# The Quark Mass Gap in Strong Magnetic Fields

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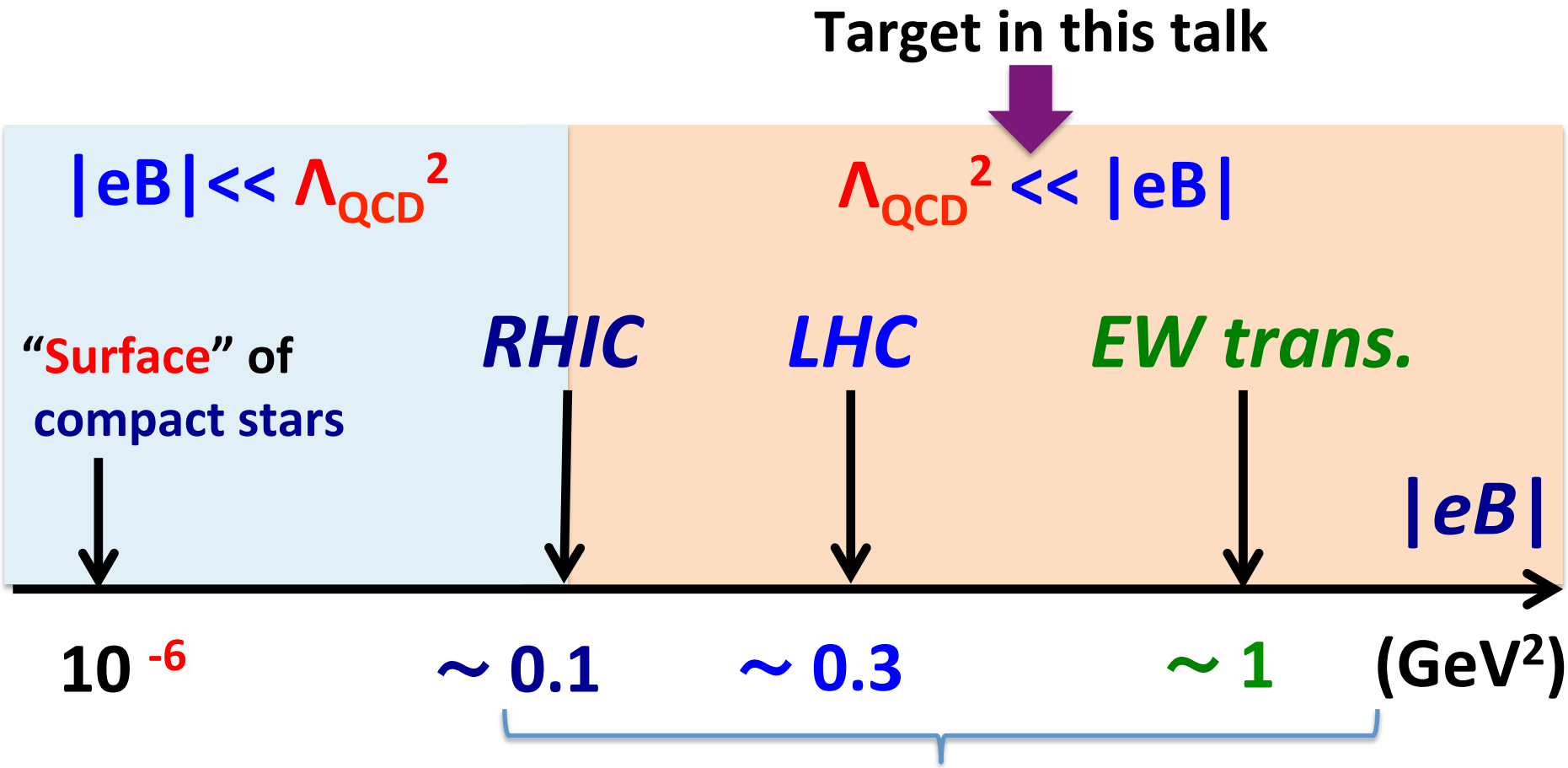
**with Nan Su (Bielefeld)**

**T.K. & Nan Su PLB720 (2013), arXiv: 1211.7318**

**T.K. & Nan Su PLB726 (2013), arXiv: 1305.4510**

# Strength of B-field




Lattice data are available, ( both for *full* and *quenched* )

waiting for theoretical explanations.

# Contents

**0 ) Basics : fermions in strong mag. fields**

**1 ) Models vs Lattice : how models fail**

**2 ) Quenched QCD in strong mag. fields**

Quark mass gap: QCD vs NJL, QED, .....

Toy (*confining*) model considerations

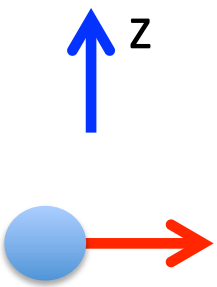
**3 ) Few comments on unquenched QCD**

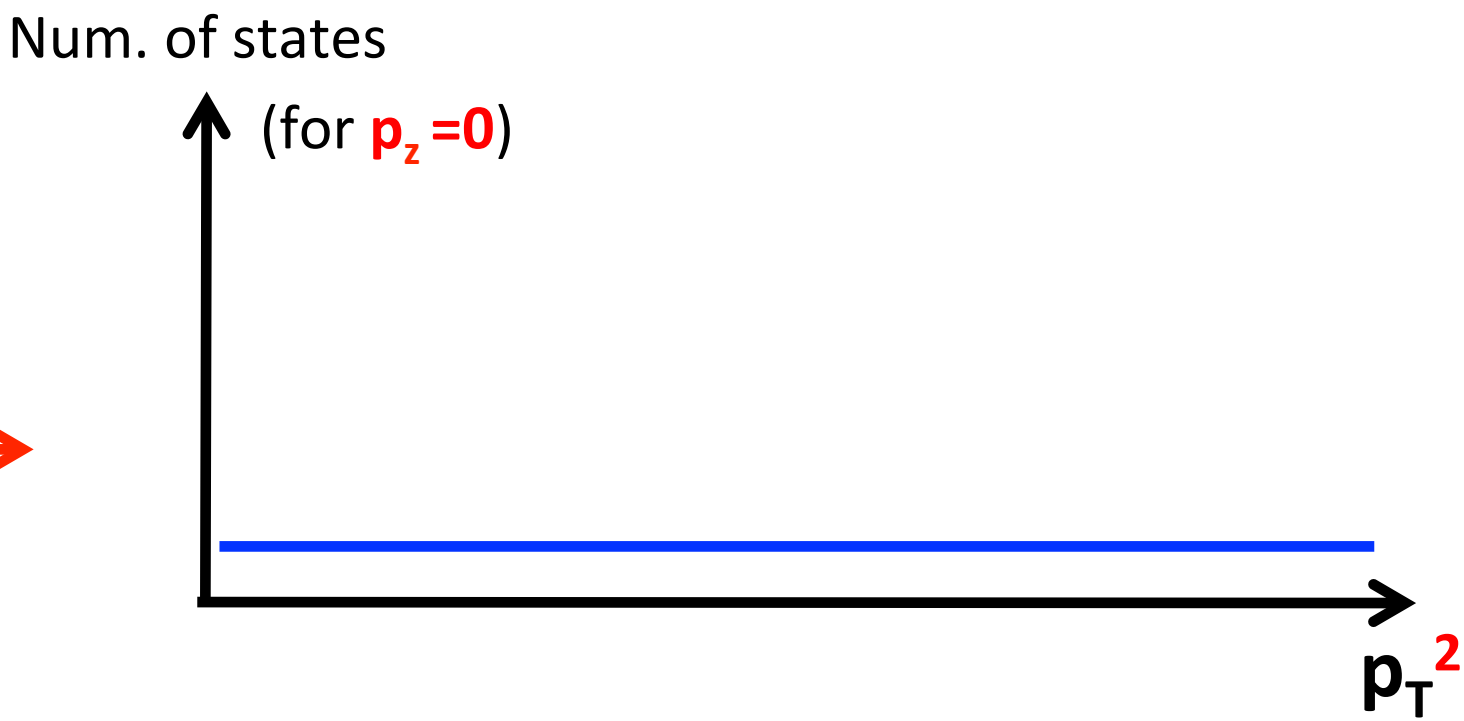
**(or 1/Nc corrections)**

# Quantum mechanics in mag. fields

(spinless, free particles)

**B = 0**

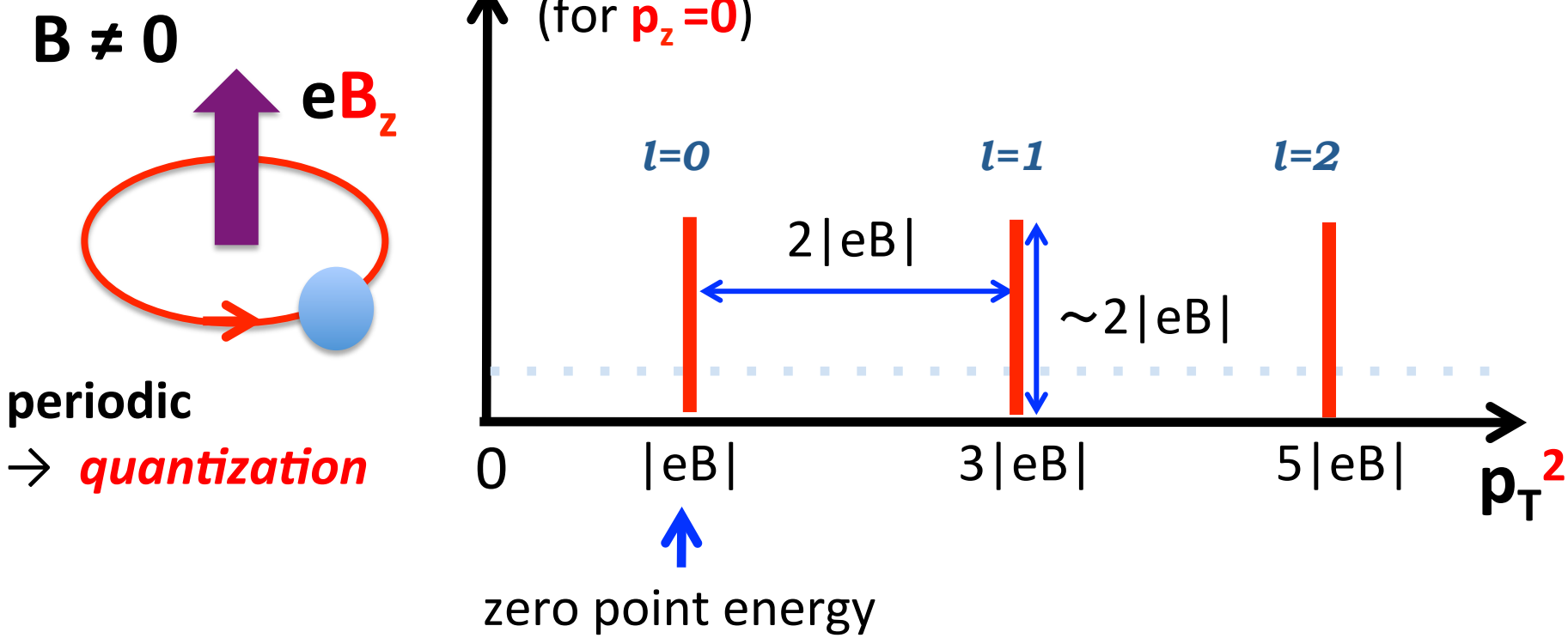


# Quantum mechanics in mag. fields

(spinless, free particles)

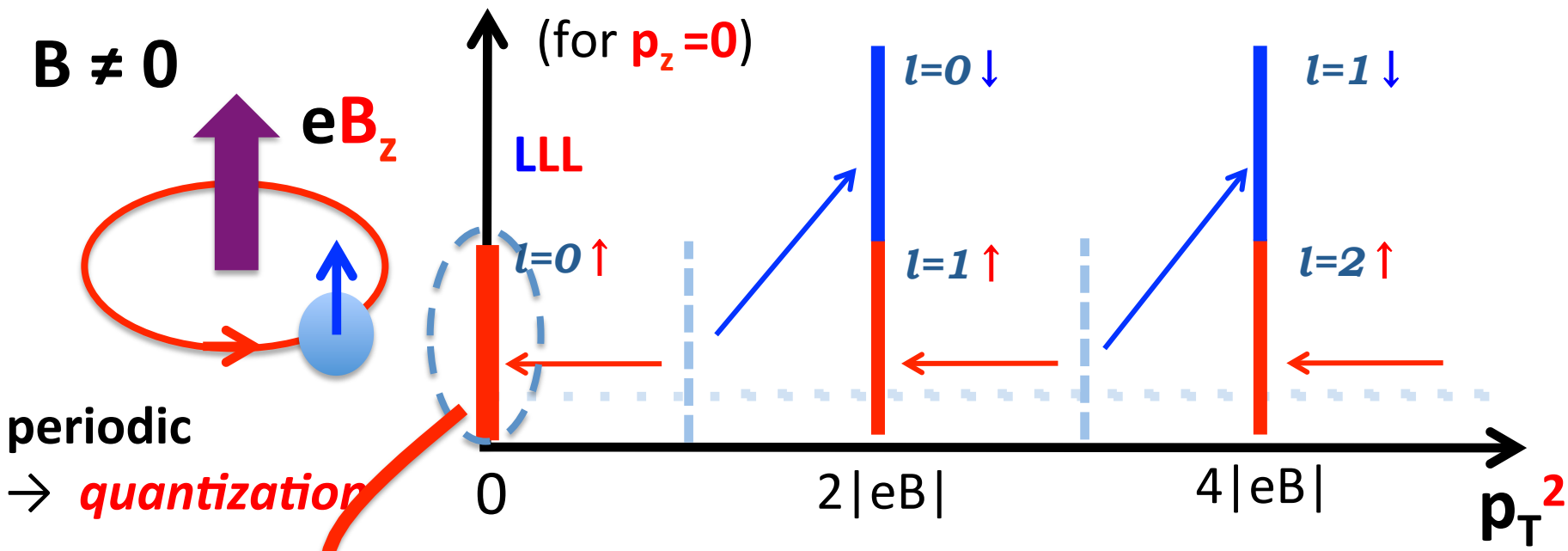
(orbital levels)



# Quantum mechanics in mag. fields

(spin 1/2, free particles)

