v_n from protons shattering a holographic shockwave

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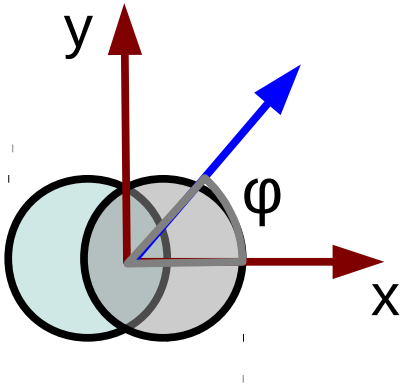
What this is not about :

- hydro
- CGC *(see R.V.: 1312.0113)*
- shockwave – shockwave collisions

based on J. Noronha & A.D.: 1401.4467

Angular asymmetries v_n



$$v_n = \langle \cos n\phi \rangle$$

[really via 2-particle cumulant:

$$(v_n\{2\})^2 = \left\langle e^{in(\phi_1-\phi_2)} \right\rangle = \langle |v_2|^2 \rangle$$

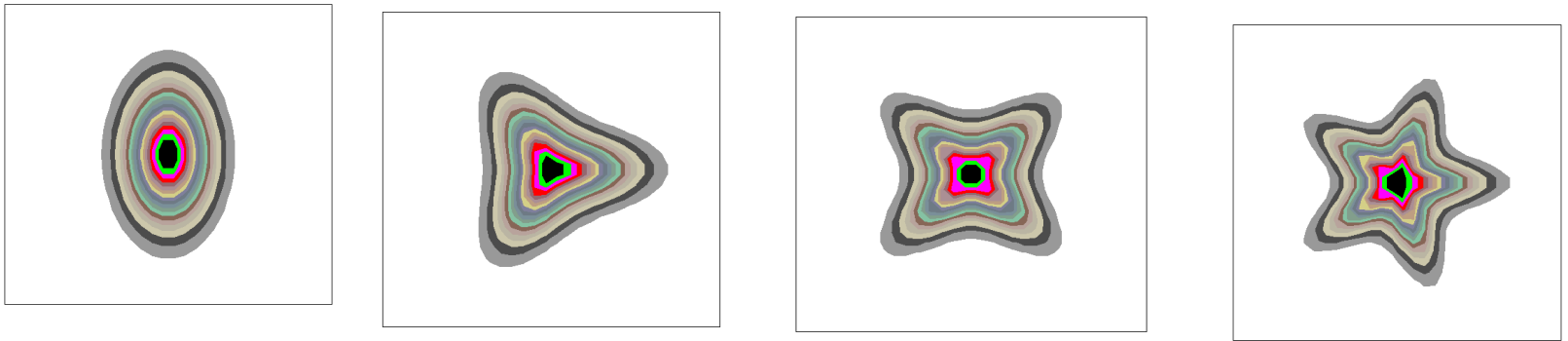
or 4-particle cumulant

$$(v_n\{4\})^4 = 2\left\langle |v_n|^2 \right\rangle^2 - \langle |v_n| \rangle^4$$

etc.]

