Why does black hole describe deconfinement phase?

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M.H.-Hyakutake-Nishimura-Takeuchi, PRL (2009)
M.H.-Hyakutake-Ishiki-Nishimura, Science (2014)
M.H.-Maltz-Susskind, hep-th (2014)

June 13, 2014 @ Brookhaven

Friday 13 June

Change to local time

13 JUN 2014 - 13:00 Local time **GROUP A** Estadio das Dunas Natal	MEXICO	12:00	CAMEROON
13 JUN 2014 - 16:00 Local time **GROUP B** Arena Fonte Nova Salvador	SPAIN	15:00	NETHERLANDS
13 JUN 2014 - 18:00 Local time **GROUP B** Arena Pantanal Cuiaba	CHILE	18:00	AUSTRALIA

I will try my best to finish this seminar in an hour.

Maldacena's conjecture: deconfining phase = black hole

SYM　　　　STRING

$1/\lambda$　　α'/R_{BH}^2

$g_{YM}^2 \sim 1/N$　　g_s

$\lambda=\infty, N=\infty$ corresponds to supergravity.

assumed to be correct without proof,
and applied to QGP

Is it correct only at large-N, strong coupling?

(supergravity, or Einstein gravity)

Or correct including $1/\lambda$ and $1/N$ corrections?

(superstring theory)

If correct, why? Can we understand it intuitively?

I want to answer to these questions, because

(1) I want to understand quantum gravity.

(2) I want to understand thermalization of QGP.

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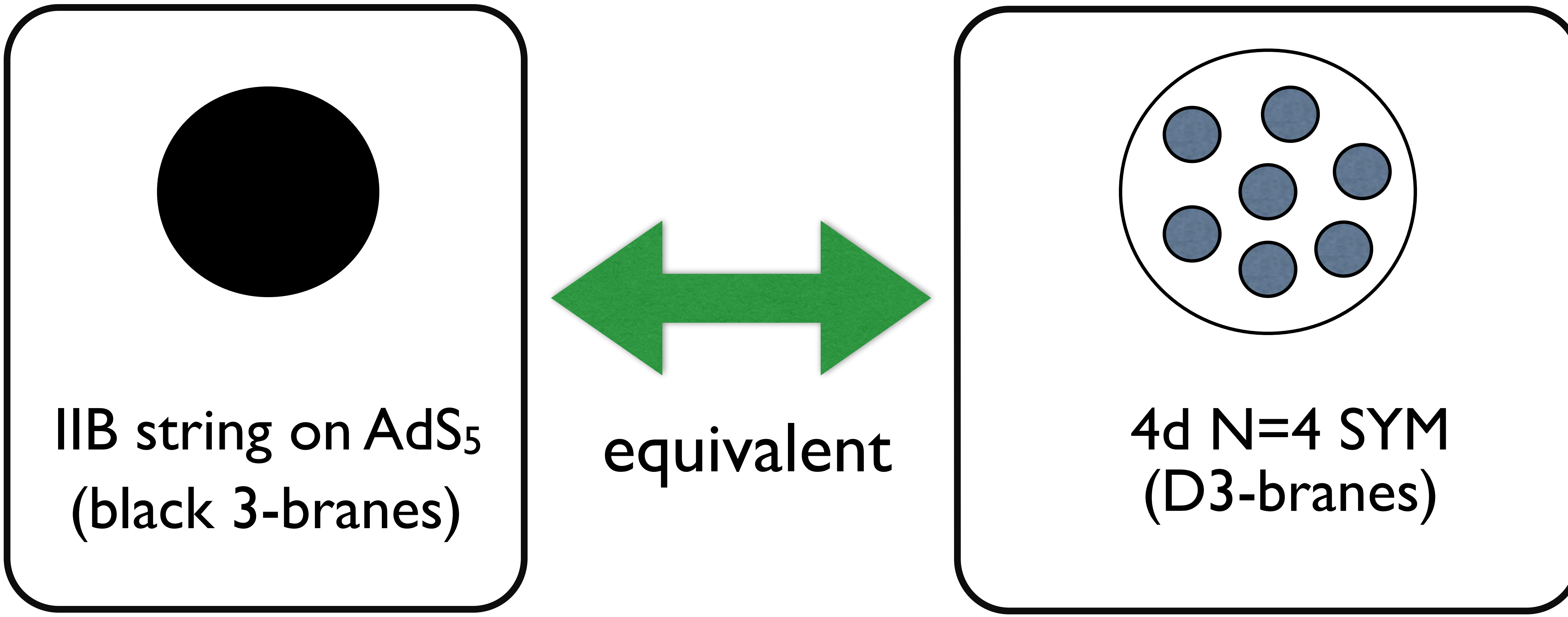
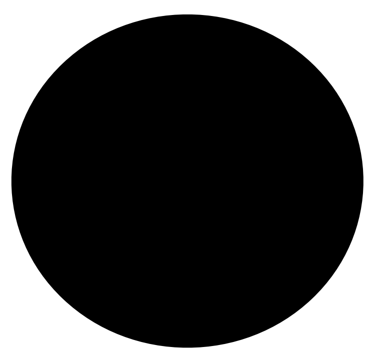


(Maldacena1997)

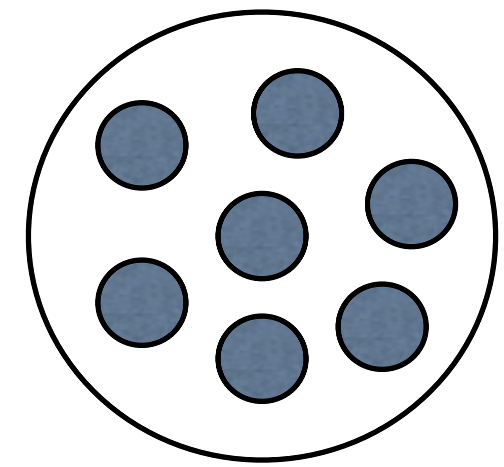
Black p-brane = bunch of Dp-branes

(+ strings between them)



IIA/IIB string around black p-brane (near horizon)

equivalent



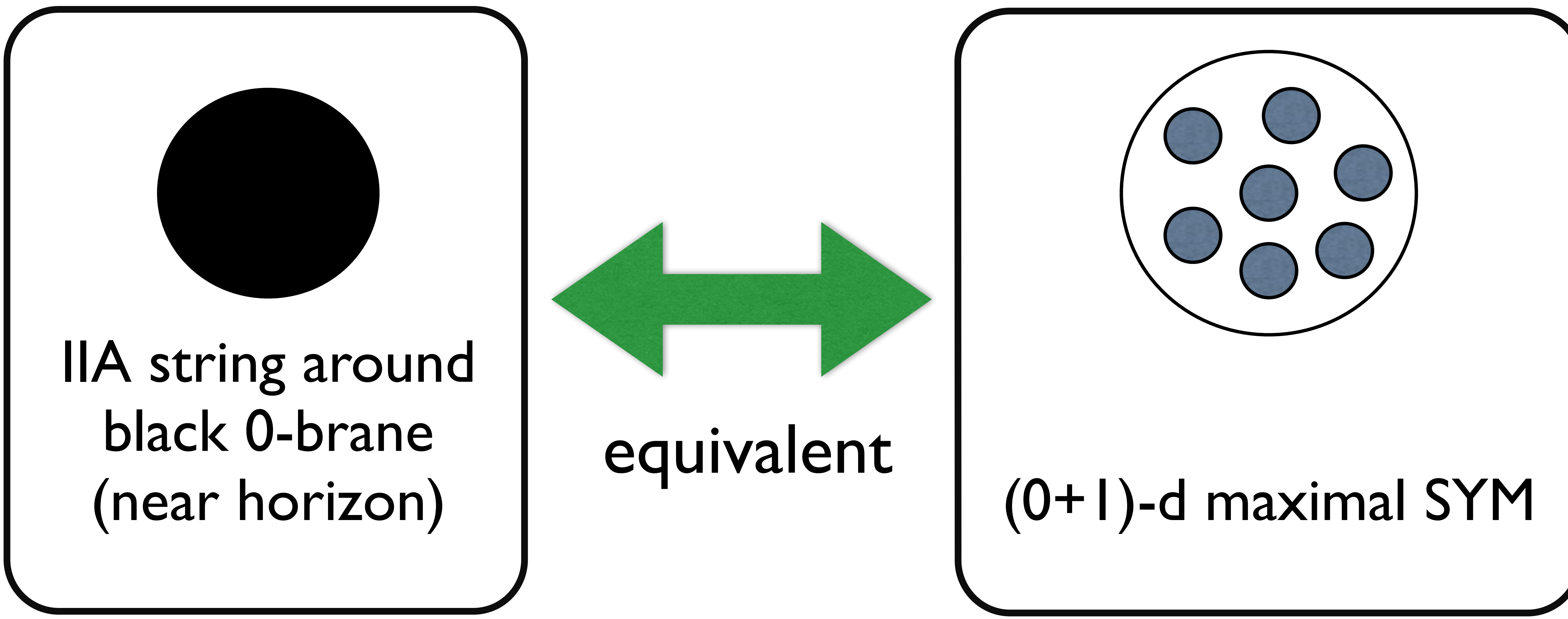
(p+1)-d maximal SYM

N Dp-branes → U(N) SYM

(Itzhaki-Sonnenschein-Maldacena-Yankielowicz 1998)

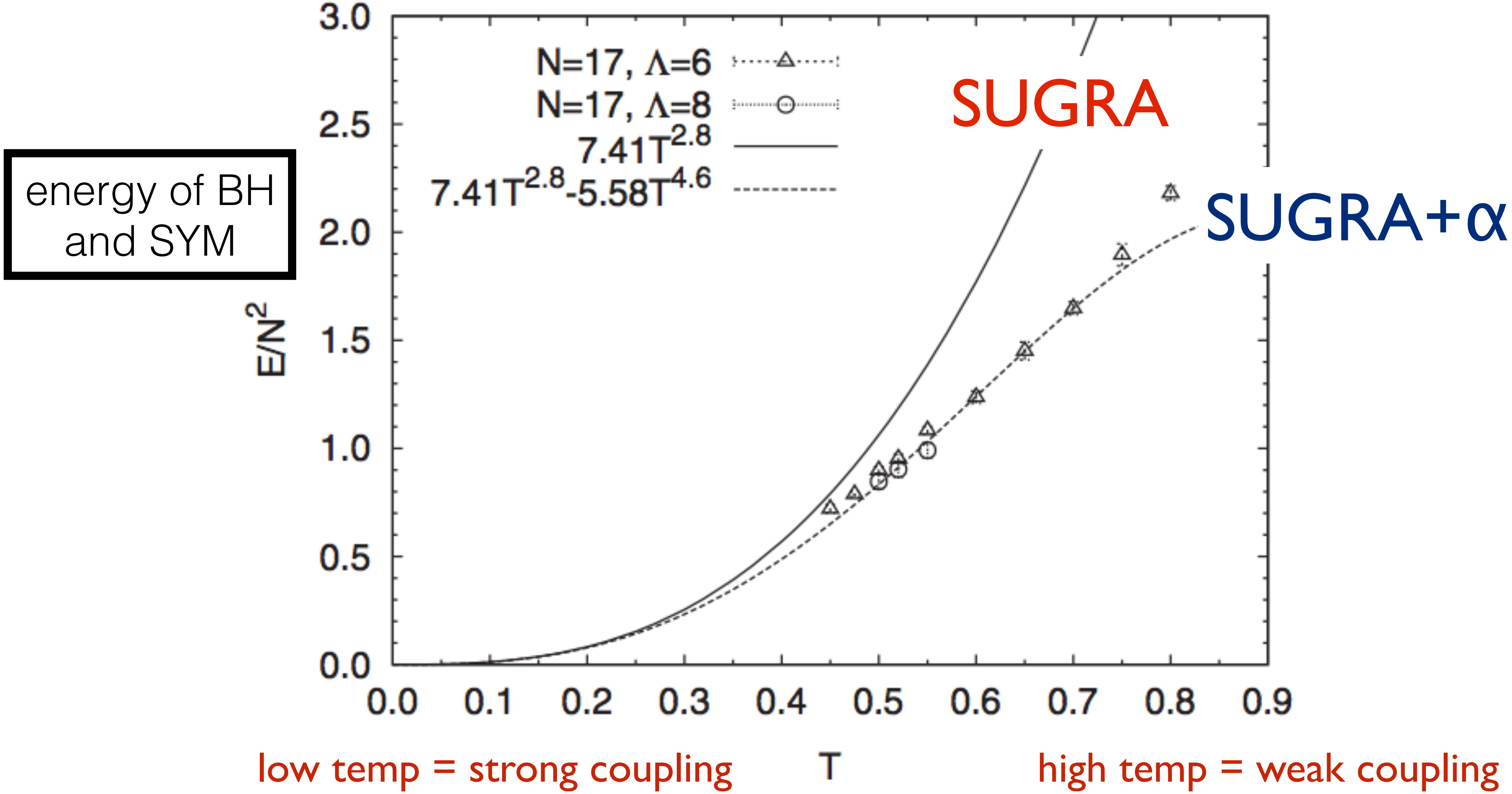
Black hole = bunch of D0-branes

(+ strings between them)



(Itzhaki-Sonnenschein-Maldacena-Yankielowicz 1998)

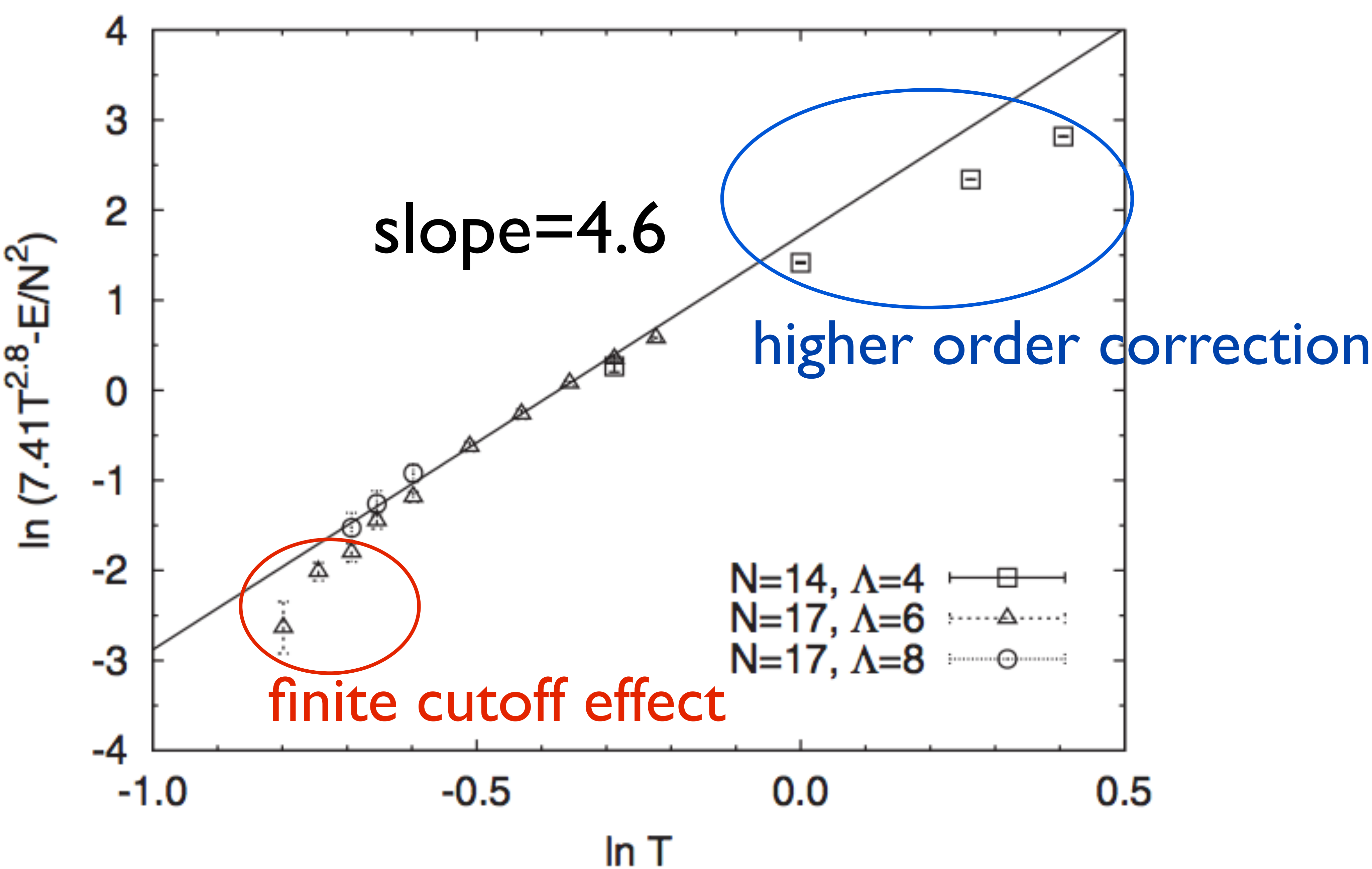
Quantitative test is possible by studying SYM numerically.



