Initial State Correlations, Entanglement Entropy, and all that

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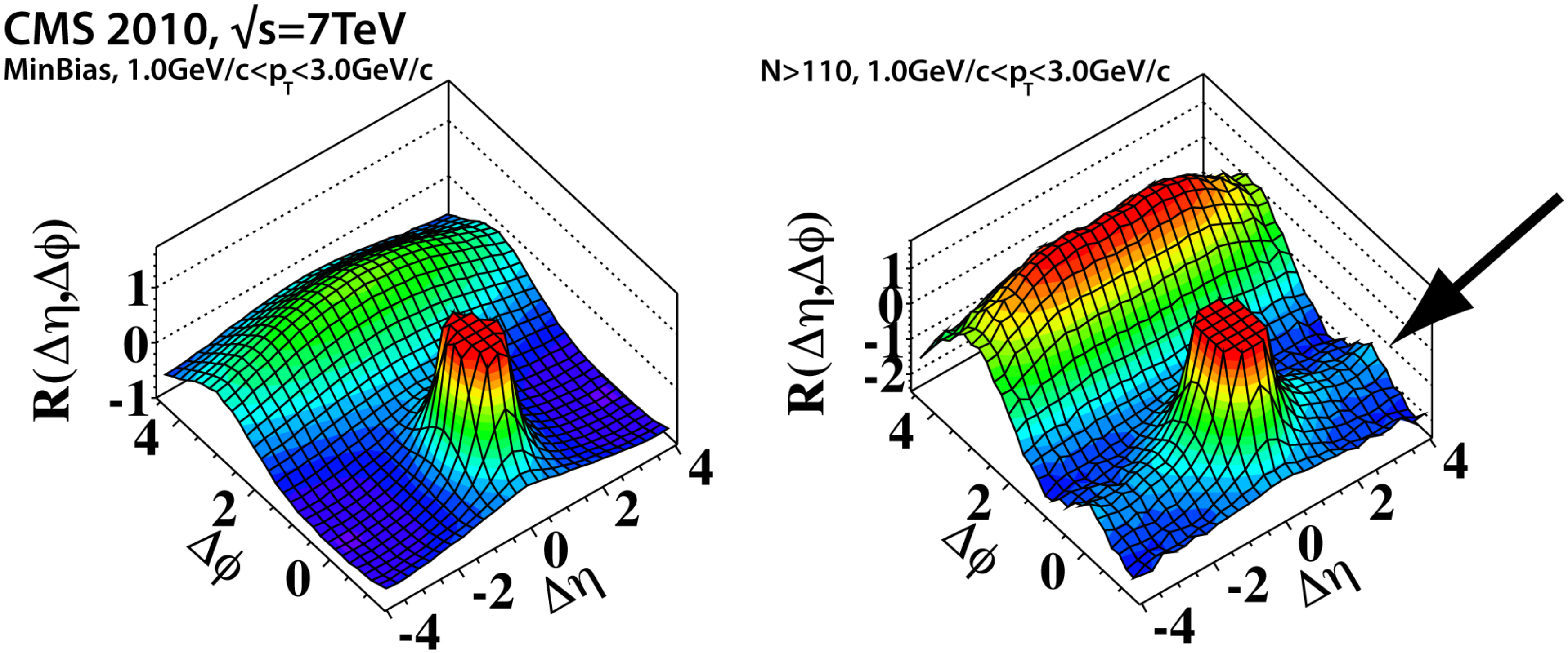
Alex Kovner and ML; Phys.Rev. D84 (2011) 094011, Phys.Rev. D83 (2011) 034017; Int.J.Mod.Phys. E22 (2013) 1330001; arXiv:1506.05394 (PRD)

T. Altinoluk, N. Armesto, G. Beuf, A. Kovner and ML arXiv:1503.07126

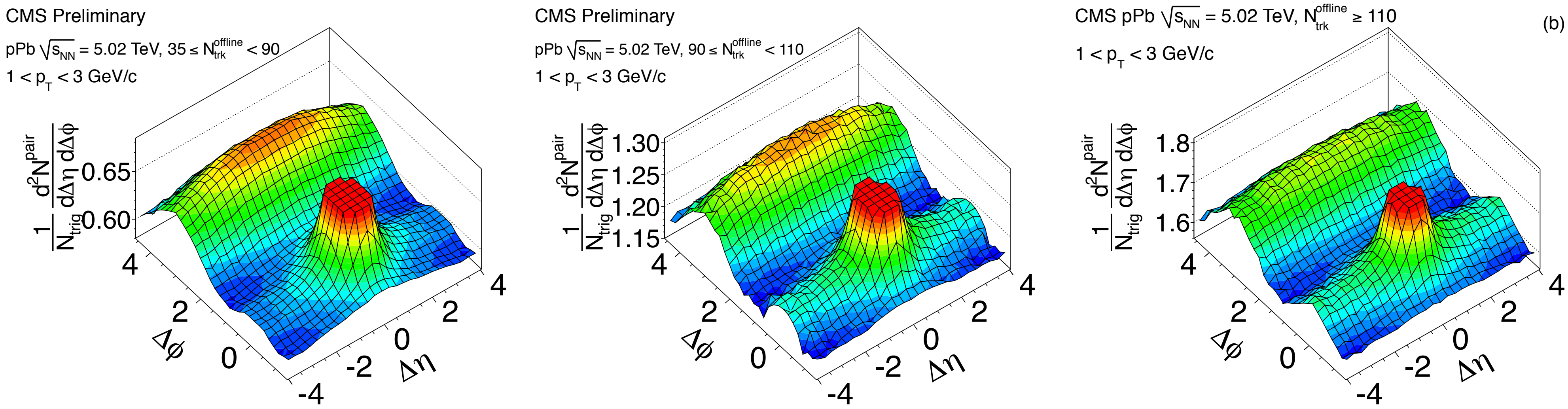
"RIDGE" - ANGULAR CORRELATIONS

Two particle correlations in $p - p$: long range in rapidity, near-side angular correlations



"High multiplicity" collisions with over a hundred charged particles produced

Multiplicity dependence of the ridge in pPb



BIg Questions

- **Origin of angular collimation?**
  Could be many. For sure explosive "wind" from hydro would lead to some.

- **Origin of long range rapidity correlations?**
  Causality: correlations exist in early stage of the collision (like in cosmology)

- **Do we see a sign of universality between $p-p$ and $p-A$ and $A-A$?**
  Hopefully Yes! High energy QCD implies this universality. In all experiments the effect emerges only when high densities are involved (color glass condensate (CGC))

- **Do we see a collective phenomenon (QGP?) in $p-p$ or $p-A$?**
  Many indications, but we are not sure yet ...

Our Goal

To discuss some general features of gluon production at high energy.

We need to compute correlations in two-gluon inclusive production rate

$$\left[\frac{d^2N}{d^2p\, d\eta\, d^2k\, d\xi} - \frac{dN}{d^2k\, d\xi}\,\frac{dN}{d^2p\, d\eta}\right] \Big/ \frac{dN}{d^2k\, d\xi}\,\frac{dN}{d^2p\, d\eta}$$

For dilute on dense (DIS), we do have QCD-derived formulae for multi-gluon production

Here I talk about only one source for the observed phenomena:
INITIAL CONDITIONS (CGC)

High Energy Scattering: CGC-type approach

Target **Projectile**

$$\langle T| \quad \rightarrow \qquad\qquad \leftarrow \quad |P\rangle$$

S**-matrix:**

$$\mathbf{S(Y)} \;=\; \langle \mathbf{T}\,\langle\,\mathbf{P}|\;\; \hat{\mathbf{S}}(\rho^{\mathrm{t}},\,\rho^{\mathrm{p}})\;\; |\mathbf{P}\rangle\,\mathbf{T}\rangle$$

CGC-type averaging

$$\mathbf{S(Y)} \;=\; \int \mathbf{D}\rho^{\mathrm{p}}\,\mathbf{D}\rho^{\mathrm{t}}\;\; \mathbf{S}[\rho^{\mathrm{p}},\,\rho^{\mathrm{t}}]\;\; \mathbf{W}^{\mathrm{p}}_{\mathbf{Y}-\mathbf{Y_0}}[\rho^{\mathrm{p}}]\;\; \mathbf{W}^{\mathrm{t}}_{\mathbf{Y_0}}[\rho^{\mathrm{t}}]$$

$\mathbf{W}^{\mathrm{p,t}}$ **are probability distributions, subject to high energy evolution equations**