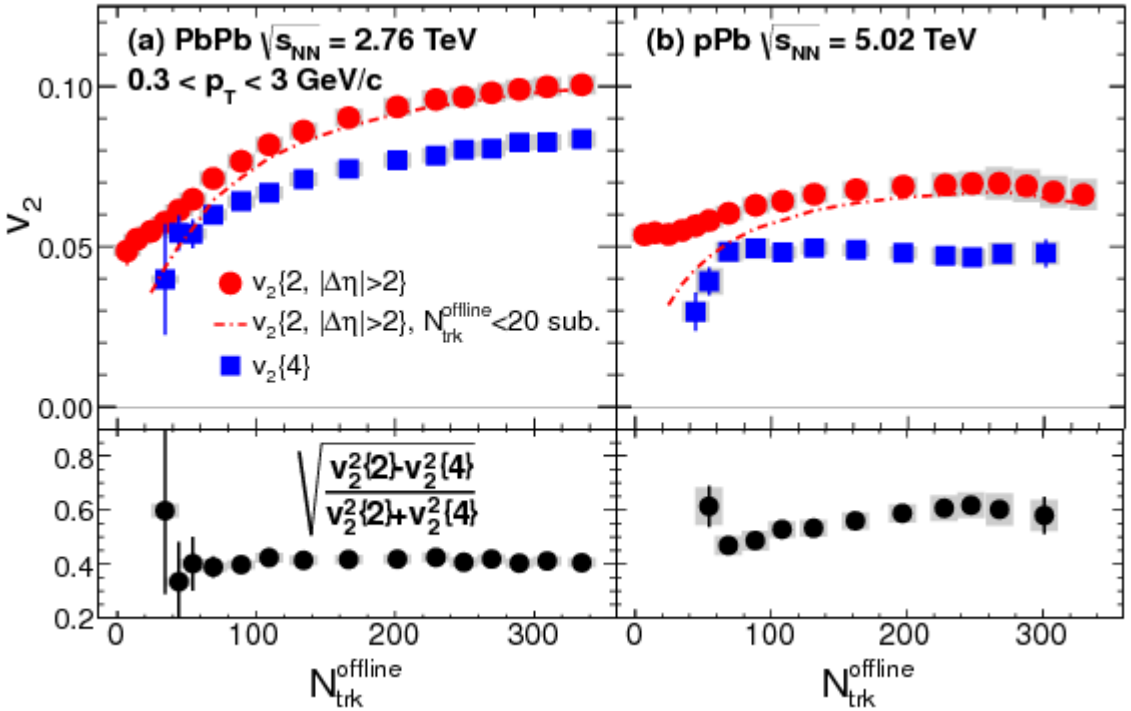
Borghini, Dinh, Ollitrault, PRC63 (2001) 054906

- for even n = 2m: $\langle \cos n\phi \rangle = +\langle \cos n(\phi+\pi) \rangle$ **P = +**
- for odd n = 2m+1: $\langle \cos n\phi \rangle = -\langle \cos n(\phi+\pi) \rangle$ **P = –**

