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Vanishing Longitudinal Cross Sections and Operator Schwinger Terms

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We show that a vanishing total cross section for virtual longitudinal photons on a hadronic target is compatible with the presence of nontrivial operator-valued Schwinger terms appearing in the one-particle matrix element of the commutator of two electromagnetic currents. However, this requires a violation of either Bjorken scaling or the usual subtraction assumptions on virtual Compton amplitudes.

Many authors have discussed connections between Schwinger terms and the total absorption cross section for virtual longitudinal photons.[1,2] The primary conclusion of these discussions is that if there are no q-number Schwinger terms, then the Bjorken scaling function for longitudinal photons must vanish. In this note we prove by explicit counterexample that the converse is not true. We show that a longitudinal cross section which identically vanishes for both timelike and spacelike photons is compatible with the occurrence of q-number Schwinger terms in the one-particle matrix element of the equal-time commutator of two electromagnetic currents. However, a vanishing longitudinal cross section does place some constraints on the form of the Schwinger term.

We begin with some definitions. We work with the one-particle matrix element of the commutator of two electromagnetic currents:

$$h_{\mu\nu}(x,p) = \langle p|\{[j_\mu(x), j_\nu(0)] - \langle 0|[j_\mu(x), j_\nu(0)]|0\rangle\}|p\rangle . \tag{1}$$

Here $j_\mu(x)$ is the electromagnetic current operator, $|0\rangle$ is the vacuum state ($\langle 0|0\rangle = 1$), and $|p\rangle$ is a one-particle state of momentum p_μ. We give $|p\rangle$ unit mass ($p^2=1$) and normalize covariantly: $\langle p'|p\rangle = 2p_0(2\pi)^3\delta^3(p'-p)$. Throughout this paper we work to lowest nontrivial order in the electric charge. Microcausality states that $[j_\mu(x), j_\nu(0)]$ vanishes for $x^2<0$. This means that $\delta(x_0)h_{\mu\nu}(x,p)$ can have support only at $\vec{x}=0$. By Schwinger term we mean any term in $[j_\mu(x), j_\nu(0)]$ proportional to a first or higher derivative of $\delta^3(x)$ at $x_0=0$. By q-number or operator Schwinger term we mean any such derivative of a δ-function term surviving after the subtraction of the vacuum expectation value indicated in Eq. (1). In momentum space we define

$$W_{\mu\nu}(q,p) = \int d^4x\, e^{iq\cdot x} h_{\mu\nu}(x,p) . \tag{2}$$

If we average over the spins of $|p\rangle$ we can write

$$W_{\mu\nu}(q,p) = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)W_1(\nu,q^2) + \left(p_\mu - \frac{q_\mu p\cdot q}{q^2}\right)\left(p_\nu - \frac{q_\nu p\cdot q}{q^2}\right)W_2(\nu,q^2), \tag{3}$$

where $\nu = p\cdot q$ is the lab energy of q. We have the crossing property $W_i(-\nu,q^2) = -W_i(\nu,q^2)$ and the support property $W_i(\nu,q^2)=0$ for $q^2 < -2|\nu|$. In terms of W_1 and W_2, the transverse and longitudinal virtual photoabsorption cross sections σ_T and σ_L are given by[3]

$$\sigma_T(\nu, q^2) = -\frac{1}{4m(\nu^2 - q^2)^{1/2}} W_1(\nu, q^2),$$

$$\sigma_L(\nu, q^2) = \frac{1}{4m(\nu^2 - q^2)^{1/2}|q^2|} \times [(-q^2 W_1(\nu, q^2) + (\nu^2 - q^2) W_2(\nu, q^2)] . \quad (4)$$

Both σ_L and σ_T are non-negative for $q^2 < 0$.

We now present our model commutator in momentum space,

$$W_{\mu\nu} = [g_{\mu\nu}(q^2 - \nu^2) - q^2 p_\mu p_\nu - q_\mu q_\nu + \nu(p_\mu q_\nu + p_\nu q_\mu)] \frac{1}{2\nu} \theta(4a^2\nu^2 - (q^2 - b)^2), \quad (5)$$

where a and b are parameters satisfying $0 < a \leq 1$, $b \geq 0$. This $W_{\mu\nu}$ corresponds to

$$W_1 = \frac{q^2 - \nu^2}{2\nu} \theta(4a^2\nu^2 - (q^2 - b)^2),$$

$$W_2 = -\frac{q^2}{2\nu} \theta(4a^2\nu^2 - (q^2 - b)^2) \quad (6)$$

or

$$\sigma_T = \frac{(\nu^2 - q^2)^{1/2}}{8\nu} \theta(4a^2\nu^2 - (q^2 - b)^2),$$

$$\sigma_L = 0 . \quad (7)$$

Clearly $W_{\mu\nu}(q)$ has the correct crossing properties and support properties in momentum space. To show causality, we simply note that

$$\frac{1}{2\nu} \theta(4a^2\nu^2 - (q^2 - b)^2) = \int_{-1}^{1} d\beta \int_0^\infty d\mu\, \delta(q^2 + 2\nu\beta - \mu)\epsilon(\nu)\delta(\mu - b)\theta(a^2 - \beta^2) . \quad (8)$$