M.H.-Hyakutake-Nishimura-Takeuchi, PRL 2009

($\lambda^{-1/3}T$: dimensionless effective temperature)

Maldacena conjecture is correct at finite coupling & temperature!



M.H.-Hyakutake-Nishimura-Takeuchi, PRL 2009

1/N correction

Dual gravity prediction (Y. Hyakutake 2013)

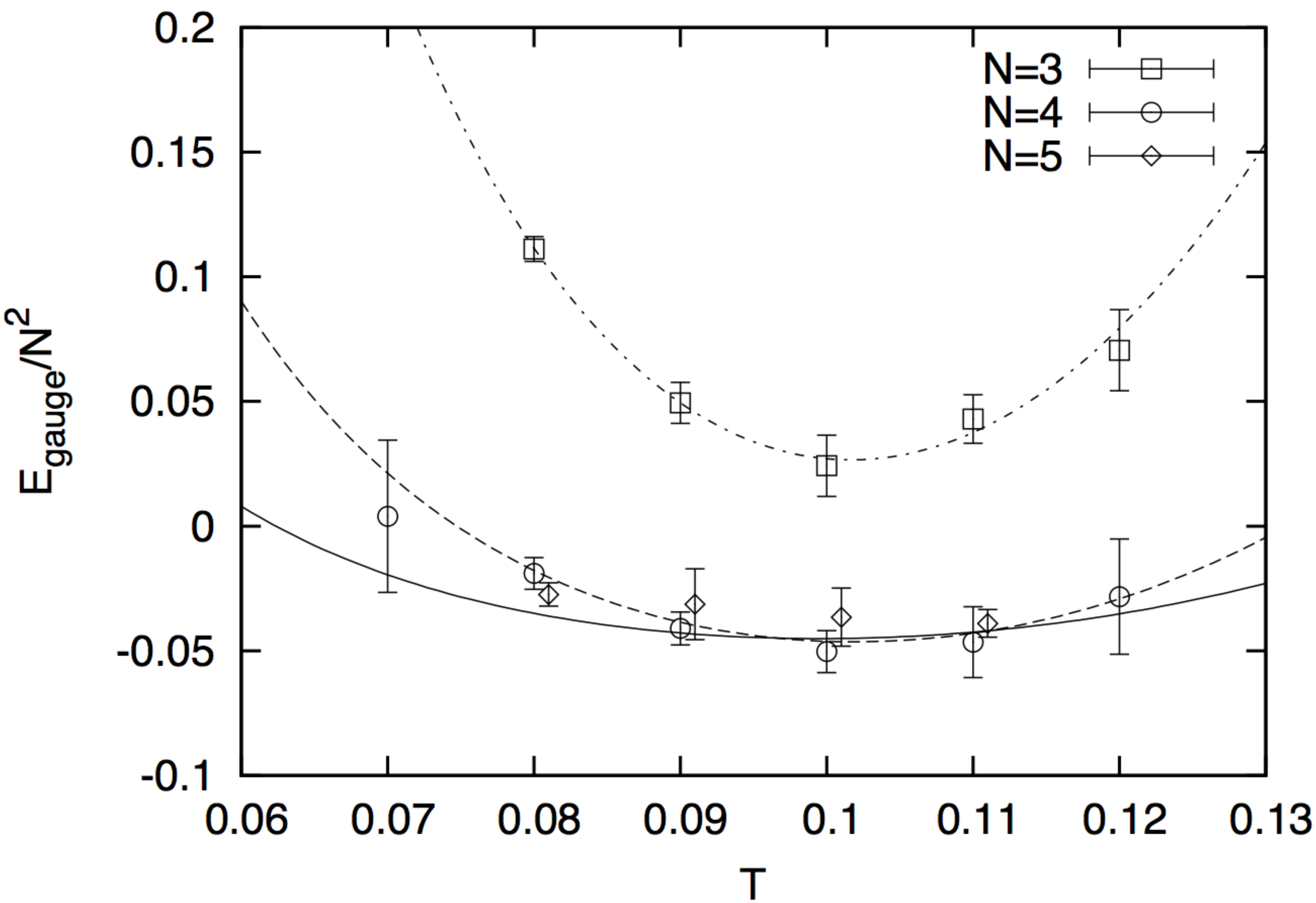
$$E/N^2 = 7.41T^{2.8} - 5.58T^{4.6} + \ldots.$$

$$+(1/N^2)(-5.77T^{0.4} + aT^{2.2} + \ldots)$$

$$+(1/N^4)(bT^{-2.6} + cT^{-2.0} + \ldots)$$

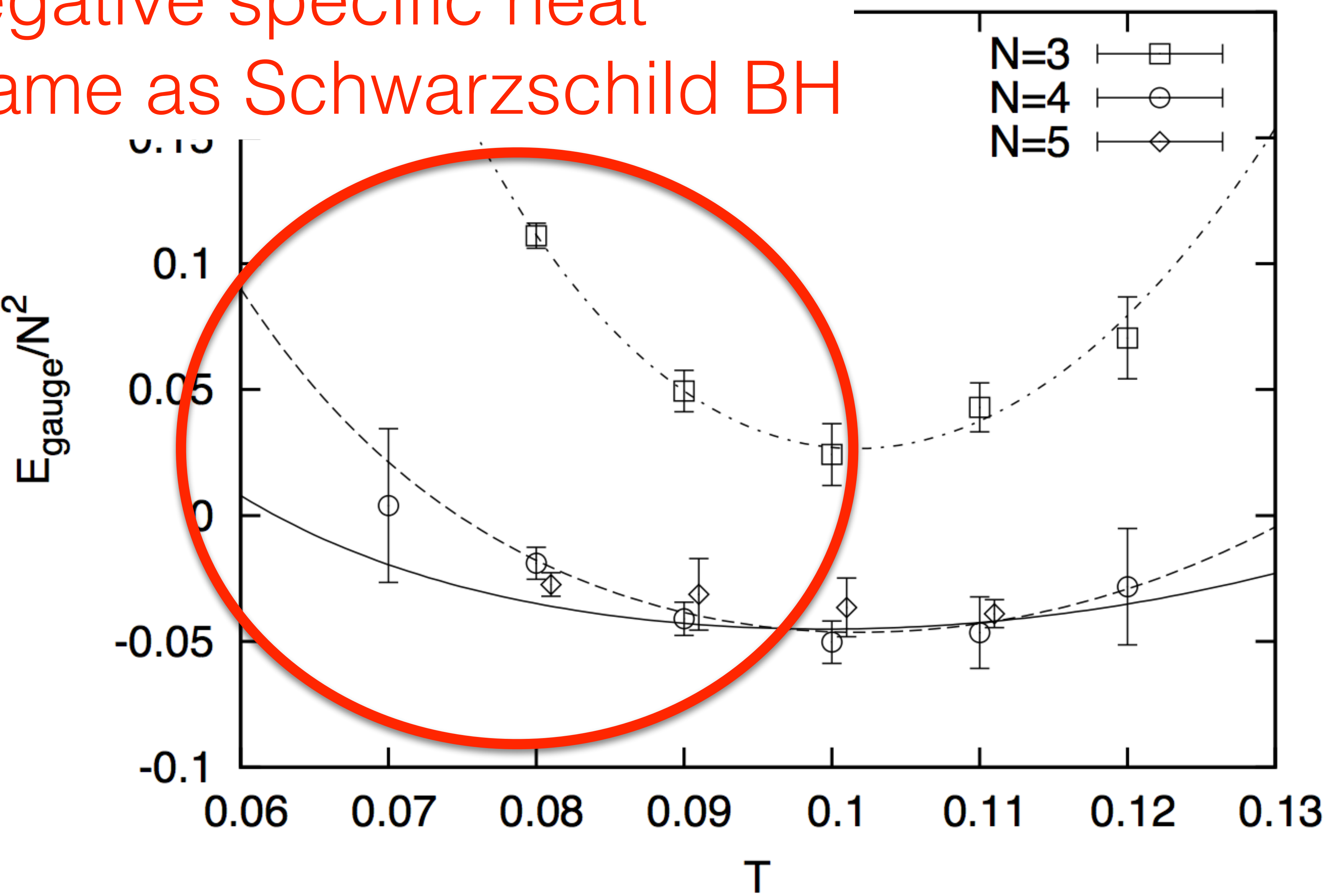
$$+\ldots..$$

Can it be reproduced from YM?



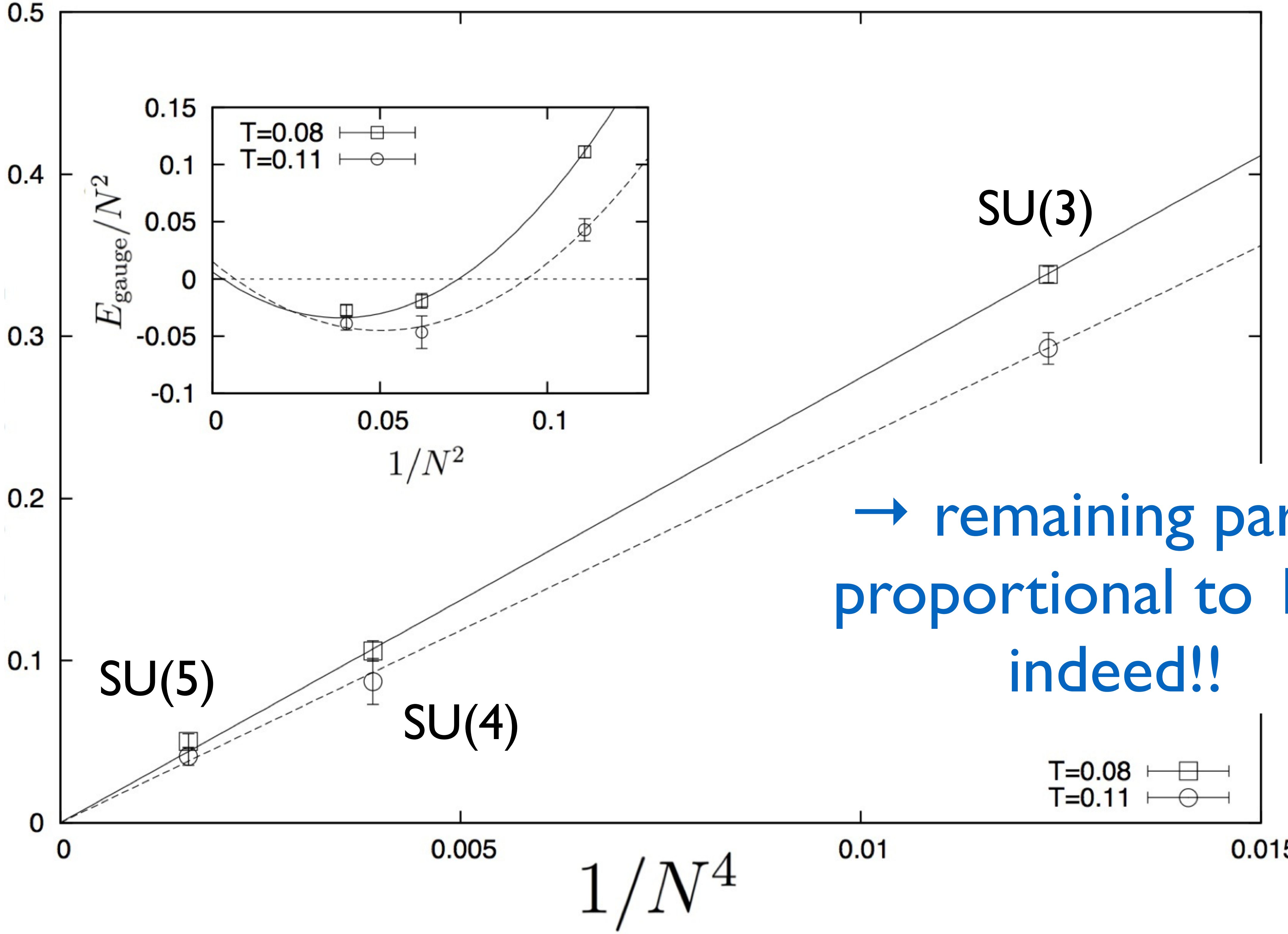
M.H.-Hyakutake-Ishiki-Nishimura, Science 2014

negative specific heat
→ the same as Schwarzschild BH



M.H.-Hyakutake-Ishiki-Nishimura, Science 2014

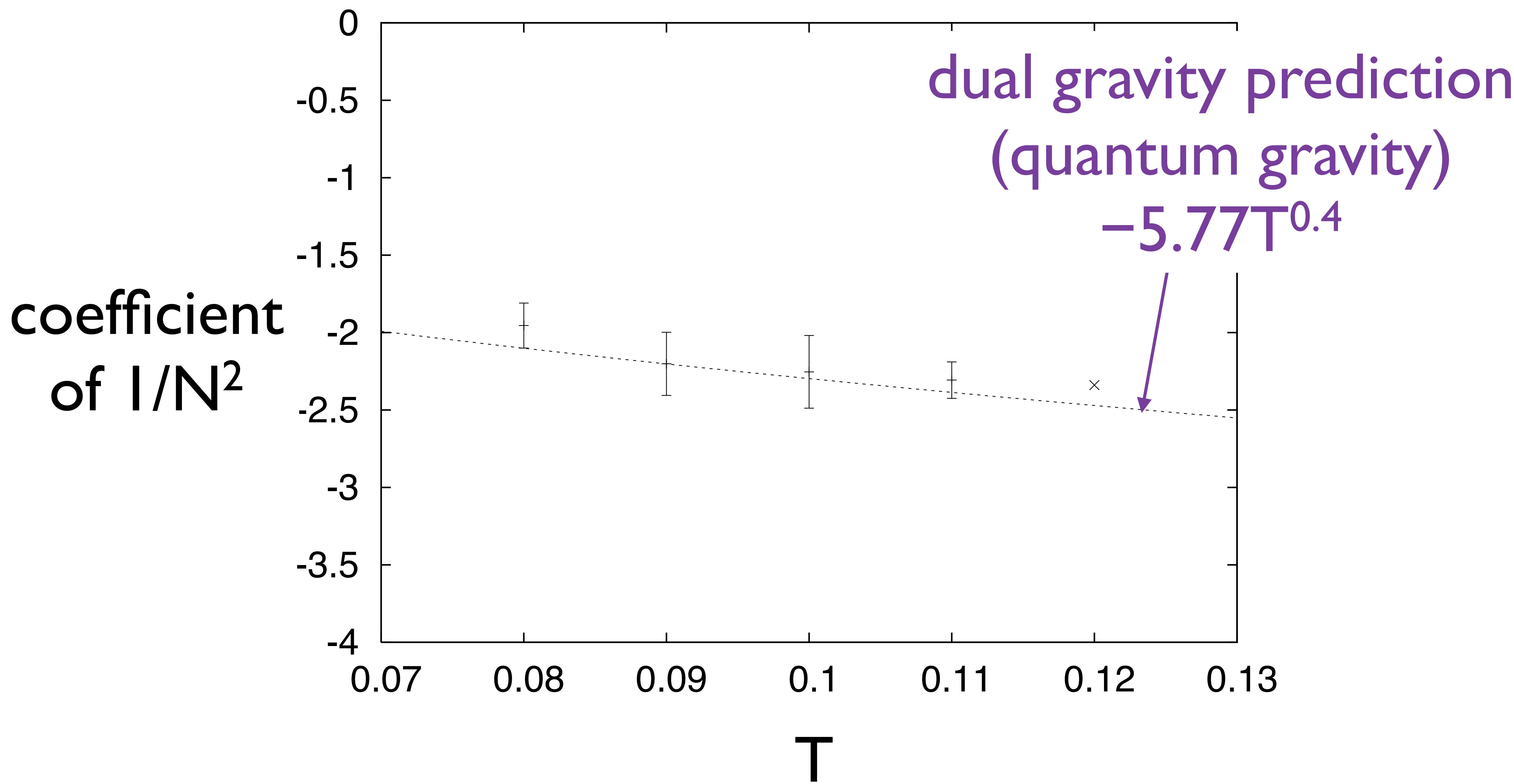
E/N^2 - $(7.41T^{2.8}$-$5.77T^{0.4}/N^2)$ vs. $1/N^4$



→ remaining part is proportional to $1/N^4$ indeed!!