For any other observable $\mathcal{O}$

$$\langle\mathcal{O}\rangle_{\mathbf{P,T}} \;=\; \int \mathbf{D}\rho^{\mathrm{p}}\,\mathbf{D}\rho^{\mathrm{t}}\;\; \mathcal{O}_{\mathbf{Y_0}}[\rho^{\mathrm{p}},\,\rho^{\mathrm{t}}]\;\; \mathbf{W}^{\mathrm{p}}_{\mathbf{Y}-\mathbf{Y_0}}[\rho^{\mathrm{p}}]\;\; \mathbf{W}^{\mathrm{t}}_{\mathbf{Y_0}}[\rho^{\mathrm{t}}]$$

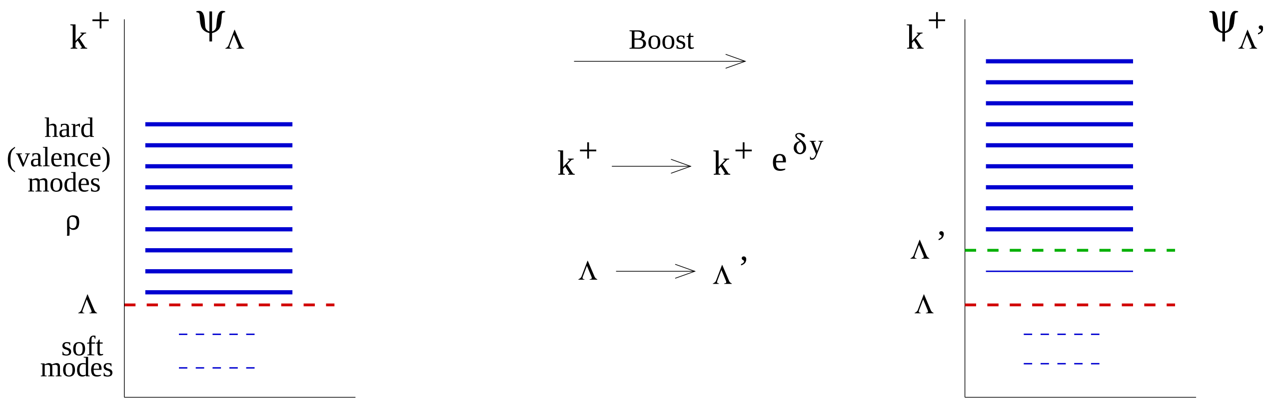
Projectile/Target averaging

We have to specify $\mathbf{W}^{\mathrm{p}}$ and $\mathbf{W}^{\mathrm{t}}$ by modelling them at some initial rapidities and then evolve to a desired an effective high energy Hamiltonian H^{HE}, which is yet to be derived.

McLerran-Venugopalan model for dense systems:

$$\mathbf{W}^{\mathbf{MV}}[\rho] = \mathcal{N} \exp\left[-\int_{\mathbf{k}} \frac{\mathbf{1}}{\mathbf{2}\mu^{\mathbf{2}}(\mathbf{k})} \rho(\mathbf{k})\rho(-\mathbf{k})\right]$$

Light Cone Wave Function



Hard particles with $k^+ > \Lambda$ scatter off the target. In the eikonal approximation, the scattering amplitude is independent of k^+. Hard (valence) modes are described by the valence density $\rho(x_\perp)$. Note rapidity independence!

Soft modes are not many. They do not contribute much to the scattering amplitude.

The boost opens a window above Λ with the width $\sim \delta y$. The window is populated by soft modes, which became hard after the boost. These newly created hard modes do scatter off the target.

In the dilute limit $\rho \sim 1$; gluon emission $\sim \alpha_s \rho$, LO = one gluon, NLO = 2 gluons

In the dense limit $\rho \sim 1/\alpha_s$, we have $\alpha_s \rho \sim 1$, and the number of gluons in the window can be very large.

Denote soft glue creation and annihilation operators as $\mathbf{a}$ and $\mathbf{a}^\dagger$.

$$\mathbf{H}_{\mathbf{QCD}} = \mathbf{H}(\rho,\, \mathbf{a},\, \mathbf{a}^\dagger)$$

Hadron wave function in the soft gluon Fock space

$$|\boldsymbol{\Psi}\rangle_{\mathbf{Y_0}} = |v\rangle = |\rho\rangle_{\mathbf{valence}} \otimes |\mathbf{0_a}\rangle_{\mathbf{soft}}$$

The evolved wave function

$$|\boldsymbol{\Psi}\rangle_{\mathbf{Y}} = \boldsymbol{\Omega}_{\mathbf{Y}}(\rho,\, \mathbf{a})\, |\boldsymbol{\Psi}\rangle_{\mathbf{Y_0}}\,;$$

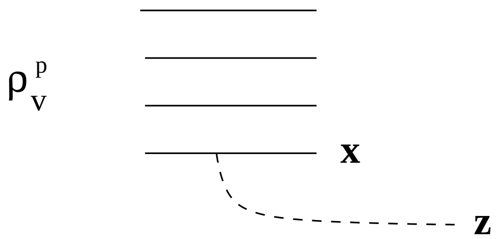
or equivalently

$$\boldsymbol{\Omega}^\dagger\, \mathbf{H}\, \boldsymbol{\Omega} = \mathbf{H}_{\mathbf{diagonal}}$$

The major challenge is to find $\boldsymbol{\Omega}$ that does the job

Gluon coherent field operator in the dilute limit

$$\Omega_Y(\rho \to 0) \equiv C_Y = \mathrm{Exp}\left\{ i \int d^2z\, b_i^a(z) \int_{e^{Y_0}\Lambda}^{e^{Y}\Lambda} \frac{dk^+}{\pi^{1/2}|k^+|^{1/2}} \left[a_i^a(k^+, z) + a_i^{\dagger a}(k^+, z) \right] \right\}$$



Linear evolution means $\delta\rho \propto \rho^p$

Emission amplitude is given by the Weizsaker-Williams field

$$b_i^a(z) = \frac{g}{2\pi} \int d^2x \frac{(x - z)_i}{(x - z)^2} \rho^a(x)$$

The operator $\mathbf{C}$ dresses the valence charges by a cloud of the WW gluons

In the dense regime: $\Omega(\rho \sim 1/\alpha_s) = \mathbf{C}\,\mathbf{B}$ $\qquad$ $\mathbf{B}$ is a Bogolyubov operator

$$\mathbf{B} = \exp[\,\Lambda(\rho)\,(a^2 + a^{\dagger\,2}) + \cdots]$$

$\mathbf{B}$ defines quasiparticles above the WW background

Altinoluk, Kovner, ML, Peressutti, Wiedemann (2007-2009)

Given the evolution of the hadronic wave function we can calculate evolution of an arbitrary observable $\hat{\mathcal{O}}(\rho)$

The evolution of the expectation value

$$\frac{d\,\langle v|\hat{\mathcal{O}}|v\rangle}{d\,Y} = \lim_{Y\to Y_0}\frac{\langle v|\Omega^\dagger_Y\,\hat{\mathcal{O}}[\rho+\delta\rho]\,\Omega_Y|v\rangle \ - \ \langle v|\hat{\mathcal{O}}[\rho]|v\rangle}{Y\ -\ Y_0} = -\int D\rho\,W[\rho]\,H^{HE}[\rho]\ \mathcal{O}[\rho]$$

Charge density due to newly produced gluon

$$\delta\rho^{\mathbf{a}}(\mathbf{x}) = \int_{\mathbf{e}^{\mathbf{Y_0}}\mathbf{\Lambda}}^{\mathbf{e}^{\mathbf{Y}}\mathbf{\Lambda}}\mathbf{dk}^+\,\mathbf{a}_{\mathbf{i}}^{\dagger\mathbf{b}}(\mathbf{k}^+,\mathbf{x})\,\mathbf{T}^{\mathbf{a}}_{\mathbf{bc}}\,\mathbf{a}^{\mathbf{c}}_{\mathbf{i}}(\mathbf{k}^+,\mathbf{x})$$

$\mathbf{H}^{\mathbf{KLWMIJ}} \ = \ \mathbf{H}^{\mathbf{HE}}(\rho\ \to\ \mathbf{0})$ A. Kovner and M.L., Phys.Rev.D71:085004, 2005

$\mathbf{H}^{\mathbf{JIMWLK}} \ = \ \mathbf{H}^{\mathbf{HE}}(\rho\ \to\ \infty)$ - Jalilian Marian, Iancu, McLerran, Leonidov, Kovner (1997-2002)

Balitsky-Kovchegov (BK) is the large $\mathrm{N_c}$ version of JIMWLK

Density Matrix of soft modes

The wave function coming into the collision region at time $t = 0$

$$|\mathbf{\Psi_{in}}\rangle \;=\; \mathbf{\Omega_Y}\,|\rho, \mathbf{0_a}\rangle\;.$$

Standard CGC formalism: first integrate out the soft modes and then average over ρ

Q: Can we learn something if we do in the opposite order? A: probably Yes.