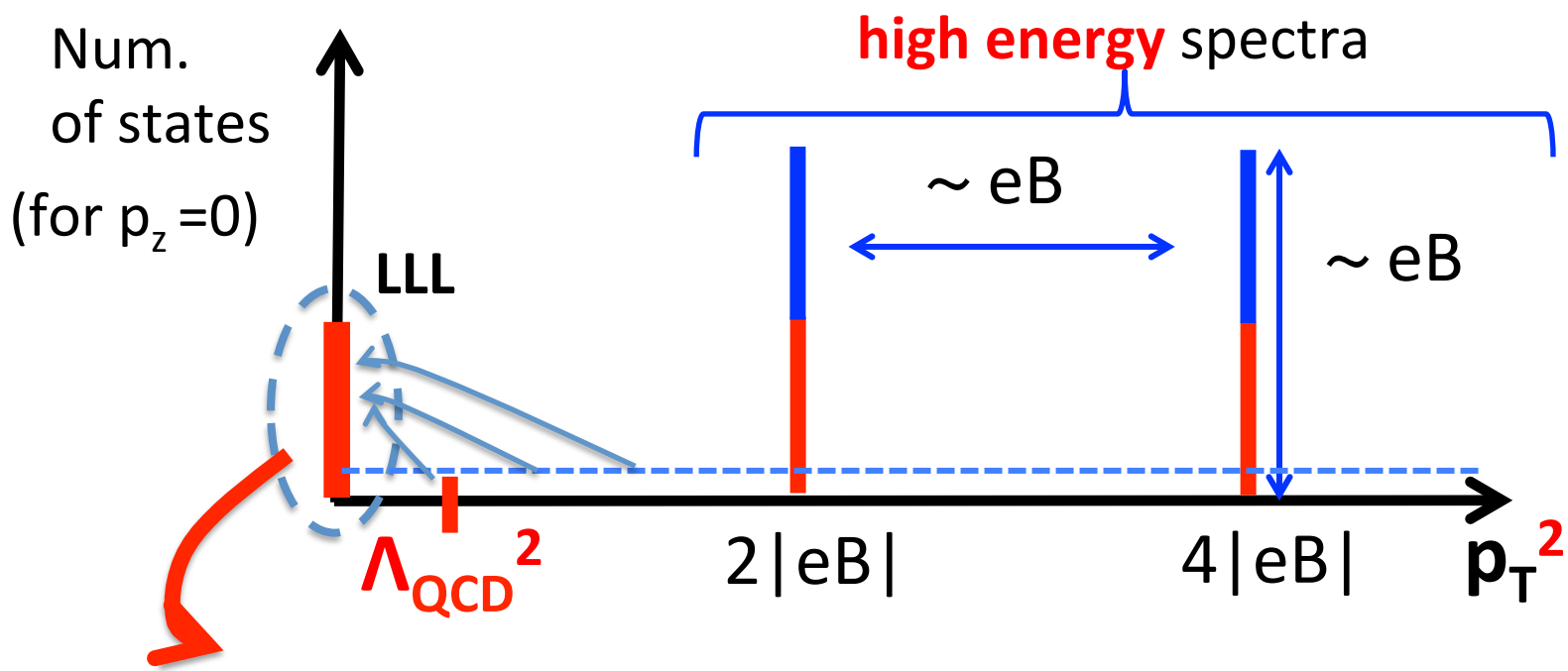
(orbital + Zeeman splitting)




periodic
→ *quantization*

- ***Energy : B-indep. (at tree level)***

# Enhanced IR phase space for quarks




***More quarks can participate in non-pert. dynamics :***

- **Bigger Chiral Condensate ~ *Magnetic Catalysis***

  **(Like BCS, SSB can happen even for arbitrarily small coupling)**

- **Larger impact on gluon dynamics** **(larger screening, ...)**

# *The reasons to study mag.* QCD

**1, Theories can be confronted with the lattice results.**

(*No sign problem, systematic studies*)

**2, Simple qualitative problems are still available (see below)**

- **They discriminate models from QCD .**

**3, Suitable for studies of non-thermal fluct. of quarks :**

(*Quantum*, T=0)

**→ Extremely important for studies of cold quark matter.**

- **Test of 1/Nc : Back-reaction to the gluon sector,**
- **Quantum phase transition, ...**

# 1, Models vs Lattice : How it fails

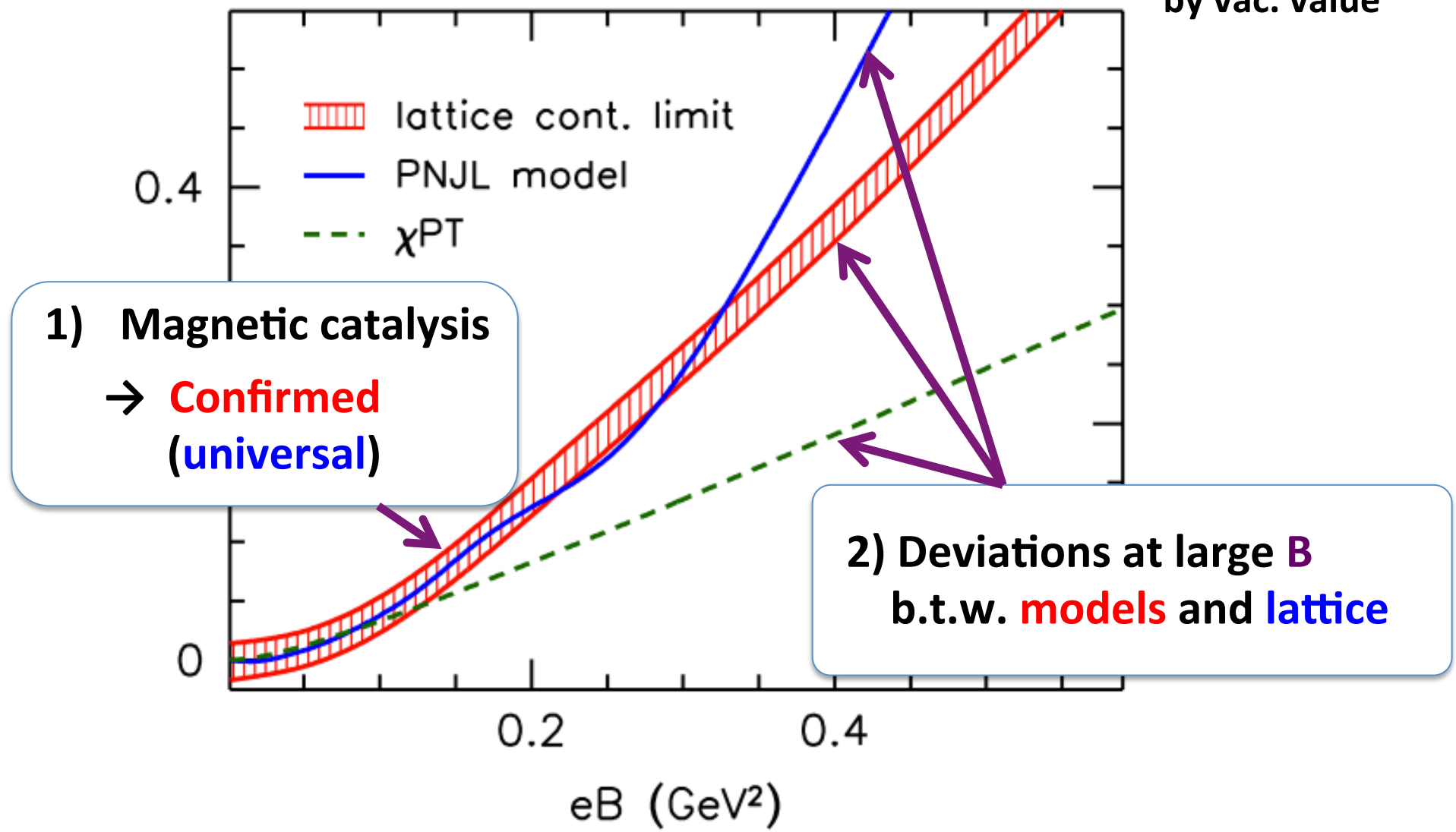
*Lattice setup* : uniform B & QED part quenched

non-uniform configs. (such as B-vortices)
are omitted in these setup.

# Models vs Lattice, 1 : *ChSB (T=0)*

(Bali et al, 2012) $\langle\bar{\psi}\psi\rangle(B) - \langle\bar{\psi}\psi\rangle(B=0)$

normalized by vac. value



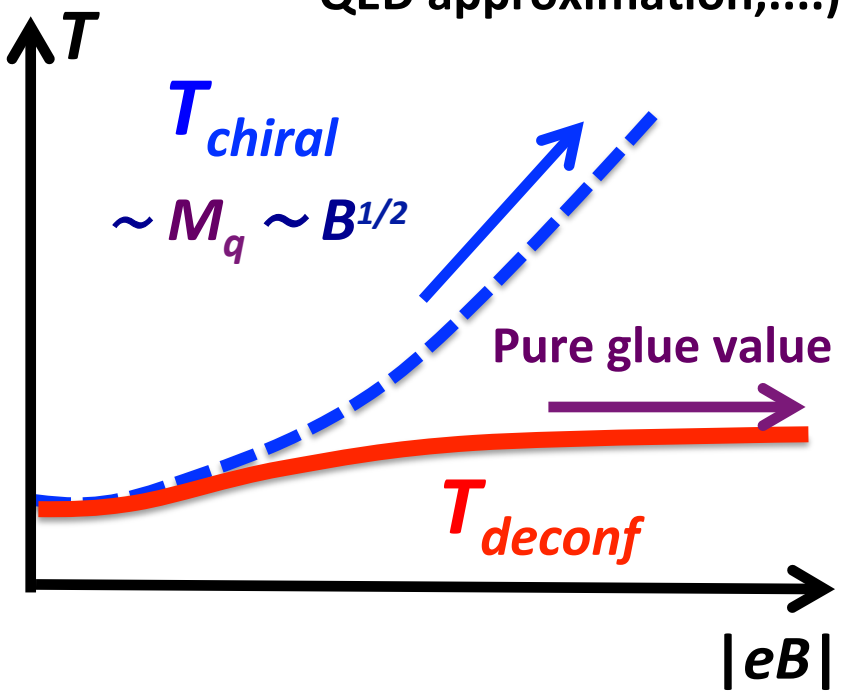

1) Magnetic catalysis

→ Confirmed (universal)

2) Deviations at large B b.t.w. models and lattice

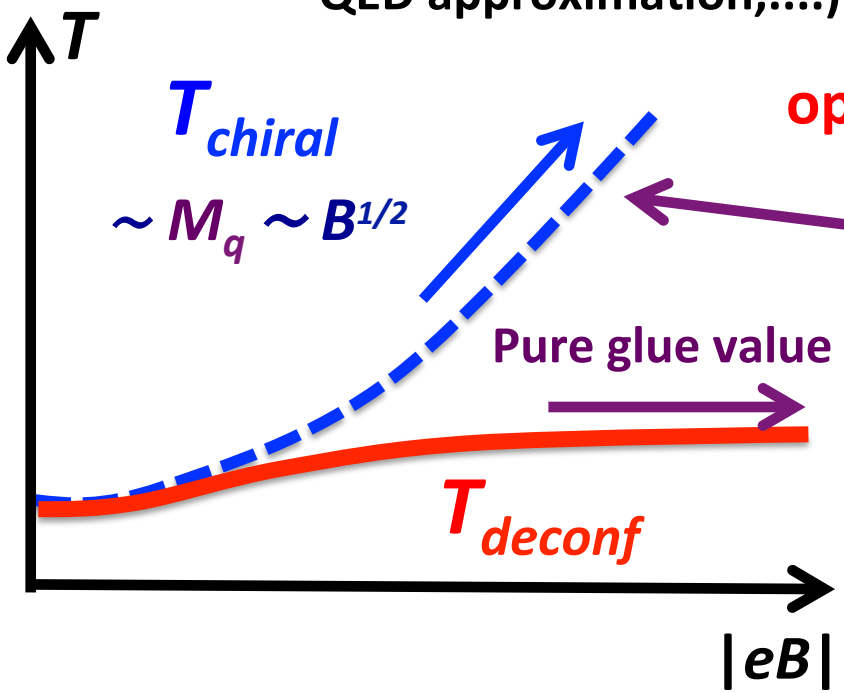
# Models vs Lattice, 2: $T_c(B)$

Models (PNJL, PQM,
QED approximation,....)

