Fragmentation Functions and Fragmenting Jet Functions in Quarkonium Production

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Review of Quarkonium Production Theory

Double Parton Fragmentation Functions

Heavy Quarkonium Fragmenting Jet Functions

Review of Quarkonium Production Theory

Color-Singlet Model (pre-1995)

$$\sigma(pp \to J/\psi + X) = f_{g/p} \otimes f_{g/p} \otimes \sigma[gg \to c\bar{c}(^3S_1^{(1)}) + X]\,|\psi_{c\bar{c}}(0)|^2$$

calculable in QCD pert. theory

known from

$\bar{c}c$ pair: same quantum numbers as J/ψ

$$\Gamma[J/\psi \to \ell^+\ell^-]$$

Suffers from theoretical inconsistencies when applied to χ_{cJ}

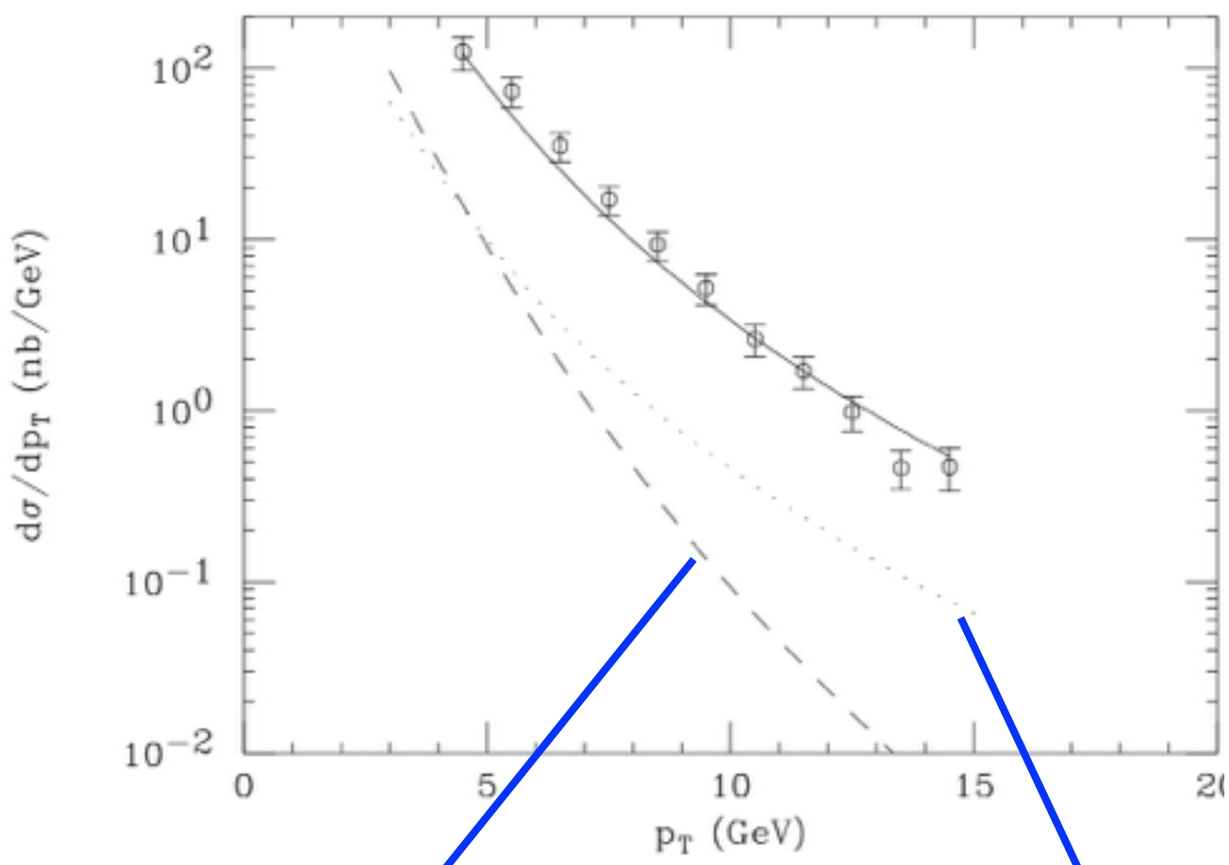
$$\Gamma[\chi_{cJ} \to \text{hadrons}] = |\psi'_{c\bar{c}}(0)|^2 \sigma[c\bar{c}(^3P_J^{(1)}) \to ggg, gq\bar{q}]$$

Not IR Safe

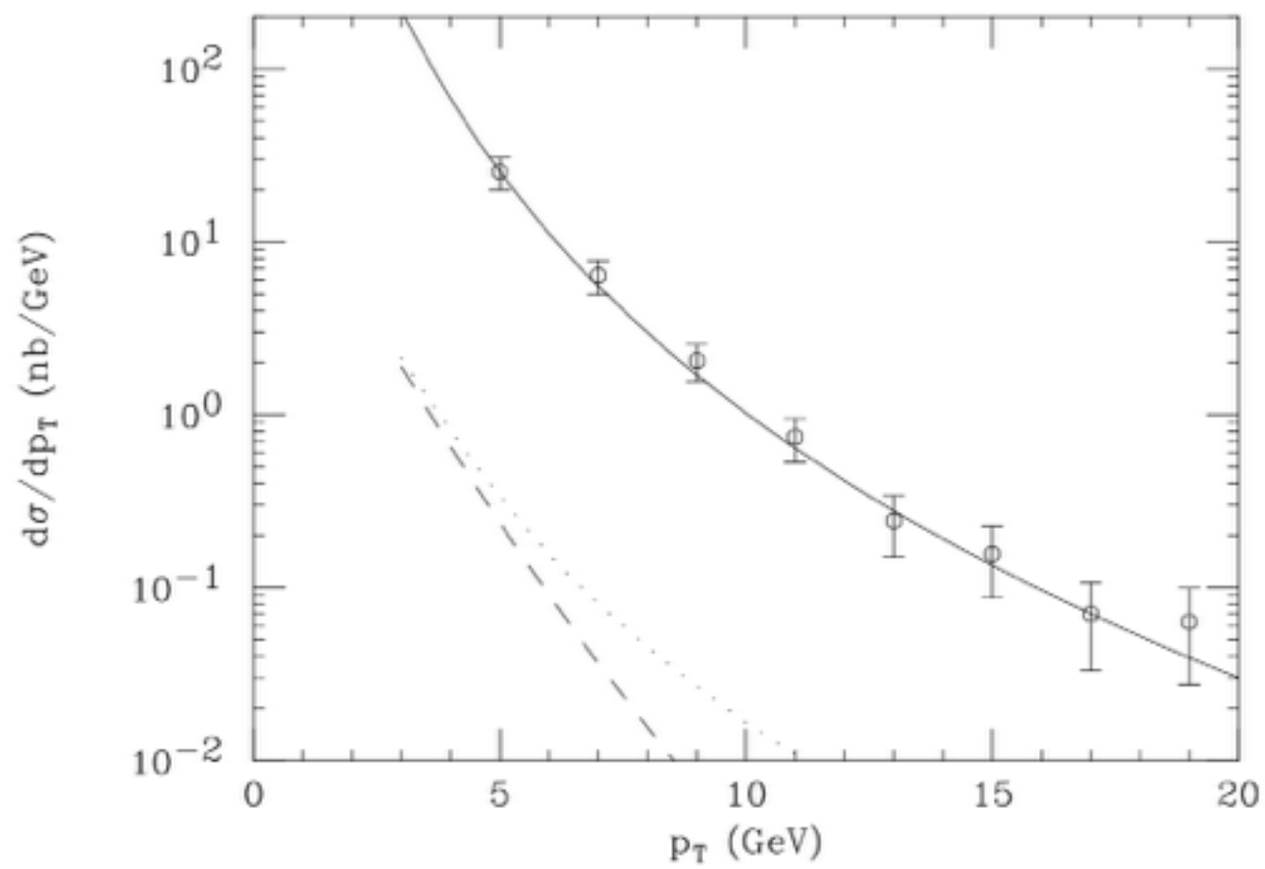
J/ψ production at Tevatron (1996)

CSM badly underpredicts J/ψ and ψ' production at large p_T

J/ψ



ψ'



CSM (LO)

CSM (Fragmentation)

Braaten, Fleming, Yuan, 1996

Non-Relativistic QCD (NRQCD) Factorization Formalism

(Bodwin, Braaten, Lepage)

$$\sigma(gg \to J/\psi + X) = \sum_n \sigma(gg \to c\bar{c}(n) + X)\langle \mathcal{O}^{J/\psi}(n)\rangle$$

$$n - {}^{2S+1}L_J^{(1,8)}$$

double expansion in α_s, v

NRQCD long-distance matrix element (LDME)

$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[1]})\rangle \sim v^3$$

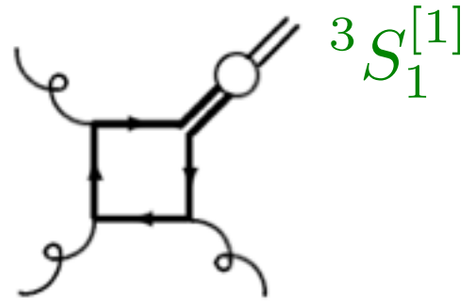
CSM - lowest order in v

$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]})\rangle, \langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]})\rangle, \langle \mathcal{O}^{J/\psi}({}^3P_J^{[8]})\rangle \sim v^7$$

color-octet mechanisms

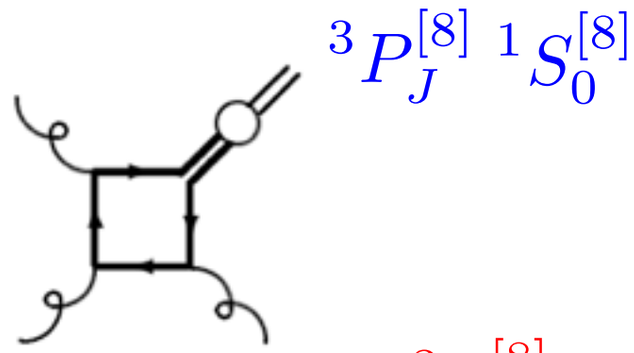
Mechanisms contributing to J/ψ production

CSM (LO)



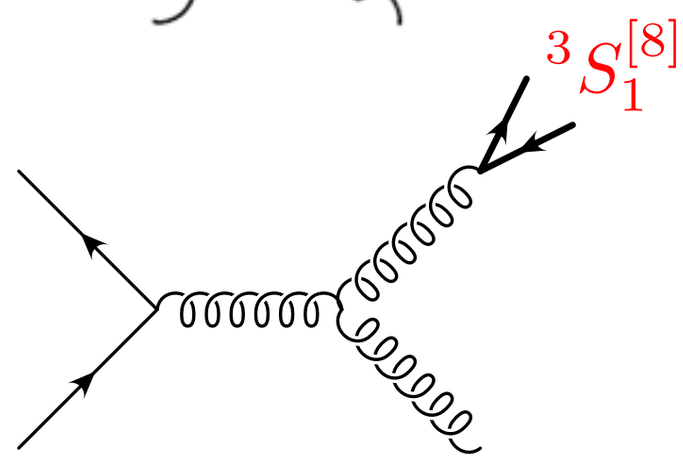
$$\frac{d\sigma}{dp_\perp^2} \sim \alpha_s^3 v^3 \frac{m_Q^4}{p_\perp^8}$$

COM (LO)



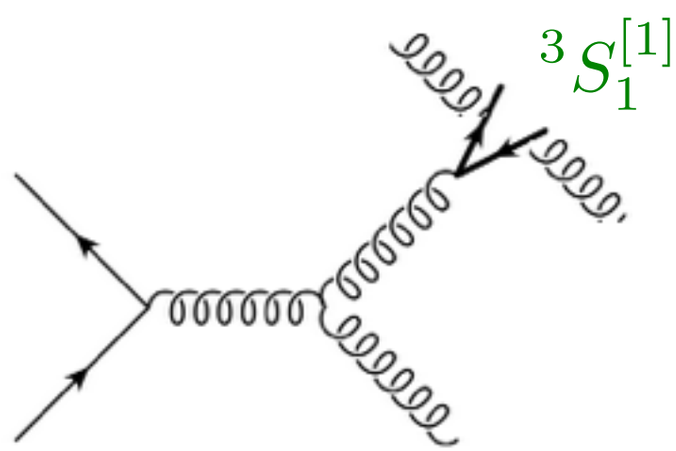
$$\alpha_s^3 v^7 \frac{m_Q^2}{p_\perp^6}$$

COM
(Fragmentation)

