Exploring Quark Gluon Plasma with Realistic Hydrodynamic Evolution

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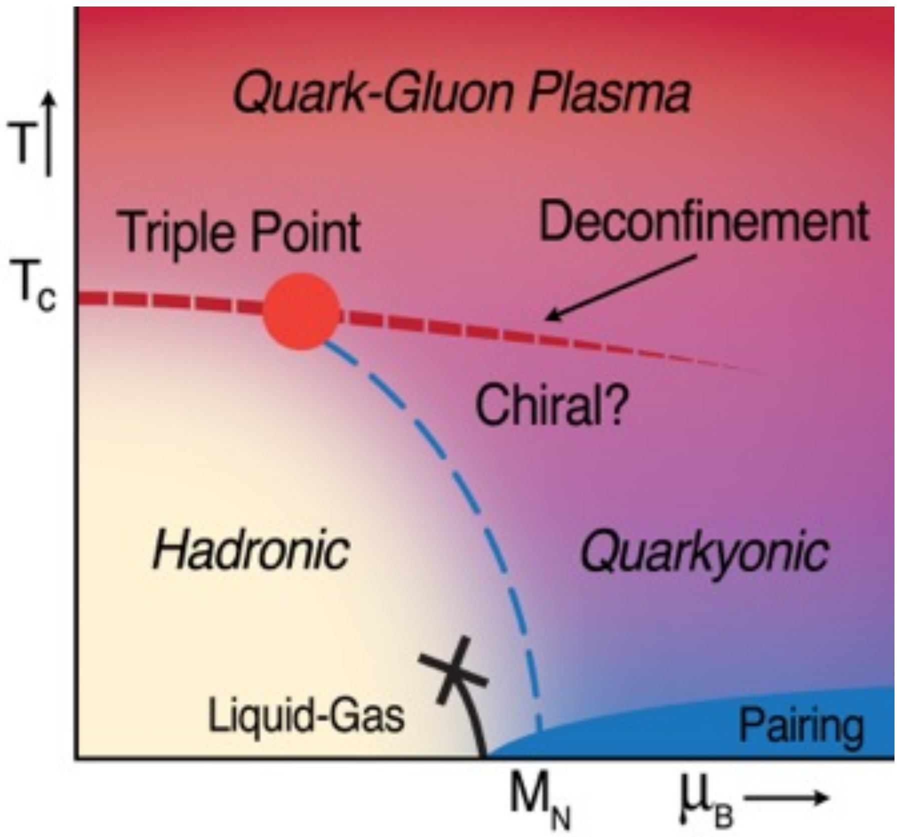
- Ho-ung Yee and YY, arXiv:1311.2574
- YY, arXiv:1312.4434

Nuclear theory seminar, BNL, Feb. 28th .

Quark-Gluon Plasma

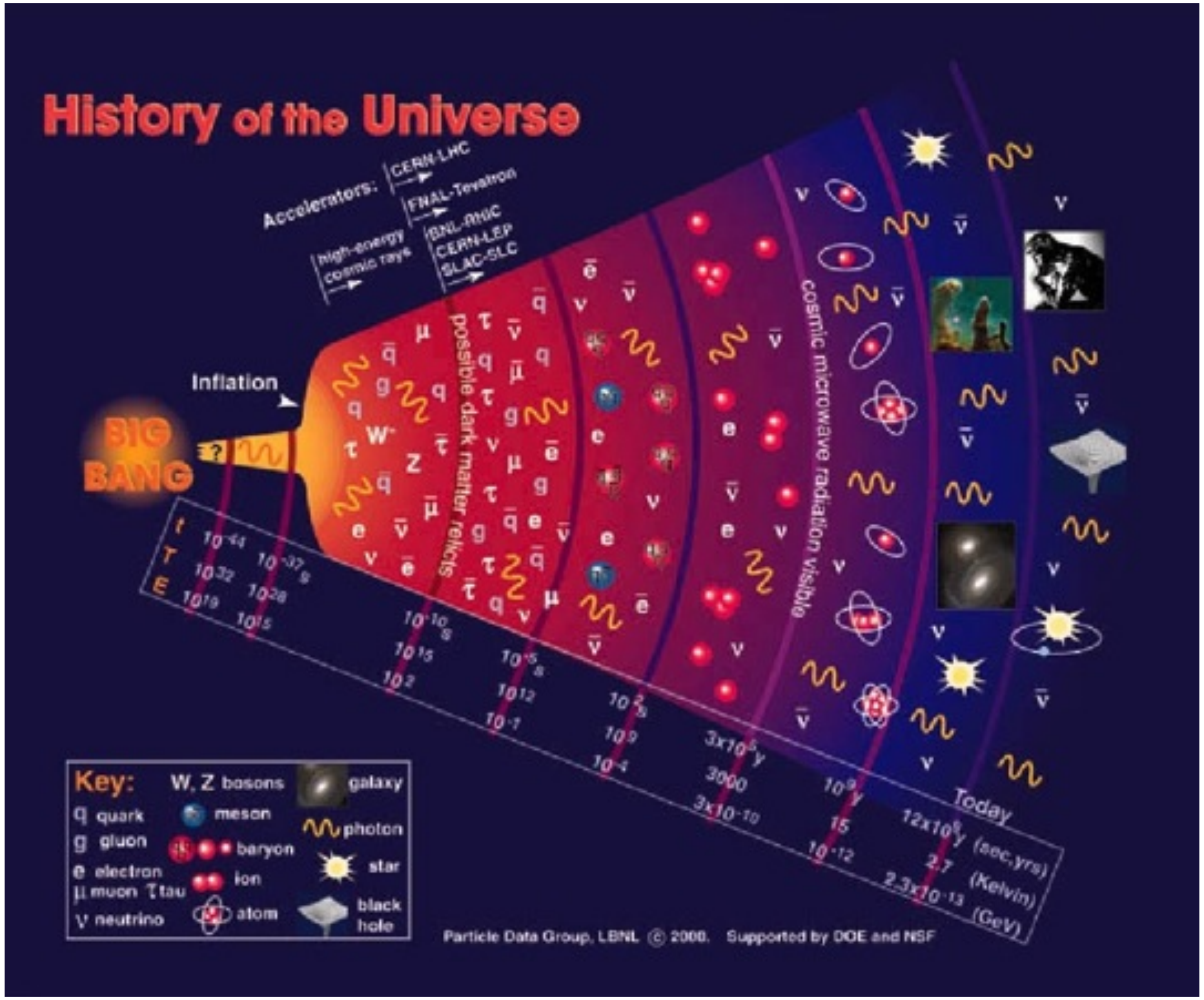
- Quark-gluon plasma(QGP) is a phase of QCD matter with temperature above transition temperature T_c .



- Where to find quark-gluon plasma?

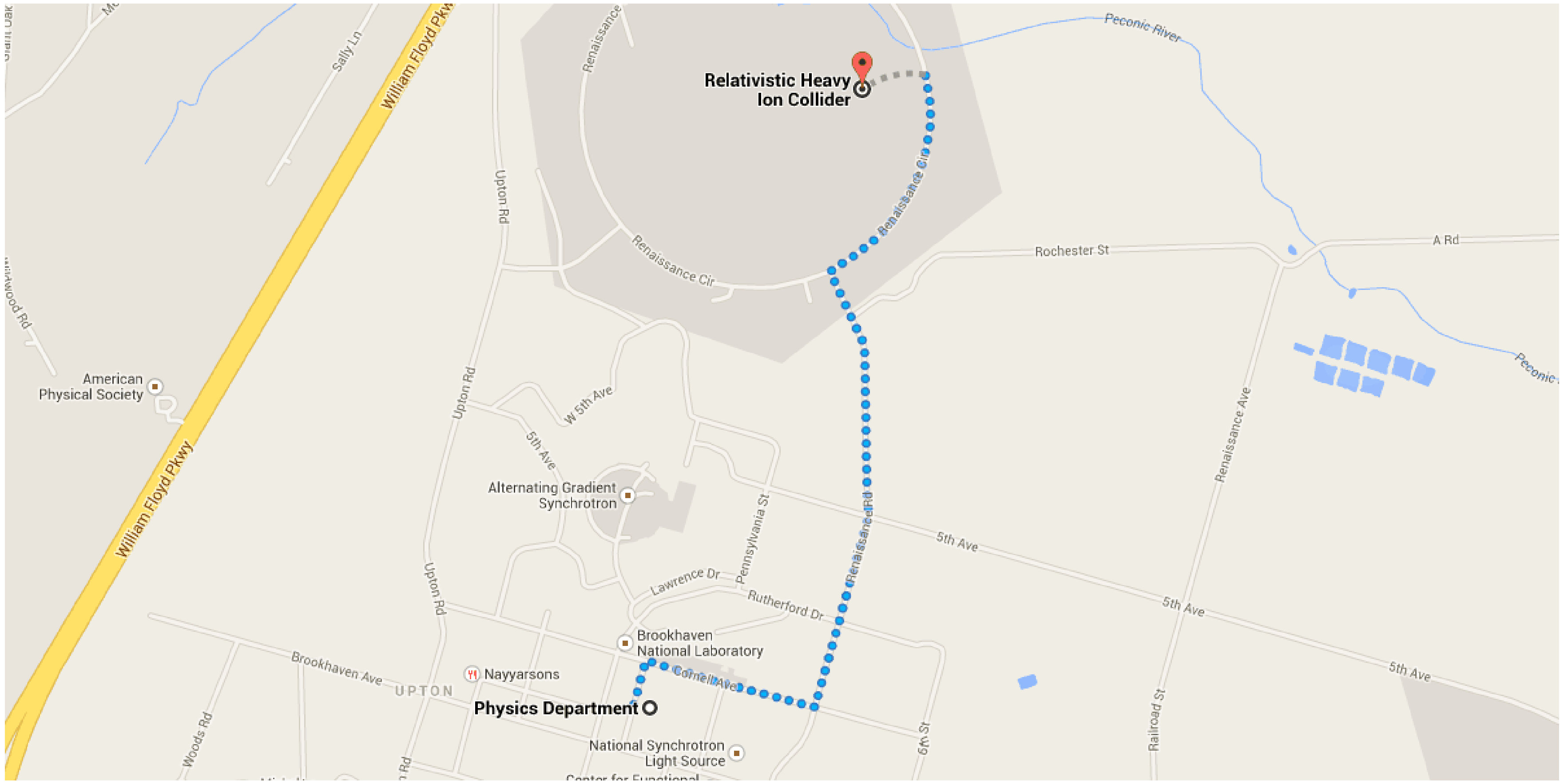
Where to find Quark-Gluon Plasma(QGP)?Option I:

- Traveling back to the early universe.



