Ivan Vitev

Applications of Soft Collinear Effective Theory to Hadronic and Nuclear Collisions

Nuclear Theory Seminar, BNL, May 2015

Upton, NY

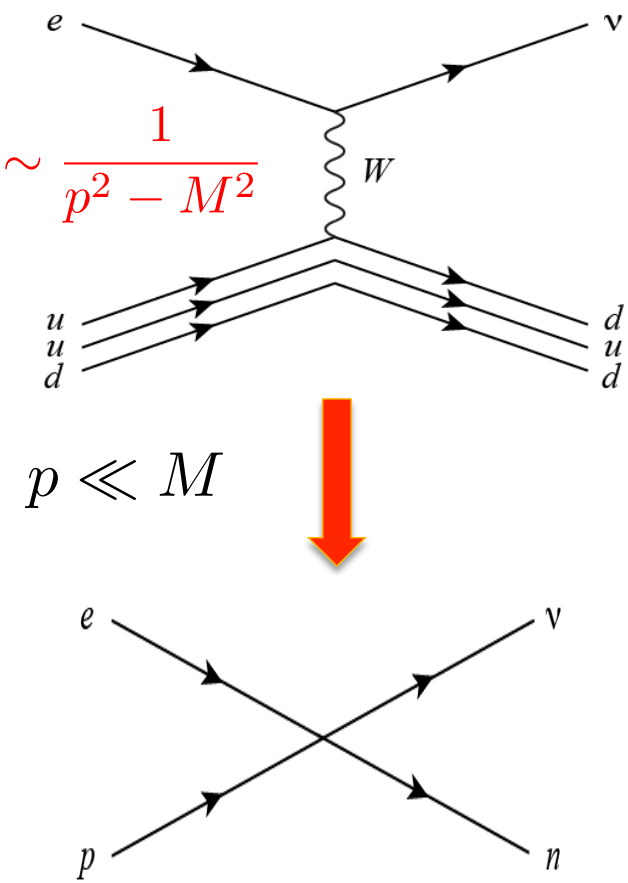
Outline of the talk

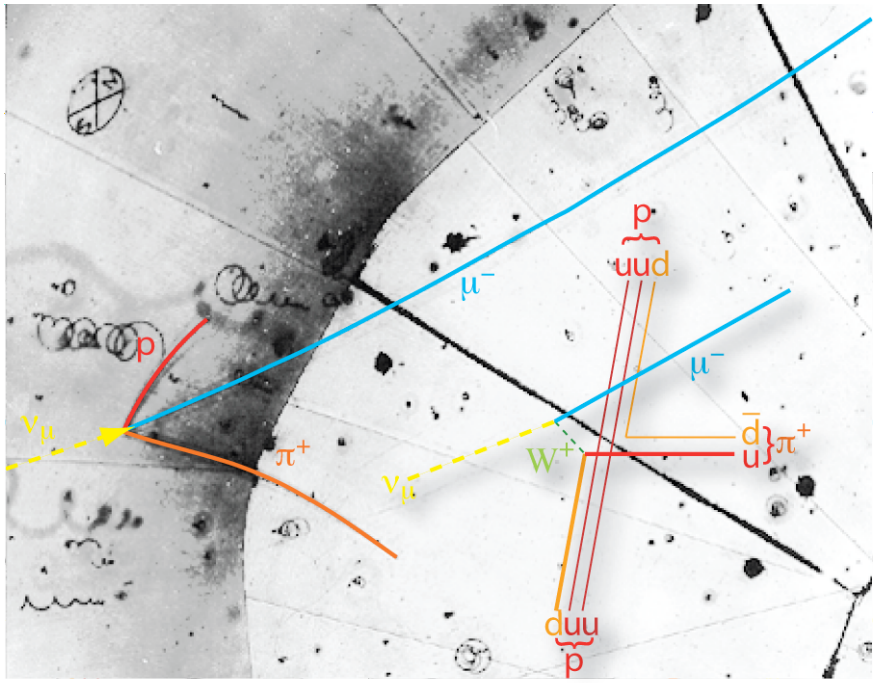
- A brief introduction to effective field theories (EFTs)
- An effective theory for jet propagation in matter - $SCET_G$. Current status and derivation of in-medium splitting functions
- Application of $SCET_G$ to heavy ion phenomenology. An evolution approach to inclusive particle quenching
- Application of SCET to hadronic jet shapes. Resummation to NLL accuracy
- Medium modification of jet shapes in $SCET_G$
- Conclusions

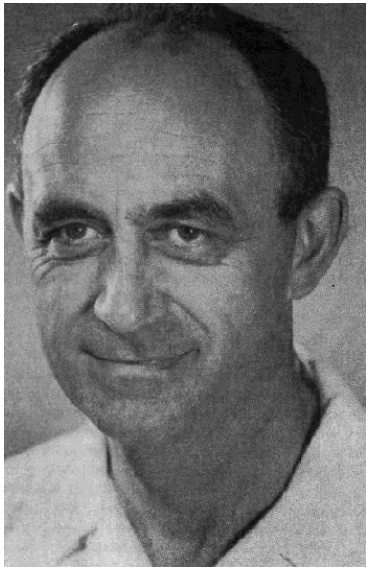
Introduction

The Fermi interaction

- The first, probably best known, effective theory is the Fermi interaction



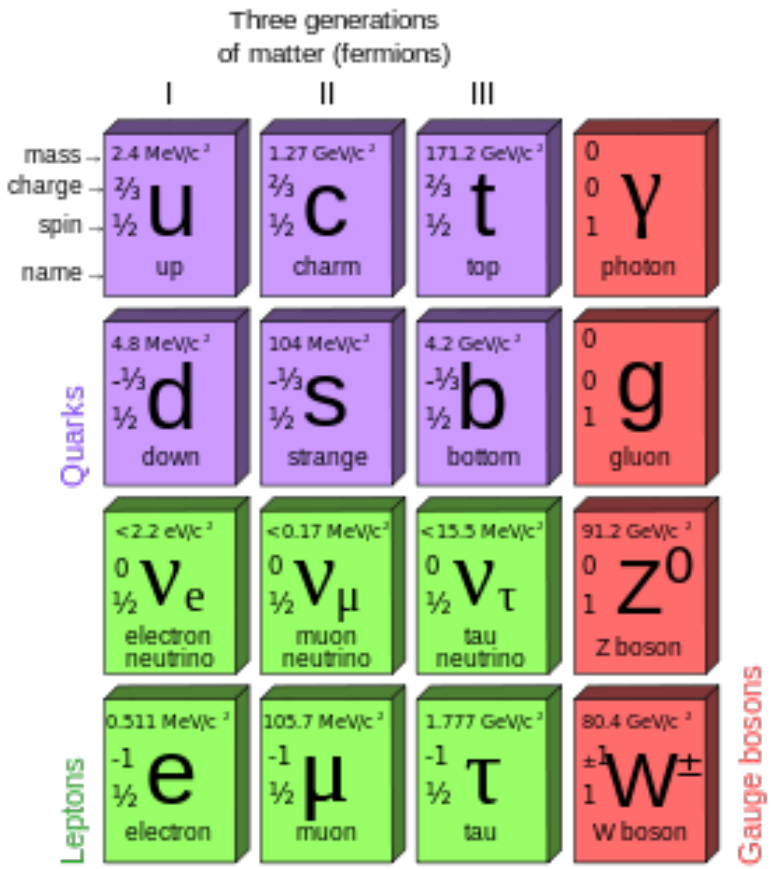




E. Fermi
(Nobel Prize)

- First direct observation of the neutrino, Nov. 1970

Effective field theories

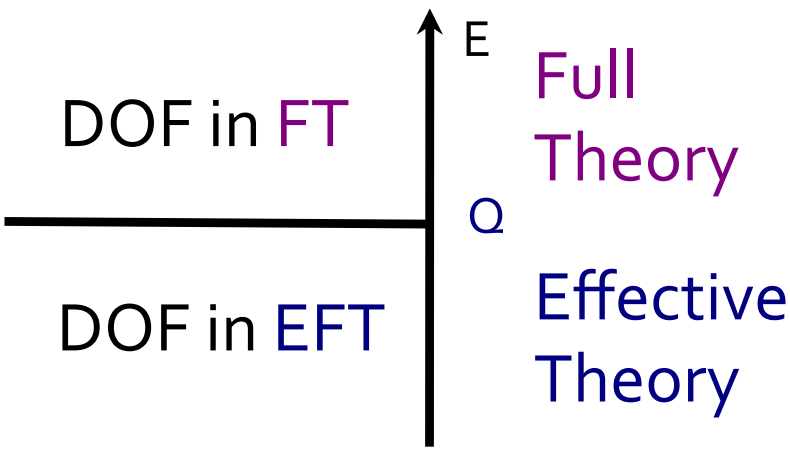


- Powerful framework based on exploiting symmetries and controlled expansions for problems with a natural separation of energy or distance scales.

- Particularly well suited to QCD and nuclear physics

- Effective theories are ubiquitous. The Standard Model is likely a low energy EFT of a theory at a much higher scale

Examples of effective field theories [EFTs]



- Focus on the significant degrees of freedom [DOF]. Manifest power counting

	Q	power counting	DOF in FT	DOF in EFT
Chiral Perturbation Theory (ChPT)	Λ_{QCD}	p/Λ_{QCD}	q, g	K,π
Heavy Quark Effective Theory (HQET)	m_b	Λ_{QCD}/m_b	ψ,A	h_v,A_s
Soft Collinear Effective Theory (SCET)	Q	$p_\perp/Q$	ψ,A	ξ_n,A_n,A_s

SCET formulation

- Modes in SCET

C. Bauer et al. (2001)

D. Pirol et al. (2004)

Collinear quarks, antiquarks	$\xi_n,\ \bar{\xi}_n$
Collinear gluons, soft gluons	$A_n,\ A_s$

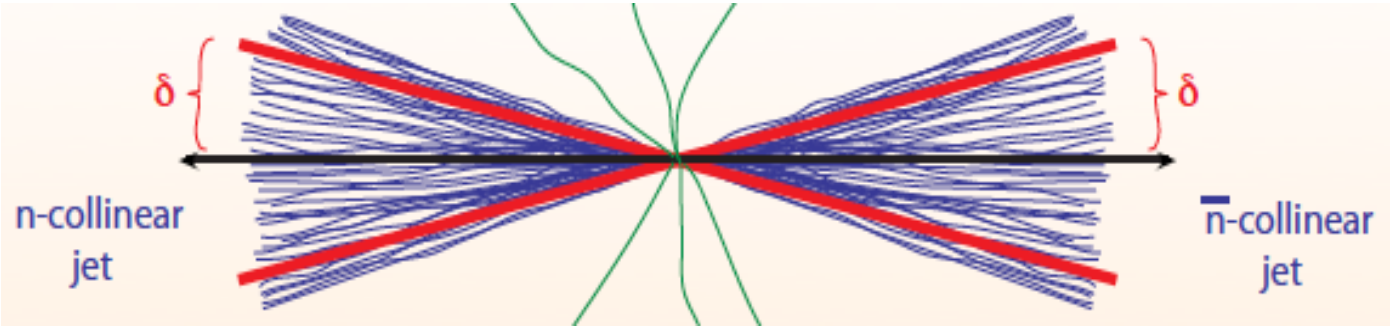
Soft quarks are eliminated through the equations of motion

SCET $_{II}$

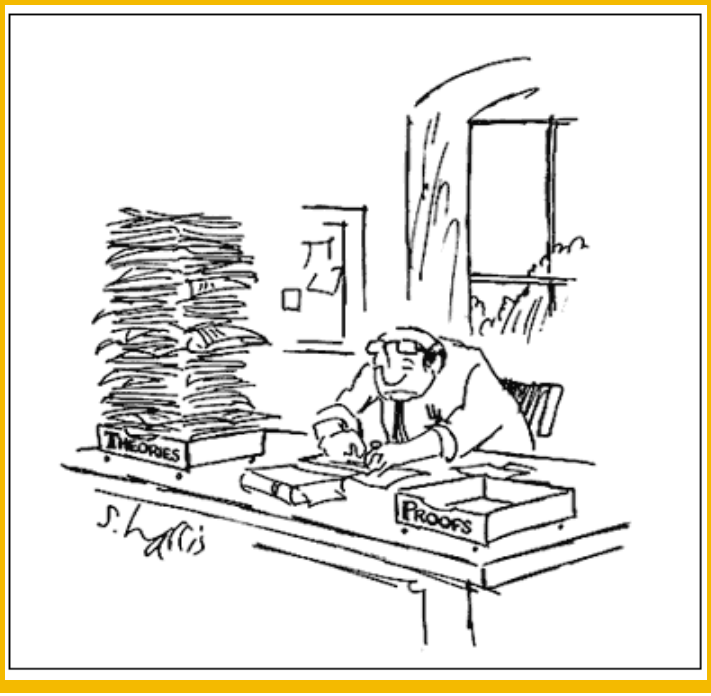
modes	$p^\mu = (+,-,\perp)$	p^2	fields
collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$	$\xi_n,\ A_n^\mu$
soft	$Q(\lambda, \lambda, \lambda)$	$Q^2\lambda^2$	$q_s,\ A_s^\mu$

- Other formulations, e.g. SCET$_I$ and ultrasoft particles

- Especially suited for jet physics

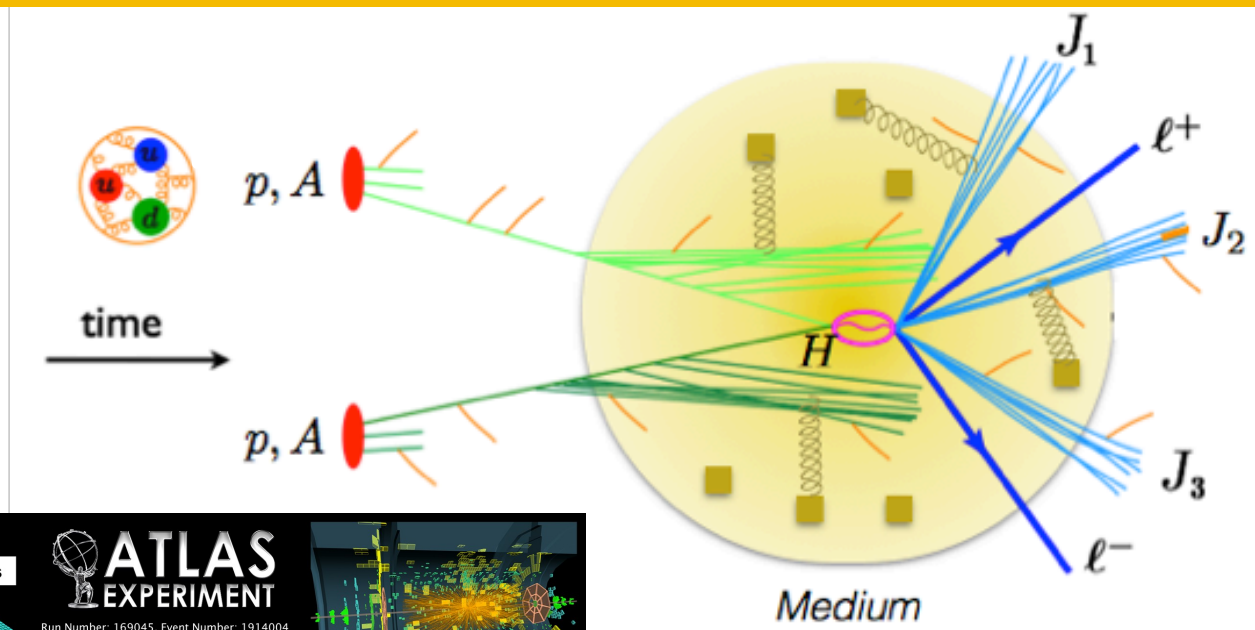


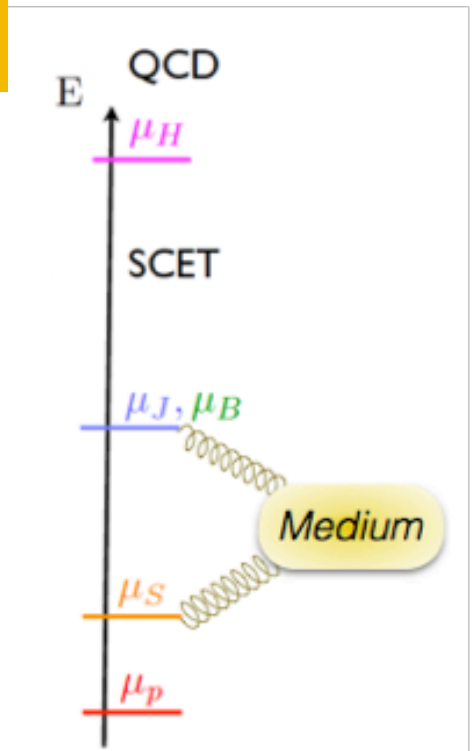
Soft Collinear Effecyive Theory with Glaubers

