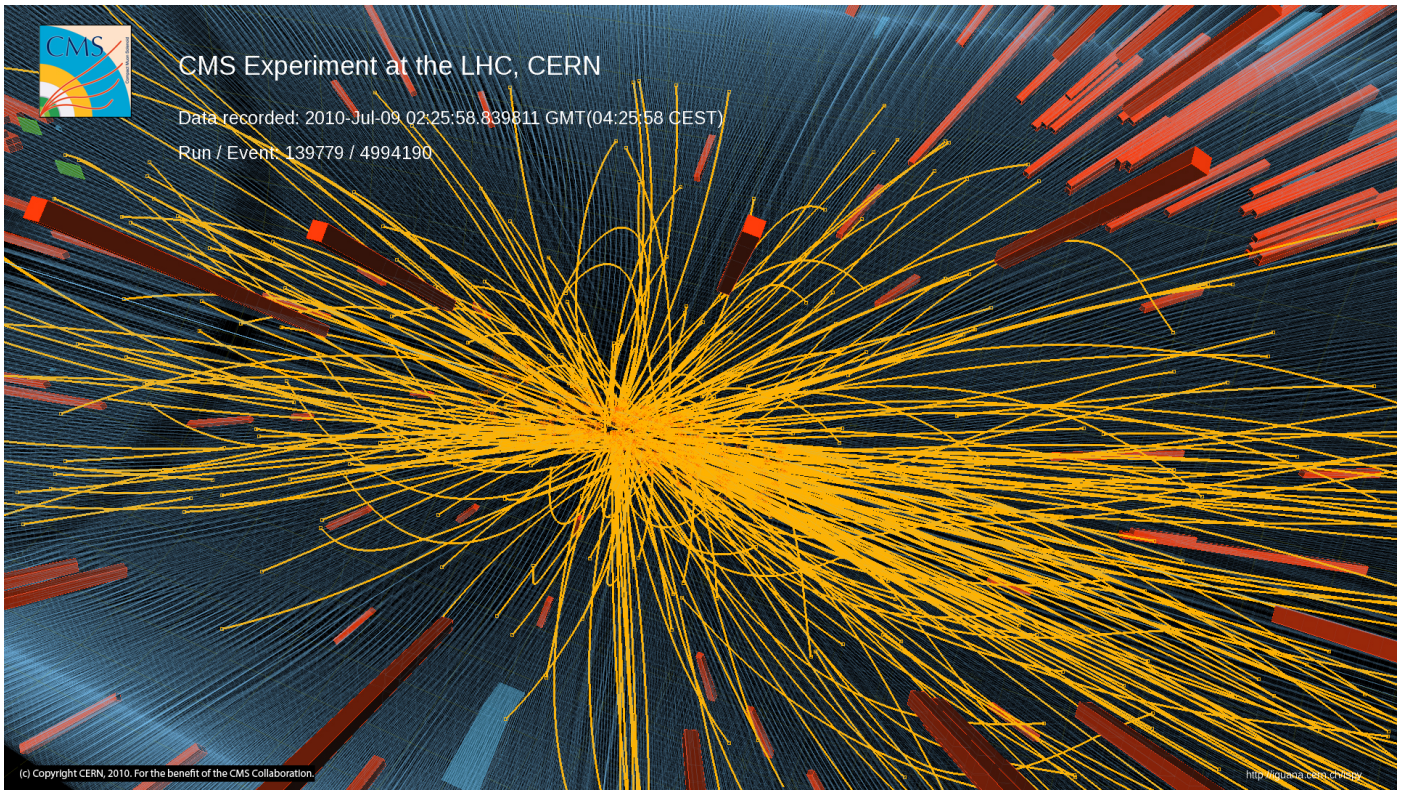
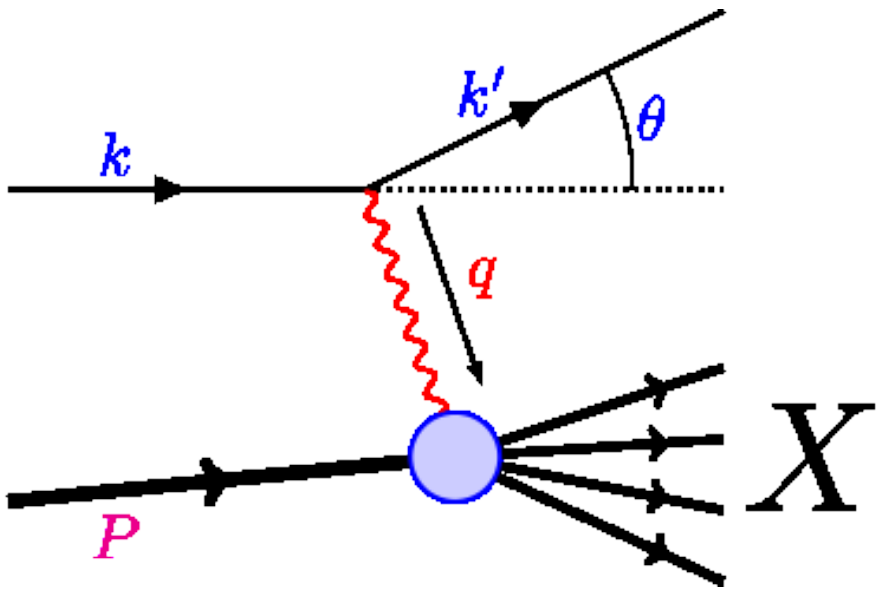
HIGHER HARMONICS FROM INTRINSIC MULTI-PARTICLE PRODUCTION



Kevin Dusling
Physical Review Letters

Brookhaven National Laboratory
March 28, 2014

Deep Inelastic Scattering



Photon Virtuality:

$$Q^2 = 4E_k E_{k'} \sin^2\left(\frac{\theta}{2}\right)$$

Energy Transfer:

$$y = \frac{E_k - E_{k'}}{E_k}$$

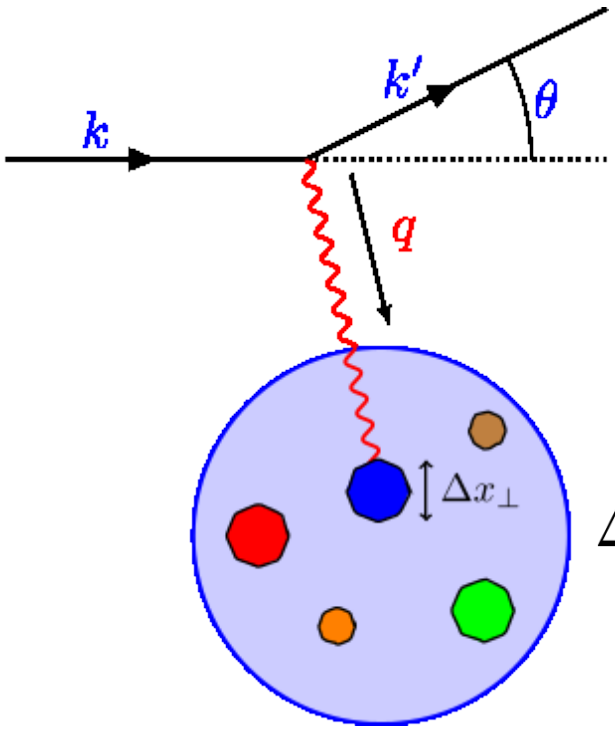
Bjorken x variable:

$$x_{Bj} = \frac{Q^2}{ys}$$

$$\frac{d^2\sigma^{ep\to eX}}{dxdQ^2} = \frac{4\pi\alpha^2}{xQ^4}\left[\left(1 - y + \frac{y^2}{2}\right)F_2(x,Q^2) - \frac{y^2}{2}F_L(x,Q^2)\right]$$

DIS: Interpretation

Q: Transverse Spatial Resolution



$$\Delta x_\perp \sim \frac{1}{Q}$$

x_{Bj}: 1/ Interaction time

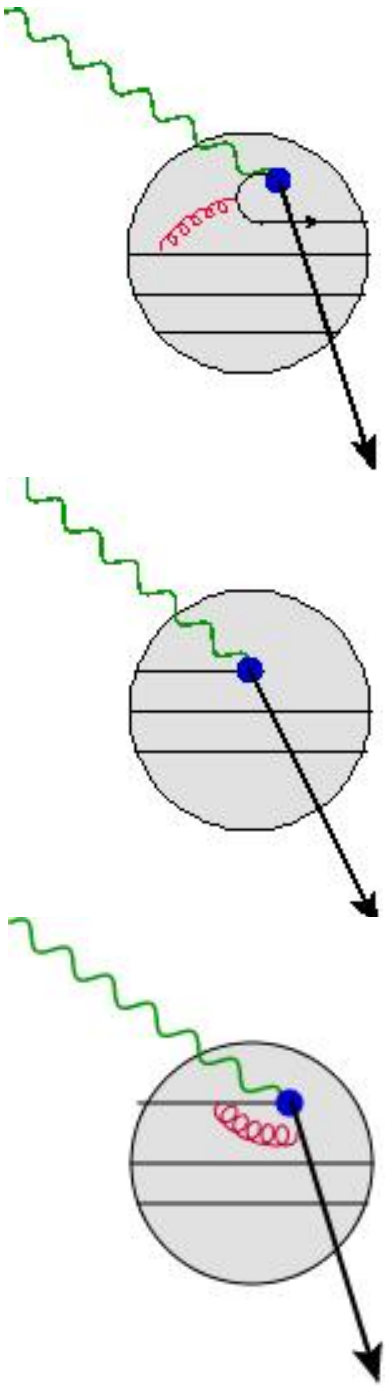
$$t_{\rm DIS} \sim \frac{x_{Bj} P}{Q^2} \ll t_{\rm parton} \sim \frac{1}{\Lambda_{\rm QCD}} \frac{P}{m}$$

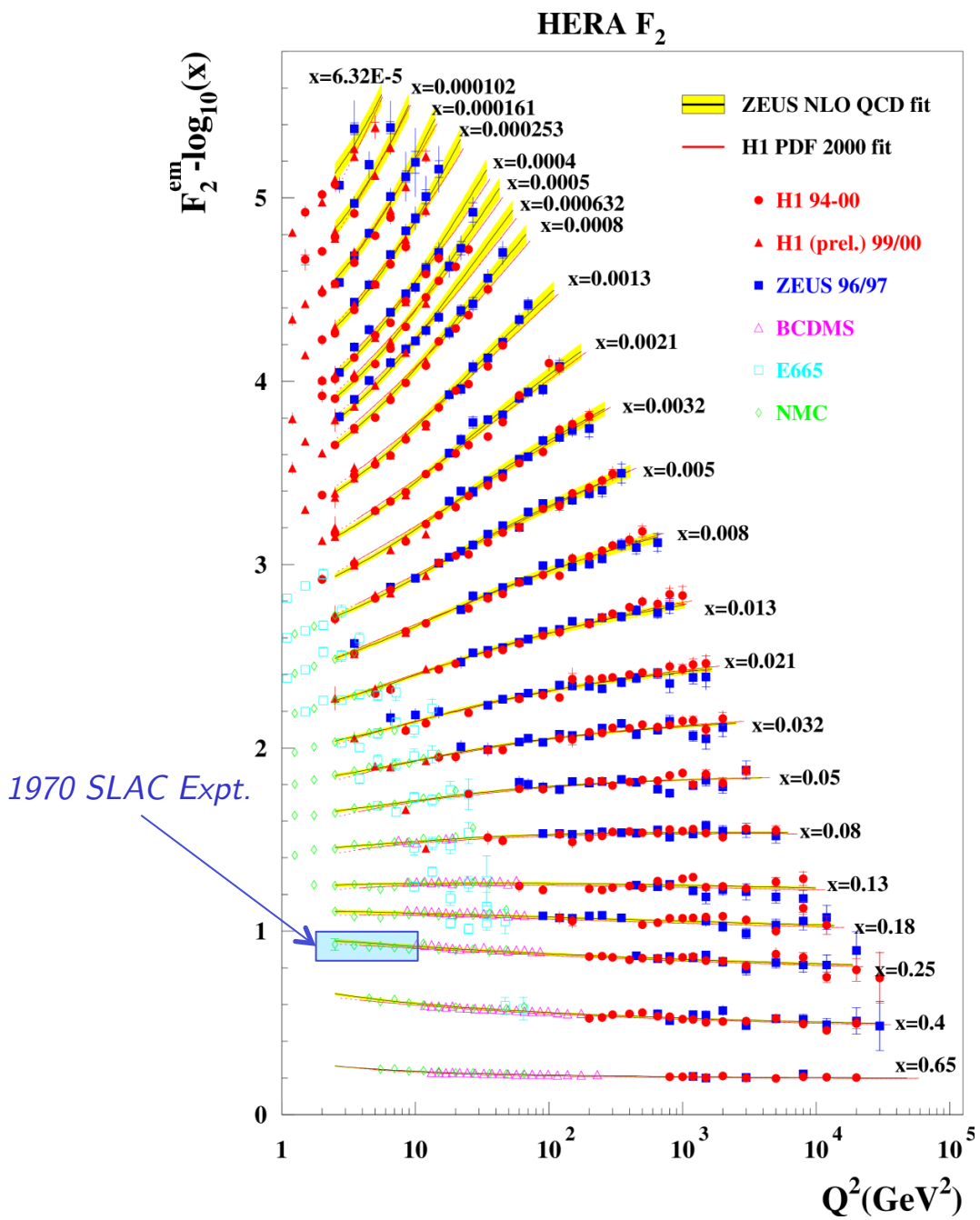
partons are free during interaction

x_{Bj}: 2/ fraction of momentum carried by struck quark

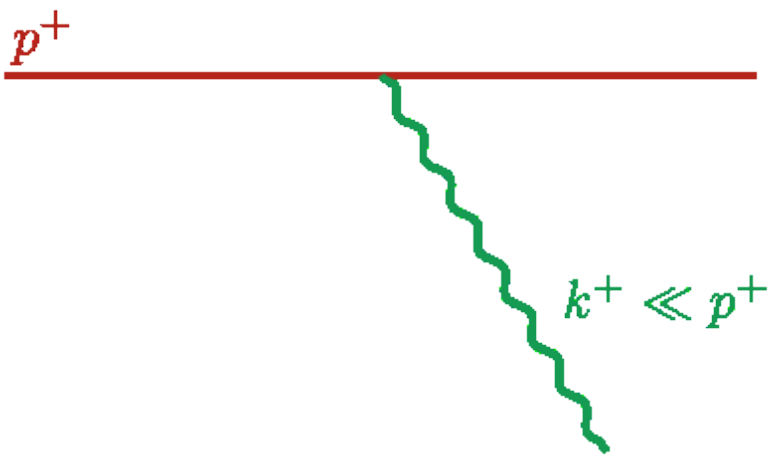
For Q=20 GeV we are probing distances 1/100th the proton size !