Where to find Quark-Gluon Plasma(QGP)?Option II:

Directions from **Physics Department** to **Relativistic Heavy Ion Collider** **Walk 1.6 mi, 30 min**



○ **Physics Department**

Pennsylvania St, Upton, NY 11973

Use caution - may involve errors or sections not suited for walking

⚠ This route has restricted usage or private roads.

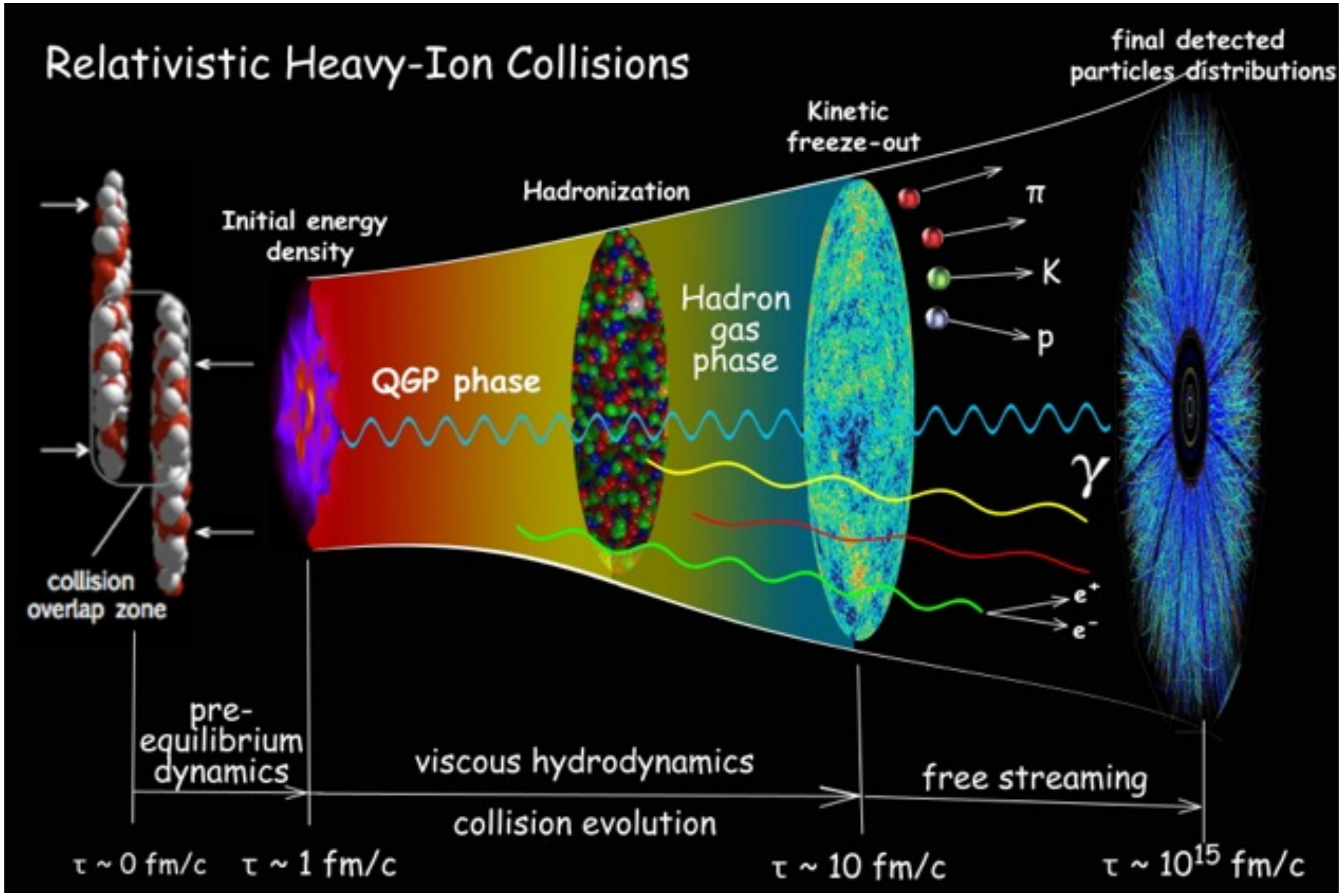
↑ 1. Head **north** on **Pennsylvania St** toward **Cornell Ave**

⚠ Restricted usage road

417 ft

Heavy-Ion collisions and hydrodynamics

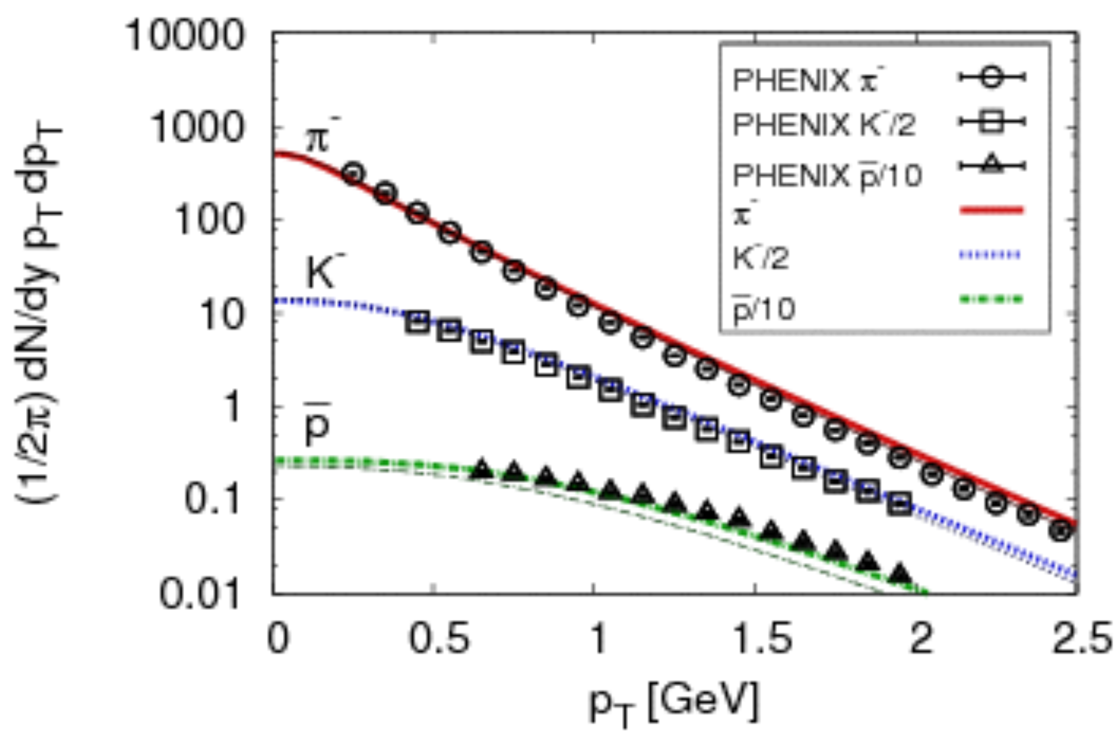
- The thermal equilibrium stage of heavy-ion collisions is well-described by relativistic hydrodynamics.



Hydrodynamics and heavy-ion collisions

- Hydrodynamics:
  - Low energy and low momentum effective theory.
  - Degree of freedom: conserved quantities such as energy density, momentum density, charge density.
  - Dynamics: conservation equations. $\partial_\mu T^{\mu\nu} = 0 \quad \partial_\mu J^\mu = 0$

- Relativistic hydrodynamic simulations provide a decent description of particle spectrum in the experiment.



