Second order hydrodynamic coefficients for the non-conformal QGP from holography

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with Stefano Finazzo, Romulo Rougemont, and Hugo Marrochio

JHEP 1502 (2015) 051, arXiv:1412.2968

Nuclear Physics & RIKEN Theory Seminar, BNL, March 2015

OUTLINE

- Motivation: Perfect fluidity of large systems/small systems

- Non-conformal relativistic hydrodynamics at 2^{nd} order

- Strongly coupled non-conformal plasma from holography

- Holographic calculation of 2^{nd} order hydro coefficients for the non-conformal plasma.

- Conclusions and outlook

Quark-Gluon Plasma (QGP):

The hottest, the tiniest, and the most perfect fluid ever made

“Perfect” fluidity in “large” systems !!!

Very small $\frac{\eta}{s} \sim 2 \times \frac{1}{4\pi}$

A theoretician's toolkit includes

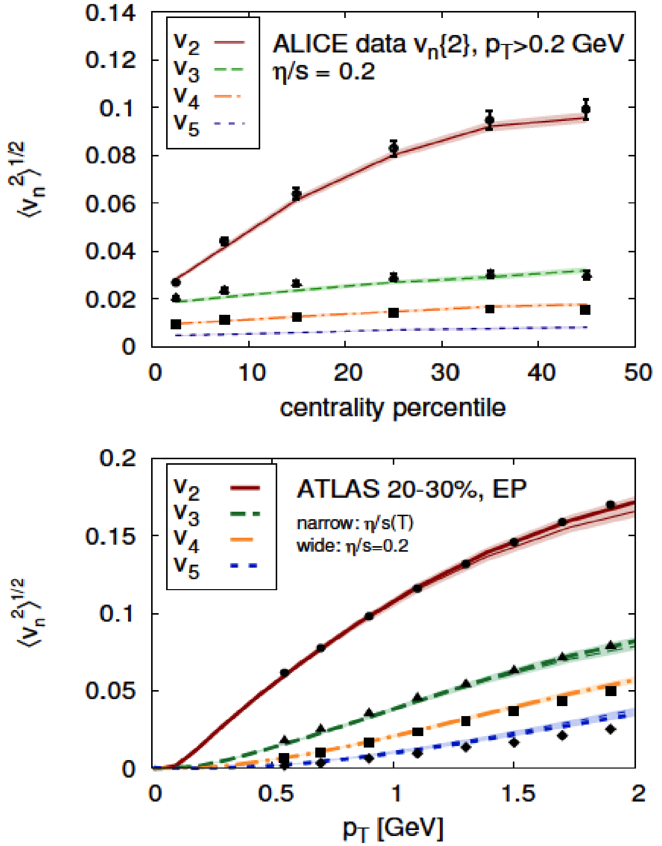
- Initial conditions for energy-momentum tensor

- Evolution using 2nd order viscous hydrodynamics

- Lattice EOS + hadron resonance gas

- Hadron cascade for later stages of evolution

2.76 TeV, Pb+Pb at LHC



Gale et al, PRL 110, 012302 (2013)

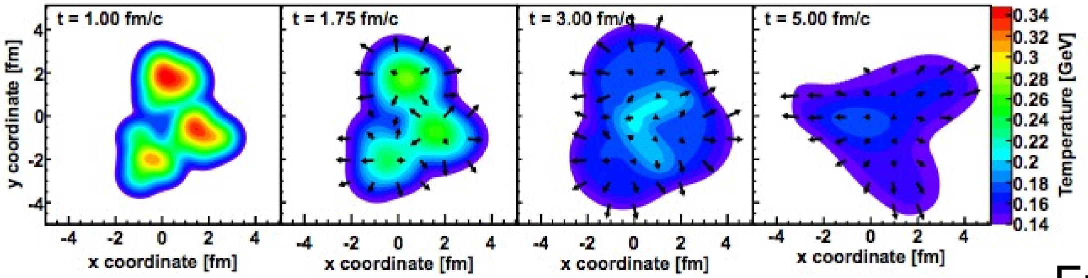
“Perfect” fluidity in “small” systems ??

p+A, d+A, 3He+A, ...

If hydro worked so well before, let us push the hell out of it to see what happens ...

Workshop at BNL last week:

3He+A



From Jamie Nagle's talk

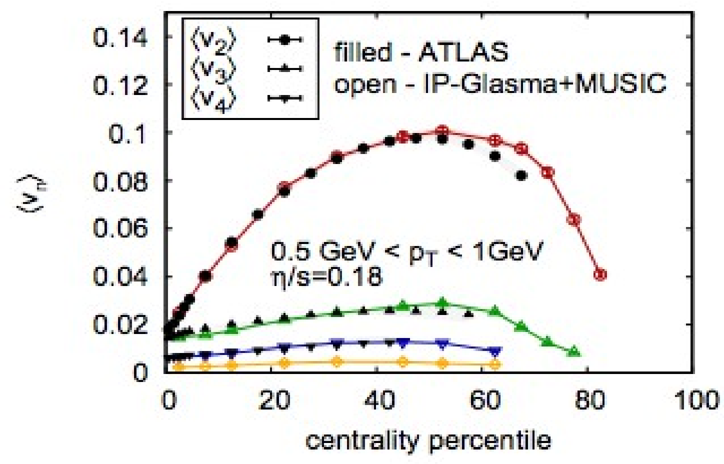
Nagle et al., Phys.Rev.Lett. 113 (2014)
11, 112301Â

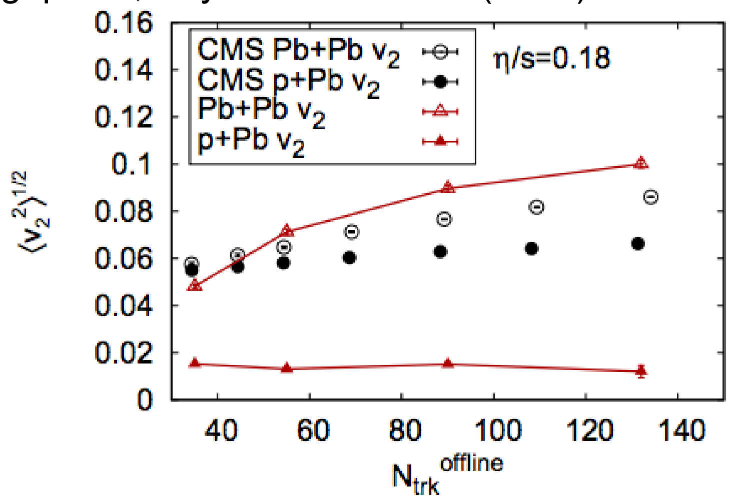
“Perfect” fluidity in “small” systems ??

There are some issues to work out in the hydro description of p+A ...

Pb+Pb

Schenke, Venugopalan, Phys.Rev.Lett. 113 (2014) 102301





Collectivity in small systems may be sensitive to several factors, e.g:

- the shape of the proton + other IS effects ...

- higher order transport coefficients (usually neglected also in Pb+Pb)

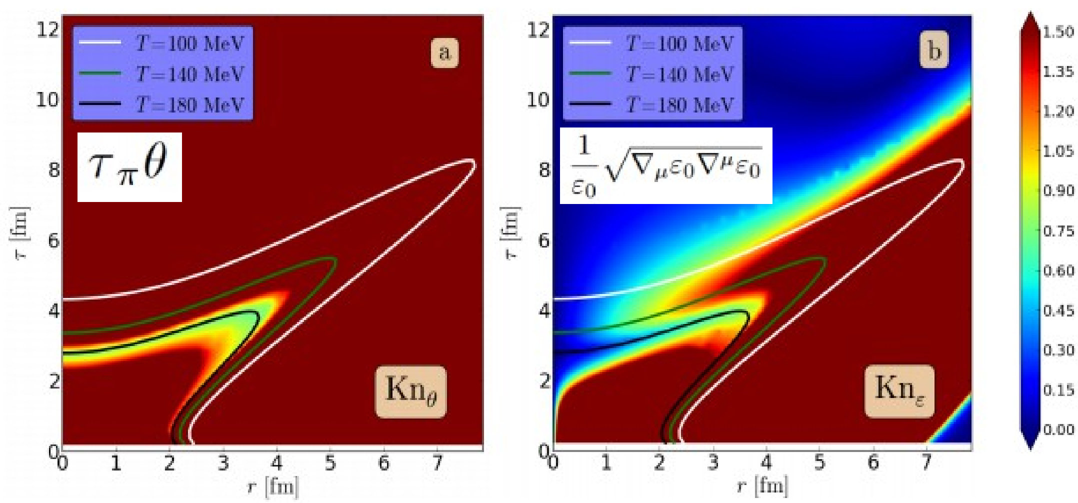
“Perfect” fluidity in “small” systems ??

There are some issues to work out in the hydro description of p+A ...

<u>Gabriel Denicol's talk last week:</u> **“pA” = Edge of validity of hydro ??**

Knudsen numbers – pA

Niemi and Denicol, arXiv:1404.7327



For a full η/s(T), hydrodynamics is basically out of its domain of validity

“Perfect” fluidity in “small” systems ??