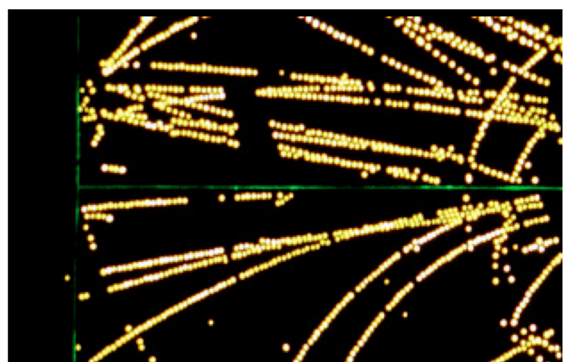
M.H.-Hyakutake-Ishiki-Nishimura, Science 2014

Maldacena conjecture is correct
at finite-N !



M.H.-Hyakutake-Ishiki-Nishimura, Science 2014

Not Even Wrong



Home   Frequently Asked Questions

This Week's Hype

Posted on May 25, 2014 by woit

Peter Woit's "This week's Hype"
on May 25, 2014

Maldacena's conjecture is correct
at finite temperature,
including $1/\lambda$ and $1/N$ corrections,
at least to the next-leading order.

So you can use it for learning about QGP at finite-N!
&
You can apply your knowledge about QGP to quantum gravity!

But why does it hold? We want to understand it intuitively,
so that we can understand physics behind it.

It should give us new perspective for both QGP and BH.

microscopic descriptions of the black hole (black brane)

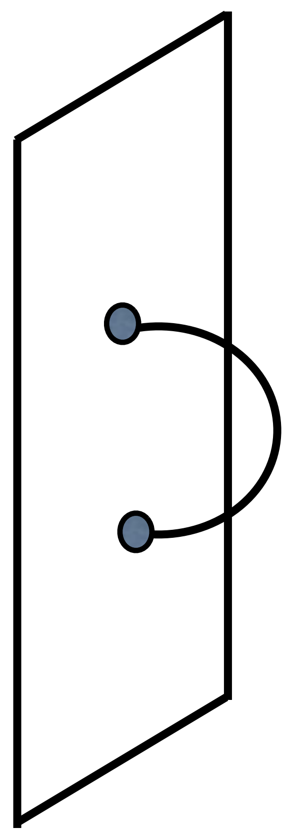
(1) D-branes + open strings

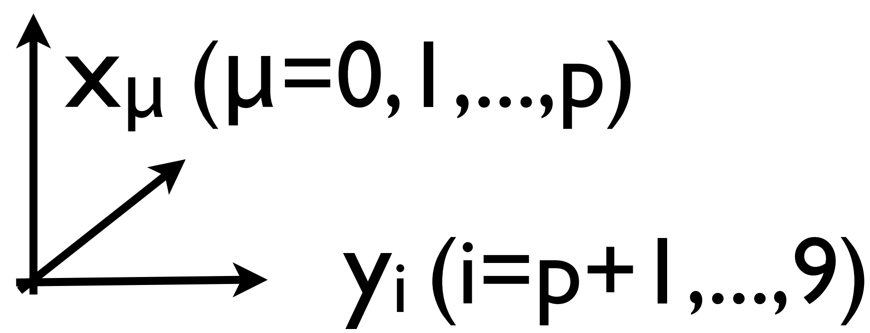
Polchinski, …

(2) condensation of closed strings

Susskind, Horowitz-Polchinski, …

Yang-Mills from D-brane





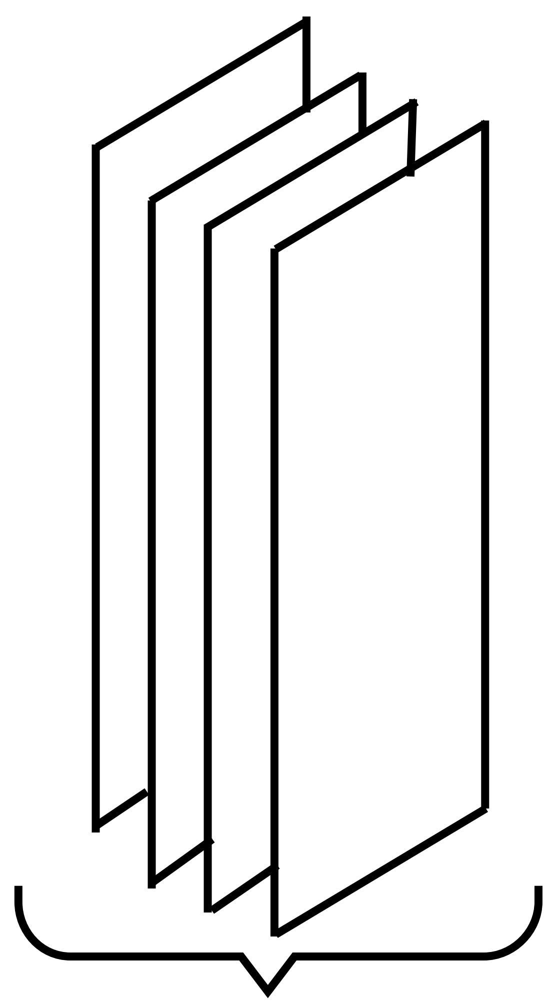
Massless d.o.f.

$A_\mu(x_\mu)$: Gauge field

$\Phi(x_\mu)$: Adjoint scalar field
(coordinate of the brane)

(and adjoint fermions)

Yang-Mills from D-brane



N Dp-branes

A_μ and Φ become
N×N matricex

(i,j)-component
= string connecting
i-th and j-th D-branes

➔ U(N) Super Yang-Mills

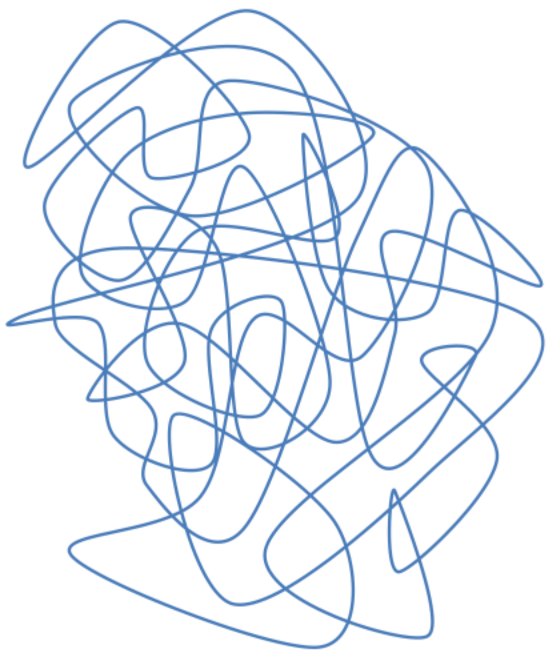
(more generally, the Dirac-Born-Infeld action)

bunch of many D-branes (N>>1)
= black brane

(large-N → heavy and big → classical gravity)

Black hole from closed string

(e.g. Susskind 1993)

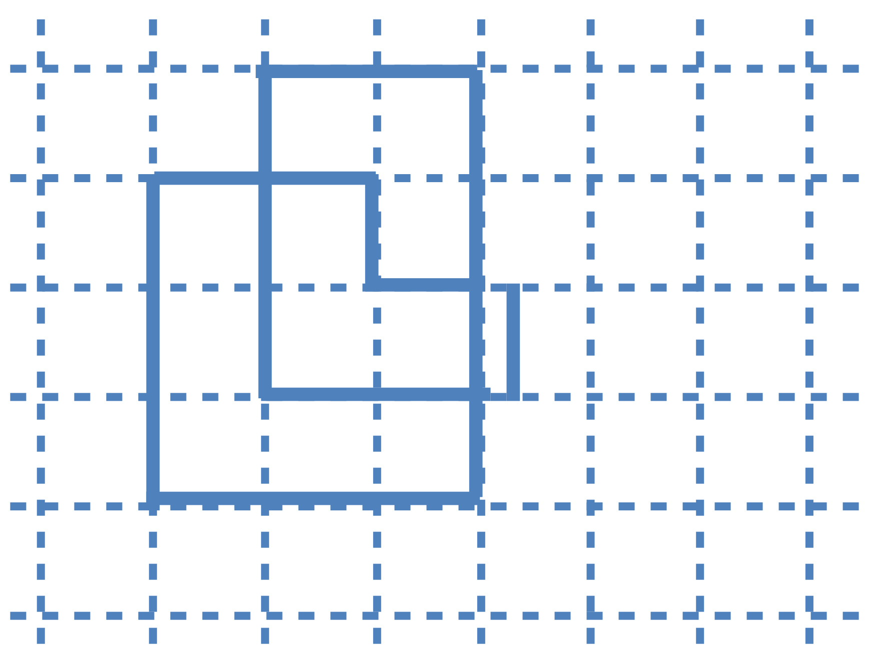


Consider a long, winding string with length L.

energy = tension $\times$ L

entropy $\sim$ L

when L >> 1, huge energy and entropy are packed in a small region → <u>*black hole*</u>



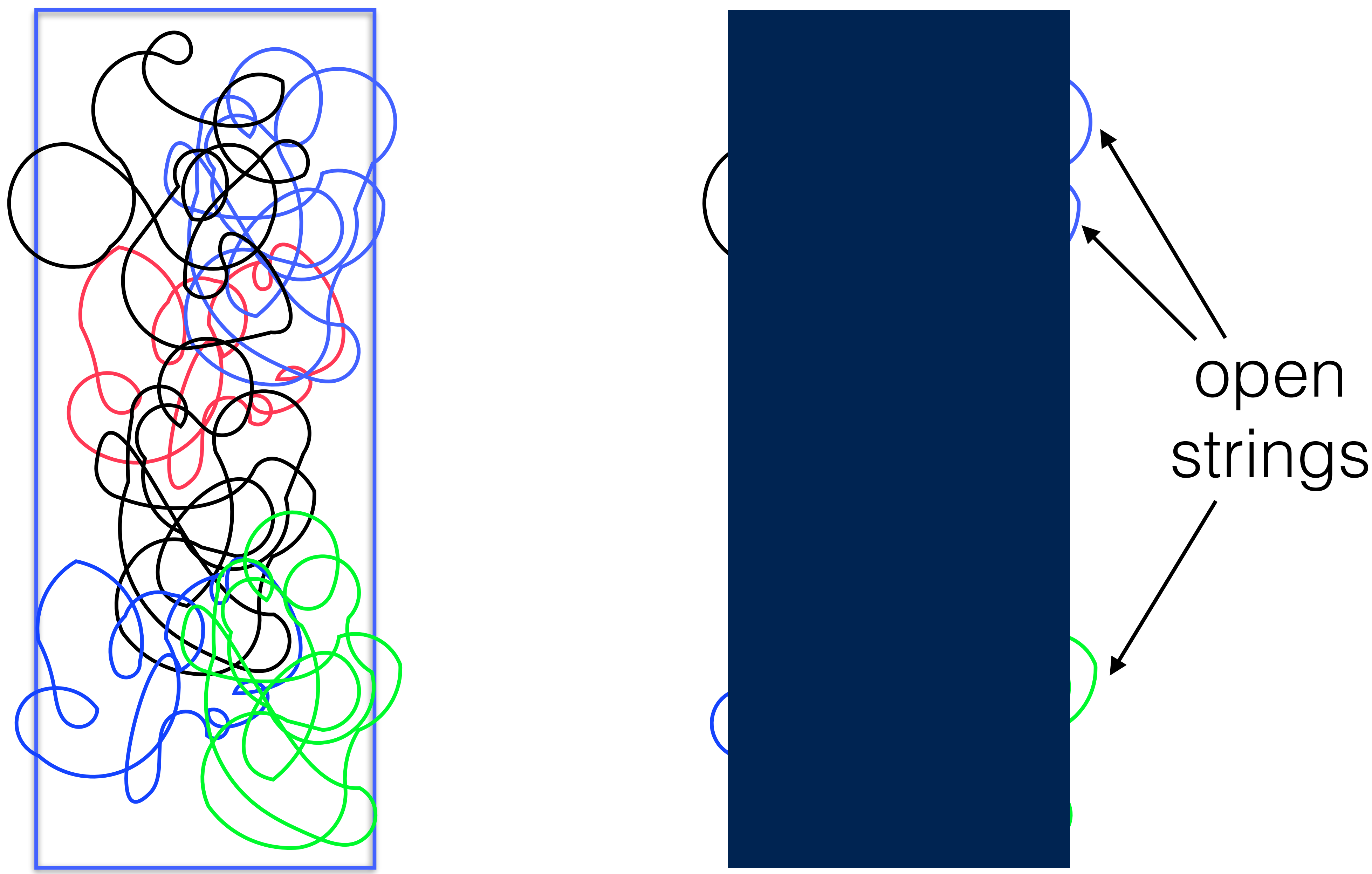
On D-dim square lattice,

# of possible shapes $\sim (2D\text{-}1)^L$

entropy $\sim L \times \log(2D\text{-}1)$

How are they related?

long, winding strings = black brane + open strings



The meaning of N (# of D-branes) becomes clear later.