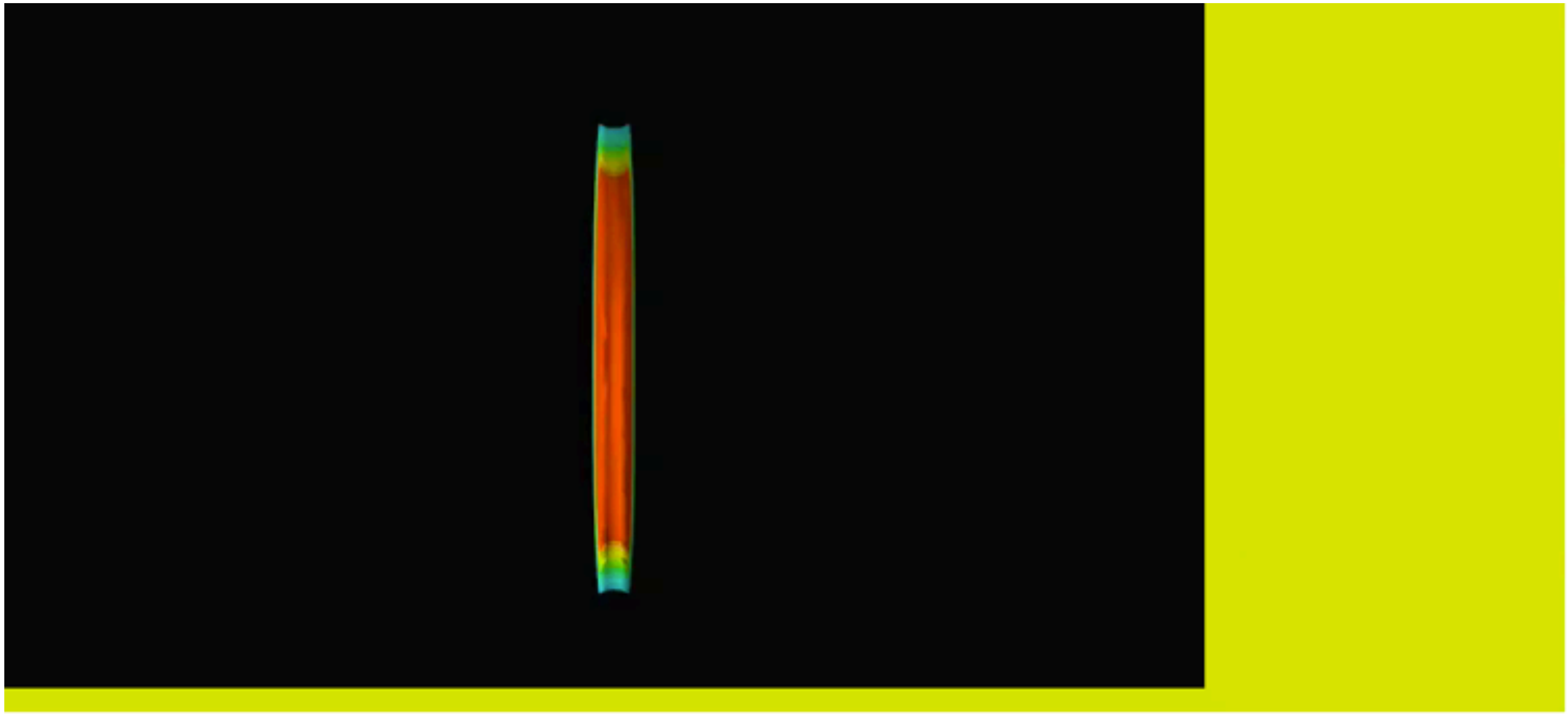
(Results of 3d hydro. simulations by McGill group)

We know hydrodynamic evolution of the fireball !

- Given a fluid cell labelled by its temporal and spatial coordinates, its temperature flow profile are known:

$$T(t,\vec{x}), u^{\mu}(t,\vec{x})$$

(Animation by Bjoern)

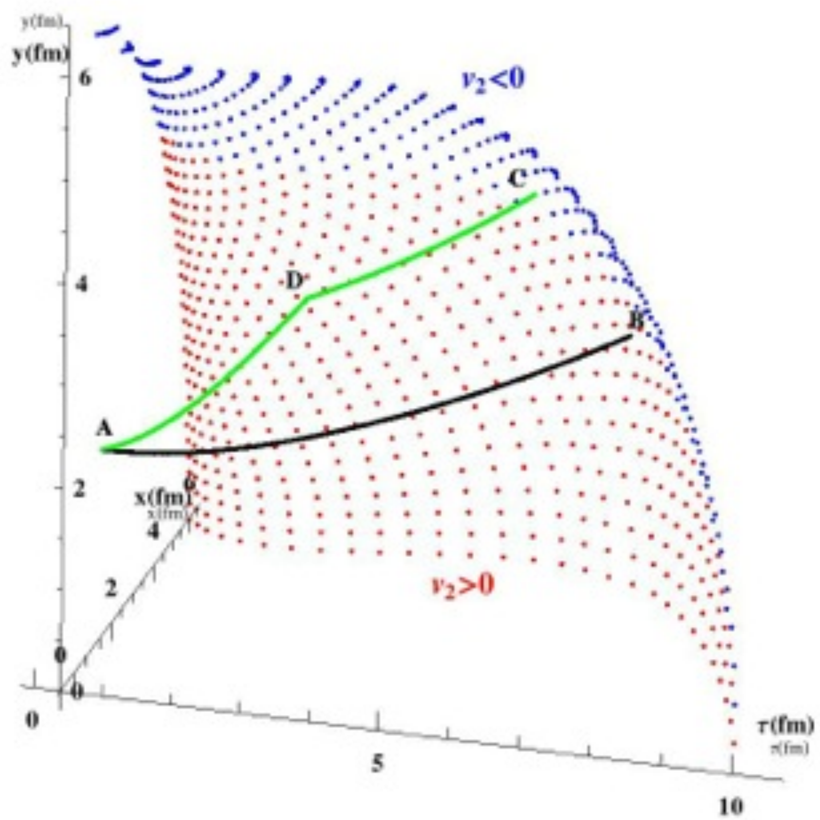


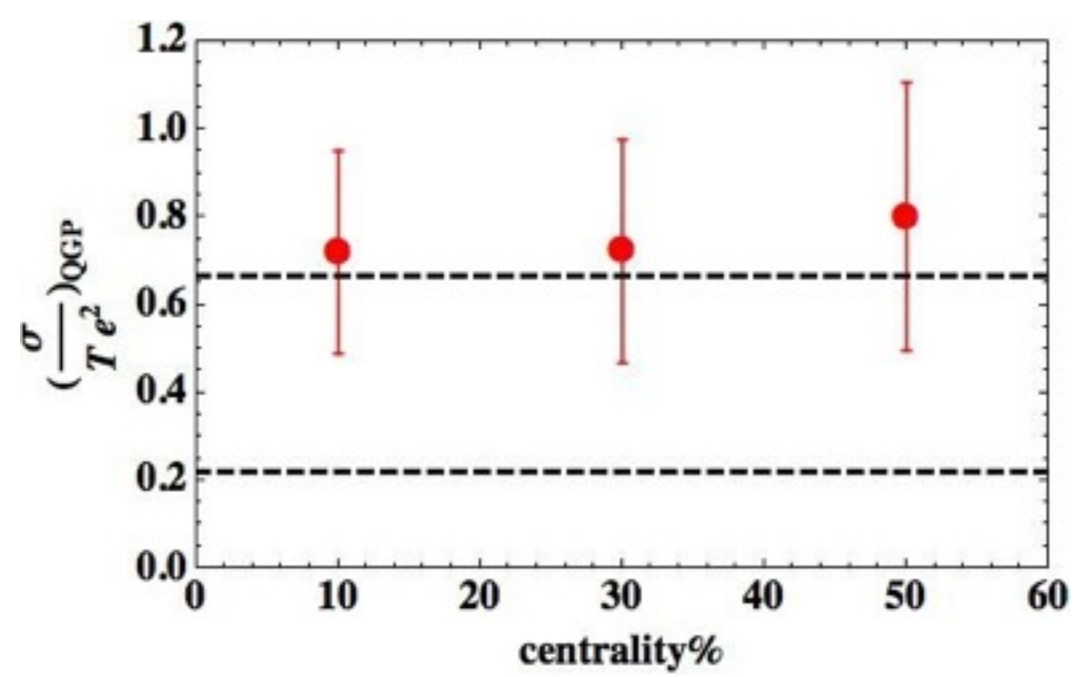
Exploring properties of QGP with realistic hydrodynamic background

- Taking temperature-flow profile as known background, how much information one could infer from the experiment?

- This talk: two examples.