- Clearly, the presence of collectivity in small systems brings “standard” hydro to its edge.

- Add to that the idea that the QGP is strongly interacting in this temperature regime and the plot thickens considerably.

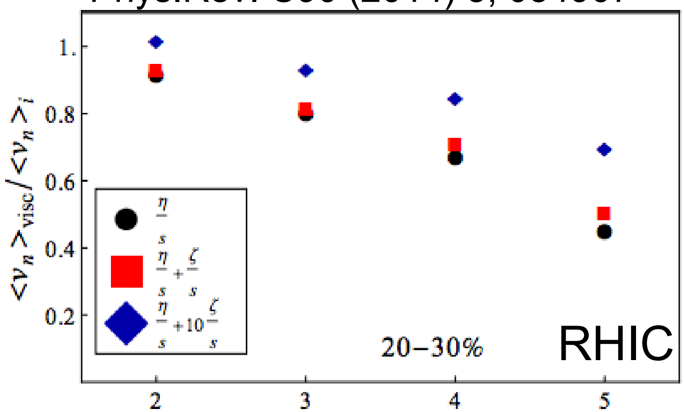
- Previous “knowledge” about the effect of T-dependent transport coefficients in such violently expanding systems must be revised.

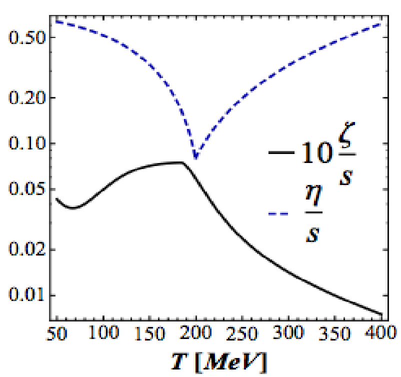
- In fact, even in large systems such as Au+Au or Pb+Pb, some simplifying model “assumptions” are simply not applicable ...

Fallacy: Bulk viscosity is too small or irrelevant in heavy ion collisions

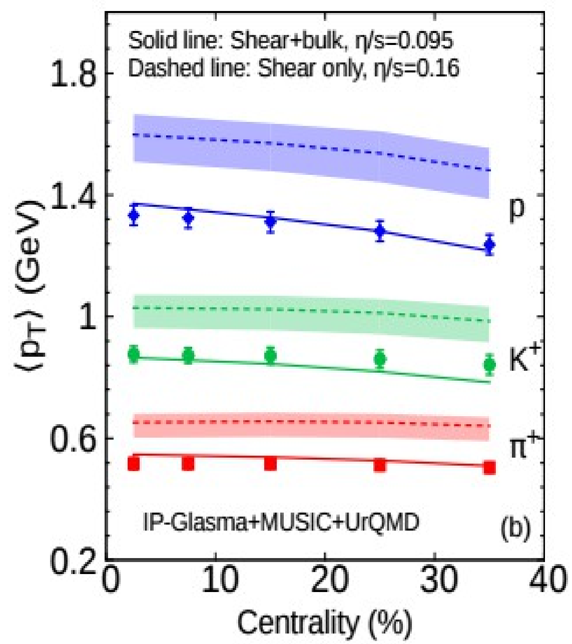
Even a smallish bulk viscosity can make a difference in large systems ...

Noronha-Hostler et al.,
Phys.Rev. C90 (2014) 3, 034907

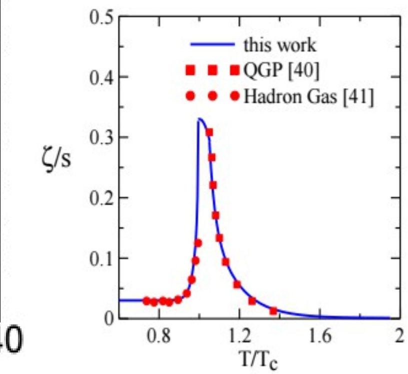




Ryu et al. (BNL+McGill), arXiv:1502.01675



LHC



- The large expansion rate in pA makes bulk viscosity effects even more important than in AA (Denicol's talk last week) !!!

- Hydrodynamics is a nonlinear theory with bulk and shear channels coupled both indirectly (via the flow) and directly in the hydro equations (more on that later).

- Thus, non-conformal effects in hydrodynamical calculations should be taken into account.

- However, there is a large theoretical uncertainty in the size of 1st order and 2nd order T-dependent hydro coefficients in the strongly coupled, non-conformal QGP.

In this talk, I will show how holography (aka gauge/gravity duality) can be used to determine the value of 13 transport coefficients in the QGP!!!

Non-conformal relativistic hydrodynamics at 2nd order in gradients

Energy-momentum conservation

$$\nabla_\mu T^{\mu\nu} = 0$$

(* zero chemical potential*)

$$T^{\mu\nu} = \varepsilon\, u^\mu u^\nu + P\Delta^{\mu\nu} + \pi^{\mu\nu} + \Delta^{\mu\nu}\Pi$$

Perfect fluid (inviscid) part

Dissipative part

$$\pi^{\mu\nu} = \Delta^{\mu\nu\alpha\beta}T_{\alpha\beta}$$

$$u_\mu \pi^{\mu\nu} = 0$$

Spatial projector

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

Flow velocity

$$u_\mu u^\mu = -1$$

How does one find $\pi^{\mu\nu}$ and Π ???

Non-conformal relativistic hydrodynamics at 2nd order in gradients

$$\theta = \nabla_\mu u^\mu$$

$$\sigma_{\mu\nu} = 2\Delta^{\alpha\beta}_{\mu\nu}\nabla_\alpha u_\beta$$

$$D = u^\mu \nabla_\mu$$

Energy-momentum conservation

$$D\varepsilon + (\varepsilon + P + \Pi)\theta + \frac{1}{2}\pi_{\mu\nu}\sigma^{\mu\nu} = 0$$

$$(\varepsilon + P + \Pi)Du^\mu + \nabla^\mu_\perp(P + \Pi) + \Delta^\mu_\nu \nabla_\alpha \pi^{\alpha\nu} = 0$$

To first order in gradients: $\pi^{\mu\nu} = -\eta\sigma^{\mu\nu}\,, \qquad \Pi = -\zeta\theta\,,$ Navier-Stokes

$$\eta, \zeta \geq 0.$$

- At 1st order, shear and bulk channels only couple to each other indirectly via flow.

- Relativistic Navier-Stokes theory is acausal and linearly unstable.

Relativistic Hydrodynamics as an effective Theory

Knudsen number/field: $$K_n \sim \frac{\ell_{micro}}{L_{macro}} \ll 1$$

For dilute gases

$$\partial u \,, \partial \varepsilon \sim 1/L_{macro} \qquad \ell_{micro} \sim \ell_{MFP} \sim \frac{1}{n\sigma}$$

Gradient expansion
(or Knudsen series expansion) →

Assumption:
Independent variables → $\varepsilon \,, u^{\mu}$

$$\pi^{\mu\nu} = \sum_{i=0}^{N} \lambda_i \, A_i^{\mu\nu}$$

$$A_i^{\mu\nu} \sim \partial^i \varepsilon \,, \partial^i u \sim (1/L_{macro})^i$$

$$\lambda_i \sim \ell_{micro}^i$$

(analogous argument for bulk)

Traceless, symmetric, transverse

Relativistic Hydrodynamics as an effective Theory

What about $\mathcal{O}(K_n^2)$???

Here things get a bit more complicated because the equations of motion

$\nabla_\mu T^{\mu\nu} = 0$ + leading order expression $\pi^{\mu\nu} \sim \eta\sigma^{\mu\nu}$ $\Longrightarrow$ $(\partial\varepsilon)^2 \sim K_n^3$

affect the Knudsen number power counting scheme.

- Still, 2nd order gradient expansion theories can be obtained, for instance, for dilute gases using the Chapman-Enskog method.

- In the case of conformal systems (where bulk viscosity is zero), Weyl invariance determines the general structure of the theory (BRSSS, JHEP 04 (2008) 100).

Non-conformal relativistic hydrodynamics at 2nd order in gradients

We use Romatschke's version of 2nd order gradient expansion for a non-conformal QGP in curved spacetime (Class. Quant. Grav. 27 (2010) 025006)

Shear channel:

$$\pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + \eta\tau_\pi\left(D\sigma^{\langle\mu\nu\rangle} + \frac{\theta}{3}\sigma^{\mu\nu}\right) + \kappa\left(\mathcal{R}^{\langle\mu\nu\rangle} - 2u_\alpha u_\beta \mathcal{R}^{\alpha\langle\mu\nu\rangle\beta}\right)$$
$$+\lambda_1\sigma^{\langle\mu}_\lambda\sigma^{\nu\rangle\lambda} + \lambda_2\sigma^{\langle\mu}_\lambda\Omega^{\nu\rangle\lambda} - \lambda_3\Omega^{\langle\mu}_\lambda\Omega^{\nu\rangle\lambda}$$
$$+2\kappa^* u_\alpha u_\beta\mathcal{R}^{\alpha\langle\mu\nu\rangle\beta} + \eta\tau_\pi^*\sigma^{\mu\nu}\frac{\theta}{3} + \lambda_4\nabla^{\langle\mu}\ln s\,\nabla^{\nu\rangle}\ln s\,,$$

Bulk channel:

$$\Pi = -\zeta\theta + \zeta\tau_\Pi D\theta + \xi_1\sigma_{\mu\nu}\sigma^{\mu\nu} + \xi_2\theta^2$$
$$+\xi_3\Omega_{\mu\nu}\Omega^{\mu\nu} + \xi_4\nabla^\perp_\mu\ln s\,\nabla^\mu_\perp\ln s + \xi_5\mathcal{R} + \xi_6 u^\mu u^\nu\mathcal{R}_{\mu\nu}\,,$$

17 temperature dependent different transport coefficients!!!!