The age of HERA





Gluon Radiation

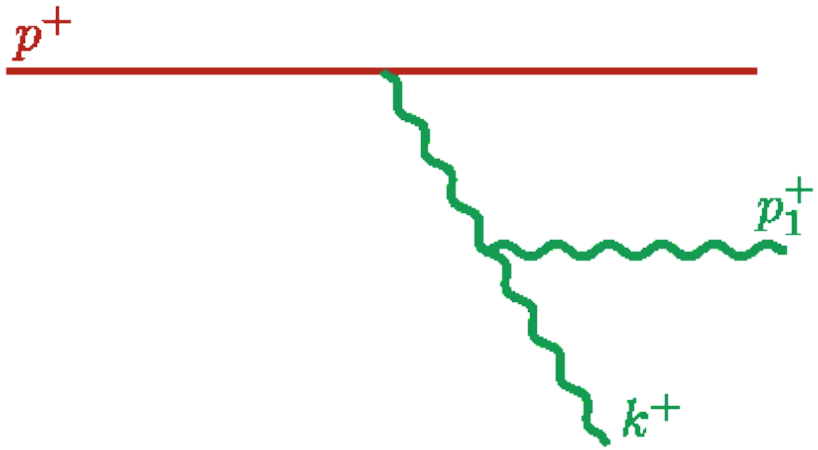


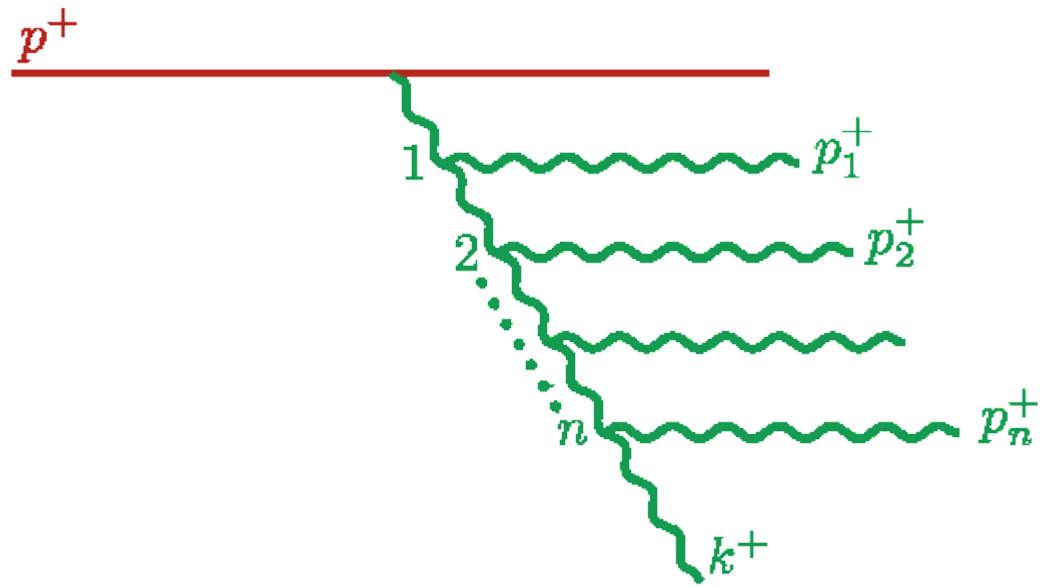
$$d\sigma = \frac{\alpha_s}{4\pi} 2C_F \left[1 + \left(1 - \frac{k}{E}\right)^2\right] \frac{dk}{k} \frac{dk_\perp^2}{k_\perp}$$

The cost to radiate an additional gluon is:

$$\frac{\alpha_s N_c}{\pi} \int_{k^+}^{p^+} \frac{dp_1^+}{p_1^+} = \frac{\alpha_s N_c}{\pi} \ln \frac{p^+}{k^+} = \bar{\alpha}_s \ln \frac{1}{x}$$

Gluon Radiation





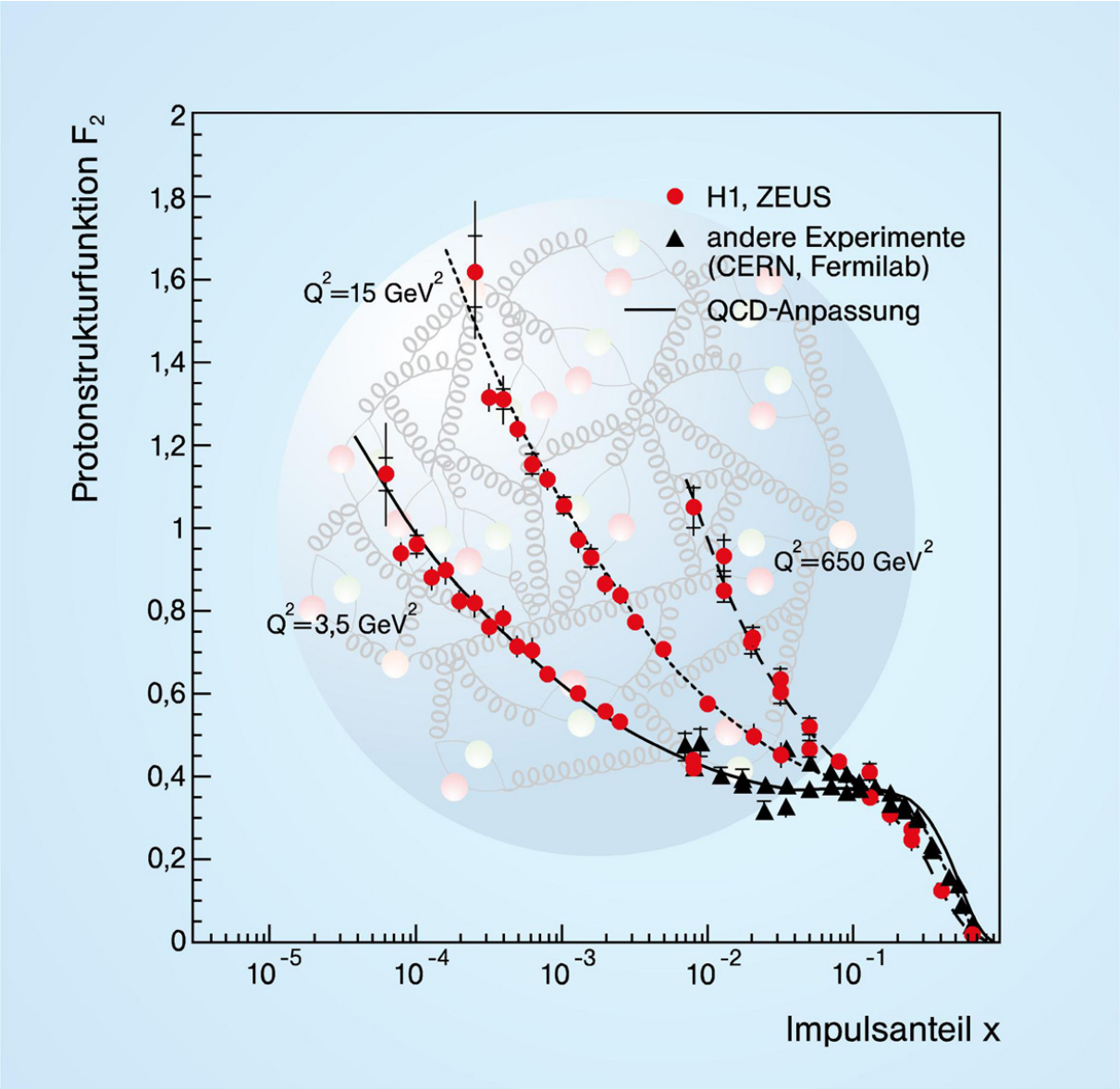
$$\sim \bar{\alpha}_s \ln \frac{1}{x} \qquad\qquad \sim \frac{1}{n!}\left(\bar{\alpha}_s \ln \frac{1}{x}\right)^n$$

When $\bar{\alpha}_s \ln(1/x) \sim 1$ *we need to perform a re-summation (BFKL)*

$$xG(x,Q^2) \sim e^{\omega \bar{\alpha}_s \ln\ 1/x = \frac{1}{x^{\omega \bar{\alpha}_s}}} \qquad \omega = 4 \ln 2$$

Find a rapid rise at small x

The rise at small x



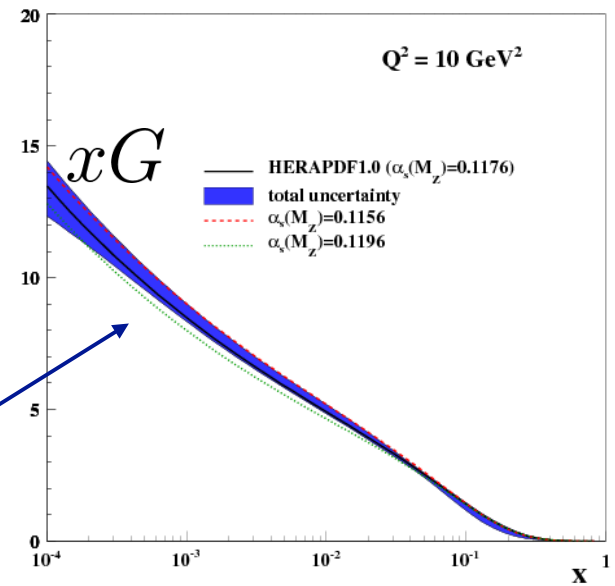
This cannot continue to rise like this forever!

Criteria for Gluon Saturation

1. *Transverse gluon density:* $\rho \sim \dfrac{A\, xG}{S_\perp} \sim A^{1/3}\, xG$