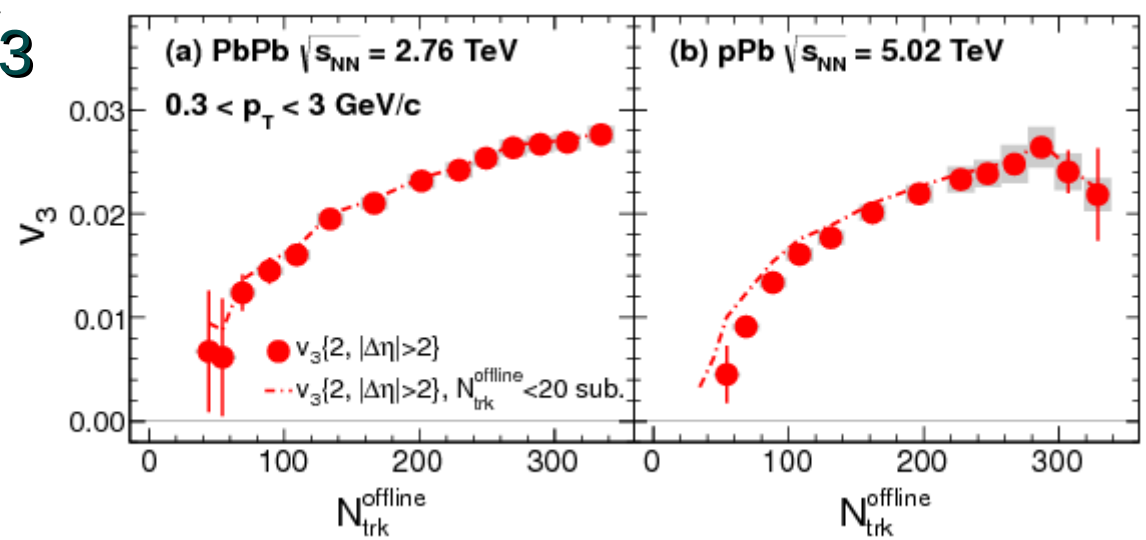


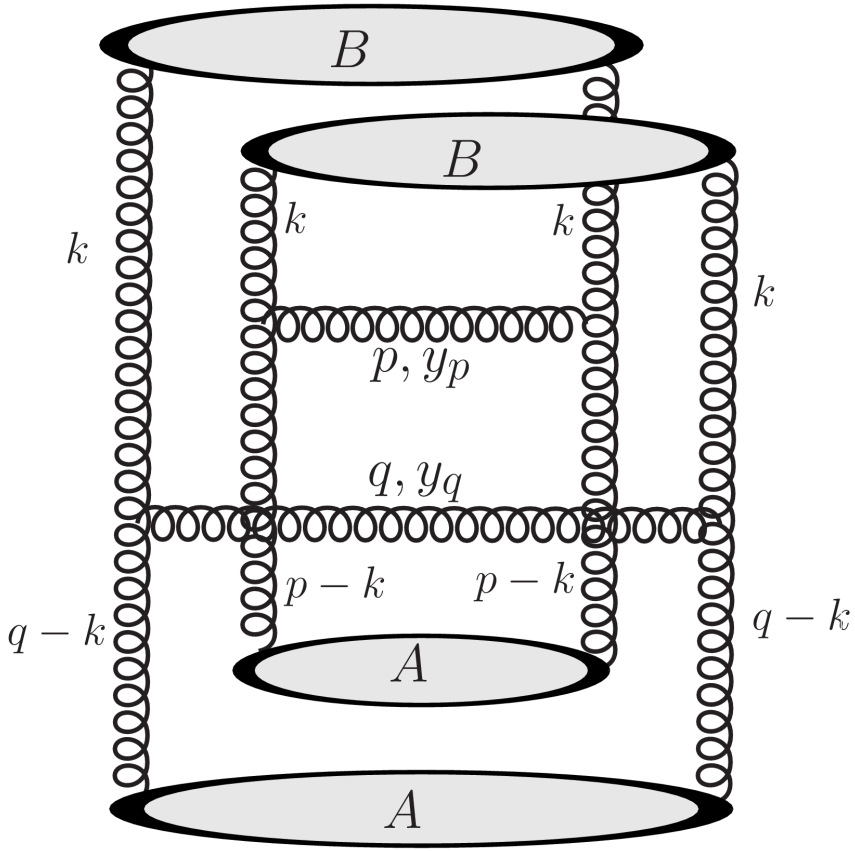
CMS data (p+Pb, high mult.)

v_2



v_3





+ perm.

Depends on angle $\angle(p_\perp, q_\perp)$, not flat in ϕ !

(A.D. : RBRC X-mas review 2009;
RBRC Workshop "Progress in High-pt Physics", March 17—19, 2010)

However, even under $\phi \to \phi + \pi$

Kovner & Lublinsky 1211.1928
Kovchegov & Wertepny, 1212.1195
A.D., Lappi, McLerran, 1310.7136

(true also in holographic shockwave for NG action $S_{NG} \sim \int \sqrt{\det g}$;
NS-NS 2-form can give P-odd moments though)

for pA / CGC :
A.D. & J. Jalilian-Marian,
PRL (2002)

quark scattering off target field, $p_T/p^- \ll 1$

$$V(x_\perp) = \mathcal{P}\,\exp\; ig \int dx^- A^+(x_\perp)$$

$$\langle \mathrm{out}, q | \mathrm{in}, p \rangle \equiv \bar{u}(q)\,\tau(q,p)\,u(p)$$

$$\tau(q,p) = 2\pi\delta(p^- - q^-)\,\gamma^- \int d^2\vec{x}\,[V(\vec{x}) - 1]\,e^{i(\vec{p}-\vec{q})\cdot\vec{x}}$$

$$\frac{d\sigma_{qA}}{d^2\vec{b}\,d^2\vec{q}} = \frac{1}{(2\pi)^2}\int d^2\vec{x}\,e^{-i\vec{q}\cdot\vec{x}}$$

$$\left(\left\langle \frac{1}{N_c}\mathrm{tr}\,\left(W(\vec{x},\vec{b}) - V(\vec{b}-\vec{x}/2) - V^\dagger(\vec{b}+\vec{x}/2)\right)\right\rangle + 1\right)$$

