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Way to Test Causality in the ρ-Meson System*

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We show how to determine experimentally if the ρ meson is a normal pole in the second sheet of the energy plane as opposed to a noncausal pole in the first sheet. This determination uses electromagnetic corrections to the reaction $\pi^- p \to \pi^+\pi^- n$. We find that for certain kinematics the ρ mesons produced in this reaction will appear to be shifted in mass in a direction controlled by the position of the ρ pole. For an 8–10-GeV beam energy, 4% of the events will be shifted by roughly 1–2 MeV.

The ρ meson is the most thoroughly studied of all boson resonances. In those experiments in which it has been seen, the ρ peak can be generally fitted with a simple pole of the form $1/[im_\rho\Gamma_\rho + (s - m_\rho^2)]$ in the amplitude.[1] Here s denotes the square of the total energy of the decay products of the ρ in their center-of-mass frame. The mass and width of the ρ meson are given by m_ρ and Γ_ρ, respectively. However, in every one of these experiments it is actually the absolute value squared of this pole that is fitted. Thus, $1/[im_\rho\Gamma_\rho + (s - m_\rho^2)]$ could be replaced with $1/[im_\rho\Gamma_\rho - (s - m_\rho^2)]$, and with a change of the phase of the background the data could be equally well fitted. The correct form to use is theoretically determined by the fact that the π-π scattering amplitude must be analytic in the upper half s plane.[2] An experimental verification of the correct form would be interesting as a test of the theoretical assumptions, primarily causality, which go into proving analyticity. This will test causality for boson systems over distances of the order of the speed of light times the ρ-meson lifetime, or about 10^{-13} cm. The aim of this paper is to show how, through a study of electromagnetic corrections, the position of the ρ pole is an experimentally measurable quantity.

Before proceeding let us discuss the situation with regard to baryonic resonances. Forward π-N dispersion relations have been tested to lab energies of 20 GeV.[3] This indicates that causality violations in the π-N system cannot have a range much larger than 10^{-15} cm.[4] This also means that all the prominent N^* resonances have their poles where theoretically expected. In order to make this test, it was necessary to know the phase of the amplitude. At first sight this would seem impossible since the cross sections are given by the absolute value squared of the amplitude. It is the interference of the strong-interaction amplitude with the electromagnetic contribution due to single-photon exchange which provides the experimental handle needed to unambiguously determine the phase of the forward strong amplitude.

The method we propose to determine the position of the ρ pole is analogous to this use of Coulomb interference to test π-N dispersion relations. We study an interference effect between a purely strong amplitude and an electromagnetic correction. Since $\pi\pi$ scattering is currently unfeasible experimentally, we focus our attention on the reaction

$$\pi^- + p \to \pi^- + \pi^+ + n\,. \tag{1}$$

Figure 1(a) shows the contribution of the ρ in the $\pi^+\pi^-$ final state. Figure 1(b) shows the electromagnetic contribution we are considering. Even though this correction has a factor of $\alpha = 1/137$ associated with it, this small factor can be partially compensated for by the photon propagator when the photon is near its mass shell. By restricting ourselves to such kinematics, this particular radiative correction can become observable while all others remain small.[5] We will show that this correction will shift the experimental position of the ρ peak in a direction dependent on the position of the ρ pole.

In order to study the effect of this correction, we must do some kinematics. Define the particle momenta as

q_μ = 4-momentum of the incident π^-,

p_μ = 4-momentum of the target p,

q'_μ = 4-momentum of the final π^-,

q^+_μ = 4-momentum of the final π^+,

n_μ = 4-momentum of the final n.

