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QUARK CONFINEMENT *

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ABSTRACT

I review current theoretical evidence for the coexistence of asymptotic freedom and quark confinement in a non-Abelian gauge theory of the strong interaction.

It is rather incongruous to discuss quark confinement at a session on quark searches. Indeed, many theorists find the idea of unconfined quarks so repulsive that they directly reject any evidence to the contrary. This prejudice stems from the observed copious production of all conventional strongly interacting particles. Only by having quarks be very heavy could we understand why present synchrotrons have not provided us with separated quark beams. However, a large quark mass would represent an extravagent new parameter in hadronic physics and would make it difficult to understand the remarkable successes of the naive non-relativistic quark model. It has become easier to imagine exact quark imprisonment rather than an approximate confinement that breaks down at some unknown energy.

An underlying gauge field forms the basis of most theoretical models of confinement. Quarks are coupled to a vector "gluo-electric" field through a generalization of Gauss's law $\vec{\nabla} \cdot \vec{G} = \rho$ where ρ is the quark density. In theories with a non-Abelian symmetry this equation is embelished with internal symmetry indices and extra source terms from the charged gluon field. Confinement in an automatic consequence of Gauss's law if the theory does not have massless gauge bosons in its spectrum. Without a massless field to support a Coulombic spreading the conserved flux of $\vec{G}$ must form itself into "flux tubes" which can only end on the other sources.[1] The only finte energy states are neutral clusters of quarks joined by these tubes with finite energy per unit length. At large separation this yields the conventional linearly rising interquark potential.

The behavior of hypothetical magnetic monopoles in a superconducting medium represents a simple example of this phenomenon. Here $\vec{G}$ represents the magnetic field and can pass through the medium only in the quantized form of "vortices" or "tubes" of magnetic flux which only end on sources carrying monopole charge. Modules of a strong interaction dynamics based on this idea have been proposed, although in complexity they appear somewhat contrived.[2]

The desire for an economical theory has led to an essentially universal enthusiasm for a Yang Mills theory of non-Abelian gauge mesons interacting with the quarks. This elegant generalization of electrodynamics endows the gauge bosons with an internal symmetry, and these "gluons" are charged with respect to each other. The confinement conjecture presumes an inherent instability of a theory of massless gluons. Consequently, a spreading gluonic field will automatically draw itself into the flux tubes necessary for confinement. A semiclassical analysis based on classical solutions to Euclidian

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Yang-Mills equations has suggested a possible mechanism for this flux collapse.[3]

If it occurs in four-dimensional space-time, confinement must be a non-perturbative phenomenon. Indeed, simple renormalization group arugments show that K, the energy per unit length of the flux tube, must exhibit an essential singularity in the bare coupling constant

$$K \sim \frac{1}{a^2} (g_0^2)^{(-\beta_1/\beta_0^2)} \exp\left(-\frac{1}{\beta_0 g_0^2}\right) \tag{1}$$

Here g_0 is the base coupling constant, defined with a cutoff of length a imposed to remove ultraviolet divergences. The numerical constants β_0 and β_1 are the first terms in a perturbative expansion of the Gell-Mann Low function

$$a \frac{\partial}{\partial a} g_0 = \beta_0 g_0^3 + \beta_1 g_0^5 + 0(g_0^7) \tag{2}$$

Equation (1) should be valid in the limit of cutoff removal by taking a to zero. The important consequence of Eq. (1) is the impossibility of any perturbation calculation of K.

A non-perturbation treatment requires an ultraviolet cutoff that is not based on the Feynman expansion. The most extensively studied such regulator is the lattice proposed by Wilson.[4] Here the cutoff parameter a is the lattice spacing, which is to be taken eventually to zero. Before this is done, the path integral defining the quantum theory is formally equivalent to a partition function for a system of variables on a four dimensional crystal. In this analogy the bare coupling constant squared corresponds to temperature. Applying high temperature series techniques to this system, Wilson showed that in the strong coupling limit the theory describes quarks connected by strings with a finite energy per unit length. In other words, confinement is automatic in the lattice theory for strong enough coupling. However, a low temperature expansion at weak coupling reproduces conventional Feynman perturbation theory. This series is at best asymptotic, but its existance suggests a possible low temperature phase of free quarks and massless gluons. As this is a behavior qualitatively distinct from confinement, one expects at least one phase transition separating the high and low temperature domains, if the free quark phase exists. Balian, Drouffe, and Itzzkson have argued that such a confinement-nonconfinement phase transition will occur in large space-time dimensionality.[5]