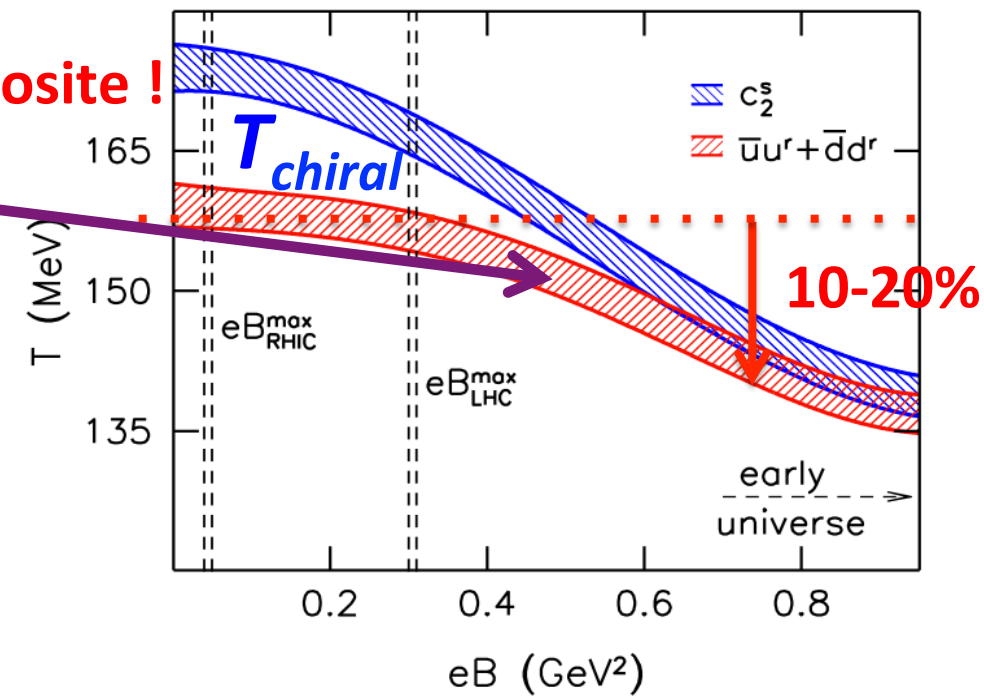


# Models vs Lattice, 2: $T_c(B)$

**Models** (PNJL, PQM, QED approximation,....)




**Lattice (full), phys. pion mass**
(Bali et al, 2012)



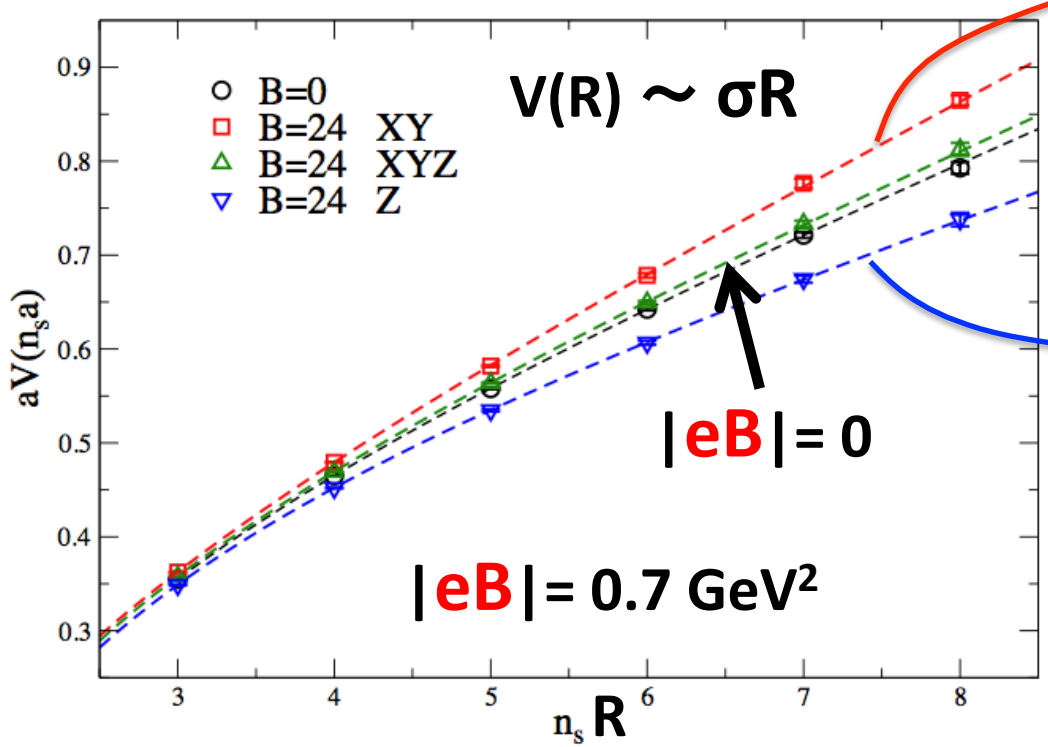

***"Inverse"*** **mag. catalysis**
( $T_{deconf}$ also decreases)

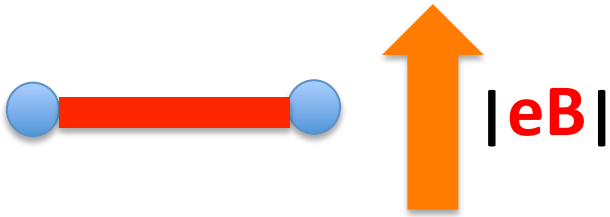
***Enhanced ChSB*, but *reduction of Tc***

# Models vs Lattice, 3: *String tension*

## *Heavy quark potential (T=0)*

Lattice, (2+1) phys. pion (Bonati et al, 2014)




σ → 10 % enhancement

σ → 10 % " reduction "

**hard** to explain *if* $M \sim |eB|^{1/2}$ (models)

( because *back-reaction* is suppressed )

# Field theory bases : quark part

*" Ritus bases for non-int. fermions in B "*

**1) Choose the gauge for EM fields : e.g.)** $A_2^{\rm em} = Bx_1$

**2) Apply "*spin projection*" :**

**($\sigma_z$ : spin)**

$$\psi_\pm \equiv \mathcal{P}_\pm \psi \qquad \mathcal{P}_\pm = \frac{1 \pm \mathrm{i}\gamma_1\gamma_2\,\mathrm{sgn}(e_f B)}{2}$$

**3) Expand by proper spatial wavefunctions :**

$$\psi_\pm(x) = \sum_{l=0} \int \frac{\mathrm{d}^2 p_L \mathrm{d}p_2}{(2\pi)^3}\, \psi^\pm_{l,p_2}(p_L)\, \underline{H_l\left(x_1 - \frac{p_2}{B}\right)}\, \mathrm{e}^{-\mathrm{i}p_2 x_2}\, \mathrm{e}^{-\mathrm{i}p_L x_L}$$

$$p_L \equiv (p_0, p_z)$$

***Harmonic oscillator w.f. with***

$$m\omega = |eB|$$

# Field theory bases : quark part

**The action for the LLL (n=0):** $\chi = \psi_+^{l=0}$

$$\mathcal{S}_{\rm LLL} = \int_{p_L,p_2} \bar{\chi}_{p_2}(p_L)\,(-{\rm i}\not{p}_L + m)\,\chi_{p_2}(p_L)$$

**( No B-dep. ! )**

**for the n-th LLs :** $\psi_n = \psi_+^{l=n} + \psi_-^{l=n-1}$

$$\mathcal{S}_{\rm nLL} = \int_{p_L,p_2} \bar{\psi}_{n,p_2}(p_L)\left(-{\rm i}\not{p}_L + {\rm i}\,{\rm sgn}(eB)\sqrt{2n|eB|}\gamma_2 + m\right)\psi_{n,p_2}(p_L)$$

**The propagators :**

diagonal

$$\langle \psi_{n,p_2}(p_L)\bar{\psi}_{n',p_2'}(p_L')\rangle = S_n^{\rm 2D}(p_L) \times \delta_{nn'}\delta(p_2 - p_2')\delta^2(p_L - p_L')$$

**(1+1)-dimensional for each index "n"**

**( *depend only on* $p_L$ )**

# Important formula

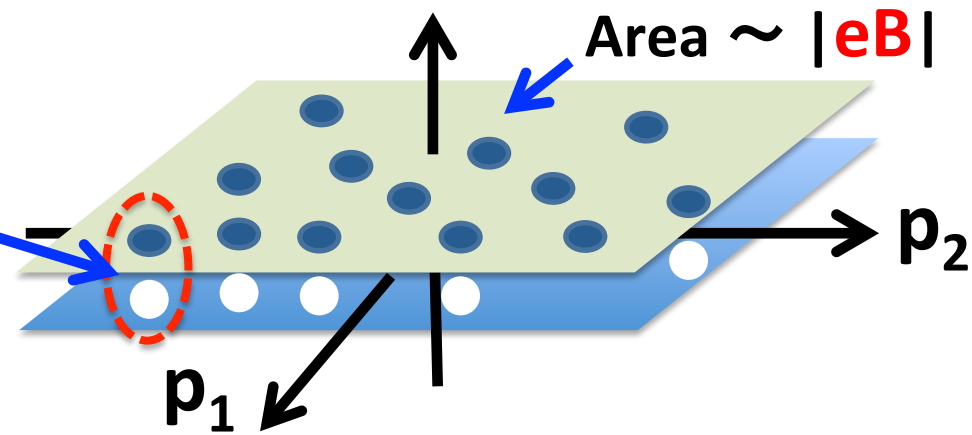
$$\langle\bar{\psi}(x)\psi(x)\rangle_{\underline{4\mathrm{D}}} = -\frac{|eB|}{2\pi}\int_{p_L}\mathrm{tr}\left(S_\chi^{2\mathrm{D}}(p_L)+\sum_{n=1}S_n^{2\mathrm{D}}(p_L)\right)$$

$$\langle\bar{\psi}(x)\psi(x)\rangle_{4\mathrm{D}} = \underline{\frac{|eB|}{2\pi}}\,\langle\bar{\psi}(x_L)\psi(x_L)\rangle_{\underline{2\mathrm{D}}}$$

(*holds even after self-energies are included*)

**Intuitively,**

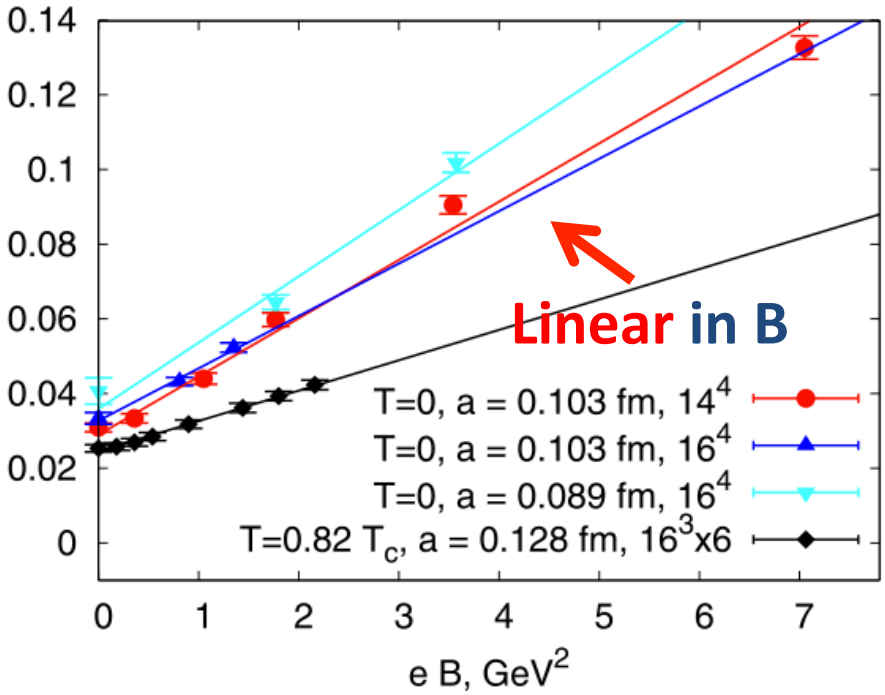
$\langle\bar{\psi}\psi\rangle_{\underline{2\mathrm{D}}}$ =



# Chiral condensate on the lattice

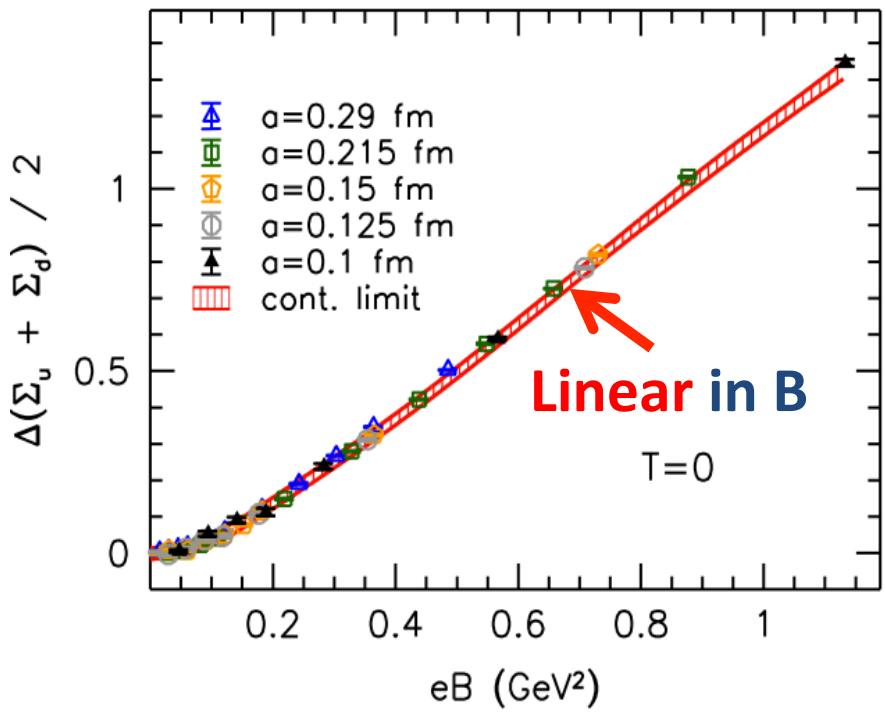
## Quenched $SU_c(2)$

Buividovich et al, 2010




## Full $SU_c(3)$

Bali et al, 2011, 2012




( B-indep. )

$$\langle\bar{\psi}\psi\rangle_{4D} = \frac{|eB|}{2\pi}\langle\bar{\psi}\psi\rangle_{2D} \Rightarrow \boxed{\langle\bar{\psi}\psi\rangle_{2D} \sim c_0\Lambda_{QCD} + \cdots}$$

# Problems in most of models...

**( The NJL, QED-like treatments, Sakai-Sugimoto models,.... )**

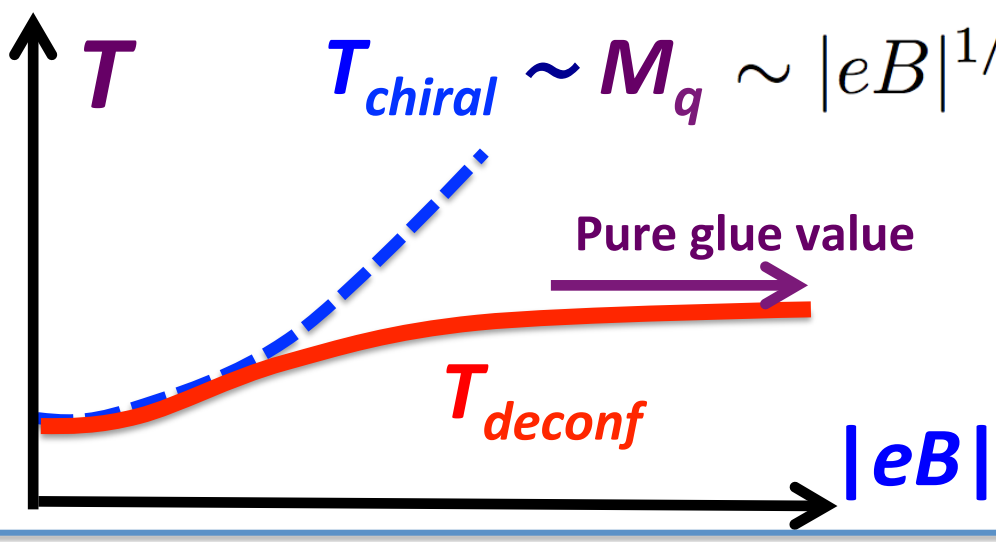
## Problem 1) *B-dep. of the chiral condensate*

$$M_q \sim |eB|^{1/2} \Rightarrow \langle \bar{\psi}\psi \rangle_{2\mathrm{D}} \sim |eB|^{1/2}$$

$$\langle \bar{\psi}\psi \rangle_{4\mathrm{D}} \sim |eB|^{3/2}$$ ≠ **lattice data**