If we average over nuclear spins, this process is kinematically determined by five Lorentz invariants. We choose to define the independent invariants

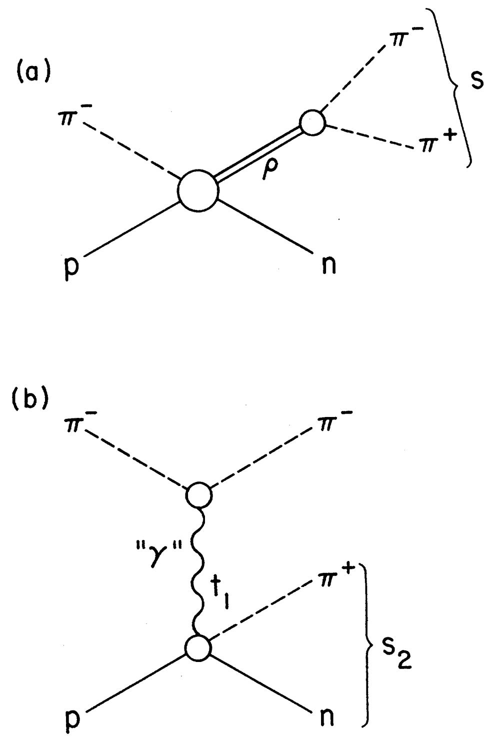

FIG. 1. (a) The contribution of the ρ meson to $\pi^- p \to \pi^+\pi^- n$. (b) The electromagnetic correction considered in the text.

$$\begin{aligned} s &= (p+q)^2 , \\ s_1 &= (q'+q^+)^2 , \\ s_2 &= (q^+ + n)^2 , \\ t_1 &= (q'-q)^2 , \\ t_2 &= (p-n)^2 . \end{aligned} \tag{2}$$

Since we are studying the ρ meson, we shall always keep s_1 near the square of the ρ mass. This means that we should expect the contribution of the process in Fig. 1(a) to the amplitude to have the form

$$T_{1(a)} \approx -(q^+ - q')_\mu T_\mu(\pi^- p \to \rho^0 n) \frac{g_{\rho^0\pi^+\pi^-}}{i m_\rho \Gamma_\rho + (s_1 - m_\rho^2)} , \tag{3}$$

where $\epsilon_\mu T_\mu(\pi^- p \to \rho^0 n)$ denotes the amplitude for ρ production with ρ polarization ϵ. We will discuss later the noncausal case with the ρ pole above the real axis.

The contribution from the process in Fig. 1(b) should be

$$T_{1(b)} = -(q+q')_\mu T_\mu(\text{“}\gamma\text{”} p \to \pi^+ n)(-e)/t_1 . \tag{4}$$

Here "γ" denotes the virtual photon of mass squared t_1 and $\epsilon_\mu T_\mu(\text{“}\gamma\text{”} p \to \pi^+ n)$ is the amplitude for pion production by virtual photons of polarization ϵ.

We now need to relate $T_\mu(\pi^- p \to \rho^0 n)$ and $T_\mu(\text{“}\gamma\text{”} p \to \pi^+ n)$. To do this we shall use the vector-dominance model.[6] This model has had many qualitative successes, and quantitatively it works to within a factor of about 2 for virtual photons of small mass.[7] We shall assume here that the model gives a rough estimate of the relative magnitude and phase. Our predictions for the electromagnetic effects will thus be only qualitative; however, we will show a basic qualitative difference between the structure of the cross section between causal and noncausal theories.

Let us consider $|t_1| \ll m_\rho^2$ and $s \gg m_\rho^2$ to simplify the kinematics. Vector dominance directly relates the process "γ"$p \to \pi^+ n$ as a function of t_1 to the process $\rho^0 p \to \pi^+ n$ for "physical" ρ^0 mesons on the mass shell:

$$\begin{aligned} T_\mu(\text{“}\gamma\text{”} p \to \pi^+ n) &\approx -e g_{\gamma\rho} \frac{1}{t_1 - m_\rho^2} T_\mu(\rho^0 p \to \pi^+ n) \\ &\approx \frac{e g_{\gamma\rho}}{m_\rho^2} T_\mu(\rho^0 p \to \pi^+ n) . \end{aligned} \tag{5}$$