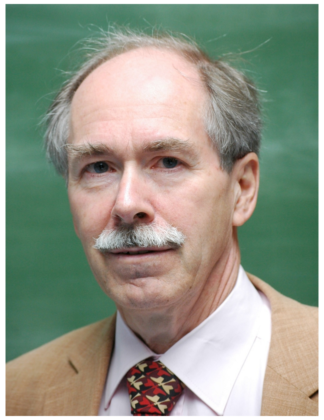
Gauge theory description

confining phase: ’t Hooft, 1974

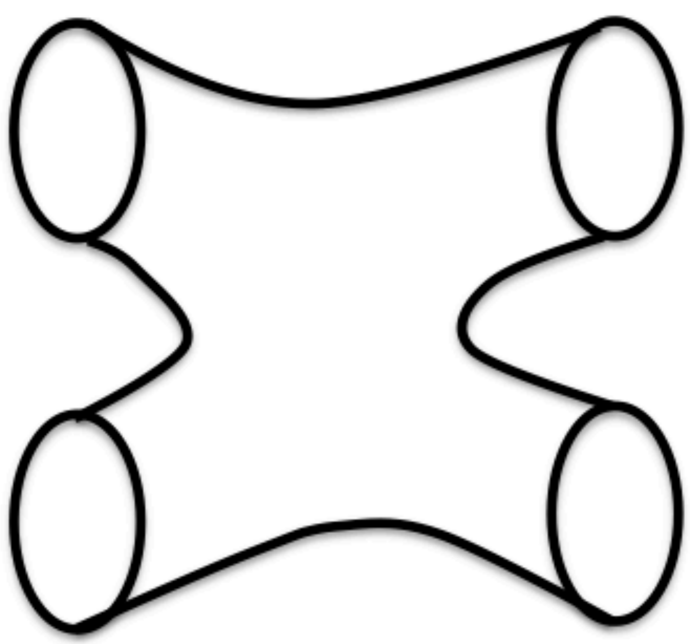
deconfining phase: M.H.-Maltz-Susskind, 2014

Strings out of YM:

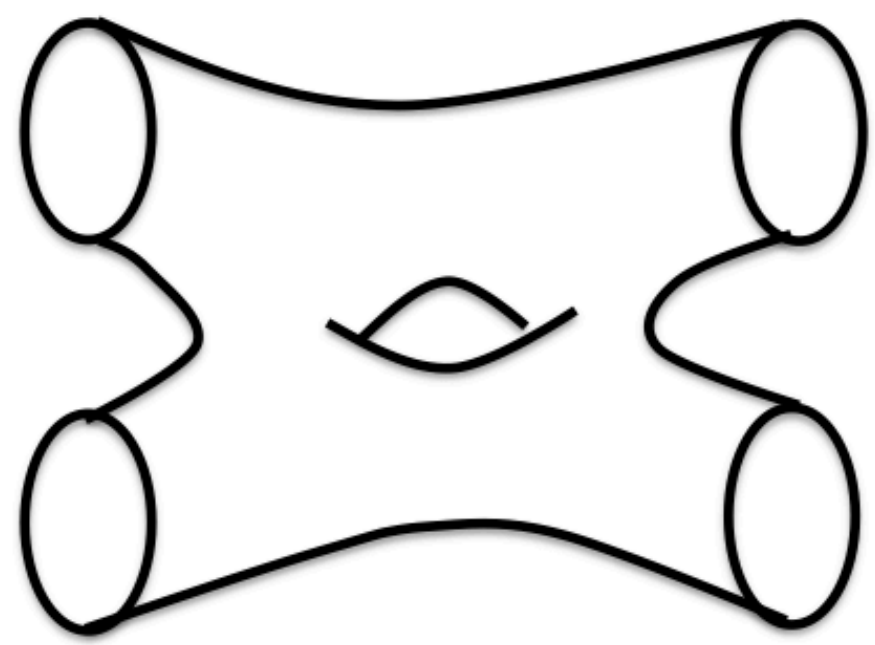
’t Hooft’s argument for the <u>confining</u> phase



scattering of strings



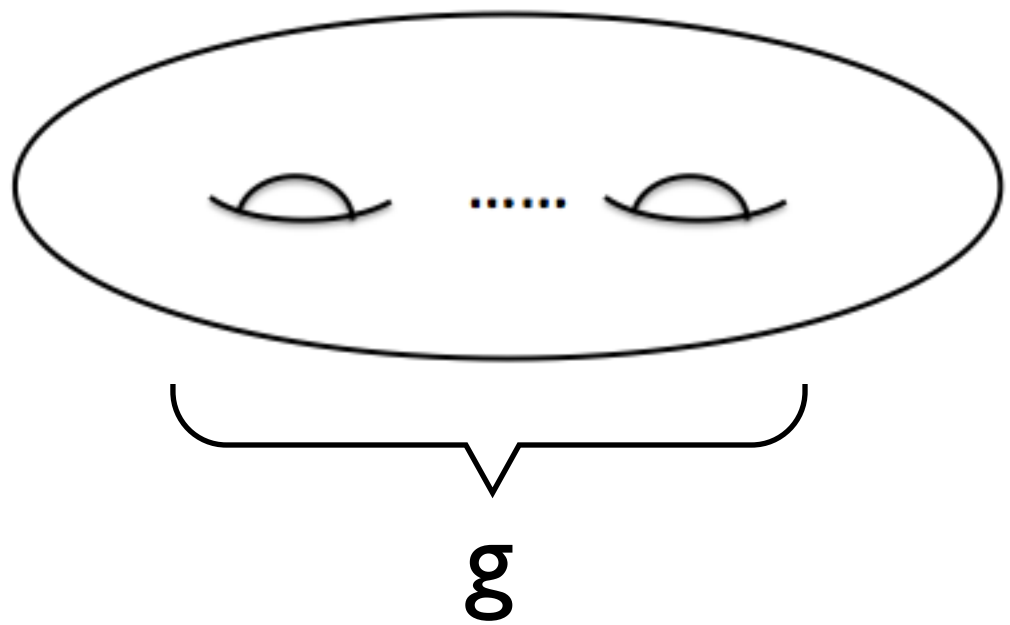
tree



one-loop

$\sim g_s^2$

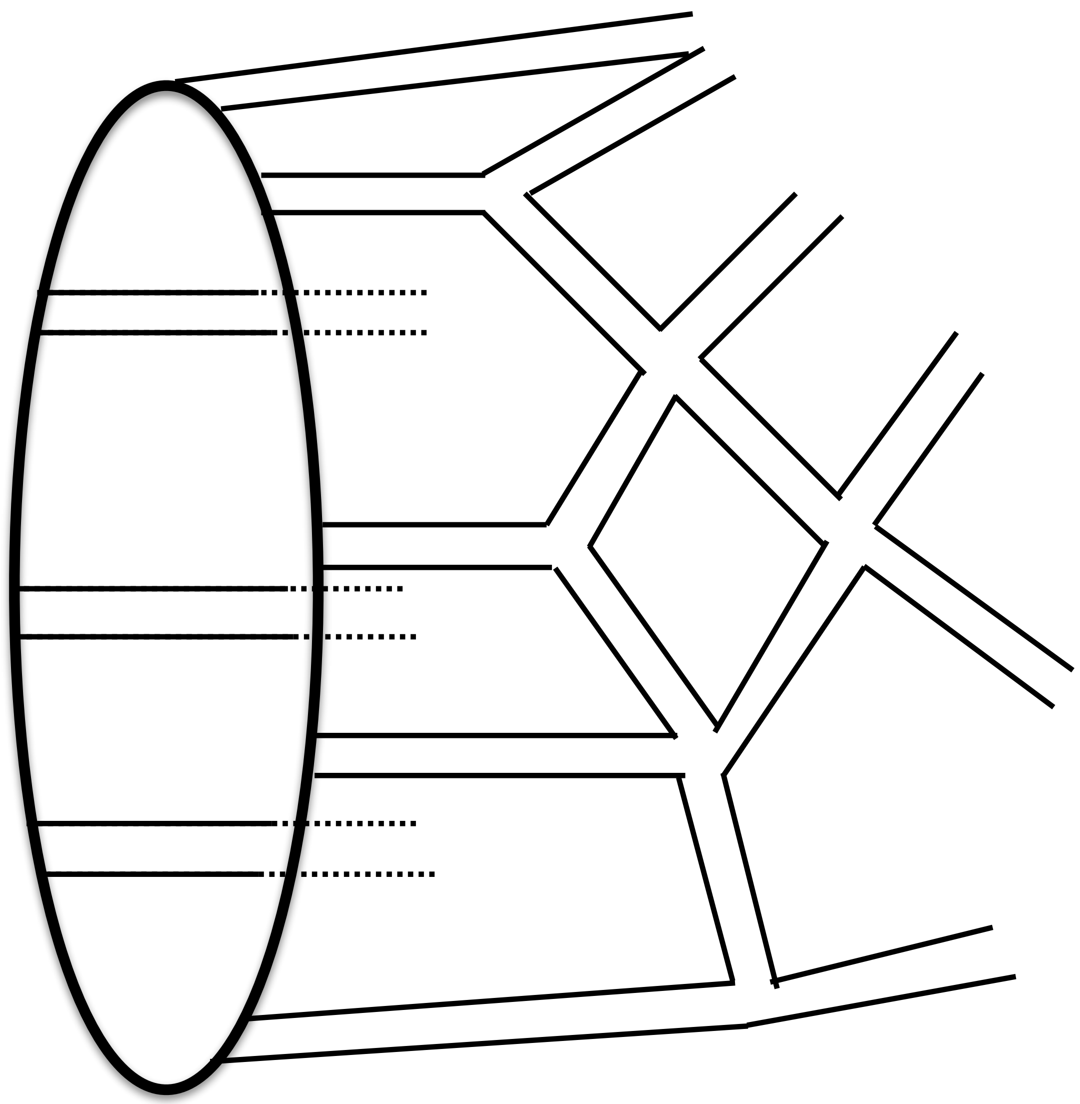
$\boxed{\text{g closed string loops} \rightarrow \text{genus g surface}}$



$\sim g_s{}^{2g}$

One takes into account the quantum effect order by order, by increasing g one by one.
→ *perturbative* formulation

Main idea



Feynman diagram
= “fishnet” made of gluons
= string worldsheet

How can they be related
without ambiguity?

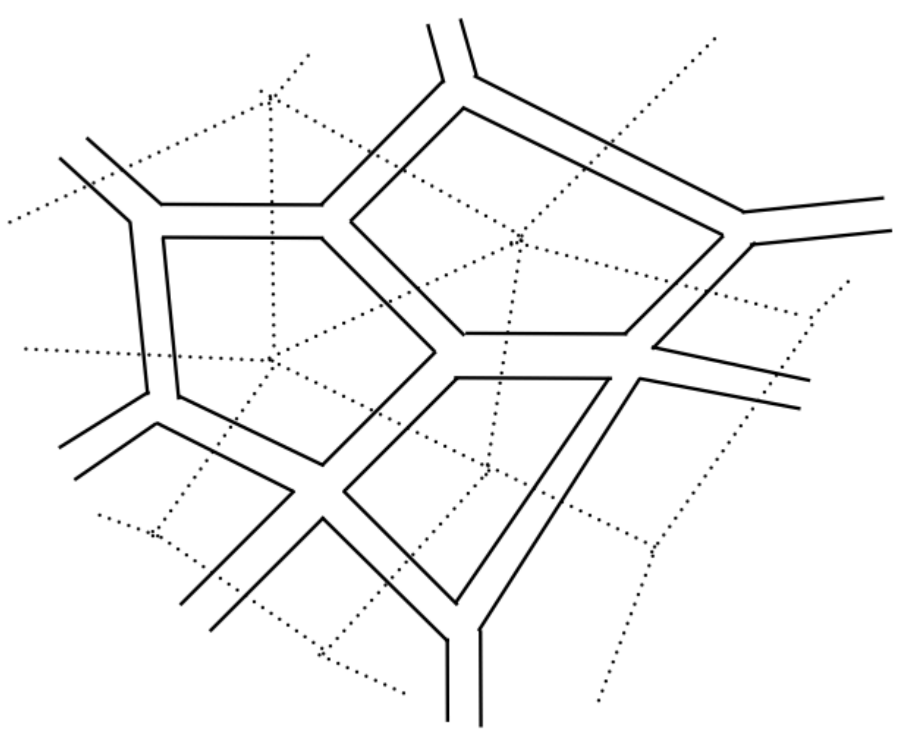
Wilson loop = creation operator of closed string

Main idea

$\boxed{\text{Feynman diagram}}$ “fish net”

$$=$$

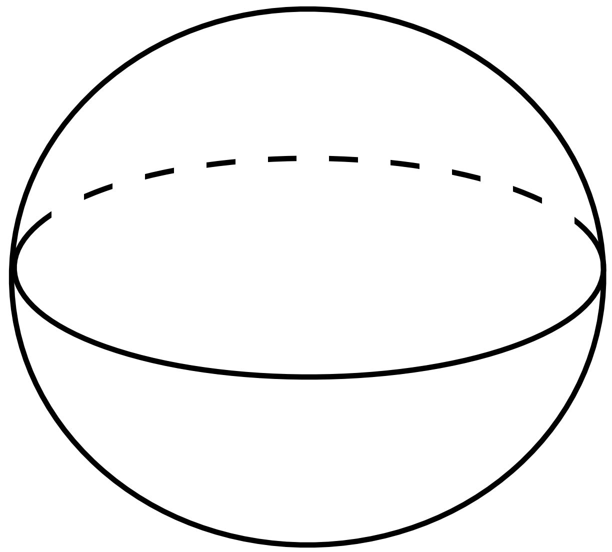
$\boxed{\text{triangulation/quadrangulation of string worldsheet}}$



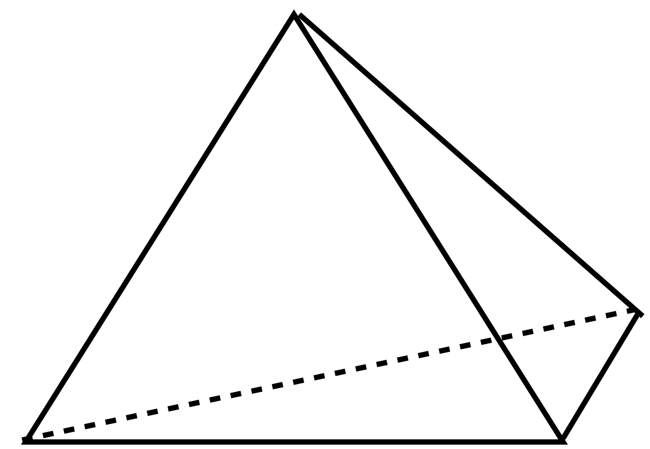
$\boxed{\text{1/N expansion}}$

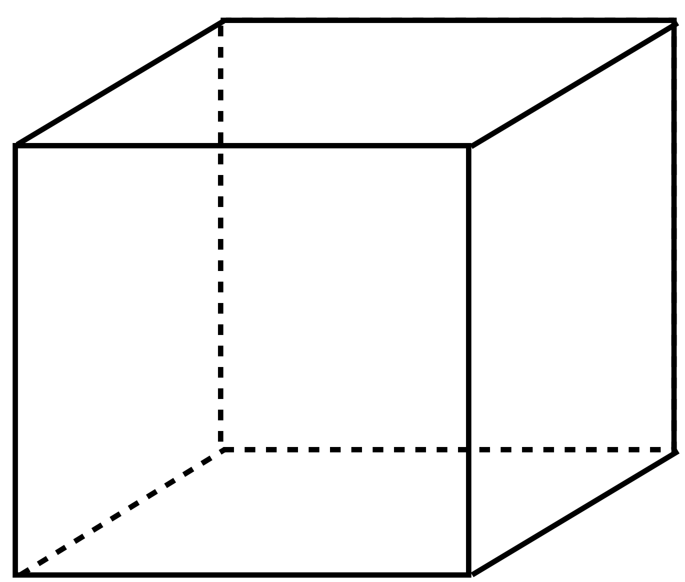
$$=$$

$\boxed{\text{genus expansion}}$



two-sphere (g=0)