## Problem 2) *" B-T " phase diagram*

$$T_{chiral} \sim M_q \sim |eB|^{1/2}$$




≠ **lattice data**

# Claim 1

***Within*** **the domain of B** ***studied on the lattice*****,**

**the quark mass gap should be :** $M_q \sim \Lambda_{\rm QCD}$

**If so,**

$$T_{\rm chiral}(B) \sim M \sim \Lambda_{\rm QCD}$$

**( instead of** $\sim |eB|^{1/2}$ **)**

**Then we have a better chance to explain**

**reduction of $T_c$ & B-dep. of gluonic quantities**

**( Fluctuations effects or back-reaction from quark to gluon sectors are operative )**

# 2, Quenched QCD in strong mag. fields

We *separate* issues of *fluctuations* such as

back reaction from quark to gluon sector,

mesonic fluctuations,

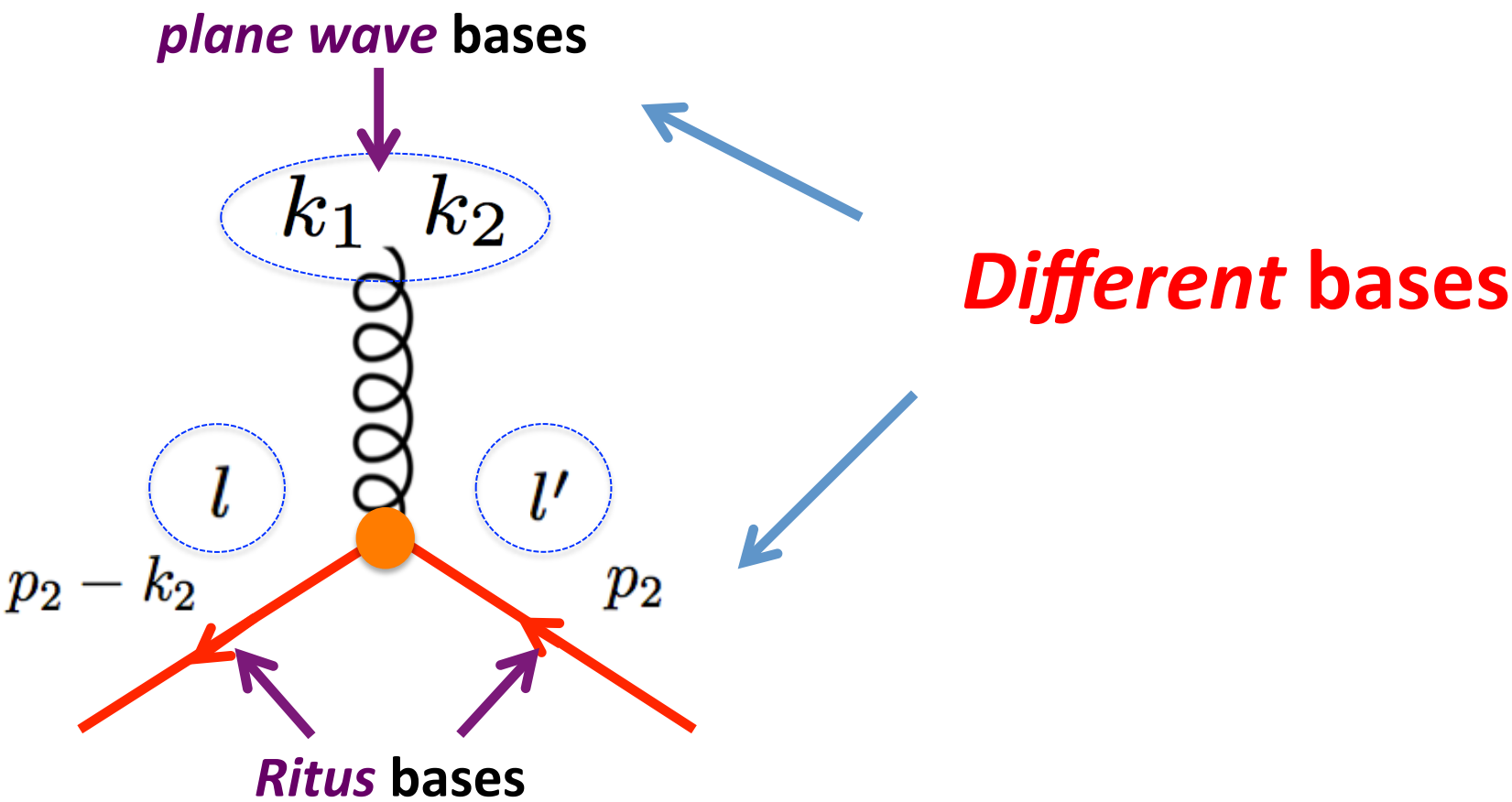
etc., etc.,.....

Unrealistic, but discussions become much clearer

# Couplings b.t.w. different LLs

$$S_{\rm int} = \int_x \bar{\psi}(x)\gamma_\mu t_a \psi(x)\, A^a_\mu(x)$$

**4D Gluons couple to different LLs.**



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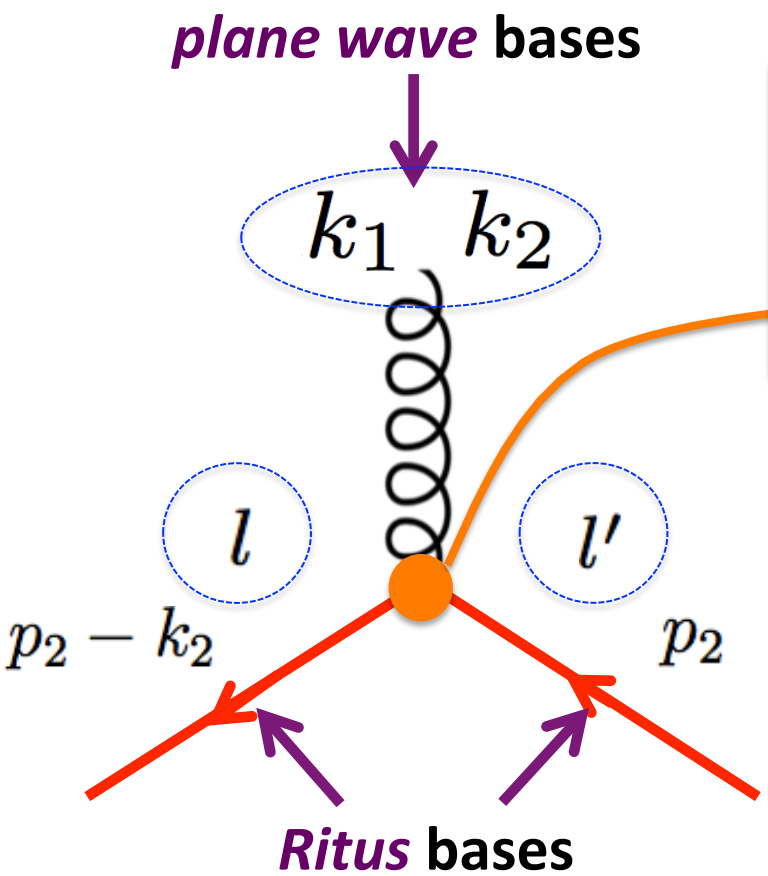



" **form factor** "

$$\Delta l = |l - l'|$$

$$I_{l,l'}(\vec{k}_\perp) \propto \left(\frac{k_\perp^2}{2|eB|}\right)^{\frac{\Delta l}{2}} \mathrm{e}^{-k_\perp^2/4|eB|}$$

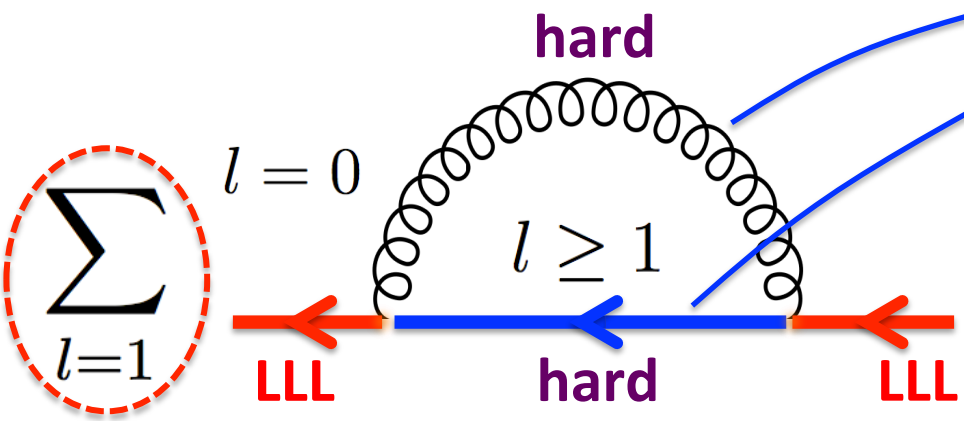
**For $\Delta l \neq 0$ processes :**

**small overlap with soft gluons**

**( Only $\Delta l = 0$ process are dangerous )**

# LLL mass gap : 3-distinct contributions

## 1) Coupling with $l \geq 1$




**"Perturbative"**

under control for

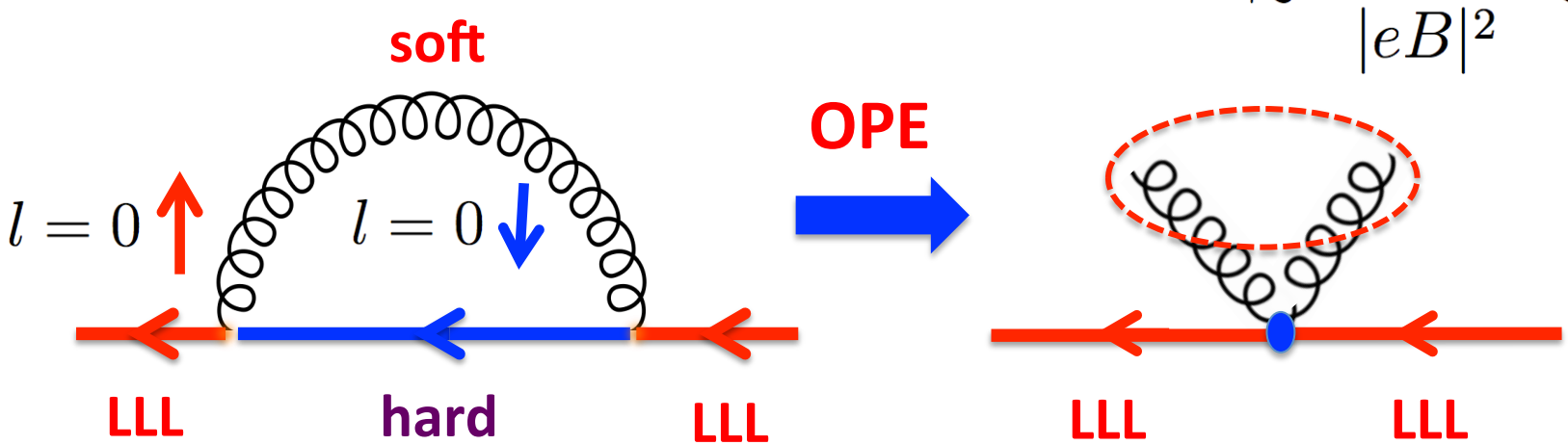
$$|eB| \geq (0.1 - 0.3)\,\mathrm{GeV}^2$$

& very small B-dep.

T.K., Nan Su (2013)

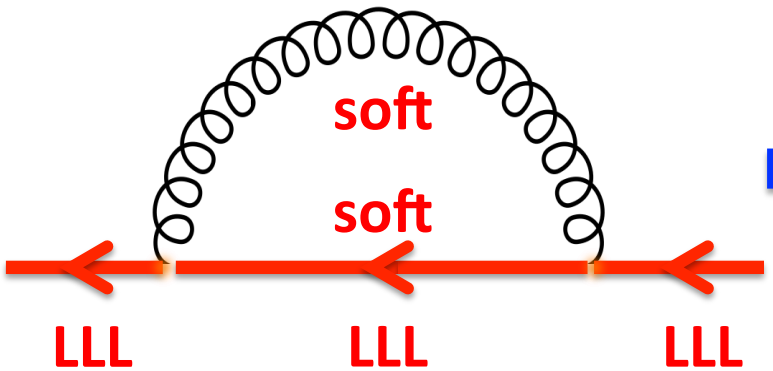
## 2) Coupling with 1st LL but $l = 0 \downarrow$

$$\sim \frac{m\langle G^2 \rangle}{|eB|^2} \ll \Lambda_{\mathrm{QCD}}$$



# LLL mass gap : 3-distinct contributions

## 3) Couplings within LLLs




Everything must be treated

"Non-perturbatively"

Natural framework → Schwinger-Dyson eq.

with

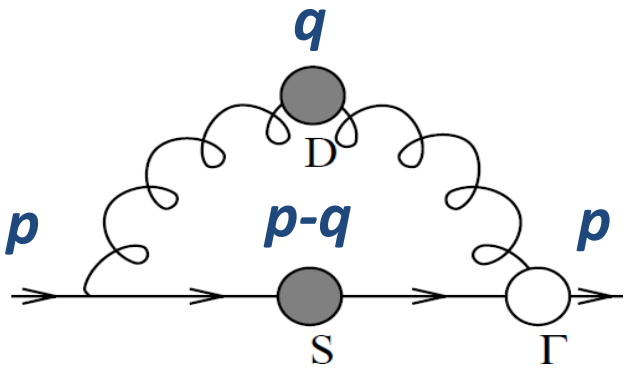
Non-perturbative " *force* "

e.g.) full gluon propagator × full vertex for quenched QCD

# Structure of the Schwinger-Dyson eq.

1) No explicit B-dep. for the LLL

2) No $p_T$-dep. → " factorization "



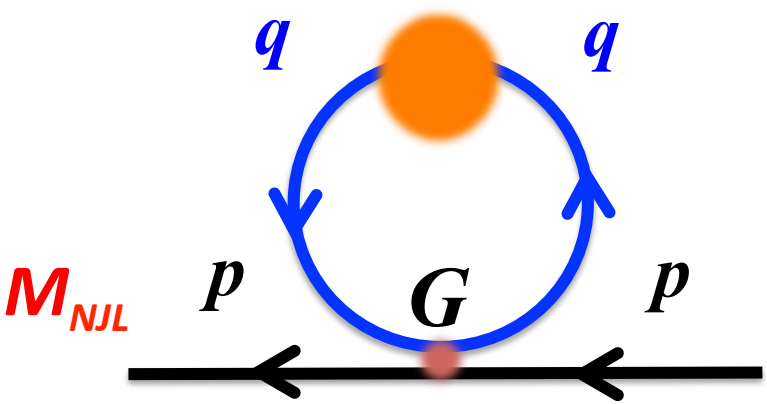

$$M(p_L) \sim \int_{q_L} S^{2D}_{LLL}(p_L - q_L; M) \int_{q_\perp} e^{-\frac{q_\perp^2}{2|eB|}} D^{4D}_{NP}(q_L, q_\perp)$$

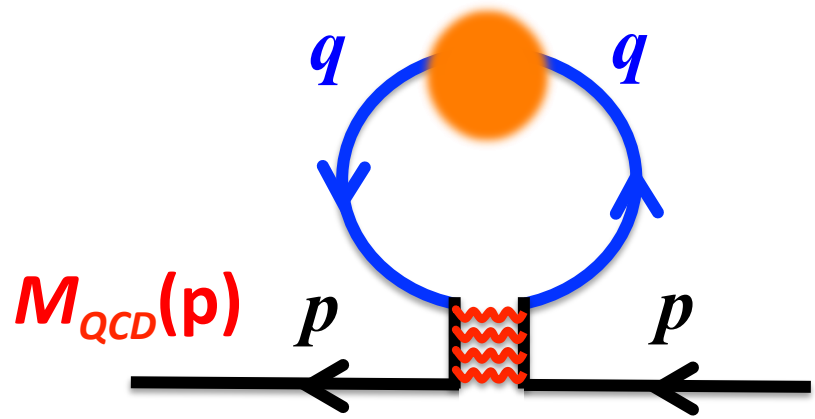
Form factor
for "ΔL = 0 process"

2D " *smeared* " force

(origin of B-dep.)