Define the reduced density matrix of soft modes

$$\hat{\rho} \;=\; \int \mathbf{D}\rho\,\mathbf{W}[\rho]\,|\mathbf{\Psi_{in}}\rangle\,\langle\mathbf{\Psi_{in}}|$$

"Dilute/Dense mix approximation": $\Omega = C$ and $W = W^{MV}$ (Gaussian), $\hat{\rho}$ is computable analytically

T. Altinoluk, N. Armesto, G. Beuf, A. Kovner and ML, arXiv:1503.07126

$$\hat{\rho} = \sum_n \frac{1}{n!} e^{-\frac{1}{2}\phi_i M_{ij}\phi_j} \left[\prod_{m=1}^{n} M_{i_m j_m} \phi_{i_m} |0\rangle \langle 0| \phi_{j_m} \right] e^{-\frac{1}{2}\phi_i M_{ij}\phi_j}$$

Here we have introduced compact notations:

$$\phi_{\mathbf{i}} \equiv \left[\mathbf{a}_{\mathbf{i}}^{\dagger\mathbf{a}}(\mathbf{x}) + \mathbf{a}_{\mathbf{i}}^{\mathbf{a}}(\mathbf{x}) \right] \; ; \qquad \mathbf{M_{ij}} \equiv \frac{\mathbf{g^2}}{\mathbf{4\pi^2}} \int_{\mathbf{u,v}} \mu^{\mathbf{2}}(\mathbf{u},\mathbf{v}) \frac{(\mathbf{x}-\mathbf{u})_{\mathbf{i}}}{(\mathbf{x}-\mathbf{u})^{\mathbf{2}}} \frac{(\mathbf{y}-\mathbf{v})_{\mathbf{j}}}{(\mathbf{y}-\mathbf{v})^{\mathbf{2}}} \delta^{\mathbf{ab}}$$

M **bears two polarisation, colour, and coordinate indices, collectively denoted as $\{ij\}$.**

Bose Enhancement

Easy to show that correlators in this $\hat\rho$ Wick factorize in terms of two basic elements:

$$tr[\hat\rho a_a^{\dagger i}(k) a_b^j(p)] = (2\pi)^2 \delta_{ab}\ \delta^{(2)}(k-p)\ g^2\mu^2(p)\ \frac{p^i p^j}{p^4}$$

$$tr[\hat\rho a_a^{i}(k) a_b^j(p)] = tr[\hat\rho a_a^{\dagger i}(k) a_b^{\dagger j}(p)] = -(2\pi)^2 \delta_{ab}\ \delta^{(2)}(k+p)\ g^2\mu^2(p)\ \frac{p^i p^j}{p^4}$$

Correlator of aaaa enters double gluon production:

$$tr[\hat\rho a_a^{\dagger i}(k_1) a_b^{\dagger j}(k_2) a_a^i(k_1) a_b^j(k_2)] = S^2(N_c^2-1)^2 \left\{ \frac{g^4\mu^2(k_1)\mu^2(k_2)}{k_1^2 k_2^2} \right.$$

$$\left. + \frac{1}{S(N_c^2-1)} \left[\delta^{(2)}(k_1-k_2) + \delta^{(2)}(k_1+k_2)\right] \frac{g^4\mu^4(k_1)}{k_1^4} \right\}$$

The first term is the “classical” square of the density.

The last term is a Bose enhancement contribution $\rightarrow$ Glasma Graphs

Entanglement Entropy

Alex Kovner and ML, arXiv:1506.05394

Initial wave function: Entanglement Entropy of soft modes

$$\sigma^{\mathbf{E}} \;=\; -\,\mathbf{tr}[\hat{\rho}\ln\hat{\rho}]$$

How to calculate $\ln$? The "replica trick":

$$\ln\hat{\rho} = \lim_{\epsilon\to\mathbf{0}}\frac{\mathbf{1}}{\epsilon}\left(\hat{\rho}^{\epsilon}-\mathbf{1}\right)$$

Calculate ρ^N and take $N \to 0$. N copies of the field - replicas.

The result

$$\sigma^{\mathbf{E}} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{tr}\left\{\ln\frac{\mathbf{M}}{\pi}+\sqrt{\mathbf{1}+\frac{\mathbf{4M}}{\pi}}\ln\left[\mathbf{1}+\frac{\pi}{\mathbf{2M}}\left(\mathbf{1}+\sqrt{\mathbf{1}+\frac{\mathbf{4M}}{\pi}}\right)\right]\right\}$$

Translationally invariant limit ($\mu = const$):

$$\mathbf{M^{ab}_{ij}(p) = g^2\mu^2\frac{p_ip_j}{p^4}\delta^{ab}}$$

For small M, or the UV contribution

$$\sigma^{\mathbf{E}}_{\mathbf{UV}} = \mathbf{tr}\left[\frac{\mathbf{M}}{\pi}\ln\frac{\pi\mathbf{e}}{\mathbf{M}}\right] = -\frac{\mathbf{N_c^2-1}}{\pi}\mathbf{S}\int_{\mathbf{p^2>\frac{Q_s^2}{g^2}}}\frac{\mathbf{d^2p}}{(\mathbf{2}\pi)^\mathbf{2}}\frac{\mathbf{Q_s^2}}{\mathbf{g^2p^2}}\ln\frac{\mathbf{Q_s^2}}{\mathbf{eg^2\,p^2}}$$

where $\mathbf{Q_s^2} = \frac{\mathbf{g^4}}{\pi}\mu^2$ σ^E is formally UV divergent

$$\sigma^{\mathbf{E}}_{\mathbf{UV}} = \frac{\mathbf{Q_s^2}}{\mathbf{4}\pi\mathbf{g^2}}(\mathbf{N_c^2-1})\mathbf{S}\left[\ln^2\frac{\mathbf{g^2\Lambda^2}}{\mathbf{Q_s^2}} + \ln\frac{\mathbf{g^2\Lambda^2}}{\mathbf{Q_s^2}}\right]$$

The large M, IR contribution is

$$\sigma^{\mathbf{E}}_{\mathbf{IR}} \simeq \frac{\mathbf{1}}{\mathbf{2}}\mathbf{tr}[\ln\frac{\mathbf{e^2M}}{\pi}] = \frac{\mathbf{N_c^2-1}}{\mathbf{2}}\mathbf{S}\int_{\mathbf{p_2<\frac{Q_s^2}{g^2}}}\frac{\mathbf{d^2p}}{(\mathbf{2}\pi)^\mathbf{2}}\ln\frac{\mathbf{e^2\,Q_s^2}}{\mathbf{g^2p^2}} = \frac{\mathbf{3(N_c^2-1)}}{\mathbf{8}\pi\mathbf{g^2}}\mathbf{SQ_s^2}$$

Properties of σ^E.

$$\sigma \approx \sigma^{\mathbf{E}}_{\mathbf{UV}} + \sigma_{\mathbf{IR}} = \frac{\mathbf{S Q_s^2}}{\mathbf{4\pi g^2}}(\mathbf{N_c^2} - \mathbf{1}) \left[\ln^2 \frac{\mathbf{g^2 \Lambda^2}}{\mathbf{Q_s^2}} + \ln \frac{\mathbf{g^2 \Lambda^2}}{\mathbf{Q_s^2}} + \frac{\mathbf{3}}{\mathbf{2}} \right]$$

UV divergent: the divergence is cutoff physically at $\mathbf{\Lambda \sim M e^{Y_0} \gg M}$, where eikonal approximation breaks down.

σ^E **is not extensive in rapidity:** only one longitudinal mode (rapidity independent) is entangled with valence degrees of freedom.

Similar to "topological entropy": insensitive to boundary region between the modes.

But not quite what we would like to know.