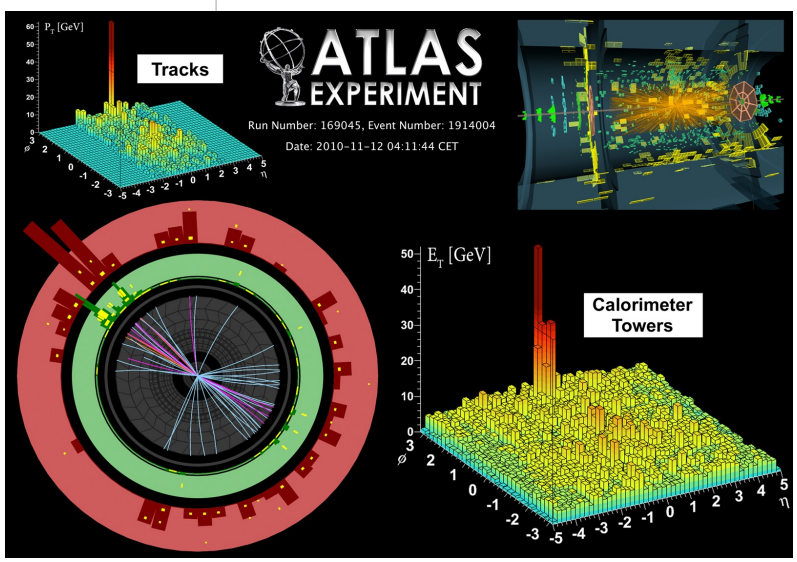


The big picture

- QCD in the medium remains a multiscale problem







- Factorization, with modified J, B, S

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Aad et al. (2010)

Ovanesyan et al. (2011)

The jet scattering kinematics

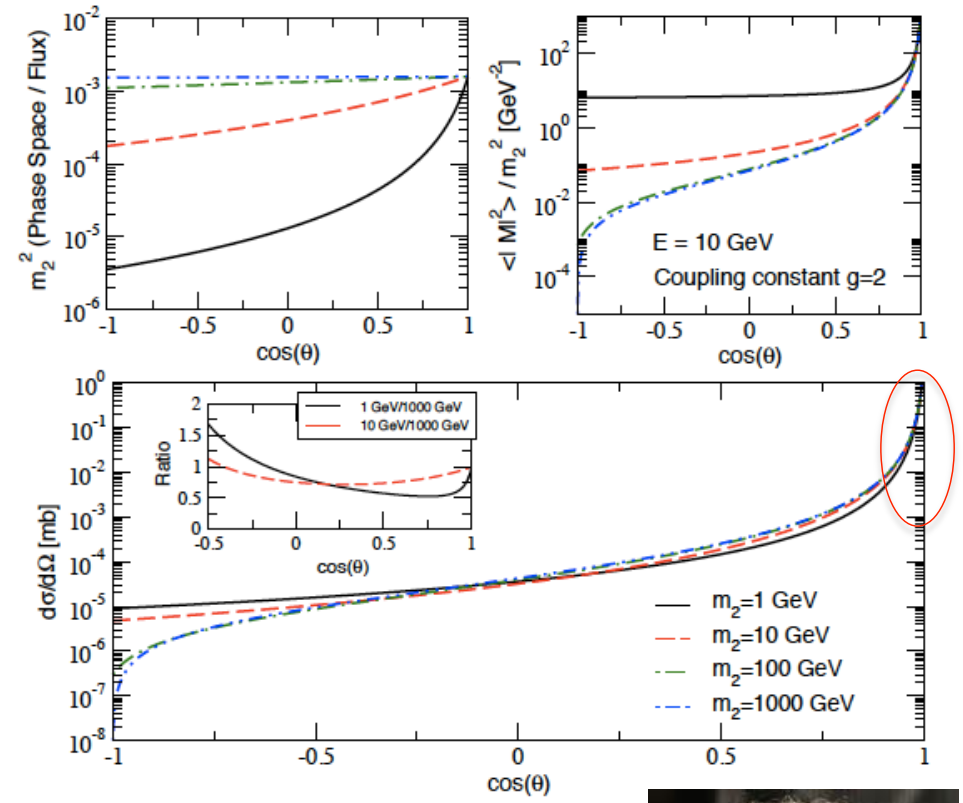
- Kinematics and channels

t – jet broadening and energy loss
s– isotropisation
u - backward hard scattering

- Fully dynamic medium recoil, cross section reduction (5% – 15%). Completely dominated by forward scattering

$$\frac{d\sigma}{d\Omega} \to \frac{d\sigma}{d^2\mathbf{q}_\perp} = \frac{C_2(R)C_2(T)}{d_A}\frac{|v(\mathbf{q}_\perp; E, m_1, m_2)|^2}{(2\pi)^2}$$

- Galuber gluon / Glauber mode



$$q = (\lambda^2, \lambda^2, \lambda)Q$$

A. Idilbi et al. (2008)

The Glauber gluon Lagrangian

- An effective theory of jet propagation in matter

Effective potential

$$\mathcal{L}_{\rm G}\left(\xi_n, A_n, \eta\right) = \sum_{p,p',q} {\rm e}^{-i(p-p'+q)x} \left(\bar{\xi}_{n,p'} \Gamma^{\mu,a}_{\rm qqA_G} \frac{\bar{\not{n}}}{2} \xi_{n,p} - i\Gamma^{\mu\nu\lambda,abc}_{\rm ggA_G} \left(A^c_{n,p'}\right)_\lambda \left(A^b_{n,p}\right)_\nu \right) \bar{\eta}\, \Gamma^{\delta,a}_{\rm s}\, \eta\, \Delta_{\mu\delta}(q)$$

- Feynman rules for different sources and gauges

Gauge	Object	Collinear source	Static source	Soft source
	p	$[\lambda^2, 1, \boldsymbol{\lambda}]$	$[1, 1, \boldsymbol{\lambda}]$	$[\lambda, \lambda, \boldsymbol{\lambda}]$
	$a_{\boldsymbol{p}}, a^\dagger_{\boldsymbol{p}}$	λ^{-1}	$\lambda^{-3/2}$	$\lambda^{-3/2}$
	$u(p)$	1	1	$\lambda^{1/2}$
	$\bar{u}(p_2)\gamma_\nu u(p_1)$	$[\lambda^2, 1, \boldsymbol{\lambda}]$	$[1, 1, \boldsymbol{\lambda}]$	$[\lambda, \lambda, \boldsymbol{\lambda}]$
R_ξ	$A^\mu(x)$	$[\lambda^4, \lambda^2, \boldsymbol{\lambda}^3]$	$[\lambda^2, \lambda^2, \boldsymbol{\lambda}^3]$	$[\lambda, \lambda, \boldsymbol{\lambda}]$
	$\Gamma_{\rm qqA_G}$	Γ^μ_1	Γ^μ_1	Γ^μ_1
	$\Gamma_{\rm ggA_G}$	$\Sigma^{\mu\nu\lambda}_1$	$\Sigma^{\mu\nu\lambda}_1$	$\Sigma^{\mu\nu\lambda}_1$
	$\Gamma_{\rm s}$	$\Gamma^\mu_1\,(n \leftrightarrow \bar{n})$	Γ^μ_3	Γ^μ_4
$A^+ = 0$	$A^\mu(x)$	$[0, \lambda^2, \boldsymbol{\lambda}^3]$	$[0, \lambda^2, \boldsymbol{\lambda}]$	$[0, \lambda, \mathbf{1}]$
	$\Gamma_{\rm qqA_G}$	Γ^μ_1	$\Gamma^\mu_1 + \Gamma^\mu_2$	$\Gamma^\mu_1 + \Gamma^\mu_2$
	$\Gamma_{\rm ggA_G}$	$\Sigma^{\mu\nu\lambda}_2$	$\Sigma^{\mu\nu\lambda}_2$	$\Sigma^{\mu\nu\lambda}_2$
	$\Gamma_{\rm s}$	$\Gamma^\mu_2\,(n \leftrightarrow \bar{n})$	Γ^μ_3	Γ^μ_4

$$\Gamma^{\mu,a}_1 = igT^a\, n^\mu\, \frac{\bar{\not{n}}}{2}\,,$$

$$\Gamma^{\mu,a}_2 = igT^a \frac{\gamma^\mu_\perp \not{p}_\perp + \not{p}'_\perp \gamma^\mu_\perp}{\bar{n}\cdot p} \frac{\bar{\not{n}}}{2}\,,$$

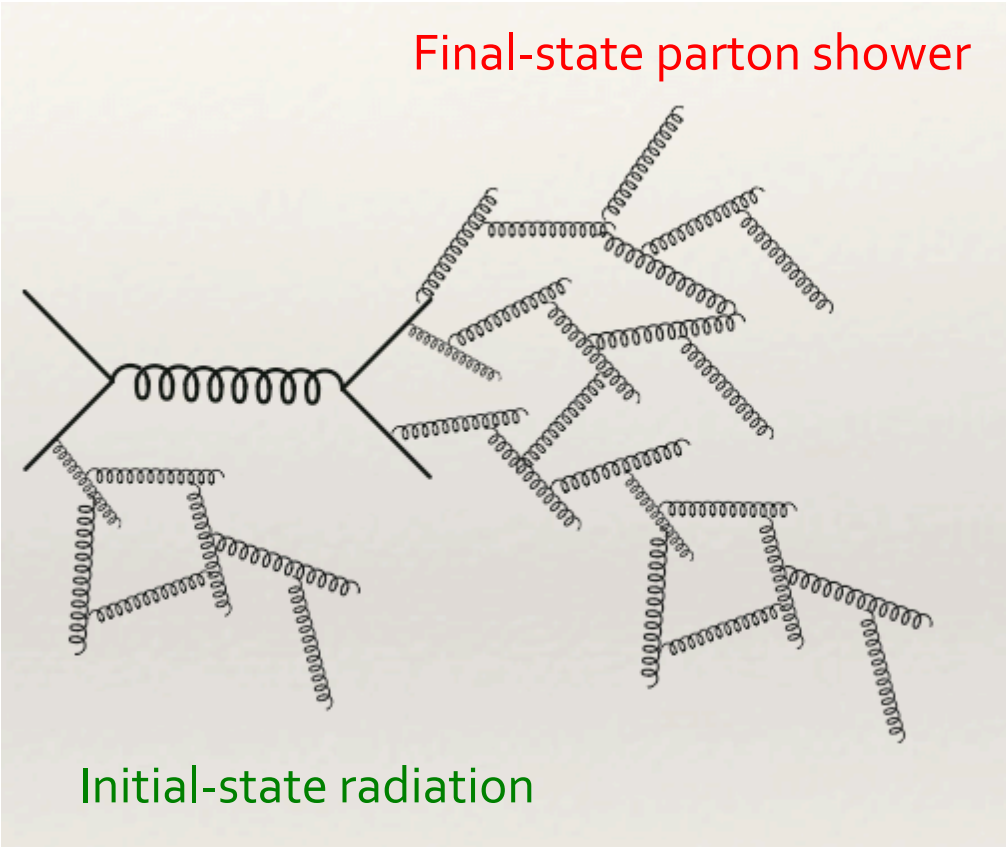
$$\Gamma^{\mu,a}_3 = igT^a\, v^\mu\,,$$

$$\Gamma^{\mu,a}_4 = igT^a\, \gamma^\mu\,,$$

$$\Sigma^{\mu\nu\lambda,abc}_1 = gf^{abc}\, n^\mu \left[g^{\nu\lambda}\, \bar{n}\cdot p + \bar{n}^\nu \left(p'^\lambda_\perp - p^\lambda_\perp\right) - \bar{n}^\lambda \left(p'^\nu_\perp - p^\nu_\perp\right) - \frac{1-\frac{1}{\xi}}{2} \left(\bar{n}^\lambda p^\nu + \bar{n}^\nu p'^\lambda\right) \right]\,,$$

G. Ovanesyan et al. (2011)

Logs, legs and loops

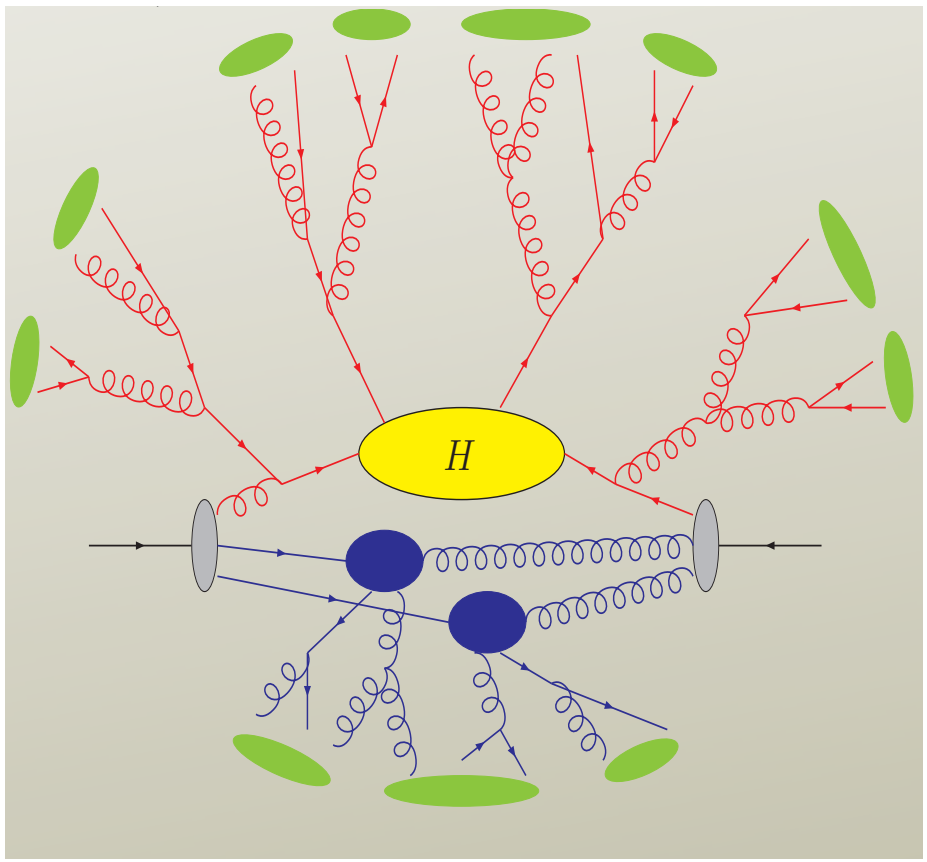


- In the description of high energy processed significant effort has been devoted to understand the logs, legs and loops

- Log - ratios of mass and energy scales, phase space, cuts. Goal is to resum
- Legs – the formation of parton shower, branchings, evolution
- Loops – virtual corrections. Goal is to include, find automated way to do some of the loops

- The are connected, one of the goals of the talk is to see if some of the technology can be ported to heavy ion collisions

The vacuum splitting kernels



- Splitting functions are related to beam (B) and jet (J) functions in SCET

$$A_{q\to qg} = \langle J|\, T\, \bar{\chi}_n(x_0)\, \mathrm{e}^{iS}\, |q(\boldsymbol{p})g(\boldsymbol{k})\rangle$$
$$A_{g\to q\bar{q}} = \langle J|\, T\, \mathcal{B}^{\lambda c}(x_0)\, \mathrm{e}^{iS}\, |q(\boldsymbol{p})\bar{q}(\boldsymbol{k})\rangle$$
$$A_{g\to gg} = \langle J|\, T\, \mathcal{B}^{\lambda c}(x_0)\, \mathrm{e}^{iS}\, |g(\boldsymbol{p})g(\boldsymbol{k})\rangle$$

$$\Gamma_W^{\alpha,a}(k) = gT_r^a \frac{\bar{n}^\alpha}{k^+ + i\epsilon}$$

Gribov et al. (1972)

G. Altarelli et al. (1977)

Y. Dokshitzer (1977)

- In the vacuum we have the DGPAL splitting kernels that factorize from the hard scattering cross section and are process independent