Now time reversal and an isospin rotation give

$$T_\mu(\rho^0 p \to \pi^+ n) = T_\mu(\pi^- p \to \rho^0 n) . \tag{6}$$

If we apply vector dominance to the pion form factor we can determine $g_{\gamma\rho}$ by the statement $F_\pi(0) = 1$. This gives

$$g_{\gamma\rho} = m_\rho^2 / g_{\rho\pi^+\pi^-} . \tag{7}$$

Combining these equations gives

$$T_\mu(\text{“}\gamma\text{”} p \to \pi^+ n) = (e/g_{\rho\pi^+\pi^-}) T_\mu(\pi^- p \to \rho^0 n) . \tag{8}$$

In order to be used in Eq. (2), Eq. (8) should be evaluated at a center-of-mass energy squared of s_2, whereas $T_\mu(\pi^- p \to \rho^0 n)$ occurring in Eq. (3) is evaluated at center-of-mass energy squared of s. By Regge theory $T_\mu(\pi^- p \to \rho^0 n)$ in the high-energy limit at fixed t_2 should go directly as the center-of-mass energy squared raised to the $\alpha(t_2)$ power, where $\alpha(t_2)$ is the dominant exchanged Regge trajectory. Since the reaction should be dominated by small t_2, we have

$$T_\mu(\text{“}\gamma\text{”} p \to \pi^+ n; s_2) \approx \frac{e(s_2/s)^{\alpha(0)}}{g_{\rho\pi^+\pi^-}} T_\mu(\pi^- p \to \rho^0 n; s) . \tag{9}$$

Here the additional argument of the T's is the center-of-mass energy squared. Note that the amplitudes have the same phase under this assumption of a single important exchange.

This argument is more complicated at finite energies if two Regge trajectories with different values of $\alpha(t_2 \approx 0)$ are both important. Then the amplitudes in Eq. (9) could differ by a phase. At moderately large s, this will be a problem if the lower trajectory has an anomalously large contribution. The relevant trajectories here are the A_2 with $\alpha(0)\approx\frac{1}{2}$ and the π with $\alpha(0)\approx 0$. Because for small t_2 we are close to the pion pole, we can have such an anomalous contribution from π exchange. For this reason either we can consider s to be sufficiently large so that most of the observed events can be described by A_2 exchange, or we can consider only moderately large s and consider small t_2 so that pion exchange dominates. We take the latter choice because Wolf[8] has shown that one-pion exchange can quite well describe experimental data on $\pi^- p\to\rho^0 n$ up to lab momenta of at least 8 GeV/c and to squared momentum transfers of at least 0.5 GeV^2. Furthermore, most events lie in this momentum-transfer range. This suggests doing this experiment at about 8–10 GeV, and assuming pion dominance of the t_2 exchange. Again, as we can only make qualitative predictions, this need only be qualitatively true. With this assumption, Eq. (9) becomes

$$T_\mu(\text{“}\gamma\text{”}p\to\pi^+n;\,s_2)\approx(e/g_{\rho\pi^+\pi^-})T_\mu(\pi^-p\to\rho^0n;\,s)\,. \tag{10}$$

If we use current conservation, the $(q+q')_\mu$ appearing in Eq. (4) can be replaced with $2q'_\mu$. If we now assume that the ρ always couples to a conserved current, the $(q^+ - q'_\mu)$ in Eq. (3) can be replaced by $-2q'_\mu$. Combining these statements and Eqs. (3), (4), and (10), we can express the T matrix for process (1) approximately by

$$T\approx\frac{2g_{\rho^0\pi^+\pi^-}q'_\mu T_\mu(\pi^-p\to\rho^0n;\,s)}{im_\rho\Gamma_\rho+(s_1-m_\rho^{\,2})}\times\left(1+\frac{e^2}{g_{\rho\pi^+\pi^-}{}^2}\,\frac{im_\rho\Gamma_\rho+(s_1-m_\rho^{\,2})}{t_1}\right). \tag{11}$$

Squaring this gives our result

$$d\sigma\approx d\sigma_S\left(1+\frac{2e^2}{g_{\rho\pi^+\pi^-}{}^2}\,\frac{s_1-m_\rho^{\,2}}{t_1}\right). \tag{12}$$

Here $d\sigma_S$ is what the cross section would be without electromagnetic interactions. We have dropped terms of order e^4.