We need to address scattering process

Semi-inclusive reactions

The wave function coming into the collision region at time $t = 0$

$$|\mathbf{\Psi_{in}}\rangle \; = \; \mathbf{\Omega_Y} \, |\rho, \mathbf{0_a}\rangle \; .$$

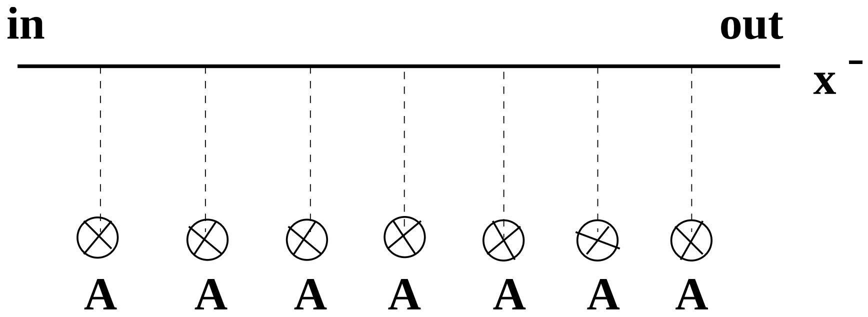
The system emerges from the collision region with the wave function

$$|\mathbf{\Psi_{out}}\rangle \; = \; \hat{\mathbf{S}} \, \mathbf{\Omega_Y} \; |\rho, \mathbf{0_a}\rangle \; .$$

The system keeps evolving after the collision to the asymptotic time $t \to +\infty$, at which point the measurement of an observable $\hat{\mathcal{O}}$ is made

$$\langle \hat{\mathcal{O}} \rangle_{\mathbf{P,T}} \; = \; \langle \;\; \langle \mathbf{0_a} | \mathbf{\Omega_Y^\dagger} \; (\mathbf{1} \; - \; \hat{\mathbf{S}}^\dagger) \, \mathbf{\Omega_Y} \;\; \hat{\mathcal{O}} \;\; \mathbf{\Omega_Y^\dagger} \;\; (\mathbf{1} \; - \; \hat{\mathbf{S}}) \, \mathbf{\Omega_Y} \, | \mathbf{0_a} \rangle \; \rangle_{\mathbf{P,T}}$$

Eikonal scattering approximation



Eikonal scattering is a color rotation

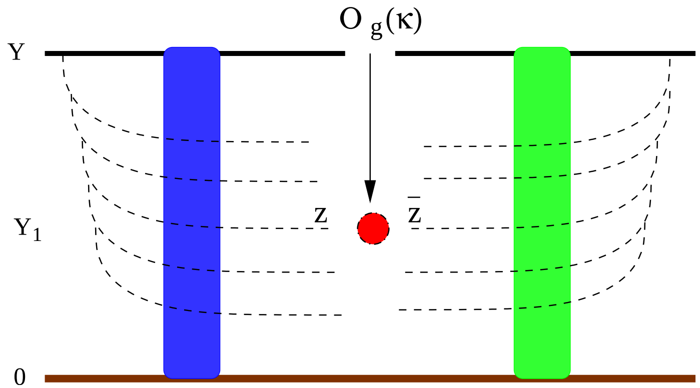
Eikonal factor does not depend on rapidity

In the light cone gauge ($\mathbf{A}^+ = 0$) the large target field component is $\mathbf{A}^- = \alpha^{\mathbf{t}}$.

$$\mathbf{S}(\mathbf{x}) = \mathcal{P}\exp\left\{\mathbf{i}\int \mathbf{dx}^+\,\mathbf{T}^{\mathbf{a}}\,\alpha^{\mathbf{a}}_{\mathbf{t}}(\mathbf{x},\mathbf{x}^+)\right\}. \qquad \text{''}\boldsymbol{\Delta}\text{''}\,\alpha^{\mathbf{t}} = \rho^{\mathbf{t}} \quad (\mathbf{YM})$$

$$|\mathbf{in}\rangle = |\mathbf{z},\mathbf{b}\rangle\,; \qquad |\mathbf{out}\rangle = |\mathbf{z},\mathbf{a}\rangle\,; \qquad |\mathbf{out}\rangle = \mathbf{S}\,|\mathbf{in}\rangle$$

Single inclusive gluon production



The observable

$$\hat{\mathcal{O}}_g \;\sim\; \mathbf{a}_i^{\dagger\, a}(\mathbf{k})\, \mathbf{a}_i^{a}(\mathbf{k})$$

$$\frac{\mathbf{dN}}{\mathbf{d^2kd}\eta} = \langle\, \sigma(\mathbf{k})\, \rangle_{\mathbf{P,T}}$$

After soft gluon averaged using dilute projectile approximation ($\boldsymbol{\Omega} \to \mathbf{C}$)

$$\sigma(\mathbf{k}) = \int_{\mathbf{z},\bar{\mathbf{z}},\mathbf{x_1},\bar{\mathbf{x}}_1} \mathbf{e}^{\mathrm{ik(z-\bar{z})}} \frac{(\bar{\mathbf{z}}-\bar{\mathbf{x}}_1)_i}{(\bar{\mathbf{z}}-\bar{\mathbf{x}}_1)^2} \frac{(\mathbf{x}_1-\mathbf{z})_i}{(\mathbf{x}_1-\mathbf{z})^2} \left\{ \rho(\mathbf{x}_1)[\mathbf{S}^\dagger(\mathbf{x}_1)-\mathbf{S}^\dagger(\mathbf{z})][\mathbf{S}(\bar{\mathbf{x}}_1)-\mathbf{S}(\mathbf{z})]\rho(\bar{\mathbf{x}}_1) \right\}$$

Entropy production

$$\sigma^{P} = \frac{1}{2}\langle \mathrm{tr}\left\{ \ln\frac{M^{P}}{\pi} + \sqrt{1+\frac{4M^{P}}{\pi}}\ln\left[1+\frac{\pi}{2M^{P}}\left(1+\sqrt{1+\frac{4M^{P}}{\pi}}\right)\right]\right\}\rangle_{T}$$

with

$$M^{P} \equiv g^{2}\int_{u,v}\mu^{2}(u,v)\frac{(x-u)_{i}}{(x-u)^{2}}\frac{(y-v)_{j}}{(y-v)^{2}}\left[(S(u)-S(x))(S^{\dagger}(v)-S^{\dagger}(y))\right]^{ab}$$

T-averaging is complicated Let expand σ^{P} around $\bar{M} \equiv \langle M^{P}\rangle_{T}$ (dilute projectile limit)

$$\bar{M} == \delta^{ab}\frac{Q_{P}^{2}\pi}{g^{2}}\int_{z}\frac{(x-z)_{i}}{(x-z)^{2}}\frac{(y-z)_{j}}{(y-z)^{2}}[P_{A}(x,y)+1-P_{A}(x,z)-P_{A}(z,y)]$$

P_A - S-matrix of an adjoint dipole Q_p - saturation momentum of the projectile.

$\bar{M}$ is *almost* single inclusive gluon, but it is not summed over ij

$$\sigma^{\mathbf{P}} = \mathbf{tr}\left[\frac{\bar{\mathbf{M}}}{\pi}\ln\frac{\pi\mathbf{e}}{\bar{\mathbf{M}}}\right] - \frac{\mathbf{1}}{\mathbf{2}\pi}\,\mathbf{tr}\left[\left\{\langle(\mathbf{M}^{\mathbf{P}} - \bar{\mathbf{M}})\ (\mathbf{M}^{\mathbf{P}} - \bar{\mathbf{M}})\rangle_{\mathbf{T}}\right\}\bar{\mathbf{M}}^{-1}\right]\ldots.$$

First term is *almost* $-n\ln n$, where n is a multiplicity per unit rapidity ($dN/d\eta$)

it depends on the production probabilities of longitudinally and transversely (with respect to the direction of their transverse momentum) polarized gluons separately

Second term - *almost* correlated part of double inclusive gluon production.

Correlations between gluons decrease entropy of the produced state. consistent with the view of entropy as measuring disorder in the final state.