✦ Story I: Effects of chiral anomaly ✦ Story II: Transport properties of QGP





Story I: chiral anomaly in heavy-ion collisions

Chiral anomaly and hydrodynamics

- The constitute relation of a chiral fluid is modified by chiral anomaly:

$$J^\mu_V = n_V u^\mu + \Delta J^\mu_{V,\mathrm{anom}} + \text{dissipative terms}$$

$$\Delta J^\mu_{V,\mathrm{anom}} = \frac{\mu_A}{2\pi^2} B^\mu \qquad \left(B^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} u_\nu F_{\alpha\beta}\right) \qquad \partial_\mu J^\mu_A = \frac{1}{2\pi^2} \vec{E}\cdot\vec{B}$$

- Nielsen-Ninomia argument(1983):

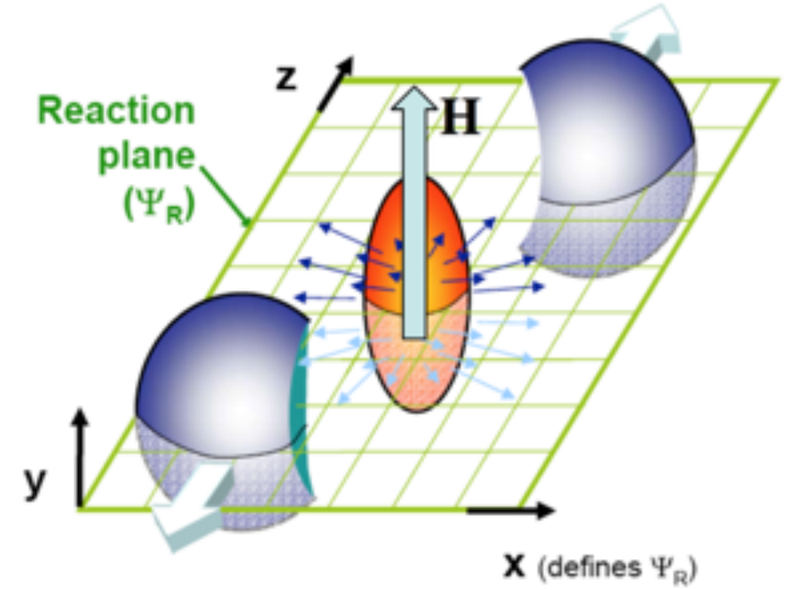
$$\mu_A \frac{dn_A}{dt} = \frac{\mu_A}{2\pi^2} \vec{E}\cdot\vec{B} = \vec{E}\cdot\Delta\vec{J}_{V,\mathrm{anom}}$$

Energy $=$ Work

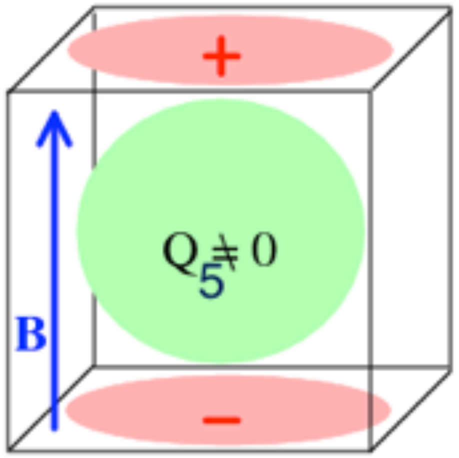
- Microscopically: effective velocity of a chiral fermi is modified by Berry curvature(e.g. M. Stephanov, YY PRL 2012).

- Similarly: $$\Delta J^\mu_{A,\mathrm{anom}} = \frac{\mu_V}{2\pi^2} B^\mu$$

Chiral anomaly and heavy-ion collisions



- Basic ingredients for the anomalous current induced by anomaly: magnetic field, and axial/ vector charge density .
- Strong magnetic field is created in heavy-ion collisions. $eB \sim m_\pi^2 \sim 10^{17}\,\mathrm{Gauss}$

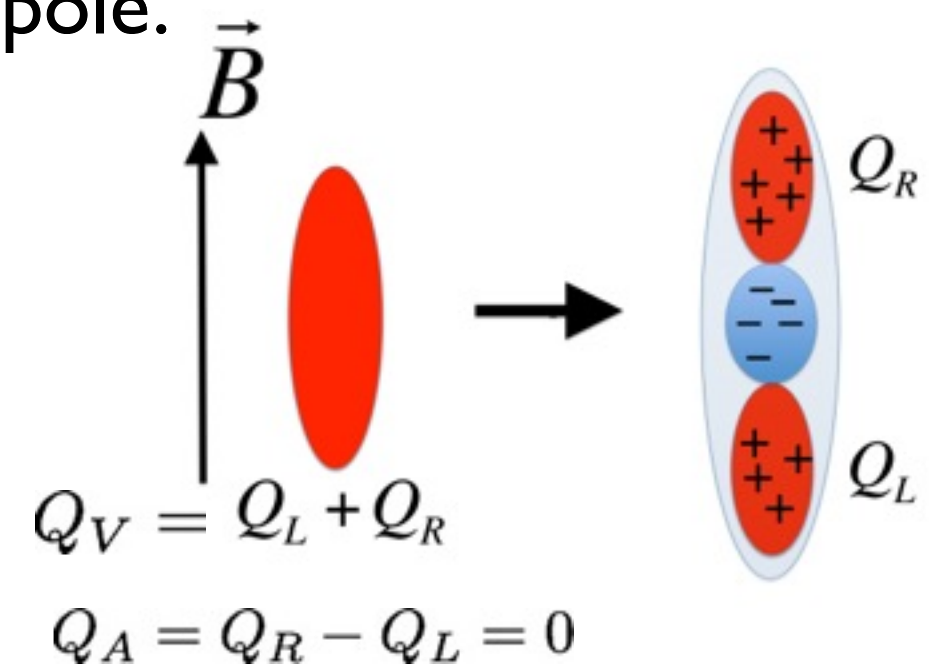


- Axial charges are generated by topological fluctuations.
- Experimentalist could select events with non-zero vector charges.

Anomaly induced charge asymmetry

- Anomalous current leads to charge quadrupole.

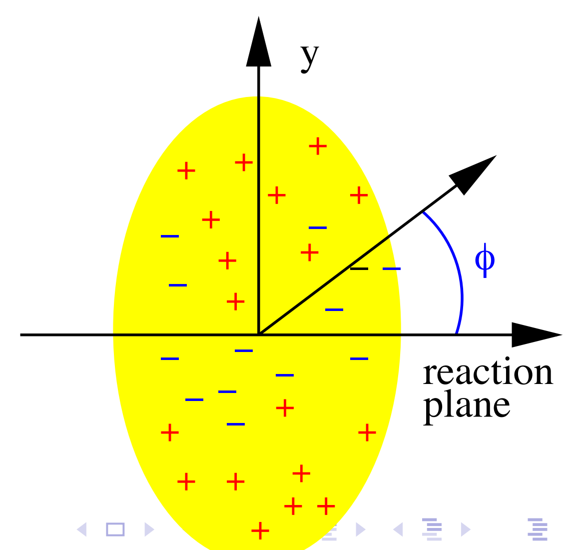
$$\Delta J^{\mu}_{R,\mathrm{anom}} = \frac{\mu_R}{4\pi^2} B^{\mu} \qquad \Delta J^{\mu}_{L,\mathrm{anom}} = -\frac{\mu_L}{4\pi^2} B^{\mu}$$



- There will be more positive charges around the polar than around equator.

Around the polar: $\cos(2\phi) < 0$

Around the equator: $\cos(2\phi) > 0$



- Quadrupole distribution leads to charge asymmetry in pion elliptic flow:

$$v_2^-(\pi) > v_2^+(\pi)$$

Charge-dependent in experiment

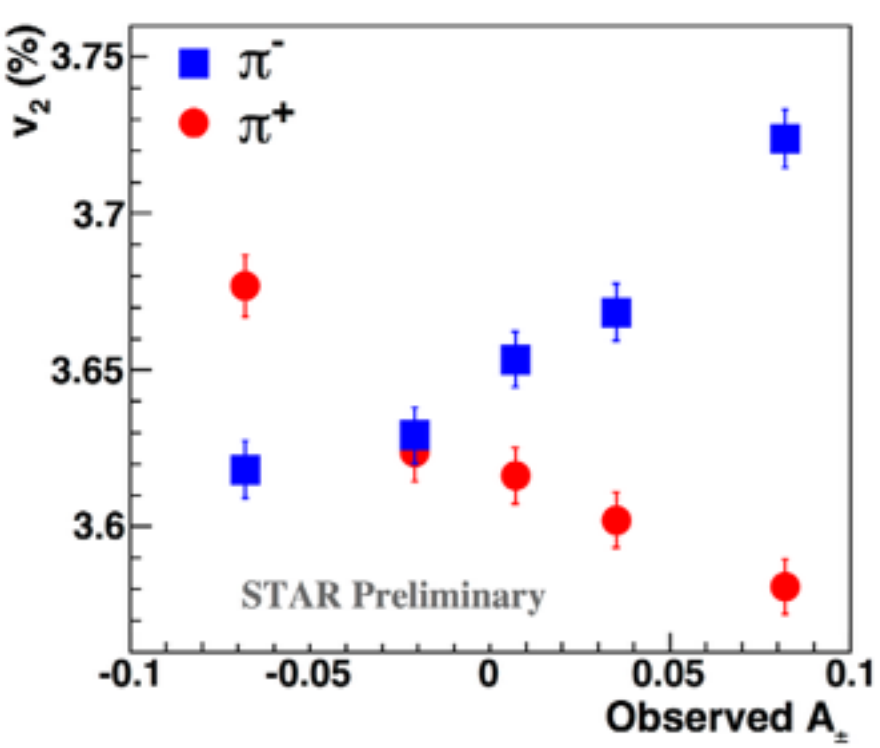
- In experiment, $v_2^-(\pi) - v_2^+(\pi) > 0$ and the difference is approximately proportional to charge asymmetry.