Non-conformal relativistic hydrodynamics at 2nd order in gradients

Note the increase in the number of T-dependent coefficients:

- 0^{th} order (ideal fluid): $c_s^2 = dP/d\varepsilon$

- 1^{st} order (Navier-Stokes): $c_s^2 = dP/d\varepsilon$, η and ζ

- 2^{nd} order theory: $c_s^2 = dP/d\varepsilon$

(can be computed on the lattice)

$\kappa, \kappa^*, \lambda_3, \lambda_4, \xi_3, \xi_4, \xi_5, \xi_6$ ⟶ Determined via **Euclidean** 2 and 3-point functions

$\eta, \zeta, \tau_\pi, \tau_\pi^*, \tau_\Pi, \lambda_1, \lambda_2, \xi_1,$ and ξ_2 ⟶ Associated with dissipative properties (**imaginary parts** of retarded 2 and 3-point functions)

Note that $\kappa, \kappa^*, \xi_5,$ and ξ_6 do not contribute to EOM in flat spacetime.

Non-conformal relativistic hydrodynamics at 2nd order in gradients

- Even in flat spacetime, there are still 13 temperature dependent transport coefficients to be computed.

- At 2nd order, qualitatively new terms appear involving

Vorticity	Vorticity + shear coupling	Shear + bulk coupling terms	
$\lambda_3 \Omega^{\langle\mu}_{\ \lambda} \Omega^{\nu\rangle\lambda}$ $\xi_3 \Omega_{\mu\nu} \Omega^{\mu\nu}$	$\lambda_2 \sigma^{\langle\mu}_{\ \lambda} \Omega^{\nu\rangle\lambda}$	$\eta \tau_\pi^* \, \sigma^{\mu\nu} \, \dfrac{\theta}{3}$	$\xi_1 \sigma_{\mu\nu} \sigma^{\mu\nu}$

- Now, shear and bulk channels interact directly via EOM.

- Approximate conformal invariance at early stages is broken by time evolution.

Non-conformal relativistic hydrodynamics at 2nd order in gradients

- This 2nd order gradient expansion theory, however, is still **acausal and unstable.**

JHEP 1502 (2015) 051, arXiv:1412.2968

- Common "trick" (BRSSS) is to employ a type of Israel-Stewart-like resummation:

Use lowest order relations: $\sigma^{\mu\nu} \to -\pi^{\mu\nu}/\eta \text{ and } \theta \to -\Pi/\zeta$

To promote $\pi^{\mu\nu}$ and Π to independent dynamical variables that obey relaxation-like equations of motion.

Israel-Stewart-like, 2nd order hydrodynamic equations

This gives in this case (in flat spacetime): JHEP 1502 (2015) 051, arXiv:1412.2968

Shear channel:

$$\tau_\pi \left(D\pi^{\langle\mu\nu\rangle} + \frac{4\theta}{3}\pi^{\mu\nu} \right) + \pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + \frac{\lambda_1}{\eta^2}\pi_\lambda^{\langle\mu}\pi^{\nu\rangle\lambda} - \frac{\lambda_2}{\eta}\pi_\lambda^{\langle\mu}\Omega^{\nu\rangle\lambda} - \lambda_3\Omega_\lambda^{\langle\mu}\Omega^{\nu\rangle\lambda}$$
$$+\tau_\pi\,\pi^{\mu\nu}\,D\,\ln\left(\frac{\eta}{s}\right) + \tau_\pi^*\pi^{\mu\nu}\,\frac{\Pi}{3\zeta} + \lambda_4\nabla^{\langle\mu}\ln s\,\nabla^{\nu\rangle}\ln s$$

Bulk channel: $$\tau_\Pi\,(D\Pi + \Pi\theta) + \Pi = -\zeta\theta + \frac{\xi_1}{\eta^2}\pi_{\mu\nu}\pi^{\mu\nu} + \frac{\xi_2}{\zeta^2}\Pi^2 + \xi_3\Omega_{\mu\nu}\Omega^{\mu\nu}$$
$$+\tau_\Pi\,\Pi\,D\,\ln\left(\frac{\zeta}{s}\right) + \xi_4\nabla_\mu^\perp\ln s\,\nabla_\perp^\mu\ln s\,.$$

These equations can be **causal and linearly stable**. They can be implemented in current numerical hydro codes.

Israel-Stewart-like, 2nd order hydrodynamic equations

- These equations are similar to other sets of equations found via kinetic theory, taking into account a different power counting scheme.

Denicol et al., Phys.Rev. D85 (2012) 114047

- This type of "UV completion" preserves the number of transport coefficients found in the gradient expansion.

- Asymptotic behavior is guaranteed to coincide with 2nd order gradient expansion.

- Shear and bulk channels after resummation obey their own differential equations.

Vorticity	Vorticity + shear coupling	**Shear + bulk coupling terms**	
$\lambda_3 \Omega^{\langle\mu}_{\lambda}\Omega^{\nu\rangle\lambda}$ $\xi_3 \Omega_{\mu\nu}\Omega^{\mu\nu}$	$\frac{\lambda_2}{\eta}\pi^{\langle\mu}_{\lambda}\Omega^{\nu\rangle\lambda}$	$\tau^*_\pi \pi^{\mu\nu}\frac{\Pi}{3\zeta}$	$\frac{\xi_1}{\eta^2}\pi_{\mu\nu}\pi^{\mu\nu}$

- Clearly, this particular UV completion (which leads to relaxation equations for the dissipative currents) is not unique.

Ex: $\ddot{x} + \gamma \dot{x} + x = f(t)$ and $\gamma \dot{x} + x = f(t)$ $\xrightarrow{\text{same}}$ $x_{asymp}(t) \sim f(t)$ + ...

- Some of the these transport coefficients have been computed by Denicol et al. in kinetic theory for a dilute gas with a constant cross section.

Phys.Rev. D 85 (2012) 114047, Phys.Rev. D 89 (2014) 7, 074010

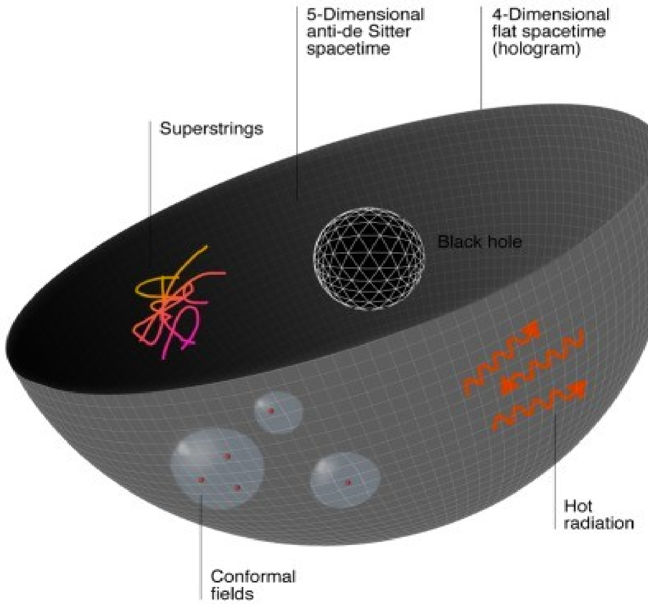
- **In the rest of the talk I will sketch the calculation of these non-conformal transport coefficients in holography.**

- Details of the calculations can be found in JHEP 1502 (2015) 051, arXiv:1412.2968

- Parametrizations for the temperature dependence of all the 2nd order transport coefficients computed in this work can be found in this paper.

Holography (gauge/string duality)

Maldacena 1997; Witten 1998; Gubser, Polyakov, Klebanov 1998



Holography in 2010's

Holography in 1970's

Strong coupling limit of large N QFT
in 4 dimensions

↕

Classical gravity in in 5 dimensions

Holography becomes reliable (or under control) when:

I) The coupling of the QFT, say, λ , is $\lambda \gg 1$

II) The number of d.o.f./volume, N, is very large, i.e., N >> 1.

$\lambda \gg 1$ 4-dimensional QFT with d.of./vol $\to \infty$	$\Rightarrow$	**HOLOGRAPHY**	$\Rightarrow$	$\sim 1/\lambda \ll 1$ 5-dimensional classical gravity

- In this talk I will **assume** holography is a property shared among strongly coupled QFT's at large N and quantum gravity.

- This is the **modern understanding** of the duality and it motivates its applications in different systems ranging from particle physics to condensed matter physics.