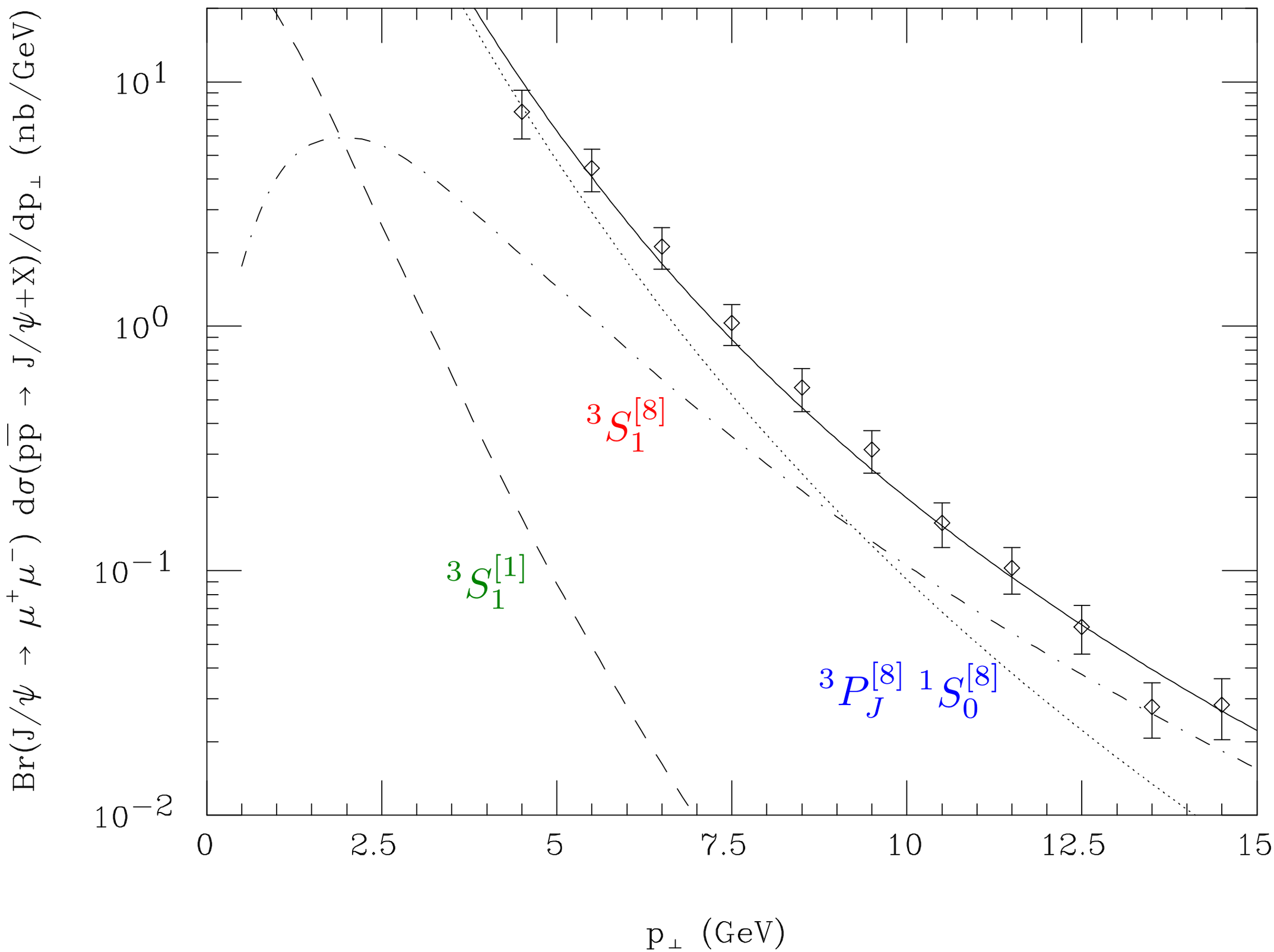


$$\alpha_s^3 v^7 \frac{1}{p_\perp^4}$$

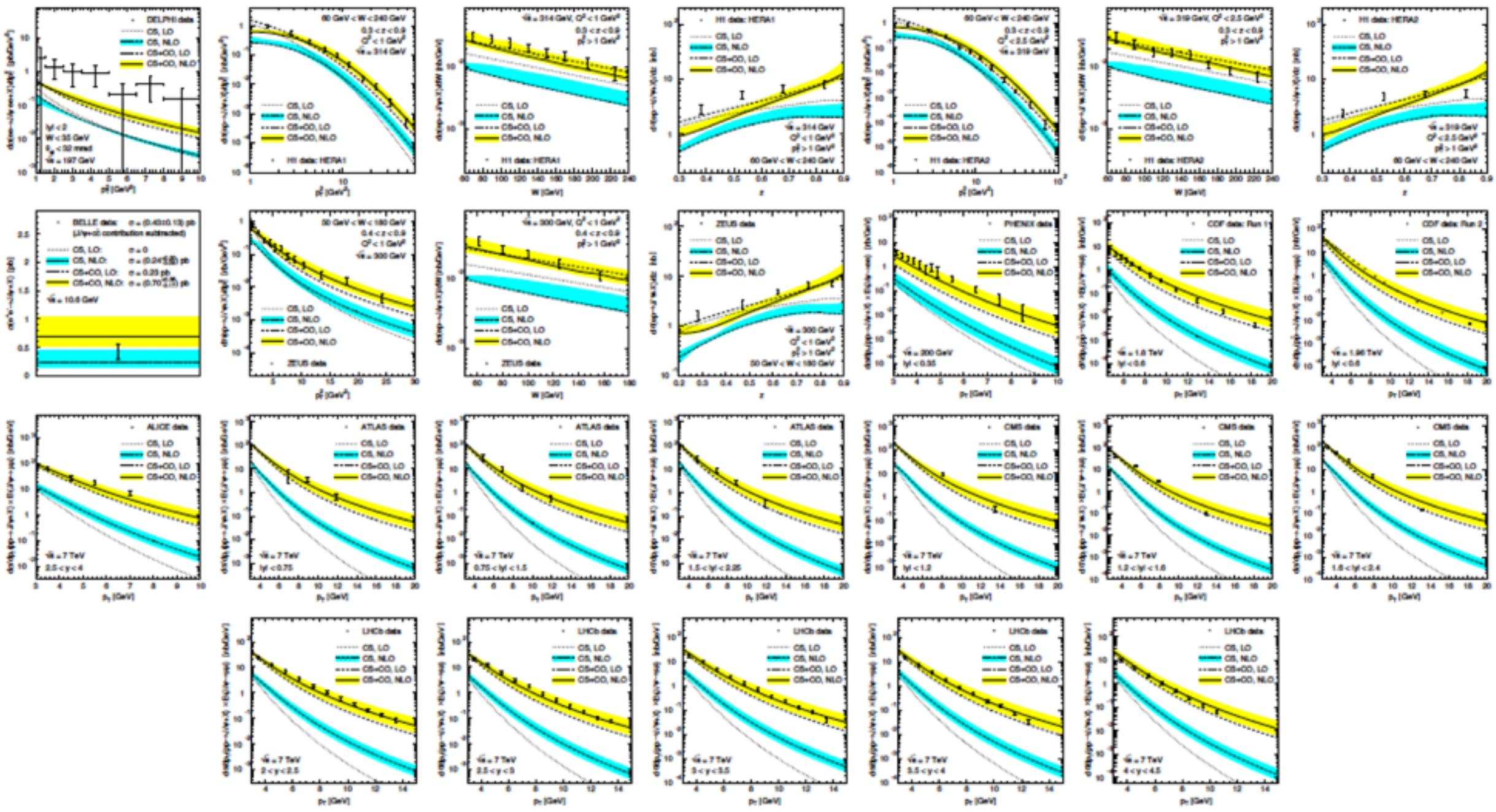
CSM
(Fragmentation)



$$\alpha_s^5 v^3 \frac{1}{p_\perp^4}$$


$Br(J/\psi \to \mu^+\mu^-)\ d\sigma(p\bar{p} \to J/\psi + X)/dp_\perp$ (nb/GeV)
10^1
10^0
10^{-1}
10^{-2}
0
2.5
5
7.5
10
12.5
15
$p_\perp$ (GeV)
$^3S_1^{[8]}$
$^3S_1^{[1]}$
$^3P_J^{[8]}$ $^1S_0^{[8]}$

Global Fits with NLO CSM + COM

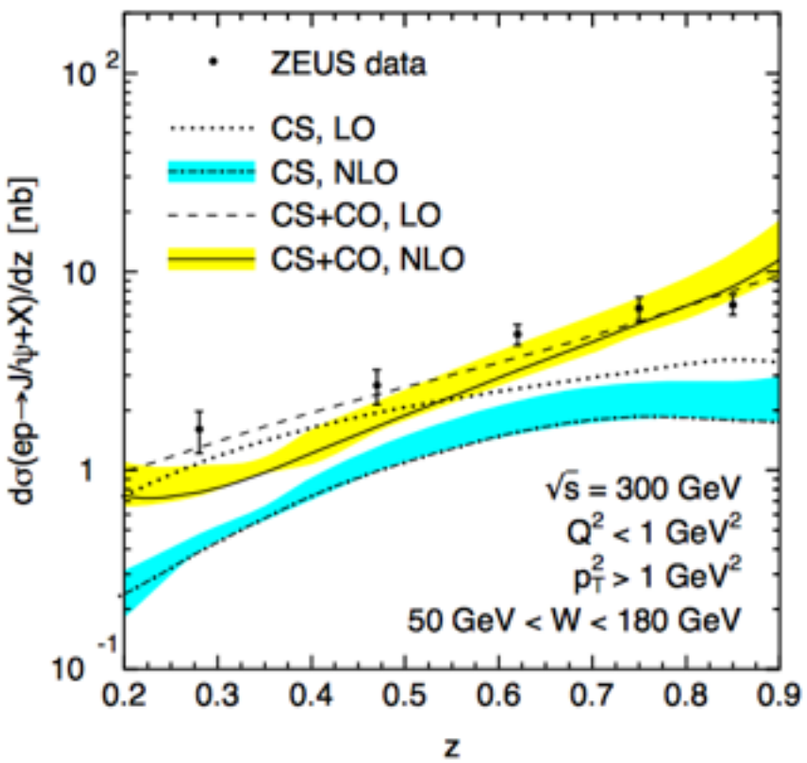


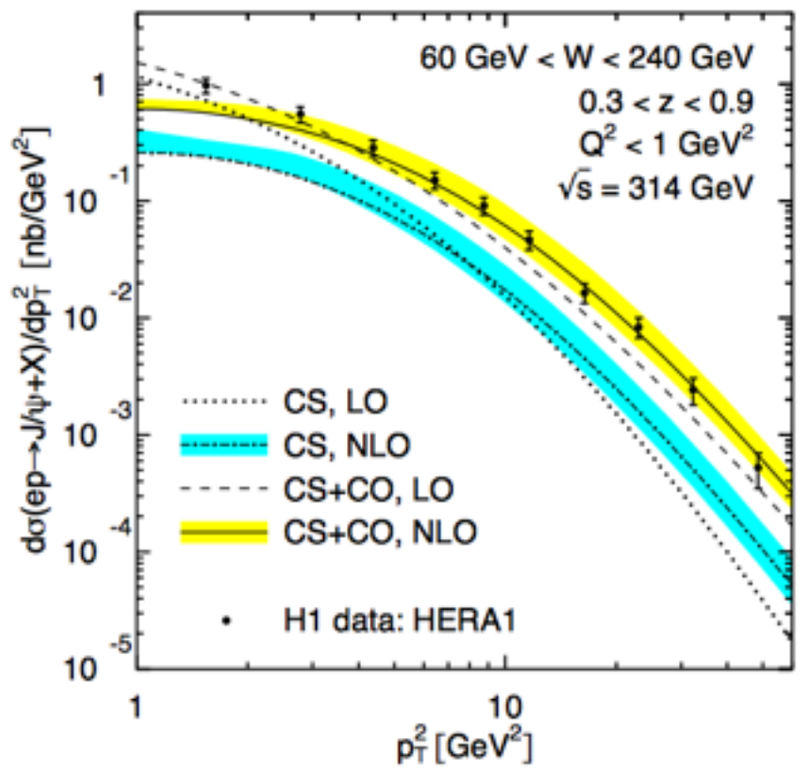
$$e^+e^-, \gamma\gamma, \gamma p, p\bar{p}, pp \to J/\psi + X$$