For a parametrically large number of produced particles ($\alpha_s dN/d\eta \sim 1$), the entropy is parametrically of order $1/\alpha_s$

"Temperature" of produced system

We can naturally define temperature through:

$$\mathbf{T^{-1} \;=\; \frac{d\sigma}{dE_T}}$$

$$\mathbf{E_\perp \;\propto\; \int d^2k\, |k|\, M^P(k) \;\propto\; (N_c^2 - 1)\, S\, \frac{Q_P^2}{g^2} Q_T}$$

Keeping only mean field term in the entropy:

$$\mathbf{T \;=\; \frac{\pi}{2}\, \langle k_T \rangle}$$

.

$$\mathbf{\langle k_T \rangle \;=\; E_\perp / N_{total}} \qquad\qquad \mathbf{N_{total} \;=\; \int d^2k\, M^P(k)}$$

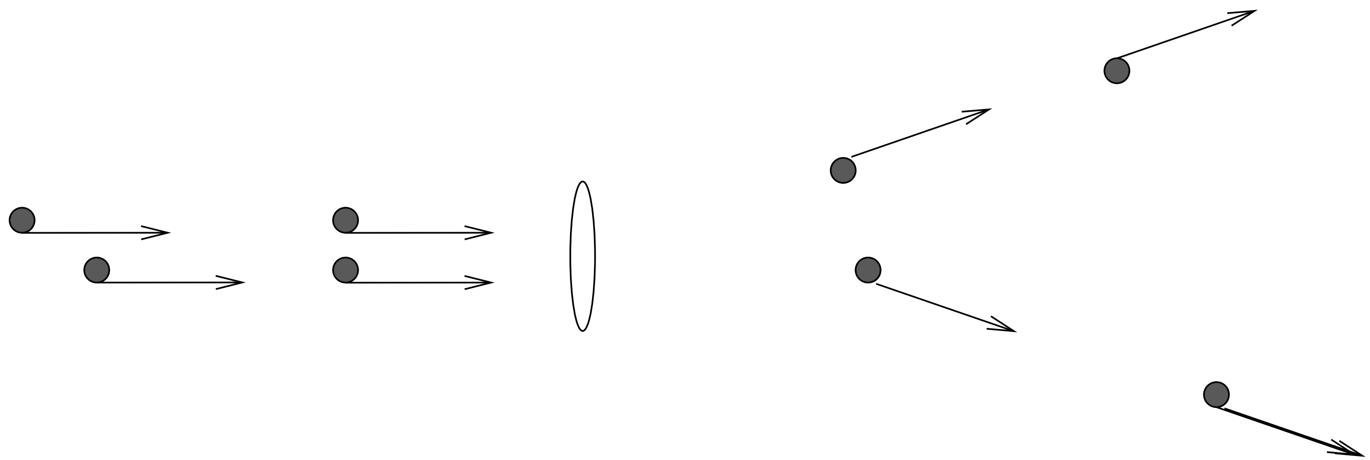
Naive picture of correlated gluon production

Long range rapidity correlations come for free with boost invariance

Incoming $|P\rangle$ is approximately boost invariant: exactly the same gluon distribution at Y_1 and Y_2.
What happens at Y_1, happens also at Y_2: If it is probable to produce a gluon at Y_1, it is also probable to produce a gluon at Y_2.

But exactly by the same logic there must be angular correlations:
Gluons scatter on exactly the same target
If the first gluon is most likely to be scattered to the right, the second gluon at the same impact parameter will be also scattered to the right

Eikonal scattering is rapidity independent!



Double inclusive gluon production

$$\mathcal{O} \;=\; \mathbf{a}^\dagger(\mathbf{k})\,\mathbf{a}(\mathbf{k})\;\mathbf{a}^\dagger(\mathbf{p})\,\mathbf{a}(\mathbf{p})$$

$$\frac{\mathbf{dN}}{\mathbf{d^2pd^2kd}\eta\mathbf{d}\xi} \;=\; \sigma_4 \;=\; \langle\sigma(\mathbf{k})\;\sigma(\mathbf{p})\rangle_{\mathbf{P,T}}$$

Configuration by configuration
(for fixed configuration of projectile charges ρ and fixed target fields S)

$\sigma(\mathbf{k})$ is a real function of k, which has a maximum at some value $\mathbf{k} = \mathbf{q_0}$. Then the two gluon production probability configuration by configuration has a maximum at

$$\mathbf{k} \;=\; \mathbf{p} \;=\; \mathbf{q_0} \simeq \mathbf{Q_s}$$

The value of $\mathbf{q_0}$ depends on configuration, but the fact that $\mathbf{k} \;\simeq\; \mathbf{p}$ does not.

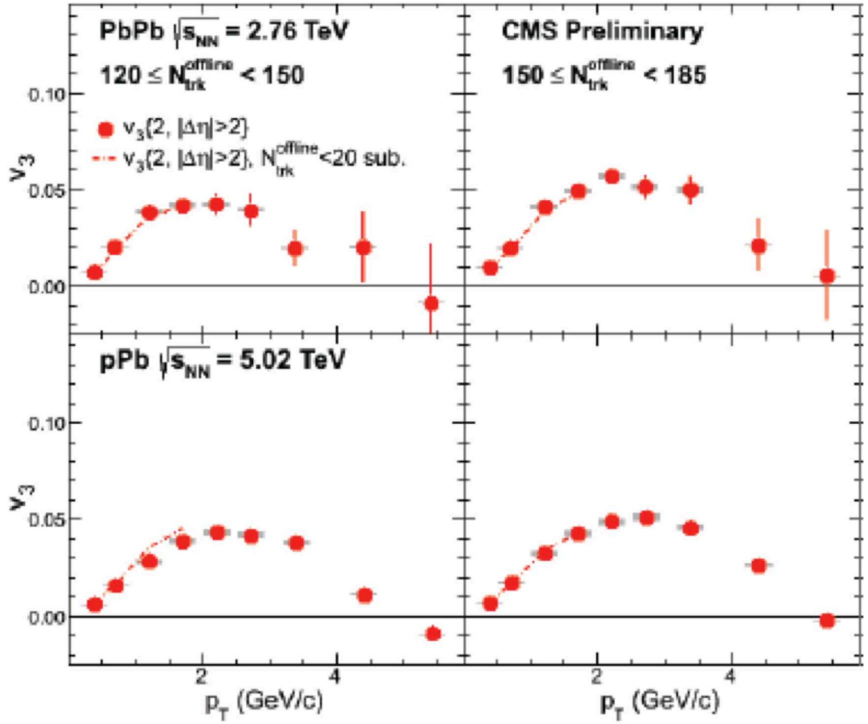
This is the near side correlation!

Is the maximum of σ_1 unique?

No, σ_1 is symmetric under $\mathbf{k} \to -\mathbf{k}$ and thus has two maxima at $\mathbf{q_0}$ and $-\mathbf{q_0}$

This means that σ^4 has a symmetry $\mathbf{k}, \mathbf{p} \to -\mathbf{k}, \mathbf{p}$ and therefore has maxima at relative angles $\phi = 0$ and $\phi = \pi$

The maximum at $\phi = \pi$ is very difficult to distinguish experimentally.

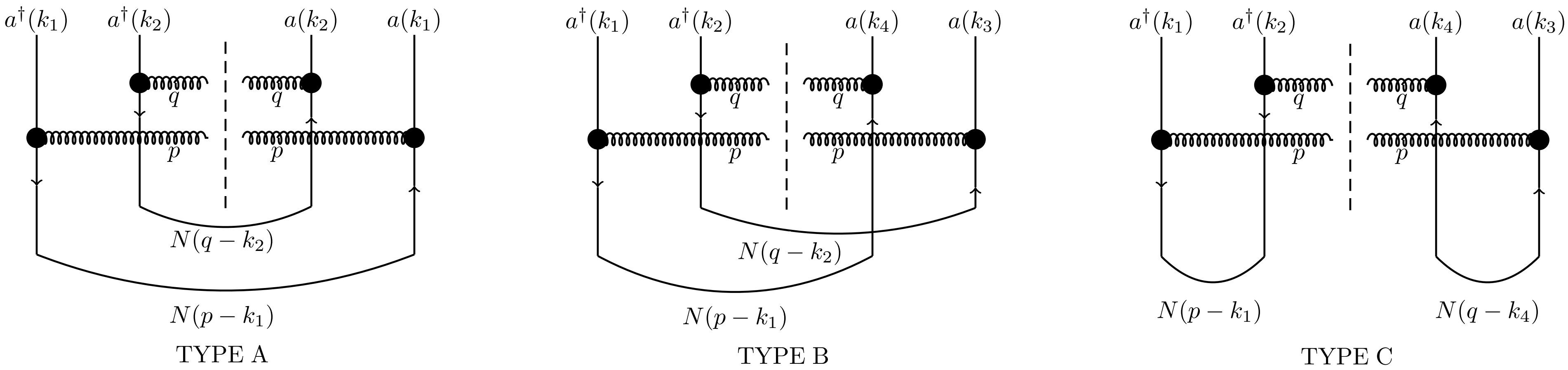


After all there is some asymmetry between $\mathbf{0}$ and π angles

The v_3 story:

Projectile/Target averaging

• **"Glasma graphs"** A. Dumitru, F. Gelis, J. Jalilian-Marian, T. Lappi: Phys.Lett. B697 (2011) 21 (arXiv:1009.5295), **followed by quite successful quantitative effort to describe all data by** K. Dusling and R. Venugopalan, Phys.Rev.Lett. 108 (2012) 262001



Type B+Type C="upside down" Type A + "suppressed"

The resulting correlations are N_c suppressed. The physics of the "Glasma graphs" is

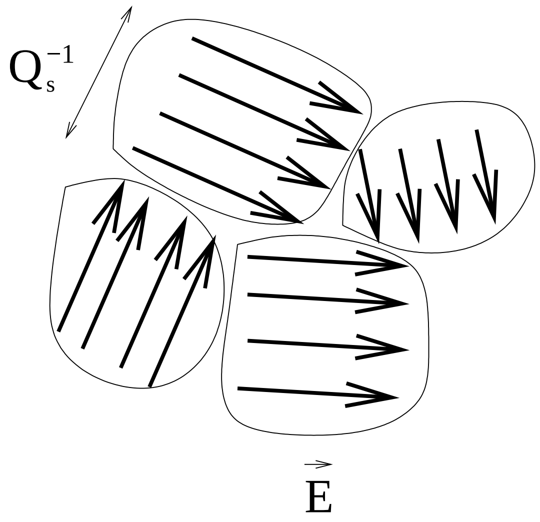
Initial state Bose enhancement $\rightarrow$ correlation in the final state.

T. Altinoluk, N. Armesto, G. Beuf, A. Kovner and ML, arXiv:1503.07126.

- **Local anisotropy** A. Kovner and M. L: PRD83 (2011) 034017; IJMP E Vol. 22 (2013) 1330001

$$\mathbf{tr}[\mathbf{S}^\dagger_{\mathbf{x}}\,\mathbf{S}_{\mathbf{y}}]/\mathbf{N_c} \;=\; \mathbf{Exp}[\,-\,(\vec{\mathbf{r}}\,\vec{\mathbf{E}}(\vec{\mathbf{b}}))^2]\,;$$

$$\vec{\mathbf{E}}(\vec{\mathbf{b}}) \;=\; \sum \vec{\mathbf{E}}_0(\vec{\mathbf{b}})\,\mathbf{e}^{-\,\mathbf{d}^2\,\mathbf{Q_s^2}}\,; \qquad \mathbf{E_0} \,=\, \mathbf{Q_s}$$



$\vec{\mathbf{r}} = \vec{\mathbf{x}} - \vec{\mathbf{y}}$ is a vector of the dipole moment.

$\langle \mathbf{tr}[\mathbf{S}^\dagger_{\mathbf{x}}\,\mathbf{S}_{\mathbf{y}}]\mathbf{tr}[\mathbf{S}^\dagger_{\mathbf{u}}\,\mathbf{S}_{\mathbf{v}}]\rangle$ has non-trivial angular correlations and the effect is a leading N_c.

A. Dumitru, A. Giannini, NPA 933 (2014) 212

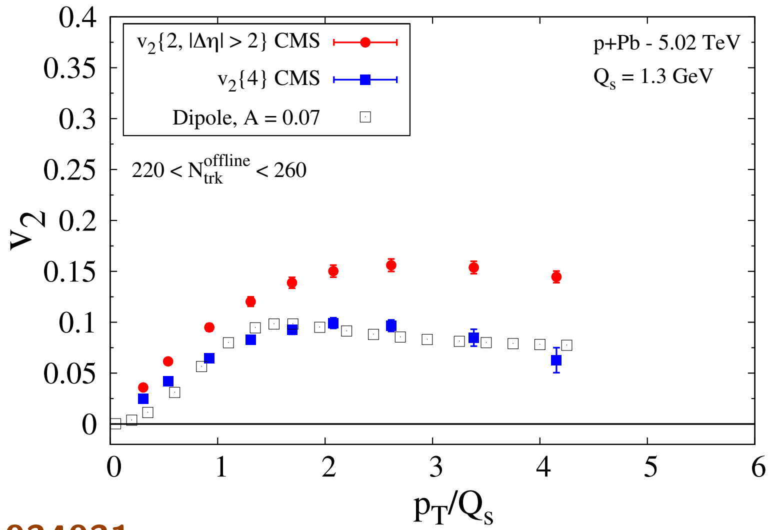
A. Dumitru, L. McLerran and V. Skokov, PLB 743, 134 (2015)

A. Dumitru and V. Skokov, PRD 91, 7, 074006 (2015)

A. Dumitru, A. V. Giannini and V. Skokov, arXiv:1503.03897,

V. Skokov, PRD 91, 5, 054014 (2015)

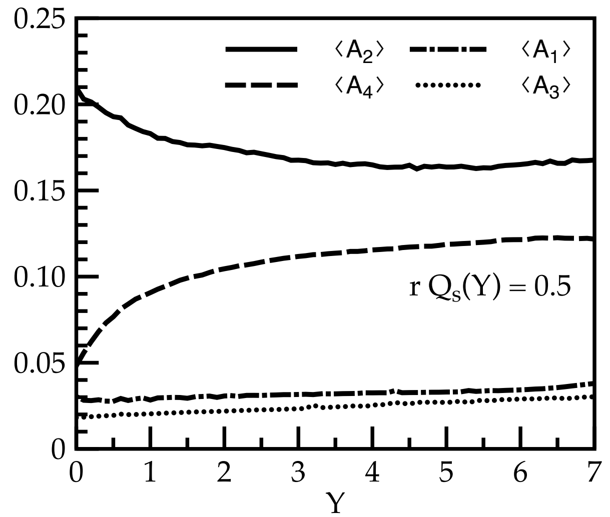
T. Lappi, PLB 744, 315 (2015)



- **Density variation** E. Levin a A. Rezaeian, PRD84 (2011) 034031

A word about high energy evolution

- With increase of rapidity, we found an exponentially fast isotropization with the exponent $\lambda_A \simeq 0.6$.
- Observed correlations must arise dynamically. Pomeron loops are needed



Quite an opposite result (presumably $1/N_c^2$ effect)

A. Dumitru, A. V. Giannini and V. Skokov, arXiv:1503.03897

$$\frac{\mathbf{N(r,\theta,Y)}}{\langle \mathbf{N(r,Y)}\rangle} = \mathbf{1} + \sum_{\mathbf{n}} \mathbf{A_n(r,Y)}\cos(\mathbf{2n}\theta)$$

Target correlations $\langle tr[S^\dagger S]\, tr[S^\dagger S]\rangle_T$ from the BK equation

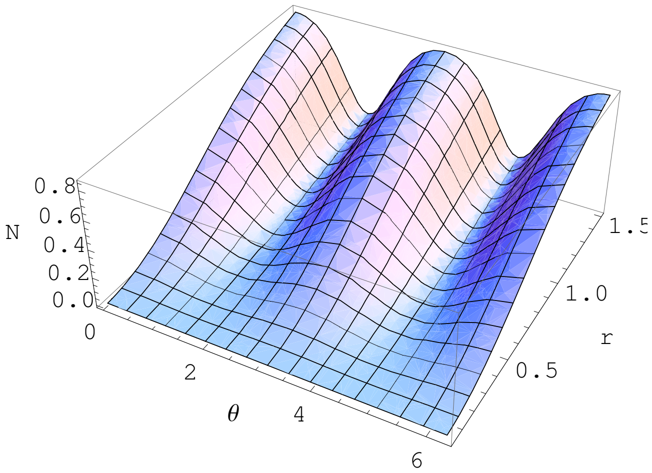
BKe for imaginary part of the dipole scattering amplitude $\mathbf{N(\vec{r}) = 1 \; - \; tr[S^\dagger_x\, S_y]/N_c}$

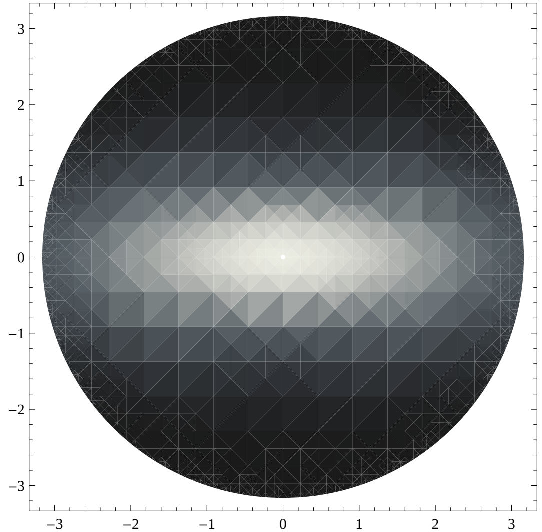
$$\partial_{\mathbf{Y}}\, \mathbf{N(\vec{r})} \; = \; \frac{\mathbf{C_F}\, \alpha_{\mathbf{s}}}{\mathbf{2}\,\pi} \int \mathbf{d^2 \vec{r}'} \frac{\mathbf{\vec{r}^2}}{\mathbf{\vec{r}'^{\,2}\, (\vec{r} - \vec{r}')^2}} \; [\mathbf{N(\vec{r}')} \; + \; \mathbf{N(\vec{r} - \vec{r}')} \; - \; \mathbf{N(\vec{r})} \; - \; \mathbf{N(\vec{r}')\, N(\vec{r} - \vec{r}')}]$$