Splitting kernel results

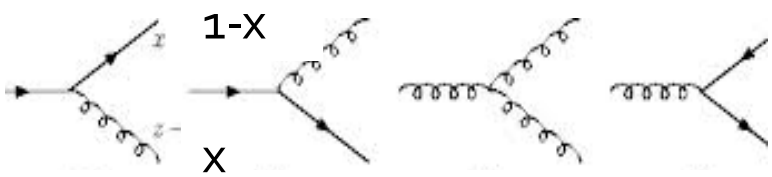
- Explicitly verified the gauge invariance and factorization in QCD

$$\left(\frac{dN}{dx\, d^2\mathbf{k}_\perp}\right)_{q\to qg} = \frac{\alpha_s}{2\pi^2} C_F \frac{1+(1-x)^2}{x}\frac{1}{\mathbf{k}_\perp^2}, \quad (\ldots|_+ + A\delta(x))$$

$$\left(\frac{dN}{dx\, d^2\mathbf{k}_\perp}\right)_{g\to gg} = \frac{\alpha_s}{2\pi^2} 2C_A\left(\frac{1-x}{x}+\frac{x}{1-x} + x(1-x)\right)\frac{1}{\mathbf{k}_\perp^2}, \quad (\ldots|_+ + B\delta(x))$$

$$\left(\frac{dN}{dx\, d^2\mathbf{k}_\perp}\right)_{g\to q\bar{q}} = \frac{\alpha_s}{2\pi^2} T_R \left(x^2+(1-x)^2\right)\frac{1}{\mathbf{k}_\perp^2}$$

$$\left(\frac{dN}{dx\, d^2\mathbf{k}_\perp}\right)_{q\to gq} = \left(\frac{dN}{dx\, d^2\mathbf{k}_\perp}\right)_{q\to qg} (x\to 1-x)$$



Reversed convention

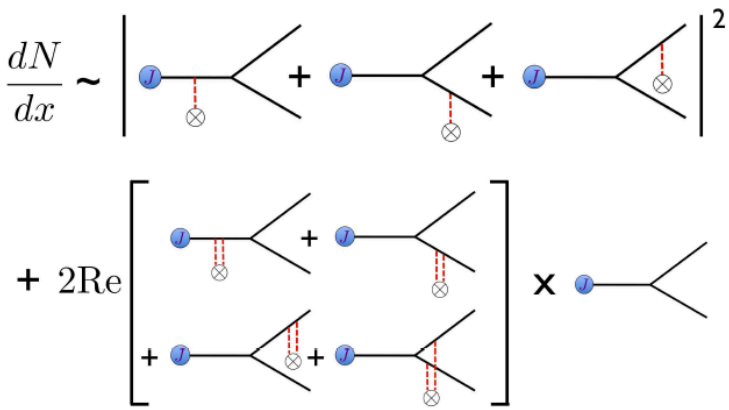
- The singular pieces A, B can be obtained form flavor and momentum conservation sum rules

$$\int_0^1 P_{qq}(x)\,\mathrm{d}x = 0,$$

$$\int_0^1 \left[P_{qq}(x)+P_{qg}(x)\right](1-x)\,\mathrm{d}x = 0,$$

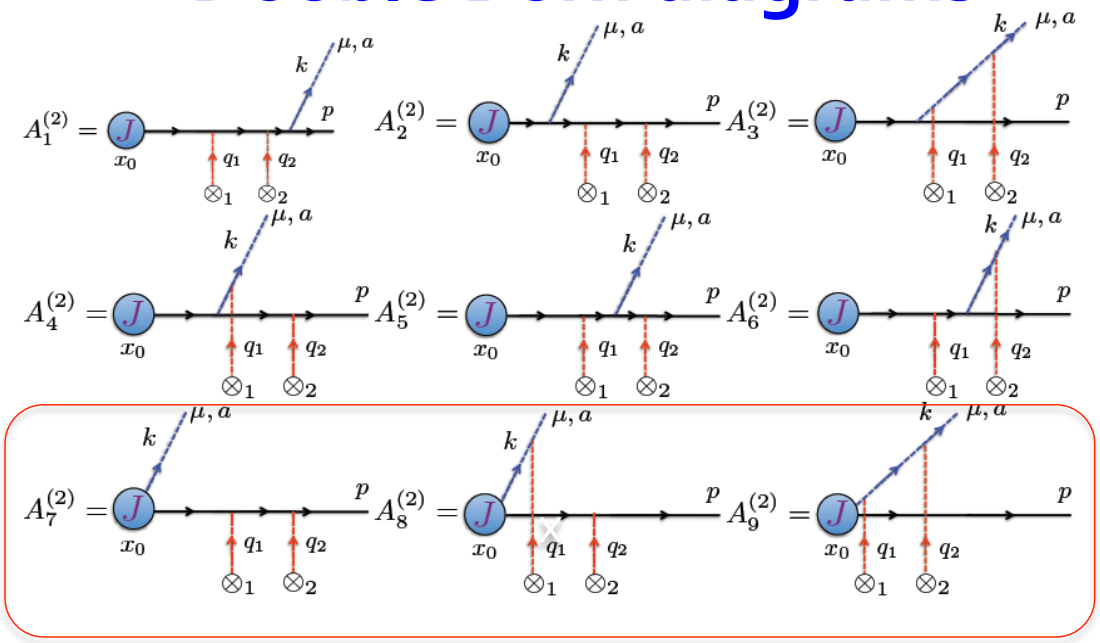
$$\int_0^1 \left[2n_f P_{gq}(x)+P_{gg}(x)\right](1-x)\,\mathrm{d}x = 0.$$

In-medium splitting kernels

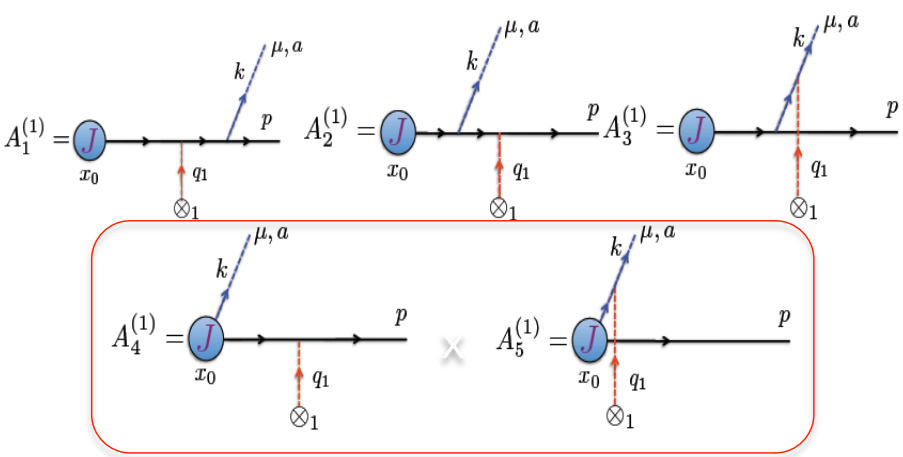


- The cross section

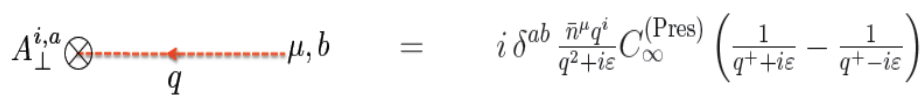
Double Born diagrams



Single Born diagrams



- Transverse Wilson line



A. Idilbi et al. (2010)

In-medium parton splittings and properties

- Direct sum

$$\frac{dN(tot.)}{dxd^2k_\perp} = \frac{dN(vac.)}{dxd^2k_\perp} + \frac{dN(med.)}{dxd^2k_\perp}$$

- Derived using $SCET_G$
- Factorize form the hard part
- Gauge-invariant
- Depend on the properties of the medium

G. Ovanesyan et al. (2012)

$$\left(\frac{dN}{dxd^2\boldsymbol{k}_\perp}\right)_{q\to qg} = \frac{\alpha_s}{2\pi^2} C_F \frac{1+(1-x)^2}{x} \int \frac{d\Delta z}{\lambda_g(z)} \int d^2\mathbf{q}_\perp \frac{1}{\sigma_{el}} \frac{d\sigma_{el}^{\text{medium}}}{d^2\mathbf{q}_\perp} \Bigg[-\left(\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}\right)^2 + \frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}\cdot\left(\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2} - \frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2}\right)$$
$$\times\left(1-\cos[(\Omega_1-\Omega_2)\Delta z]\right) + \frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2}\cdot\left(2\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2} - \frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2} - \frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}\right)\left(1-\cos[(\Omega_1-\Omega_3)\Delta z]\right)$$
$$+\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}\cdot\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2}\left(1-\cos[(\Omega_2-\Omega_3)\Delta z]\right) + \frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}\cdot\left(\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2} - \frac{\boldsymbol{D}_\perp}{\boldsymbol{D}_\perp^2}\right)\cos[\Omega_4\Delta z]$$
$$+\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}\cdot\frac{\boldsymbol{D}_\perp}{\boldsymbol{D}_\perp^2}\cos[\Omega_5\Delta z] + \frac{1}{N_c^2}\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}\cdot\left(\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2} - \frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}\right)\left(1-\cos[(\Omega_1-\Omega_2)\Delta z]\right)\Bigg].$$

$$N.B.\ x \to 1-x \qquad A,\ldots D,\Omega_1\ldots\Omega_5 - functions(x,k_\perp,q_\perp)$$

$$\left(\frac{dN}{dxd^2\boldsymbol{k}_\perp}\right)_{\left\{\begin{smallmatrix} g\to q\bar{q} \\ g\to gg\end{smallmatrix}\right\}} = \left\{\begin{matrix} \frac{\alpha_s}{2\pi^2} T_R\left(x^2+(1-x)^2\right) \\ \frac{\alpha_s}{2\pi^2} 2C_A\left(\frac{x}{1-x}+\frac{1-x}{x}+x(1-x)\right)\end{matrix}\right\} \int d\Delta z \left\{\begin{matrix}\frac{1}{\lambda_q(z)} \\ \frac{1}{\lambda_g(z)}\end{matrix}\right\} \int d^2\mathbf{q}_\perp \frac{1}{\sigma_{el}} \frac{d\sigma_{el}^{\text{medium}}}{d^2\mathbf{q}_\perp}$$
$$\times\Bigg[2\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}\cdot\left(\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2} - \frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}\right)\left(1-\cos[(\Omega_1-\Omega_2)\Delta z]\right) + 2\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2}\cdot\left(\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2} - \frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}\right)\left(1-\cos[(\Omega_1-\Omega_3)\Delta z]\right)$$
$$+\left\{\begin{matrix}\frac{1}{N_c^2-1} \\ -\frac{1}{2}\end{matrix}\right\}\Bigg(2\left(\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2} - \frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}\right)\cdot\left(\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2} - \frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2}\right) + 2\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}\cdot\left(\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2} - \frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2}\right)\cos[(\Omega_1-\Omega_2)\Delta z]$$
$$+2\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2}\cdot\left(\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2} - \frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}\right)\cos[(\Omega_1-\Omega_3)\Delta z] + 2\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2}\cdot\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}\cos[(\Omega_2-\Omega_3)\Delta z]$$
$$-2\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}\cdot\left(\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2} - \frac{\boldsymbol{D}_\perp}{\boldsymbol{D}_\perp^2}\right)\cos[\Omega_4\Delta z] - 2\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}\cdot\frac{\boldsymbol{D}_\perp}{\boldsymbol{D}_\perp^2}\cos[\Omega_5\Delta z]\Bigg)\Bigg].$$

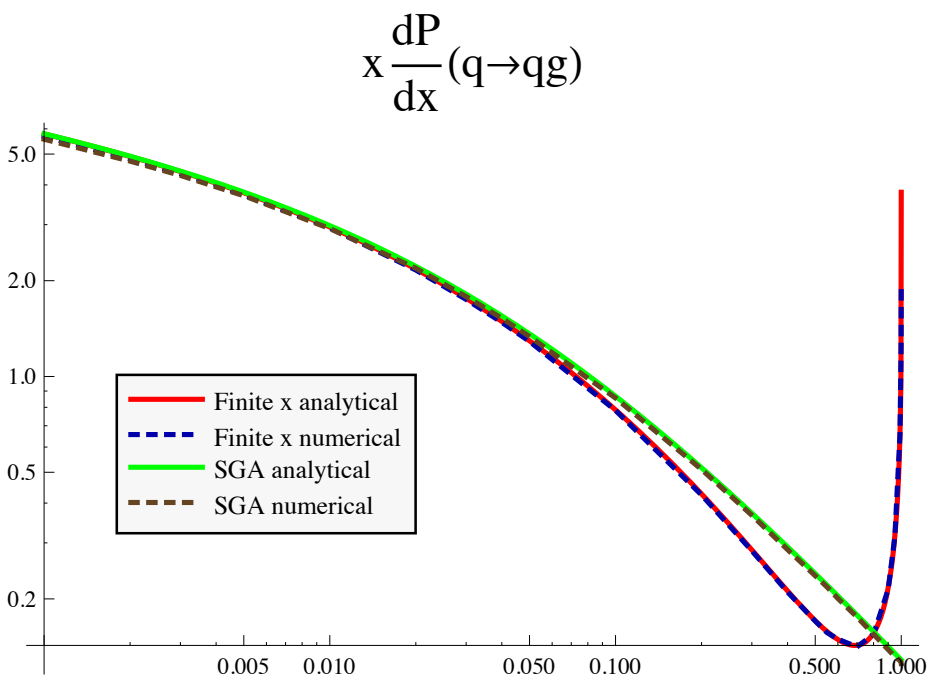
The soft-gluon energy loss limit

Take the small-x soft gluon emission limit

$$x\left(\frac{dN}{dx}\right)_{\left\{\begin{matrix} q\to qg \\ g\to gg \end{matrix}\right\}} = \frac{\alpha_s}{\pi^2}\left\{\begin{matrix} C_F[1+\mathcal{O}(x)] \\ C_A[1+\mathcal{O}(x)] \end{matrix}\right\}\int\frac{d\Delta z}{\lambda_g(z)}\int d^2\mathbf{k}_\perp d^2\mathbf{q}_\perp\frac{1}{\sigma_{el}}\frac{d\sigma_{el}^{\text{medium}}}{d^2\mathbf{q}_\perp}$$

M Gyulassy et al . (2000)

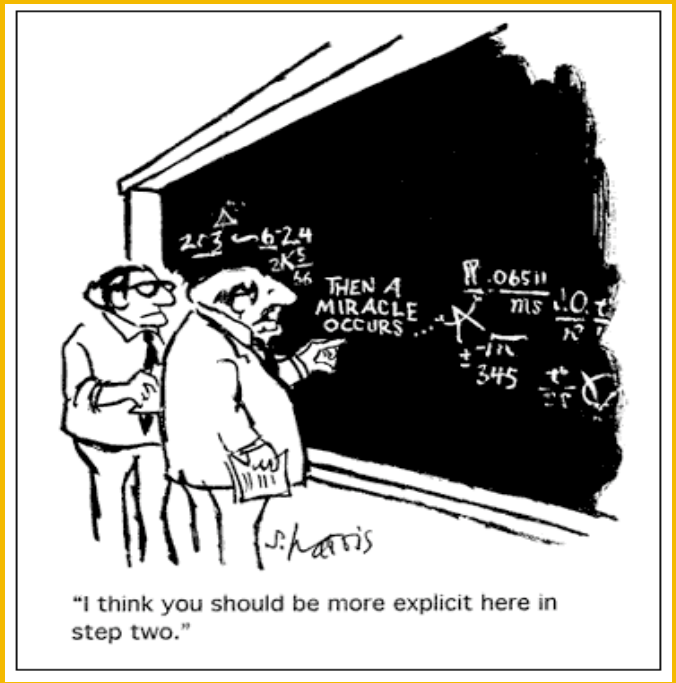
$$\times\frac{2\boldsymbol{k}_\perp\cdot\boldsymbol{q}_\perp}{\boldsymbol{k}_\perp^2(\boldsymbol{k}_\perp-\boldsymbol{q}_\perp)^2}\left[1-\cos\frac{(\boldsymbol{k}_\perp-\boldsymbol{q}_\perp)^2}{xp_0^+}\Delta z\right].$$



The result reduces to the GLV energy-loss differential intensities

- Only 2 medium-induced splittings survive
- There is no flavor mixing
- Results can be interpreted as energy loss

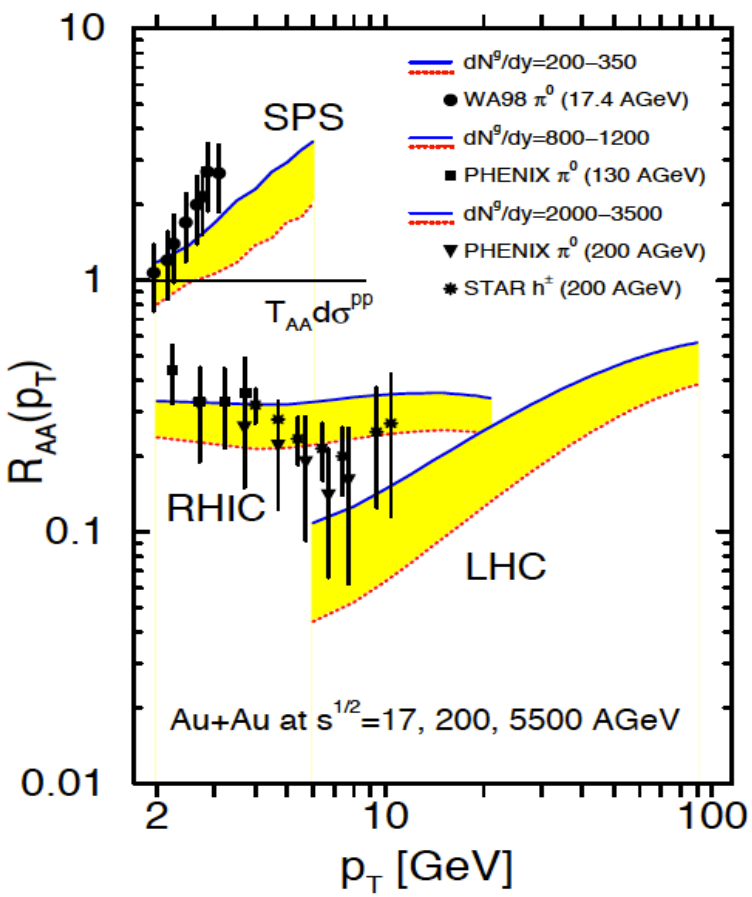
Application to inclusive hadron suppression



"I think you should be more explicit here in step two."