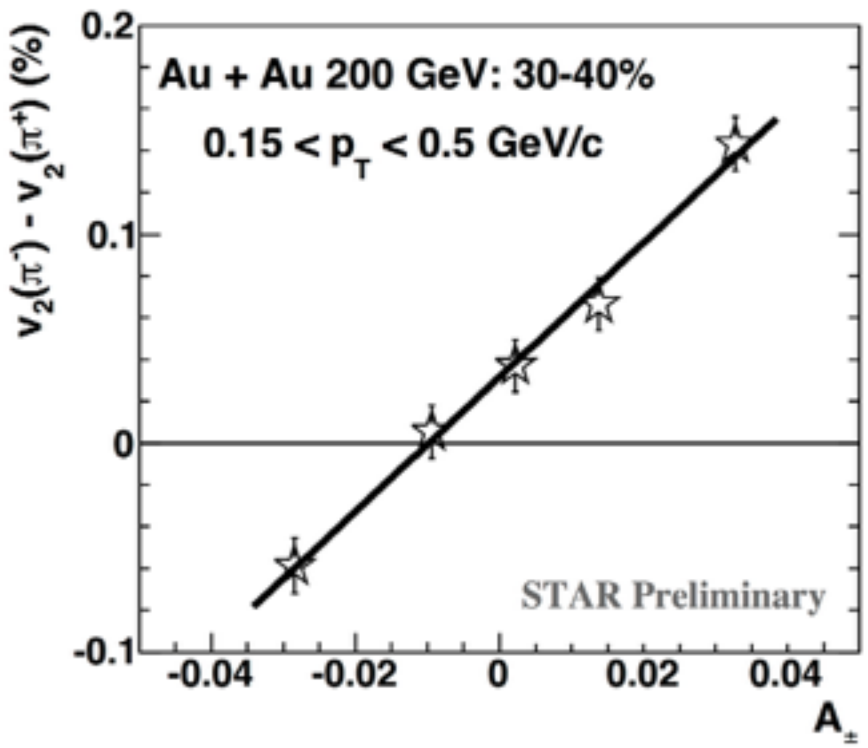
$$\frac{d^3N}{p_\perp dp_\perp d\phi_p dY}\Big|_{Y=0} = N^\pm(p_\perp)\left(1 + 2v_2^\pm(p_\perp)\cos(2\phi_p) + \dots\right)$$

$$v_2^- - v_2^+ = r\,A_\pm \qquad A_\pm = \frac{N^+ - N^-}{N^+ + N^-}$$

(relative charge difference)

(Slope parameter)





How much would chiral anomaly contribute to the data?

- The effects of anomaly is qualitatively consistent with the data.

- Quantitatively, how much would chiral anomaly contribute to the data?

- Two groups have opposite conclusions:

  - Burnier-Kharzeev-Liao-Yee(2012), solving anomalous equation in static temperature and flow background.

  - Hongo-Hirono-Hirano(2013), solving full set of hydro. equation with separating the initial effects and the effects of evolution.

- To resolve the controversial, a realistic computation is imperative.

Realistic implementation of chiral charge transportation

- Ho-ung, Yee and I revisited this problem(1311.2574). The slope parameter can be interpreted as a response to charge asymmetry. We then treat charge density as a probe.

- The back-reaction from charge density to the temperature flow profile can be neglected. We solve anomalous hydrodynamic equation linearized around the realistic hydrodynamic background. (hydro. background from P&R Romatschke).

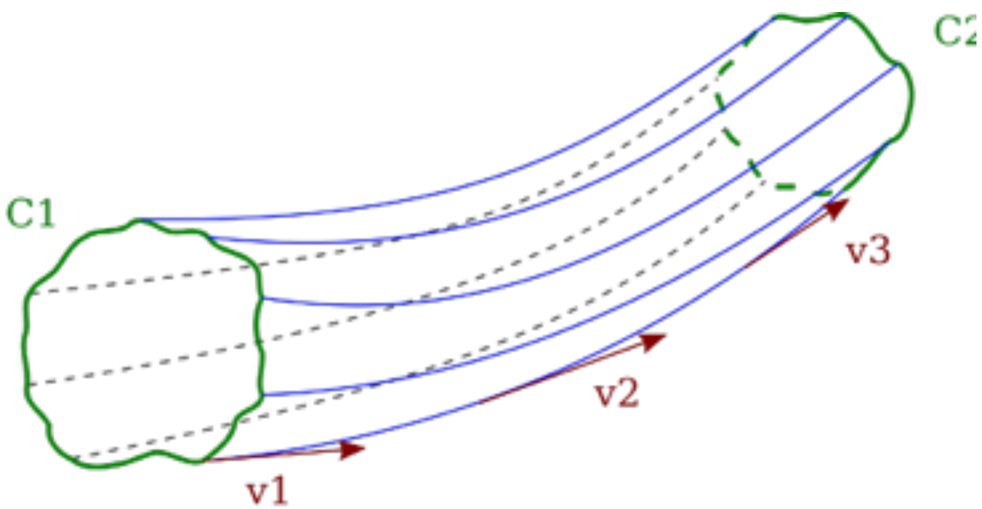
- We choose the initial condition in such way that the slope parameter is zero in the absence of chiral anomaly.

- A lot of numerics? No if we formulate the problem in a physical intuitive way.

Ideal hydrodynamics without anomaly and streamline

- What does current conservation imply for a normal fluid?

$$\partial_\mu \left(J^\mu_{\mathrm{norm}}\right) = \partial_\mu \left(n u^\mu\right) = u^\mu \partial_\mu \left(n\right) + n \left(\partial^\mu u_\mu\right) = 0$$



- Total charge is fixed in a fluid cell along the streamline. Change of charge density is compensated by the expansion of the volume.

Streamlines of anomalous hydrodynamics

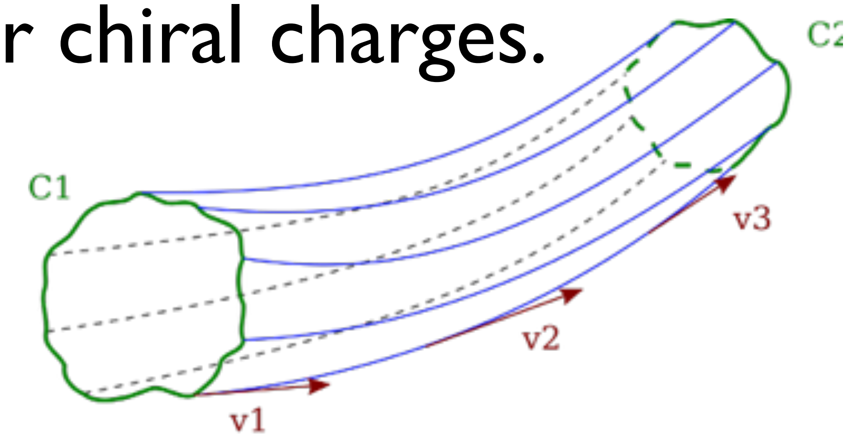
- Rewrite the constitute relation of chiral current similar to normal current as much as possible.

$$J^\mu_{L,R} = n_{L,R} u^\mu \mp \frac{qN_c}{4\pi^2}\mu_{L,R} B^\mu = n_{L,R}\left(u^\mu \mp u^\mu_{\text{anom}}\right) = \tilde{n}_{L,R}\tilde{u}^\mu_{L,R}$$

$$\partial_\mu J^\mu_{L,R} = 0 \qquad\qquad u^\mu_{\text{anom}} = \frac{qN_c B^\mu}{4\pi\chi}(n_{L,R} = \chi\mu_{L,R})$$

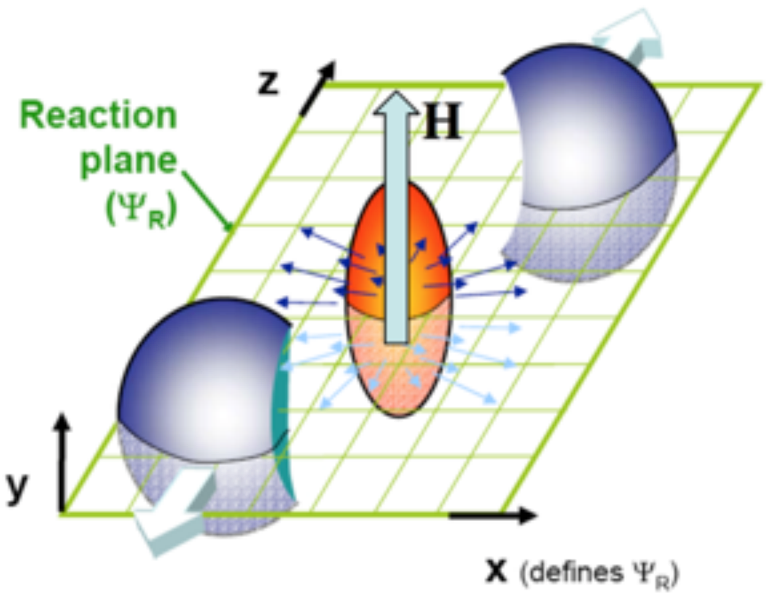
- Solving anomalous hydrodynamic equation is translated into the determination of streamline for chiral charges.