$$S = \frac{N}{4\lambda}\int d^4x Tr F_{\mu\nu}^2$$

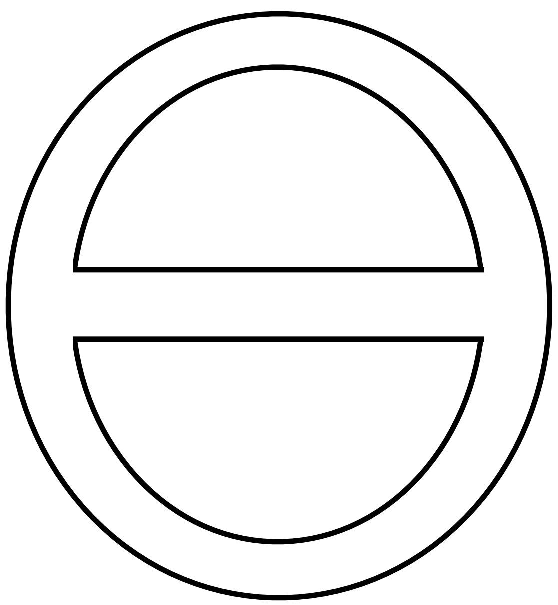
(U(N) gauge group)

vertex $\sim$ N

index loop $\sim$ N

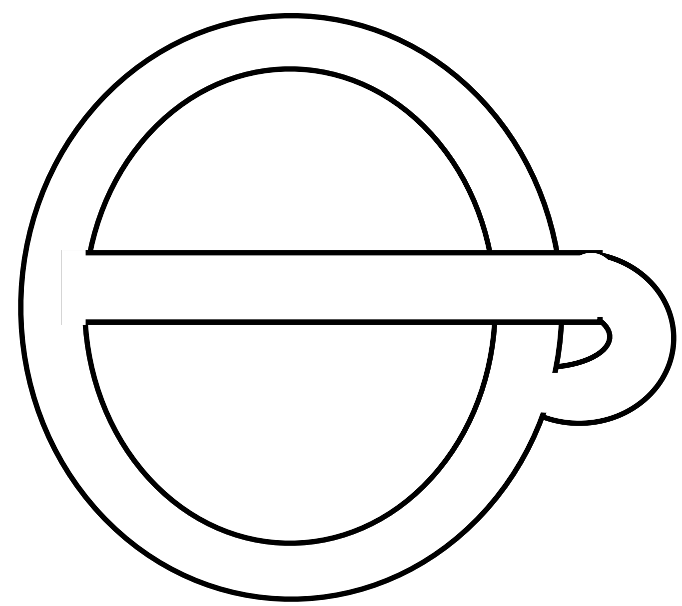
propagator $\sim$ 1/N

planar diagram

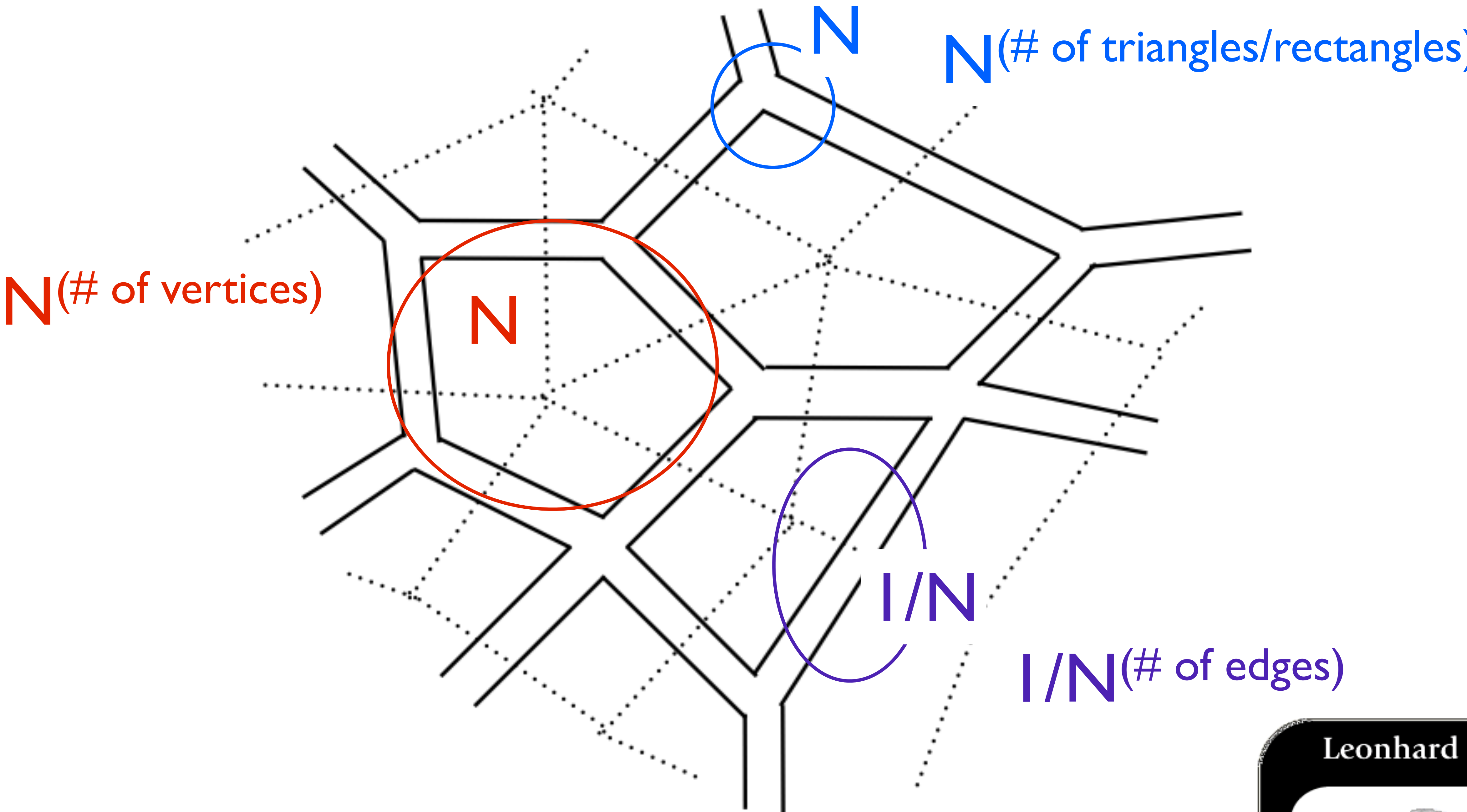


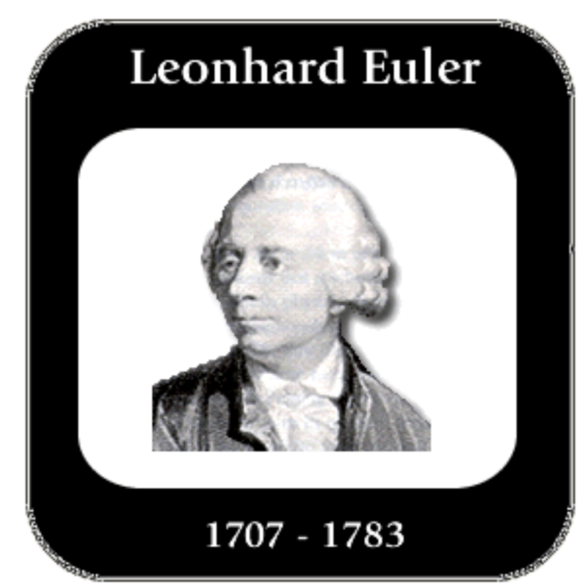
$N^2 \times N^{-3} \times N^3 = N^2$

nonplanar diagram
(genus one)



$N^2 \times N^{-3} \times N^1 = N^0$



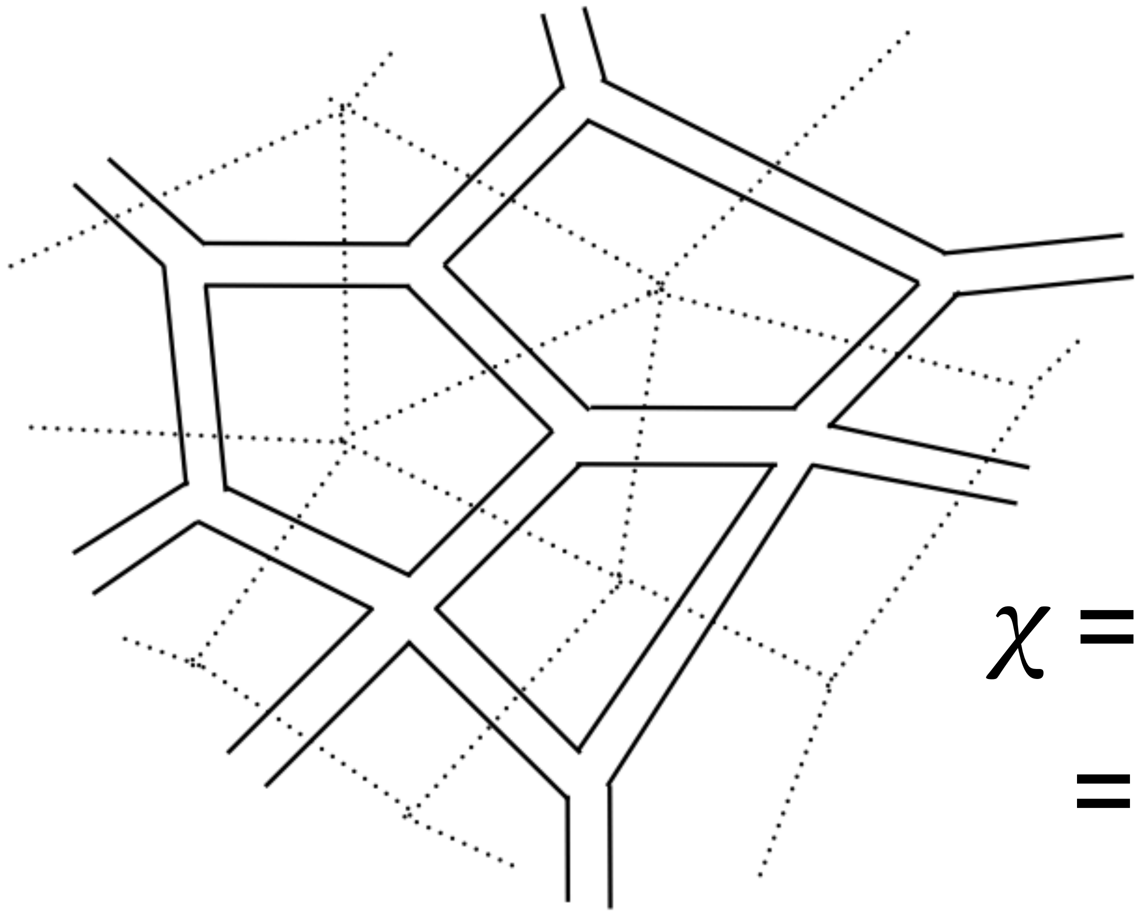


$$N^{(\text{\# of vertices})} \times 1/N^{(\text{\# of edges})} \times N^{(\text{\# of triangles/rectangles})} = N^{\chi}$$

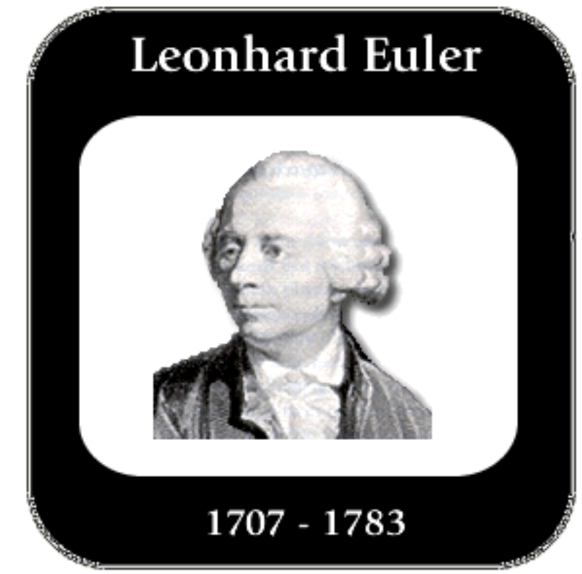
vertex $\sim$ N $\sim$ triangle/rectangle

index loop $\sim$ N $\sim$ vertex

propagator $\sim$ 1/N $\sim$ edges



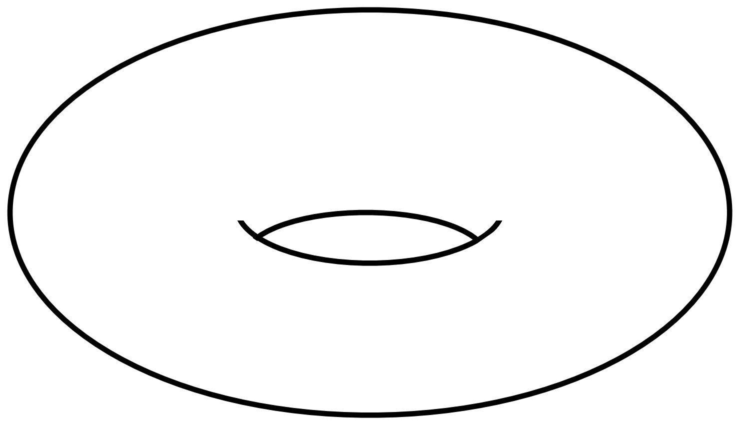
$\sim N^{\chi}$



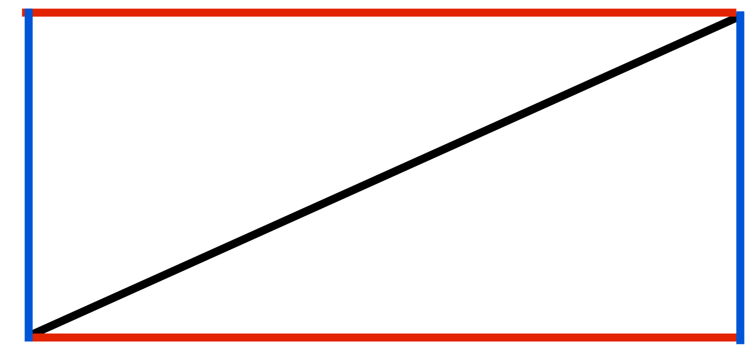
χ = Euler number

= (# vertices) − (# propagators) + (# index loops)