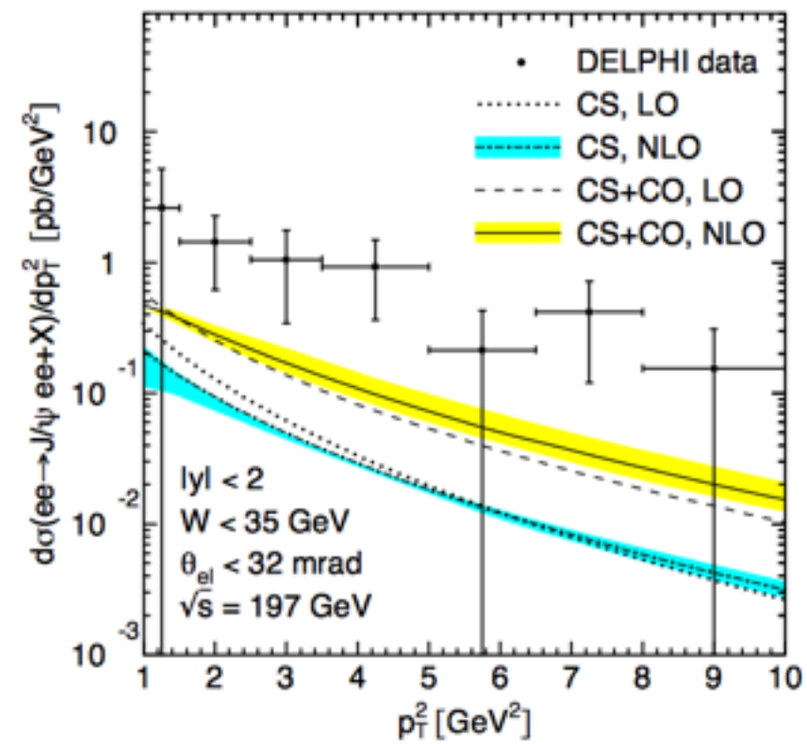
fit to 194 data points, 26 data sets,

Butenschoen and Kniehl, PRD 84 (2011) 051501, arXiv:1105.0820

NLO: CSM + COM Required to Fit Data







$$ep \to J/\psi + X$$

$$\gamma^*\gamma^* \to J/\psi + X$$

Status of NRQCD approach to J/ψ Production

NLO: COM + CSM required for most processes

extracted LDME satisfy NRQCD v-scaling

$$\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]})\rangle = 1.32 \text{ GeV}^3$$

$\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]})\rangle$	$(4.97 \pm 0.44) \times 10^{-2}$ GeV^3
$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\rangle$	$(2.24 \pm 0.59) \times 10^{-3}$ GeV^3
$\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]})\rangle$	$(-1.61 \pm 0.20) \times 10^{-2}$ GeV^5

$$\chi^2_{\text{d.o.f.}} = 857/194 = 4.42$$

Outstanding Problems

Polarization

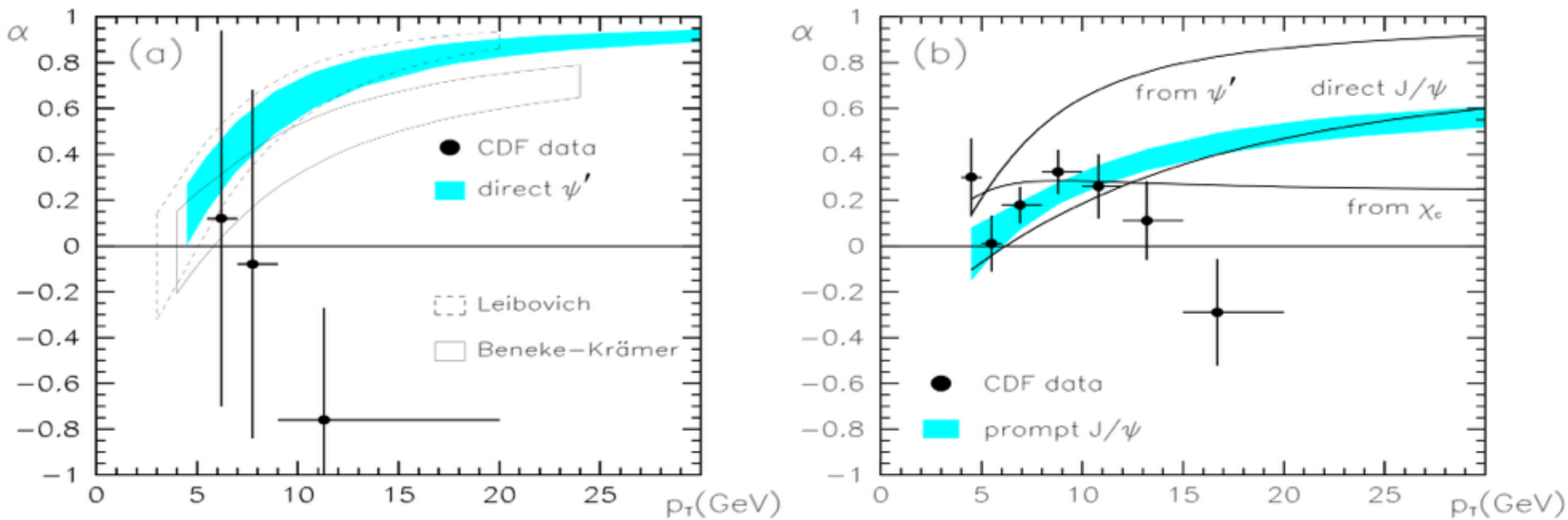
Convergence

resummation of $\log(p_\perp/m_Q)$

LHC: $p_\perp \sim 70\,\mathrm{GeV}\,(J/\psi)$ $\quad p_\perp \sim 50\,\mathrm{GeV}\,(\Upsilon)$

Polarization Puzzle

$^3S_1^{[8]}$ fragmentation at large p_T predicts transversely polarized J/ψ, ψ'

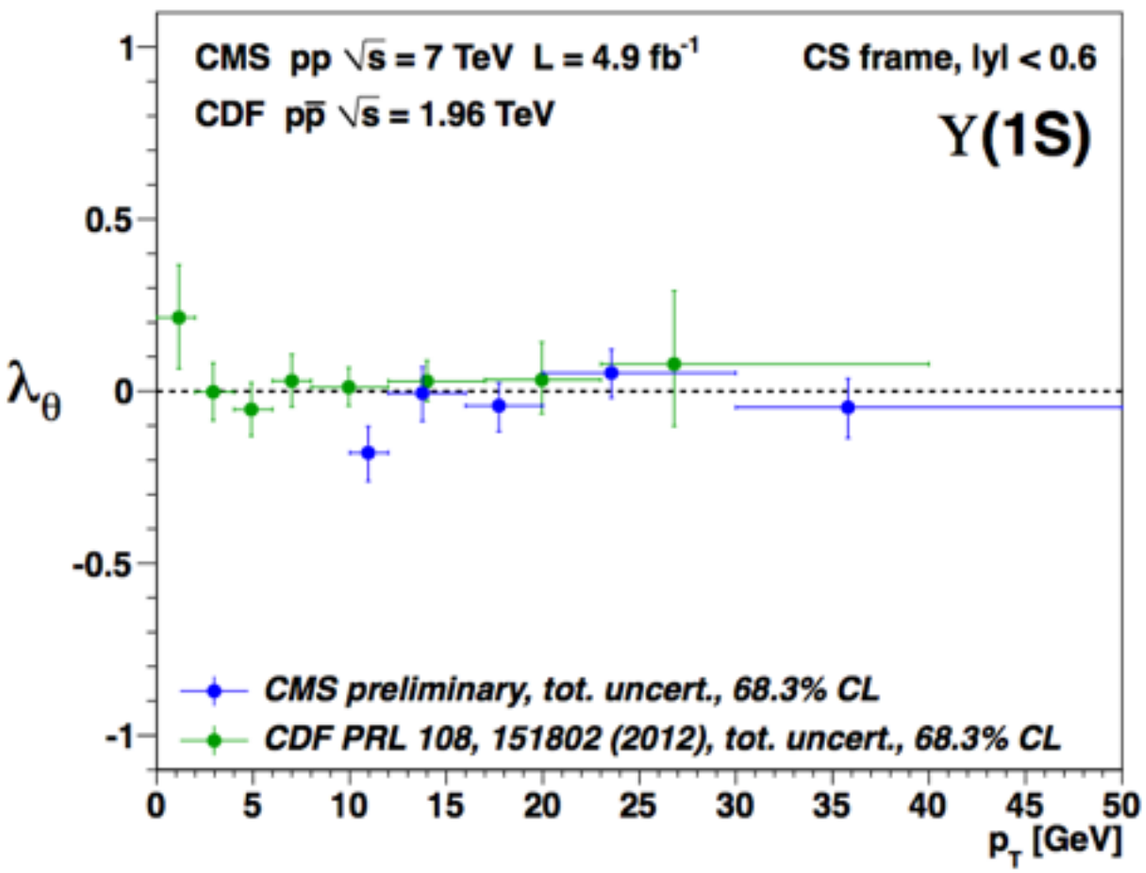


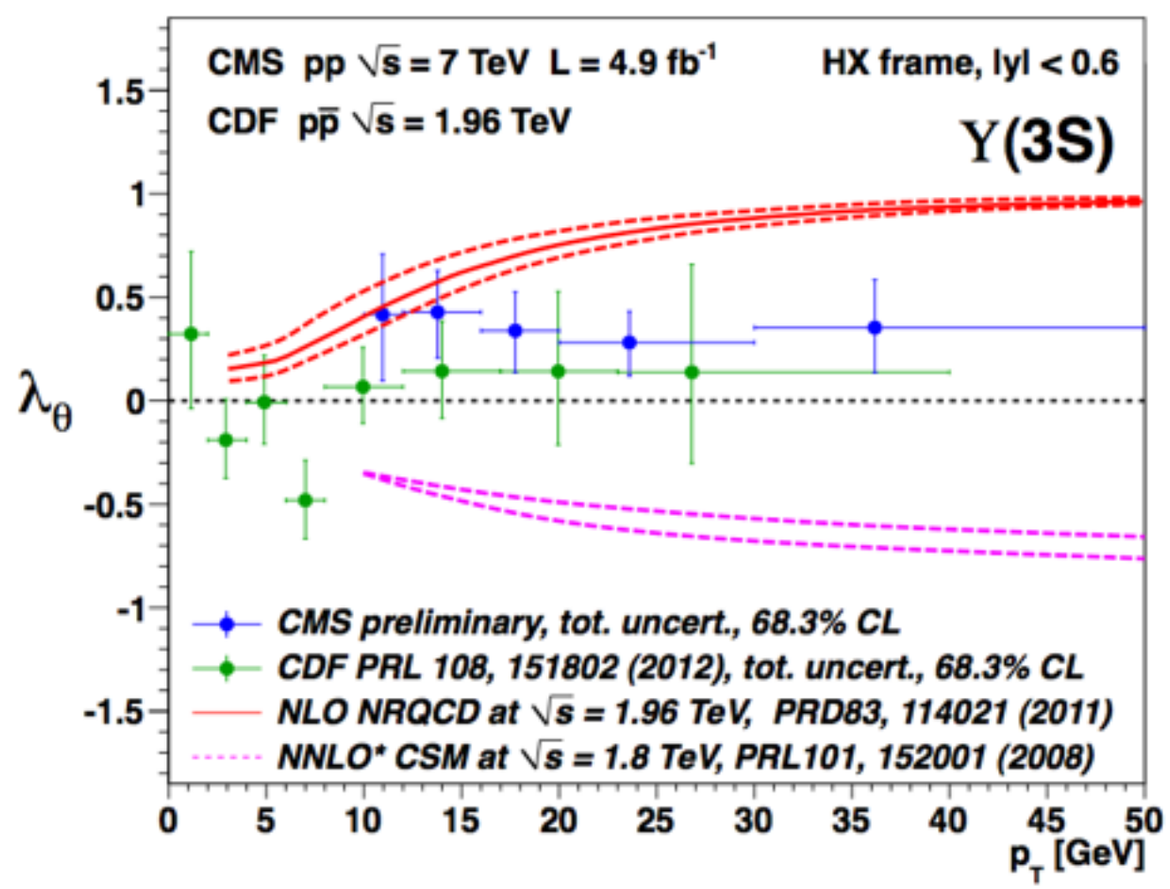
ψ'