This is a special case of the Deser-Gilbert-Sudarshan representation[4] which in turn is a special case of the Dyson representation[5] for Fourier transforms of functions vanishing outside the light cone. The additional momentum factors defining $W_{\mu\nu}(q)$ in Eq. (5) correspond to derivatives in position space implying that $W_{\mu\nu}(q)$ itself has a causal Fourier transform. Knowing that $h_{\mu\nu}(x, p)$ is causal, it is easy to find the equal-time commutator; we look at the $0i$ component ($i \in \{1, 2, 3\}$):

$$\delta(x_0) h_{0i}(x, p) = \frac{ai}{\pi} [(p_0^2 - 1)\partial_i - p_i \vec{\nabla} \cdot \vec{p}] \delta^4(x), \quad (9)$$

clearly exhibiting the q-number Schwinger terms.

Having presented the example, we now make several comments.

(1) In our example

$$\lim_{\nu \to \infty} \sigma_T(\nu, q^2) \Big|_{q^2 \text{fixed}} = \text{constant} . \quad (10)$$

Such a behavior is expected in a simple diffraction or Pomeranchukon exchange picture. The fact that the asymptotic cross section is independent of q^2 is inessential; we have made models where $\sigma_T(\nu = \infty, q^2)$ is a nonconstant function of q^2.

(2) Bjorken[6] has conjectured the scaling behavior

$$\lim_{\nu \to \infty} W_1(q^2, \nu) \Big|_{2\nu/q^2 = \omega} = f_1(\omega),$$

$$\lim_{\nu \to \infty} \nu W_2(q^2, \nu) \Big|_{2\nu/q^2 = \omega} = f_2(\omega) . \quad (11)$$

Our model violates this conjecture for $|\omega| > 1/a$ since

$$\lim_{\nu \to \infty} W_1(q^2, \nu) \Big|_{|\omega| > 1/a} = \infty , \quad (12)$$

with a similar statement on νW_2. The parameter a is arbitrary in the range $0 < a \leq 1$, implying that the value of ω above which Bjorken scaling breaks down can be made as large as desired. Of course, in our example the longitudinal structure function scales in a trivial manner,

$$W_L \equiv W_1 - \frac{(\nu^2 - q^2)}{q^2} W_2 = 0 .$$

(3) The singularity in σ_T at $\nu = 0$, $q^2 = b$ occurs in the unphysical region. Thus the amplitude can be modified in a neighborhood of this point to give a finite σ_T without changing the commutator in any way. Also note that the step in σ_T at $q^2 = b \pm 2a\nu$ can be smoothed out by smearing over a range of a.

(4) By rotational invariance, if the equal-time commutator is well-defined and has only first derivative of δ-function terms, it must have the form (assuming the spin states of $|p\rangle$ are averaged over)

$$\delta(x_0) h_{0i}(x, p) = i[C(p_0)\partial_i + D(p_0) p_i (\vec{\nabla} \cdot \vec{p})] \delta^4(x) . \quad (13)$$

Our example is of this form with

$$C(p_0) = \frac{a}{\pi}(p_0^2 - 1),$$

$$D(p_0) = -\frac{\alpha}{\pi} . \quad (14)$$

It can be readily verified that in our example $\delta(x_0) h_{\mu\nu}(x, p)$ has no more than first derivative of $\delta^4(x)$ for all μ, ν. This implies, as shown in Ref. 2, that for all μ, ν we have $\delta(x_0) h_{\mu\nu}(x, p)$ completely determined by $C(p_0)$, $D(p_0)$, $d/dp_0 C(p_0)$, and $d/dp_0 D(p_0)$.

(5) The sum rule[1,2]

$$\int_{-\vec{q}^2}^{\infty} dq^2 \frac{\sigma_L(\nu, q^2)}{|q^2|} \Big|_{\text{fixed lab} \vec{q}} = \frac{\pi C(1)}{2|\vec{q}|} \quad (15)$$

implies that $C(p_0)$ vanishes in the rest frame of p. This sum rule in the Bjorken scaling limit has been shown to imply that vanishing q-number Schwinger terms require a vanishing longitudinal scaling function[1,2]; however, the vanishing of σ_L only implies that $C(p_0)$ vanishes at $p_0=1$.