= (# triagnles/quadrangles) − (# edges)+ (# vertices)



torus



triangulation of torus

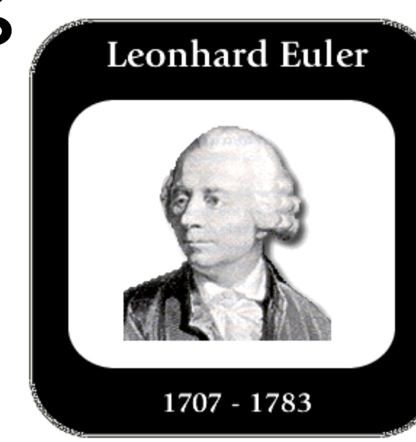
$\boxed{\text{Euler number}}$

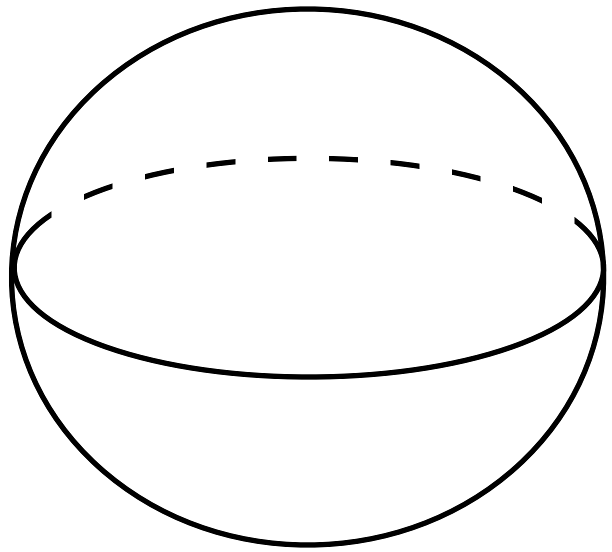
$$\chi = (\#\text{triangles}) - (\#\text{sides}) + (\#\text{vertices}) = 2 - 3 + 1 = 0$$

more generally,

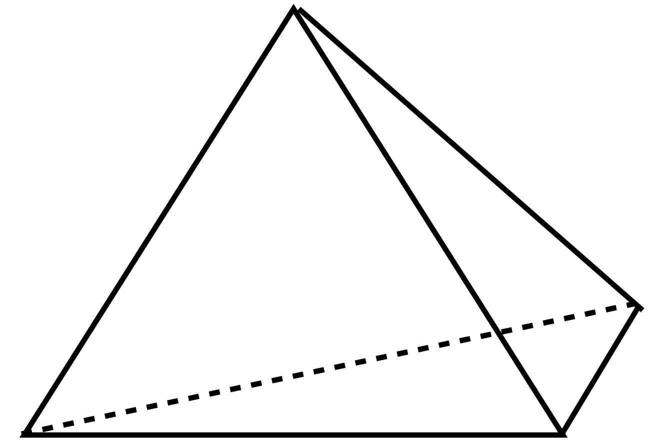
$$\chi = (\#\text{triangles}) - (\#\text{sides}) + (\#\text{vertices}) = 2 - 2g$$

where $\quad g = (\#\text{genus})$



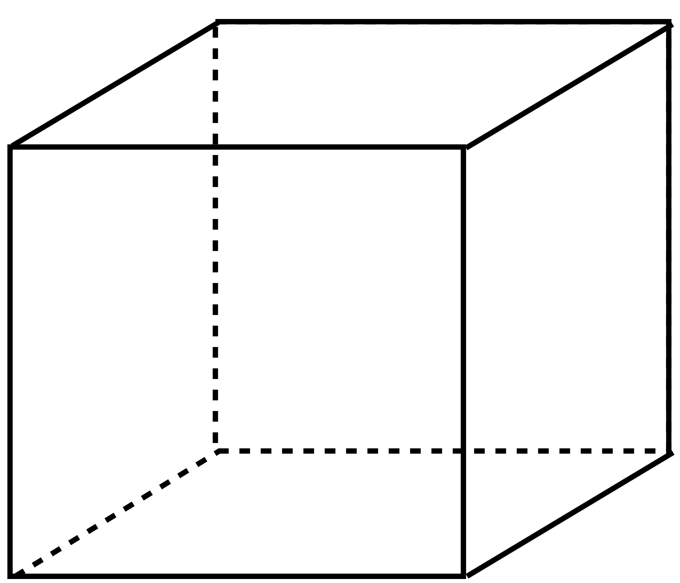


two-sphere (g=0)



4 triangles
6 sides
4 vertices

$$\boxed{4-6+4 = 2 = 2-2g}$$

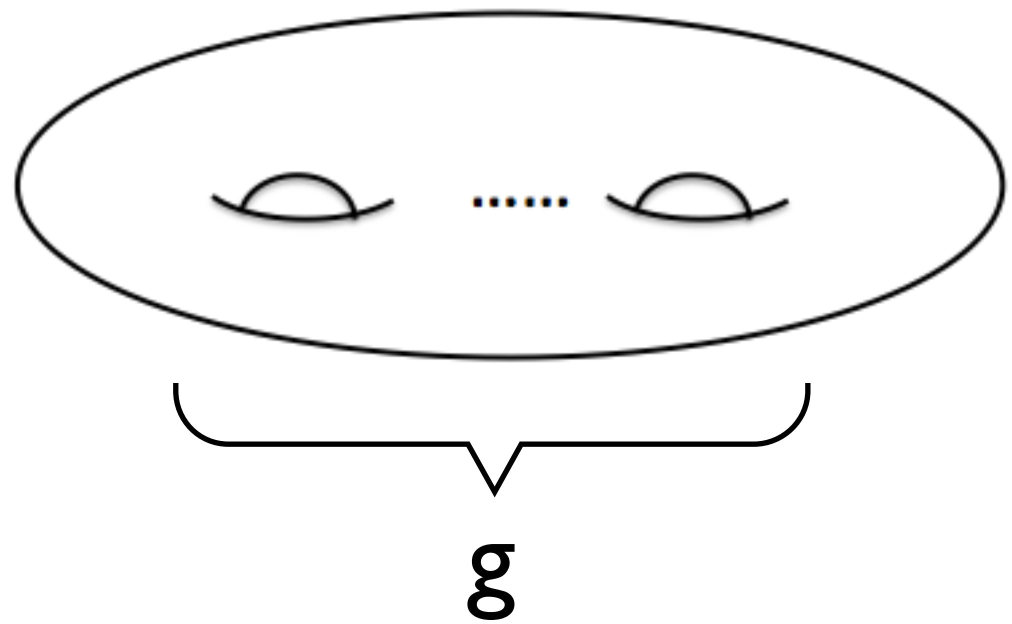


6 squares
12 sides
8 vetices

$$\boxed{6-12+8 = 2 = 2-2g}$$

genus-g diagram = diagram which can be drawn on genus-g surface

$\Rightarrow$ $\boxed{\text{g closed string loops}}$



$(1/N)^{2g-2} = g_s^{2g-2}$

$1/N = g_s$

$\boxed{\text{Yang-Mills gives } \textit{nonperturbative} \text{ formulation of string theory.}}$

$\boxed{\text{large-N limit is free string theory.}}$

Lattice gauge theory description at strong coupling

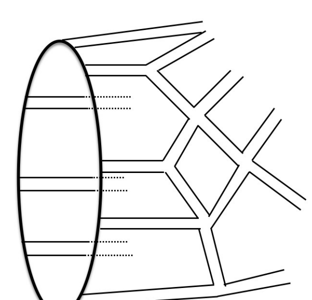
Understand it by using the Hamiltonian formulation of lattice gauge theory (Kogut-Susskind, 1974)

$$H = \frac{\lambda N}{2}\sum_{\vec{x}}\sum_{\mu}\sum_{\alpha=1}^{N^2}\left(E^{\alpha}_{\mu,\vec{x}}\right)^2 + \frac{N}{\lambda}\sum_{\vec{x}}\sum_{\mu<\nu}\left(N - \mathrm{Tr}(U_{\mu,\vec{x}}U_{\nu,\vec{x}+\hat{\mu}}U^{\dagger}_{\mu,\vec{x}+\hat{\nu}}U^{\dagger}_{\nu,\vec{x}})\right)$$

$$[E^{\alpha}_{\mu,\vec{x}}, U_{\nu,\vec{y}}] = \delta_{\mu\nu}\delta_{\vec{x}\vec{y}}\cdot\tau^{\alpha}U_{\nu,\vec{y}}$$

$$\sum_{\alpha=1}^{N^2}\tau^{\alpha}_{ij}\tau^{\alpha}_{kl} = \frac{\delta_{il}\delta_{jk}}{N^2}$$

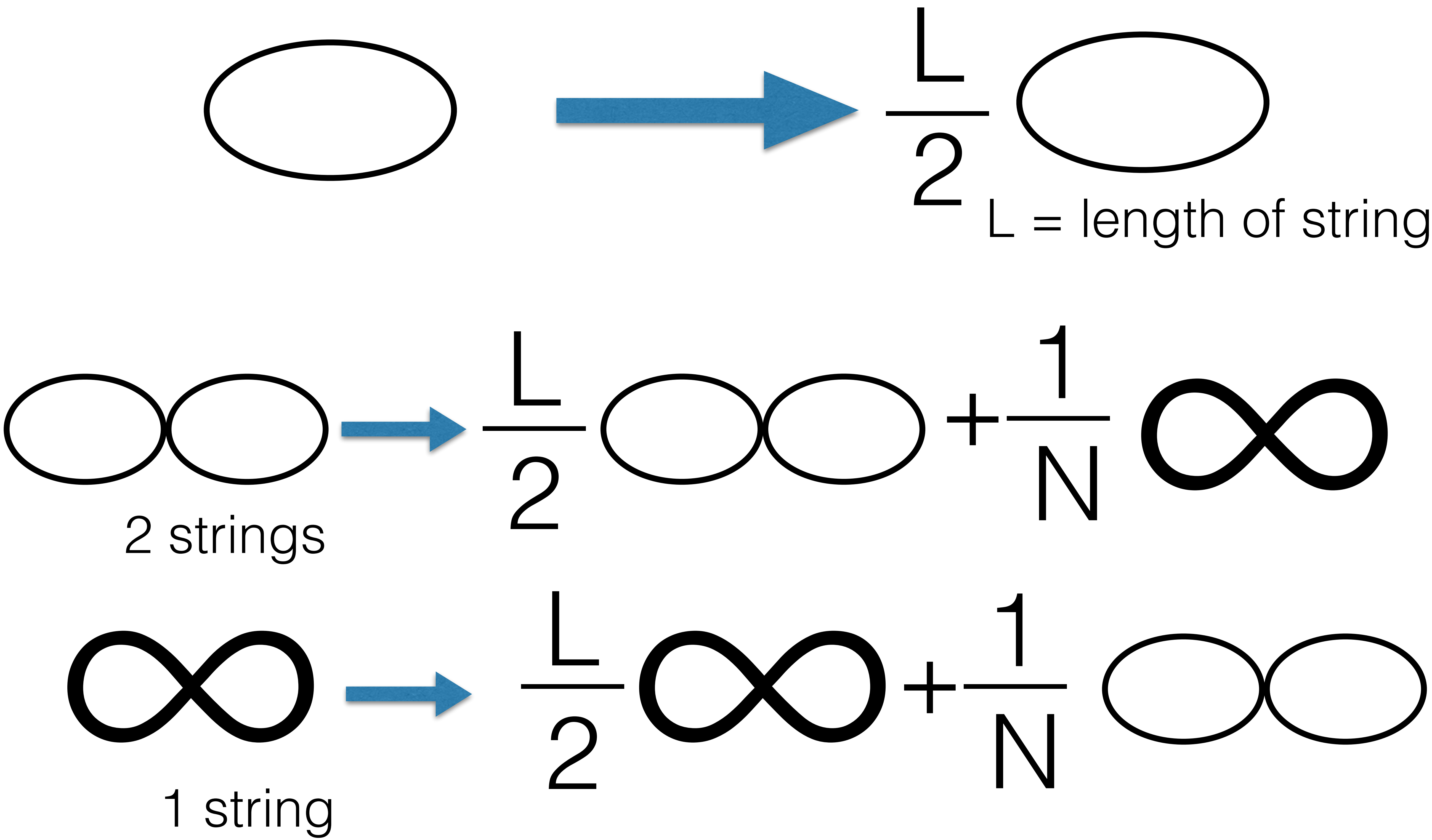
Hilbert space is expressed by <u>Wilson loops.</u> (closed string)



strong coupling limit

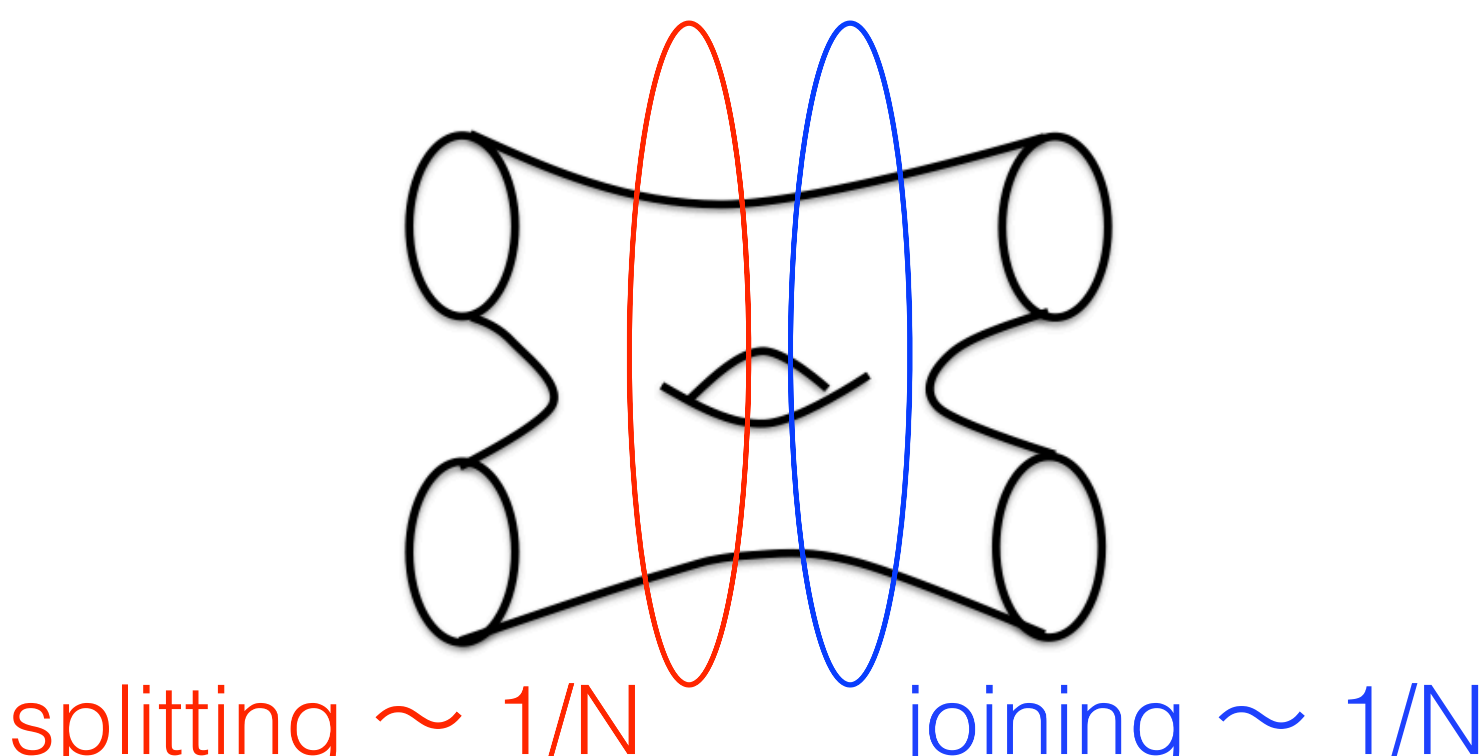
$$H = \frac{N}{2}\sum_{\vec{x}}\sum_{\mu}\sum_{\alpha=1}^{N^2}(E^{\alpha}_{\mu,\vec{x}})^2$$

(λ=1 for simplicity)



$$W_C = \mathrm{Tr}(U_{\mu,\vec{x}} M_C) \qquad W_{C'} = \mathrm{Tr}(U_{\mu,\vec{x}} M_{C'})$$

$$\begin{aligned} &H|W_C, W_{C'}\rangle \\ &= \frac{\lambda(L+L')}{2}|W_C, W_{C'}\rangle \\ &\quad +\lambda N \sum_\alpha \mathrm{Tr}(\tau^\alpha U_{\mu,\vec{x}} M_C) \cdot \mathrm{Tr}(\tau^\alpha U_{\mu,\vec{x}} M_{C'})|0\rangle \\ &= \frac{\lambda(L+L')}{2}|W_C, W_{C'}\rangle + \frac{\lambda}{N}\mathrm{Tr}(U_{\mu,\vec{x}} M_C U_{\mu,\vec{x}} M_{C'})|0\rangle \end{aligned}$$