# Self-energy : NJL, QM models vs QCD

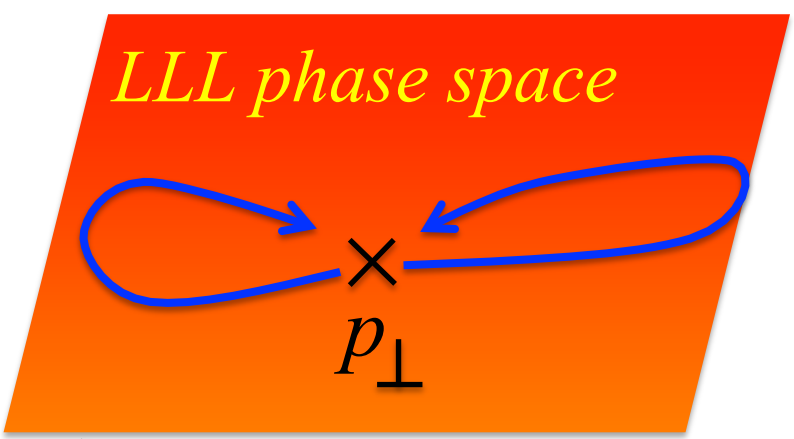


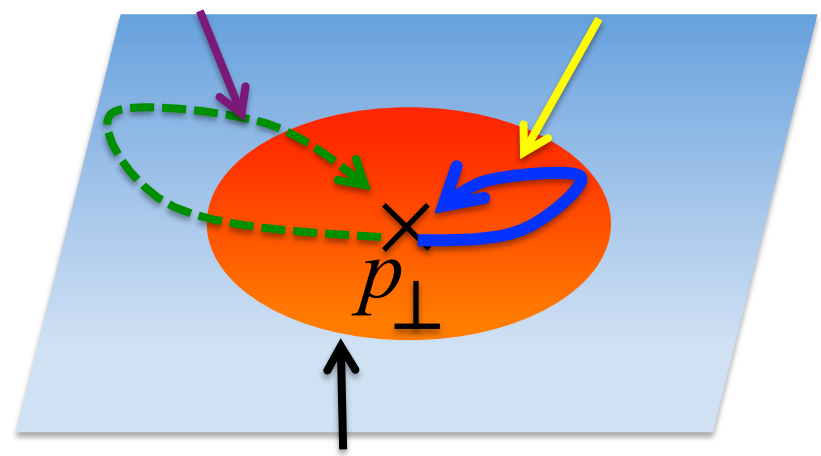




all the self-int. have equal strength $G$

weak coupling

strong coupling




area ~ $|eB|$

area ~ $\Lambda_{QCD}^2 << |eB|$

# Comparison of forces, 1

$$\int_{q_\perp} \underline{e^{-\frac{q_\perp^2}{2|eB|}}}\, D_{\rm NP}^{\rm 4D}(q) \sim \int_0^{\underline{\sim|eB|}} dq_\perp^2\; D_{\rm NP}^{\rm 4D}(q)$$

origin of B-dep.

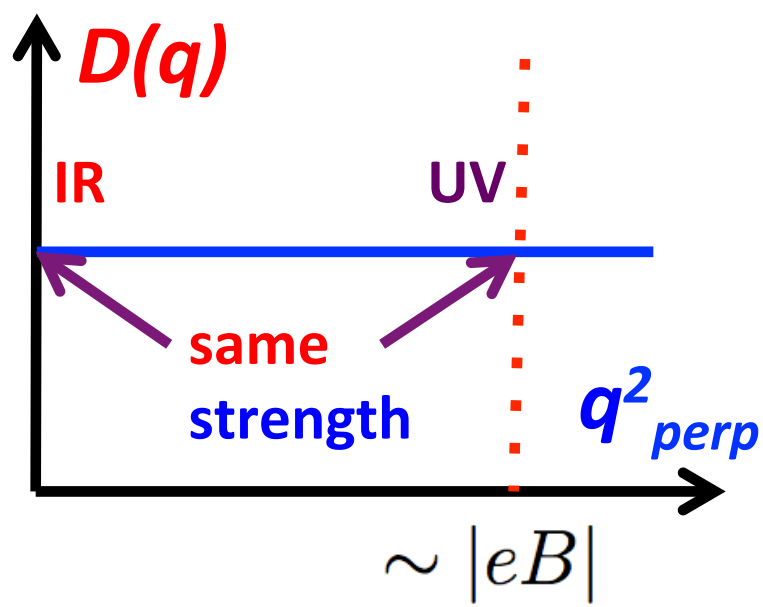
## 1) Contact interactions (NJL, etc.)

$$\sim \int_0^{\sim|eB|} dq_\perp^2 \text{ const.}$$

$$\Rightarrow \boxed{\sim \underline{|eB|} \times \text{const.}}$$

2D Force is strongly B-dep.

$$M \sim |eB|^{1/2}$$



# Comparison of forces, 2

$$\int_{q_\perp} \underline{e^{-\frac{q_\perp^2}{2|eB|}}}\, D_{\rm NP}^{\rm 4D}(q) \sim \int_0^{\underline{\sim|eB|}} dq_\perp^2\; D_{\rm NP}^{\rm 4D}(q)$$

origin of B-dep.

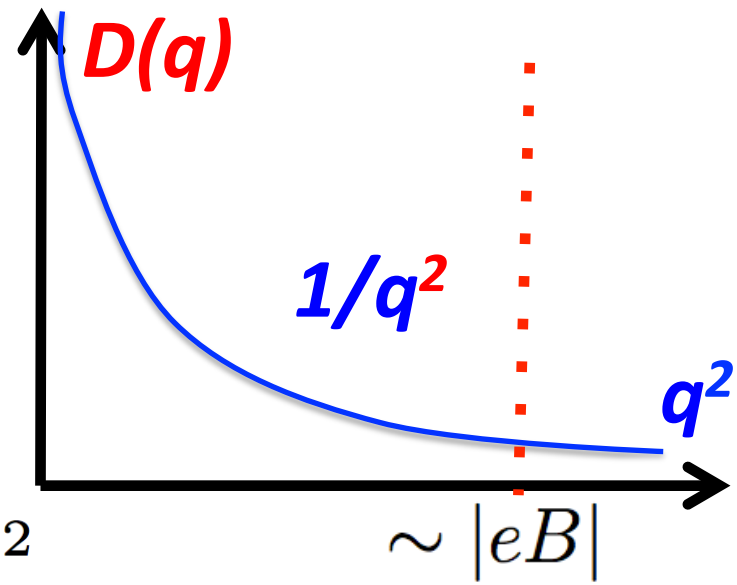
---

## 2) QED case ( $1/q^2$ force : *running is negligible* )

$$\sim \int_0^{\sim|eB|} dq_\perp^2\; \frac{1}{q_\perp^2 + q_L^2}$$

$$\Rightarrow \boxed{\sim \ln \frac{q_L^2}{\underline{|eB|}}}$$

2D Force is still marginally B-dep.



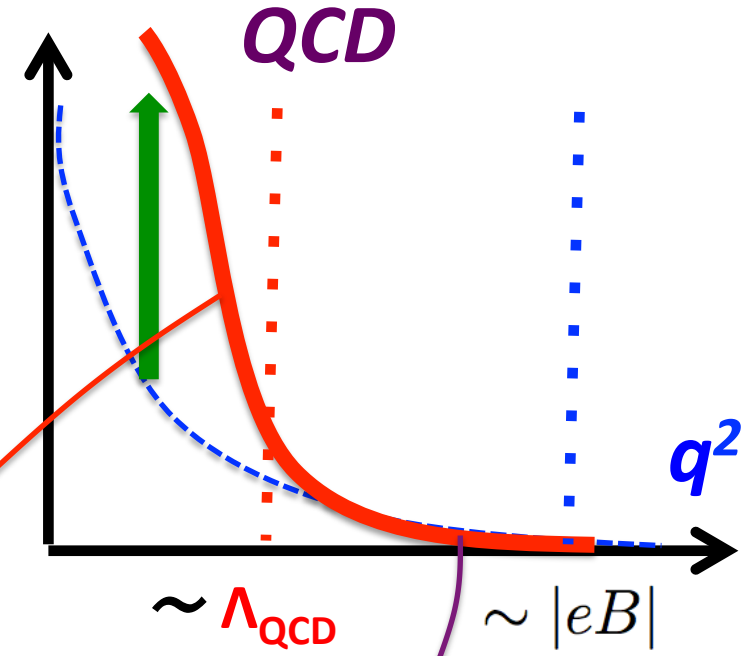

$$M \sim |eB|^{1/2}\, \underline{e^{-O(1)/\alpha^{1/2}}}$$

(exponentially small)

# Comparison of forces, 3

**Now suppose: QCD force has strong "*IR enhancement*"**

$$\int_{q_\perp} e^{-\frac{q_\perp^2}{2|eB|}} D^{4D}(q_L, q_\perp)$$




**For small $q_{perp}$ $\sim \Lambda_{QCD}$ :**

**we can set :** $e^{-\frac{q_\perp^2}{2|eB|}} \sim 1$

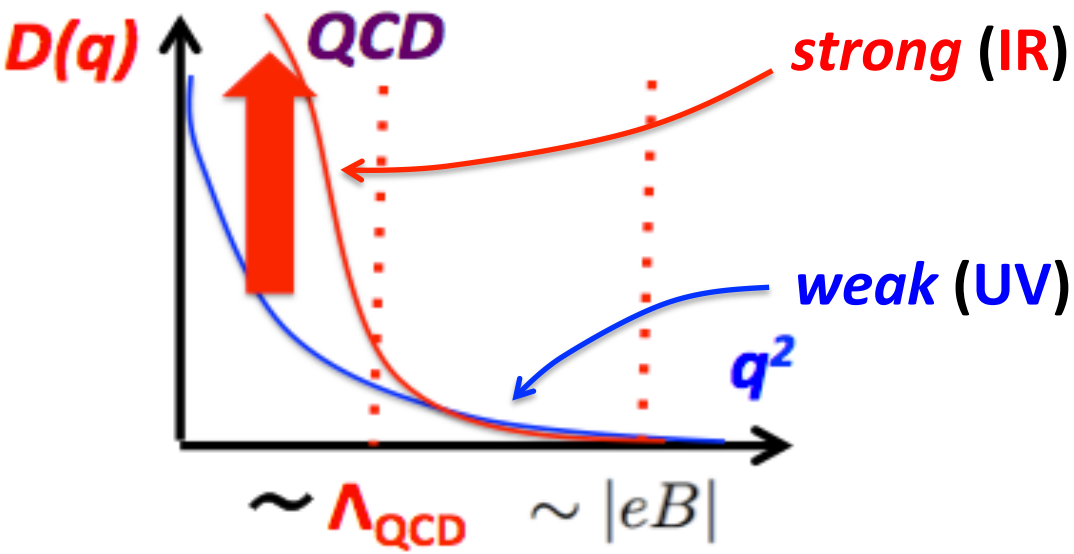
$$\sim \int_0^{\sim\Lambda_{QCD}^2} dq_\perp^2 D^{4D}(q_L, q_\perp)$$ + ***small B-dep. corrections***

**The dominant part :**
***"nearly B-indep."***

$$M \sim \Lambda_{QCD}$$

# Claim 2

**More and more "*IR enhancement*" *of forces*, then less and less *B-dep.* of *the quark mass gap*.**




$$M \sim \Lambda_{\rm QCD}$$

**The key is "*contrast*" between *IR* and *UV* behaviors.**

# Example) a *toy* model study

## " *Linear rising* " *potential* for color charges

$$D_{\mu\nu} = C_F \times g_{\mu 0} g_{\nu 0} \times \frac{\sigma}{(\vec{p}^2)^2}$$

*string tension*

- **Motivated by Coulomb gauge studies.**

  (ref: Gribov, Zwanziger)

- **The model has " *IR enhancement* ".**

- ***Confining*, in the sense that**

  **"No $q\bar{q}$ continuum in the meson spectra."**

- ***Oversimplifications* : *No $1/p^2$ tail, No color mag. int.*, etc.**

- ***We will solve eqs. within " rainbow ladder "***

# Schwinger-Dyson eq. for the LLL

e.g.) scalar part

$$M(p_L) = \int_{q_L} \gamma_0 \, S^{2\mathrm{D}}_{\mathrm{LLL}}(p_L - q_L; M) \, \gamma_0 \otimes \boxed{\int_{q_\perp} \mathrm{e}^{-\frac{q_\perp^2}{2|eB|}} \, D^{4\mathrm{D}}_{00}(q)}$$

for large B

$$\int_0^\infty \mathrm{d}q_\perp^2 \, \frac{\sigma \, \mathrm{e}^{-\frac{q_\perp^2}{2|eB|}}}{(q_\perp^2 + q_z^2)^2} \quad \Rightarrow \quad \frac{\sigma}{q_z^2} - \cancel{\frac{\sigma}{q_z^2 + \underline{2|eB|}}}$$

(confining in 2D)

The *B-dependence* dropped out, and we get

$$M(p_L) \simeq \int_{q_L} \gamma_0 \, S^{2\mathrm{D}}_{\mathrm{LLL}}(p_L - q_L; M) \, \gamma_0 \times \frac{\sigma}{q_z^2}$$

SD-eq. for *'t Hooft model* ($QCD_2$) in $A_z = 0$ gauge

(the *Bethe-Salpeter* eq. can be also reduced to $QCD_2$)

# *Few comments on unquenched QCD*

## *Now imagine back-reaction from quark to gluon sector :*

**"Thermal" quark fluctuations)**

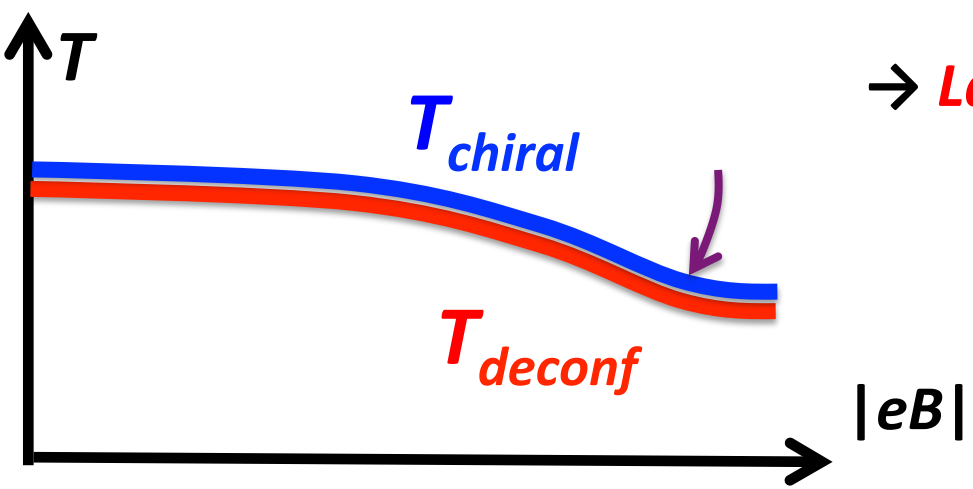
phase space — Boltzmann factor

**B = 0 )** $$\sim \int \mathrm{d}^3\vec{p}\ \mathrm{e}^{-\frac{E}{T}} \sim \underline{\Lambda_{\mathrm{QCD}}^3} \times \mathrm{e}^{-\frac{\Lambda_{\mathrm{QCD}}}{T}}$$

**enhanced** ↓ **No big change** ↓

**B ≠ 0 )** $$\sim \underline{|eB|\,\Lambda_{\mathrm{QCD}}} \times \mathrm{e}^{-\frac{\Lambda_{\mathrm{QCD}}}{T}}$$

→ ***Larger back-reaction*** **at larger B**



# *Summary*

1) Magnetized QCD is a good "laboratory".