Mathematical Definition of the Holographic Duality

$$\boxed{Z_{\mathrm{QFT}}[J_i] \;=\; Z_{\mathrm{QG}}[\Phi[J_i]]}$$

(properly renormalized)

$Z_{\mathrm{QFT}}[J_i]$ Partition function of the QFT as a function of the sources

$Z_{\mathrm{QG}}[\Phi[J_i]]$ Partition function of the gravitational theory in AdS

The bulk fields play the role of the coupling constants of the QFT that are now promoted to dynamical fields on the higher dimensional spacetime where the extra dimension is the RG scale.

- What about asymptotically free gauge theories such as QCD?

- Large N is important to avoid quantum loops in the gravity description (fine).

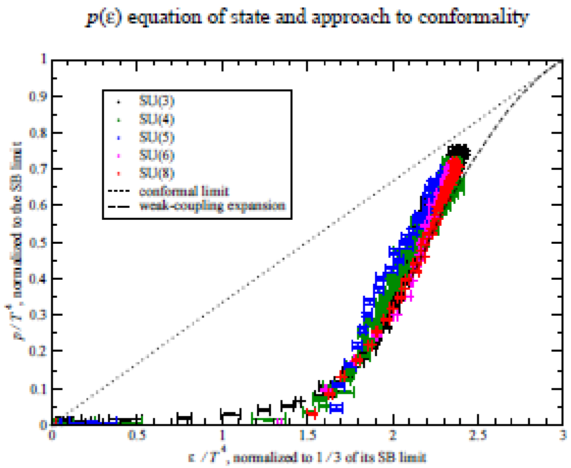
- Strong coupling is of fundamental importance to make sure that single trace operators that are not related to conserved global symmetries have large anomalous dimensions.

- One still needs to show that this universal reasoning is applicable to confining theories such as the strong coupling limit of large N QCD.

- **I believe that it is, at least when it comes to its hydrodynamic behavior. Still working on it (ask me about this later this year) ...**

Effective holographic theory for strongly coupled thermal QCD

Pure glue lattice data indicates that Nc = 3 >> 1



M. Panero, arXiv:0907.3719 [hep-lat]

- Minimal set of fields in 5d bulk holography that describe the basics of a non-conformal, strongly interacting plasma:

Metric plus a dynamical scalar field !!!

Gubser et al. 2008
Kiritsis et al, 2008
Noronha, 2009

$$S_{\mathrm{ES}}^{(\mathrm{bulk})} = \frac{1}{16\pi G_5}\int_{\mathcal{M}_5} d^5x\sqrt{-g}\left[R - \frac{(\partial_M\Phi)^2}{2} - V(\Phi)\right]$$

Φ is the scalar field and $V(\Phi)$ is the scalar potential

- Start with a nontrivial UV fixed point – strongly interacting CFT.

- Add the deformation due to a relevant scalar operator, which gives a nontrivial IR behavior for the theory.

- This generic model has been used in many instances to understand generic non-conformal behavior at strong coupling from holography.

- The scalar potential is an **input** of the theory (ex nusquam)

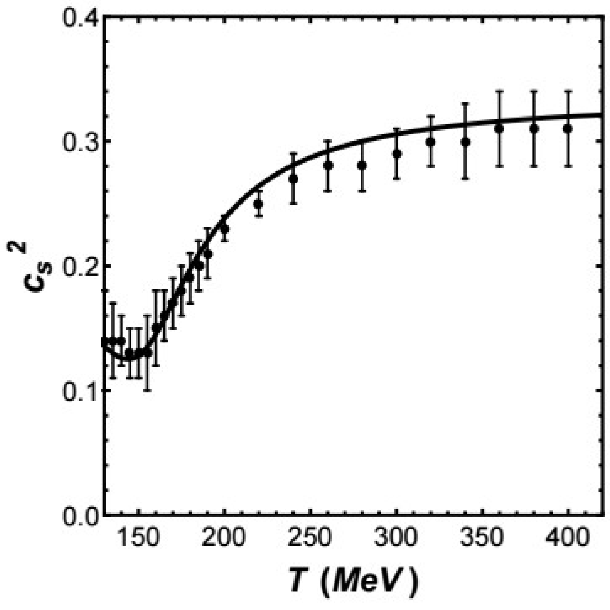
$$V(\Phi) = \frac{-12\cosh\gamma\Phi + b_2\Phi^2 + b_4\Phi^4 + b_6\Phi^6}{L^2}$$

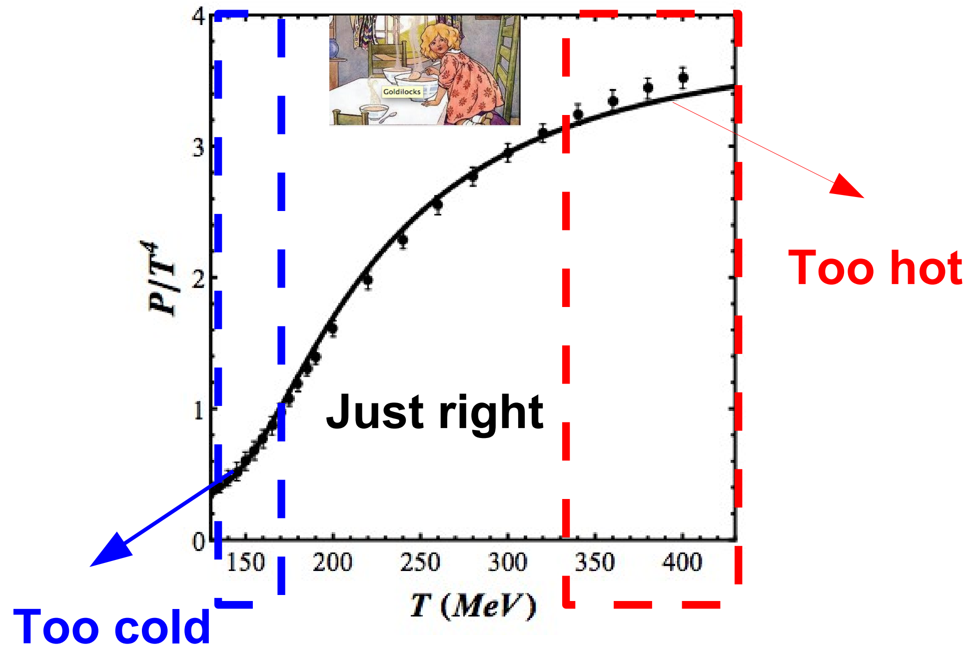
$$\gamma = 0.606,\ b_2 = 0.703,\ b_4 = -0.1,\ b_6 = 0.0034$$

and it is completely fixed by requiring that the model fits current lattice QCD data.

Holographic description of QGP thermodynamics

Lattice data from Borsanyi et al, JHEP 08 (2012) 053.





5d bulk metric
(Gubser gauge)

$$ds^2 = e^{2A(\Phi)}\left(-h(\Phi)dt^2 + dx_i^2\right) + e^{2B(\Phi)}\frac{d\Phi^2}{h(\Phi)}$$

- One needs to solve Einstein's equations coupled to the dynamical scalar to find the black brane background corresponding to the equilibrium plasma.

- Temperature dependent transport coefficients are then predictions of the model.

- Clearly, other forms for the scalar potential will lead to black branes that can also describe lattice QCD thermodynamics equally well.

- However, we have checked that once the speed of sound is fixed (i.e., the EOS) the transport coefficients do not change.

- In other words, the transport coefficients are completely defined by the equilibrium properties of the black brane + holography.

- In this sense, our results for the transport coefficients provide the **answer for a holographic non-conformal plasma with equilibrium properties similar to QCD**.

Shear viscosity

Due to the universality of isotropic black brane horizons (KSS PRL 2004), this non-conformal model has

$$\boxed{\eta/s = 1/(4\pi)}$$

Kubo formula

$$\eta = -\lim_{q\to 0}\lim_{\omega\to 0} \mathrm{Im}\left[\frac{\partial G_R^{xy,xy}(\omega,q)}{\partial\omega}\right]$$

$$G_R^{xy,xy}(\omega,\vec{q}) = -i\int_{\mathbb{R}^{1,3}} d^4x\, e^{i(\omega t-\vec{q}\cdot\vec{x})}\,\theta(t)\langle[\hat{T}^{xy}(t,\vec{x}),\hat{T}^{xy}(0,\vec{0})]\rangle$$

- This is in the ballpark of the values considered in hydro simulations.