Anisotropic initial conditions at some initial rapidity $Y_0 = \ln 10^2$.

$$\mathbf{N(Y_0, \vec{r})} \; = \; \mathbf{1} \; - \; \mathbf{Exp[\, -\, a\, r^2\, xg^{LOCTEQ6}(x_0, 4/r^2)\, F(\theta)];} \qquad\qquad \mathbf{a} \; = \; \frac{\alpha_{\mathbf{s}}(\mathbf{r^2})\,\pi}{\mathbf{2\, N_c\, R^2}}$$

$$\mathbf{F(\theta)} \; = \; \mathbf{1 - A} \; + \; \mathbf{2\, A\, cos^2(\theta)} \qquad\qquad \mathbf{A} \; = \; \mathbf{3/4}$$



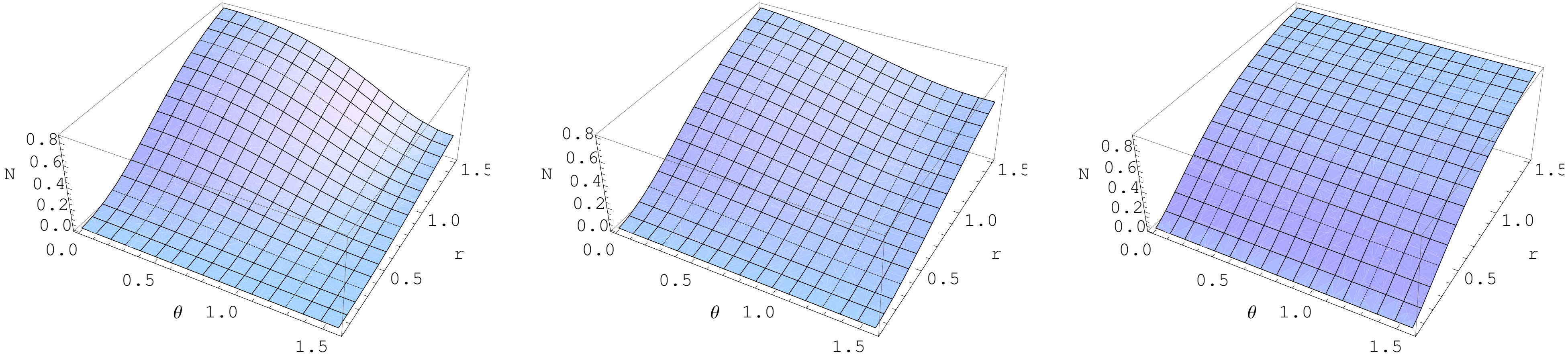


$\mathbf{W}[\delta] = \mathbf{1/2\pi}$, **constant for any δ ranging from 0 to 2π.**

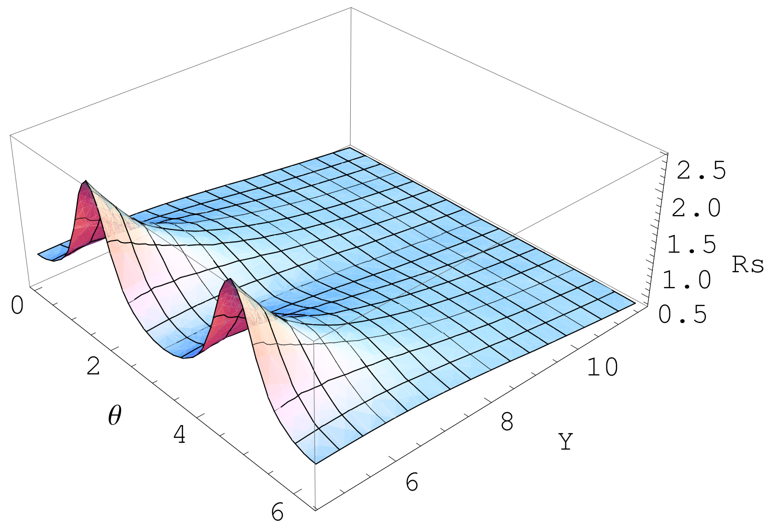
$$\langle \mathbf{F} \rangle_\delta = \int_0^{2\pi} \mathbf{d}\delta\, \mathbf{F}(\theta + \delta)\, \mathbf{W}[\delta] = \mathbf{1}$$

We are interested in the two-dipole correlator $\langle \mathbf{N}(\mathbf{Y}, \mathbf{r_1}, \theta_1, \delta)\, \mathbf{N}(\mathbf{Y}, \mathbf{r_2}, \theta_2, \delta) \rangle_\delta$.

Single configuration solution



the saturation scale $N(Y, R_S, \theta) = 1/2$

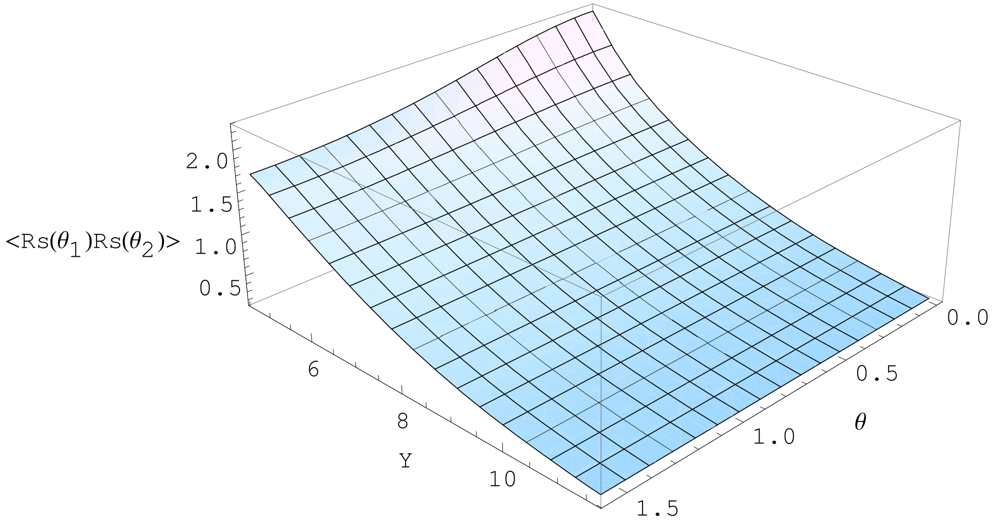


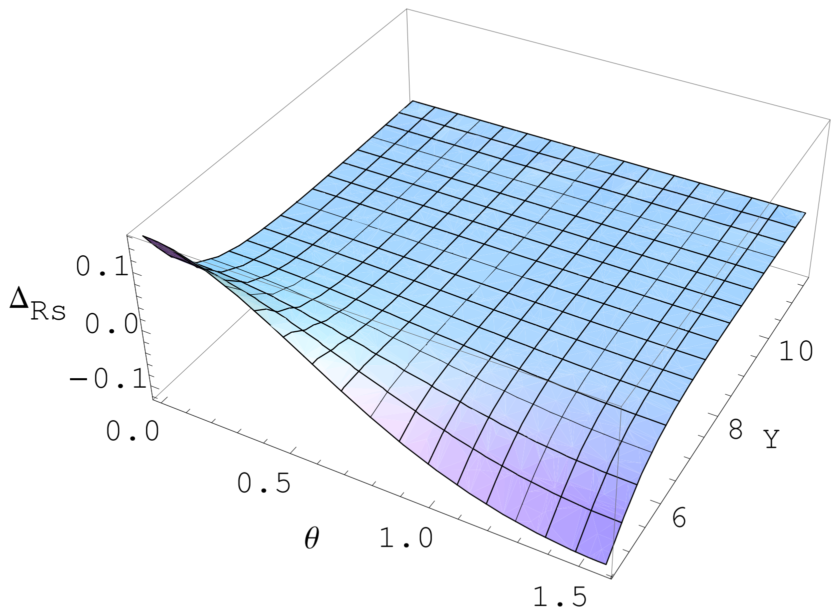
Very fast isotropization!