Ultimately we are interested in the continuum limit of the theory. In the language of solid state physics, this requires taking the bare coupling constant to a critical value so that correlation scales, i.e., physical Compton wavelengths, become large relative to the cutoff represented by the lattice spacing. The perturbative renormalization group indicates one such critical point at vanishing bare coupling. A continuum limit at this point yields the phenomenon of asymptotic freedom; the effective renormalized coupling will go to zero when defined on decreasing length scales. This phenomenon allows perturbative predictions of scaling phenomena in high momentum transfer processes.

To have asymptotic freedom in the same phase that Wilson's expansion demonstrates confinement, four dimensional space time must be inadequate to exhibit the deconfining phase transition mentioned above. Based on an approximate analysis of Migdal and Kadanoff, current lore is that four dimensions are critical for gauge theories.[6] In more than four dimensions all gauge groups should exhibit a spin-wave phase transition whereas for less than four dimensions any continuous gauge group will always confine. In exactly four dimensions only Abelian groups should show a non-trivial phase structure; indeed, this is necessary if Wilson's formalism is to describe quantum electrodynamics, the prototype of all gauge theories.

Recently from two rather different techniques strong evidence has appeared supporting the "standard" picture of the phase structure of lattice gauge theory. The first technique is to extrapolate the strong coupling series into a region where weak coupling predictions should apply.[7] In being able to smoothly join the weak and strong coupling behavior, one obtains evidence for the lack of a phase transition separating these regimes. These methods have been quite successful in identifying the parameters characterizing this matching; in particular they agree to the Λ parameter discussed below.

The other technique supporting the standard picture is Monte Carlo simulation. Considering the path integral as a partition function for a statistical system at a given temperature, a Monte Carlo procedure generates a sequence of configurations which are typical of an ensemble in thermal equilibrium. This is done by making random changes in the gauge fields in such a way that the probability of obtaining any configuration C is proportional to the Boltzmann factor

$$P(c) \sim e^{-\beta S(c)} \qquad (3)$$

where $S(c)$ is the action associated with the gauge field configuration and β is the inverse temperature or inverse coupling squared.

As the entire lattice is stored in the computer memory, one can measure any desired correlation function. One is effectively doing experiments on a four dimensional crystal. For SU(2) gauge theory my crystals have been up to 10^4 sites in size while for SU(3) I have so far been limited to 6^4.

In Figure 1 I show the results of thermal cycles on the internal energy of several of the models.[8] The internal energy P is the expectation value of the action density and is normalized so that at infinite temperature it has value 1 and at zero temperature it vanishes. By slowly increasing the temperature from cold to hot and then reducing it, regions of slow convergence will appear as hysteresis effects. These are hints of phase transitions, where the convergence time should diverge on an infinite lattice. The figure shows SU(2) gauge theory in both four and five dimensions as well as the SO(2) theory of electrodynamics in four dimensions. I include SU(2) in five dimensions to illustrate the criticality of four dimensions. The signals of phase transitions in the four dimensional SO(2) and the five dimensional SU(2) models are clear whereas the four dimensional SU(2) model appears much smoother.

To provide more support for the lack of a transition in the four dimensional non-Abelian case, I have studied the interaction between external sources with quark quantum numbers. This is done by measuring