Improving upon traditional E-loss

- While still LO, it predicted in 2002, 2006 – the R_{AA} at high p_T for both RHIC and LHC

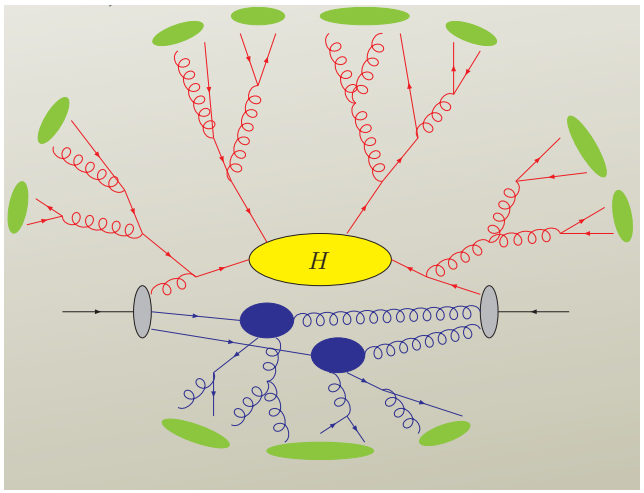


Include the quenched parton and the radiative gluon fragmentation

- Difficult to make connection to the standard LO, NLO, ...; LL, NLL ... pQCD approach (higher orders and resummation)
- There is considerable model dependence and it is difficult to systematically improve this approach

I. Vitev et al. (2002)

Evolution of the fragmentation functions



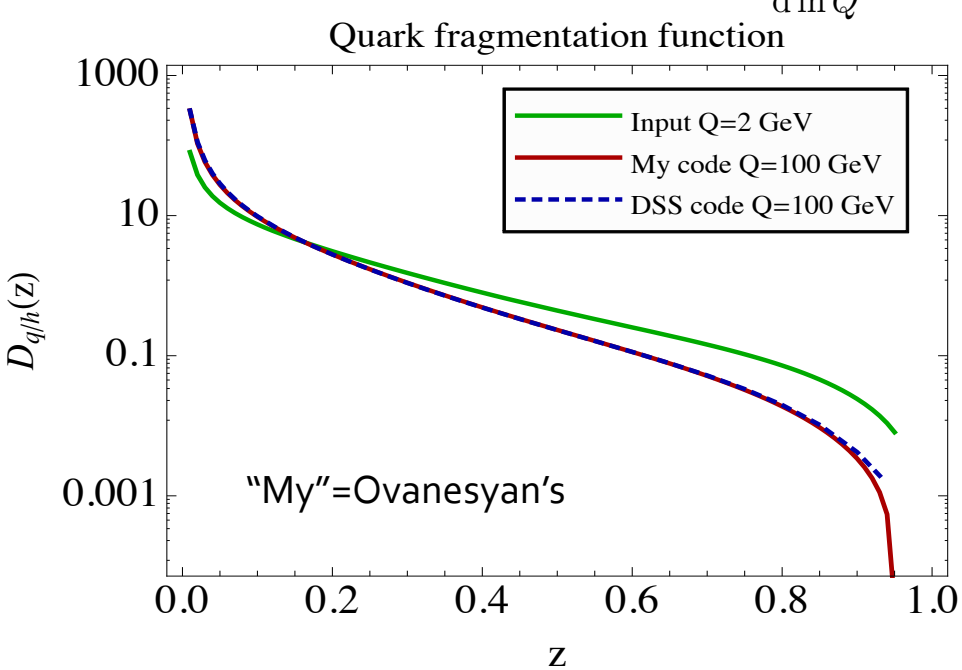
- Yield LLA or MLLA (LL')

$$\frac{\mathrm{d}D_q(z,Q)}{\mathrm{d}\ln Q} = \frac{\alpha_s(Q^2)}{\pi}\int_z^1 \frac{\mathrm{d}z'}{z'}\left\{P_{q\to qg}(z',Q)D_q\left(\frac{z}{z'},Q\right) + P_{q\to gq}(z',Q)D_g\left(\frac{z}{z'},Q\right)\right\},$$

$$\frac{\mathrm{d}D_{\bar{q}}(z,Q)}{\mathrm{d}\ln Q} = \frac{\alpha_s(Q^2)}{\pi}\int_z^1 \frac{\mathrm{d}z'}{z'}\left\{P_{q\to qg}(z',Q)D_{\bar{q}}\left(\frac{z}{z'},Q\right) + P_{q\to gq}(z',Q)D_g\left(\frac{z}{z'},Q\right)\right\},$$

$$\frac{\mathrm{d}D_g(z,Q)}{\mathrm{d}\ln Q} = \frac{\alpha_s(Q^2)}{\pi}\int_z^1 \frac{\mathrm{d}z'}{z'}\left\{P_{g\to gg}(z',Q)D_g\left(\frac{z}{z'},Q\right)\right.$$

$$\left. + P_{g\to q\bar{q}}(z',Q)\left(D_q\left(\frac{z}{z'},Q\right) + \bar{q}\ \text{term}\ \right)\right\}.$$



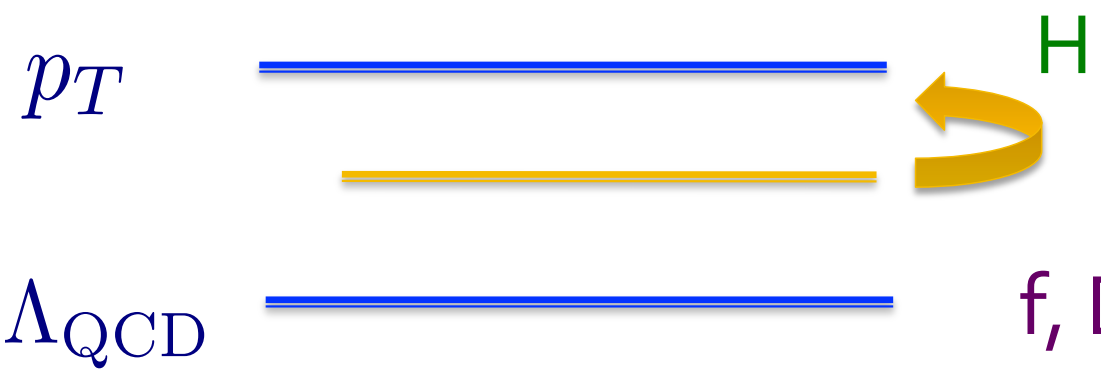
- Vacuum evolution frozen under Q_0~ 1GeV
- In the medium: effective thermal masses, finite α_s

de Florian et al. (20xx)

Application of the evolution/ resummation approach

- The goal is to evaluate the nuclear modification and the related cross sections

$$R_{AA}(p_T) = \frac{H(\mu, p_T) \otimes f(\mu) \otimes f(\mu) \otimes D^{\rm med}(\mu)}{H(\mu, p_T) \otimes f(\mu) \otimes f(\mu) \otimes D(\mu)}$$



We have choices.

The simplest choice is:

$$\mu = p_T$$

- Run the DGLAP evolution with the medium induced splitting kernels
- Really important to find connection to proper E-loss applications and understand the uncertainty form the choice of method

Vacuum evolution in the soft gluon limit

$$P_{q\to qg} = \frac{2C_F}{x_+} + \left(\frac{2C_F}{x}\, g[x,Q,L,\mu]\right)_+ ,$$

$$P_{g\to gg} = \frac{2C_A}{x_+} + \left(\frac{2C_A}{x}\, g[x,Q,L,\mu]\right)_+ ,$$

$$P_{g\to q\bar{q}} = 0,$$

$$P_{q\to gq} = 0,$$

- If a connection is to be found between the energy loss and the evolution approach, it is in the soft gluon limit

Equations decouple

$$\frac{\mathrm{d}D_{h/c}(z,Q)}{\mathrm{d}\ln Q} = \frac{\alpha_s}{\pi}\int_z^1 \frac{\mathrm{d}z'}{z'}\,\left[P_{c\to cg}(z',Q)\right]_+ D_{h/c}(z/z',Q).$$

$$\frac{\mathrm{d}D_{h/c}(z,Q)}{\mathrm{d}\ln Q} = 2C_R\frac{\alpha_s}{\pi}\left\{\int_z^1 \mathrm{d}z'\,\frac{1}{1-z'}\left[\frac{1}{z'}\,D_{h/c}(z/z',Q) - D_{h/c}(z,Q)\right] + D_{h/c}(z,Q)\,\ln(1-z)\right\}$$

$$\left(1+z\frac{\partial}{\partial z}\right) D_{h/c}(z,Q) \approx [1-n(z)]\, D_{h/c}(z,Q)$$

- Using the "+" function, expanding around z' = 1 and relating derivatives to local slope we obtain the evolution to LLA

$$D_{h/c}(z,Q) = \mathrm{e}^{-2C_R\frac{\alpha_s}{\pi}\left[\ln\frac{Q}{Q_0}\right]\{[n(z)-1](1-z)-\ln(1-z)\}}\, D_{h/c}(z,Q_0).$$

Medium-modified evolution of the fragmentation functions

- Using the same techniques. The vacuum and the medium induced evolution factorize

$$\frac{\mathrm{d}\ln D_{h/c}^{\mathrm{med.}}(z,Q)}{\mathrm{d}\ln Q} = [\cdots]_{\mathrm{vac.}} - [n(z)-1]\left\{\int_0^{1-z}\mathrm{d}z'\,z'Q\frac{dN}{dz'dQ}(z',Q)\right\} - \int_{1-z}^{1}\mathrm{d}z'Q\frac{dN}{dz'dQ}(z',Q)\,.$$

$$D_{h/c}^{\mathrm{med.}}(z,Q) = \mathrm{e}^{-2C_R\frac{\alpha_s}{\pi}\left[\ln\frac{Q}{Q_0}\right]\{[n(z)-1](1-z)-\ln(1-z)\}}\,D_{h/c}(z,Q_0)$$

$$\times e^{-[n(z)-1]\left\{\int_0^{1-z}\mathrm{d}z'\,z'\int_{Q_0}^{Q}dQ'\frac{dN}{dz'dQ'}(z',Q')\right\}-\int_{1-z}^{1}\mathrm{d}z'\int_{Q_0}^{Q}dQ'\frac{dN}{dz'dQ'}(z',Q')}$$

$$= D_{h/c}(z,Q)e^{-[n(z)-1]\left\langle\frac{\Delta\tilde{E}}{E}\right\rangle_z-\langle N^{\tilde{g}}\rangle_z}\,.$$

- *The main result*: direct relation between the evolution and energy loss approaches first established here

$$\left\langle\frac{\Delta\tilde{E}}{E}\right\rangle_z = \int_0^{1-z}\mathrm{d}z'\,z'\int_{Q_0}^{Q}dQ'\frac{dN}{dz'dQ'}(z',Q') = \int_0^{1-z}\mathrm{d}z'\,z'\frac{dN}{dz'}(z') \qquad \to_{z\to 0}\left\langle\frac{\Delta E}{E}\right\rangle,$$

$$\langle N^{\tilde{g}}\rangle_z = \int_{1-z}^{1}\mathrm{d}z'\int_{Q_0}^{Q}dQ'\frac{dN}{dz'dQ'}(z',Q') = \int_{1-z}^{1}\mathrm{d}z'\frac{dN}{dz'}(z') \qquad \to_{z\to 1}\langle N^g\rangle\,.$$

G. Ovanesyan et al. (2014)

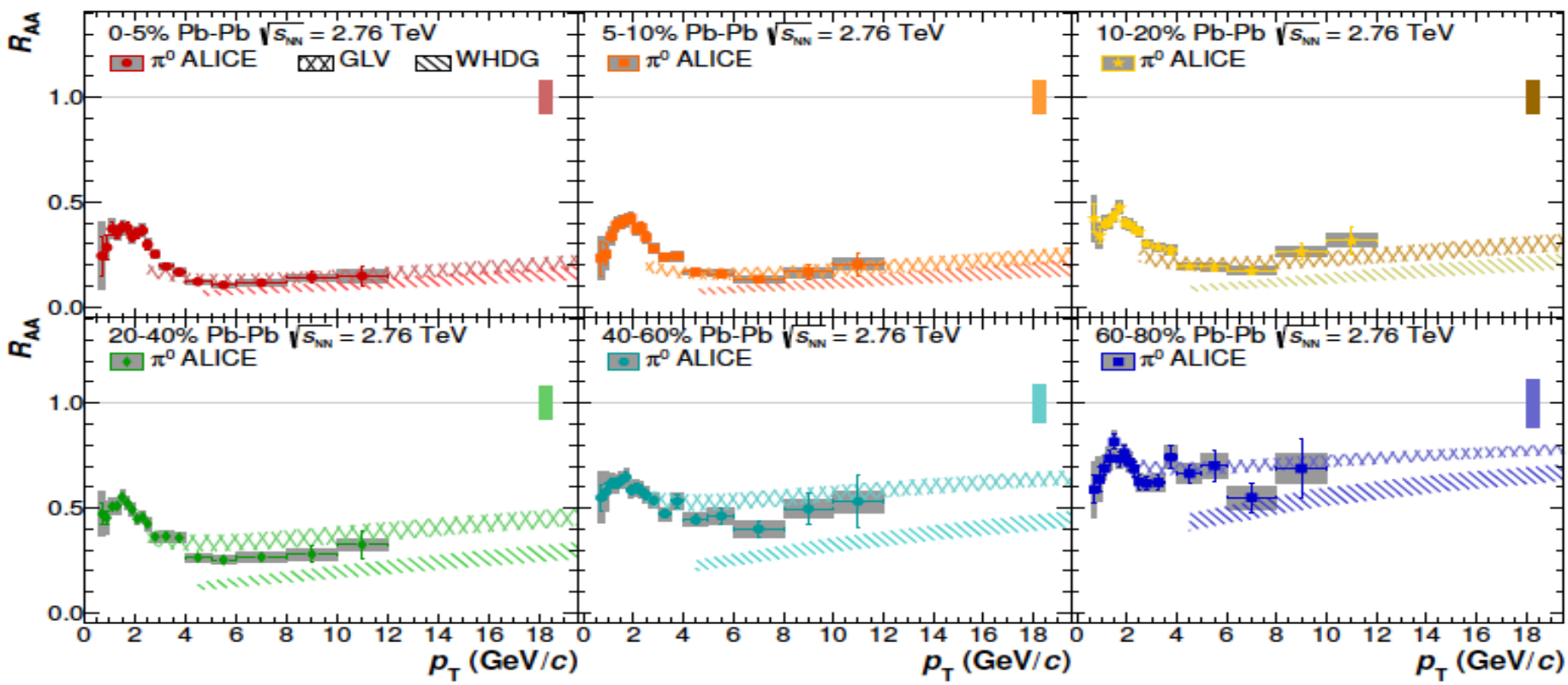
Reminder – traditional energy loss phenomenology

$$\int_0^1 d\epsilon\, P(\epsilon) = 1\,, \quad \int_0^1 d\epsilon\, \epsilon\, P(\epsilon) = \left\langle \frac{\Delta E}{E} \right\rangle \qquad D_c^{\text{quench}}(z) = \int_0^{1-z} d\epsilon\, \frac{P_c(\epsilon)}{(1-\epsilon)} D_c\left(\frac{z}{1-\epsilon}\right)$$

M. Gyulassy et al. (2002)

ALICE Collab. (2014)