(6) The example allows us to verify two sum rules from Ref. 2:

$$\int_{-\vec{q}^2}^{\infty} dq^2 \sigma_L(\nu, q^2)\frac{|q^2|}{q^2}\bigg|_{\vec{q}\text{ fixed}} = \tfrac{1}{2}\pi[D(1)+\tfrac{1}{2}C'(1)]\,|\vec{q}\,| + \frac{A(1)}{|\vec{q}\,|}, \tag{16}$$

$$\int_{-\vec{q}^2}^{\infty} dq^2 \sigma_T(\nu, q^2)\bigg|_{\vec{q}\text{ fixed}} = \tfrac{1}{4}\pi C'(1)\,|\vec{q}\,| + \frac{\pi}{2|\vec{q}\,|}A(1). \tag{17}$$

Here the function $A(p_0)$ is defined by

$$\int d^4x\, \delta(x_0)\partial_0 h_{ij}(x,p) = i[A(p_0)g_{ij} + B(p_0)p_i p_j]\delta^4(x) \tag{18}$$

and $C'(1)$ denotes

$$\frac{d}{dp_0}C(p_0)\bigg|_{p_0=1}.$$

Note that a vanishing longitudinal cross section implies

$$\tfrac{1}{2}C'(1) = -D(1) \tag{19}$$

and

$$A(1)=0. \tag{20}$$

Note also that absence of q-number Schwinger terms would imply the sum rule

$$\int_{-\infty}^{\infty} \sigma_T(\nu=\infty, q^2)dq^2 = 0. \tag{21}$$

(7) If we try to find the light-cone commutator

$$\delta(x_0+x_3)h_{\mu\nu}(x,p) \tag{22}$$

we discover that it is undefined. This perhaps is expected since light-cone commutators are related to the limit $p_0\to\infty$ and $\lim_{p_0\to\infty}C(p_0)=\infty$.

(8) We can show that there can be no q-number Schwinger terms if we assume (1) Bjorken scaling for all ω, (2) the usual dispersion relations for the virtual Compton amplitudes hold for both finite q^2 and in the scaling limit, and (3) $\sigma_L(\nu, q^2)=0$. To see this we first write the covariant and gauge-invariant amplitude[7]

$$\begin{aligned} T^*_{\mu\nu} &= i\int d^4x\, e^{iq\cdot x}\theta(x_0)\langle p|[j_\mu(x), j_\nu(0)]|p\rangle \\ &\quad + 2C(p_0)(g_{\mu\nu}-g_{\mu 0}g_{\nu 0}) - 2D(p_0)(p_\mu - p_0 g_{\mu 0})(p_\nu - p_0 g_{\nu 0}) + p_1(q^2, p\cdot q)(q^2 g_{\mu\nu} - q_\mu q_\nu) \\ &\quad + p_2(q^2, p\cdot q)[(p\cdot q)^2 g_{\mu\nu} + q^2 p_\mu p_\nu - (p\cdot q)(p_\mu q_\nu + p_\nu q_\mu)] \\ &= V_1(q^2, p\cdot q)(q^2 g_{\mu\nu} - q_\mu q_\nu) + V_2(q^2, p\cdot q)[(p\cdot q)^2 g_{\mu\nu} + q^2 p_\mu p_\nu - (p\cdot q)(p_\mu q_\nu + p_\nu q_\mu)], \end{aligned} \tag{23}$$

where p_1 and p_2 are polynomials in their arguments. Usual Regge lore gives an unsubtracted dispersion relation for V_2 and a once-subtracted dispersion relation for V_1. Taking the limit $q_0\to i\infty$ of this equation with the dispersion representations for V_1 and V_2 yields the result that $C(p_0)$ is a constant and $D(p_0)$ vanishes. Coupling this result with equation (15) and the assumption that $\sigma_L=0$ gives $C(p_0)=0$.

We emphasize that all three above assumptions are necessary to ensure against q-number Schwinger terms. Our example shows that condition (1) is necessary. It is not difficult to find examples that show the necessity of the other conditions. In addition, it is not sufficient to replace condition (3) with its Bjorken limit.

In conclusion, we have proven by explicit example the compatibility of nonvanishing q-number Schwinger terms in $\delta(x_0)\langle p|[j_\mu(x), j_\nu(0)]|p\rangle$ with vanishing longitudinal virtual photon total cross sections. We also have transverse cross sections constant asymptotically in energy at fixed virtual photon mass. Our example does, however, violate Bjorken scaling for $|\omega|=|2\nu/q^2|$ larger than an arbitrary parameter in the model. This nonscaling behavior is required by our other assumptions.

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Minimally Coupled Spin-Two Field

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The minimally coupled spin-2 wave equation is examined and is shown to lead to the correct number of constraints. The difference between the equation used here and that considered by Velo and Zwanziger is made explicit.

