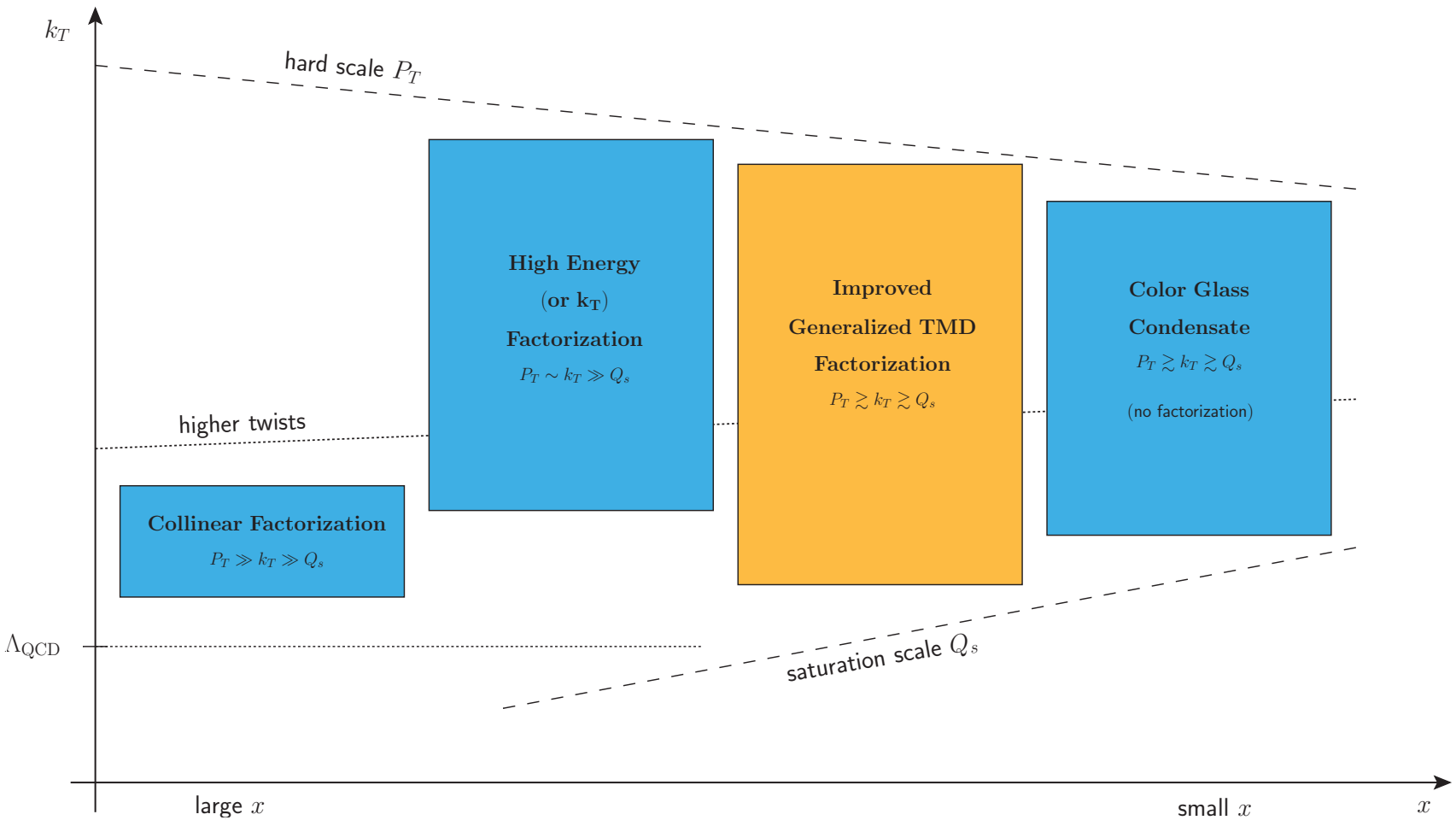
High energy factorization



Three scales involved:

- hard scale P_T (of the order of the average transverse momentum of jets)
- transverse momentum imbalance k_T
- saturation scale Q_s (increasing with energy)

High energy factorization



Three scales involved:

- hard scale P_T (of the order of the average transverse momentum of jets)
- transverse momentum imbalance k_T
- saturation scale Q_s (increasing with energy)

High energy factorization

Improved Generalized TMD factorization

[P.K., K. Kutak, C. Marquet, E. Petreska, S. Sapeta, A. van Hameren, arXiv:150342]

$$\frac{d\sigma_{AB}}{dP.S.} \sim \sum_{i=1,2} \left[f_{q/B} \otimes \Phi^{(i)}_{gq\to qg} \otimes K^{(i)}_{gq\to qg} + f_{g/B} \otimes \left(\Phi^{(i)}_{gg\to q\bar{q}} \otimes K^{(i)}_{gg\to q\bar{q}} + \Phi^{(i)}_{gg\to gg} \otimes K^{(i)}_{gg\to gg} \right) \right]$$

$$K^{(1)}_{gq\to gq} = -\frac{\bar{u}\left(\bar{s}^2+\bar{u}^2\right)}{2\bar{t}\hat{t}\hat{s}}\left(1+\frac{\bar{s}\hat{s}-\bar{t}\hat{t}}{N_c^2\,\bar{u}\hat{u}}\right) \qquad \Phi^{(1)}_{gq\to gq} = \mathcal{F}^{(1)}_{qg}$$