Strings out of YM: deconfining phase

M.H.-Maltz-Susskind, 2014

Berkowitz-M.H.-Hayden-Maltz-Susskind, in progress

- interaction (joining/splitting) is 1/N-suppressed

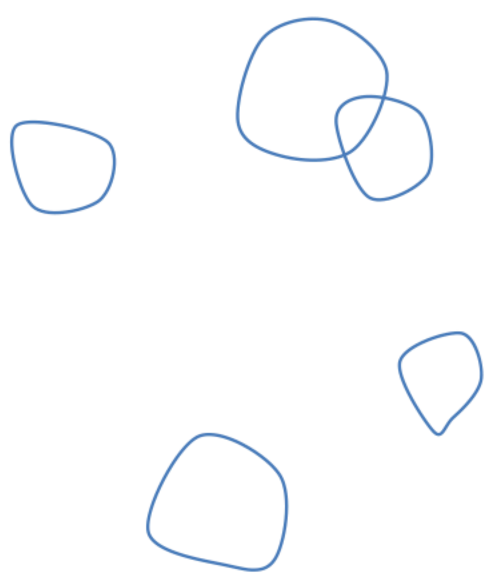
"large-N limit is the theory of free string"

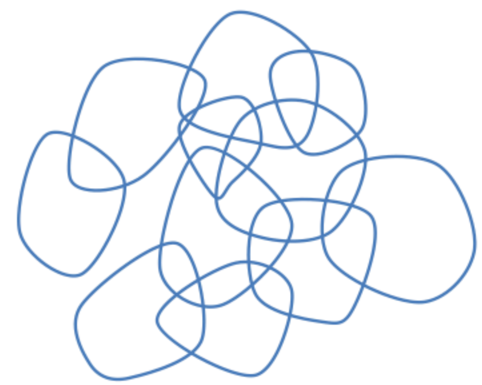
- It is true when L is $O(N^0)$. ($\rightarrow$confining phase)

- In deconfinement phase, total length of the strings is $O(N^2)$ $\rightarrow$ number of intersections increases with N$\rightarrow$interaction is **not** negligible

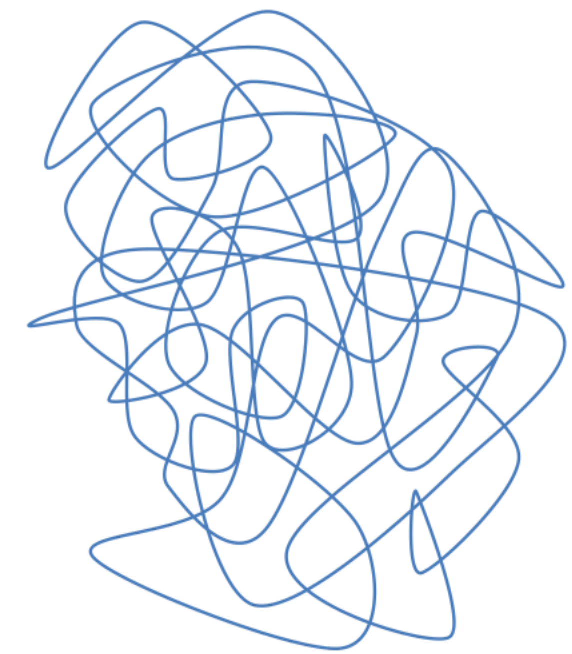
large-N limit is still very dynamical!

confining phase
= gas of short strings



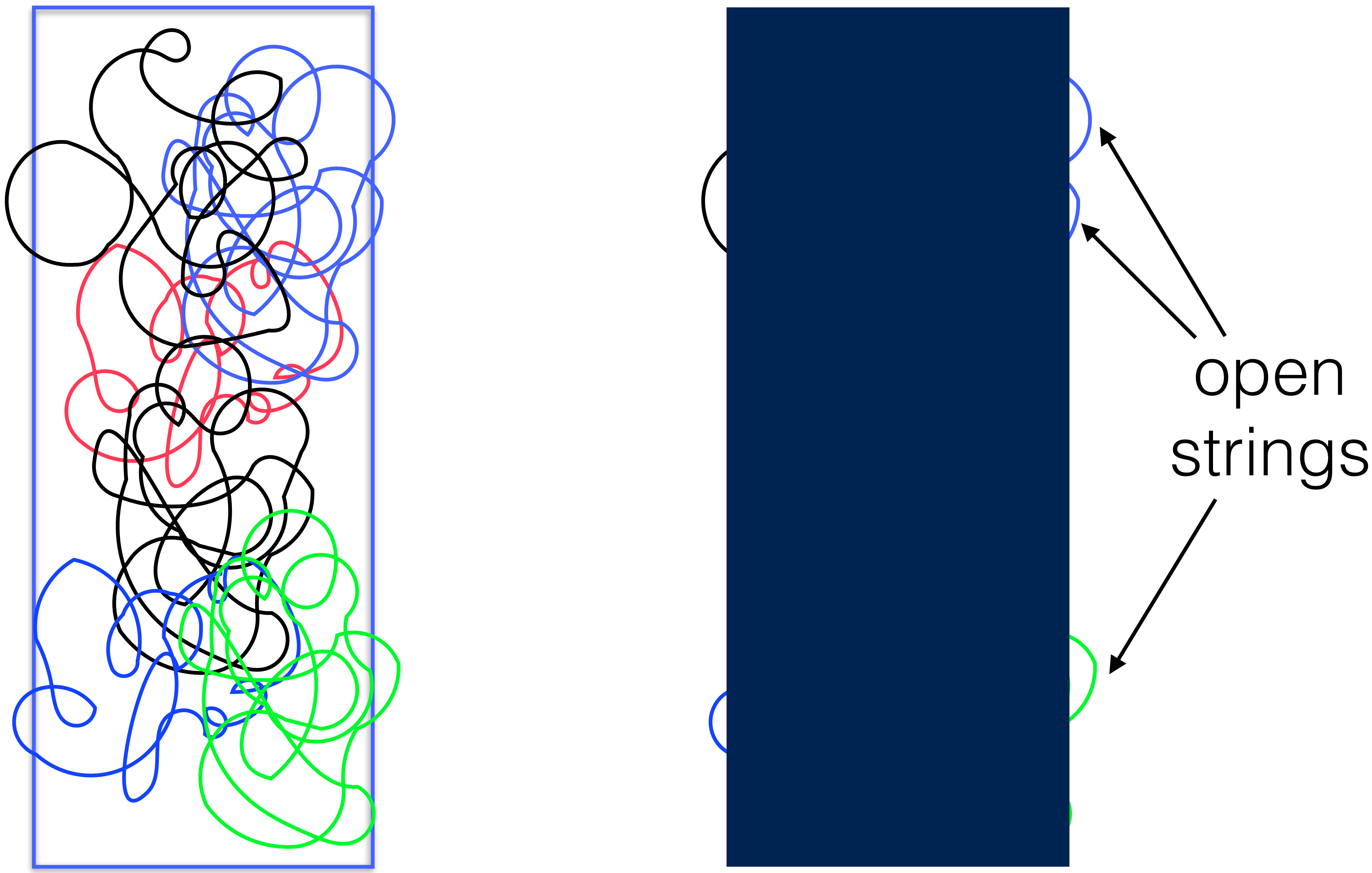


long and winding string,
which is interpreted as BH,
appears



as the density of strings increase,
interaction between strings
becomes important,and…

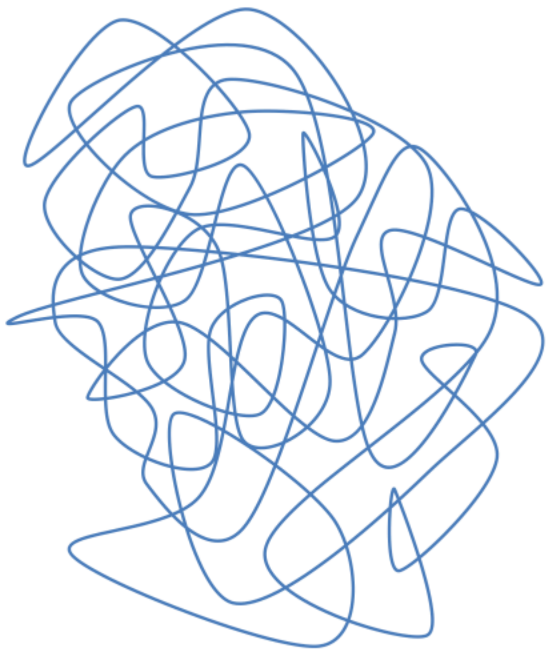
long, winding *QCD*-strings = black brane + open *QCD*-strings



open strings = Wilson lines, which have N color d.o.f at endpoints

→ black brane is made from N Dp-branes

D-dim square lattice at strong coupling



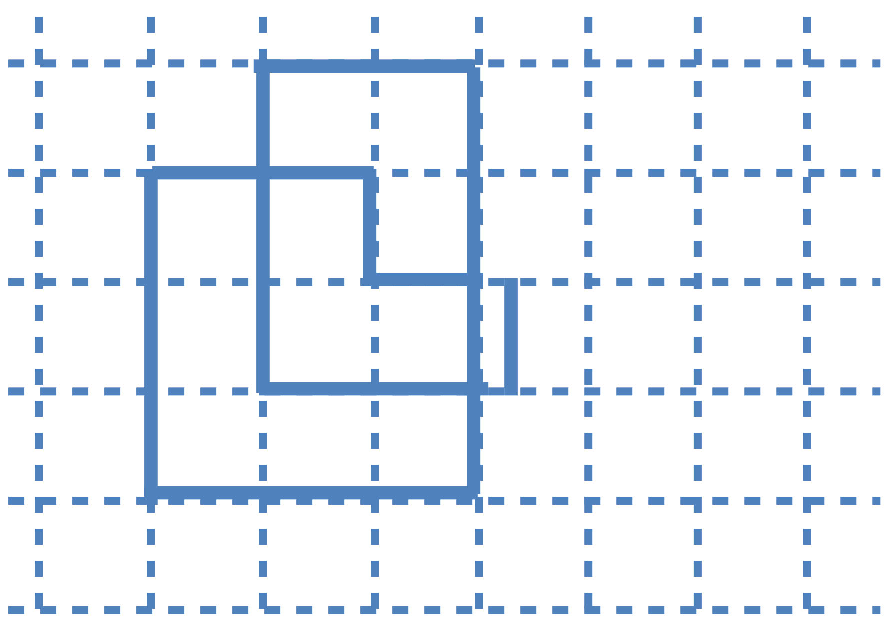
deconfining phase = long string

huge mass and entropy are packed in a small region → BH

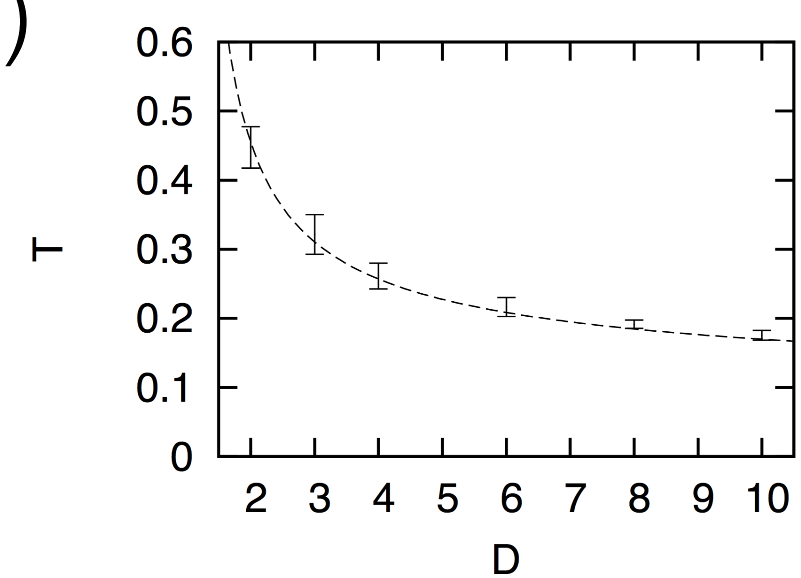
$$E = L(T)/2,\ S = L(T)\times\log(2D-1)$$

$$F = E-TS = L(T)\times(1/2 - T\times\log(2D-1))$$

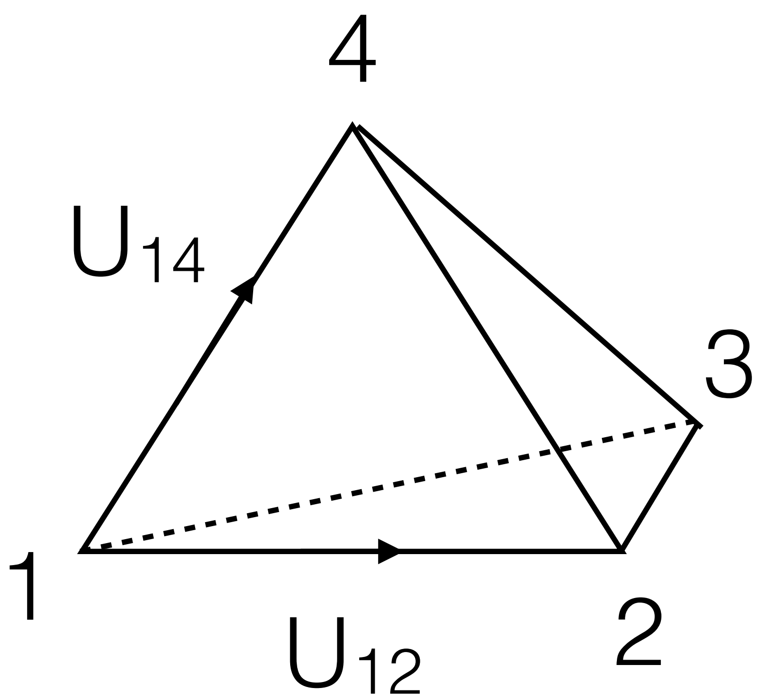
$L \sim N^2$



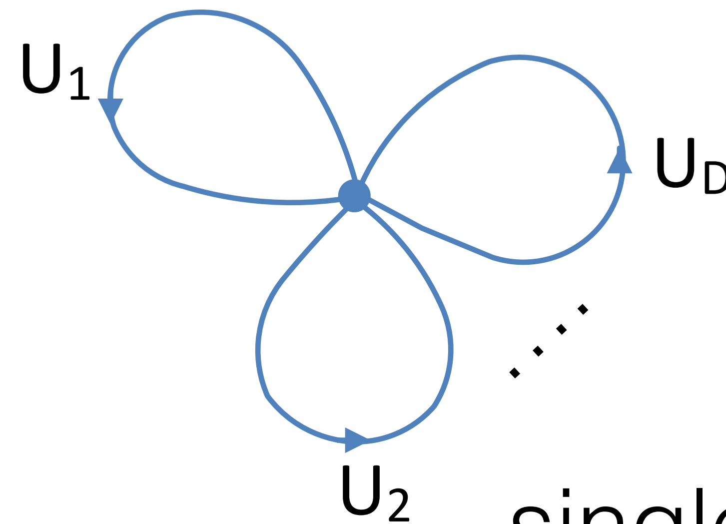
$$T_C = \frac{1}{2\log(2D-1)}$$



matrix models at strong coupling



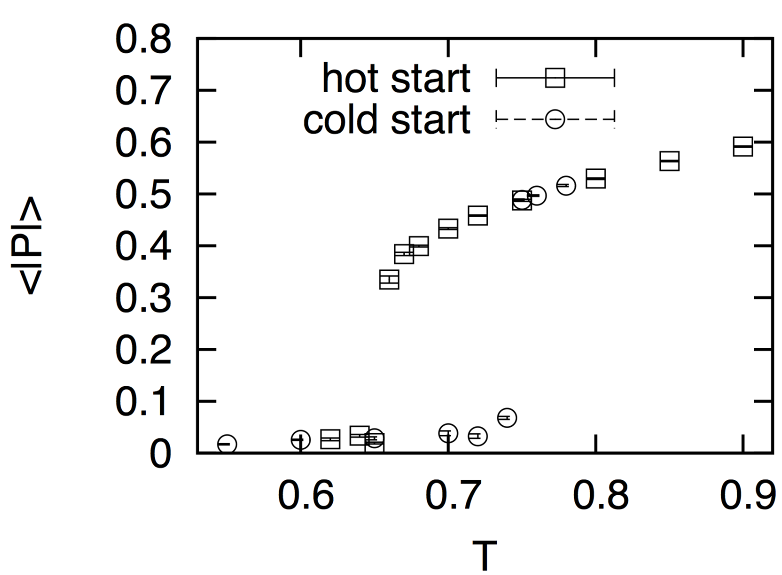
tetrahedron $T_c = \frac{1}{2\log 2} = 0.72\ldots$