- Note, however, that his clearly fails away from "the Goldilocks temperature zone"

Bulk viscosity

The Kubo formula is $$\zeta = -\frac{4}{9}\lim_{\omega\to 0}\frac{1}{\omega}\,\mathrm{Im}\left[G_R(\omega,\vec{q}=\vec{0})\right]$$

Defined using the retarded correlator

$$G_R(\omega,\vec{q}) \equiv -i\int_{\mathbb{R}^{1,3}} d^4x\, e^{i(\omega t-\vec{q}\cdot\vec{x})}\theta(t)\left\langle\left[\frac{1}{2}T_a^a(t,\vec{x}),\frac{1}{2}T_b^b(0,\vec{0})\right]\right\rangle$$

This is computed holographically by considering fluctuations of spatial components of the metric $\psi \equiv h_x^x = e^{-2A(\phi)}h_{xx}$ with infalling boundary conditions at the horizon

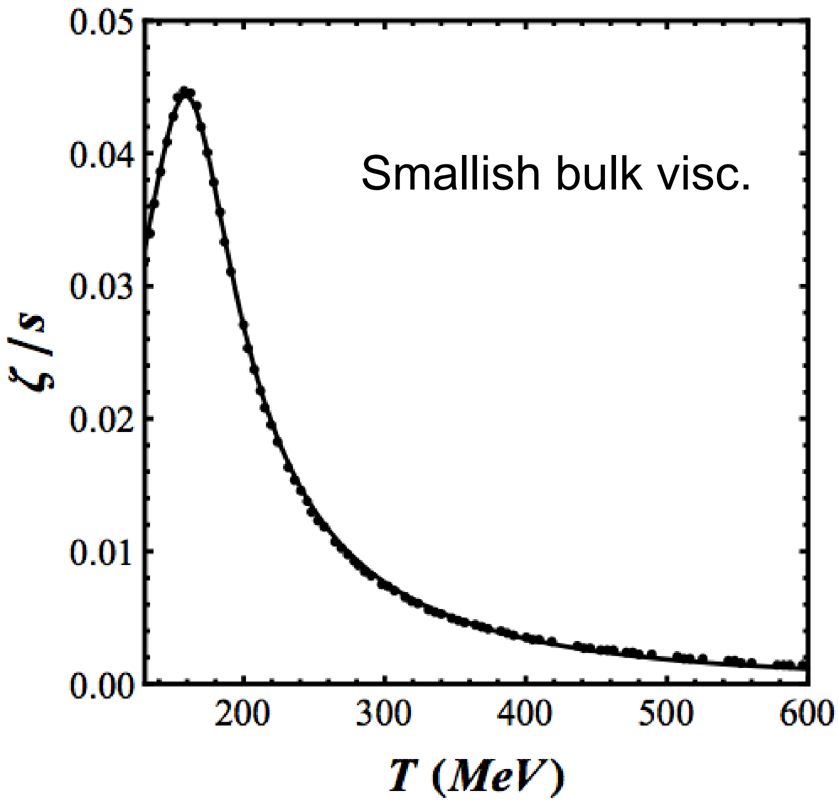
$$\psi'' + \left(\frac{1}{3A'} + 4A' - 3B' + \frac{h'}{h}\right)\psi' + \left(\frac{e^{-2A+2B}}{h^2}\omega^2 - \frac{h'}{6hA'} + \frac{h'B'}{h}\right)\psi = 0,$$

Bulk viscosity

Infalling boundary conditions: $\psi(\phi \to \phi_H) \approx C e^{i\omega t} |\phi - \phi_H|^{-\frac{i\omega}{4\pi T}}$

General formula

$$\frac{\zeta}{s} = \frac{\eta}{s} |C|^2 \frac{V'(\phi_H)^2}{V(\phi_H)^2}$$



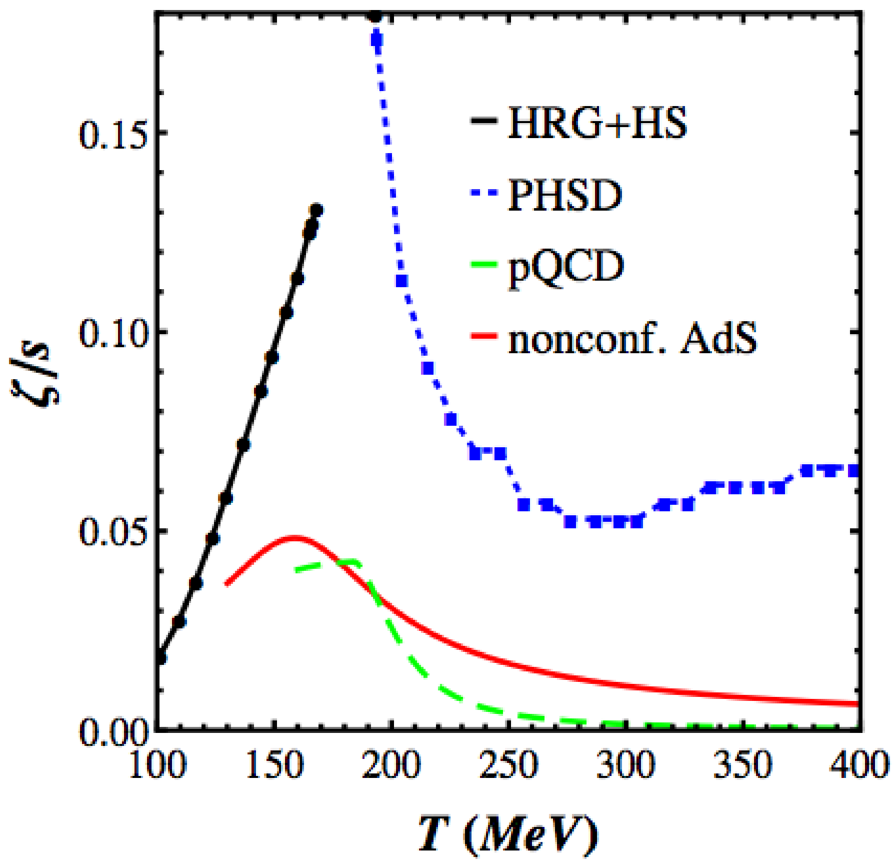
Parametrization for hydro

$$\frac{\zeta}{s}\left(x = \frac{T}{T_c}\right) = \frac{a}{\sqrt{(x-b)^2 + c^2}} + \frac{d}{x^2 + e^2}$$

$T_c = 143.8$ MeV

a	b	c	d	e
0.01162	1.104	0.2387	-0.1081	4.870

Bulk viscosity

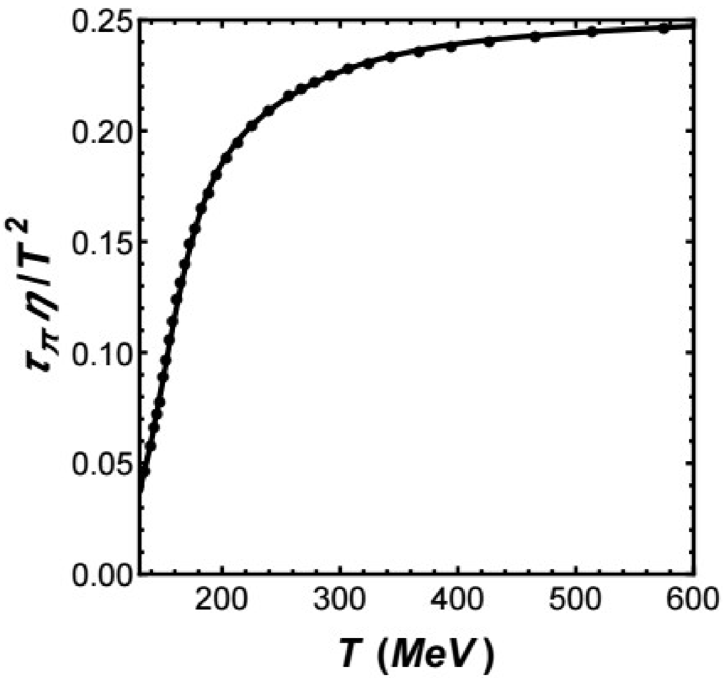


The bulk viscosity from holography seems to give the smallest estimate for this transport coefficient of the QGP.

Results for the 2nd order transport coefficients:

Shear relaxation time

In our paper JHEP 1502 (2015) 051, arXiv:1412.2968 we have shown in detail how to compute the shear relaxation time (a general method was given)



$$\tau_\pi = \frac{1}{2\eta}\left(\lim_{q\to 0}\lim_{\omega\to 0}\frac{\partial^2 G_R^{xy,xy}(\omega,q)}{\partial\omega^2} - \kappa + T\frac{d\kappa}{dT}\right)$$

Parametrization for hydro

$$\tau_\pi \eta / T^2\left(x = \frac{T}{T_c}\right) = \frac{a}{1 + e^{b(c-x)} + e^{d(e-x)} + e^{f(g-x)}}$$