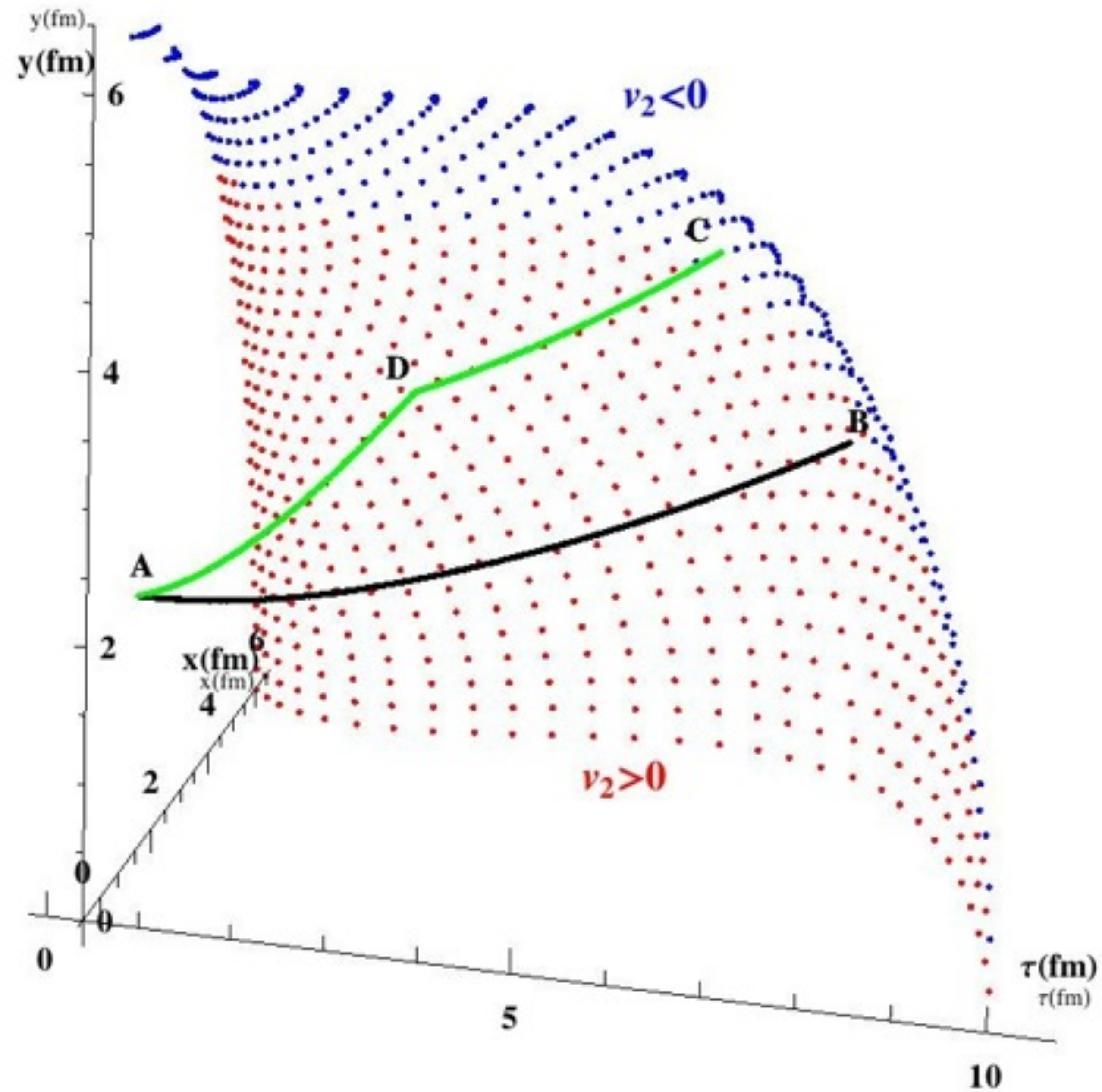
$$\frac{dX^\mu(\lambda)}{d\lambda} = \tilde{u}^\mu_{L,R}(X^\mu)$$



- The streamlines for left and right charge are different due to chiral anomaly.

Streamline in heavy-ion collisions

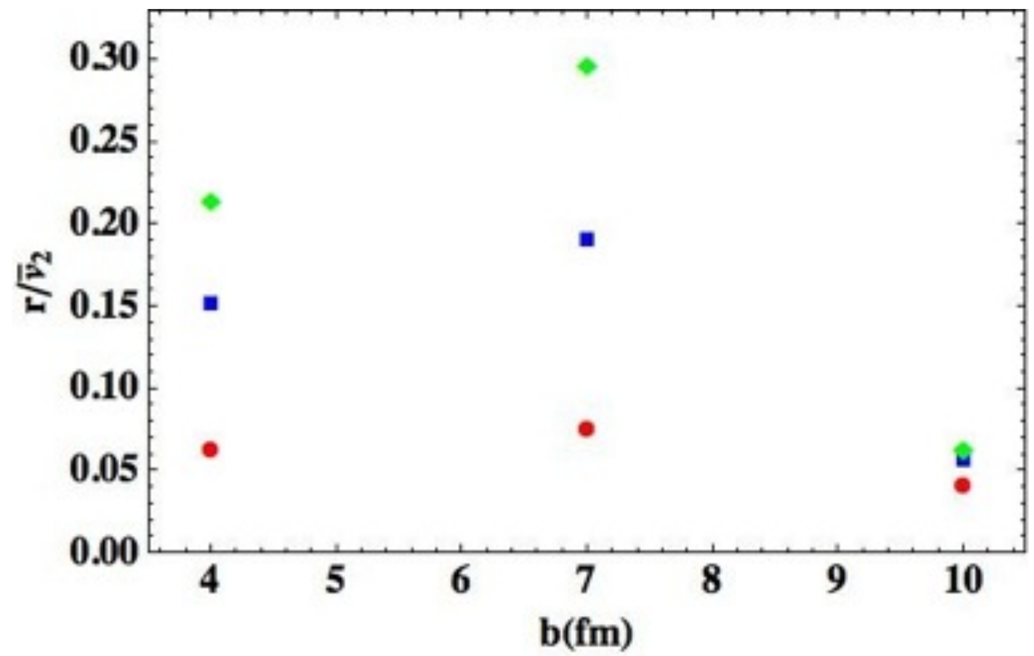




- Anomalous fluid velocity changes the streamline of chiral charges. This leads to $v_2^-(\pi) - v_2^+(\pi) > 0$.

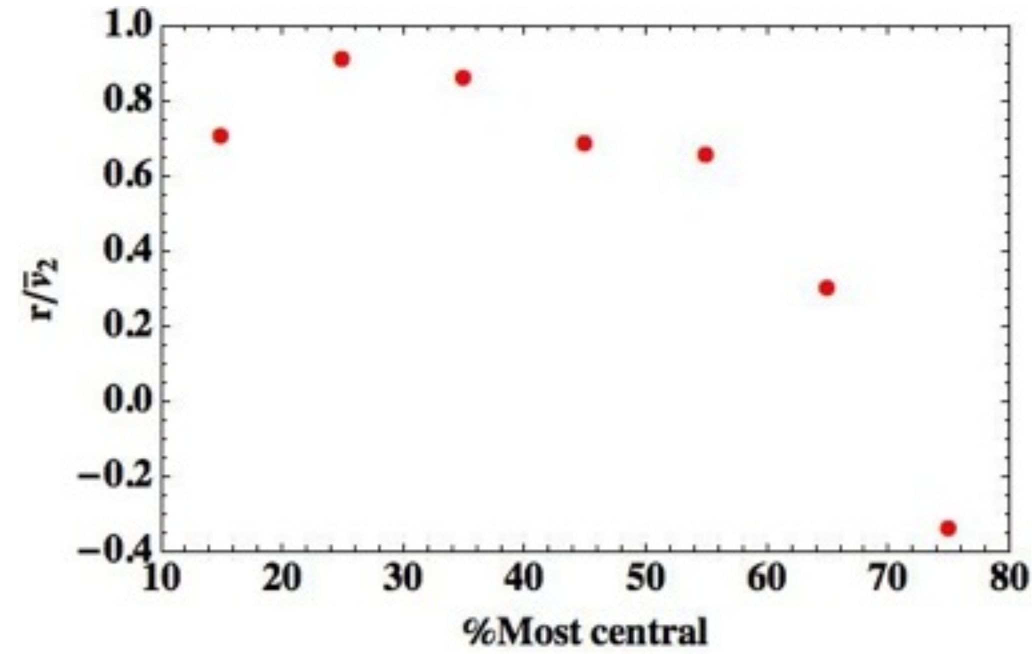
Results

- Chiral anomaly will give sizable contribution to the data if life time of magnetic field is a few fm.



Red, blue, green dots are corresponding to

$\tau_B = 2,\ 4,\ 6\ \mathrm{fm}$



STAR data

- Otherwise, it is more likely that the difference in elliptic flow it is due to initial distribution.(See Bzdak-Bozek(2013) for other sources of contributions)

Summary of Story I:

- We have developed a physically intuitive method to solve anomalous hydrodynamics in realistic background.

- As an application, we have studied charged dependent elliptic in experiment.

- The method based streamline gives a transparent picture on the chiral charge transportation in heavy-ion collisions.

- We could solve the anomalous hydrodynamic equation with arbitrary initial conditions.