J/ψ

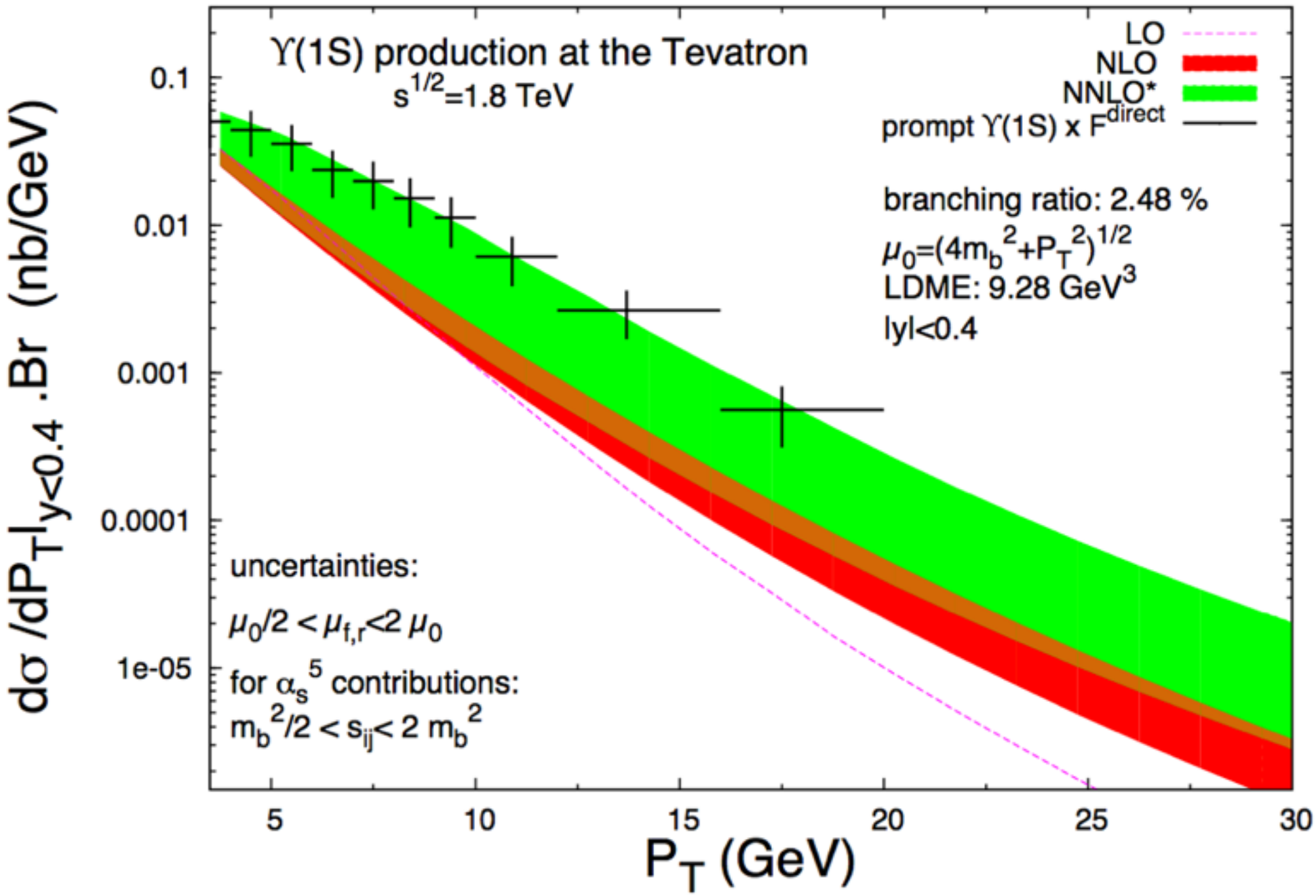
Braaten, Kniehl, Lee, 1999

Polarization of Υ(1S), Υ(3S) at LHC





Convergence of NRQCD Calculations



NNLO* - incomplete NNLO calculation

P. Artoisenet, et. al., 2008

Recent Attempts to Resolve J/ψ Polarization Puzzle

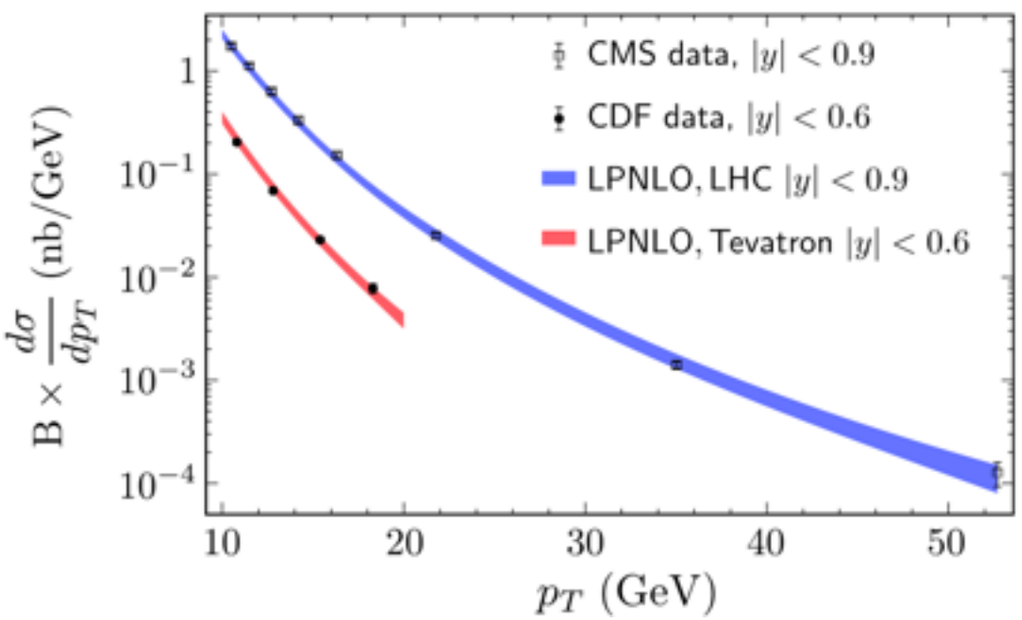
i) large p_t production at CDF

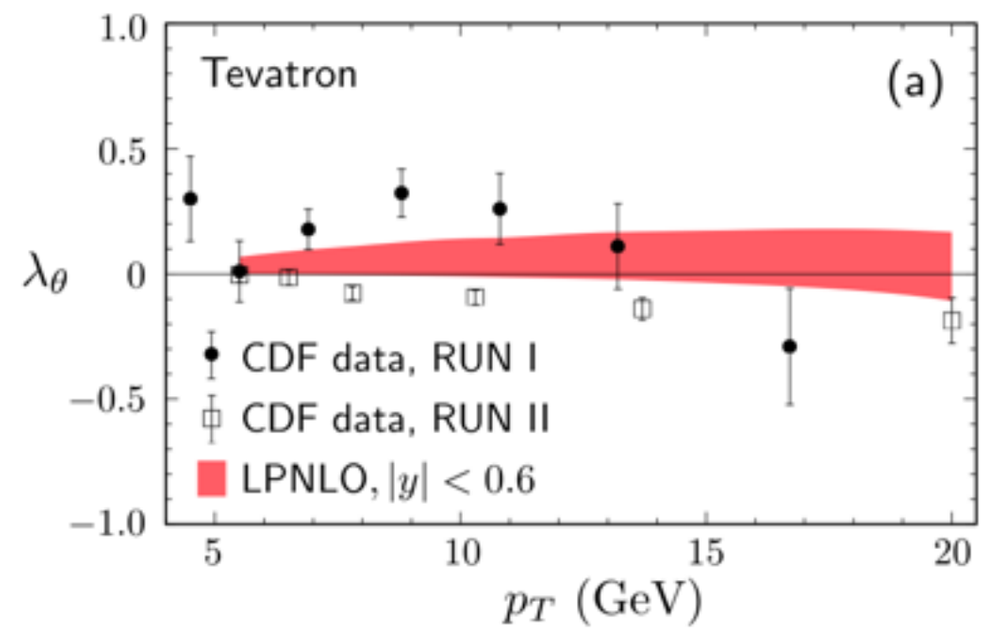
ii) resum logs of p_t/m_c using AP evolution

iii) fit COME to p_t spectrum, predict basically no polarization

Bodwin, et. al. arXiv:1403.3612

Chao, et. al. PRL 108, 242004 (2012)





Extracted COME **inconsistent** with global fits

$$\langle \mathcal{O}^{J/\psi}(^1S_0^{(8)}\rangle = 0.099 \pm 0.022\,\mathrm{GeV}^3$$

$$\langle \mathcal{O}^{J/\psi}(^3S_1^{(8)}\rangle = 0.011 \pm 0.010\,\mathrm{GeV}^3$$

$$\langle \mathcal{O}^{J/\psi}(^3P_0^{(8)}\rangle = 0.011 \pm 0.010\,\mathrm{GeV}^5$$

Improving Understanding of Quarkonium Production

need systematic expansion in m_Q/p_T as well as α_s, v :

double parton fragmentation functions

Independent extractions of COME at high p_T:

Heavy quarkonium fragmenting jet functions (FJF's)

Reformulating NRQCD Expansion for large P_T

Kang, Qiu, Sterman, PRL 108 (2012)102002, Nucl. Phys. Proc. Suppl. 214 (2011) 39

expand first in $m_Q/p_\perp$ quarks, are light-like particles in this limit

$$d\sigma_{A+B\to H+X}(p) \approx \sum_f d\hat{\sigma}_{A+B\to f+X}(p_f = p/z) \otimes D_{H/f}(z, m_Q)$$

single parton fragmentation $\sim \frac{1}{p_\perp^4}$

$$+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta)/2z, p(1\pm\zeta')/2z)$$

double parton fragmentation $\sim \frac{m_Q^2}{p_\perp^6}$

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$$+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta)/2z, p(1\pm\zeta')/2z) \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z, \zeta, \zeta', m_Q)$$

double parton fragmentation $\sim \frac{m_Q^2}{p_\perp^6}$

Double parton fragmentation function (DPFF)

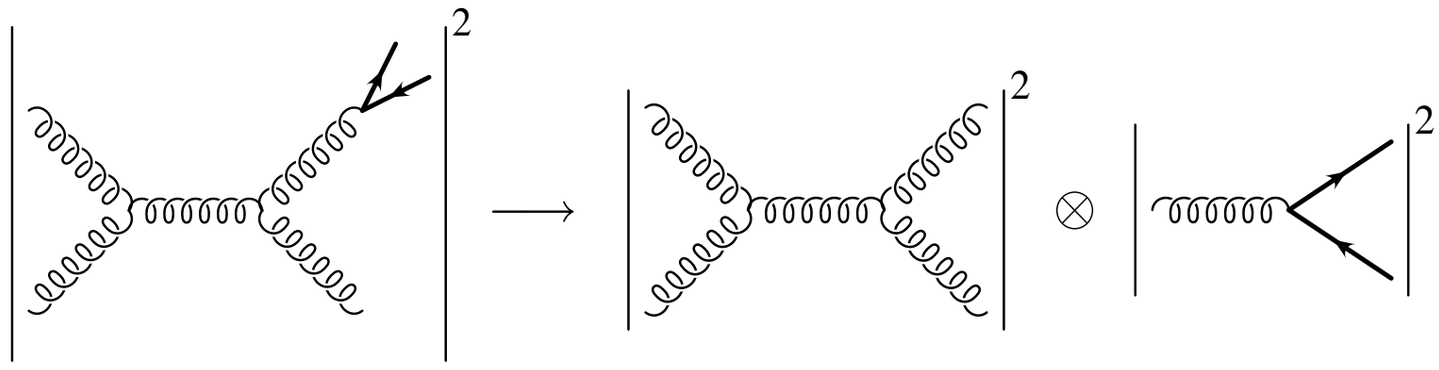
perturbatively calculate the DPFF in terms of NRQCD LDME

accounts for ~70% of NLO CSM correction

yields largely longitudinal polarized J/ψ

Resum $\ln(p_\perp/m_Q)$ in single parton fragmentation

$$\frac{d\hat{\sigma}}{dp_\perp}(ij \to J/\psi + X)_{\text{octet}} \overset{p_\perp \to \infty}{\longrightarrow} \int dz \frac{d\hat{\sigma}}{dp_\perp}(ij \to g(p_\perp/z) + X) D_{g\to J/\psi}(z)$$

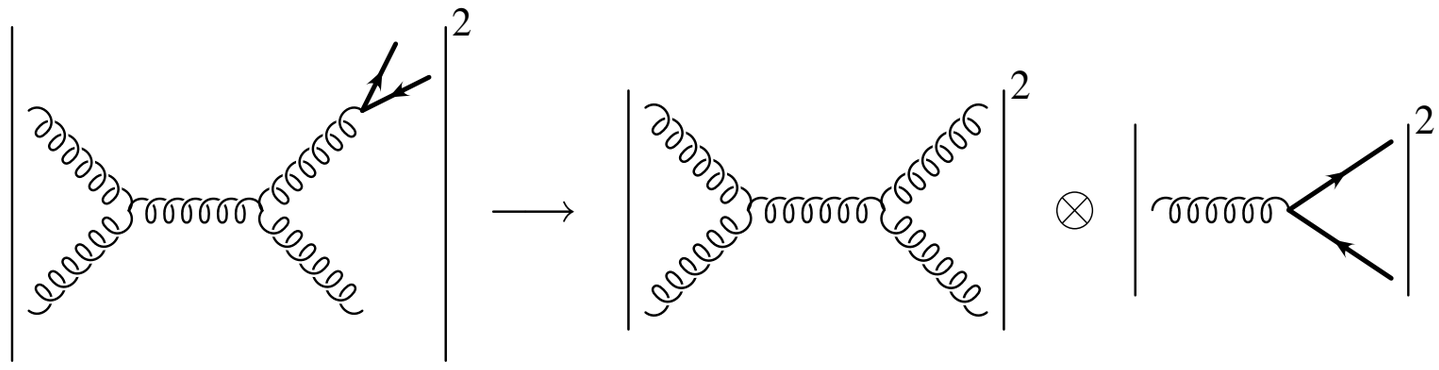


use DGLAP $\mu \frac{dD_{g\to\psi_Q}}{d\mu}(z,\mu) = \frac{\alpha_s(\mu)}{\pi} \int_z^1 \frac{dy}{y} P_{gg}(y) D_{g\to\psi_Q}(\frac{z}{y},\mu)$

Resum $\ln(p_\perp/m_Q)$ in double parton fragmentation?