2. *Recombination cross-section:* $\sigma_{gg\to g} \sim \dfrac{\alpha_S}{Q^2}$

3. *Saturation Criteria:* $\rho\sigma_{gg\to g} \gtrsim 1$

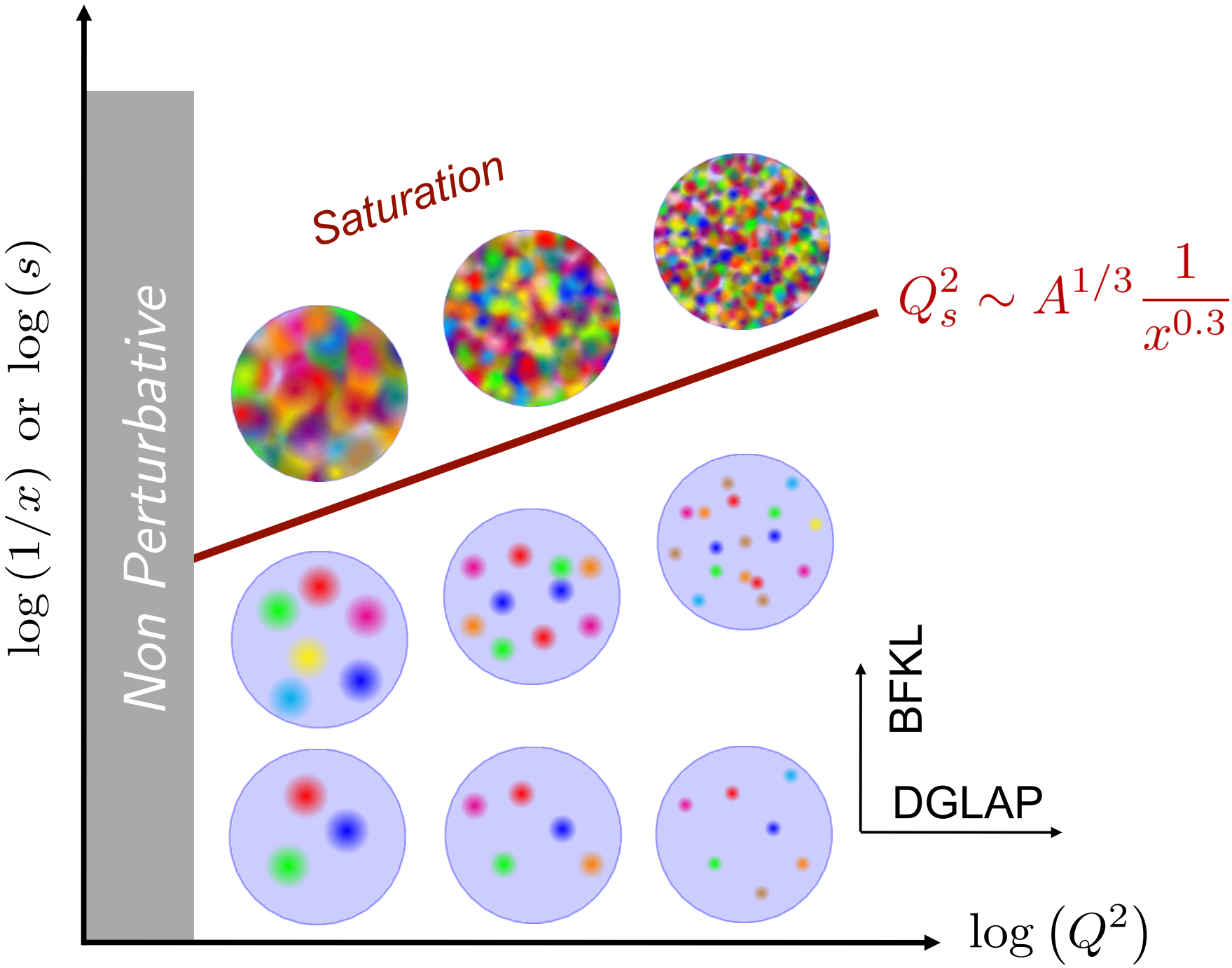


$$Q_s^2 \sim A^{1/3}\, xG \sim A^{1/3} x^{-0.3}$$

Saturation scale is a new momentum scale in problem

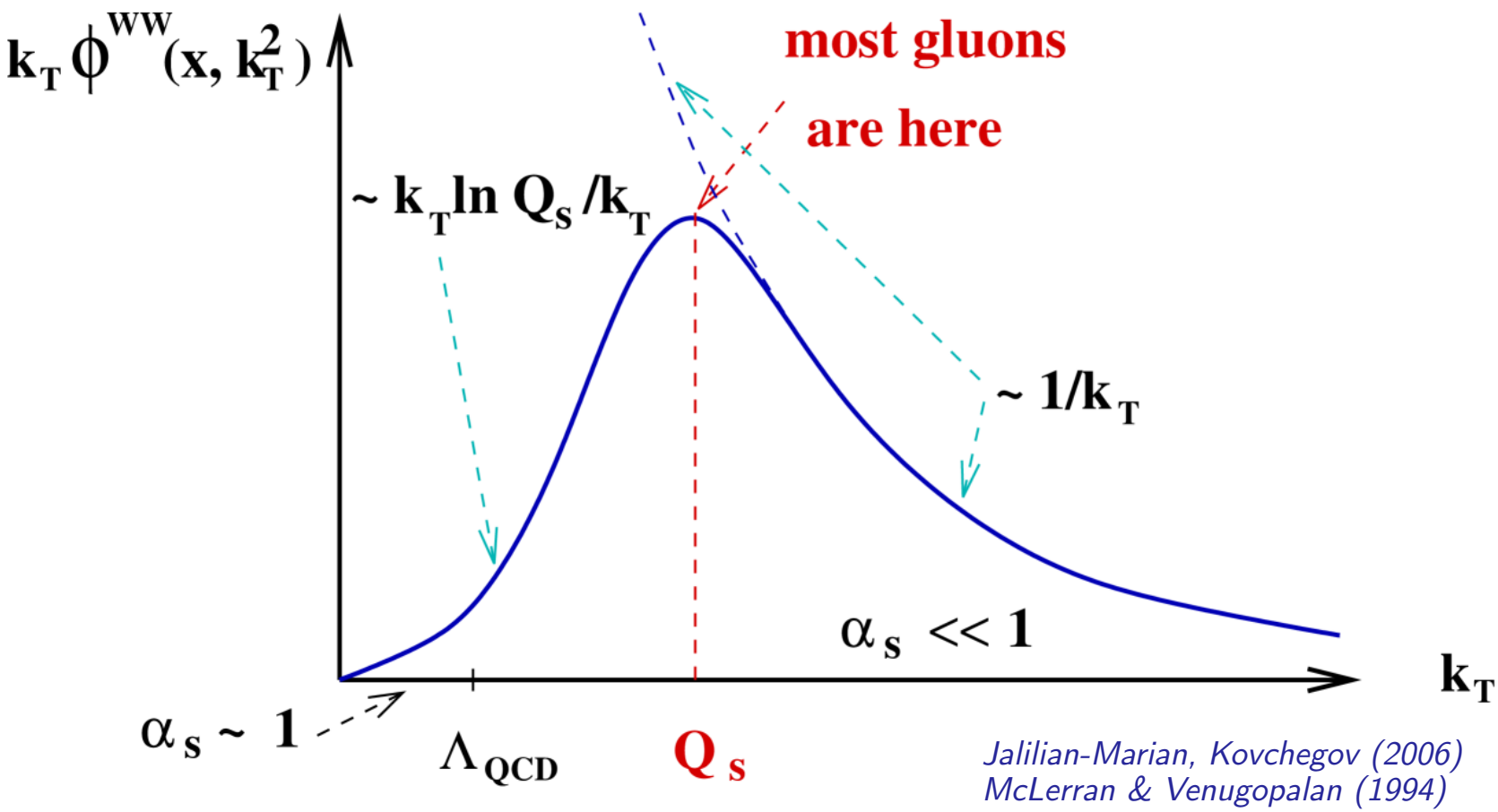
Gribov, Levin, Ryskin (1983)

High Energy Landscape of QCD



Gluon Saturation

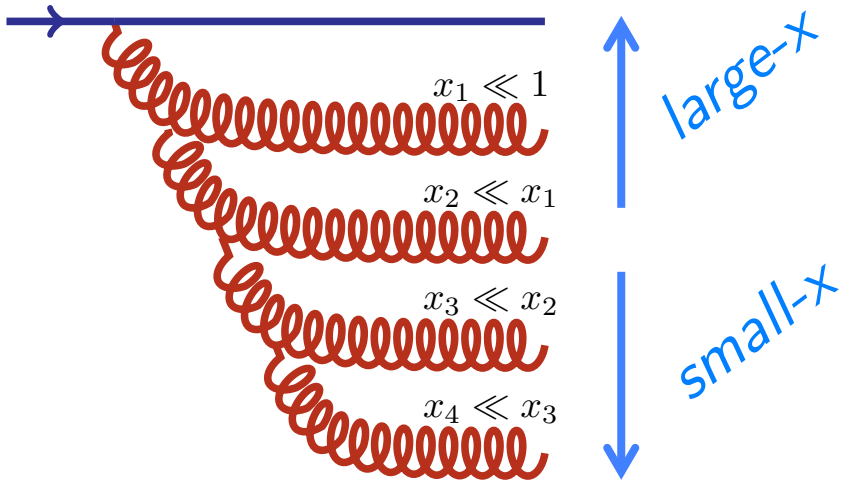
A large nucleus, $A \gg 1$, results in a high occupation of gluons, $\rho \sim A^{1/3}$ and therefore the gluon-field can be treated classically.



Jalilian-Marian, Kovchegov (2006)
McLerran & Venugopalan (1994)

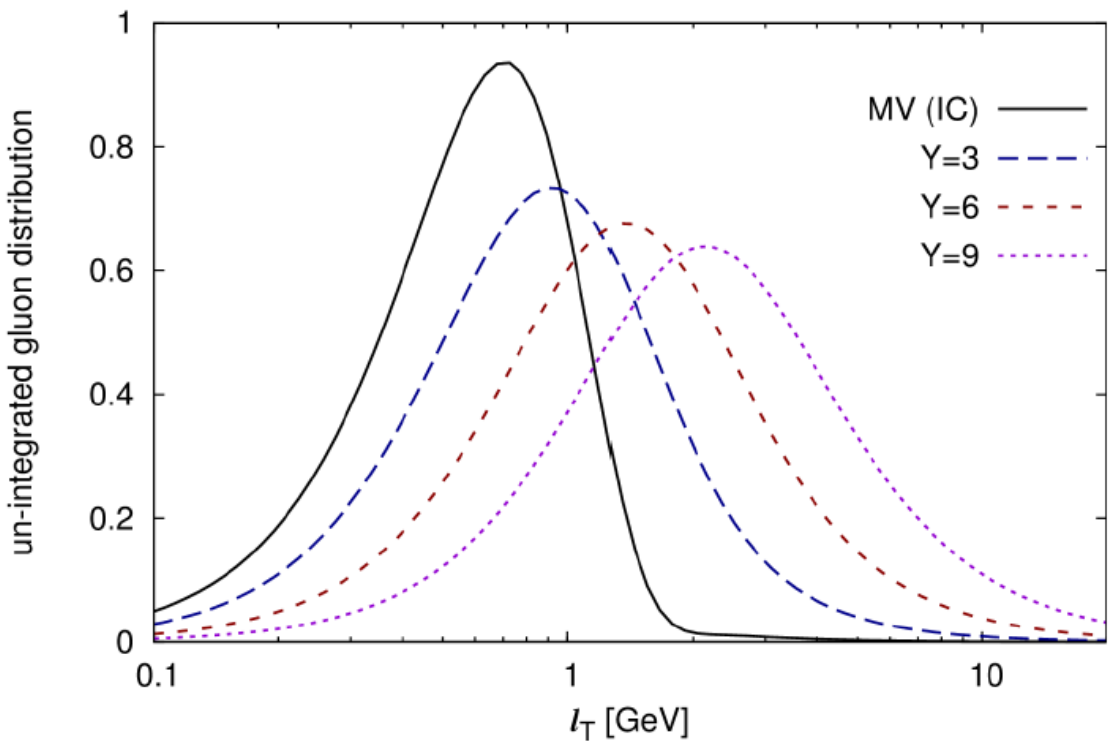
Power counting changes considerably ...

Many-body high energy QCD: The Color Glass Condensate (CGC)



Observables must be independent from how the large-x and small-x degrees of freedom are separated:
Functional Renormalization Group equation

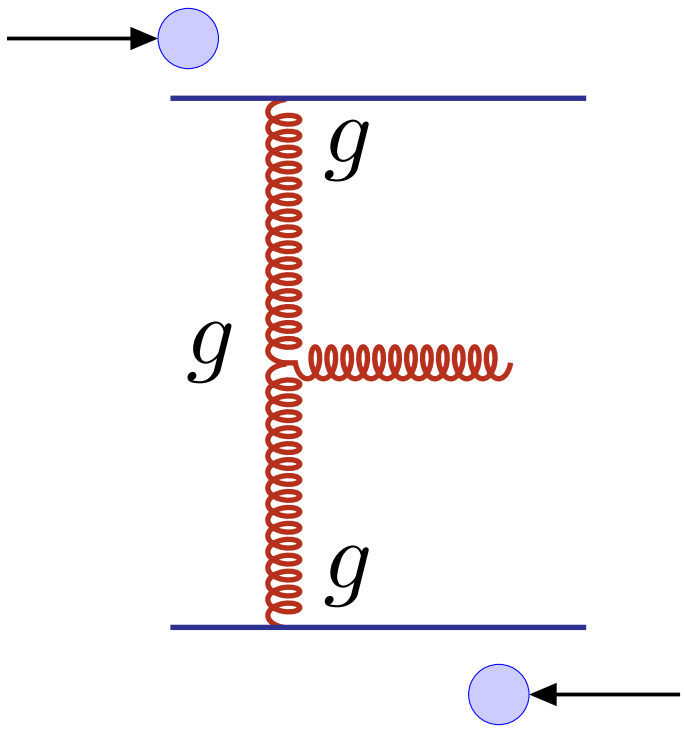
(JIMWLK - Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner).



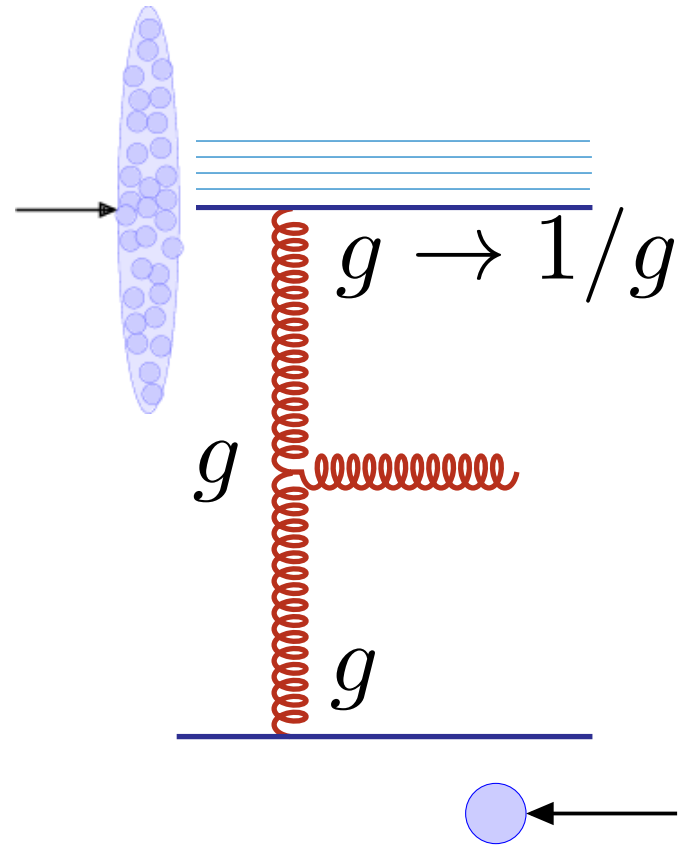
For reviews see:
McLerran, Lect. Notes Phys.583:291-334 (2002), arXiv:hep-ph/0104285

Gelis, Iancu, Jalilian-Marian, Venugopalan: Ann. Rev. Nucl. Part. Sci. (2010), arXiv: 1002.0333