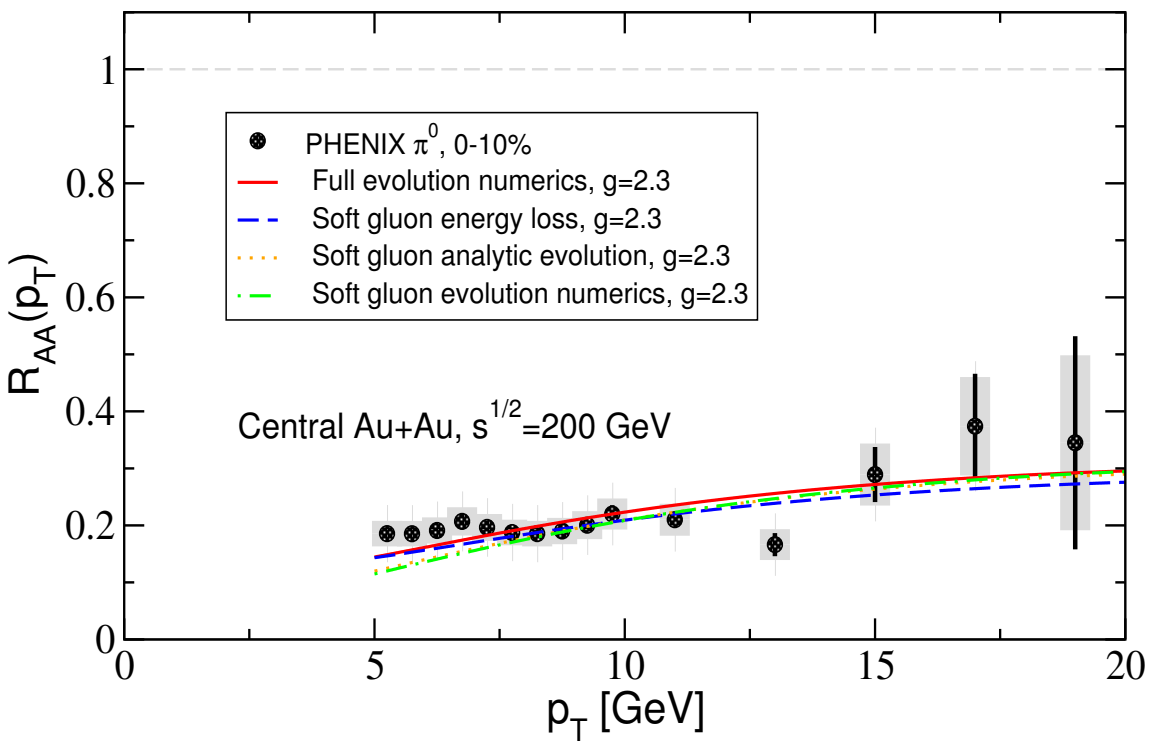
- Successfully predicted the energy and transverse momentum dependence of R_{AA}



Uncertainty from the implementation of the approach

- Four approaches to compare

- Full numerical solution to the in-medium DGLAP, full splitting kernels
- Full numerical solution to the in-medium DGLAP, soft gluon limit of the splitting kernels
- Analytic solution to the in-medium DGLAP evolution in the soft limit
- Traditional energy loss with muti-gluon emission

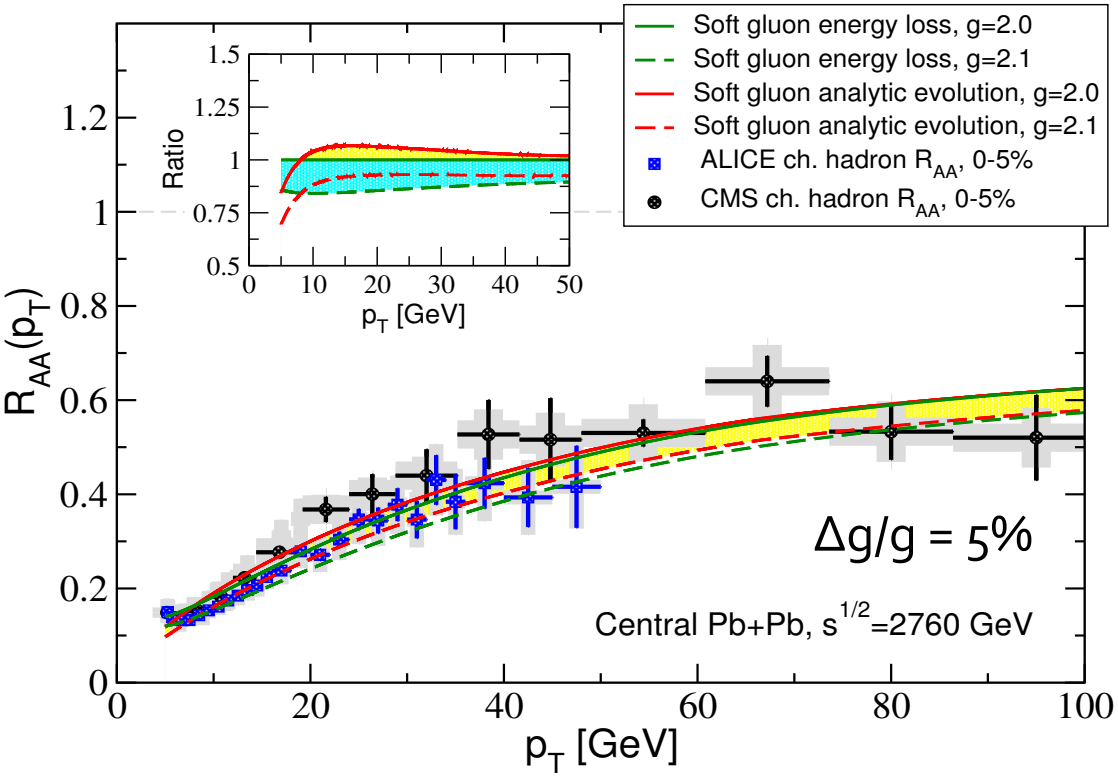


- Differences are minimal between implementations for inclusive hadrons
- Validates more than a decade of hadron suppression phenomenology

Numerical results: E-loss vs analytic evolution in the soft gluon limit

- All approaches include final-state effects only

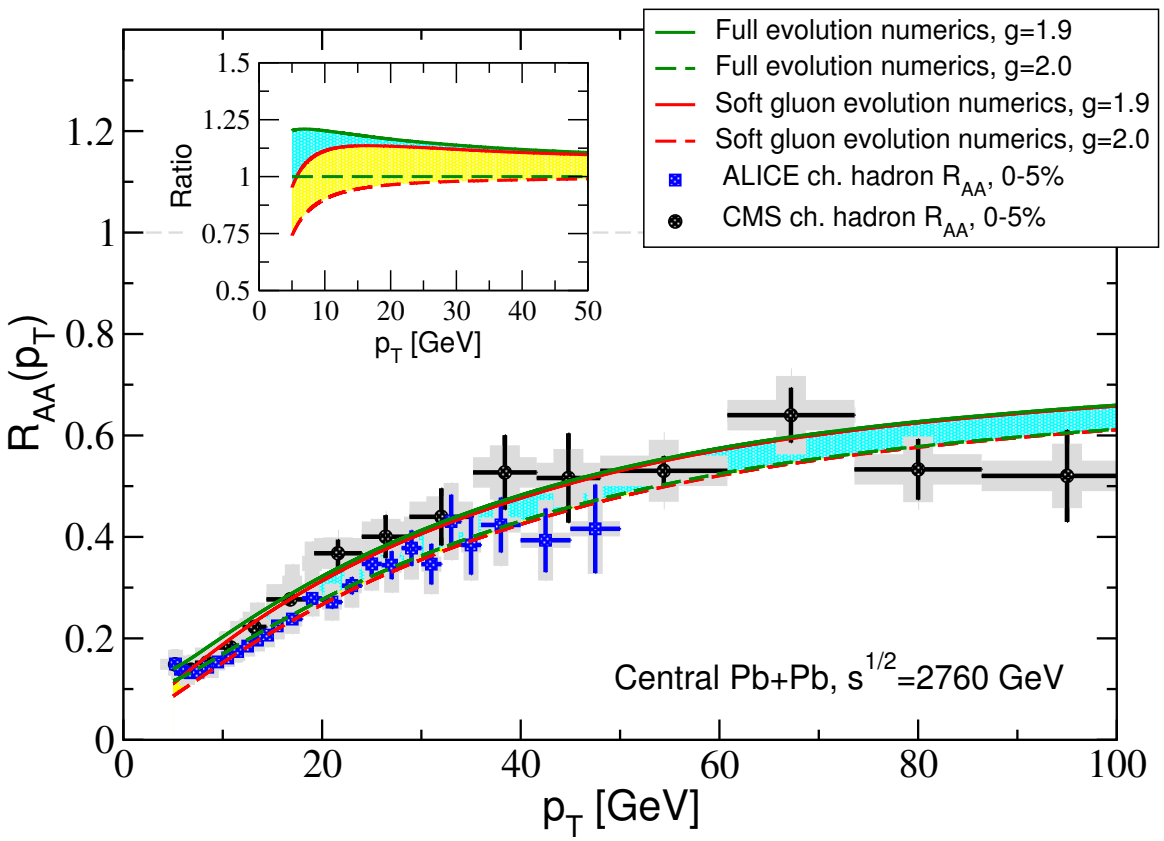
Note that we have ignored other nuclear matter effects: Cronin effect, cold nuclear matter energy loss, power corrections/shadowing. At the LHC final-state effects dominate



The coupling between the jet and the medium can be constrained to 5%, The scattering cross sections/ properties of the medium to 20% (~g^4)

Numerical results: full-x vs small-x evolution

- Implement the fully numerical solution of the DGLAP evolution equations: the full splitting kernels and the soft gluon limit (small x)



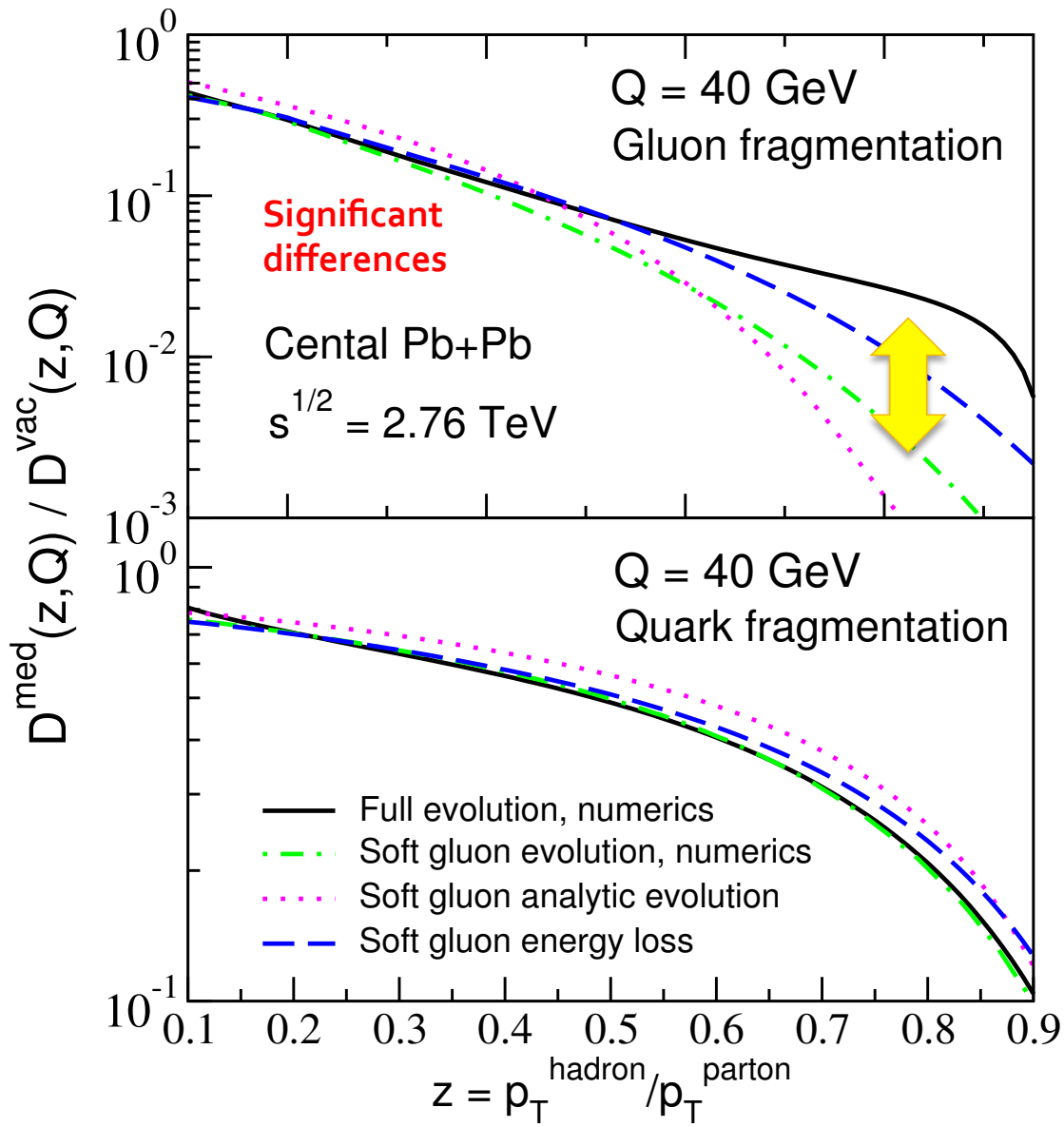
- The coupling between the jet and the medium can be constrained to the same accuracy - 5%
- Full evolution woks slightly better at low virtualities

Future directions

"Modified" fragmentation functions

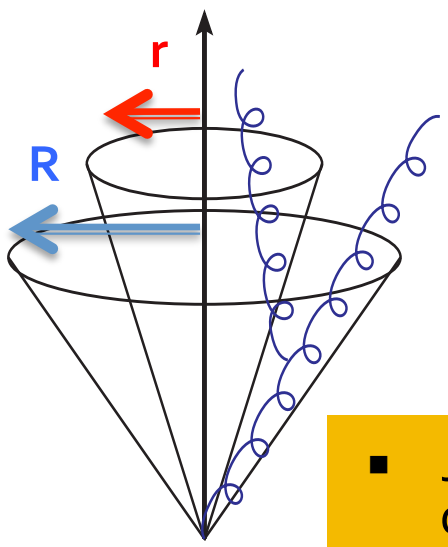
- Dominated by quark fragmentation, small differences, inclusive measurements

- Points the directions where significant improvement can be expected – longitudinal and transverse structure of jets, tagged jets and dijets



Jet shape resummation in SCET

Applications of SCET to jet shapes



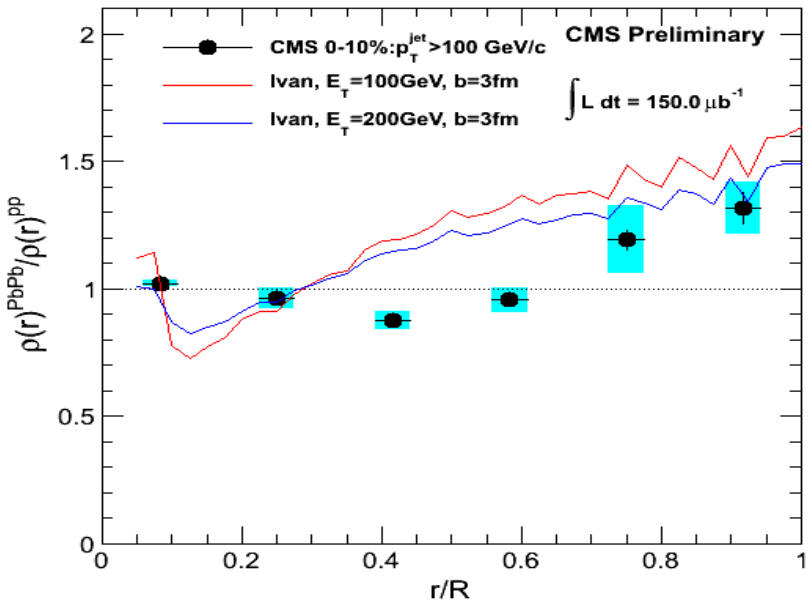
Jet shapes

$$\Psi_{\rm int}(r;R) = \frac{\sum_i (E_T)_i \Theta(r-(R_{\rm jet})_i)}{\sum_i (E_T)_i \Theta(R-(R_{\rm jet})_i)},$$

$$\psi(r;R) = \frac{d\Psi_{\rm int}(r;R)}{dr}.$$

- Jet shapes reflect the energy density inside the jet and the structure of the parton shower