Since t_1 is negative, the term multiplying $d\sigma_S$ in Eq. (12) decreases the cross section for $s_1>m_\rho^{\,2}$ and increases it for $s_1<m_\rho^{\,2}$. This would be observed as a shift in the experimental maximum of the cross section as a function of s_1. This shift is of magnitude

$$\Delta s_{1\,\max}=\frac{e^2}{g_{\rho\pi^+\pi^-}{}^2}\,\frac{m_\rho^{\,2}\Gamma_\rho^{\,2}}{t_1}, \tag{13}$$

where we again keep only the lowest order in e^2. To arrive at Eq. (13) we assume that all rapid variations in the cross section as a function of s_1 when s_1 is near $m_\rho^{\,2}$ arise from the ρ pole. Any slowly varying background will give a shift in $s_{\max}$ from $m_\rho^{\,2}$, but should not alter Eq. (13) substantially.

Let us now consider the possibility that the ρ meson is noncausal and has its pole in the upper half s_1 plane. For such a noncausal ρ we merely replace $1/[im_\rho\Gamma_\rho+(s-m_\rho^2)]$ with $1/[im_\rho\Gamma_\rho-(s-m_\rho^2)]$ in all of the above equations. We should comment on the choice of over-all sign in front of these expressions. We would like to preserve the optical theorem for $\pi\pi$ elastic scattering. This means that the imaginary part of the $\pi\pi$ elastic amplitude is determined from the total cross section. Therefore, any modifications we make in the amplitude should change only the real part. This suggests taking T into $-T^*$ as indicated in the beginning of this paragraph. Since we are discussing exotic things such as a violation of causality, perhaps unitarity and the consequent optical theorem should be modified as well. Such changes could alter any conclusions regarding a noncausal ρ. However, in order to make any progress at all, we consider only a simple violation of causality and assume the optical theorem is still valid.

Making the above modification of the ρ pole, Eq. (9) becomes

$$d\sigma_N\approx d\sigma_S\left(1-\frac{2e^2}{g_{\rho\pi^+\pi^-}{}^2}\,\frac{s_1-m_\rho^{\,2}}{t_1}\right). \tag{14}$$

Here $d\sigma_N$ denotes the cross section in our particular noncausal model. Clearly now the observed ρ peak is shifted to higher energy, instead of lower as before. This makes the suggested experiment clear: One should measure the position of the ρ maximum in s_1 as a function of t_1 and look for a shift at small t_1. Theory predicts a shift to lower energies; a shift to higher energies would indicate something is wrong with the theory.

Before making numerical estimates let us make some comments. Presumably one could find many other dynamical mechanisms yielding a shift in the observed ρ peak as a function of t_1. However, the effect discussed here is singled out because the photon pole in the t_1 channel gives a shift going as $1/t_1$. Thus, the effect appears for t_1 small and is essentially absent for large t_1. Other shift mechanisms will presumably be smooth as $t_1\to 0$. As a consequence of this strong t_1 dependence, difficulties in extracting the true ρ mass and width[9] are irrelevant. We need only look for a shift in the experimental maximum of the cross section.

At this point we must note that since we are working near the edge of the physical region, caution is necessary to avoid possible kinematic effects such as the Deck effect.[10] To minimize these effects it is essential that comparable distributions in s, s_2, and t_2 be taken while t_1 is varied and the maximum in s_2 is observed. It should be possible to accomplish this with an appropriate weighting of the data. In addition, data with small s_2, where resonances in the s_2 channel may give rise to rapidly changing phases, should not be used in the analysis.

Although we have used the vector-dominance model to obtain our result, we feel that the shift will occur and that its direction is reasonably model-independent. The only way for the vector-dominance model to predict the wrong direction for the shift would be for it to make a phase error of the order of π. We feel that such a violent violation of the model is unlikely in the light of its successes.