Power counting in p+A



$$\mathcal{O}\left(g^6\right)$$



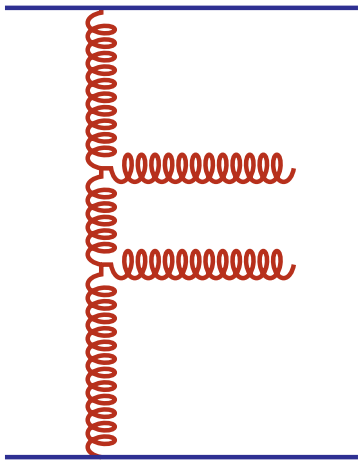
$$\mathcal{O}\left(g^2\right)$$

Can obtain saturation by large A, small x or high energy

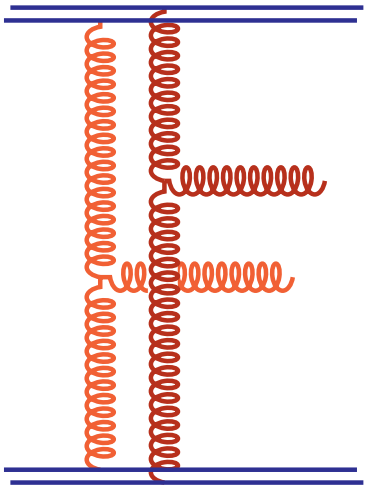
Power counting in QCD: multiparticle production

"jet" graph | *"glasma" graph*

Low color density (min. bias events)

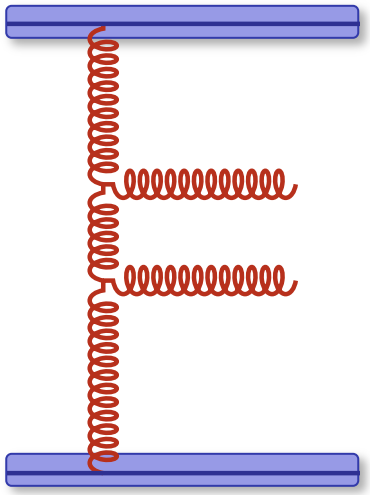


$\mathcal{O}\left(g^8\right)$

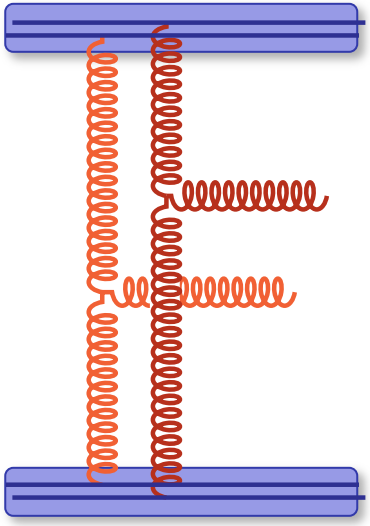


$\mathcal{O}\left(g^{12}\right)$

High color density ("central" events)



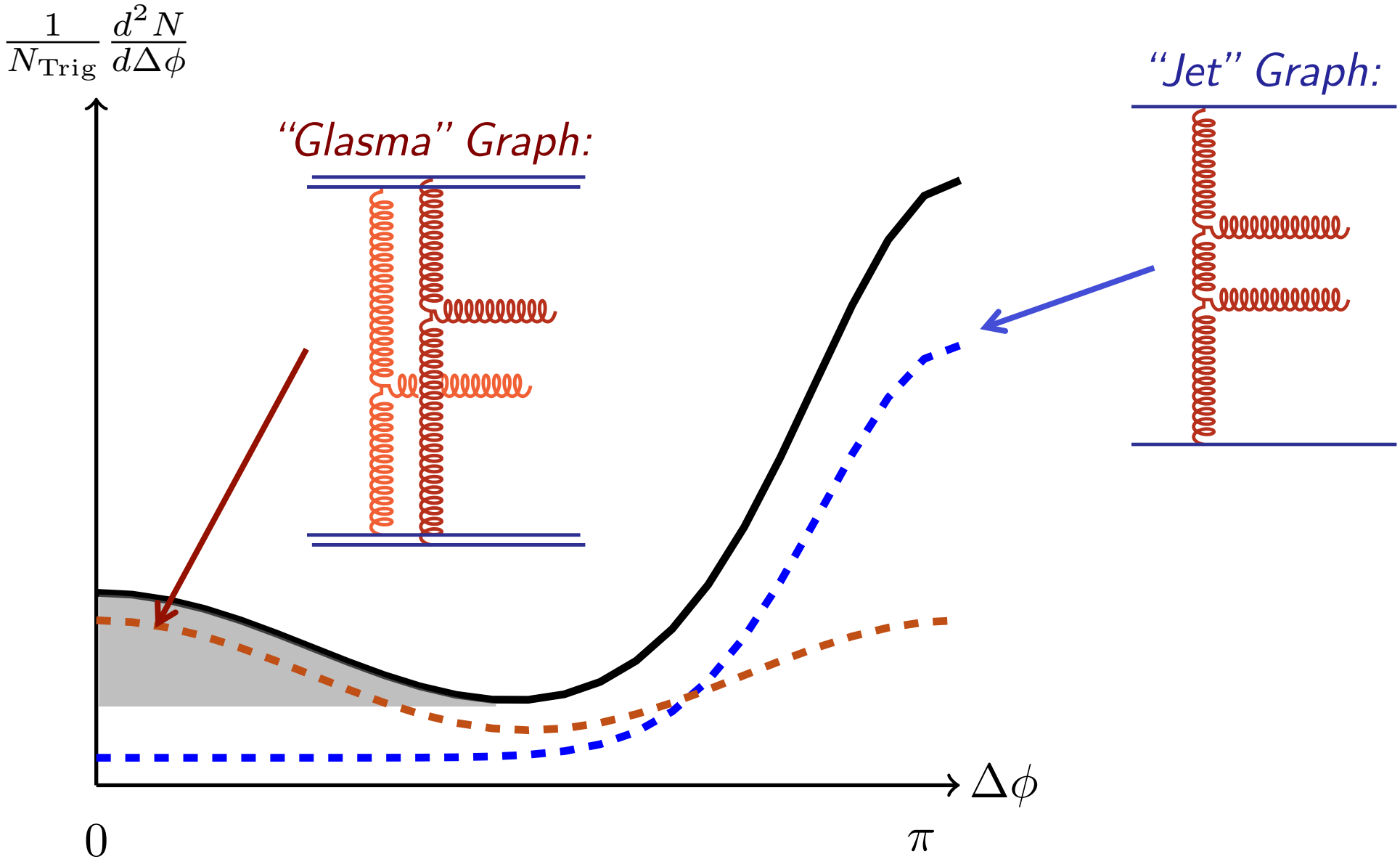
$\mathcal{O}\left(g^0\right)$



$\mathcal{O}\left(1/g^4\right)$

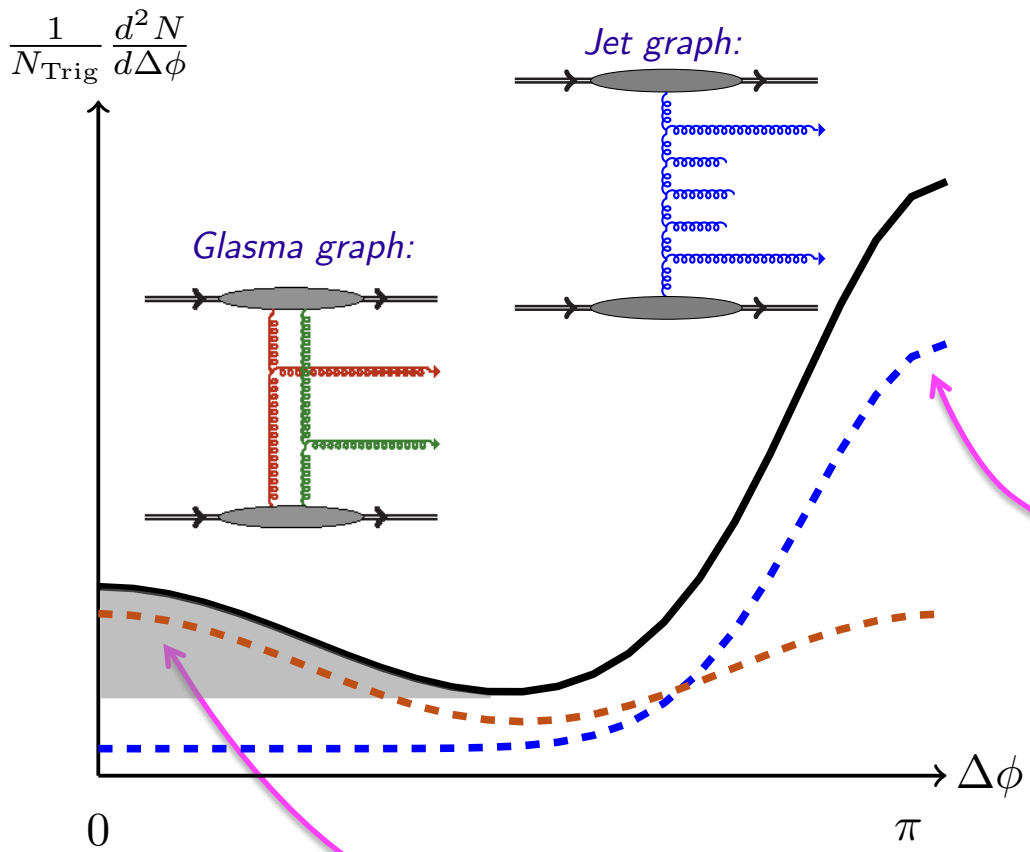
Expect g^{16} enhancement of glasma graph! *How can this be seen?*

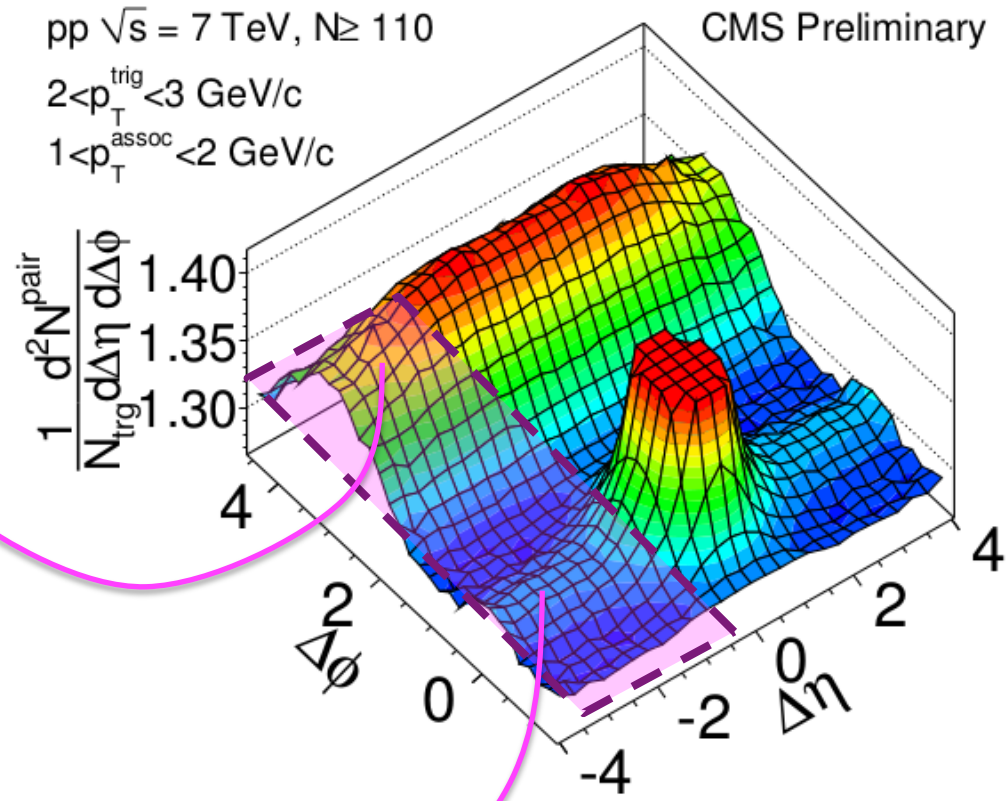
Di-hadron production



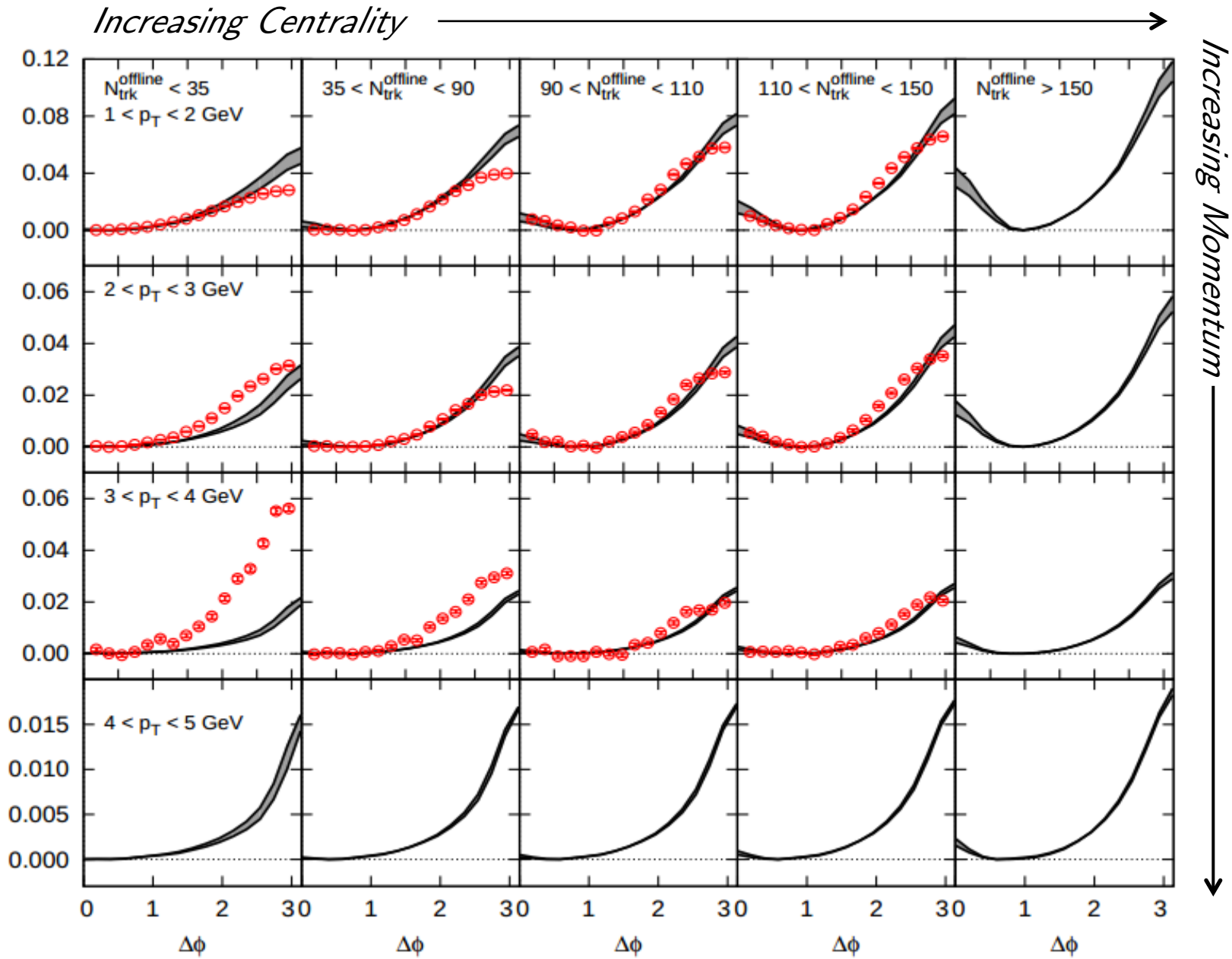
Any thing like this in the data?