Early prediction and "comparison" to CMS data



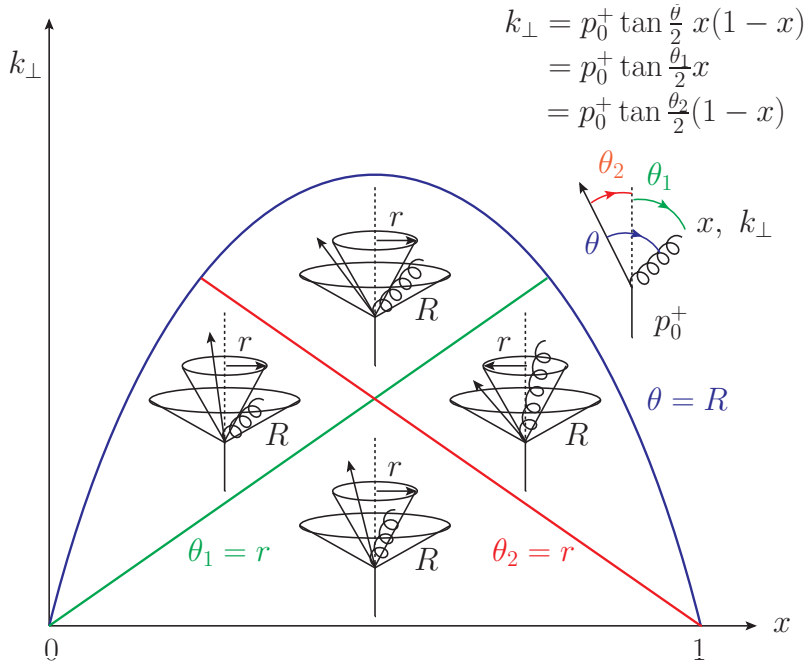
$$L = \log \frac{r}{R}$$

$$C(\bar{n}\cdot p,\mu) = 1 + a_s[L^2 + L + 1] + a_s^2[L^4 + L^3 + L^2 + L + 1] + a_s^3[L^6 + L^5 + L^4 + L^3 + L^2 + L + 1]$$

LL NLL

- Small and intermediate r/R - this is the region where we hope we can improve using SCET resummation techniques and full $SCET_G$ medium-induced splittings

Phase space for the jet energy distribution



- To first non-trivial order, the phase space for the jet shape contributions is tractable

Y.-T. Chien et al. (2014)

- Define a jet energy function

$$J_\omega(E_r,\mu) = \sum_{X_c} \langle 0|\bar{\chi}_\omega(0)|X_c\rangle\langle X_c|\chi_\omega(0)|0\rangle \delta(E_r - \hat{E}^{<r}(X_c))$$

- Need the distribution of the average energy

$$J_\omega^{E_r}(\mu) = \int dE_r E_r\ J_\omega(E_r,\mu)$$

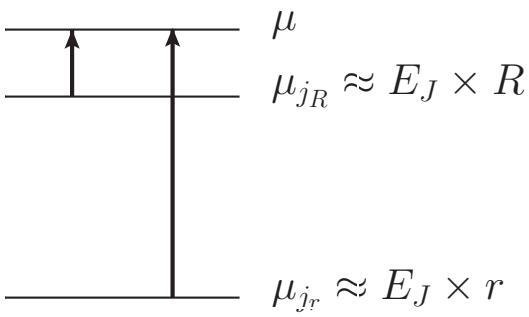
$$\Psi_\omega(r) = \frac{\langle E_r\rangle_\omega}{\langle E_R\rangle_\omega} = \frac{J_\omega^{E_r}(\mu)/J_\omega(\mu)}{J_\omega^{E_R}(\mu)/J_\omega(\mu)} = \frac{J_\omega^{E_r}(\mu)}{J_\omega^{E_R}(\mu)}$$

- Integral jet function

$$\frac{2}{\omega} J_\omega^{qE_r}(\mu) = \alpha_s \left[a \ln^2 \frac{\omega^2 \tan^2\frac{r}{2}}{\mu^2} + b \ln \frac{\omega^2 \tan^2\frac{r}{2}}{\mu^2} + \text{finite} \right]$$

NLL calculation of jet shapes

- We use SCET resummation techniques and $\text{SCET}_{\text{G.}}$



We start form the natural scales that eliminate all large logarithms in the fixed order calculation and evolve to a common scale [resumming ln(r/R)]

$$\frac{dJ_\omega^{qE_r}(\mu)}{d\ln\mu} = \left[-C_F\Gamma_{\text{cusp}}(\alpha_s)\ln\frac{\omega^2\tan^2\frac{R}{2}}{\mu^2} - 2\gamma^q(\alpha_s)\right] J_\omega^{qE_r}(\mu)$$

$$\frac{dJ_\omega^{gE_r}(\mu)}{d\ln\mu} = \left[-C_A\Gamma_{\text{cusp}}(\alpha_s)\ln\frac{\omega^2\tan^2\frac{R}{2}}{\mu^2} - 2\gamma^g(\alpha_s)\right] J_\omega^{gE_r}(\mu)$$

- To resum the jet shape to NLL accuracy

$$\Gamma_{\text{cusp}}(\alpha_s) = \left(\frac{\alpha_s}{4\pi}\right)\Gamma_0 + \left(\frac{\alpha_s}{4\pi}\right)^2\Gamma_1 + \cdots ,$$

$$\gamma(\alpha_s) = \left(\frac{\alpha_s}{4\pi}\right)\gamma_0 + \left(\frac{\alpha_s}{4\pi}\right)^2\gamma_1 + \cdots .$$

Order	Γ_{cusp}	γ	β
NLL	2-loop	1-loop	2-loop

NLL	1-loop	2-loop
β	$\beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_f$	$\beta_1 = \frac{34}{3}C_A^2 - \frac{20}{3}C_A T_F n_f - 4C_F T_F n_f$
Γ_{cusp}	$\Gamma_0 = 4$	$\Gamma_1 = 4\left[\left(\frac{67}{9} - \frac{\pi^2}{3}\right)C_A - \frac{20}{9}T_F n_f\right]$
γ	$\gamma_0^q = -3C_F,\ \gamma_0^g = -\beta_0$	

Solution to the resummed jet shape

- Define

$$S(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha \frac{\Gamma_{\rm cusp}(\alpha)}{\beta(\alpha)} \int_{\alpha_s(\nu)}^{\alpha} \frac{d\alpha'}{\beta(\alpha')} ,$$

$$A_i(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha \frac{\gamma^i(\alpha)}{\beta(\alpha)} , \quad A_\Gamma(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha \frac{\Gamma_{\rm cusp}(\alpha)}{\beta(\alpha)} ,$$

- Jet function

$$J_\omega^{iE_r}(\mu) = J_\omega^{iE_r}(\mu_{j_r}) \exp\left[-2C_i S(\mu_{j_r},\mu) + 2A_i(\mu_{j_r},\mu)\right] \left(\frac{\omega^2 \tan^2\frac{R}{2}}{\mu_{j_r}^2}\right)^{C_i A_\Gamma(\mu_{j_r},\mu)}$$

- Integral jet shape

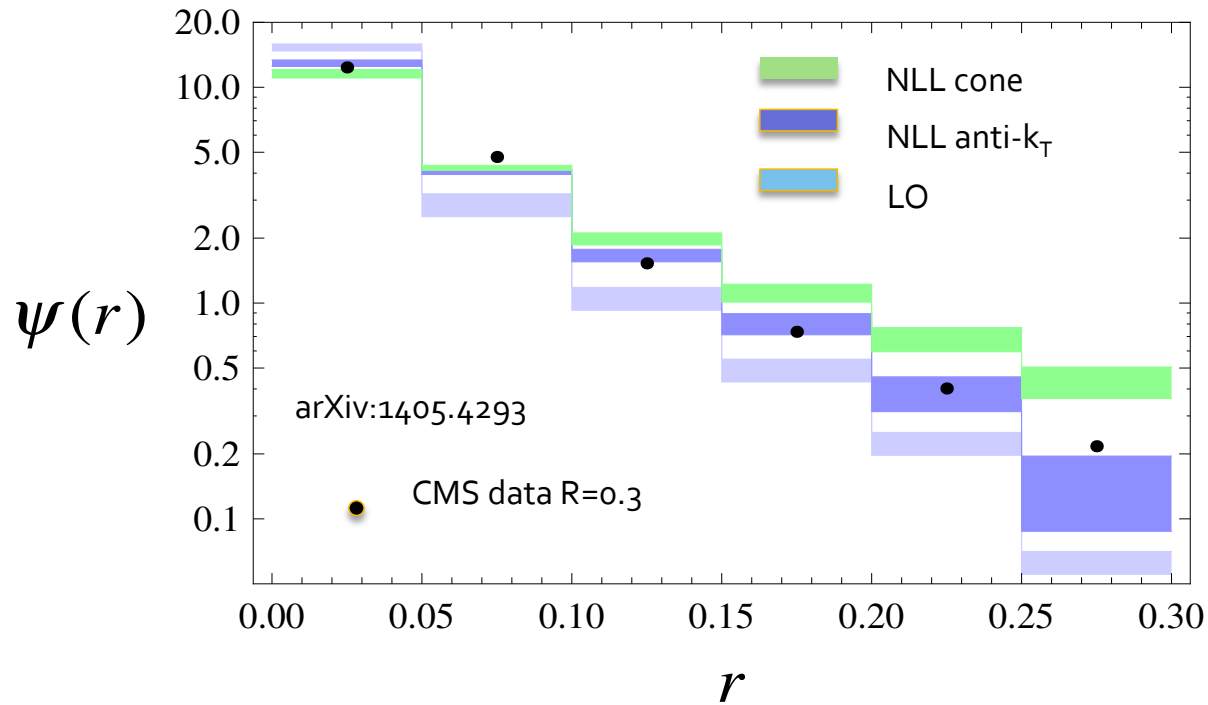
$$\Psi_\omega^i(r) = \frac{J_\omega^{iE_r}(\mu)}{J_\omega^{iE_R}(\mu)} = \frac{J_\omega^{iE_r}(\mu_{j_r})}{J_\omega^{iE_R}(\mu_{j_R})} \exp[-2C_i S(\mu_{j_r},\mu_{j_R}) + 2A_i(\mu_{j_r},\mu_{j_R})] \times \left(\frac{\mu_{j_r}^2}{\omega^2 \tan^2\frac{R}{2}}\right)^{C_i A_\Gamma(\mu_{j_R},\mu_{j_r})}$$

Y.-T.Chien et al. (2014)

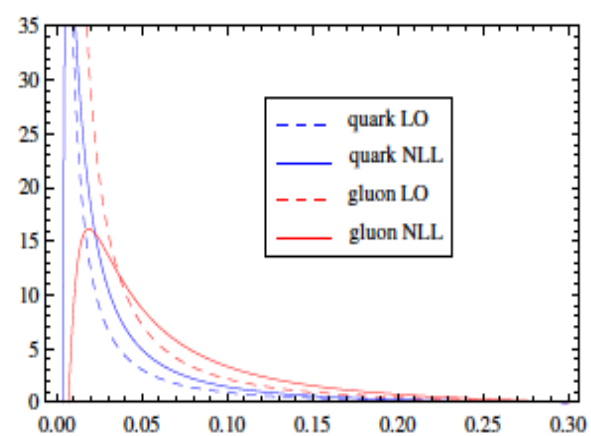
We start form the natural scales that eliminate all large logarithms in the fixed order calculation and evolve to a common scale [resumming ln(r/R)]

Numerical NLL results in p+p collisions

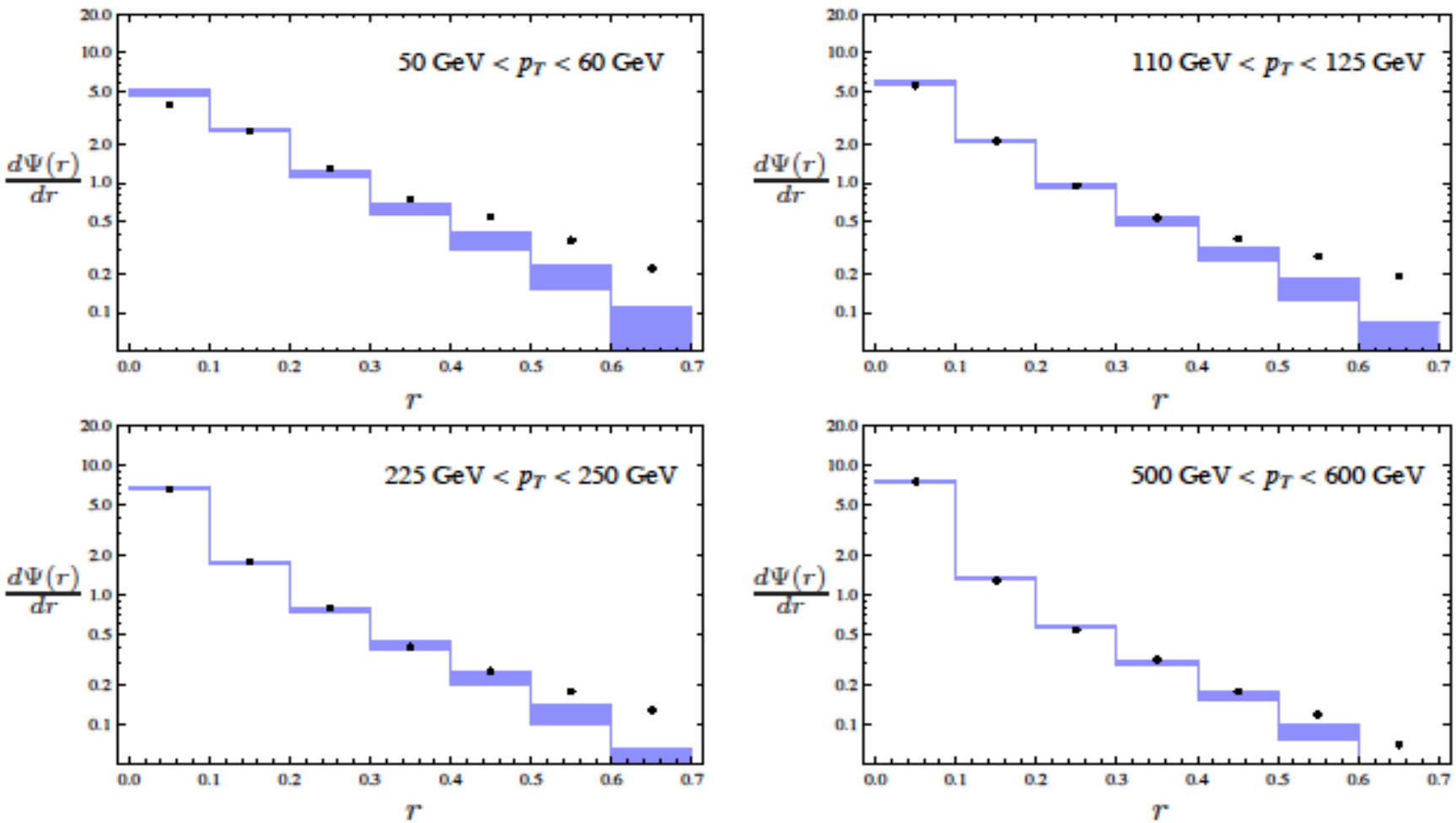
- We derived the algorithm dependence of the jet shapes (anti)k_T vs cone
- Significant improvement over fixed order calculation