$W(\vec{x})$ is light-like Wilson loop,

$$= V(\vec{b}-\vec{x}/2)\,V^\dagger(\vec{b}+\vec{x}/2)$$

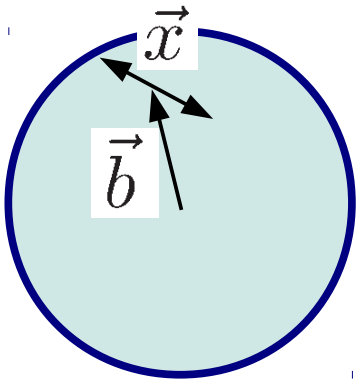
in covariant gauge, “dipole”

CGC approach :

- evaluate <tr W> in MV model or from BK solution
- here: in holographic shockwave background
- in the present context, this is but an intuitive ansatz; projectile q/g weakly coupled to target field

Next :

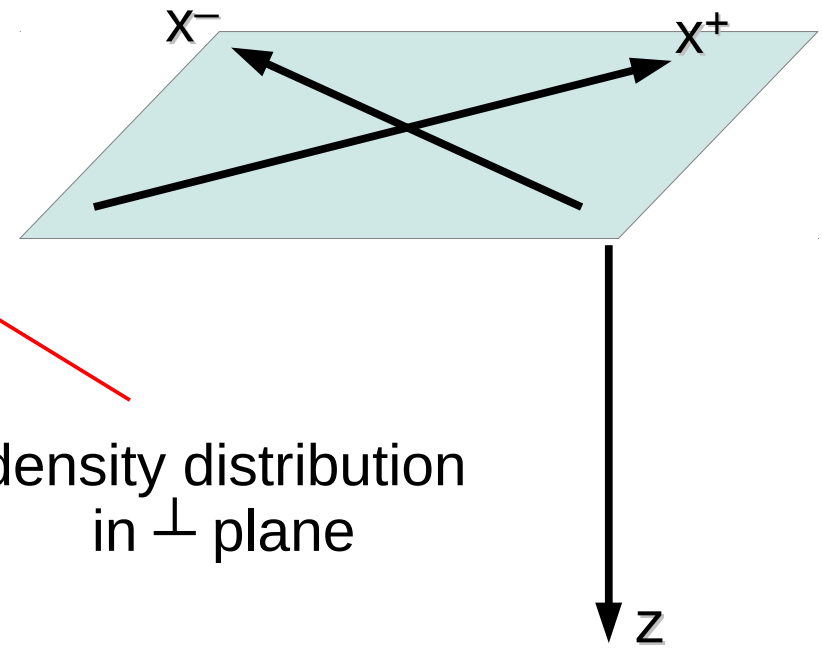
- holographic shockwave
  ($N_c \to \infty$, $\lambda \gg 1$; note this implies $g \to 0$;
  gravity: $L^2/\alpha' = \sqrt{\lambda} \gg 1$)
- computation of light-like Wilson loop



Note :

- rotational symmetry spontaneously broken by dipole vector $\vec{x} = \vec{b} + \frac{\vec{x}}{2} - \left[\vec{b} - \frac{\vec{x}}{2}\right]$
- shift in transv. coordinate of quark from amplitude to c.c.

shockwave travelling along x^+ axis :



$$\langle \hat{T}_{--}(x^-, \vec{x}) \rangle = \frac{N_c^2}{2\pi^2}\, p^+\, \delta(x^-)\, \mu^2\, f(\vec{x})$$

support ~ $A^{1/3}$

~ $A^{1/3}$

density distribution in ⊥ plane

aside:

$$\int d^2x\, dx^- \langle \hat{T}_{--} \rangle \sim N_c^2\, p^+ \mu^2 A^{2/3}$$

$$\int dx^- \sqrt{\delta(x^-)} \to \int dx^- \sqrt{\frac{\Theta(x^-)\,\Theta(a - x^-)}{a}} = \sqrt{a} \sim A^{1/6}$$

source $J(\vec{x})\delta(z - 1/\mu)$ in the bulk
(for the above T_ _ on the boundary) :

$$f(\vec{x}) = \int d^2\vec{x}\,' \frac{J(\vec{x}\,')}{(1 + \mu^2 |\vec{x} - \vec{x}\,'|^2)^3}$$