2) To explain data for chiral condensate & $T_c(B)$,

   $M_q(B)$ should be $\sim \Lambda_{QCD}$, instead of $\sim |eB|^{1/2}$ .

3) IR enhancement of QCD forces was the key.

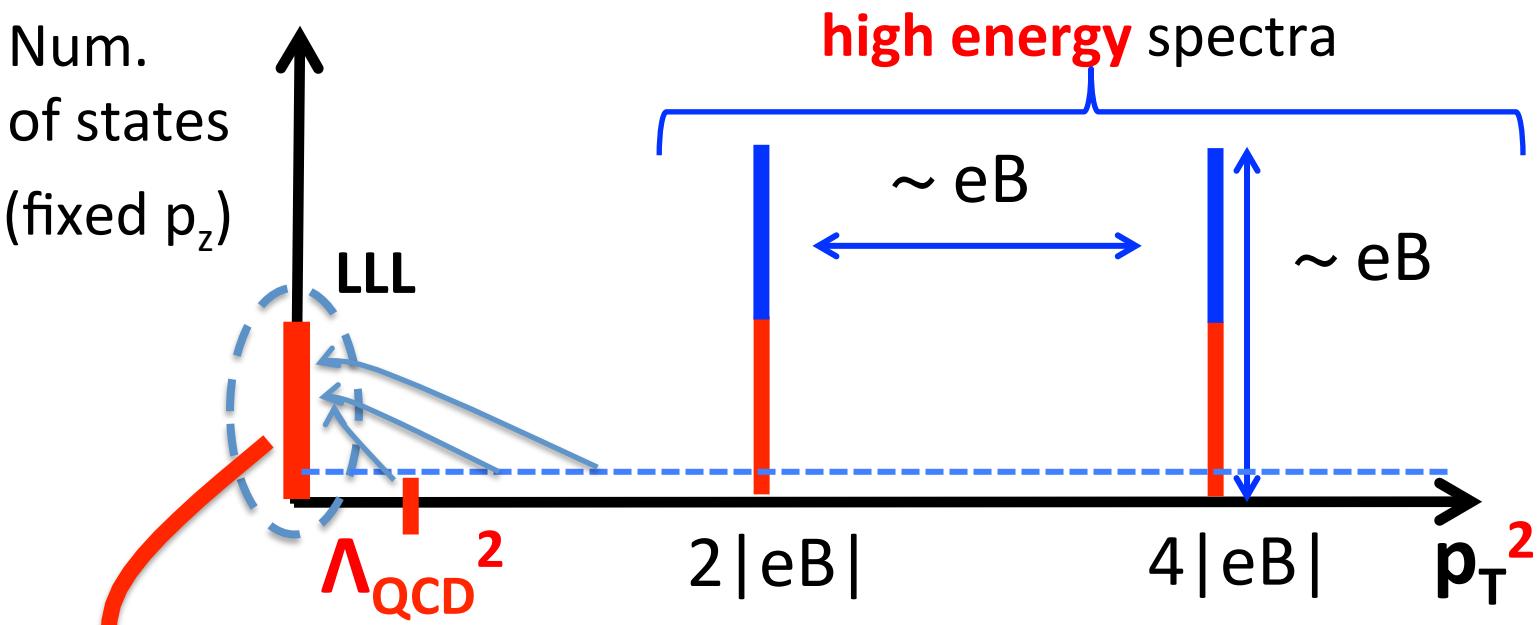
4) With $M_q(B) \sim \Lambda_{QCD}$ , fluctuations are now operative.

(FRG with $M_q(B) \sim |eB|^{1/2}$ does not explain the inverse mag. catalysis)

( see, Kamikado-Kanazawa '14 )

# Backup

# The IR phase space for quarks

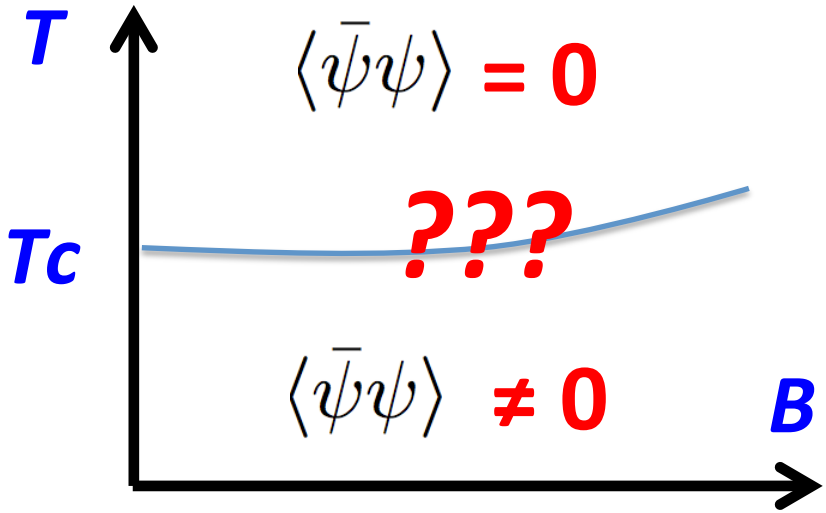



***More quarks can stay at low energy.***

***Size* of *IR phase space of quarks* can be *controlled* by *B*.**

# Magnetized QCD : Basics, 8

## *The QCD phase diagram in (B-T) planes.*



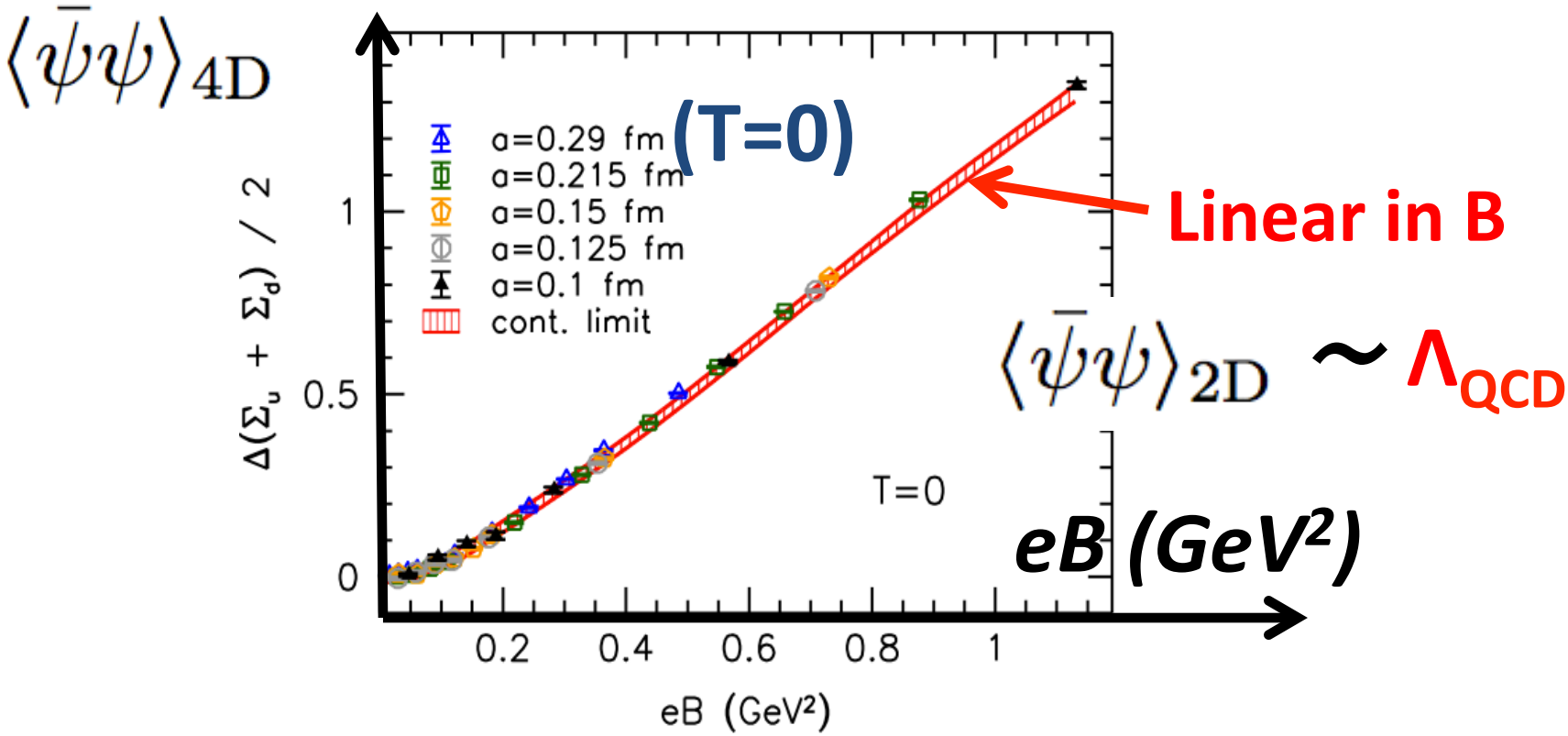

***Tc* : *Dissociation temperature of condensate.***

$$\langle\bar{\psi}\psi(x)\rangle_B^{4\mathrm{D}} \sim \underline{|eB|} \times \underline{\langle\bar{\psi}\psi(t,z)\rangle_B^{2\mathrm{D}}}$$

***Tc* is determined by dissociation of** $\langle\bar{\psi}\psi\rangle_{2\mathrm{D}}$

# On *Magnetic Catalysis*

***Lattice simulations indeed confirmed.***




***However, its B-dep. is different from theories***

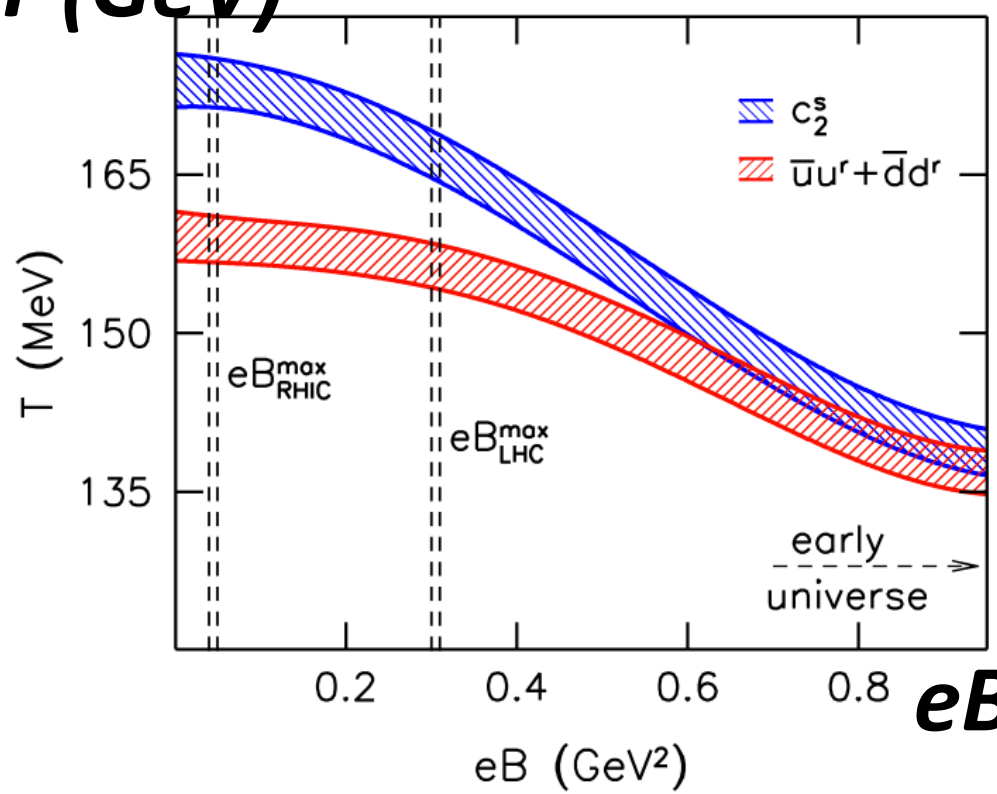
# On *B-T phase diagram*

***Theories:*** ***At larger B,***

***Dissociation of Chiral condensates***

***& De-confinement happen at larger Tc***

***T (GeV)***




***eB (GeV²)***

***However***

***B ↑ → Tc ↓***

**Opposite !!!**

# Origins of contradictions ?

***Most* Theories predict :** $\langle\bar{\psi}\psi\rangle_{2D} \sim |eB|^{1/2}$

***(NJL model or QED like calculations)***

***Then Tc behaves like :*** $Tc \sim |eB|^{1/2}$

***( $|eB|^{1/2}$ condensate does not easily dissociate)***

---

***Instead we need :*** $\langle\bar{\psi}\psi\rangle_{2D} \sim \Lambda_{QCD}$

***Then Tc behaves like :*** $Tc \sim \Lambda_{QCD}$

***( We have better chance to explain reduction of Tc )***

# *Competing* effects for larger IR phase space

$B \uparrow$

**Magnetic Catalysis for ChSB**

(Review: Shovkovy 12)

**Larger Screening effects for gluons**

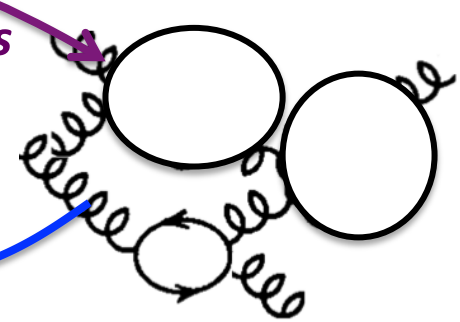
(Miransky-Shovkovy 02)