Anatomy of a proton-proton collision



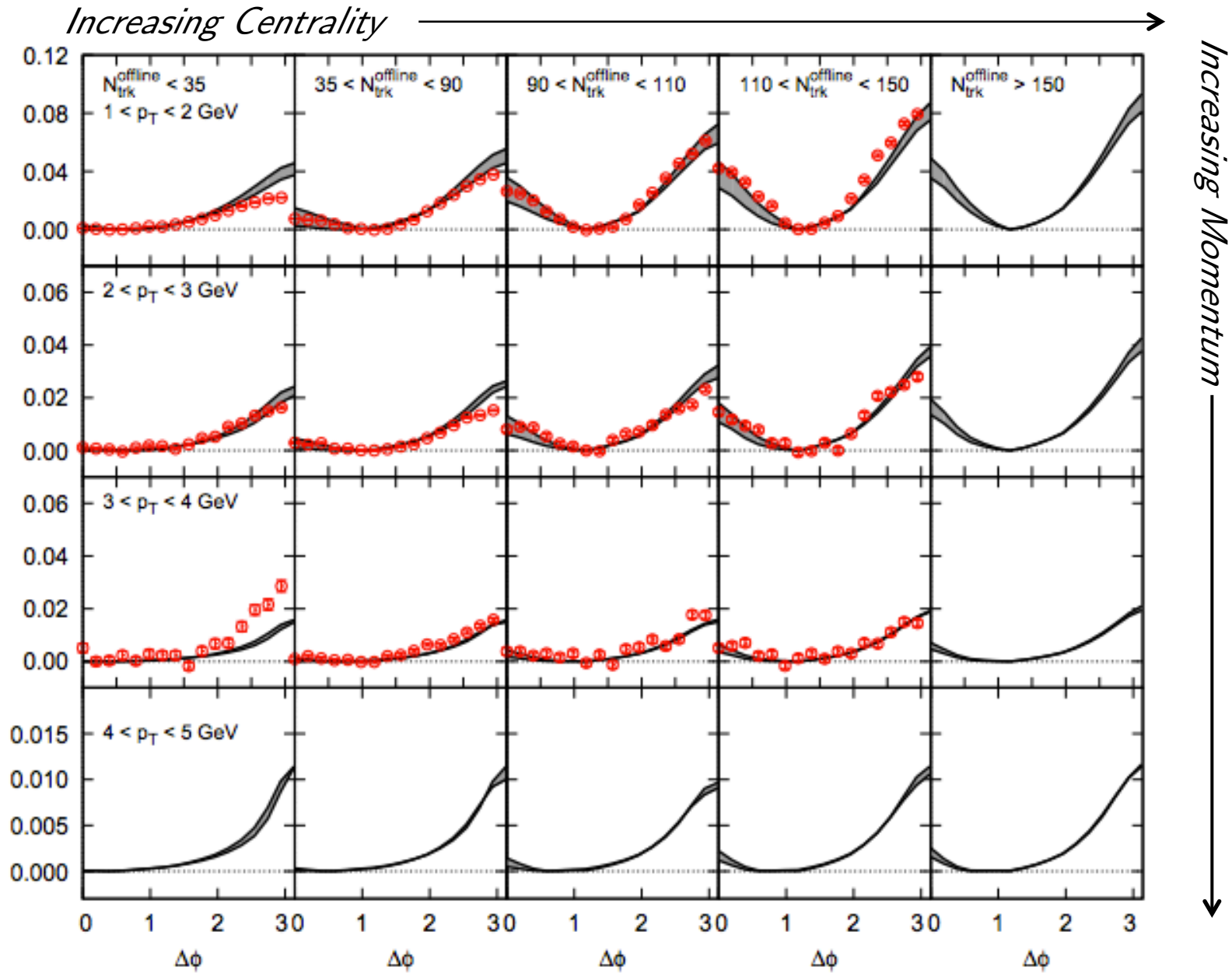


Proton-Proton systematics



Dusling, Venugopalan, Phys.Rev. D87 (2013) 094034, arXiv:1302.7018

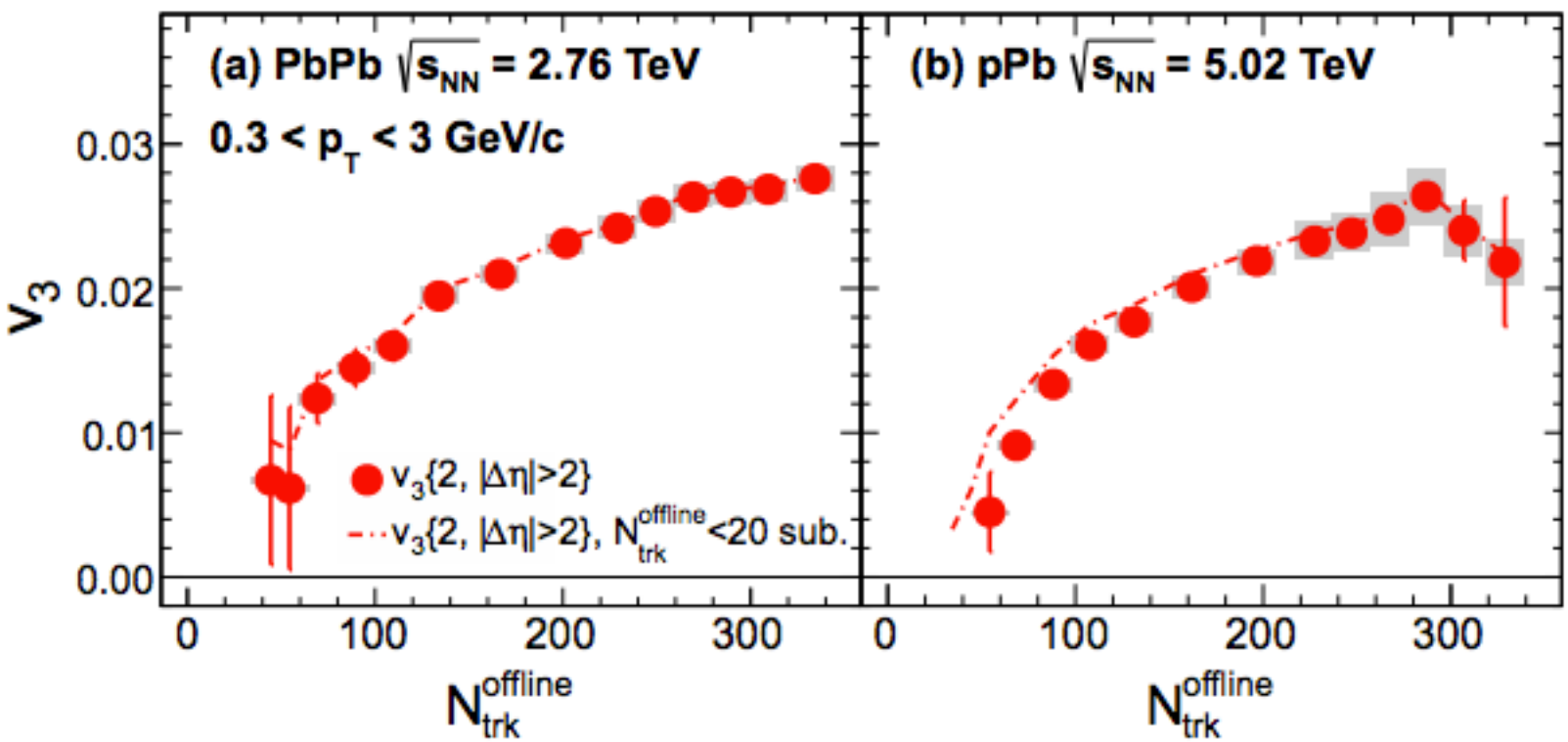
Proton-Lead systematics



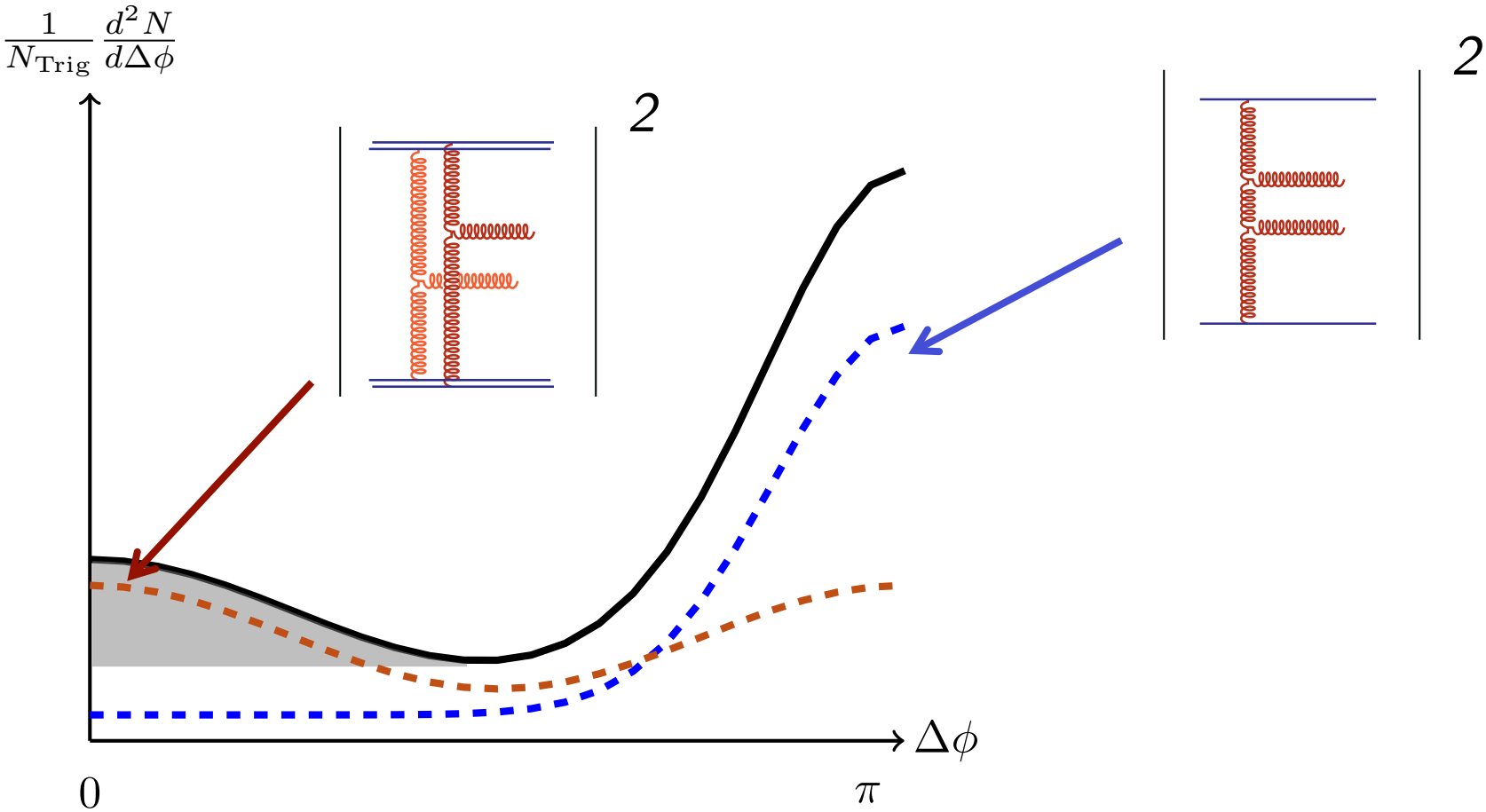
Dusling, Venugopalan, Phys.Rev. D87 (2013) 094034, arXiv:1302.7018

Part II: The v3 puzzle

$$\frac{1}{N_{\rm trig}}\frac{d^2N}{d\Delta\phi} = a_0 + 2\sum_{n=1} a_n \cos(n\Delta\phi)$$



Di-hadron production



What about:

$2\mathrm{Re}\left\{ \quad \times \quad {}^{*} \right\}$

Particle production and LSZ

The amplitude for single gluon production

$$\langle \mathbf{p}, \lambda, a | \Omega_{\rm in} \rangle = i\epsilon^{\lambda}_{\mu}(\mathbf{p}) \int d^4x \; e^{ip\cdot x} \Box_x \langle \Omega_{\rm out} | A^{\mu}_{a}(x) | \Omega_{\rm in} \rangle$$

and for two-gluon production

$$\langle \mathbf{p}, \lambda, a; \mathbf{q}, \sigma, b | \Omega_{\rm in} \rangle =$$

$$i\epsilon^{\lambda}_{\mu}(\mathbf{p}) \int d^4x \; e^{ip\cdot x} \Box_x \; i\epsilon^{(\sigma)}_{\nu}(\mathbf{q}) \int d^4y \; e^{iq\cdot y} \Box_y \langle \Omega_{\rm out} | \mathrm{T} \left[A^{\mu}_{a}(x) A^{\nu}_{b}(y) \right] | \Omega_{\rm in} \rangle$$

Particle production and LSZ

Squaring the matrix element

$$\frac{dN}{d^2\mathbf{p}_T dy_p} = \frac{1}{2(2\pi)^3}\left(p^2\right)^2 \left\langle \mathcal{A}_a^{i*}(p)\mathcal{A}_a^i(p)\right\rangle$$

$$\begin{aligned}\frac{d^2N}{d^2\mathbf{p}_T dy_p d^2\mathbf{q}_T dy_q} &= \frac{1}{4(2\pi)^6}\left(p^2\right)^2\left(q^2\right)^2\left\langle \mathcal{A}_a^{i*}(p)\mathcal{A}_a^i(p)\mathcal{A}_b^{j*}(q)\mathcal{A}_b^j(q)\right\rangle \\ &+ \frac{1}{4(2\pi)^6}\left(p^2\right)^2\left(q^2\right)^2\left\langle 2\mathrm{Re}\left[\mathcal{A}_{\mathrm{a}}^{\mathrm{i}*}(\mathrm{p})\mathcal{G}_{\mathrm{ab}}^{\mathrm{ij}}(\mathrm{p},\mathrm{q})\mathcal{A}_{\mathrm{b}}^{\mathrm{j}*}(\mathrm{q})\right]\right\rangle \\ &+ \frac{1}{4(2\pi)^6}\left(p^2\right)^2\left(q^2\right)^2\left\langle \mathcal{G}_{ba}^{\dagger\, ji}(q,p)\mathcal{G}_{ab}^{ij}(p,q)\right\rangle\end{aligned}$$

where we defined

$$\mathcal{A}_a^\mu(x) = \langle\Omega_{\mathrm{out}}|A_a^\mu(x)|\Omega_{\mathrm{in}}\rangle$$

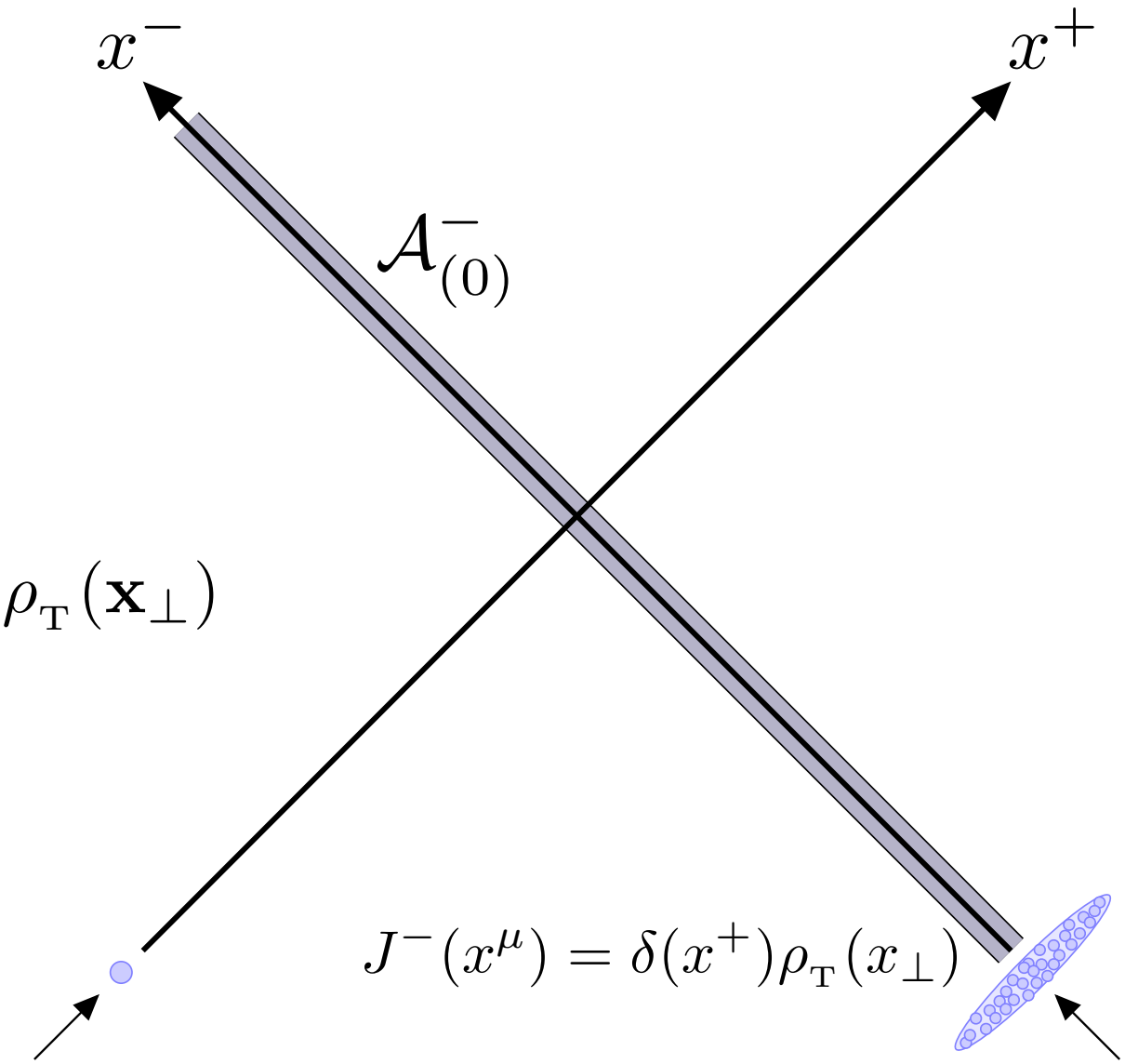
$$\mathcal{G}_{ab}^{\mu\nu}(x,y) \equiv \langle\Omega_{\mathrm{out}}|\mathrm{T}\left[A_a^\mu(x)A_b^\nu(y)\right]|\Omega_{\mathrm{in}}\rangle - \mathcal{A}_a^\mu(x)\mathcal{A}_b^\nu(y)$$

Background field of nucleus

$$\mathcal{A}^{+}_{(0)} = 0$$

$$\mathcal{A}^{i}_{(0)} = 0$$

$$\mathcal{A}^{-}_{(0)} = -\frac{1}{\partial_\perp^2}\delta(x^+)\rho_{\mathrm{T}}(\mathbf{x}_\perp)$$



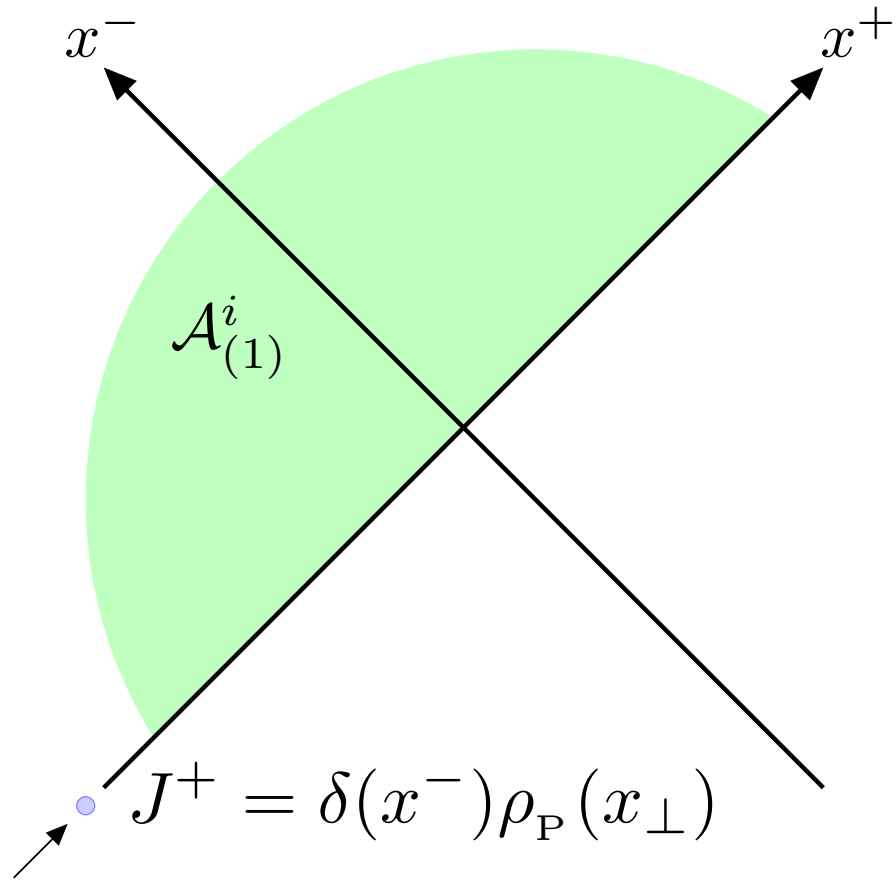
F. Gelis and Y. Mehtar-Tani, PRD 73, 034019 (2006).

Field of Proton (No Collision)

$$\mathcal{A}^{+}_{(1)} = 0$$

$$\mathcal{A}^{i}_{(1)} = \theta(x^{-})\frac{\partial^{i}}{\partial^{2}_{\perp}}\rho_{p}(\mathbf{x}_{\perp})$$

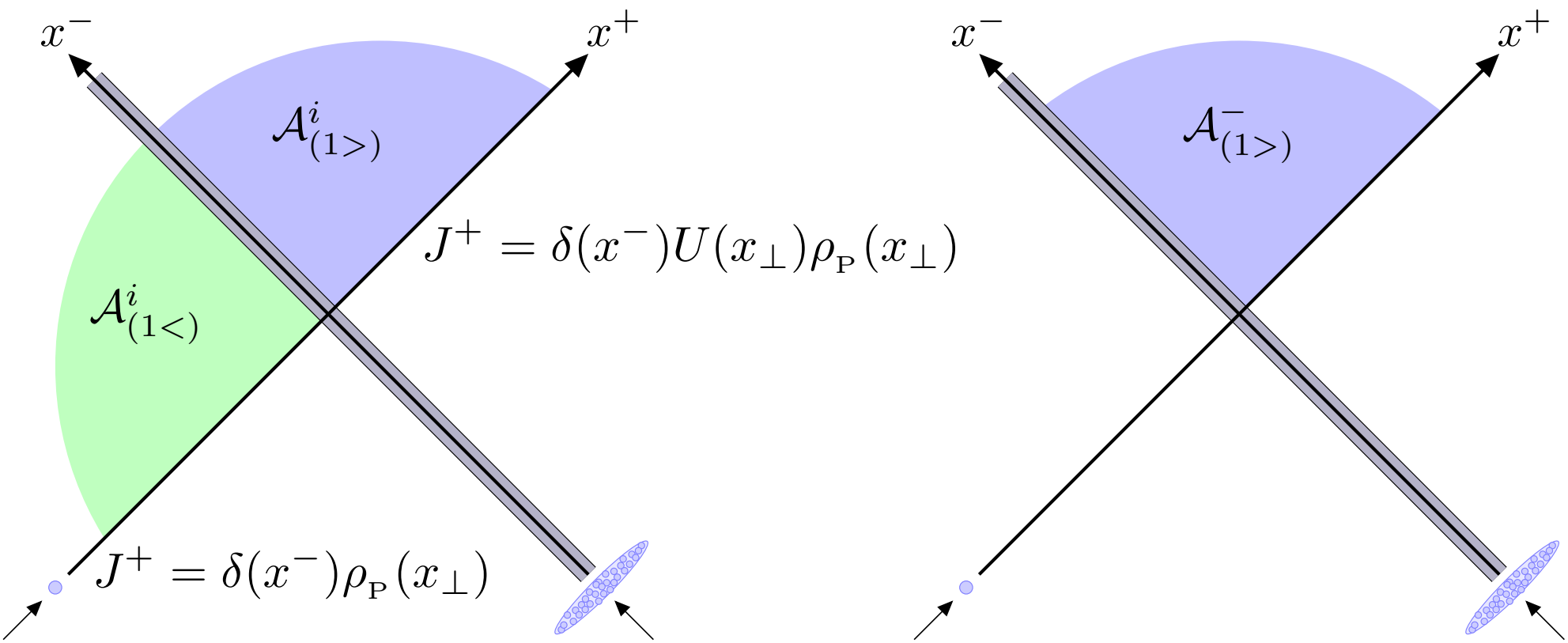
$$\mathcal{A}^{-}_{(1)} = 0$$



$$-p^{2}\mathcal{A}^{i}_{(1<)}(p) = ip^{i}\left[\frac{p^{2}}{(p^{+}+i\epsilon)(p^{-}-i\epsilon)}\right]\frac{\rho_{\mathrm{P}}(\mathbf{p}_{T})}{\mathbf{p}^{2}_{T}}$$

$$-p^{2}\mathcal{A}^{-}_{(1<)}(p) = 0$$

Fields after collision



$$-p^2\mathcal{A}^i_{(1>)}(p) = i\int_{\mathbf{k}_{1\perp}}\left[\frac{p^i}{(p^+ + i\epsilon)(p^- + i\epsilon)} - \frac{2\,k_1^i}{\mathbf{k}_{1\perp}^2}\right]U(\mathbf{p}_T - \mathbf{k}_{1\perp})\rho_{\rm P}(\mathbf{k}_{1\perp})$$

$$U(x_\perp) \equiv \mathcal{P}_{x^+}\exp\left[ig\int_{0^-}^{0^+} dx^+\mathcal{A}^-_{(0)}\right]$$

Background field propagator

The gluon propagator in a classical background field satisfies

$$-i\left[g^{\mu\nu}D^{ac}_{\rho}D^{\rho}_{cb} - D^{\mu}_{ac}D^{\nu}_{cb} + 2gf^{acb}F^{\mu\nu}_{c} - \frac{1}{\lambda}n^{\mu}n^{\nu}\delta^{ab}\right]_x \mathcal{G}^{bd}_{\nu\lambda}(x,y) = g^{\mu}_{\lambda}\delta^{ad}\delta^{(4)}(x-y)$$

This can be expanded to first order in the proton charge density (single scattering Born approx.)