Einstein eqs :

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \frac{(D-1)(D-2)}{2L^2} g_{\mu\nu} = 8\pi G_D J_{\mu\nu}$$

$$ds^2 = \frac{L^2}{z^2} \left(p^+ \delta(x^-) \mu^2 \mathcal{F}(z, \vec{x}) z^4 \, dx^{-2} - 2dx^+ dx^- + d\vec{x}^2 + dz^2 \right)$$

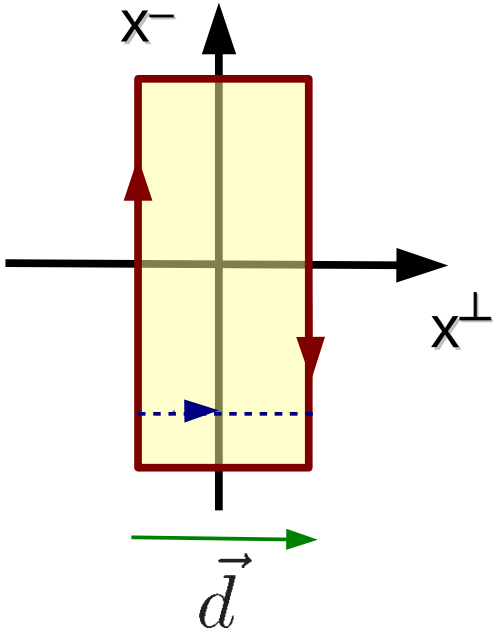
with $\lim_{z \to 0} \mathcal{F}(z, \vec{x}) = f(\vec{x})$ (no divergence at the boundary, z → 0)

[see Janik & Peschanski: hep-th/0512162
Gubser, Pufu & Yarom: 0805.1551
Albacete, Kovchegov & Taliotis: 0806.1484 (for T_ _)
…
]

Correspondence :

$$\frac{1}{N_c}\langle \mathrm{tr}\ W(\mathcal{C})\rangle = \exp\, iS_{\mathrm{NG}}(\Omega)$$

(boundary of world sheet $\partial\Omega = \mathcal{C}$)



$$iS_{\mathrm{NG}} = \frac{i}{2\pi\alpha'}\int_\Omega d^2\sigma\,\sqrt{-\det h_{ab}}$$

$$h_{ab} = g_{MN}\,\partial_a X^M \partial_b X^N$$

$$X^M(\sigma) = (x^-, \vec{b}+\sigma\vec{d}, 0, d\,u(\sigma))\,, \qquad \sigma\in[-1/2,1/2]$$

$$iS_{NG} = -\frac{\sqrt{\lambda}}{2\pi}A^{1/6}\mu\, d\int_{-1/2}^{1/2} d\sigma\,\sqrt{\mathcal{F}\left(d\,u(\sigma),\vec{b}+\sigma\vec{d}\right)}\sqrt{1+u'(\sigma)^2}$$

EoM: u(σ)≡0

$$iS_{on-shell\,NG} = -\frac{\sqrt{\lambda}}{2\pi}A^{1/6}\mu\, d\int_{-1/2}^{1/2} d\sigma\,\sqrt{f\left(\vec{b}+\sigma\vec{d}\right)}$$

Discussion :

1) infinite, uniform nucleus, $f(x_\perp) = 1$:

$$\frac{1}{N_c}\langle \mathrm{tr}\, W\rangle = \exp\left(-\frac{\sqrt{\lambda}}{2\pi}A^{1/6}\mu\, d\right) \stackrel{!}{=} e^{-Q_s d}$$

$$\longrightarrow \quad Q_s = \frac{\sqrt{\lambda}}{2\pi}A^{1/6}\,\mu \sim A^{1/3} \qquad (!)$$

(expression agrees with Albacete, Kovchegov & Taliotis, *op. cit.*)

first new result:

$\longrightarrow$ p_T distribution :

$$\frac{dN}{dy\, d^2p_T} = \frac{Q_s}{\left(p_T^2 + Q_s^2\right)^{3/2}}$$

- saturates at $p_T < Q_s$
- not applicable at $p_T \to \infty$ $\quad (\langle p_T^2\rangle = \infty)$

2) fluctuations : $f(x_\perp) = 1 + \delta f(x_\perp)$

(note: contribution to T_{--} is ~ N^2)

→ $\vec{d} \to -\vec{d}$ symmetry of NG action: P-even angular distribution

explicit example:

$$\delta f(\vec{x}) = \int \frac{d^2k}{(2\pi)^2}\, \delta f(\vec{k})\, e^{i\vec{k}\cdot\vec{x}}$$

$$\delta f(\vec{k}) = \frac{1}{2}(2\pi)^2\, \mathcal{A}(1/|\vec{k}_0|) \times \left[\delta(\vec{k}-\vec{k}_0) + \delta(\vec{k}+\vec{k}_0) + i(\delta(\vec{k}-\vec{k}_0) - \delta(\vec{k}+\vec{k}_0))\right]$$

→ $$iS_{on-shell} = -Q_s\, d \left[1 + \mathcal{A}(\ell) \frac{\sin\left(\frac{d}{2\ell}\cos\phi\right)}{\frac{d}{\ell}\cos\phi}\right]$$

$$\cos\phi \equiv \hat{\vec{d}} \cdot \hat{\vec{k}}_0$$

- **generates v_n for even n**
- **decreases with decreasing d / ℓ (ℓ = 1/k_0)**

  (v_n not from soft ℓ but ℓ ~ d)

P-odd moments :

Full string action contains metric, dilaton, NS-NS 2-form and a bunch of other p-forms...

$$S_{\mathrm{NS-NS}} = \frac{1}{4\pi\alpha'} \int_\Omega d^2\sigma \, B_{MN} \, \epsilon_{ab} \, \partial_a X^M \partial_b X^N$$

- B_{MN}: Neveu-Schwarz-Neveu-Schwarz two form, anti-sym.
- related to Odderon on gauge theory side
  (Brower, Djuric, Tan: 0812.0354; Avsar, Hatta, Matsuo: 0912.3806)
- single nucleus case: B_{MN} pure gauge so that
  $F_{MNR} = \partial_M B_{NR}$ + perm. = 0
- → does not enter EoM, shockwave solution unchanged

amplitude : $\exp\left(iS_{\rm NG} + iS_{\rm NS-NS}\right)$

real imag.

$$iS_{\rm NS-NS} = \frac{i}{2\pi\alpha'}\int_{-\infty}^{\infty} dx^- \int_0^{1/2} d\sigma$$
$$\vec{d}\cdot\left[\vec{B}_{-\vec{x}_\perp}(x^-,\vec{b}+\vec{d}\sigma,0) + \vec{B}_{-\vec{x}_\perp}(x^-,\vec{b}-\vec{d}\sigma,0)\right]$$

write $B_{-j} \sim \delta(x^-)\, C_j$ (other components = 0)

$F_{i-j} = \partial_i B_{-j} - \partial_j B_{-i} = 0$ (pure gauge)

satisfied by $C_j = \partial_j\, f(\vec{x}) = \partial_j\, \delta f(\vec{x})$

$$\longrightarrow \quad i\int_{-1/2}^{1/2} d\sigma\, \vec{d}\cdot\vec{\nabla}\, f(\sigma\vec{d}) = -2iA(\ell)\, \sin\left(\frac{d}{2\ell}\cos\phi\right)$$

Summary / Conclusion

- computed VEV of light-like Wilson loop in holographic shock wave background
- q-shockwave collision gives $dN/dpt^2 \sim 1 / (pt^2 + Qs^2)^{3/2}$
  $Qs \sim A^{1/3}$
- spont. breaking of rot. sym. by fluctuations of source (target $T_{\mu\nu}$) which couple to q.m. $|\mathcal{M}|^2$
  fluctuation scale $\ell \sim 1/pt$
- NG action (metric only) gives even v_n
  NS-NS field can give odd v_n
- for the first time <u>initial state v_3</u>
- v_n as holographic snapshot of "classical" ($\sim N^2$) target density fluctuations