Quantification of chiral magnetic effect

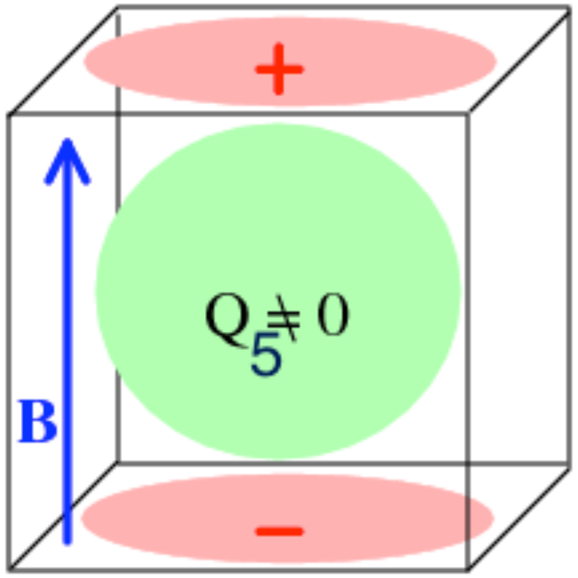
- Axial charge created in heavy-ion collisions will lead to vector current(chiral magnetic effect) thus charge dipole.

$$\Delta J^{\mu}_{V,\mathrm{anom}} = \frac{\mu_A}{2\pi^2} B^{\mu}$$

- Charge dipole implies non-trivial correlations between particle pairs(Kharzeev-McLerran-Warringa).

$$\langle \cos\left(\phi_{1,+} + \phi_{2,+} - 2\Psi_{\mathrm{RP}}\right)\rangle < 0$$

$$\langle \cos\left(\phi_{1,+} + \phi_{2,-} - 2\Psi_{\mathrm{RP}}\right)\rangle > 0$$

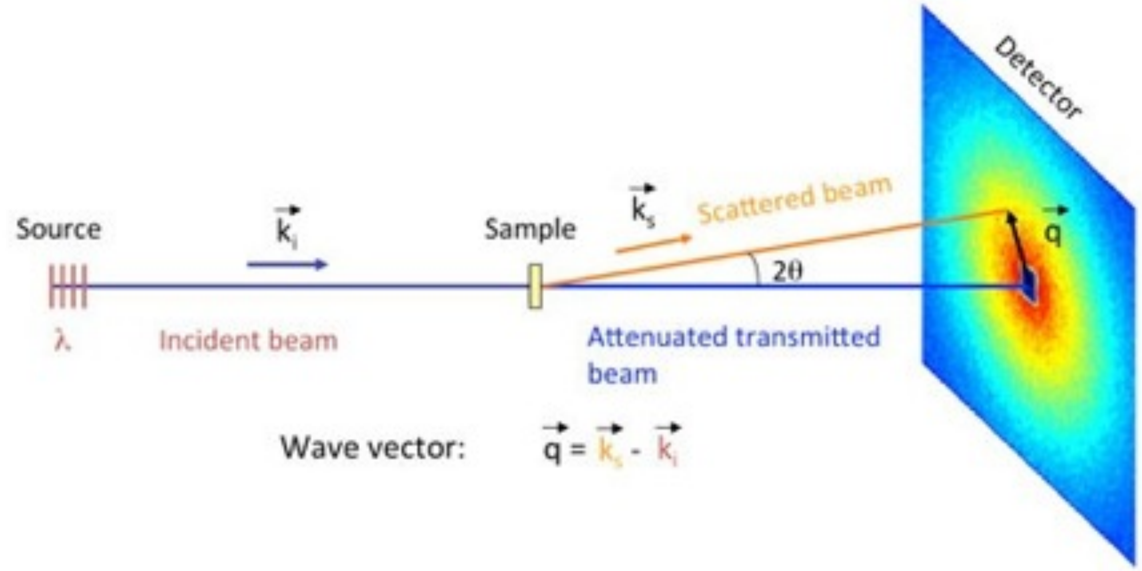


- Due to large background effects, quantify such effects in realistic background is imperative(in progress with Jinfeng, Liao).

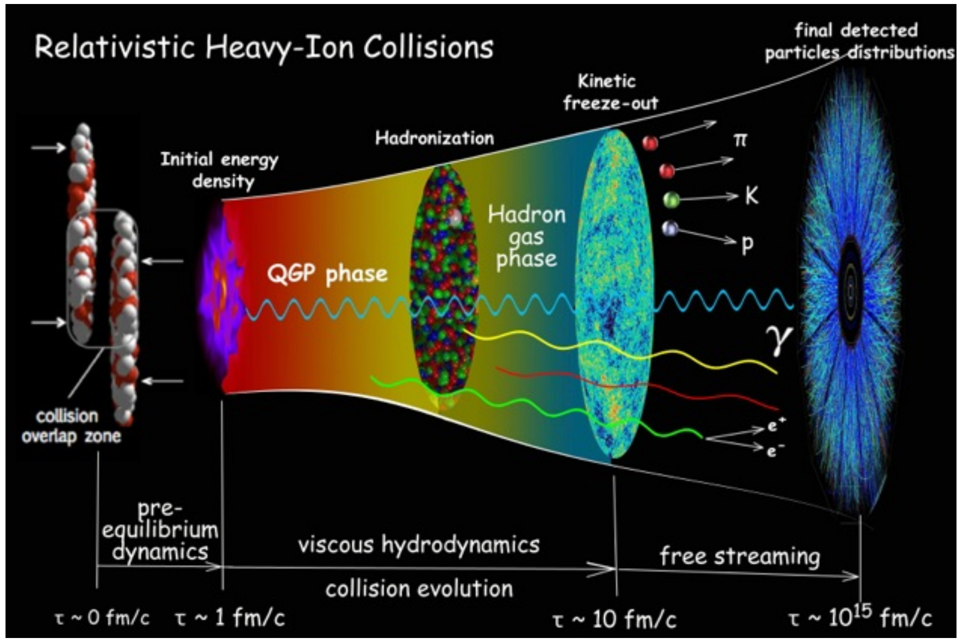
Story II: Electrical conductivity of QGP from photon production

Photon as a probe of QGP

- The properties of condensed matter can be measured by scatterings experiments.



- Quark-gluon plasma is radiating. Photon produced in heavy-ion collisions is a probe of QGP . $(\alpha_{EM} \ll \alpha_s)$



Photon as a probe of QGP

- Photon produced per volume per time is related to Green's function of QGP:

$$P_T^{ij} = \delta^{ij} - \hat{p}^i \hat{p}^j$$

$$\omega \frac{d\Gamma_\gamma}{d^3\vec{p}} = -\frac{n_B(\omega/T)}{(2\pi)^3} P_T^{ij} \mathrm{Im}\left[G_{ij}^R(\omega = |\vec{p}|)\right]$$

Retarded correlator: $G_R^{\mu\nu} \sim \langle J^\mu J^\nu \rangle$

- Photons are produced during the full evolution of the fireball with shifted frequency:

$$\frac{dN_\gamma}{p_t dp_t d\phi_p dY} = \int_{T \geq T_f} d^4x \omega_{\text{shift}} \frac{d\Gamma_\gamma}{d^3p'}\Big|_{\omega_{\text{shift}} = |\vec{p}|} \qquad \omega_{\text{shift}} = p_{\text{Lab}}^\mu u_\mu$$

Photon frequency is shifted in the rest frame of fluid.

- To study photon spectrum, we need i) theoretical understanding of photon rate and ii) hydrodynamic evolution.

Photon and thermal correlators

- Photon production in heavy-ion collisions has been studied by evolving pQCD photon rate with hydrodynamic simulations.

- QGP is strongly coupled! pQCD rate may not be applicable for photon energy below a few GeV!

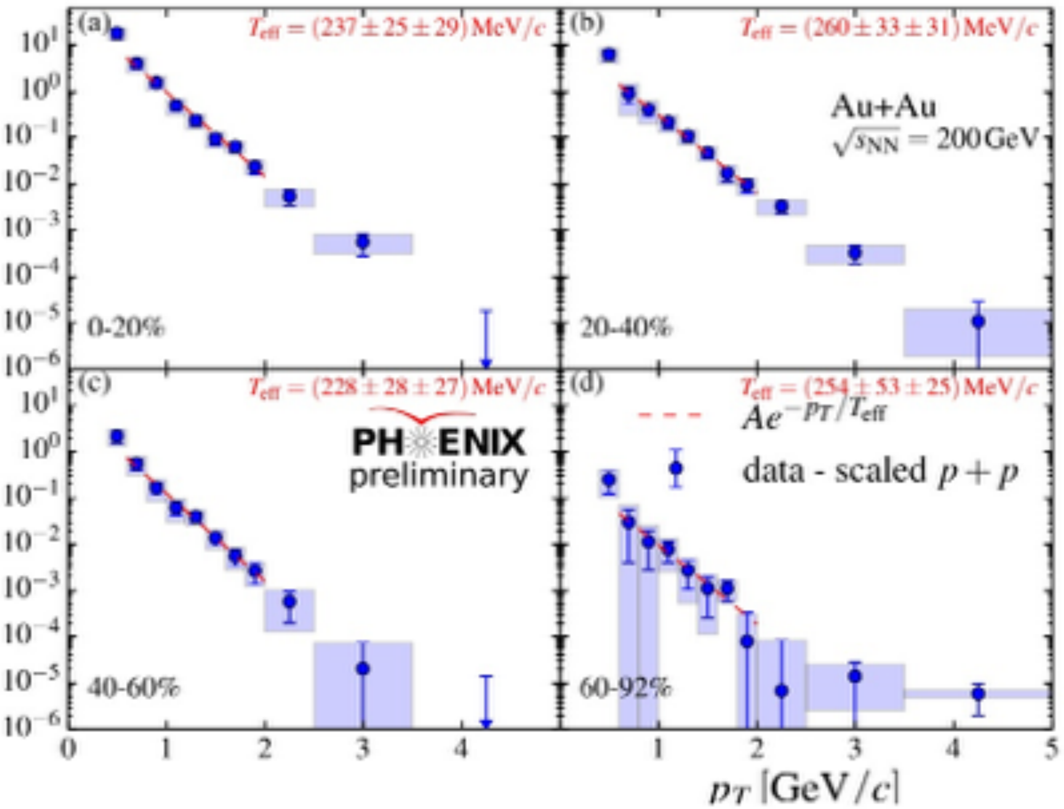
- Determination of correlation functions from microscopic theory is challenging.