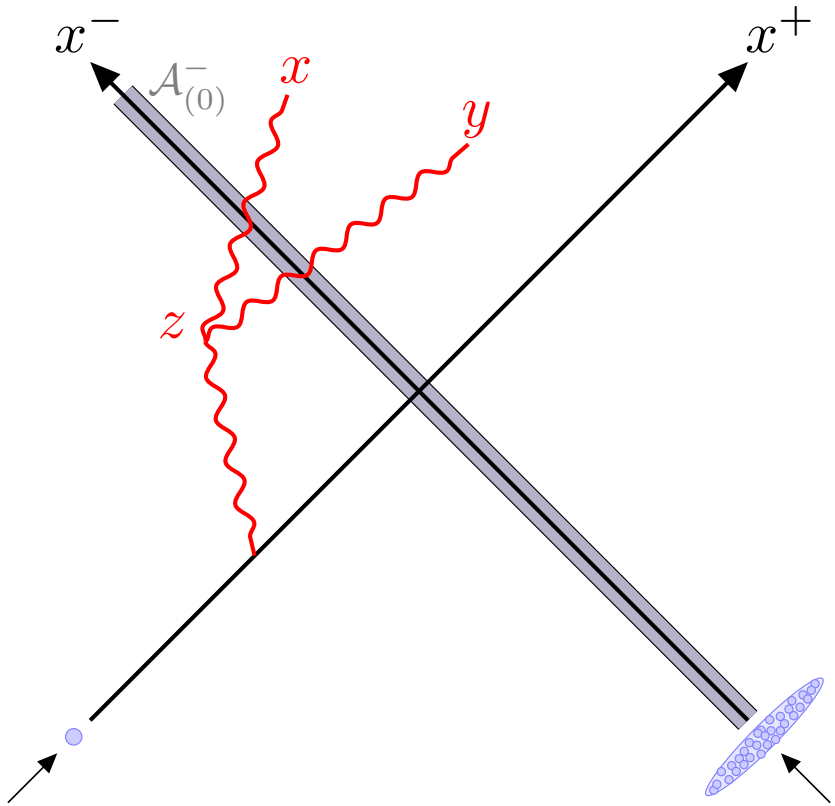
$$\delta\mathcal{G}^{\mu\nu}(x,y) \equiv \mathcal{G}^{\mu\nu}(x,y) - \mathcal{G}^{\mu\nu}_{(0)}(x,y) = -\int d^4z\, \mathcal{G}^{\mu\lambda}_{(0)}(x,z)\delta\Gamma^{(3)}_{\lambda\sigma}(z)\mathcal{G}^{\sigma\nu}_{(0)}(z,y)$$

where we defined the triple-gluon vertex

$$\delta\Gamma^{(3)}_{\mu\nu} = -ig\left\{\left[\partial_\lambda\mathcal{A}^{\lambda}_{(1)} + 2\mathcal{A}^{\lambda}_{(1)}\partial_\lambda\right]g_{\mu\nu} + \partial_\mu\mathcal{A}_{(1)\nu} - 2\partial_\nu\mathcal{A}_{(1)\mu} - \mathcal{A}_{(1)\mu}\partial_\nu - \mathcal{A}_{(1)\nu}\partial_\mu\right\}$$

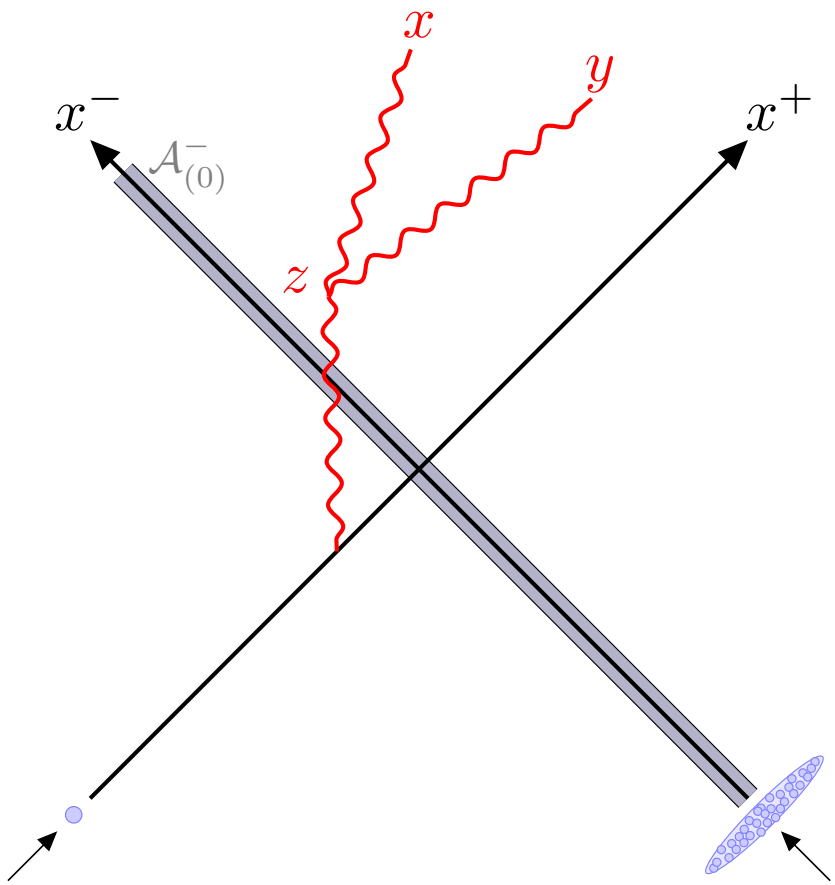
K. Fukushima, Y. Hidaka, Nucl. Phys. A813 (2008) 171-197

Di-jet Amplitude



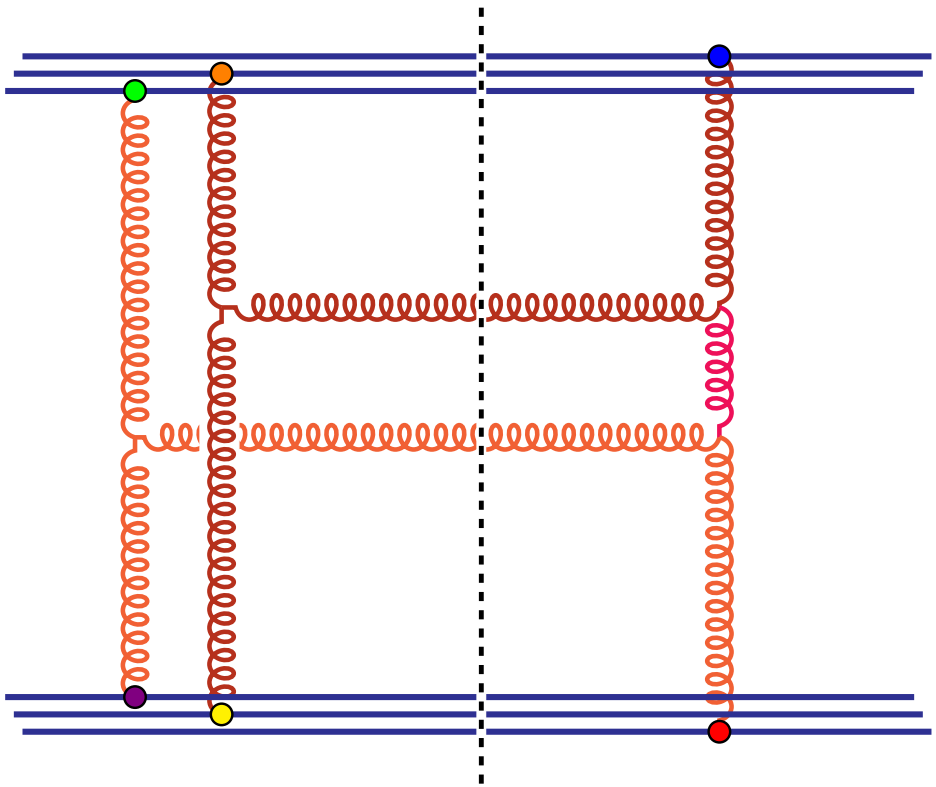
$$p^2 q^2 \delta \mathcal{G}^{ij,ab}_{(<)}(p,q) = 4ig \int_{\mathbf{k}_{1\perp},\mathbf{k}_{2\perp}} \frac{\rho^c_{\mathrm{P}}(\mathbf{k}_{2\perp})}{\mathbf{k}^2_{2\perp}} \frac{(\mathbf{p}_T - \mathbf{k}_{1\perp})^i \mathbf{k}^j_{2\perp}}{(\mathbf{p}_T - \mathbf{k}_{1\perp})^2} \left[U(\mathbf{k}_{1\perp}) T^c U^\dagger(\mathbf{k}_{1\perp} + \mathbf{k}_{2\perp} - \mathbf{p}_T - \mathbf{q}_T)\right]^{ab}$$

Di-jet Amplitude



$$p^2 q^2 \delta \mathcal{G}^{ij,ab}_{(>)}(p,q) = -4ig \int_{\mathbf{k}_{1\perp},\mathbf{k}_{2\perp}} \frac{\rho^c_{\mathrm{P}}(\mathbf{k}_{2\perp})}{\mathbf{k}^2_{2\perp}} \frac{\mathbf{p}^i_T \mathbf{k}^j_{2\perp}}{\mathbf{p}^2_T} \left[U(\mathbf{k}_{1\perp}) T^c U^\dagger(\mathbf{k}_{1\perp} + \mathbf{k}_{2\perp} - \mathbf{p}_T - \mathbf{q}_T)\right]^{ab}$$

Putting it all together ...



$$\frac{d^2N}{d^2\mathbf{p}_T dy_p d^2\mathbf{q}_T dy_q} =$$

$$\frac{32\alpha_s^2}{(2\pi)^4} f^{ade} f^{cdb} f^{agh} f^{bkl} \int_{\mathbf{k}_{1\perp},\mathbf{k}_{2\perp},\mathbf{k}_{3\perp}} \left[\frac{\mathbf{p}_T^i}{\mathbf{p}_T^2} - \frac{\mathbf{k}_{1\perp}^i}{\mathbf{k}_{1\perp}^2}\right] \cdot \left[\frac{\mathbf{p}_T^i}{\mathbf{p}_T^2} - \frac{\mathbf{k}_{2\perp}^i}{\mathbf{k}_{2\perp}^2}\right] \left[\frac{\mathbf{q}_T^j}{\mathbf{q}_T^2} - \frac{\mathbf{k}_{3\perp}^j}{\mathbf{k}_{3\perp}^2}\right] \cdot \frac{(\mathbf{q}_T + \mathbf{k}_{1\perp})^j}{(\mathbf{q}_T + \mathbf{k}_{1\perp})^2}$$

$$\times \langle \rho_{\mathrm{P}}^{g*}(\mathbf{k}_{2\perp}) \rho_{\mathrm{P}}^{k*}(\mathbf{k}_{3\perp}) \rho_{\mathrm{P}}^{c}(\mathbf{q}_T + \mathbf{k}_{1\perp}) \rangle_{\mathrm{P}} \frac{\langle \rho_{\mathrm{T}}^{h*}(\mathbf{p}_T - \mathbf{k}_{2\perp}) \rho_{\mathrm{T}}^{l*}(\mathbf{q}_T - \mathbf{k}_{3\perp}) \rho_{\mathrm{T}}^{e}(\mathbf{p}_T - \mathbf{k}_{1\perp}) \rangle_{\mathrm{T}}}{(\mathbf{p}_T - \mathbf{k}_{2\perp})^2 (\mathbf{q}_T - \mathbf{k}_{3\perp})^2 (\mathbf{p}_T - \mathbf{k}_{1\perp})^2}$$

Three point function

For a large nucleus (MV model) there is a contribution proportion to the cubic Casimir operator (albeit suppressed):

$$\begin{aligned}\langle \mathcal{O}[\rho]\rangle = & \int [d\rho]\, \mathcal{O}[\rho] \exp\left(-\frac{1}{2}\int_{\mathbf{x}_T,\mathbf{y}_T} \frac{\rho^a(\mathbf{x}_T)\rho^b(\mathbf{y}_T)}{\mu^2(y,\mathbf{x}_T-\mathbf{y}_T)} + \frac{d^{efg}}{\kappa_A}\int_{\mathbf{z}_T} \rho^e(\mathbf{z}_T)\rho^f(\mathbf{z}_T)\rho^g(\mathbf{z}_T)\right) \\ \simeq & \frac{d^{efg}}{\kappa_A}\int [d\rho] \int_{\mathbf{z}_T} \rho^e(\mathbf{z}_T)\rho^f(\mathbf{z}_T)\rho^g(\mathbf{z}_T)\mathcal{O}[\rho] \exp\left(-\frac{1}{2}\int_{\mathbf{x}_T,\mathbf{y}_T} \frac{\rho^a(\mathbf{x}_T)\rho^b(\mathbf{y}_T)}{\mu^2(y,\mathbf{x}_T-\mathbf{y}_T)}\right)\end{aligned}$$