Resum $\ln(p_\perp/m_Q)$ in single parton fragmentation

$$\frac{d\hat{\sigma}}{dp_\perp}(ij \to J/\psi + X)_{\text{octet}} \overset{p_\perp \to \infty}{\longrightarrow} \int dz \frac{d\hat{\sigma}}{dp_\perp}(ij \to g(p_\perp/z) + X) D_{g\to J/\psi}(z)$$



use DGLAP $\mu \frac{dD_{g\to\psi_Q}}{d\mu}(z,\mu) = \frac{\alpha_s(\mu)}{\pi} \int_z^1 \frac{dy}{y} P_{gg}(y) D_{g\to\psi_Q}(\frac{z}{y},\mu)$

Resum $\ln(p_\perp/m_Q)$ in double parton fragmentation?

need analogous equations for the DPFFs

goal of recent work

Soft Collinear Effective Theory (SCET)

SCET degrees of freedom $n^\mu = (1, \hat{n}), \bar{n}^\mu = (1, -\hat{n})$

collinear $(\bar{n} \cdot p, n \cdot p, p_\perp) \sim Q(1, \lambda^2, \lambda)$

soft $(\bar{n} \cdot p, n \cdot p, p_\perp) \sim Q(\lambda^2, \lambda^2, \lambda^2)$

Q $(p_\perp)$ hard scale

λ $(m_Q/p_\perp)$ expansion parameter

SCET Lagrangian: soft and collinear easily decoupled, matrix elements factorize into

Hard Parts
(Wilson coefficients match QCD onto SCET) $\sim Q$

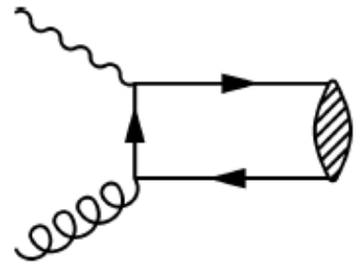
Collinear Functions
(parton distribution, jet, beam functions) $\sim Q\lambda$

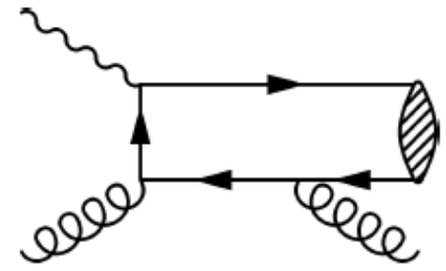
Soft Functions $\sim Q\lambda^2$

$$H \otimes J \otimes \ldots \otimes J \otimes S$$

J/ψ photoproduction near kinematic endpoints

$$\gamma p \to J/\psi + X$$

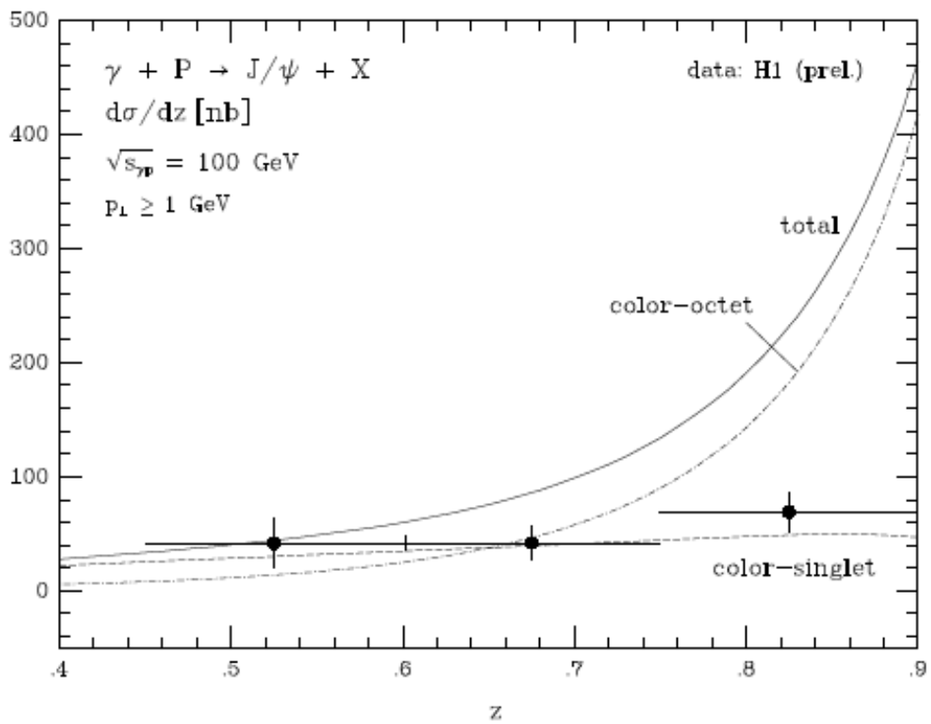




$z = E^{\psi}/E^{\psi}_{\max}$

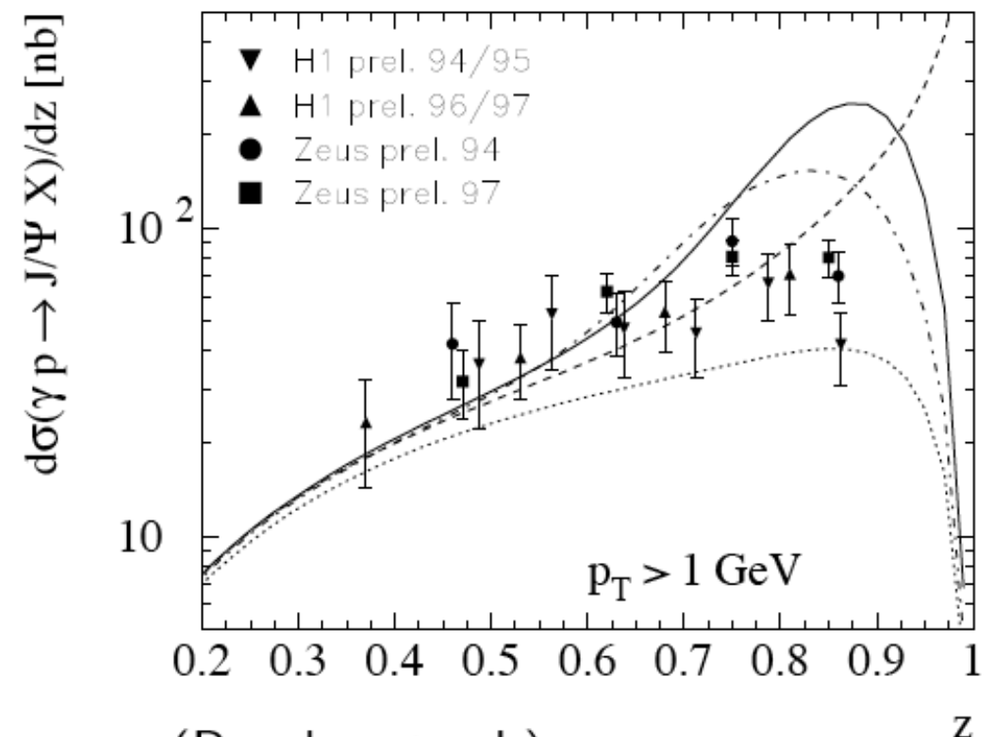
$\sigma_0 \delta(1-z)$

$-\sigma_0 \frac{C_A \alpha_s}{\pi} \frac{\ln(1-z)}{1-z} + \ldots$



(Cacciari, Kramer)

NLO CO



(Beneke, et. al.)

includes shape function,
no perturbative resummation

SCET Resummation for CO Photoproduction

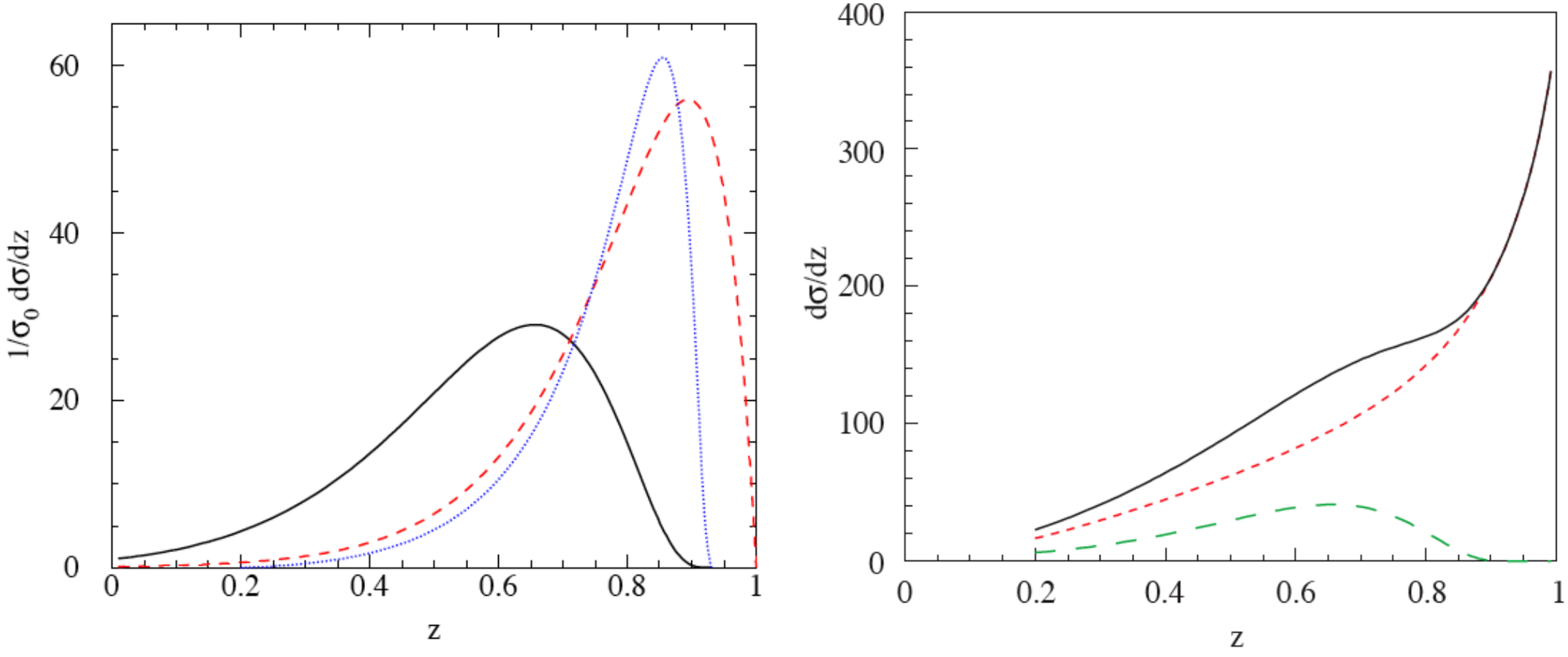
Fleming, TM, Leibovich, PRD 74 (2006) 114004

$$\frac{d\sigma}{dz} = \sigma_0 \rho \int dk^+ S^{(8,{}^1S_0)}(-\sqrt{s}(1-z)+k^+) \int_\rho^1 \frac{d\xi}{\xi} C_{II}\left(\frac{\rho}{\xi}, k^+\right) f_{g/P}(\xi)$$

shape function $Q\lambda^2 \sim \sqrt{s}(1-z)$

$Q\lambda \sim \sqrt{s(1-z)}$ beam function

Evolution eqs. for shape, beam function resum log(1-z)