$T_c = 143.8$ MeV

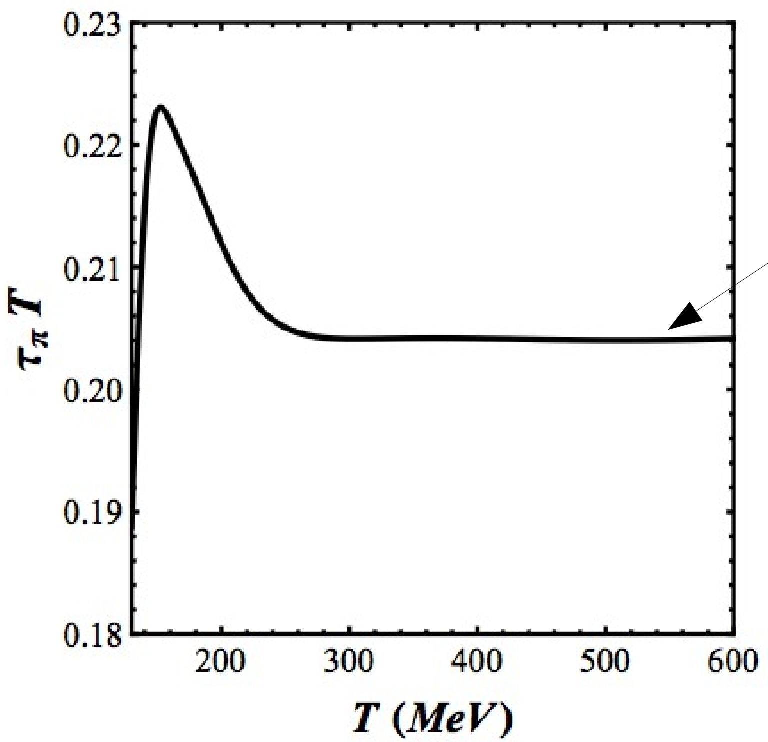
a	b	c	d	e	f	g
0.2664	2.029	0.7413	0.1717	-10.76	9.763	1.074

Results for the 2nd order transport coefficients:

Shear relaxation time

- Shear relaxation time has a small peak in the region T ~ 150 – 250 MeV

Very different than kinetic theory calculations!



CFT (SYM) value
BRSSS (2008)

Results for the 2nd order transport coefficients:

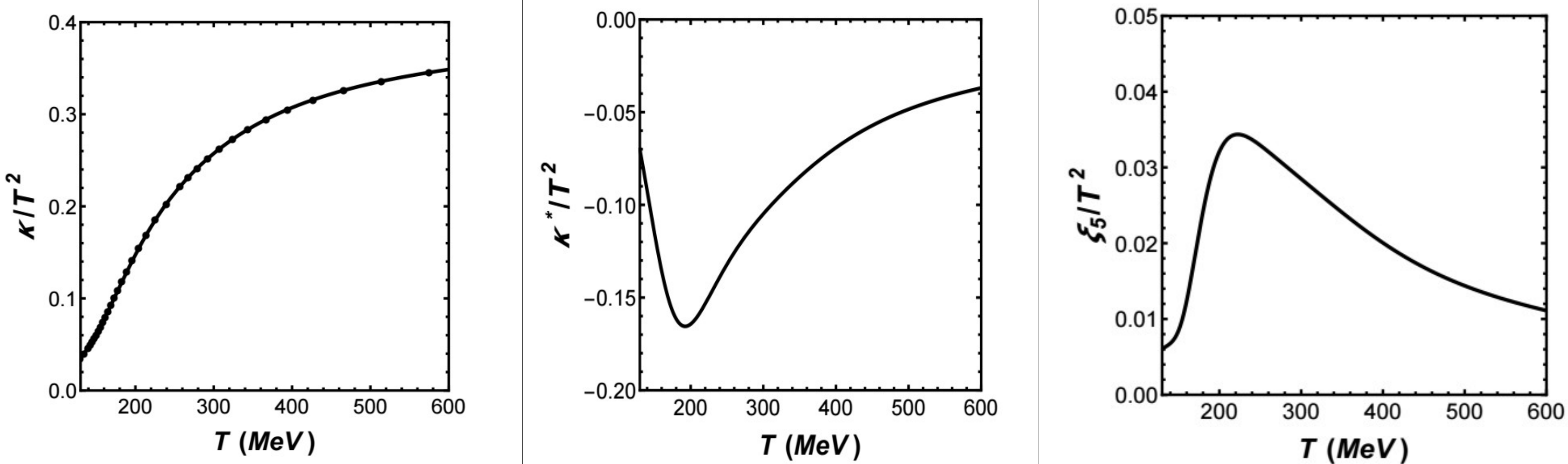
Transport coefficients for curved spacetime

PREDICTION THAT CAN BE TESTED ON THE LATTICE

$$\kappa = -\lim_{q\to 0}\lim_{\omega\to 0}\frac{\partial^2 G_R^{xy,xy}(\omega,q)}{\partial q^2}$$

$$\kappa^* = \kappa - \frac{T}{2}\frac{d\kappa}{dT}$$

$$\xi_5 = \frac{1}{2}\left(c_s^2 T\frac{d\kappa}{dT} - c_s^2\kappa - \frac{\kappa}{3}\right)$$



Results for the 2nd order transport coefficients:

coefficients λ_3 and λ_4

These coefficients are associated with the shear channel in the EOM

$$\boxed{\lambda_3 \Omega^{\langle\mu}_{\lambda} \Omega^{\nu\rangle\lambda}}$$

Require calculation of Euclidean 3-point function !!!!

(could be computed on the lattice)

$$\lambda_3 = 2\kappa^* - 4 \lim_{p_z, q_z \to 0} \frac{\partial^2}{\partial p_z \partial q_z} G_E^{xt,yt,xy}(p_t = 0, \vec{p}, q_t = 0, \vec{q})$$

$$\boxed{\lambda_4 \nabla^{\langle\mu} \ln s \, \nabla^{\nu\rangle} \ln s}$$

$$\lambda_4 = -2\kappa^* + \kappa - \frac{c_s^4}{2} \lim_{p_x, q_y \to 0} \frac{\partial^2}{\partial p_x \partial q_y} G_E^{tt,tt,xy}(p_t = 0, \vec{p}, q_t = 0, \vec{q})$$

Results for the 2nd order transport coefficients:

coefficients λ_3 and λ_4

- 3-point functions in non-conformal holography is a formidable task (work in progress).

- Our best estimate for these coefficients here consists in taking the CFT value for the 3-point functions while fully taking into account non-conformal effects in the other terms of the Kubo formulas that define them.

$$\lim_{p_z,q_z\to 0}\frac{\partial^2}{\partial p_z \partial q_z}G_E^{xt,yt,xy}(p_t=0,\vec{p},q_t=0,\vec{q})=0.$$

$$\lim_{p_x,q_y\to 0}\frac{\partial^2}{\partial p_x \partial q_y}G_E^{tt,tt,xy}(p_t=0,\vec{p},q_t=0,\vec{q})=\frac{2\kappa}{c_s^4}$$

$$\boxed{\lambda_3=-\lambda_4=2\kappa^*}$$

Vorticity+shear coupling is not small !!!

Results for the 2nd order transport coefficients:

coefficients ξ_3, ξ_4

These coefficients are associated with the bulk channel in the EOM

G. D. Moore and K. A. Sohrabi, JHEP **11** (2012) 148

$$\boxed{\xi_3 \Omega_{\mu\nu}\Omega^{\mu\nu}}$$

(could be computed on the lattice)

$$\xi_3 = \frac{3c_s^2}{2}T\left(\frac{d\kappa^*}{dT} - \frac{d\kappa}{dT}\right) + \frac{3}{2}(c_s^2 - 1)\left(\kappa^* - \kappa\right) - \frac{\lambda_4}{c_s^2} + \frac{1}{4}\left(c_s^2 T\frac{d\lambda_3}{dT} - 3c_s^2\lambda_3 + \frac{\lambda_3}{3}\right)$$

$$\boxed{\xi_4 \nabla_\mu^\perp \ln s\, \nabla_\perp^\mu \ln s}$$

(could be computed on the lattice)

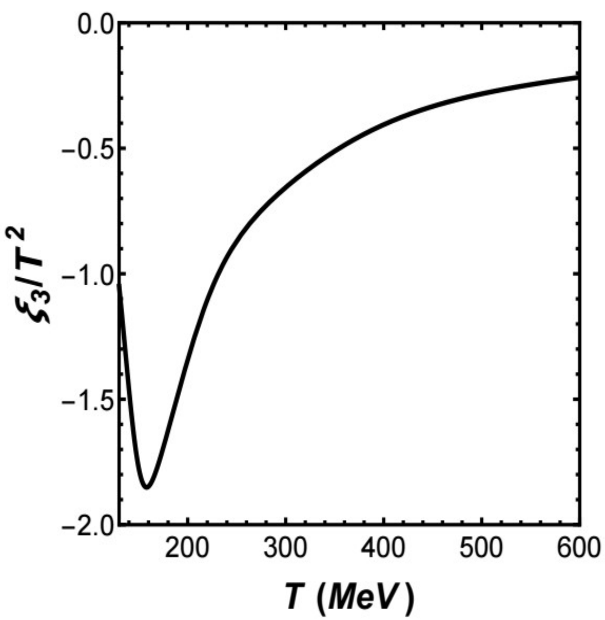
$$\xi_4 = -\frac{\lambda_4}{6} - \frac{c_s^2}{2}\left(\lambda_4 + T\frac{d\lambda_4}{dT}\right) + c_s^4(1-3c_s^2)\left(T\frac{d\kappa}{dT} - T\frac{d\kappa^*}{dT} + \kappa^* - \kappa\right) + \\ - c_s^6 T^3 \frac{d^2}{dT^2}\left(\frac{\kappa - \kappa^*}{T}\right),$$