Just t show that the fixed order is divergent



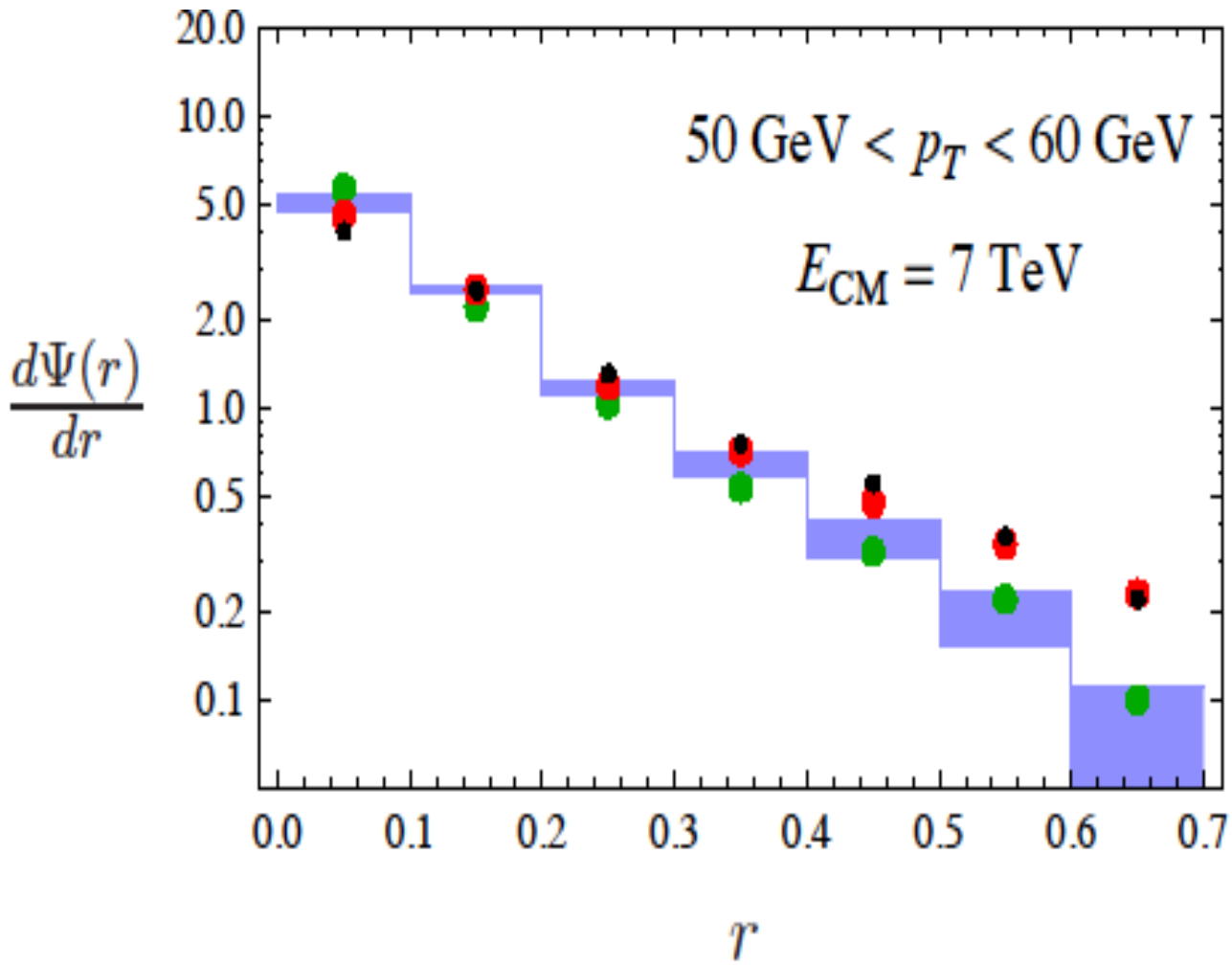
Limitations of the calculation



- Deviations at large R and/or small energy

NLL results in p+p collisions

- The calculation does not include initial-state radiation/beam functions and hadronization effects
- Power suppressed but visible at the tail of the distribution and lower p_T



Calculation of jet shapes in the medium

- A *preliminary* result

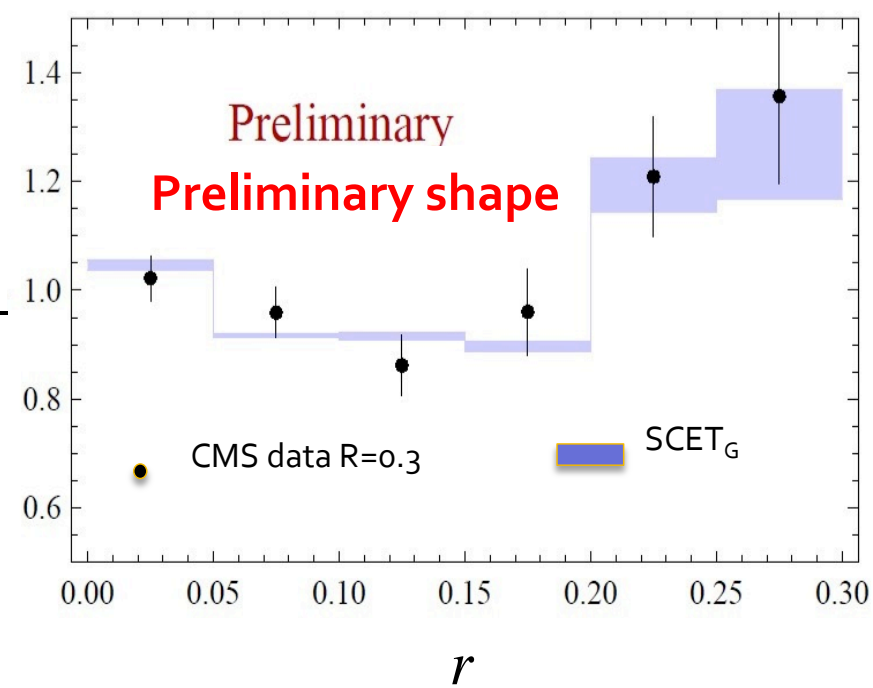
$$J^i_{E,r}(\mu) = \sum_{i,k} \int_{PS} dx dk_\perp \left[\frac{dN^{vac}_{i\to jk}}{dx d^2k_\perp} + \frac{dN^{med}_{i\to jk}}{dx d^2k_\perp} \right] E_r(x, k_\perp)$$

- Need to combine with the splitting functions evalualted numerically for the same medium geometry and expansion, present predictions for the upcoming run at the LHC

$$\Psi(r) = \frac{J^{vac}_{E,r} + J^{med}_{E,r}}{J^{vac}_{E,R} + J^{med}_{E,R}} = \frac{\Psi^{vac}(r) J^{vac}_{E,R} + J^{med}_{E,r}}{J^{vac}_{E,R} + J^{med}_{E,R}}$$

- Even with the caveats, the improvement in the calculation can be significant

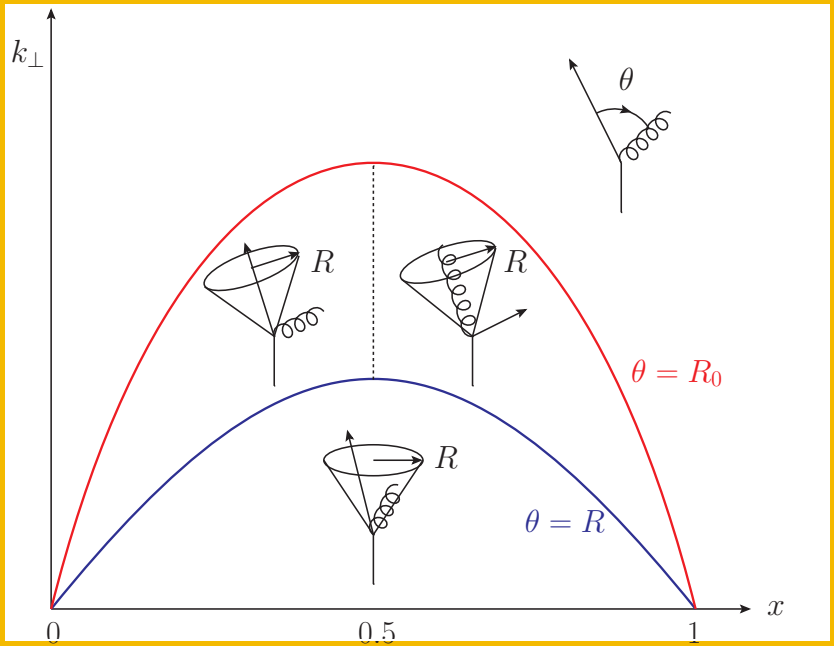
$$\frac{\psi(r)^{PbPb}}{\psi(r)^{pp}}$$



Y.-T. Chien et al. In preparation

Future directions

- Complete the calculation of the medium-modified jet shapes and move to other jet substructure observables
- Evaluate the suppression of jets beyond the naïve energy loss approximation
- Extend the theory to couple the soft and Glauber sectors and look at soft particle redistribution of the jet energy (a la CMS)



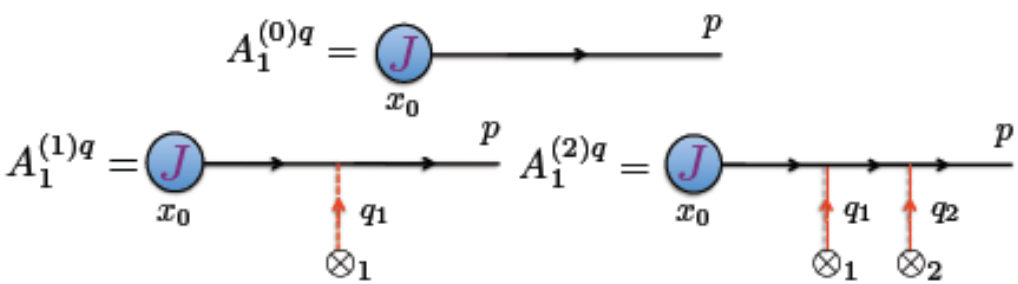
- Evaluate initial-state med-induced splitting kernels, mass etc

Conclusions

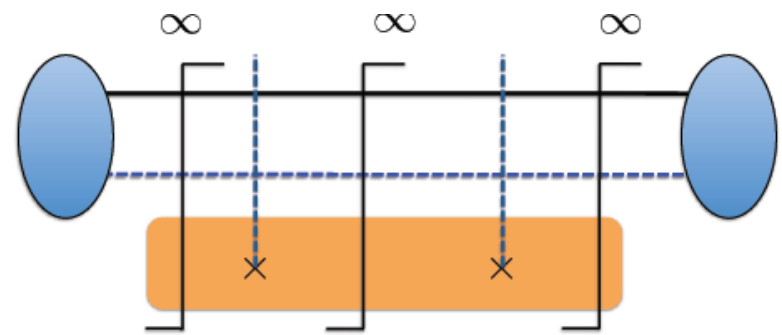
- Effective theories of QCD have enabled important conceptual and breakthroughs in our understanding of strong interactions and very significant improvement in the accuracy of the theoretical predictions
- An effective theory for jet propagation in matter $SCET_G$ was constructed (collinear and Glauber sectors). Derived all medium-induced parton splittings, proved factorization and gauge invariance for the medium-induced parton splittings.
- Established the connection between the traditional energy loss approaches and the evolution/parton shower approach to jet quenching. Allows to consistently improve the accuracy of the calculations going to higher orders and perform resummation. Demonstrated that we can constrain non-perturbative properties of the QGP to 20%.
- We identified observables where the full splitting kernels can make a significant difference. Evaluated jet shapes in p+p collisions to NLL accuracy (no IS radiation).
- We have preliminary results in A+A reactions are quite encouraging. However, we have to finish the calculation and compare to data to see of the improvements will carry through

III. Main results: jet broadening

- Jet broadening and its gauge invariance



M. Gyulassy et al. (2001)



Classes of diagrams (single Born, double Born). Reaction Operator

- General result. Will evaluate the broadening (or lack off) of jets

$$\frac{dN^{(n)}(\mathbf{p}_\perp)}{d^2\mathbf{p}_\perp} = \prod_{i=1}^{n} \int_{z_{i-1}}^{L} \frac{dz_i}{\lambda} \int d^2\mathbf{q}_{\perp i} \left[\frac{1}{\sigma_{el}(z_i)} \frac{d\sigma_{el}(z_i)}{d^2\mathbf{q}_{\perp i}} \left(e^{-\mathbf{q}_{\perp i}\cdot\vec{\nabla}_{\mathbf{p}_\perp}} \right) - \delta^2(\mathbf{q}_\perp) \right] \frac{dN^{(0)}(\mathbf{p}_\perp)}{d^2\mathbf{p}_\perp}$$

- In special cases such as constant density and the Gaussian approximation

Starting with a collinear beam of quarks/gluons

we recover

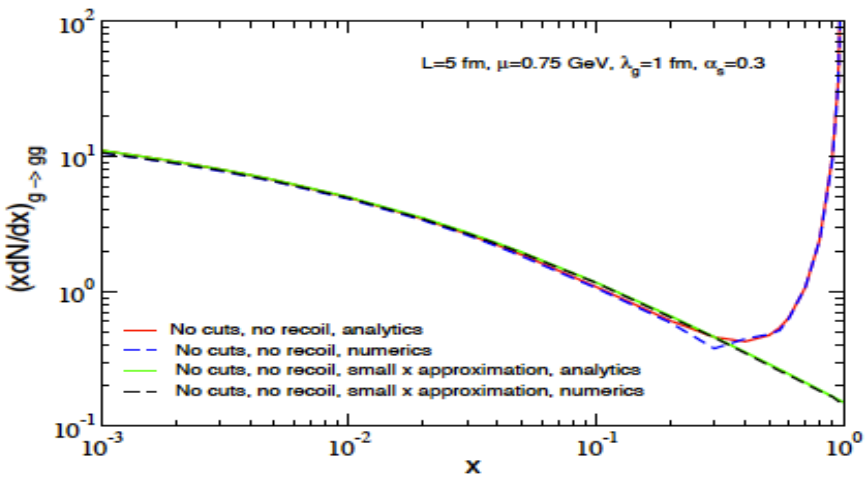
M. Gyulassy et al. (2002)

$$\frac{dN(\mathbf{p}_\perp)}{d^2\mathbf{p}_\perp} = \frac{1}{2\pi} \frac{e^{-\frac{\mathbf{p}^2}{2\chi\mu^2\xi}}}{\chi\mu^2\xi}$$