Peak shifted from z ~ 0.9 to z ~ 0.7, lower nonperturbative and perturbative resummation required

SCET and Quarkonium Production at Large P_T

Fleming, TM, Leibovich, Rothstein, PRD 86 (2012) 094012, PRD 87 (2012) 074022

SCET Factorization reproduces results of Kang, Qiu, Sterman

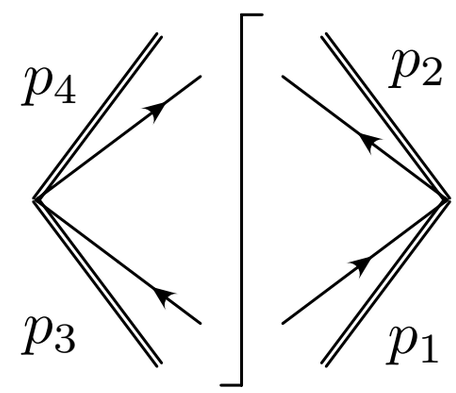
Double Parton Fragmentation Function

$$\langle 0|\bar{\chi}_{n',\omega_2'}\Gamma^{i(\mu)}\{1,T^A\}\chi_{n',\omega_1'}\mathcal{P}^H_{n',Q}\bar{\chi}_{n',\omega_4'}\Gamma^i_{(\mu)}\{1,T^A\}\chi_{n',\omega_3'}|0\rangle$$
$$= \delta(\omega_1'-\omega_2'+\omega_3'-\omega_4')\int\frac{dz}{z}\,du\,dv\,\delta(z-\frac{\bar{n}'\cdot p}{\omega_1'-\omega_2'})$$
$$\times\delta(v-1-z\frac{\omega_2'}{\bar{n}'\cdot p})\delta(u-z\frac{\omega_4'}{\bar{n}'\cdot p})D^{Q\bar{Q}}_{i\{1,8\}}(u,v,z)$$

SCET collinear quark fields

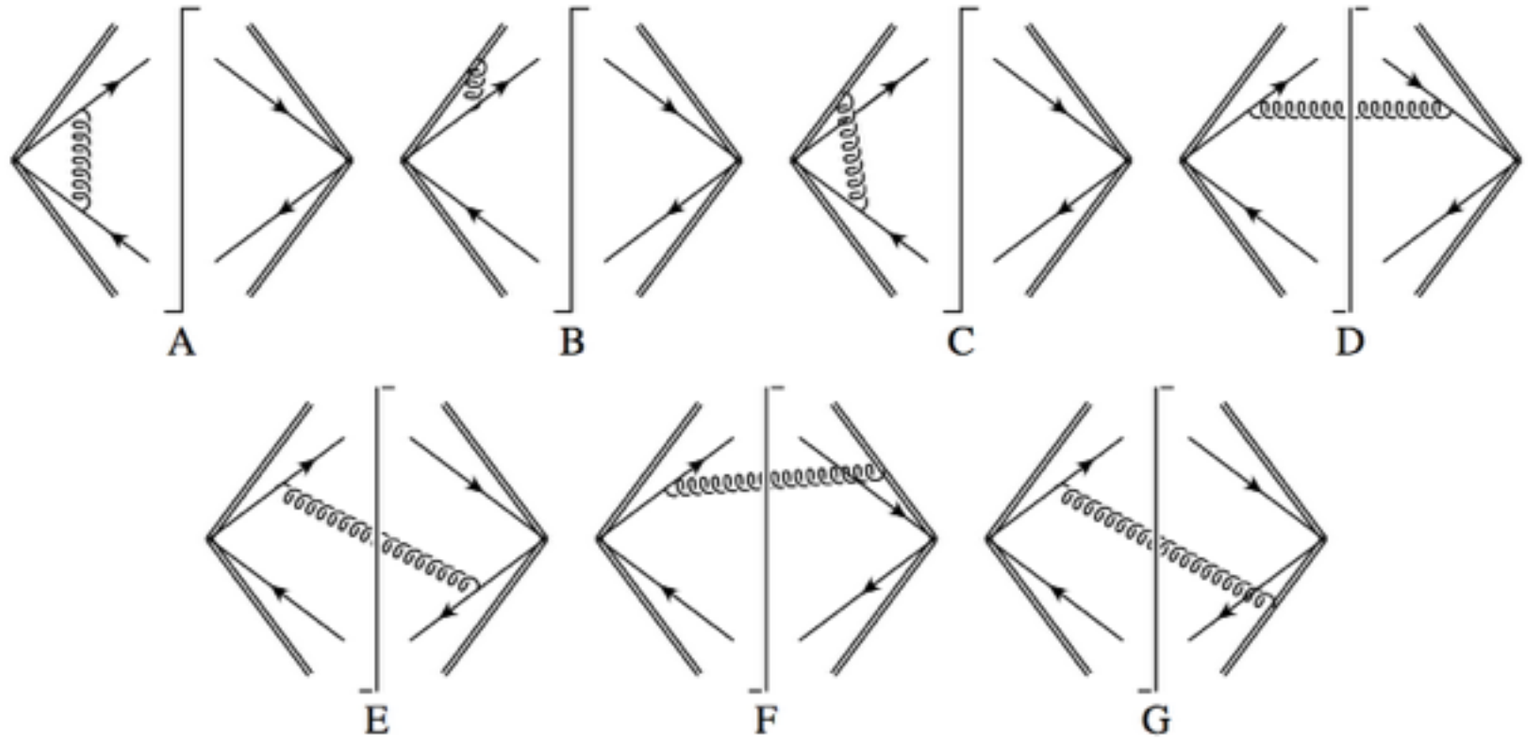
Projection operator onto final state w/ hadron H

$$D^{Q\bar{Q}}_{i(1,8)}(u,v,z)$$



$$\bar{n}\cdot p_1 = u\,\bar{n}\cdot p_{Q\bar{Q}} = \frac{u}{z}\,\bar{n}\cdot p_H \qquad \bar{n}\cdot p_4 = v\,\bar{n}\cdot p_{Q\bar{Q}} = \frac{v}{z}\,\bar{n}\cdot p_H$$

Evolution Equations



$$\mu \frac{d}{d\mu} D^{Q\bar{Q}}_{i,a}(u'', v'', z'', \mu) = -\int du' dv' \frac{dz'}{z'} \gamma_{ia,jb}(u'', u', v'', v', z''/z', \mu) D^{Q\bar{Q}}_{j,b}(u', v', z', \mu)\,,$$

ERBL evolution in variables u,v

$$\mu^2 \frac{d}{d\mu^2} D^{[1]}_{Q\bar{Q}\to H}(u, v, z; \mu) = \int_0^1 dw V(u, w; \mu) D^{[1]}_{Q\bar{Q}\to H}(w, v, z; \mu)$$

ERBL Kernel

DGLAP-like evolution for variable z

$$\mu^2 \frac{d}{d\mu^2} D^{[1]}_{Q\bar{Q}\to H}(u, v, z; \mu) = \int_0^1 dx P_{Q\bar{Q}[1]\to Q\bar{Q}[8]}(x; \mu) D^{[8]}_{Q\bar{Q}\to H}(w, v, z/x; \mu)$$

Anomalous Dimensions

$$
\begin{aligned}
\gamma_{11} \;=\; & -3\frac{\alpha_s C_F}{\pi}\delta(u-u')\delta(v-v')\delta(1-z/z') \\
& -\frac{\alpha_s C_F}{\pi}\delta(v-v')\delta(1-\frac{z}{z'})\left[\theta(u'-u)\frac{u}{u'}\frac{1}{(u'-u)_+}+\theta(u-u')\frac{\bar{u}}{\bar{u}'}\frac{1}{(u-u')_+}\right] \\
& -\frac{\alpha_s C_F}{\pi}\delta(u-u')\delta(1-\frac{z}{z'})\left[\theta(v'-v)\frac{v}{v'}\frac{1}{(v'-v)_+}+\theta(v-v')\frac{\bar{v}}{\bar{v}'}\frac{1}{(v-v')_+}\right],
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{81} \;=\; & -\frac{\alpha_s}{\pi}\theta(1-z/z')\left(\frac{z}{z'}\right)^2\left\{\left[\frac{uv'+vu'}{(1-z/z')_+}+\frac{1-z/z'}{z/z'}\right]\frac{1}{u'v'}\delta(\bar{v}-\frac{z}{z'}\bar{v}')\delta(\bar{u}-\frac{z}{z'}\bar{u}')\right. \\
& +\left[\frac{\bar{u}\bar{v}'+\bar{v}\bar{u}'}{(1-z/z')_+}+\frac{1-z/z'}{z/z'}\right]\frac{1}{\bar{u}'\bar{v}'}\delta(v-\frac{z}{z'}v')\delta(u-\frac{z}{z'}u') \\
& -\left[\frac{u\bar{v}'+\bar{v}u'}{(1-z/z')_+}+\frac{1-z/z'}{z/z'}\right]\frac{1}{u'\bar{v}'}\delta(v-\frac{z}{z'}v')\delta(\bar{u}-\frac{z}{z'}\bar{u}') \\
& \left.-\left[\frac{\bar{u}v'+v\bar{u}'}{(1-z/z')_+}+\frac{1-z/z'}{z/z'}\right]\frac{1}{\bar{u}'v'}\delta(\bar{v}-\frac{z}{z'}\bar{v}')\delta(u-\frac{z}{z'}u')\right\}.
\end{aligned}
$$

$$
\gamma_{18} = \frac{C_F}{2N_c}\gamma_{81}
$$

$$
\begin{aligned}
\gamma_{88} \;=\; & -3\frac{\alpha_s C_F}{\pi}\delta(u-u')\delta(v-v')\delta(1-z/z') \\
& +\frac{\alpha}{\pi}\frac{1}{2N_c}\delta(v-v')\delta(1-\frac{z}{z'})\left[\theta(u'-u)\frac{u}{u'}\frac{1}{(u'-u)_+}+\theta(u-u')\frac{\bar{u}}{\bar{u}'}\frac{1}{(u-u')_+}\right] \\
& +\frac{\alpha_s}{\pi}\frac{1}{2N_c}\delta(u-u')\delta(1-\frac{z}{z'})\left[\theta(v'-v)\frac{v}{v'}\frac{1}{(v'-v)_+}+\theta(v-v')\frac{\bar{v}}{\bar{v}'}\frac{1}{(v-v')_+}\right] \\
& -\frac{\alpha_s}{\pi}\theta(1-z/z')\left(\frac{z}{z'}\right)^2\left\{\frac{N_c^2-2}{2N_c}\left[\frac{uv'+vu'}{(1-z/z')_+}+\frac{1-z/z'}{z/z'}\right]\frac{1}{u'v'}\delta(\bar{v}-\frac{z}{z'}\bar{v}')\delta(\bar{u}-\frac{z}{z'}\bar{u}')\right. \\
& \qquad +\frac{N_c^2-2}{2N_c}\left[\frac{\bar{u}\bar{v}'+\bar{v}\bar{u}'}{(1-z/z')_+}+\frac{1-z/z'}{z/z'}\right]\frac{1}{\bar{u}'\bar{v}'}\delta(v-\frac{z}{z'}v')\delta(u-\frac{z}{z'}u') \\
& \qquad +\frac{1}{N_c}\left[\frac{u\bar{v}'+\bar{v}u'}{(1-z/z')_+}+\frac{1-z/z'}{z/z'}\right]\frac{1}{u'\bar{v}'}\delta(v-\frac{z}{z'}\xi)\delta(\bar{u}-\frac{z}{z'}\bar{u}') \\
& \qquad \left.+\frac{1}{N_c}\left[\frac{\bar{u}v'+v\bar{u}'}{(1-z/z')_+}+\frac{1-z/z'}{z/z'}\right]\frac{1}{\bar{u}'v'}\delta(\bar{v}-\frac{z}{z'}\bar{v}')\delta(u-\frac{z}{z'}u')\right\}.
\end{aligned}
$$