$$K^{(2)}_{gq\to gq} = -\frac{C_F}{N_c}\,\frac{\bar{s}\left(\bar{s}^2+\bar{u}^2\right)}{\bar{t}\hat{t}\hat{u}} \qquad \Phi^{(2)}_{gq\to gq} = \frac{1}{N_c^2-1}\left(-\mathcal{F}^{(1)}_{qg}+N_c^2\mathcal{F}^{(2)}_{qg}\right)$$

$$K^{(1)}_{gg\to q\bar{q}} = \frac{1}{2N_c}\,\frac{\left(\bar{t}^2+\bar{u}^2\right)\left(\bar{u}\hat{u}+\bar{t}\hat{t}\right)}{\bar{s}\hat{s}\hat{t}\hat{u}} \qquad \Phi^{(1)}_{gg\to q\bar{q}} = \frac{1}{N_c^2-1}\left(N_c^2\mathcal{F}^{(1)}_{gg}-\mathcal{F}^{(3)}_{gg}\right)$$

$$K^{(2)}_{gg\to q\bar{q}} = \frac{1}{4N_c^2C_F}\,\frac{\left(\bar{t}^2+\bar{u}^2\right)\left(\bar{u}\hat{u}+\bar{t}\hat{t}-\bar{s}\hat{s}\right)}{\bar{s}\hat{s}\hat{t}\hat{u}} \qquad \Phi^{(2)}_{gg\to q\bar{q}} = -N_c^2\mathcal{F}^{(2)}_{gg}+\mathcal{F}^{(3)}_{gg}$$

$$K^{(1)}_{gg\to gg} = \frac{N_c}{C_F}\,\frac{\left(\bar{s}^4+\bar{t}^4+\bar{u}^4\right)\left(\bar{u}\hat{u}+\bar{t}\hat{t}\right)}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}}$$

$$\Phi^{(1)}_{gg\to gg} = \frac{1}{2N_c^2}\Big(N_c^2\mathcal{F}^{(1)}_{gg}-2\mathcal{F}^{(3)}_{gg}+\mathcal{F}^{(4)}_{gg} + \mathcal{F}^{(5)}_{gg}+N_c^2\mathcal{F}^{(6)}_{gg}\Big)$$

$$K^{(2)}_{gg\to gg} = -\frac{N_c}{2C_F}\,\frac{\left(\bar{s}^4+\bar{t}^4+\bar{u}^4\right)\left(\bar{u}\hat{u}+\bar{t}\hat{t}-\bar{s}\hat{s}\right)}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}}$$

$$\Phi^{(2)}_{gg\to gg} = \frac{1}{N_c^2}\Big(N_c^2\mathcal{F}^{(2)}_{gg}-2\mathcal{F}^{(3)}_{gg}+\mathcal{F}^{(4)}_{gg} + \mathcal{F}^{(5)}_{gg}+N_c^2\mathcal{F}^{(6)}_{gg}\Big)$$

$\hat{s},\hat{t},\hat{u}$ – ordinary Mandelstam variables, $\hat{s}+\hat{t}+\hat{u}=k_T^2$

$\bar{s},\bar{t},\bar{u}$ – off-shell momentum $k_A = x_A p_A + k_T$ replaced by $x_A p_A$, $\bar{s}+\bar{u}+\bar{t}=0$

Future plans

- Loop corrections: perhaps certain recursion relations for one-loop one-leg off-shell amplitudes can be derived from Wilson line/light-front approach.
- Real corrections: a generalization of Catani-Seymour massive dipole method for NLO jet calculations. [PK, W. Slominski, Phys.Rev. D86 (2012) 094008]
- Relation of recursions from Wilson line/light-front approach to BCFW method
- Phenomenology based on the improved TMD factorization

Backup

Color-ordered off-shell helicity amplitudes

[A. van Hameren, PK, K. Kutak, JHEP 1212 (2012) 029]

In spinor formalism, the non-zero off-shell helicity amplitudes have the form of the MHV amplitudes with certain modification of spinor products:

$$\mathcal{M}_{g^*g\to gg}\left(1^*,2^-,3^+,4^+\right) = 2g^2\,\rho_1\,\frac{\langle 1^*2\rangle^4}{\langle 1^*2\rangle\langle 23\rangle\langle 34\rangle\langle 41^*\rangle}$$

$$\mathcal{M}_{g^*g\to gg}\left(1^*,2^+,3^-,4^+\right) = 2g^2\,\rho_1\,\frac{\langle 1^*3\rangle^4}{\langle 1^*2\rangle\langle 23\rangle\langle 34\rangle\langle 41^*\rangle}$$

$$\mathcal{M}_{g^*g\to gg}\left(1^*,2^+,3^+,4^-\right) = 2g^2\,\rho_1\,\frac{\langle 1^*4\rangle^4}{\langle 1^*2\rangle\langle 23\rangle\langle 34\rangle\langle 41^*\rangle}$$

where $\langle ij\rangle = \langle k_i - |k_j+\rangle$ with spinors defined as $|k_i\pm\rangle = \frac{1}{2}\left(1\pm\gamma_5\right)u\left(k_i\right)$. Modified spinor products involve only longitudinal component of the off-shell momentum $\langle 1^*i\rangle = \langle p_A\,i\rangle$. Similar expressions can be derived for quarks.