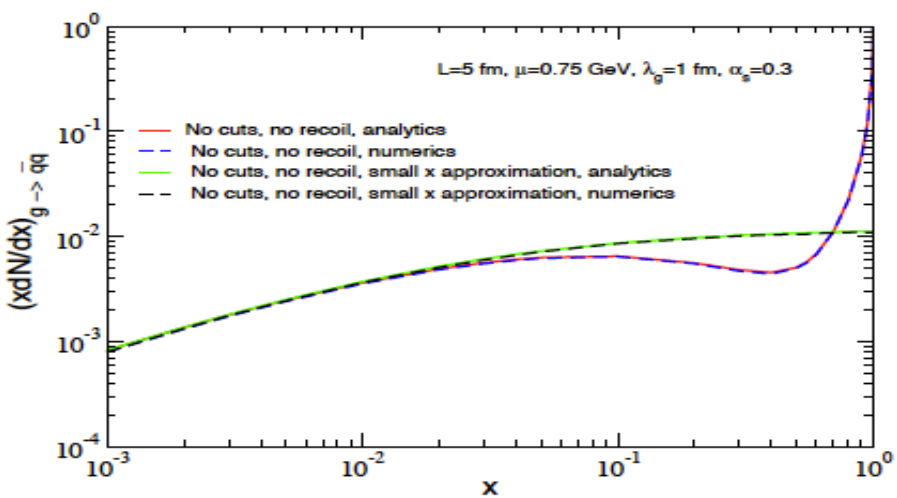
$$\chi = \frac{L}{\lambda}$$

III. Numerical examples

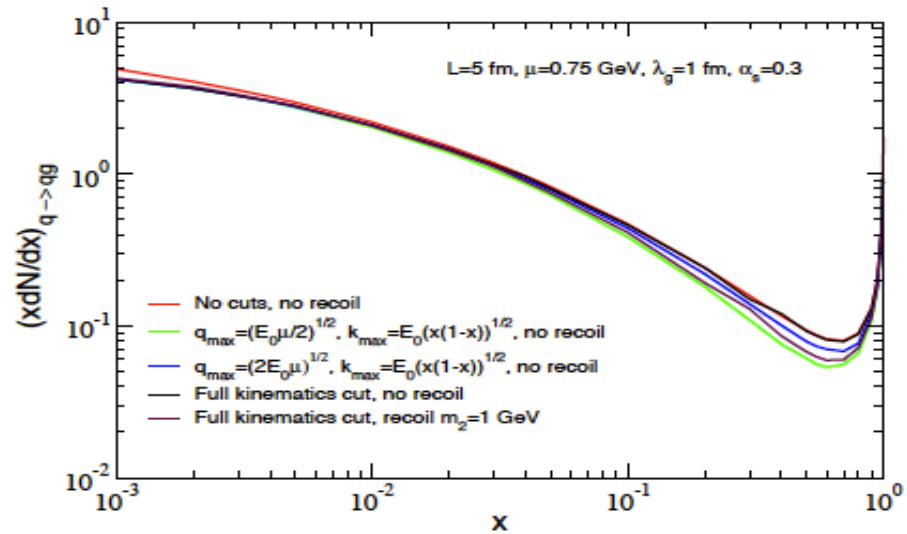
- Leading intensity term



- Sub-leading intensity term

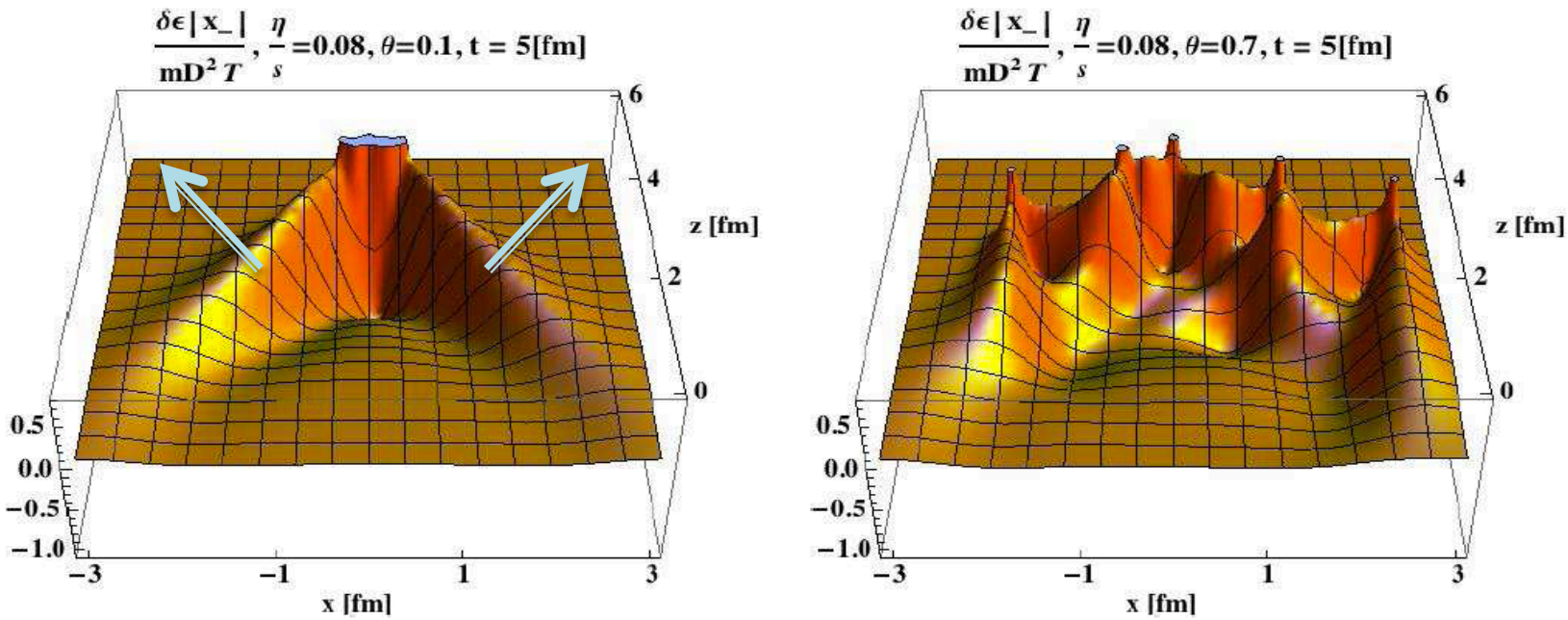


- Kinematic effects



- We have theoretical control over the in-medium splittings
- The large-x and kinematic effects are of the same order
- Will be incorporated in future phenomenological applications

III. Why are Mach cones initiated by jets unlikely



- An individual parton (or a collinear system) can produce a Mach cone on an event by event basis. Multiple events will reduce the observable effect

I. Vitev (2005)

- Typical medium-induced shower multiplicities are N^g=4 (quark) and N^g=8 (gluon) and emitted at large angles ~ 0.7 (much larger than in the vcuum)
- Each parton quickly becomes an individual source of excitation and these multiple sources wipe out any conical signature

IV. Resummation, RG equations and Higgs production at the LHC

- SCET is very effective in resumming in large infrared logarithms using Renormalization group equations

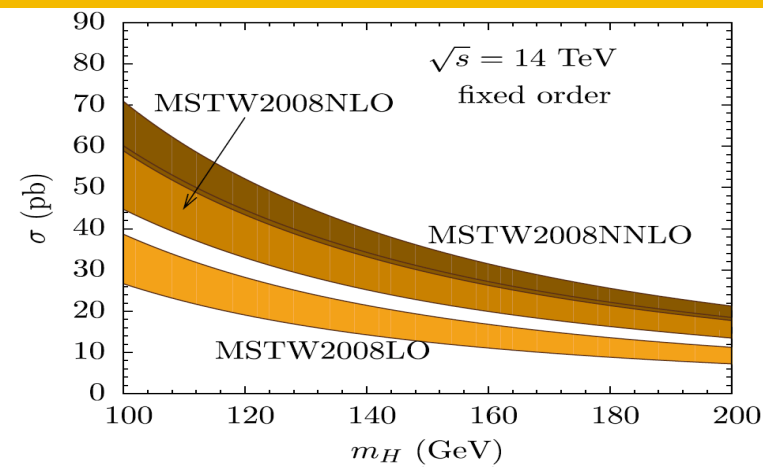
General structure of Sudakov logs

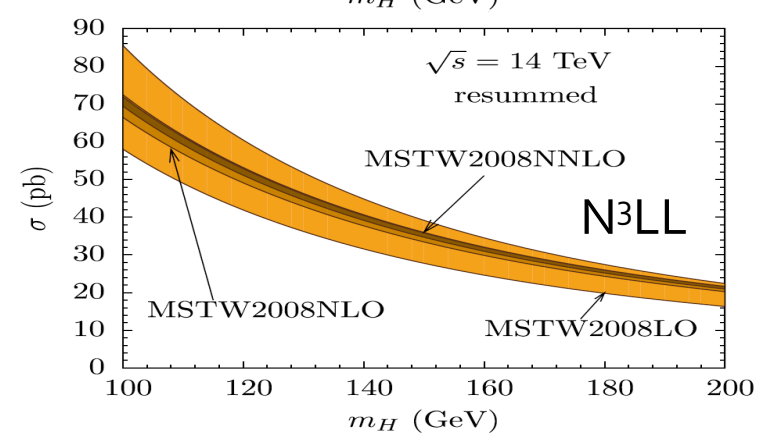
$$a_s = \frac{\alpha_s}{\pi}$$

$$L = \log\left(\frac{\mu}{\bar{n}\cdot p}\right)$$

$$C(\bar{n}\cdot p,\mu) = 1 + a_s[L^2 + L + 1] + a_s^2[L^4 + L^3 + L^2 + L + 1] + a_s^3[L^6 + L^5 + L^4 + L^3 + L^2 + L + 1]$$

LL NLL





- It can improve upon traditional techniques, such as CCS

J. Collins et al. (1985)

V. Ahrens et al. (2009)

IV. Factorization in SCET and angularities

- Factorization theorems have been proven in SCET for a number of observables: event shapes [e^+e^-], Higgs [pp], top [e^+e^-] …

  - Angularity observables: generalization of traditional event shapes

$$\tau_a = \frac{1}{Q}\sum_i |\vec{p}_i^{\,T}| \exp(-\eta_i(1-a))$$

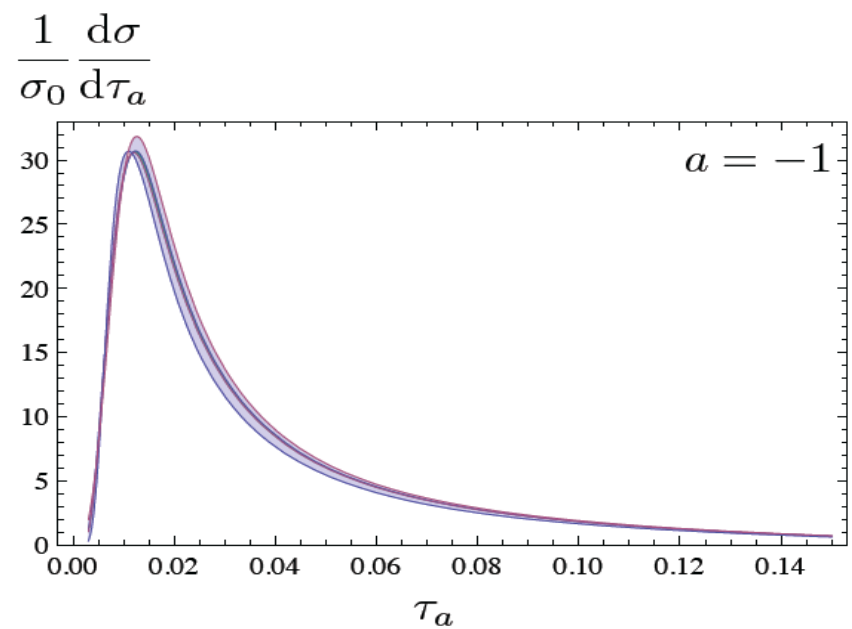
$$-\infty < a < 2$$

C. Berger et al. (2003)

- Factorized in hard function, jet functions and soft function

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}e} = H(Q)\int de_n\, J_n(e_n)\, de_{\bar{n}}\, J_{\bar{n}}(e_{\bar{n}})\, de_s\, S(e_s)\,\delta(e - e_n - e_{\bar{n}} - e_s)$$

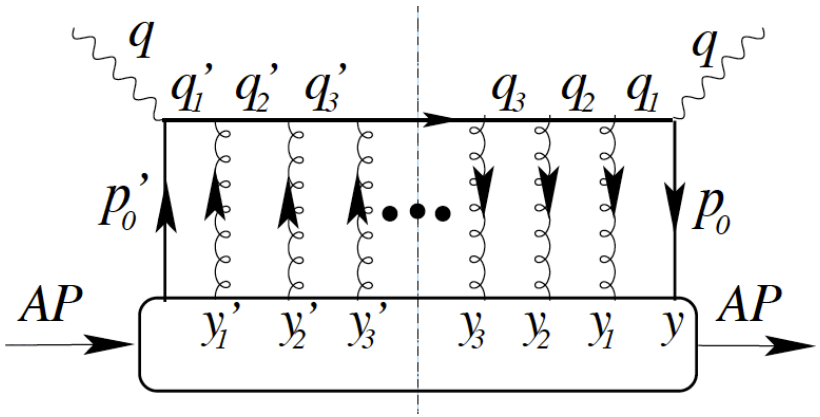
C. Bauer et al. (2008)



A. Hornig et al. (2010)

IV. Applications to nuclear collisions

- Final state parton broadening in semi-inclusive DIS.



A. Idilbi et al. (2008)

$$\frac{d^2 W^{\mu\nu}}{d^2 l_\perp} = e^{(DL^-)\nabla^2_{l_\perp}} \frac{d^2 W_0^{\mu\nu}}{d^2 l_\perp}$$

- Have argued to recover the general QCD result in the Gaussian broadening region

J. Qiu et al. (2003)

- Formulation of a transport coefficient as a Wilson line

$$\hat{q} = \langle q_\perp^2 \rangle / \lambda_g$$

$$W_F\left[y^+, y_\perp\right] \equiv P\left\{\exp\left[ig\int_0^{L^-} dy^-\, A^+(y^+, y^-, y_\perp)\right]\right\}$$

$$\sum_{m=1,n=1}^{\infty} \frac{d^2 \mathcal{A}_{nm}}{d^2 k_\perp} = \frac{\sqrt{2}}{L^3 N_c} \int dy^+ dy_\perp dy'_\perp\, e^{-ik_\perp\cdot(y_\perp - y'_\perp)} \left\langle \mathrm{Tr}\left[\left(W_F^\dagger[y^+, y'_\perp] - 1\right)\left(W_F[y^+, y_\perp] - 1\right)\right]\right\rangle$$

- Not gauge-invariant. Proof needed

F. D'Eramo et al. (2010)

SCET formulation

Energetic quarks and leptons
collinear modes

Include also soft quarks and gluons

C. Bauer et al. (2001)

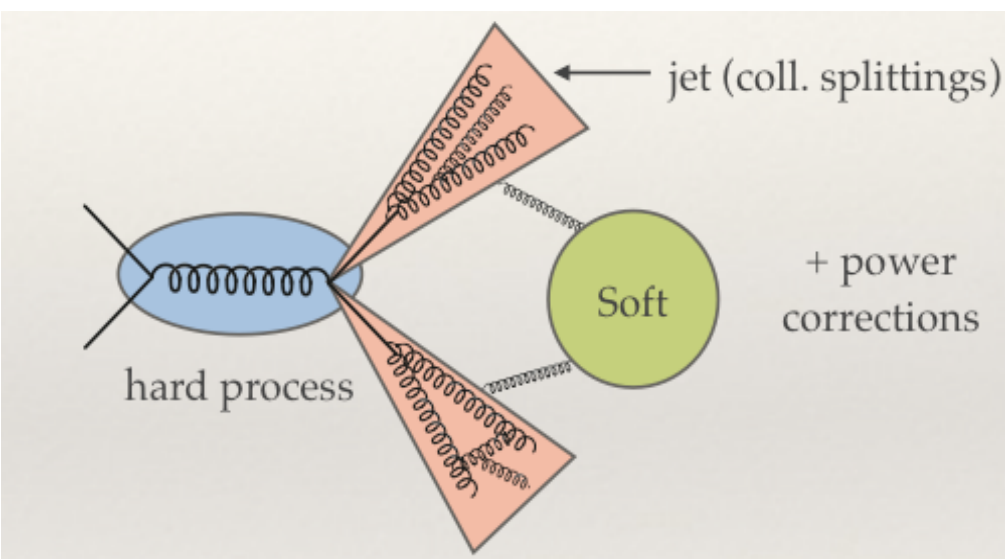
D. Pirol et al. (2004)

$$p_c = (p_+, p_-, p_\perp) \sim (\frac{\Lambda^2}{Q}, Q, \Lambda) = Q(\lambda^2, 1, \lambda)$$

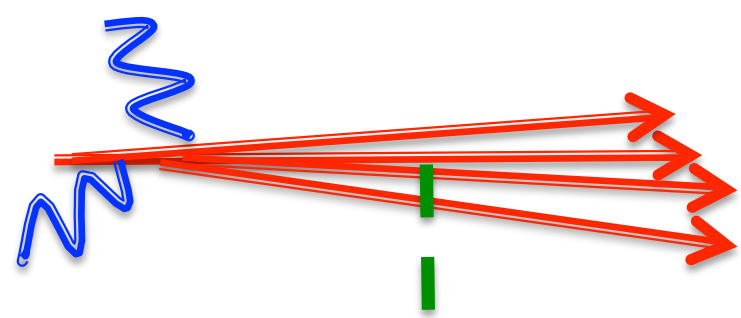
$$p_s = (p_+, p_-, p_\perp) \sim (\Lambda, \Lambda, \Lambda) = Q(\lambda, \lambda, \lambda)$$

- SCET Lagrangian to all orders in λ [Can expand to LO, NLO,…]

Collinear quarks, antiquarks	$\xi_n,\ \bar{\xi}_n$
Collinear gluons, soft gluons	$A_n,\ A_s$



- The missing mode

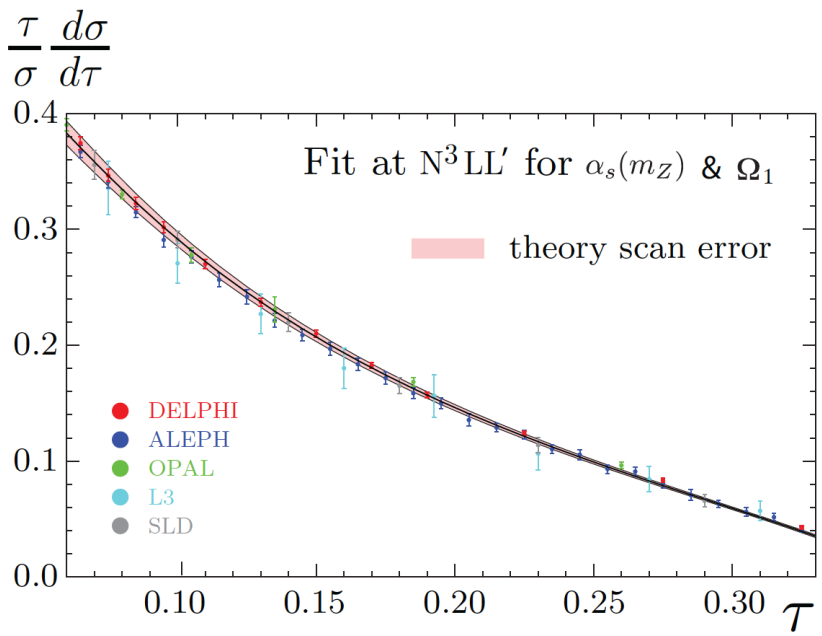


$$q \sim [\lambda^2, \lambda^2, \lambda]$$

A new mode – the Glauber gluon

Success of SCET

- Most precise $\alpha_s(m_Z)$ extraction from global thrust distribution at N³LL′

$$\alpha_s(M_Z) = 0.1135 \pm 0.0002_{\text{expt}} \pm 0.0005_{\text{hadr}} \pm 0.0009_{\text{pert}}$$

R. Abbate et al. (2010)