In this framework the three-point function is

$$\left\langle \rho^a(\mathbf{u}_T)\rho^b(\mathbf{v}_T)\rho^c(\mathbf{w}_T)\right\rangle = 6\frac{d^{abc}}{\kappa_A}\int_{\mathbf{z}_T} \mu^2(\mathbf{u}_T-\mathbf{z}_T)\mu^2(\mathbf{v}_T-\mathbf{z}_T)\mu^2(\mathbf{w}_T-\mathbf{z}_T)$$

The color charge density is related to the UGD:

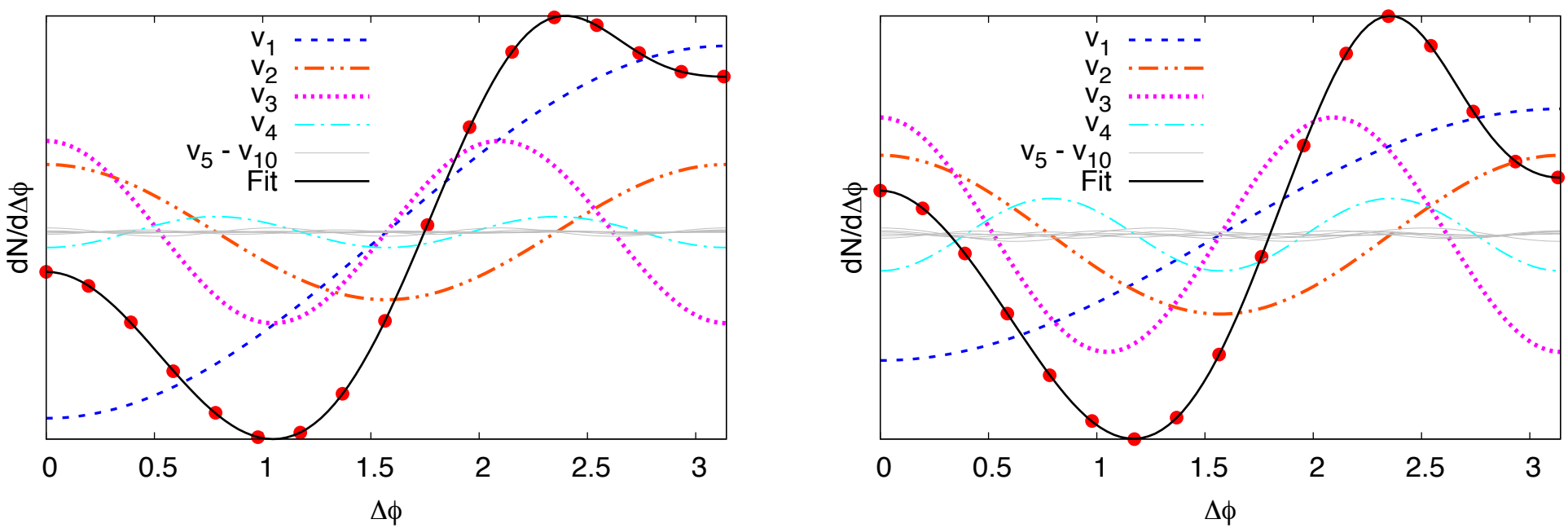
$$\begin{aligned}&\langle \rho^a(\mathbf{k}_{1\perp})\rho^b(\mathbf{k}_{2\perp})\rho^{*c}(\mathbf{k}_{3\perp})\rangle \sim \\ &\frac{d_{abc}}{\kappa_A}\delta^2(\mathbf{k}_{1\perp}+\mathbf{k}_{2\perp}-\mathbf{k}_{3\perp})\mathbf{k}_{1\perp}^2\Phi_{\mathrm{P}}(\mathbf{k}_{1\perp})\mathbf{k}_{2\perp}^2\Phi_{\mathrm{P}}(\mathbf{k}_{2\perp})\mathbf{k}_{3\perp}^2\Phi_{\mathrm{P}}(\mathbf{k}_{3\perp})\end{aligned}$$

Result for interference diagram

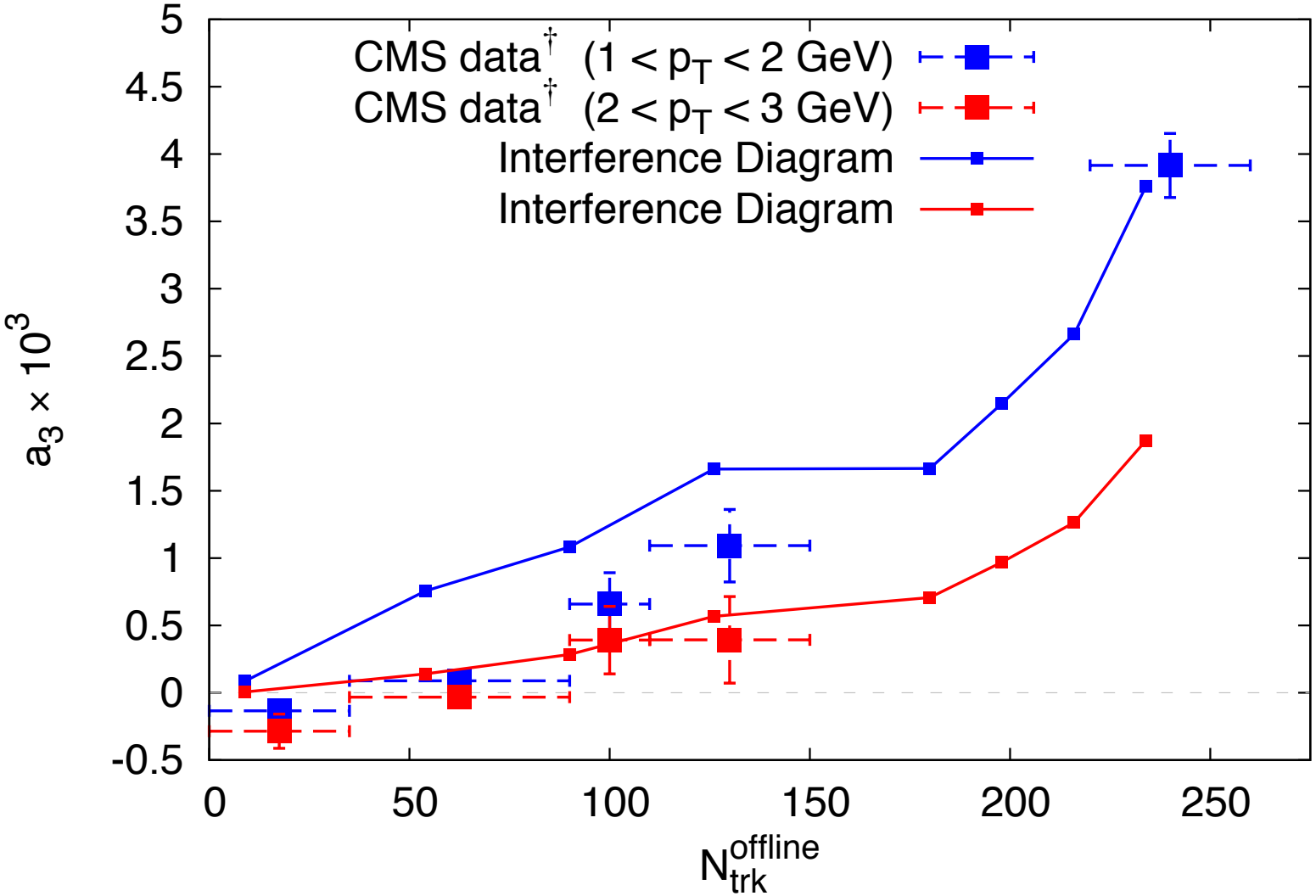
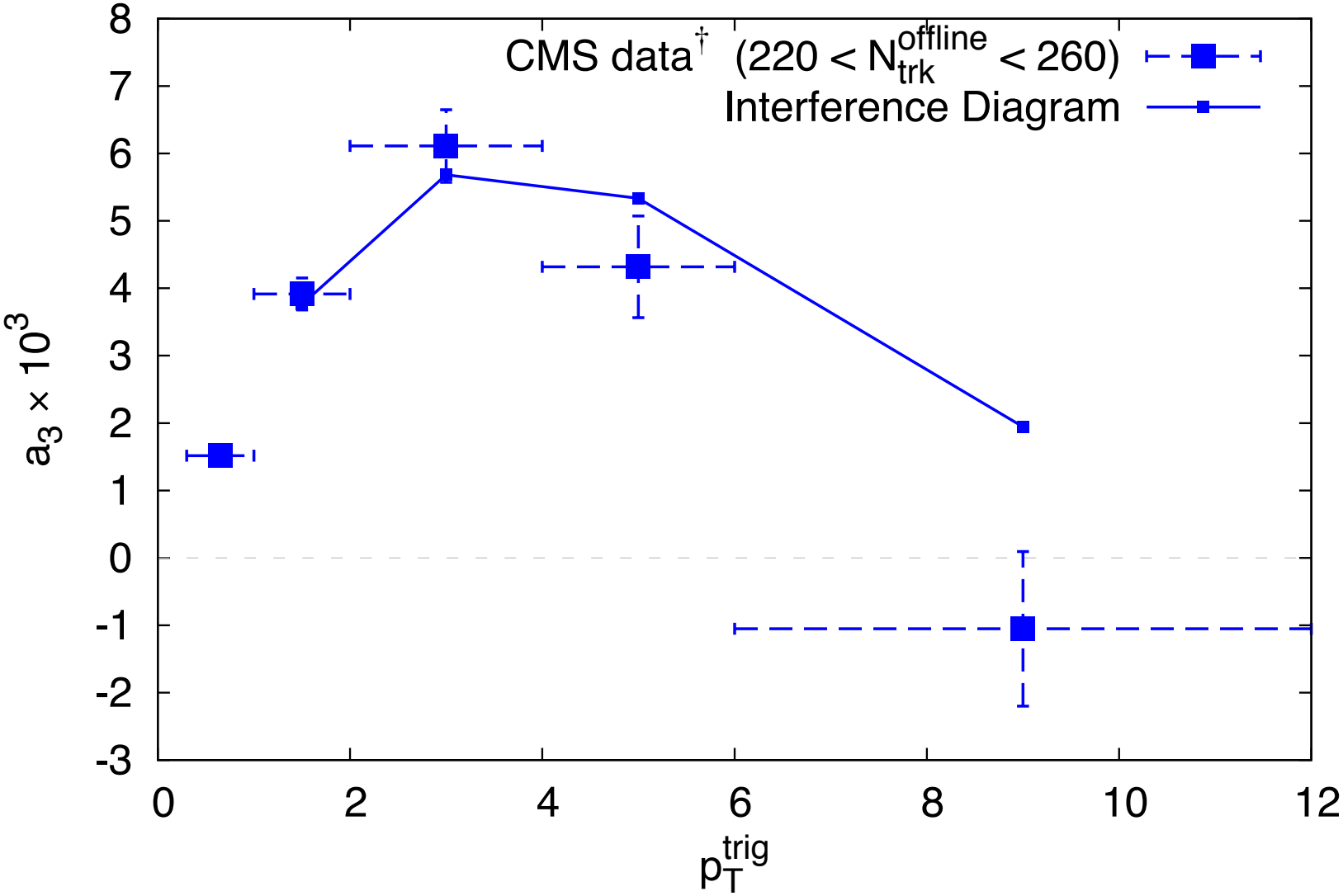
$$\frac{d^2N}{d^2\mathbf{p}_T dy_p d^2\mathbf{q}_T dy_q} = \frac{32\times 30\alpha_s^2\kappa_3^2 S_\perp}{(2\pi)^4}\int_{\mathbf{k}_{3\perp},\mathbf{k}_{4\perp}} \Phi_{\rm P}(\mathbf{k}_{3\perp})\Phi_{\rm P}(\mathbf{k}_{4\perp})\Phi_{\rm P}(\mathbf{k}_{3\perp}-\mathbf{k}_{4\perp})$$

$$\times\Phi_{\rm T}(\mathbf{p}_T+\mathbf{q}_T-\mathbf{k}_{4\perp})\Phi_{\rm T}(\mathbf{p}_T-\mathbf{k}_{3\perp})\Phi_{\rm T}(\mathbf{k}_{3\perp}-\mathbf{k}_{4\perp}+\mathbf{q}_T)$$

$$\times\mathbf{k}_{3\perp}^2\left[\frac{\mathbf{p}_T^i}{\mathbf{p}_T^2}-\frac{\mathbf{k}_{3\perp}^i}{\mathbf{k}_{3\perp}^2}\right]\cdot\left[\frac{\mathbf{k}_{4\perp}^i-\mathbf{q}_T^i}{(\mathbf{k}_{4\perp}-\mathbf{q}_T)^2}-\frac{p^i}{\mathbf{p}_T^2}\right]\left((\mathbf{k}_{4\perp}-\mathbf{k}_{3\perp})^2\frac{\mathbf{k}_{4\perp}\cdot\mathbf{q}_T}{\mathbf{q}_T^2}-\mathbf{k}_{4\perp}\cdot(\mathbf{k}_{4\perp}-\mathbf{k}_{3\perp})\right)$$



V_3 systematics



V_3 systematics