Results for the 2nd order transport coefficients:

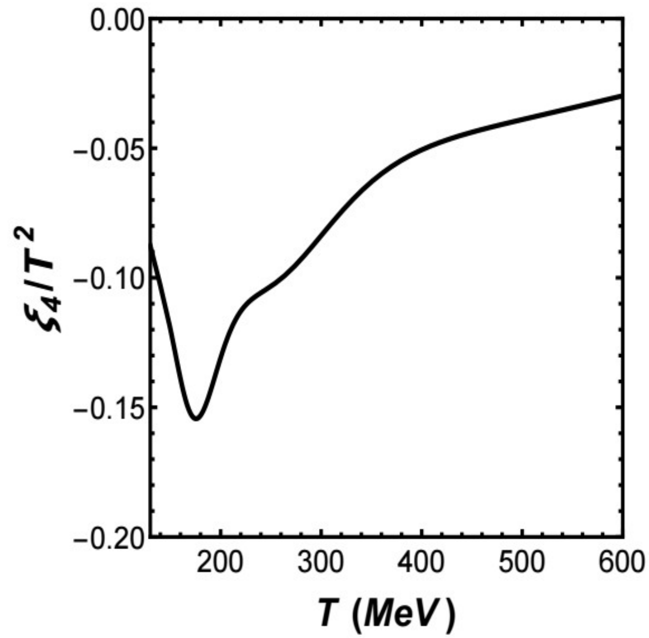
coefficients ξ_3, ξ_4

$$\xi_3 \Omega_{\mu\nu} \Omega^{\mu\nu}$$

$$\xi_4 \nabla^{\perp}_{\mu} \ln s \, \nabla^{\mu}_{\perp} \ln s$$



Vorticity effect: bulk channel



2nd order T gradient: bulk channel

Results for the 2nd order transport coefficients:

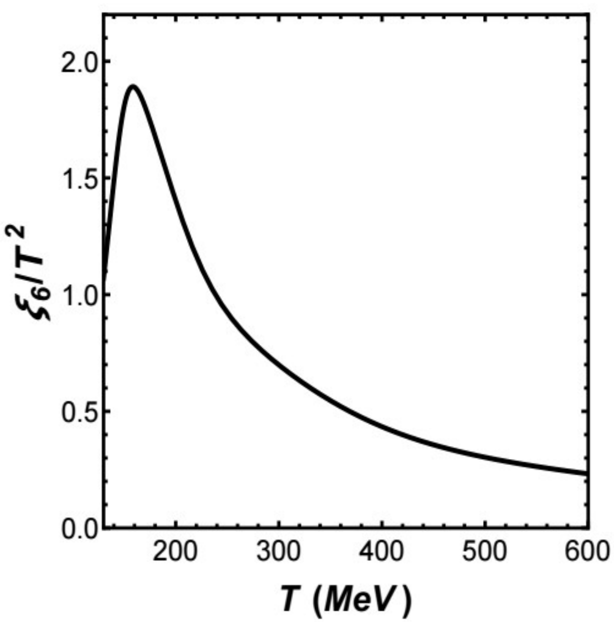
coefficient ξ_6

This coefficient is also associated with the bulk channel in the EOM

G. D. Moore and K. A. Sohrabi, JHEP **11** (2012) 148

$\boxed{\xi_6 u^\mu u^\nu \mathcal{R}_{\mu\nu}}$ Vanishes from EOM in flat space

(could be computed on the lattice)



$$\xi_6 = c_s^2\left(3T\frac{d\kappa}{dT} - 2T\frac{d\kappa^*}{dT} + 2\kappa^* - 3\kappa\right) - \kappa + \frac{4\kappa^*}{3} + \frac{\lambda_4}{c_s^2}$$

Note that ξ_3, ξ_4, ξ_6 are not small.

Vorticity+bulk coupling may be important !!!

Lattice QCD will play a role here !!!

Results for the 2nd order transport coefficients:

A lower bound estimate for τ_Π

While the calculation of the bulk relaxation time can be done using a method similar to the one we developed for the shear relaxation time, here we use:

Causality/linear stability constraint (valid for any Israel-Stewart-like theory)

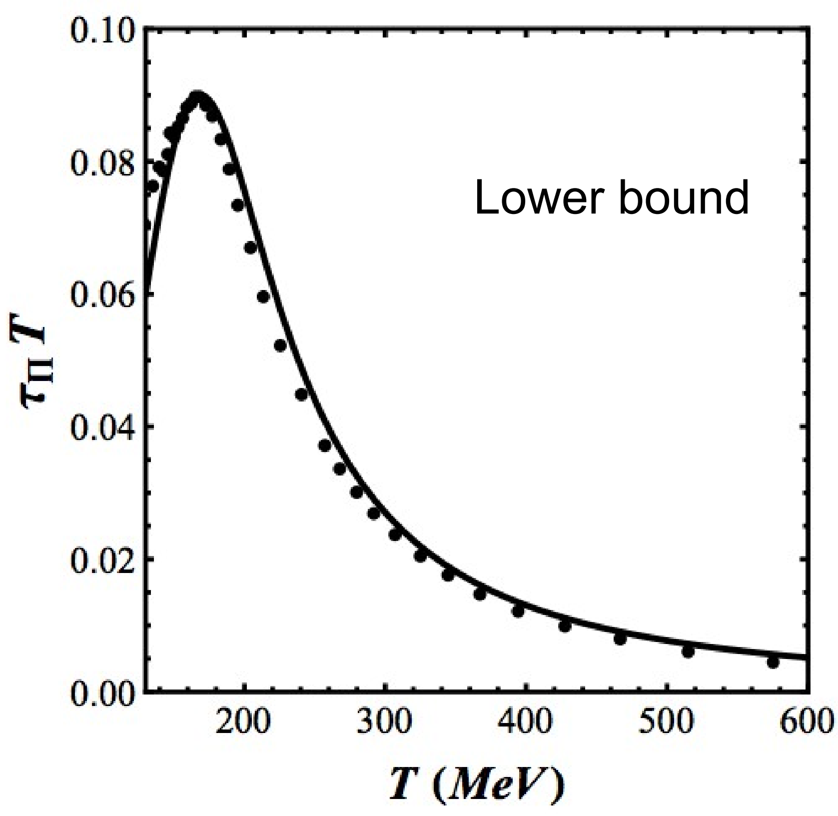
S. Pu, T. Koide and D. H. Rischke, Phys. Rev. D **81**, 114039 (2010)

$$\boxed{\frac{\zeta}{s\tau_\Pi T} + \frac{\eta}{s\tau_\pi T} \leq 1 - c_s^2}$$

This gives the smallest bulk relaxation time that this system can have.

Results for the 2nd order transport coefficients:

A lower bound estimate for τ_Π



Parametrization for hydro

$$\tau_\Pi T\left(x=\frac{T}{T_c}\right)=\frac{a}{\sqrt{(x-b)^2+c^2}}+\frac{d}{x}$$

$T_c = 143.8$ MeV

a	b	c	d
0.05298	1.131	0.3958	-0.05060

Results for the 2nd order transport coefficients:

$$\lambda_1,\ \lambda_2,\ \xi_1,\ \xi_2,\ \text{and}\ \tau_\pi^*$$

- Very little is known about these coefficients ...

For strongly coupled SYM theory:
$$\lambda_1 = 2\frac{\eta^2}{sT}, \qquad \lambda_2 = -\ln 2\frac{\eta}{\pi T}$$

$$4\lambda_1 + \lambda_2 = 2\eta\tau_\pi$$

For the non-conformal plasma constructed via dimensional reduction of a higher dimensional pure gravity action one finds (this is OK for our model)

I. Kanitscheider and K. Skenderis, JHEP **0904**, 062 (2009)

Shear + bulk coupling

$$\xi_1 = \lambda_1\left(\frac{1}{3} - c_s^2\right) \qquad \tau_\pi^* = -3\tau_\pi\left(\frac{1}{3} - c_s^2\right) \qquad \xi_2 = 2\eta\tau_\pi c_s^2\left(\frac{1}{3} - c_s^2\right)$$

Conclusions and Outlook

- The observed collectivity in small systems (e.g. high multiplicity pA collisions) provides a nice way to test the limit of applicability of relativistic hydrodynamic modeling of the QGP's spacetime evolution.

- The rapidly evolving systems in heavy ion collisions may be sensitive to higher order derivative corrections - 2nd order hydro coefficients may be relevant.

- We have computed (or given our best theoretical estimate for) the non-conformal effects on 2nd order transport coefficients of a strongly coupled plasma that has equilibrium properties similar to the QGP determined on the lattice.

- The holographic correspondence allowed for the calculation of 13 transport coefficients of the non-conformal plasma. Parametrization of the T-dependence of these coefficients was given in a way that could be readily used in hydro codes.

Conclusions and Outlook

- The size of the transport coefficients that describe the effects of vorticity in the shear and bulk channels was found to be comparable to that of the other coefficients.

- Our calculations indicate that the direct coupling of bulk of shear viscosity, which necessarily appears at 2nd order in gradients, can be relevant in a strongly coupled plasma.