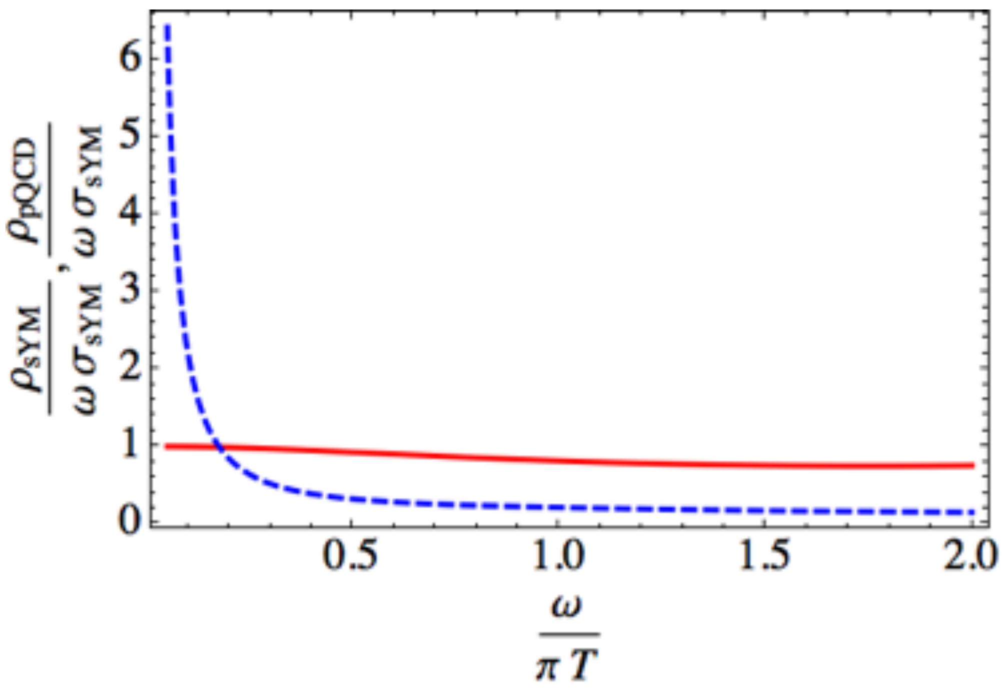
- In low frequency limit, a macroscopic description is possible!

$$P_T^{ij}\,\mathrm{Im}\left[G_{ij}^R(\omega=|\vec{p}|)\right)_{B=0}=\frac{\alpha_{\mathrm{EM}}\omega\sigma_0}{\pi^2 e^2}$$

Soft Photon and conductivity

- Lowest pt in experiment: 0.5 GeV. (by PHENIX) $\pi T_{QGP} \approx 1GeV$





(sSYM vs pQCD)

- Strongly interacting system has a wider hydrodynamic regime!
- We will use the hydrodynamic approximation to study the lowest pt photon data

$$P_T^{ij}\,\mathrm{Im}\left[G_{ij}^R(\omega = |\vec{p}|)\right)_{B=0} = \frac{\alpha_{\rm EM}\omega\sigma_0}{\pi^2 e^2}$$

Electrical conductivity of QGP

- We write down the rate

$$\frac{dN_\gamma}{2\pi p_t dp_t dY} = \frac{\alpha_{\rm EM}}{\pi^2}\int_0^{2\pi}\frac{d\phi_p}{2\pi}\int_{T\geq T_f} d^4x \frac{\omega_{\rm shift}\sigma}{\exp(\omega_{\rm shift}/T)-1} \qquad \omega_{\rm shift} = p^\mu_{\rm Lab}u_\mu$$

- We estimate conductivity at QGP temperature by computing the following ratio.

$$\langle\frac{\sigma}{e^2T}\rangle \equiv \frac{\frac{dN_\gamma}{2\pi p_t dp_t dY}}{\frac{\alpha_{\rm EM}}{\pi^2}\int_0^{2\pi}\frac{d\phi_p}{2\pi}\int_{T\geq T_f} d^4x \frac{\omega_{\rm shift}T}{\exp(\omega_{\rm shift}/T)-1}}$$

From the data

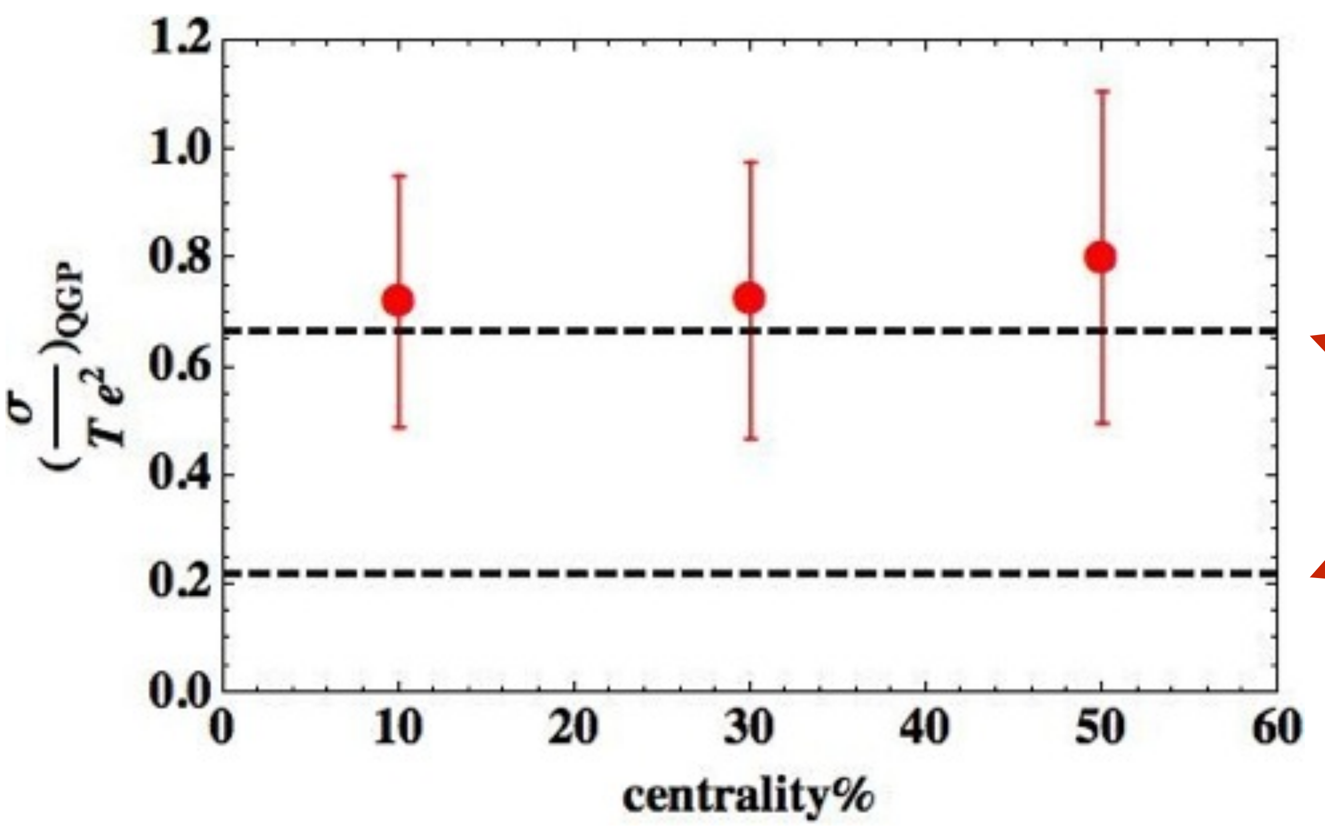
(NB: σ/T is dimensionless, similar to η/s .)

- We evolve the integral with realistic hydrodynamic background.

(The realistic hydrodynamic background is from Heinz' group, available online: https://wiki.bnl.gov/TECHQM/index.php/Main_Page)

Electrical conductivity of QGP

- PHENIX data, different for different centralities.
- Hydrodynamic background, different for different centralities.
- The ratio has a weak-dependence on centralities! Conductivity is the properties of QGP!



$$\langle \frac{\sigma}{e^2 T} \rangle \equiv \frac{\frac{dN_\gamma}{2\pi p_t dp_t dY}}{\frac{\alpha_{EM}}{\pi^2} \int_0^{2\pi} \frac{d\phi_p}{2\pi} \int_{T \geq T_f} d^4x \frac{\omega_{\text{shift}} T}{\exp(\omega_{\text{shift}}/T) - 1}}$$

Lattice extraction by Bielefeld-BNL group(2011) at T=1.45Tc

YY, ArXiv:1312.4434(PRC)

Summary of Story II

- We have estimated electrical conductivity, for the first time in literature, from soft photon data.

- Thermal correlations function at other pt window? More information from dileptons?

The spherical cow

- Theorists always use highly simplified scientific models to study complex real phenomena.

The actual cow

- In heavy-ion collisions, studying the problems in realistic environment is important!

Conclusion and outlook

- We have shown two examples on exploring the properties of QGP with realistic hydrodynamic background.
  - Effects of chiral anomaly and pion elliptic flow.
  - conductivity and photon spectrum.

- More fruitful results in future.