$\langle \bar{\psi}\psi \rangle \uparrow$

$M_q \uparrow$

suppress

suppress



$T_c \uparrow$

$T_c \downarrow$

(larger gap → approach pure glue results)

(due to reduction of non-pert. force)

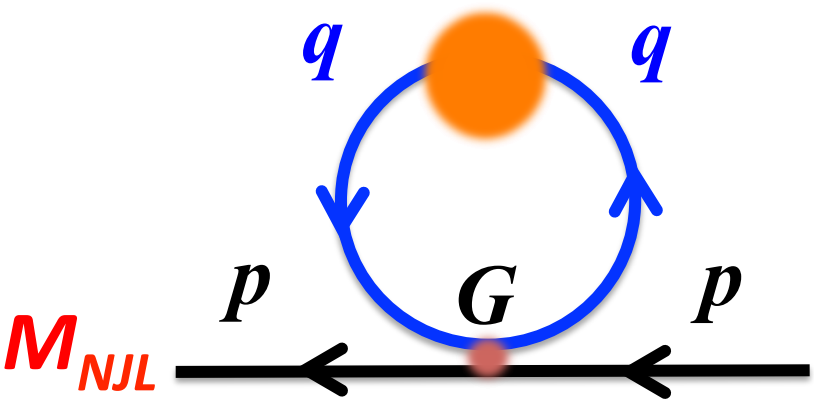
# Gap eq. (NJL case, T=0)

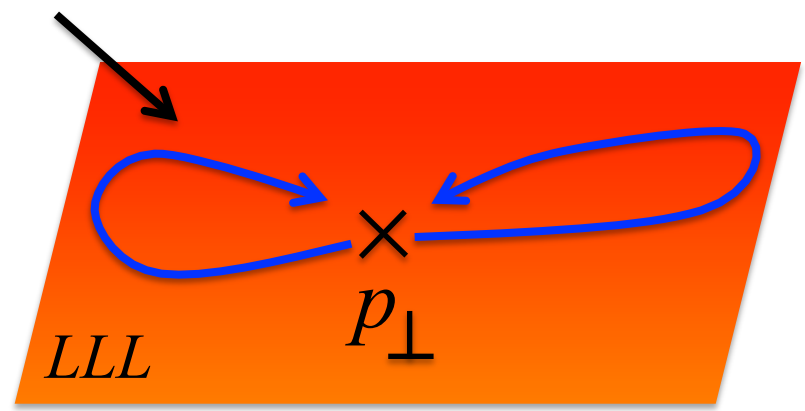
*Gap eq.*

$$M_{\rm NJL}(B) = G\,{\rm tr}\,S(x,x) \rightarrow G\frac{|eB|}{2\pi}\int\frac{{\rm d}^2q_L}{(2\pi)^2}\,{\rm tr}\,S_{\rm 2D}(q_L)$$

area ~ $|eB|$

*2D quark propagator*



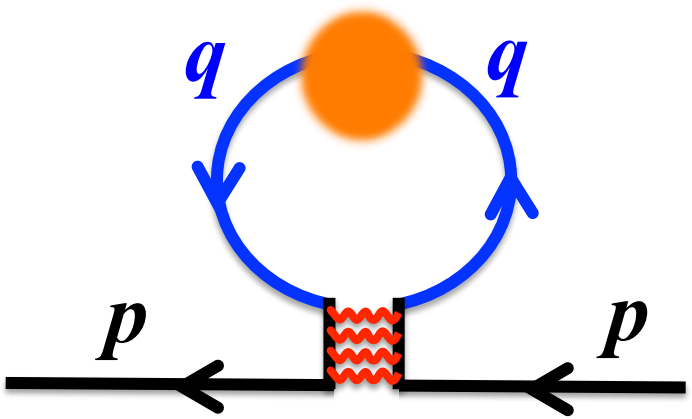

***Problem :*** **all the self-interactions have equal strength *G*.**

$$\langle\bar\psi\psi\rangle^B_{\rm NJL} \simeq -\frac{1}{G}M_{\rm NJL}(B)$$

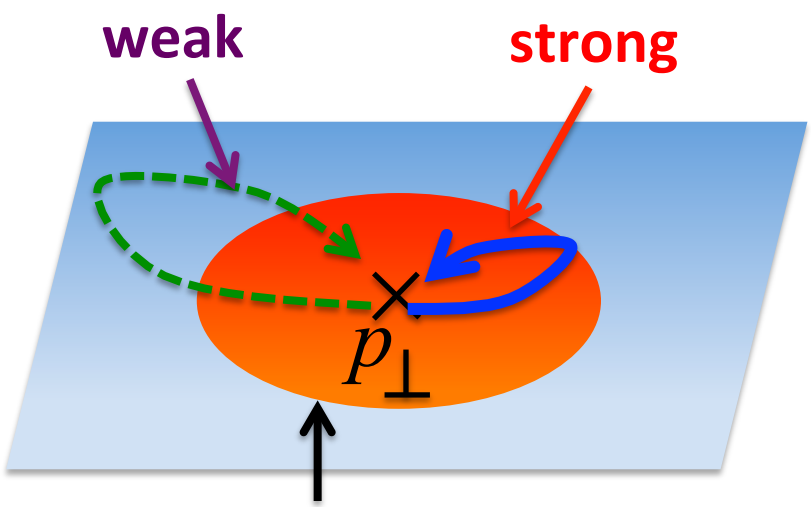
***same B-dep.***

# Gap eq. (QCD, T=0)

**Key features :** **strong in IR & weak in UV**




***M*(p)**




area ~ $\Lambda_{QCD}^2 \ll |eB|$

**The Gap eq. does NOT pick up the factor |eB| .**

(modulo weak coupling corrections)

***The Gap is solely determined by the scale $\Lambda_{QCD}$ .***

# Condensate (QCD, T=0)

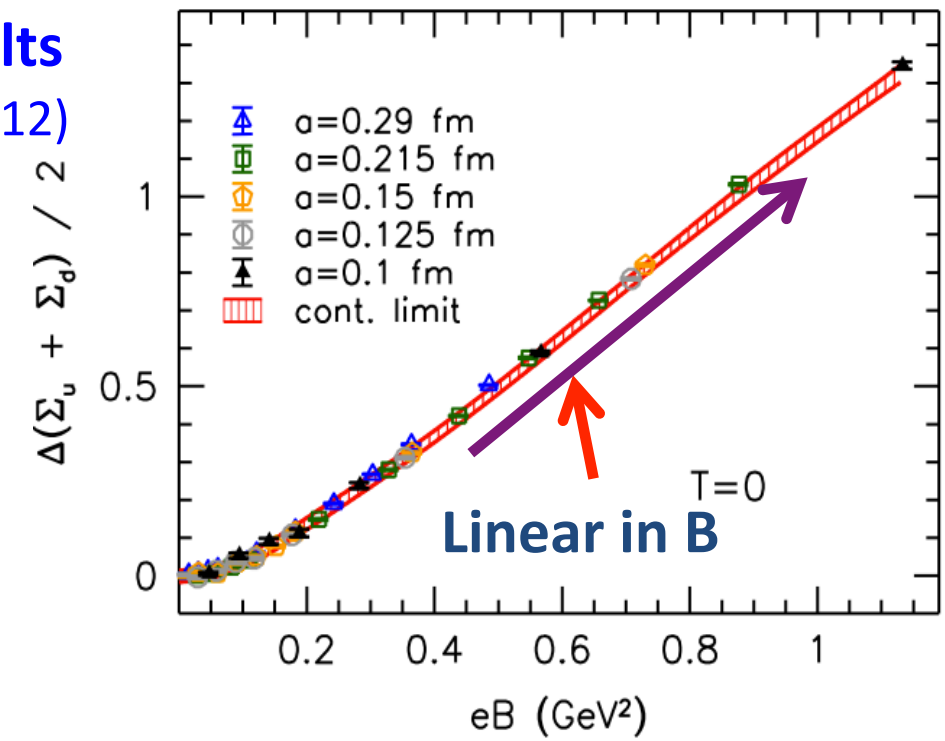
$$\langle\bar{\psi}\psi\rangle_{4\mathrm{D}} = \mathrm{tr}\, S(x,x) \rightarrow \frac{|eB|}{2\pi}\int \frac{\mathrm{d}^2 q_L}{(2\pi)^2}\, \mathrm{tr}\, S_{2\mathrm{D}}(q_L)$$

$$\Rightarrow \langle\bar{\psi}\psi\rangle_{4\mathrm{D}} \sim |eB| \times \langle\bar{\psi}\psi\rangle_{2\mathrm{D}}$$

$\sim \Lambda_{QCD}$

**Lattice results**

(Bali et al. 11,12)



# *Bethe-Salpeter eq. for the LLLs*

Consider ***meson currents*** for which
***both*** quark & anti-quark can couple to the ***LLL states***.

(Some currents **CAN NOT**, see next slide.)

*Dim. reduction can be carried out in the same way :*

*B**oth total** & **relative** momenta are **indep. of trans. momenta**.*

- *Quark & anti-quark **align** in the z-direction.*

# *Classifications of Mesons* (2-flavor case)

Expanding quark fields by the Landau levels: $\psi^f = \psi^f_{\rm LLL} + \sum_{n=1} \psi^f_n$

we can pick out currents $\bar{\psi}\Gamma\psi$ for which

**both quark & anti-quark** can decay to the **LLL.**

## *List of light mesons:*

***neutral*** $(u\bar{u}\,, d\bar{d}) \otimes (1\,, \gamma_5\,, \gamma_L\,, \gamma_L\gamma_5\,, \sigma_{LL'}\,, \sigma_{\perp\perp'})$

***charged*** $(u\bar{d}\,, d\bar{u}) \otimes (\gamma_\perp\,, \gamma_\perp\gamma_5\,, \sigma_{L\perp})$

**e.g.)**

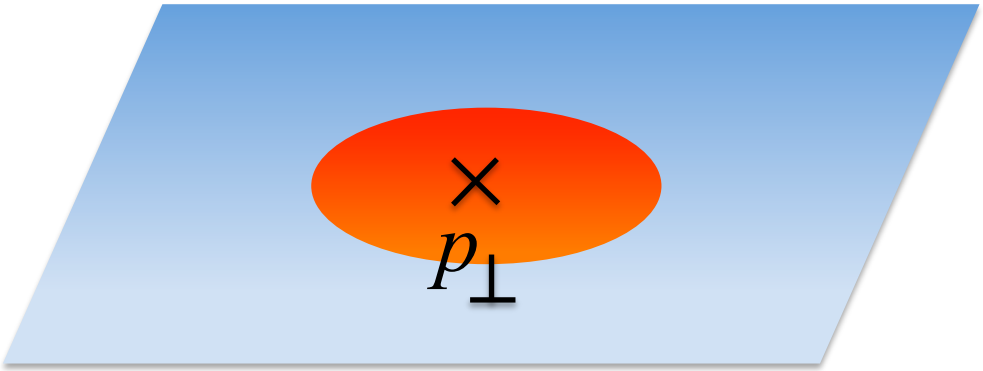
- **Neutral** pion (charged pions do NOT ).
- **Neutral**, **longitudinal** part of **vector** mesons.
- **Charged**, **transverse** part of **vector** mesons.

**.....**

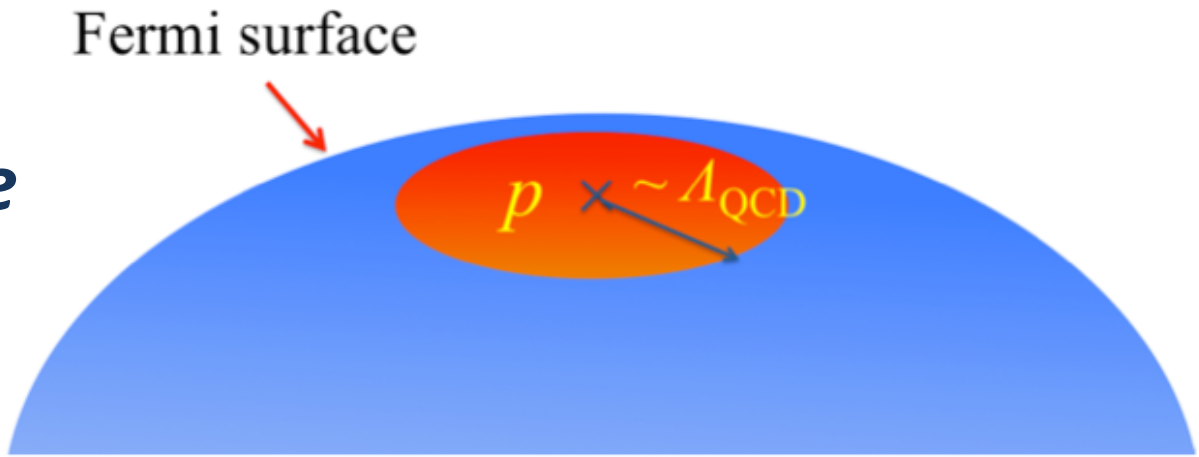
(***seems to be consistent with known lattice results.***)

# *Implications for dense QCD ?*

***Physics of the LLL***




***Physics near the Fermi surface***




**Similar modulo Fermi surface curvature**

# What's new? : History

## 1) *ChSB in mag. fields (concept) : 1989 -*

Klevansky-Lemmer (89), Suganuma-Tatsumi (90),

Gusynin-Miransky-Shovkovy (94-), .... ( for NJL, QED,... )

( *Not specific to QCD*, "*universal* aspects" of fermions at B )

## 2) *QCD in mag. fields (paradigm shift) : 2007 -*

Kharzeev-McLerran-Warringa (07), Fukushima-Kharzeev-Warringa (08),..