Experimentally we have[11]

$$e^2/g_{\rho^0\pi^+\pi^-}{}^2 \approx \tfrac{1}{270}. \tag{15}$$

This means that in order to observe this shift, it is desirable to go to the smallest value of $|t_1|$ possible. The kinematics place some constraints on $|t_1|$. If we fix $s_1 \approx m_\rho^2$ and have $s \gg m_\rho^2$ then t_1 is restricted for fixed t_2 by

$$t_1 \leq -\left(\frac{m_\pi^2}{m_\rho^2}\right)\frac{(m_\pi^2 - t_2)^2}{m_\rho^2}\left[1 + O(t_2/m_\rho^2,\, m_\pi^2/m_\rho^2)\right]. \tag{16}$$

For the sake of obtaining some numbers, assume we would like to observe a difference of about 3 MeV in $\sqrt{s}$ between the shift in the causal versus the noncausal picture. This means that we should look at those events that have

$$|t_1| \lesssim \frac{2e^2}{g_{\rho\pi^+\pi^-}{}^2}\frac{m_\rho^2\Gamma_\rho^2}{2m_\rho(3\text{ MeV})} = 0.02\text{ GeV}^2 \sim m_\pi^2. \tag{17}$$

Looking at Eq. (16) we see that this means we must keep $|t_2| \lesssim m_\rho^2$, a constraint experimentally satisfied by the majority of events.[8]

In order to estimate what fraction of the events will satisfy Eq. (17), let us express t_1 in terms of laboratory energies and the angle θ of q' with respect to the beam direction. Assuming small θ and m/q_0' gives

$$\begin{aligned} t_1 &= 2m_\pi^2 - 2q_0q_0' + 2|\vec{q}\,||\vec{q}\,'|\cos\theta \\ &\approx -\frac{\theta^2}{2}q_0q_0' - m_\pi^2\left(\frac{(q_0 - q_0')^2}{q_0q_0'}\right). \end{aligned} \tag{18}$$

In order to have t_1 of order m_π^2 with large q_0, clearly we must have q_0 and q_0' of the same order, and θ^2 must be of order m_π^2/q_0^2.

The ρ^0's produced in π^-p collisions have typical transverse momentum a small fraction of m_ρ.[8] When the ρ's decay, they give π's with longitudinal momentum of order $q_0/2$ and transverse momentum of order $m_\rho/2$. Thus, the typical π^- produced in $\pi^-p \to \pi^+\pi^-n$ when $s_1 \approx m^2$ will have an angle θ of order m_ρ/q_0. Combining this with the previous paragraph tells us that roughly $m_\pi^2/m_\rho^2 \approx 4\%$ of the events in reaction (1) will have the desired 1–2-MeV shift due to this electromagnetic correction. Note that the beam energy drops out of this ratio. Our argument may seem rough, but uncertainties in our various assumptions would cast doubt on more detailed calculations. This estimate should nevertheless serve as an indication of the accuracy needed to see this shift.

In conclusion, we have found that one can test causality in the ρ system by studing reaction (1) at about 10-GeV lab energy and looking at those events which have small t_1. These events should show a shift in the position of the observed ρ maximum to lower energy if the theory is causal and to higher energy in a noncausal theory. A rough estimate gives that 4% of the events in reaction (1) should give a shift of about 1–2 MeV. The strong variation of this shift with t_1 allows the unambiguous separation of the desired effect from other mechanisms shifting the observed ρ mass. When there exists a sufficiently detailed study of reaction (1), one should look for this effect. However, because dispersion relations work in the πN system,[3] we feel that such an experiment should not be done solely to observe this shift.

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[1]In principle, this form must be modified to be a real analytic function for s on the real axis below threshold at $s = 4m_\pi^2$. A possible form would be

$$[i\Gamma_\rho(s - 4m_\pi^2)^{1/2} + (s - m_\rho^2)]^{-1}.$$

Since at $s \sim m_\rho^2$ we are much closer to the pole than to the branch point at threshold, we drop this correction throughout this paper. Including it would make no essential change in our conclusions but would complicate the formulas.

[2]In terms of phase shifts, we are considering the possibility that the $I = J = 1$ $\pi^+\pi^-$ phase shift is rapidly falling at $s = m_\rho^2$ rather than the theoretical rapid rise.