Evolution eqs., **NRQCD DPFF's**

Y.-Q. Ma, J.W. Qiu, H. Zhang, arXiv:1311.7078

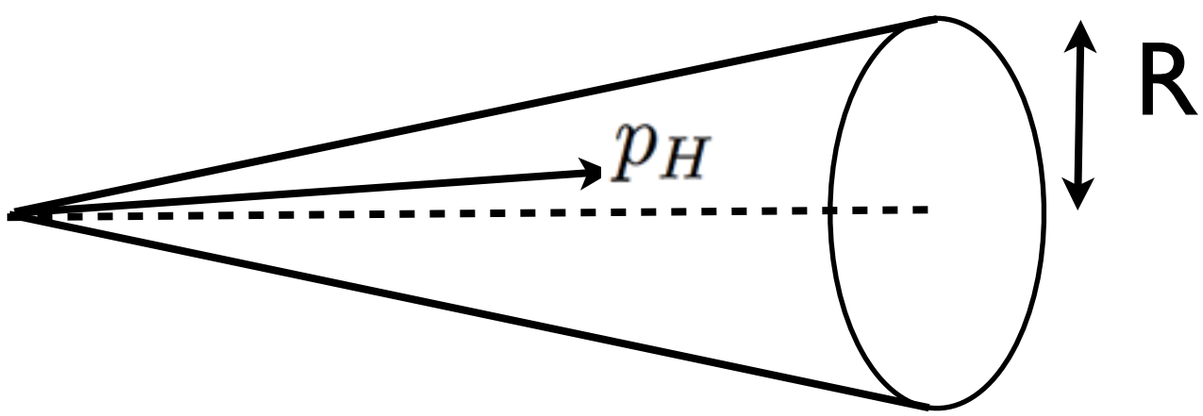
Z.-B.Kang, Y.-Q. Ma, J.W. Qiu, G. Sterman, arXiv:1401.0923

Y.-Q. Ma, J.W. Qiu, H. Zhang, arXiv:1401.0524

Fragmenting Jet Functions

Procura, Stewart, arXiv:0911.4980
Jain, Procura, Waalewijn, arXiv:1101.4953
Procura, Waalewijn, arXiv:1111.6605

jets with identified hadrons



Jet Energy: E

$$p_H^+ = z p_{\rm jet}^+$$

cross sections determined by **fragmenting jet function (FJF):**

$$\mathcal{G}_g^h(E, R, \mu, z)$$

inclusive hadron production: fragmentation functions

$$\frac{1}{\sigma_0}\frac{d\sigma^h}{dz}(e^+e^- \to h\,X) = \sum_i \int_z^1 \frac{dx}{x}\, C_i(E_{\rm cm}, x, \mu)\; D_i^h(z/x, \mu)$$

jet cross sections: jet functions

$$\frac{\mathrm{d}\sigma^h}{\mathrm{d}z}(E,R) = \int \mathrm{d}\Phi_N \mathrm{tr}[H_N S_N] \prod_\ell J_\ell$$

$$\boxed{\mathcal{G}_g^h(E,R,\mu,z) \longrightarrow D_i^h(z/x,\mu)\ ,\ J_\ell}$$

relationship to jet function:

$$\sum_h \int_0^1 \mathrm{d}z z D_j^h(z,\mu) = 1$$

$$J_i(E,R,z,\mu) = \frac{1}{2}\sum_h \int \frac{dz}{(2\pi)^3}\, z\, \mathcal{G}_i^h(E,R,z,\mu)$$

cross section for jet w/ identified hadron from jet cross section

$$\frac{\mathrm{d}\sigma^h}{\mathrm{d}z}(E,R) = \int \mathrm{d}\Phi_N \mathrm{tr}[H_N S_N] \prod_\ell J_\ell \; J_i(E,R,z,\mu)$$

$$\frac{\mathrm{d}\sigma^h}{\mathrm{d}z}(E,R) = \int \mathrm{d}\Phi_N \mathrm{tr}[H_N S_N]\, \mathcal{G}_i^h(E,R,z,\mu) \prod_\ell J_\ell \,.$$

relationship to fragmentation functions

$$\mathcal{G}_i^h(E,R,z,\mu) = \sum_i \int_z^1 \frac{\mathrm{d}z'}{z'} \mathcal{J}_{ij}(E,R,z',\mu) D_j^h\left(\frac{z}{z'},\mu\right)\left[1+\mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{4E^2\tan^2(R/2)}\right)\right]$$

matching coefficients calculable in perturbation theory

$$\frac{\mathcal{J}_{gg}(E,R,z,\mu)}{2(2\pi)^3} = \delta(1-z) + \frac{\alpha_s(\mu)C_A}{\pi}\left[\left(L^2-\frac{\pi^2}{24}\right)\delta(1-z) + \hat{P}_{gg}(z)L + \hat{\mathcal{J}}_{gg}(z)\right]$$

$$\hat{\mathcal{J}}_{gg}(z) = \begin{cases} \hat{P}_{gg}(z)\ln z & z \le 1/2 \\ \frac{2(1-z+z^2)^2}{z}\left(\frac{\ln(1-z)}{1-z}\right)_+ & z \ge 1/2. \end{cases}$$

$$L = \ln[2E\tan(R/2)/\mu].$$

scale for $\mathcal{J}_{ij}(E,R,z,\mu)$

sum rule for matching coefficients

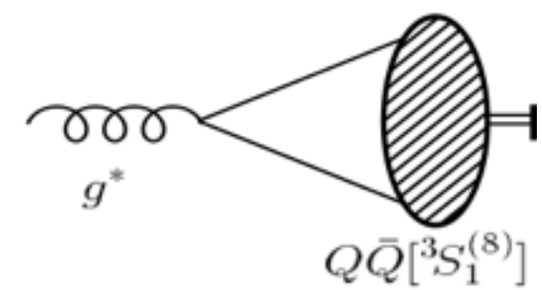
$$\sum_j \int_0^1 \mathrm{d}z\, z\, \mathcal{J}_{ij}(R,z,\mu) = 2(2\pi)^3\, J_i(R,\mu)$$

NRQCD fragmentation functions

Braaten, Yuan, hep-ph/9302307
Braaten, Chen, hep-ph/9604237
Braaten, Fleming, hep-ph/9411365

Perturbatively calculable **at the scale $2m_c$**

$$D_g^{\psi(8)}(z, 2m_c) = \frac{\pi\alpha_s(2m_c)}{3M_\psi^3}\langle O^\psi(^3S_1^{(8)})\rangle\delta(1-z)$$



$$D_g^{\psi(1)}(z, 2m_c) = \frac{5\alpha_s^3(2m_c)}{648\pi^2}\frac{\langle O^\psi(^3S_1^{(1)})\rangle}{M_\psi^3}\int_0^z dr\int_{(r+z^2)/2z}^{(1+r)/2} dy\frac{1}{(1-y)^2(y-r)^2(y^2-r)^2}$$
$$\sum_{i=0}^{2} z^i\left(f_i(r,y) + g_i(r,y)\frac{1+r-2y}{2(y-r)\sqrt{y^2-r}}\ln\frac{y-r+\sqrt{y^2-r}}{y-r-\sqrt{y^2-r}}\right),$$

Altarelli-Parisi evolution: $2m_c$ to 2E tan(R/2)

FJF in terms of fragmentation function

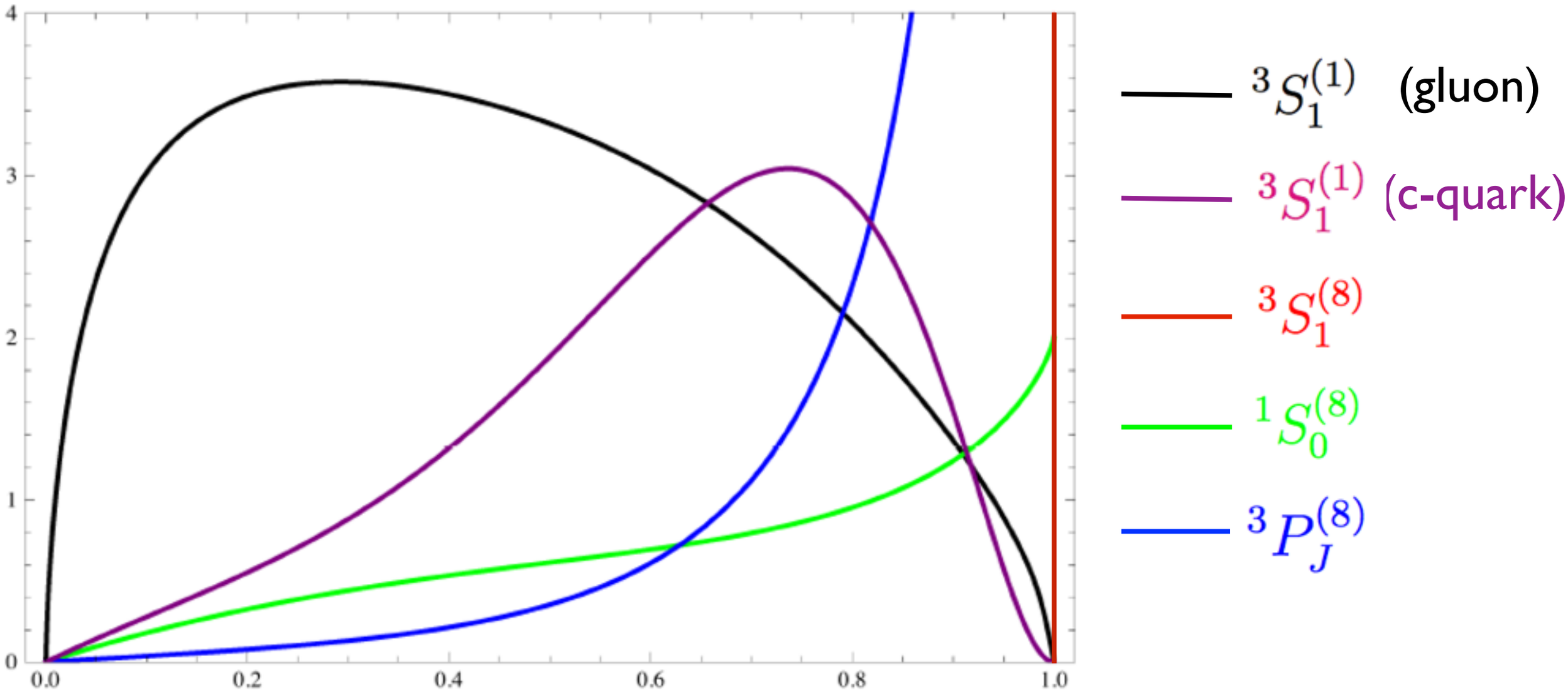
$$\begin{aligned}\mathcal{G}_g^{\psi}(E,R,z,\mu) \;=\; & D_{g\to\psi}(z,\mu)\left(1+\frac{C_A\alpha_s}{\pi}\left(L_{1-z}^2-\frac{\pi^2}{24}\right)\right)\\ & +\frac{C_A\alpha_s}{\pi}\left[\int_z^1\frac{dy}{y}\tilde{P}_{gg}(y)L_{1-y}D_{g\to\psi}\left(\frac{z}{y},\mu\right)\right.\\ & +2\int_z^1 dy\frac{D_{g\to\psi}(z/y,\mu)-D_{g\to\psi}(z,\mu)}{1-y}L_{1-y}\\ & \left.+\theta\left(\frac{1}{2}-z\right)\int_z^{1/2}\frac{dy}{y}\hat{P}_{gg}(y)\ln\left(\frac{y}{1-y}\right)D_{g\to\psi}\left(\frac{z}{y},\mu\right)\right]\end{aligned}$$

$$L_{1-z}=\ln\left(\frac{2E\tan(R/2)(1-z)}{\mu}\right)$$

For large E, FJF ~ NRQCD frag. function (at scale 2E tan(R/2))

$$\boxed{\mathcal{G}_g^h(E,R,\mu=2E\tan(R/2),z)\to D_g^{\psi}(z,2E\tan(R/2))+O(\alpha_s)}$$