( *QCD topology & Its phenomenological applications* )

## 3) *Lattice studies on ChSB & Deconf. : 2008 -*

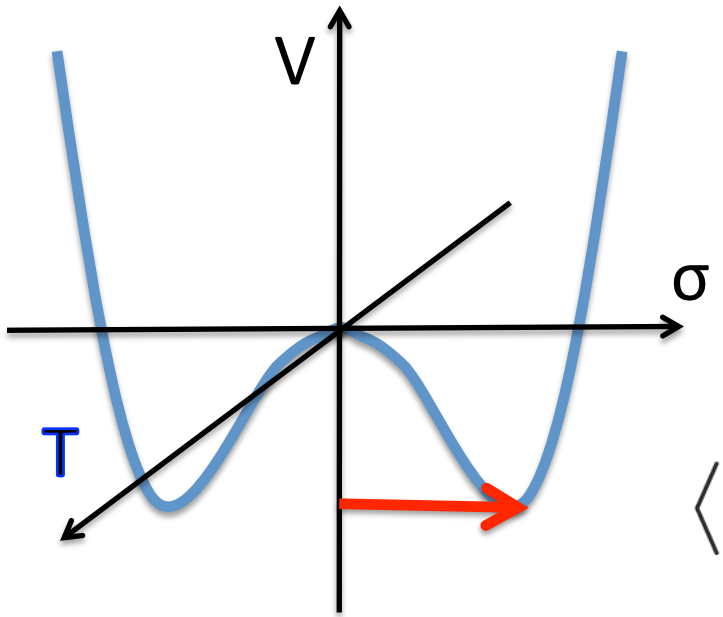
Buividovich et al. (2008) (quenched)

D'Elia-Muckherjee-Sanflippo (2010) (full, heavy pion)

Bali et al. (2012) (full, physical pion)

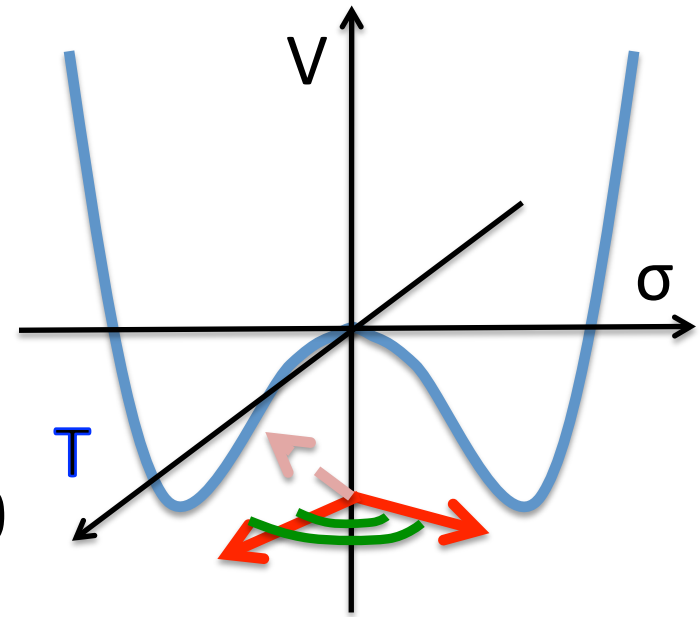
# Coleman's theorem ?

- Coleman's theorem: No Spontaneous sym. breaking in 2D




$\langle e^{i\theta} \rangle \neq 0$ (SSB)

$\langle \rho \rangle \neq 0$




$\langle e^{i\theta} \rangle = 0$ (No SSB)

IR divergence in (1+1)D phase dynamics

- Phase fluctuations belong to:

| Excitations (physical pion spectra) | ground state properties (No pion spectra) |
|---|---|

# Quasi-long range order & large Nc

- Local order parameters:

gapless modes  gapped modes

$$\langle \bar{\Psi}_+ \Psi_- \rangle \sim \langle e^{i\sqrt{4\pi/N_c N_f}\phi} \rangle \otimes \langle \mathrm{tr} g \rangle \otimes \langle \mathrm{tr} h \rangle$$

due to IR divergent phase dynamics → 0  0  finite

But this does not mean the system is in the usual symmetric phase!

- Non-Local order parameters:

$$\langle \bar{\Psi}_+ \Psi_-(x) \bar{\Psi}_- \Psi_+(0) \rangle \sim \begin{cases} e^{-m|x|} & \text{: symmetric phase} \\ \langle \bar{\Psi}_+ \Psi_- \rangle^2 & \text{: long range order} \\ |x|^{-C/N_c} & \text{: quasi-long range order} \end{cases}$$

(including disconnected pieces)

(power law)

# On the IR prescription

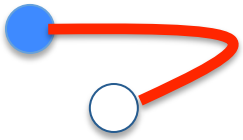
$$\frac{\sigma}{(\vec{k}^2)^2} \xrightarrow{\text{IR cut}} \frac{\sigma}{(\vec{k}^2+\Lambda_{\rm IR}^2)^2} \xrightarrow{\text{F.T.}} \underline{-\frac{\sigma}{\Lambda_{\rm IR}}} + \sigma r + O(\Lambda_{\rm IR} r^2)$$

linear potential

- Probe colored objects:

IR div.: const. from naïve IR cutoff

- Color singlet sector:



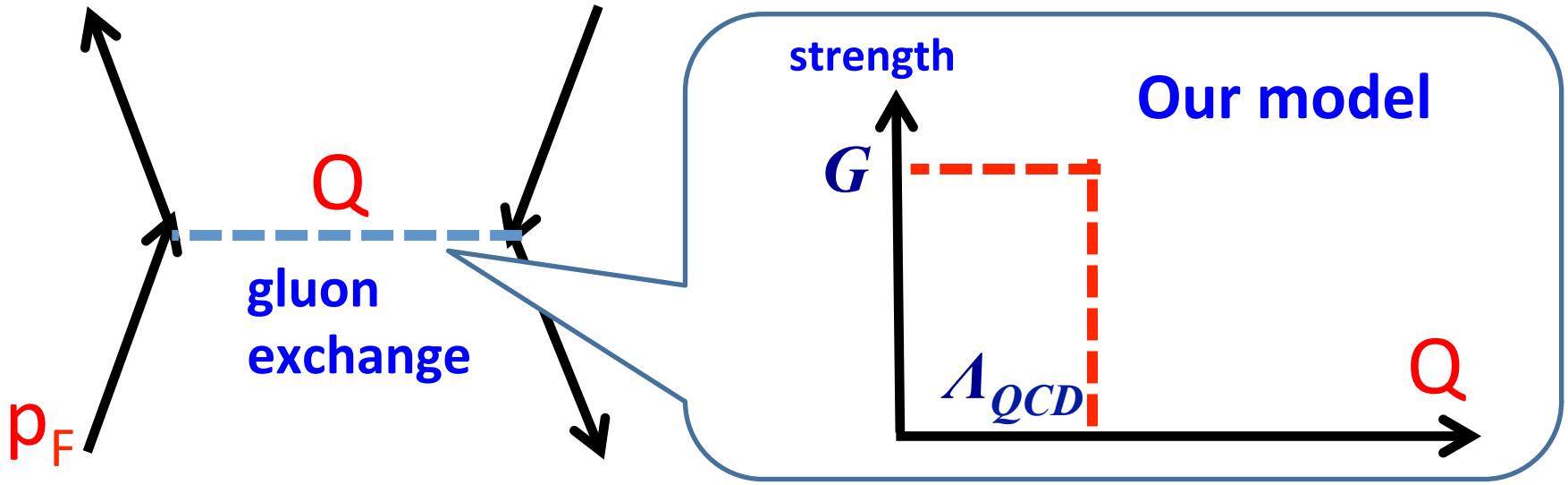
IR const. → irrelevant.
(Linear conf. without IR const.)

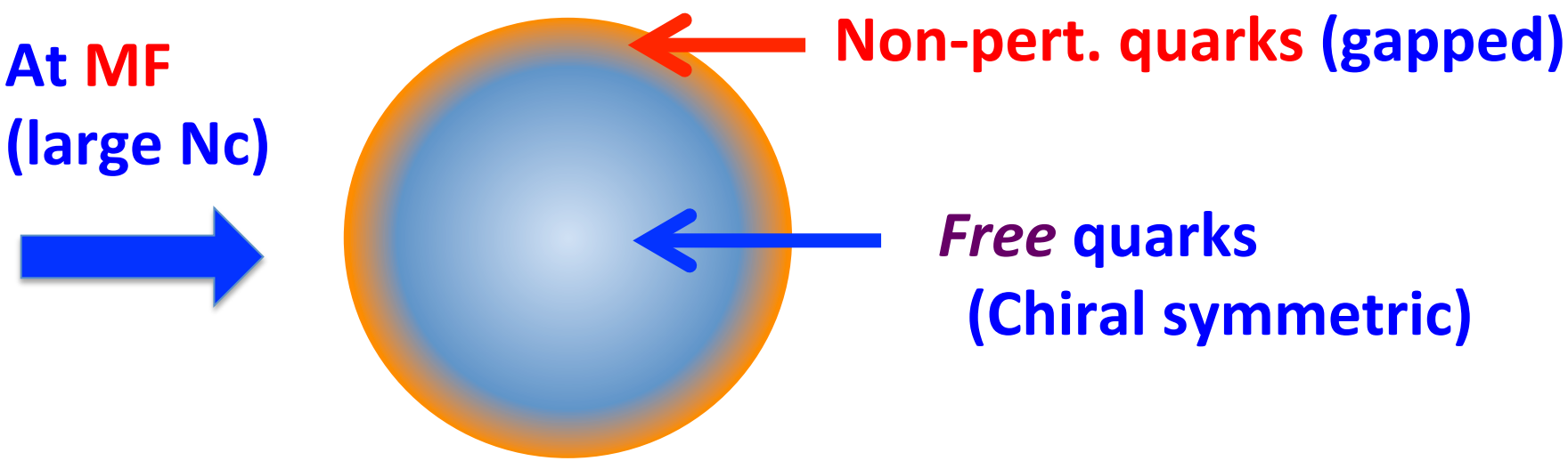
- As far as color-singlet sector is concerned,
  we can get the same results even if we drop off div. const.
  (principal value IR regulation; e.g., Coleman, Aspects of Symmetry)

- S-D eqs. → just sub-diagrams in B-S eqs.

- Div. of poles will be used as color selection rules at best.
  (Actually div. of poles may not be necessary condition: Callan-Coote-Gross76 )

# Model & consequences

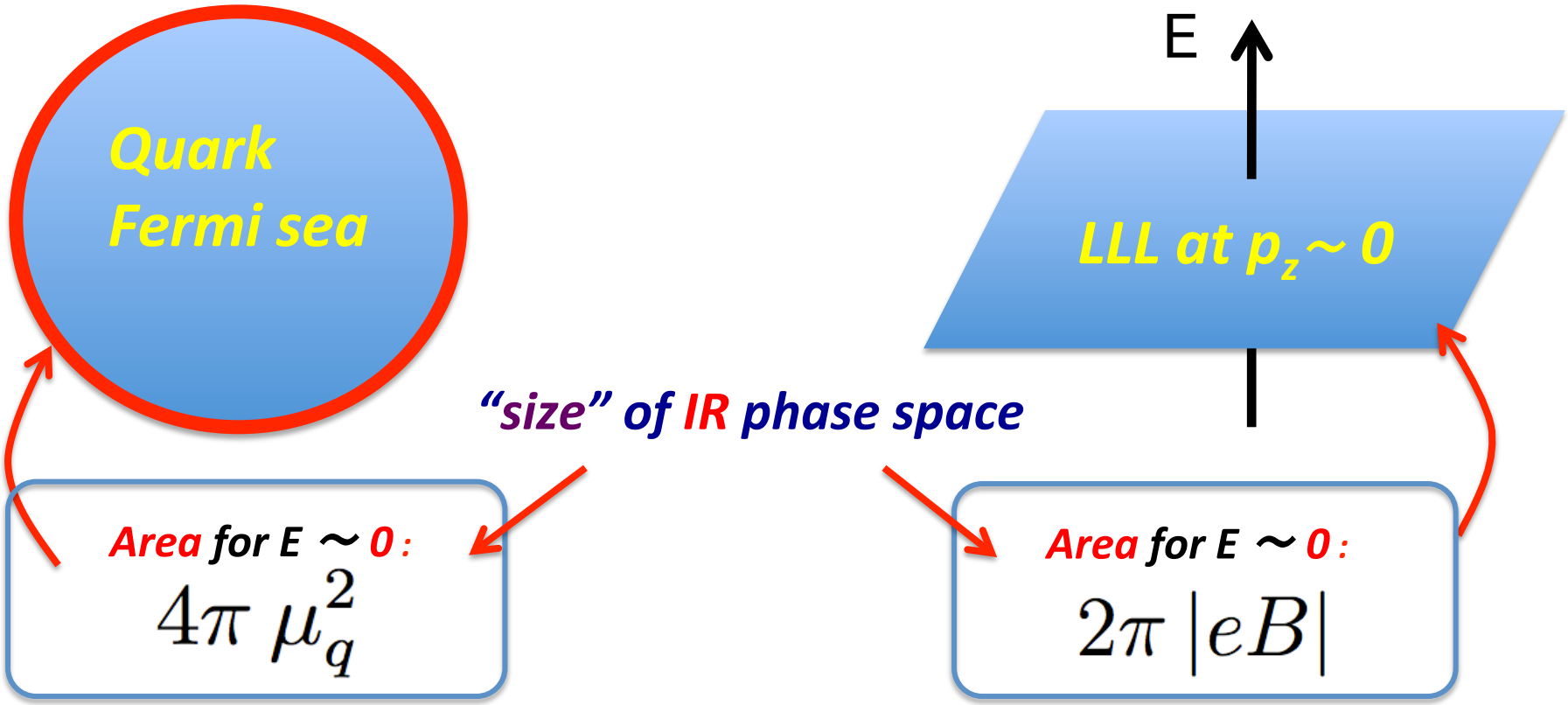


# Applications in mind : *Dense* QCD

***A vital question** in dense QCD:*

***Which μ*** **turns gluodynamics into** ***weak coupling*** **regime?**

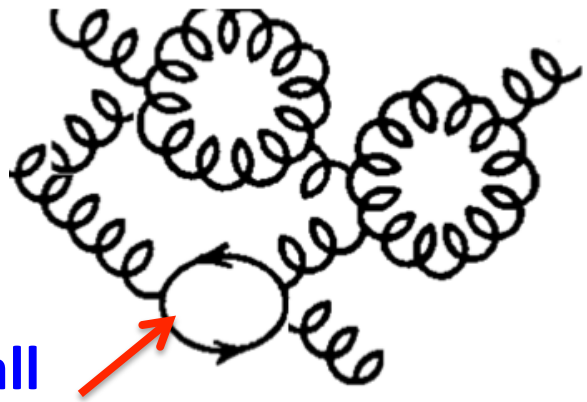



**Hopefully, we may get its rough estimate from "B-exp."**

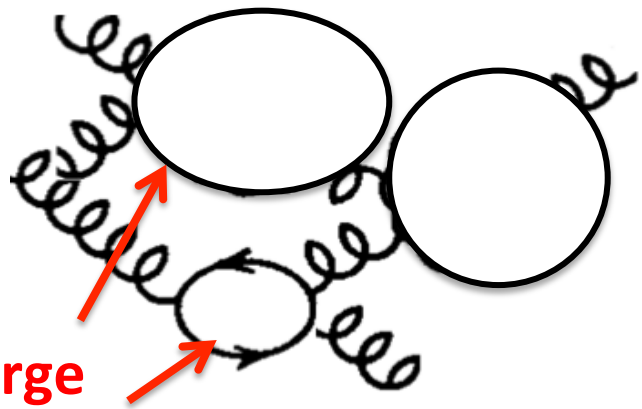
# Quarks as probes of gluodynamics

## A "*naive*" picture (*weak coupling, pert.*)

***small B*** ⟷ ***large B***



**small** fraction



**large** fraction

***We need non-pert. version of this***

***for most of phenomenologically interesting region***