In a series of papers, Velo and Zwanziger[1,2] have given an account of the difficulties encountered when interactions are introduced into Lagrangian fields for higher-spin particles ($S \geq 1$). The main defect which they find is the propagation of signals faster than the speed of light, thus violating special relativity. Their results are in accord with those of Johnson and Sudarshan,[3] whose analysis of the minimally coupled Rarita-Schwinger equation gave an indirect demonstration of the lack of covariance by showing that the equal-time anticommutator between fields is not positive definite in all Lorentz frames. In particular the two approaches give the same threshold value for the magnitude of the magnetic field at which anomalies appear. This is not unexpected since the complete propagator and the anticommutation relations are closely related.

The pathology exhibited by the minimally coupled spin-2 equation considered[2] by Velo and Zwanziger is more serious. When the interaction is switched on, an equation of constraint turns into an equation of motion, thereby increasing the number of independent field components to six, which is obviously inappropriate for describing a spin-2 particle. It is shown here that a slight alteration to their Lagrangian, which retains the minimal nature of the interaction, rids the theory of this defect.

The minimally coupled spin-2 equation used in Ref. 2 is

$$\begin{aligned} L^{\mu\nu} \equiv\; & (\pi^2 - m^2)\psi^{\mu\nu} + m^2 g^{\mu\nu}\psi + \tfrac{1}{2}(\pi^\mu\pi^\nu + \pi^\nu\pi^\mu)\psi \\ & + g^{\mu\nu}\pi_\sigma\pi_\rho\psi^{\sigma\rho} - g^{\mu\nu}\pi^2\psi - \pi_\sigma\pi^\mu\psi^{\sigma\nu} - \pi_\sigma\pi^\nu\psi^{\sigma\mu} \\ =\; & 0\,, \end{aligned} \tag{1}$$

where $\pi_\mu = p_\mu + eA_\mu$, $\psi = \psi_\mu{}^\mu$, and $\psi^{\mu\nu}$ is assumed *a priori* to be symmetric. By contracting $L^{\mu\nu}$ with $g_{\mu\nu}$ and $\pi_\mu\pi_\nu$ and comparing the two resulting equations we get

$$\begin{aligned} \tfrac{3}{2} m^4\psi = ie(& \pi_\mu\pi_\nu F_\sigma{}^\mu\psi^{\sigma\nu} + \pi_\mu F_\nu{}^\mu\pi_\sigma\psi^{\sigma\nu} \\ & + \pi_\mu F_{\nu\sigma}\pi^\nu\psi^{\sigma\mu} - \pi^\mu F_{\nu\mu}\pi^\nu\psi - \tfrac{1}{2}\pi_\mu\pi^\sigma F_\sigma{}^\mu\psi)\,, \end{aligned} \tag{2}$$

where we have used

$$\begin{aligned}[][\pi_\mu, \pi_\nu] &= ie(\partial_\mu A_\nu - \partial_\nu A_\mu) \\ &= ieF_{\mu\nu}\,. \end{aligned}$$

When no interaction is present, the right-hand side of (2) is zero, thus giving the constraint $\psi = 0$. However, (2) is an equation of motion since it involves second-order time derivatives of the wave function in the term

$$(\pi^0)^2 F_i{}^0\psi^{i0}\,.$$

The spin-2 equation we start with here is given by [4,5]

$$\Lambda^{\mu\nu}_{\sigma\rho}\psi^{\sigma\rho} = 0\,, \tag{3}$$

where

$$\begin{aligned} \Lambda^{\mu\nu}_{\sigma\rho} = & -\tfrac{1}{2}(p^2 - m^2)(g^\mu_\sigma g^\nu_\rho + g^\mu_\rho g^\nu_\sigma) \\ & - \tfrac{1}{2} a m^2 (g^\mu_\sigma g^\nu_\rho - g^\mu_\rho g^\nu_\sigma) \\ & + \tfrac{1}{2}(g^\mu_\sigma p^\nu p_\rho + g^\nu_\sigma p^\mu p_\rho + g^\mu_\rho p^\nu p_\sigma + g^\nu_\rho p^\mu p_\sigma) \\ & + \alpha(g^{\mu\nu}p_\sigma p_\rho + p^\mu p^\nu g_{\sigma\rho}) + \beta p^2 g^{\mu\nu} g_{\sigma\rho} \\ & - \gamma m^2 g^{\mu\nu} g_{\sigma\rho}\,, \end{aligned} \tag{4}$$

$\beta = \frac{1}{2}(3\alpha^2 + 2\alpha + 1)$, $\gamma = \alpha + 2\beta$, and $\alpha \neq -\frac{1}{2}$. When $a \neq 0$ the symmetry of $\psi^{\mu\nu}$ may be derived by multiplying (3) by the antisymmetric projector