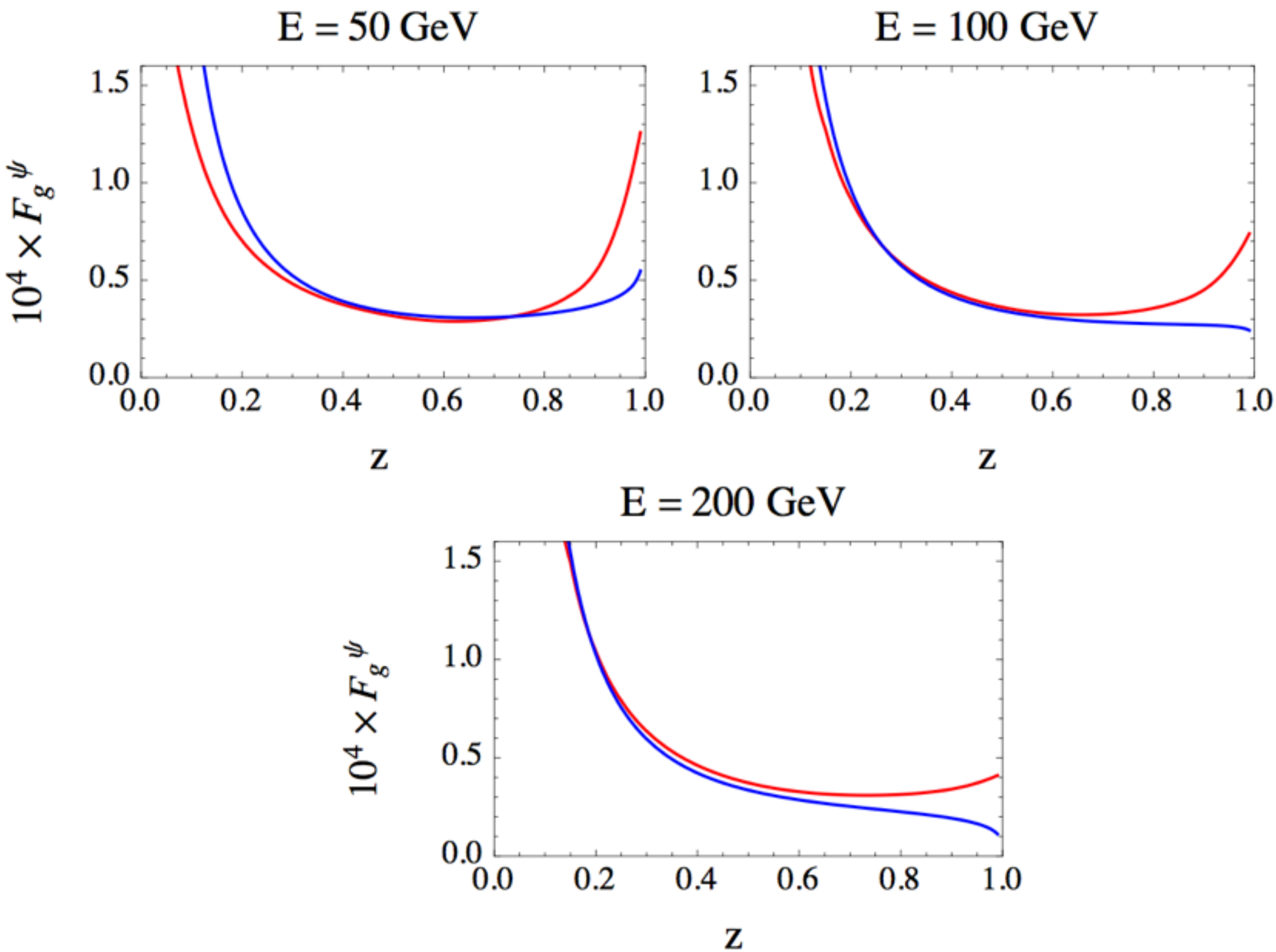
NRQCD FF's (at scale $2m_c$)



(normalization arbitrary)

Evolution to 2E tan(R/2) will soften discrepancies

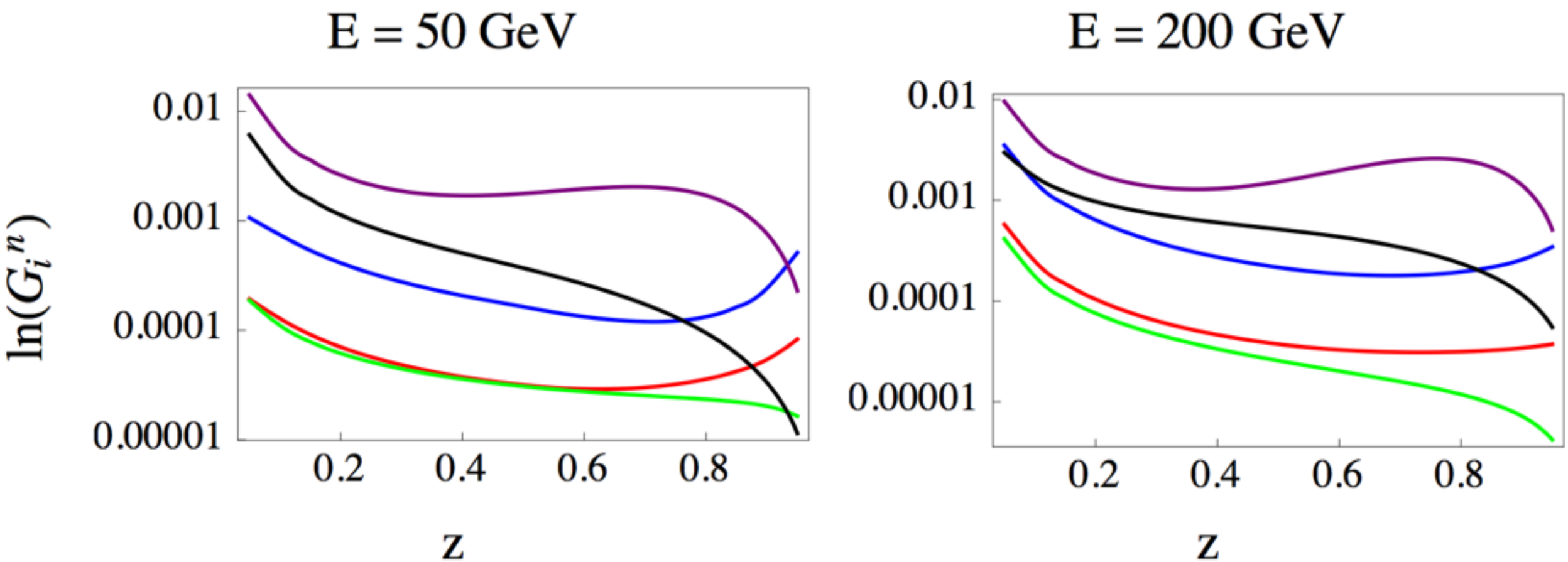
Color-Octet $^3S_1$ fragmentation function, FJF



fragmentation function

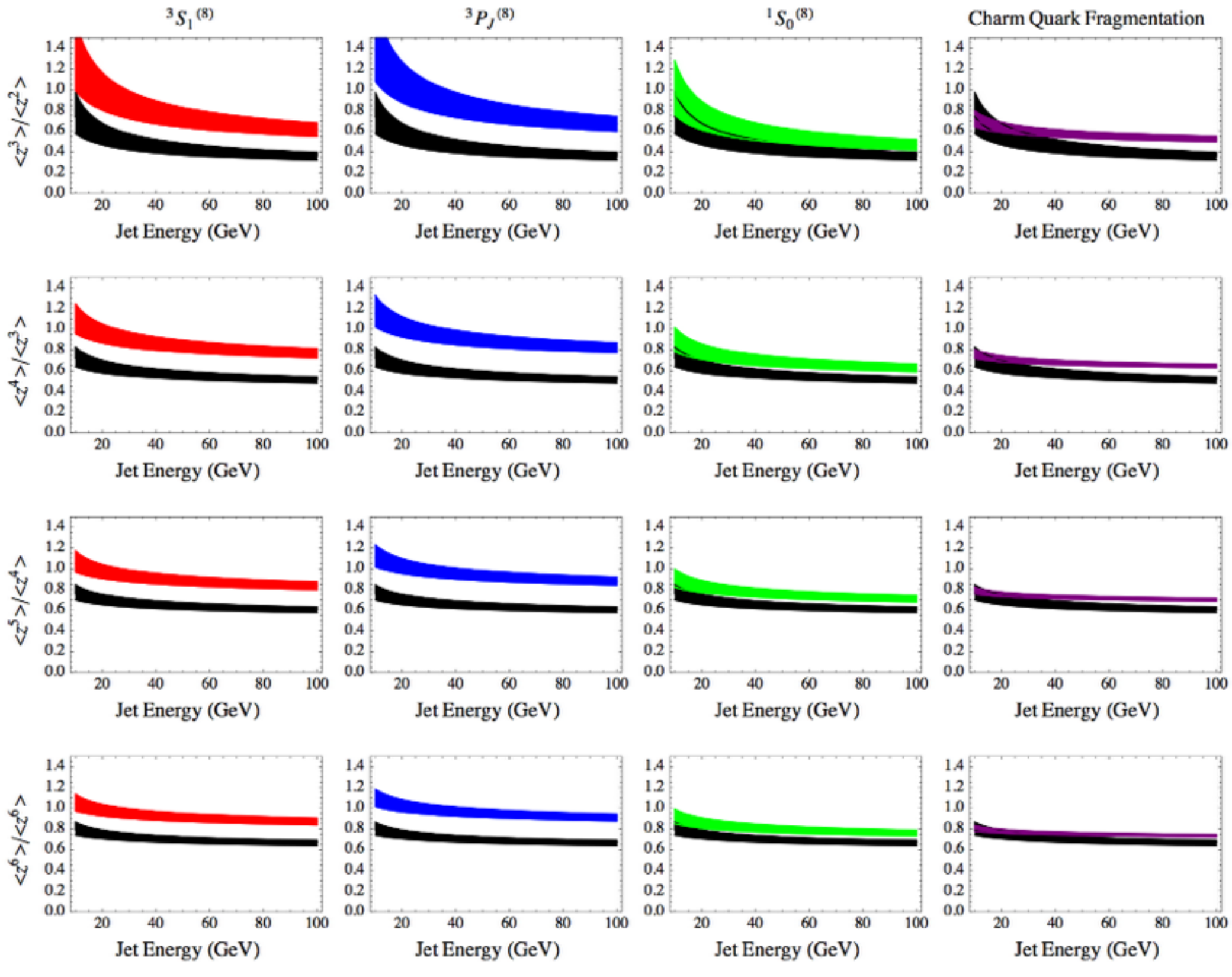
fragmenting jet function

FJF's at Different Energies



Ratios of Moments

$$E\tan(R/2) < \mu < 4E\tan(R/2)$$



Ratios of Moments

$$\left.\frac{\langle z^{n+1}\rangle}{\langle z^{n}\rangle}\right|_{^3P_J^{(8)}} \approx \left.\frac{\langle z^{n+1}\rangle}{\langle z^{n}\rangle}\right|_{^3S_1^{(8)}} > \left.\frac{\langle z^{n+1}\rangle}{\langle z^{n}\rangle}\right|_{^1S_0^{(8)}} \approx \left.\frac{\langle z^{n+1}\rangle}{\langle z^{n}\rangle}\right|_{\mathrm{c-quark}} > \left.\frac{\langle z^{n+1}\rangle}{\langle z^{n}\rangle}\right|_{^3S_1^{(1)}}$$

Conclusions

NRQCD describes much world data on quarkonium data but puzzles, esp. polarization, remain

quarkonium production: expansion in m_Q/p_T
double parton fragmentation function

existing analyses focus on inclusive p_t spectra, polarization can we find other observables distinguish various production mechanisms at high p_T?

measuring $Q\bar{Q}$ within jets, and using jet observables should provide insights into $Q\bar{Q}$ production

quarkonium fragmenting jet functions (FJFs)

Backup

fragmentation function (QCD)

$$D_q^h(z) = z \int \frac{\mathrm{d}x^+}{4\pi}\, e^{ik^- x^+/2}\, \frac{1}{4N_c}\, \mathrm{Tr} \sum_X \langle 0|\bar{\not{n}}\, \Psi(x^+, 0, 0_\perp)|Xh\rangle\langle Xh|\bar{\Psi}(0)|0\rangle\big|_{p_h^\perp=0}$$

fragmentation function (SCET)

$$D_q^h\Big(\frac{p_h^-}{\omega}, \mu\Big) = \pi\omega \int dp_h^+\, \frac{1}{4N_c}\, \mathrm{Tr} \sum_X \bar{\not{n}}\, \langle 0|[\delta_{\omega,\bar{\mathcal{P}}}\, \delta_{0,\mathcal{P}_\perp}\, \chi_n(0)]|Xh\rangle\langle Xh|\bar{\chi}_n(0)|0\rangle$$

Jet function (SCET)

$$J_u(k^+\omega) = -\frac{1}{\pi\,\omega}\, \mathrm{Im} \int d^4x\, e^{ik\cdot x}\, i\, \langle 0|\, \mathrm{T}\, \bar{\chi}_{n,\omega,0_\perp}(0)\, \frac{\bar{\not{n}}}{4N_c}\, \chi_n(x)|0\rangle$$

fragmentation jet function (SCET)

$$\mathcal{G}^h_{q,\mathrm{bare}}(s,z) = \int \mathrm{d}^4y\, e^{\mathrm{i}k^+y^-/2} \int \mathrm{d}p_h^+ \sum_X \frac{1}{4N_c}\, \mathrm{tr}\, \Big[\frac{\bar{\not{n}}}{2}\langle 0|[\delta_{\omega,\overline{\mathcal{P}}}\, \delta_{0,\mathcal{P}_\perp}\, \chi_n(y)]|Xh\rangle\langle Xh|\bar{\chi}_n(0)|0\rangle\Big]$$

$$\delta(p^+/z - P_H^+) \to \delta(p^+/z - P_H^+)\delta(p^- - s/p^+)$$

FF FJF