- A full implementation of the transport coefficients discussed here in numerical hydro is essential to assess their phenomenological relevance in heavy ion collisions.

- For instance, one could compare their effects in large (AA) x small systems (pA).

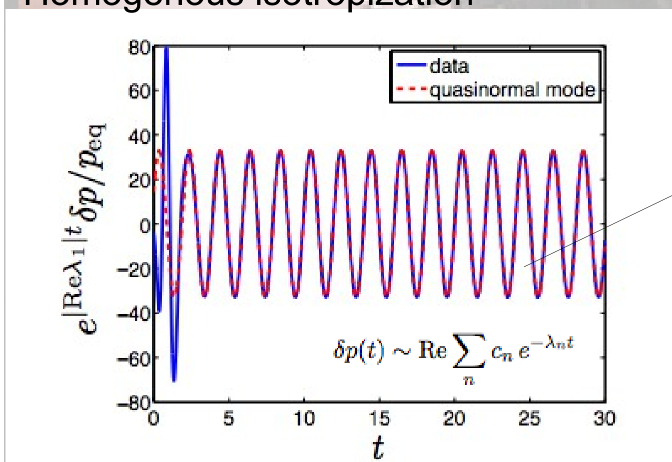
BACKUP SLIDES

- Let us assume that holography provides a good description of the hydrodynamic behavior of a strongly coupled QGP.

- If that is true, then the tower of non-hydrodynamical Quasinormal Modes (QNM) found in holography will set the near equilibrium phenomena.

- However, as shown in our paper Phys.Rev. D83 (2011) 074019 and later by Janik et al. Phys.Rev.Lett. 113 (2014) 26, 261601, holography implies that near equilibrium phenomena involves QNM ringing (e.g., Chesler-Yaffe JHEP 1407 (2014) 086)

Homogenous isotropization



This oscillatory behavior can never be described by Israel-Stewart-like theories ...

Early times ~ rapid expansion = pA

QNM ringing in pA ????

Holography is a duality between QFT and gravity

Maps quantum many body physics to classical dynamics of black hole horizons in one higher dimension

Replaces quasiparticles with geometry as the effective d.o.f.

When QFT is strongly coupled, new weakly coupled d.o.f. in the gravity theory emerge.

Emergent fields in the theory of gravity live in a dynamical spacetime with an extra dimension.

This extra dimension plays the role of an energy scale in the QFT with the motion along the extra dimension providing a geometric representation of the QFT's renormalization group (RG) flow.

Quantum many body physics problems, such as thermodynamics and transport phenomena, become equivalent to problems in classical gravity.

Universal black hole phenomena are mapped into universal behavior in QFT's

HOLOGRAPHY

Holographic dictionary

Rosetta stone

Witten, GPK, 1997

Boundary QFT				Bulk Gravity
Operator	$\mathcal{O}(x)$	$\longleftrightarrow$	$\Phi(x,r)$	Field
Spin	$s_{\mathcal{O}}$	$\longleftrightarrow$	s_Φ	Spin
Global Charge	$q_{\mathcal{O}}$	$\longleftrightarrow$	q_Φ	Gauge Charge
Scaling dimension	$\Delta_{\mathcal{O}}$	$\longleftrightarrow$	m_Φ	Mass
Source	$J(x)$	$\longleftrightarrow$	$\Phi(x,r)\|_\partial$	Boundary Value (B.V.)
Expectation Value	$\langle\mathcal{O}(x)\rangle$	$\longleftrightarrow$	$\Pi_\Phi(x,r)\|_\partial$	B.V. of Radial Momentum
Global Symmetry Group	G	$\longleftrightarrow$	G	Gauge Symmetry Group
Source for Global Current	$\mathcal{A}_\mu(x)$	$\longleftrightarrow$	$A_\mu(x,r)\|_\partial$	B.V. of Gauge Field
Expectation of Current	$\langle\mathcal{J}^\mu(x)\rangle$	$\longleftrightarrow$	$\Pi^\mu_A(x,r)\|_\partial$	B.V. of Momentum
Stress Tensor	$T^{\mu\nu}(x)$	$\longleftrightarrow$	$g_{\mu\nu}(x,r)$	Spacetime Metric
Source for Stress-Energy	$h_{\mu\nu}(x)$	$\longleftrightarrow$	$g_{\mu\nu}(x,r)\|_\partial$	B.V. of Metric
Expected Stress-Energy	$\langle T^{\mu\nu}(x)\rangle$	$\longleftrightarrow$	$\Pi^{\mu\nu}_g(x,r)\|_\partial$	B.V. of Momentum
# of Degrees of Freedom Per Spacetime Point	N^2	$\longleftrightarrow$	$\left(\frac{L}{\ell_P}\right)^{d-1}$	Radius of Curvature In Planck Units
Characteristic Strength of Interactions	λ	$\longleftrightarrow$	$\left(\frac{L}{\ell_s}\right)^d$	Radius of Curvature In String Units
QFT Partition Function with Sources $J_i(x)$	$Z_{\mathrm{QFT}_d}[J_i]$	$\longleftrightarrow$	$Z_{\mathrm{QG}_{d+1}}[\Phi_i[J_i]]$	QG Partition Function in AdS w/ $\Phi_i\|_\partial = J_i$
QFT Partition Function at Strong Coupling	$Z^{\lambda,N\gg 1}_{\mathrm{QFT}_d}[J_i]$	$\longleftrightarrow$	$e^{-I_{\mathrm{GR}_{d+1}}[\Phi[J_i]]}$	Classical GR Action in AdS w/ $\Phi_i\|_\partial = J_i$
QFT n-Point Functions at Strong Coupling	$\langle\mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\rangle$	$\longleftrightarrow$	$\left.\frac{\delta^n I_{\mathrm{GR}_{d+1}}[\Phi[J_i]]}{\delta J_1(x_1)\ldots\delta J_n(x_n)}\right\|_{J_i=0}$	Classical Derivatives of the On-Shell Classical Gravitational Action
Thermodynamic State		$\longleftrightarrow$		Black Hole
Temperature	T	$\longleftrightarrow$	T_H	Hawking Temperature $\sim$ Mass
Chemical Potential	μ	$\longleftrightarrow$	Q	Charge of Black Hole
Free Energy	F	$\longleftrightarrow$	$I_{\mathrm{GR}}\|_{(\text{on-shell})}$	On-Shell Bulk Action
Entropy	S	$\longleftrightarrow$	A_H	Area of Horizon

- Holography as a geometrization of the RG group flow of strongly coupled, large N theories has sparked a lot interest.

Polchinski, Liu, Rangamani, and many others

- In fact, recently Behr, Kuperstein and Mukhopadhyay, arXiv:1502.06619 [hep-th] argued that 4d large N, strongly coupled CFT's that undergo "highly efficient RG flows" (under very general conditions), **must have** a 5d classical gravity description.

$$\frac{\partial t^{\mu}{}_{\nu}(\Lambda)}{\partial \Lambda} = \frac{1}{\Lambda^3}\cdot\frac{1}{2}\Box t^{\mu}{}_{\nu}(\Lambda) - \frac{1}{\Lambda^5}\cdot\left(\frac{1}{4}\,\delta^{\mu}{}_{\nu}t^{\alpha}{}_{\beta}(\Lambda)t^{\beta}{}_{\alpha}(\Lambda) - \frac{7}{128}\Box^2 t^{\mu}{}_{\nu}(\Lambda)\right) - \\ +\frac{1}{\Lambda^5}\log\Lambda\cdot\frac{1}{32}\cdot\Box^2 t^{\mu}{}_{\nu}(\Lambda) + \mathcal{O}\left(\frac{1}{\Lambda^7}\log\Lambda\right).$$

$$\underline{\nabla_{(\Lambda)\mu}t^{\mu}{}_{\nu}(\Lambda) = 0}$$

$$g_{\mu\nu}(\Lambda) = \eta_{\mu\nu} + \frac{1}{\Lambda^4}\cdot\frac{1}{4}\eta_{\mu\alpha}t^{\alpha}{}_{\nu}(\Lambda) + \frac{1}{\Lambda^6}\cdot\frac{1}{24}\eta_{\mu\alpha}\Box t^{\alpha}{}_{\nu}(\Lambda) + \\ +\frac{1}{\Lambda^8}\cdot\left(\frac{1}{32}\,\eta_{\mu\alpha}t^{\alpha}{}_{\rho}(\Lambda)t^{\rho}{}_{\nu}(\Lambda) - \frac{7}{384}\,\eta_{\mu\nu}t^{\alpha}{}_{\beta}(\Lambda)t^{\beta}{}_{\alpha}(\Lambda) + \frac{11}{1536}\,\eta_{\mu\alpha}\Box^2 t^{\alpha}{}_{\nu}(\Lambda)\right) - \\ +\frac{1}{\Lambda^8}\log\Lambda\cdot\frac{1}{516}\cdot\eta_{\mu\alpha}\Box^2 t^{\alpha}{}_{\nu}(\Lambda) + \mathcal{O}\left(\frac{1}{\Lambda^{10}}\log\Lambda\right)$$

Shear viscosity

Comparison among models