Angular correlations of the saturation radius

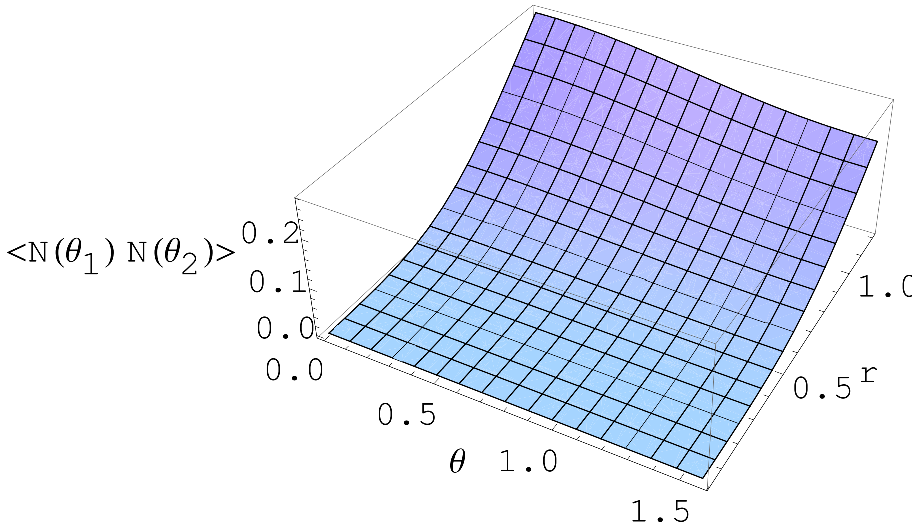
Two quantities of interest: correlator of two saturation scales $\langle \mathbf{R_s}(\theta_1)\mathbf{R_s}(\theta_2)\rangle_\delta$ and

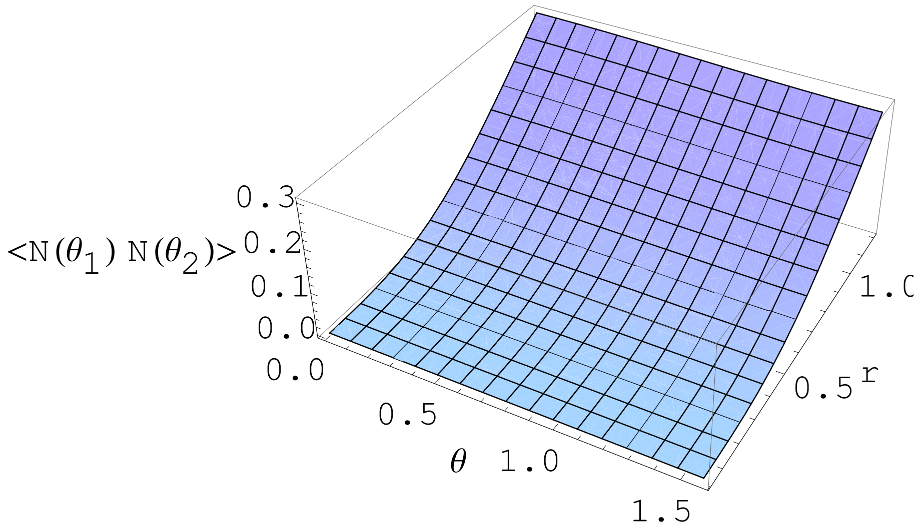
$$\mathbf{\Delta_{R_s}}(\mathbf{Y},\mathbf{r},\theta) \equiv \frac{\langle \mathbf{R_s}(\mathbf{Y},\theta_1,\delta)\,\mathbf{R_s}(\mathbf{Y},\theta_2,\delta)\rangle_\delta - \langle \mathbf{R_s}(\mathbf{Y},\theta_1,\delta)\rangle_\delta\,\langle \mathbf{R_s}(\mathbf{Y},\theta_2,\delta)\rangle_\delta}{\langle \mathbf{R_s}(\mathbf{Y},\theta_1,\delta)\rangle_\delta^{\mathbf{2}}}, \qquad \theta = \theta_1 - \theta_2$$





Angular correlations $\langle \mathbf{N}(\mathbf{Y},\mathbf{r},\theta_1)\mathbf{N}(\mathbf{Y},\mathbf{r},\theta_2)\rangle_\delta$





Again fast anizotropization

$$\textbf{Ang. correlations} \sim \mathbf{e}^{-\lambda\mathbf{Y}}, \qquad \lambda \simeq \mathbf{0.6}$$

Presumably related to the second BFKL eigenvalue

$$\omega_{\mathbf{n=0}} = \mathbf{4}\,\ln \mathbf{2}\,\bar{\alpha}_{\mathbf{s}}; \qquad \omega_{\mathbf{n=2}} = \mathbf{4}\,(\ln \mathbf{2} - \mathbf{1})\,\bar{\alpha}_{\mathbf{s}}$$

Towards correlations in symmetric collisions.

T. Altinoluk, A. Kovner, E. Levin, ML, JHEP 1404 (2014) 075

$$\frac{\mathbf{dN}}{\mathbf{d^2pd^2kd}\eta\mathbf{d}\xi} = \langle\sigma(\mathbf{k})\ \sigma(\mathbf{p})\rangle_{\mathbf{P,T}}$$

$$\sigma(\mathbf{k}) = \int_{\mathbf{z},\bar{\mathbf{z}},\mathbf{x_1},\bar{\mathbf{x}}_1} \mathbf{e}^{\mathbf{ik(z-\bar{z})}}\vec{\mathbf{f}}(\bar{\mathbf{z}}-\bar{\mathbf{x}}_1)\cdot\vec{\mathbf{f}}(\mathbf{x_1}-\mathbf{z})\ \left\{\rho(\mathbf{x_1})[\mathbf{S}^\dagger(\mathbf{x_1}) - \mathbf{S}^\dagger(\mathbf{z})][\mathbf{S}(\bar{\mathbf{x}}_1) - \mathbf{S}(\mathbf{z})]\rho(\bar{\mathbf{x}}_1)\right\}$$

Identify target Pomeron $\quad P_A^T(x,y) \equiv 1 - \langle S(x)S^\dagger(y)\rangle_T/N_c$

and projectile Pomeron as $\quad P_A^P(x,y) \sim \frac{1}{\nabla^2}(x-\bar{x})\frac{1}{\nabla^2}(y-\bar{y})\,\langle\rho(\bar{x})\rho(\bar{y})\rangle_P$

after color projection algebra and some little massage

$$\frac{d\sigma}{d\eta\, dk^2\, d\xi\, dp^2} \sim \frac{1}{k^2}\frac{1}{p^2}\int_{x,y,u,v} \cos k(x-y)\cos p(u-v)$$
$$\times\Big\{\frac{1}{4}\frac{\partial}{\partial(ij\bar{i}\bar{j})}[\bar{P}^T_A(x,y)\bar{P}^T_A(u,v)]\Delta^{ijkl}\Delta^{\bar{i}\bar{j}\bar{k}\bar{l}}\frac{\partial}{\partial(kl\bar{k}\bar{l})}[\bar{P}^P_A(x,y)\bar{P}^P_A(u,v)]$$
$$-\frac{8}{N_c^2}\frac{\partial}{\partial(ij\bar{i}\bar{j})}[\bar{N}^T_{xy}\bar{N}^T_{uv}\bar{Q}^T_{yuvx}]\Delta^{ijkl}\Delta^{\bar{i}\bar{j}\bar{k}\bar{l}}\frac{\partial}{\partial(kl\bar{k}\bar{l})}[\bar{N}^P_{yx}\bar{N}^P_{vu}\bar{Q}^P_{xvuy}]\Big\}$$

where we have defined

$$\frac{\partial}{\partial(ijkl)} \equiv \frac{\partial}{\partial x_i}\frac{\partial}{\partial y_j}\frac{\partial}{\partial u_k}\frac{\partial}{\partial v_l}$$

$$\Delta^{ijkl} \equiv \delta^{ij}\delta^{kl} + \delta^{ik}\delta^{jl} - \delta^{il}\delta^{jk}$$

Here $Q^T(yuvx) = tr[S(y)S^\dagger(u)S(v)S^\dagger(y)]$ **(quadrupole/B-Reggeon)**

The expression is manifestly symmetric with respect to target/projectile.