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[10]R. T. Deck, Phys. Rev. Letters 13, 169 (1964).

[11]The ρ width is related to $g_{\rho\pi^+\pi^-}{}^2$ by

$$\frac{g_{\rho\pi^+\pi^-}{}^2}{4\pi}=\frac{3}{2}\,\frac{\Gamma_\rho m_\rho{}^2}{[(\frac{1}{2}m_\rho)^2-m_\pi{}^2]^{3/2}}\approx 2\,.$$

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Hadronic Corrections to Goldberger-Treiman Relations for Strangeness-Carrying Currents*

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Hadronic corrections to Goldberger-Treiman relations for vector and axial-vector strangeness-carrying currents are calculated. In both cases, the corrections are found to be less than 10%. A bound for $G_{\Lambda pK}$ is set.

In the dispersion-theoretic version, the PCAC (partial conservation of axial-vector current) hypothesis assumes that the matrix element of the divergence of an axial-vector current satisfies an unsubtracted dispersion relation dominated by the lowest pseudoscalar meson. The Goldberger-Treiman relation (GTR) derived from this hypothesis provides a direct experimental test of this hypothesis itself. For a strangeness-conserving axial-vector current, it is believed that the pion dominance of the divergence of this current is a good approximation, for the pion pole is far below thresholds of other hadronic states. In fact, it is well known that many soft-pion results derived from PCAC and current-algebra assumptions are in very good agreement with experiment. However, the GTR is experimentally found to have a 10% discrepancy. Pagels[1] tried to understand this discrepancy in terms of hadronic continuum corrections but failed. Later, he[2] proposed a once-subtracted dispersion-relation version of PCAC as a remedy.

In this paper, we extend Pagels's calculation to the study of strangeness-carrying vector and axial-vector currents. There are several reasons of interest for such an investigation. First of all, we want to see how good the GTR's for strangeness-carrying currents are and how large the hadronic corrections are in cases where the lowest poles are rather close to the thresholds of the next higher states. Second, such calculations can give us some information about the values on bounds of the form factors g_Λ^A and g_Λ^V of weak hadronic currents. Such information is useful in checking the Cabibbo theory in strangeness-changing leptonic decays.

In the calculations, we have used experimental information on the coupling constants as far as possible. Where no information is presently available, we have used SU(3) estimates. Our results show that in both cases of vector and axial-vector currents, the corrections to GTR's are less than 10%. A bound for $G_{\Lambda pK}$ is also estimated.

I. STRANGENESS-CARRYING AXIAL-VECTOR CURRENT

The matrix elements of the strangeness-carrying axial-vector current A_μ^{4+i5} and its divergence between the proton and Λ states are specified by Lorentz invariance as

$$\langle p(p')|A_\mu^{4+i5}(0)|\Lambda(p)\rangle=\bar{u}_p(p')[\gamma_\mu\gamma_5F_1(t)+q_\mu\gamma_5F_2(t)+q_\nu\sigma_{\mu\nu}F_3(t)]u_\Lambda(p)\,, \tag{1}$$

$$\langle p(p')|\,\partial_\mu A_\mu^{4+i5}(0)|\Lambda(p)\rangle=i\bar{u}_p(p')\gamma_5u_\Lambda(p)D(t)\,, \tag{2}$$

where $q_\mu=(p-p')_\mu$, $t=q^2$, and $F_i(t)$, $i=1, 2, 3$ are the usual form factors. From Eqs. (1) and (2), it is obvious that

$$D(t)=(m_\Lambda+m_p)F_1(t)-tF_2(t)\,. \tag{3}$$

PCAC assumes that $D(t)$ satisfies an unsubtracted dispersion relation dominated by the K pole, i.e.,

$$D(t)=\frac{f_Km_K{}^2G_{\Lambda pK}}{m_K{}^2-t}+\frac{1}{\pi}\int_{t_0}^{\infty}\frac{\mathrm{Im}D(t')}{t'-t}dt' \tag{4}$$

$$\simeq\frac{f_Km_K{}^2G_{\Lambda pK}}{m_K{}^2-t}\,, \tag{5}$$