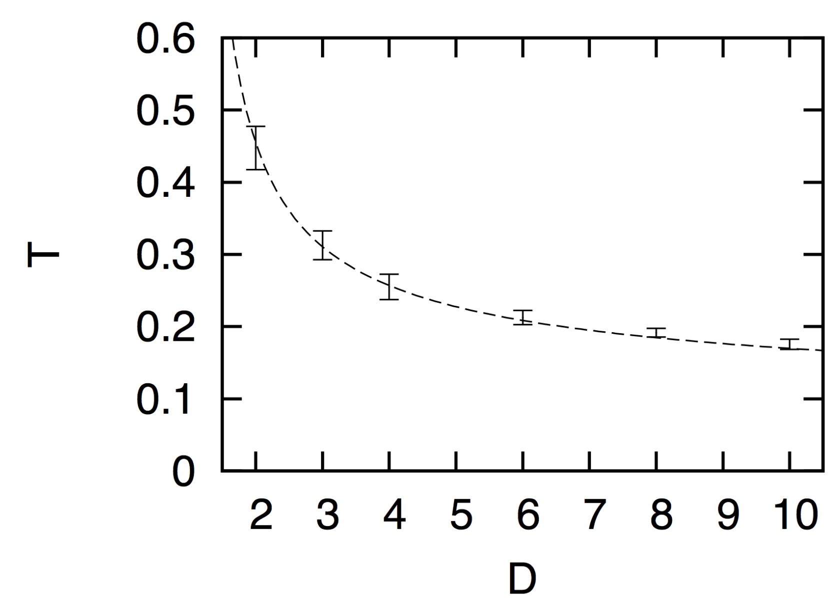


$T_c = \frac{1}{2\log(2D-1)}$

single-site with D-links (Eguchi-Kawai model)

(Equivalent to large-volume lattice via Eguchi-Kawai equivalence)





Why $L \sim N^2$?

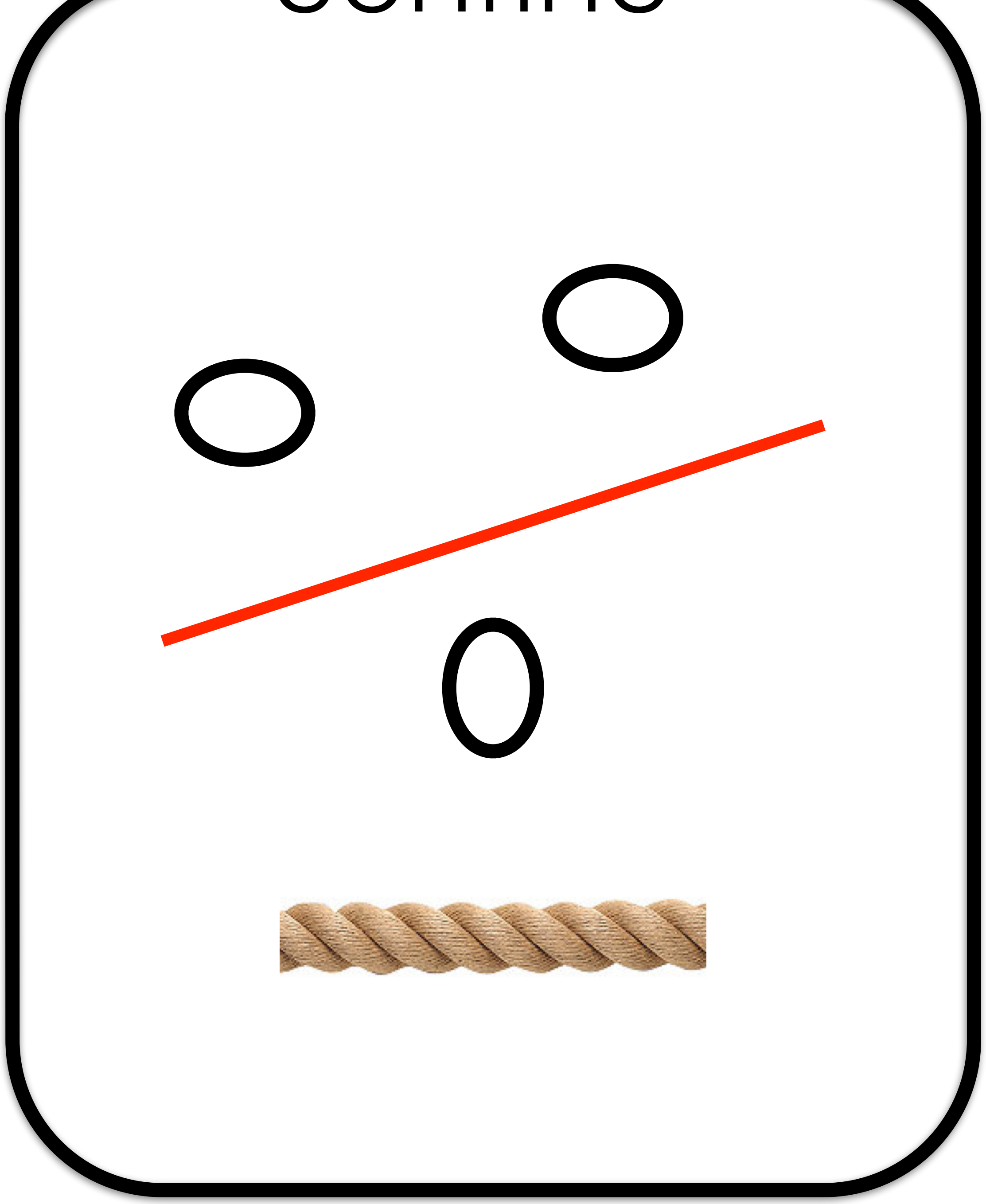
- $U(N)$ matrices has N^2 components.
- D.O.F. in unit volume $\sim$ N^2.
- Tr(UU'U''…..)

length $\gtrsim$ N^2 $\rightarrow$ factorizes to shorter traces

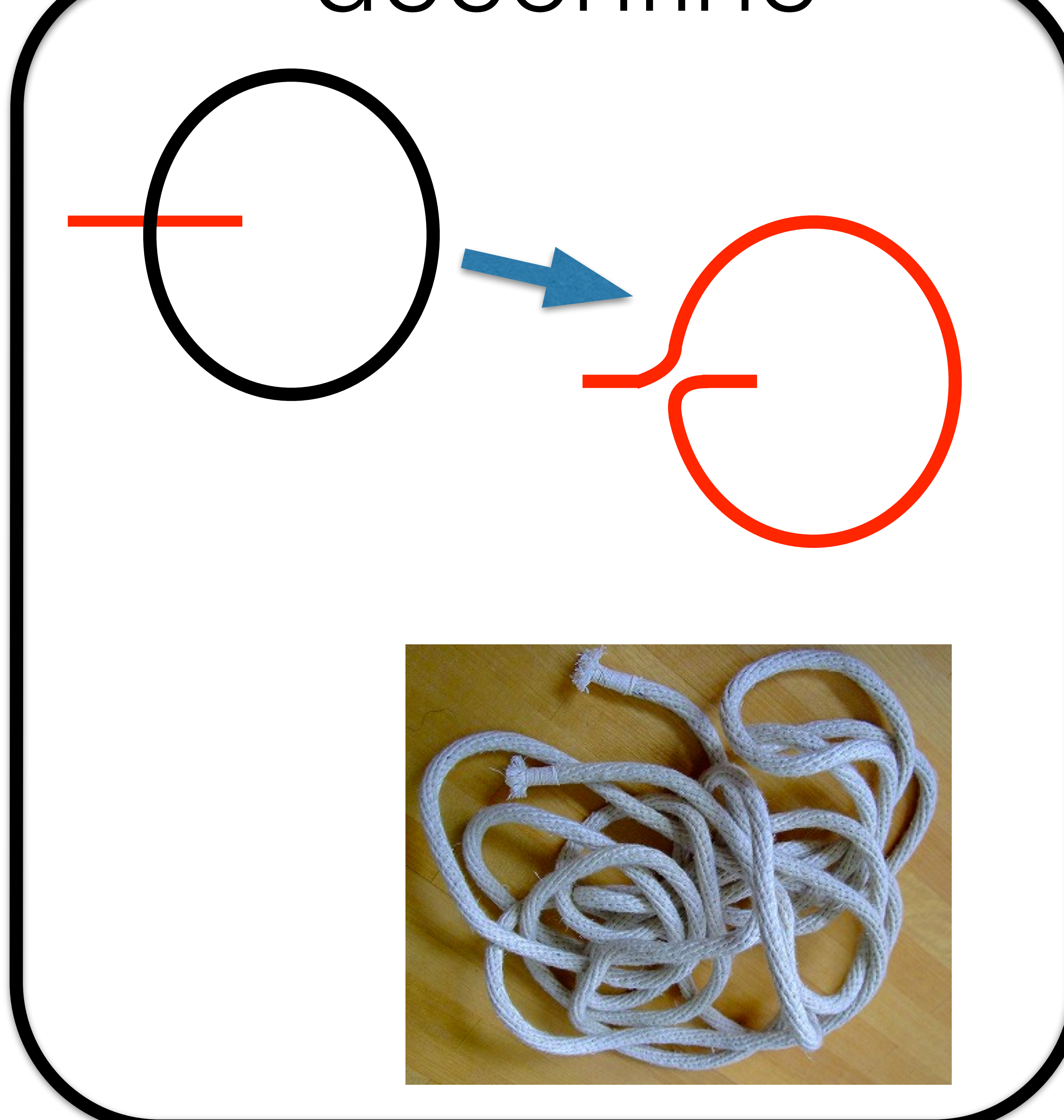
N^2 is the upper bound.
Beyond there, the counting changes;
not much gain for the entropy.

(de)confinement of probe charges

confine



deconfine



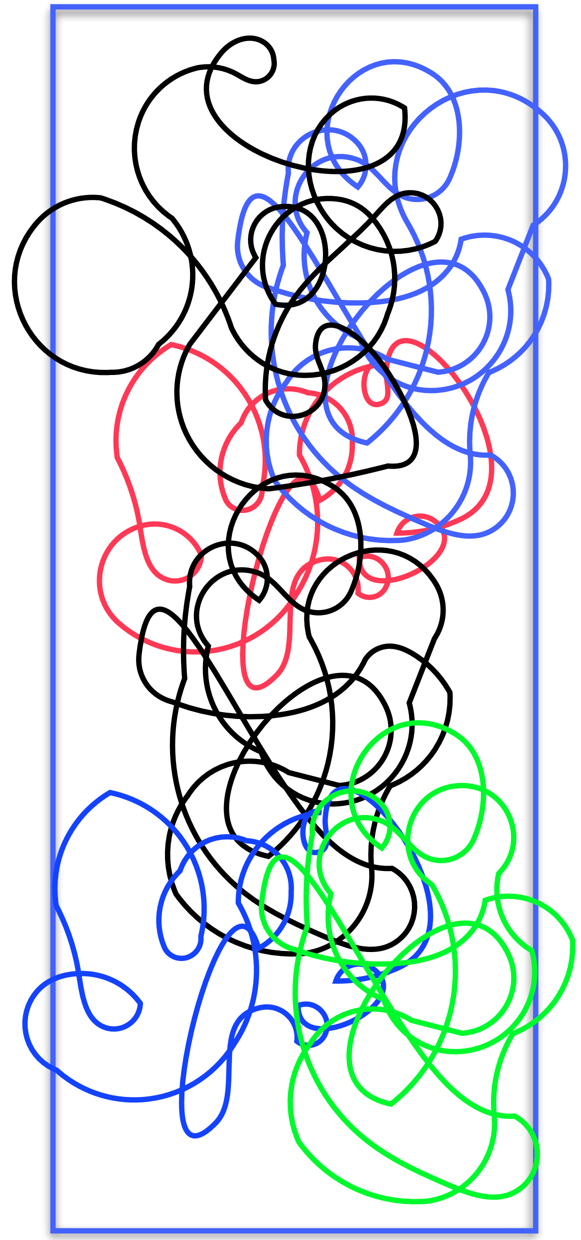
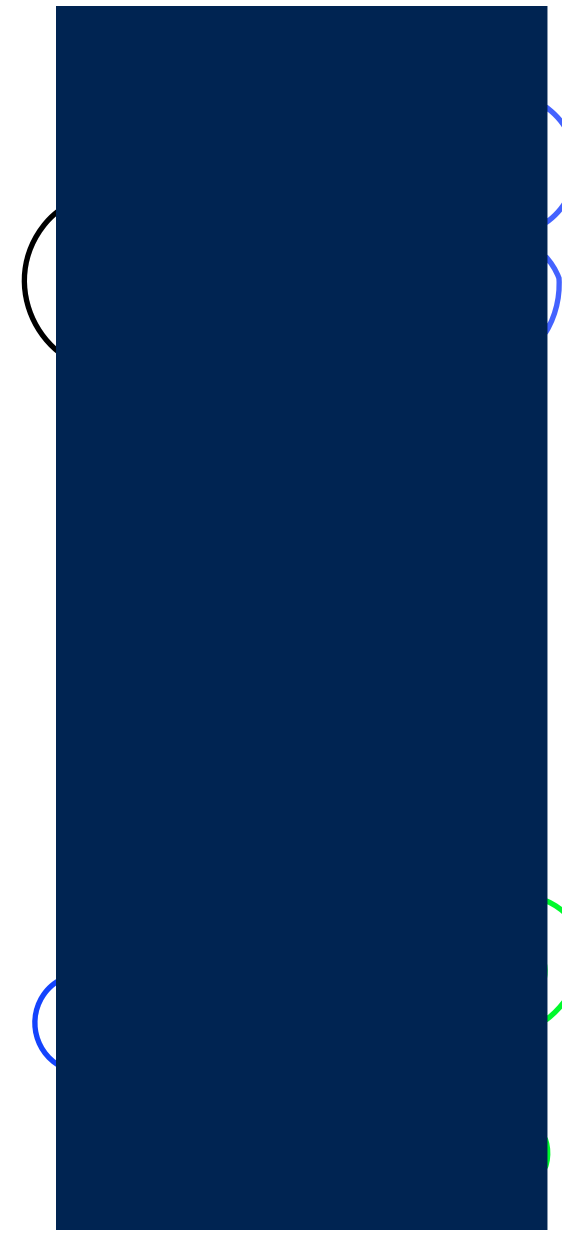
conclusion

Maldacena's conjecture is correct
at finite temperature,
including $1/\lambda$ and $1/N$ corrections,
at least to the next-leading order.

so, lattice/nuclear theorists can study
quantum gravity, by studying field theory.

Occupy Princeton

conclusion

deconfinement phase =  = 

strong coupling lattice gauge theory contains the essence.