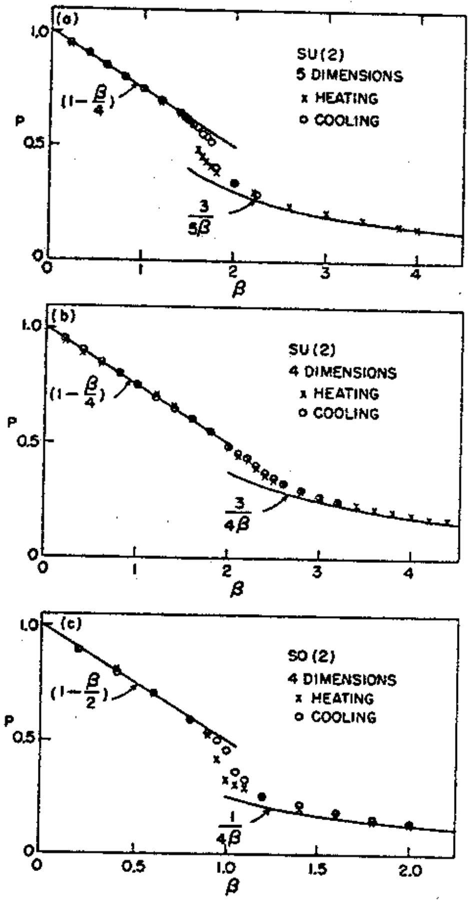

Fig. 1. Thermal cycles on (a) SU(2) gauge theory in 5 dimensions, (b) SU(2) in 4 dimensions, and (c) SO(2) in 4 dimensions. The quantity β is the inverse temperature and P is the internal energy.

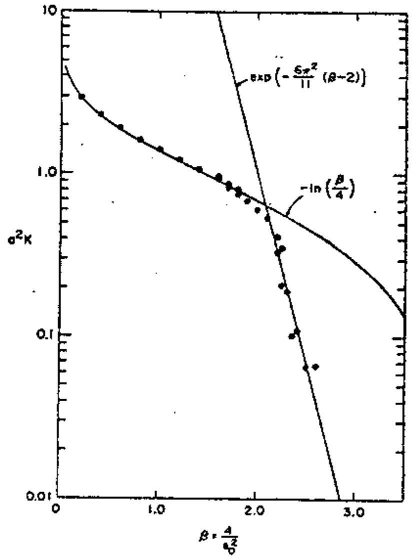

Fig. 2. The cutoff squared times the string tension as a function of β. The solid lines are the strong and weak coupling limits.

expectation values of Wilson loops, exponentials of the gauge field integrated about closed curves in the lattice. From this analysis I have extracted the coefficient K of the linear potential between widely separated quarks.[9] In Figure 2 I plot the measurements of a^2K versus the inverse temperature $\beta = \frac{4}{g_0^2}$. Also plotted are the first term in the strong coupling expansion

$$a^2K = -\log\frac{\beta}{4} + O(\beta^2) \tag{4}$$

and the asymptotic freedom prediction of Eq. (1) with the prefactor neglected and an arbitrarily chosen normalization. If a^2K for large β does indeed follow the asymptotic freedom prediction, then the linear potential will survive the continuum limit and confinement is the consequence.

One previously unknown number follows from this analysis. This is the overall normalization in Eq. (2) and is a parameter relating the short distance asymptotic freedom behavior to the long distance confining potential. Defining the parameter Λ_0 by

$$\Lambda_0 = \lim_{a\to 0} \frac{1}{a} (\beta_0 g_0^2(a))^{(-\beta_1/2\beta_0^2)} \exp\left(\frac{-1}{2\beta_0 g_0^2(a)}\right) \tag{5}$$

I find[10]

$$\Lambda_0 = (1.3 \pm .2) \times 10^{-2} \sqrt{K} \qquad SU(2) \tag{6}$$

$$\Lambda_0 = (5.0 \pm 1.5) \times 10^{-3} \sqrt{K} \qquad SU(3) \tag{7}$$

At first sight these small numbers are surprising; indeed, I argued earlier that large parameters are undesirable in stong interaction physics. However, the value of Λ_0 is strongly dependent on renormalization scheme. Perturbative calculation to one loop order can relate different schemes and Hasenfratz and Hasenfratz have recently related this Λ_0 to a more conventional Λ^{mom} defined by the three point vertex in momentum space and in Feynman gauge.[11] They find

$$\Lambda^{mom} = 57.5 \Lambda_0 \qquad SU(2) \tag{8}$$

$$\Lambda^{mom} = 83.5 \Lambda_0 \qquad SU(3) \tag{9}$$

These large factors largely cancel the small numbers in Eqs. (6) and (7).

Using the string model connection between K and the Regge slope gives $\sqrt{K} \sim 400$ MeV; consequently, for the physical group SU(3) we obtain

$$\Lambda^{mom} = 170 \pm 50 \text{ MeV} \tag{10}$$

This number is phenomenologically encouraging but I do not know what corrections arise from inclusion of virtual light quark loops.

In conclusion, lattice gauge theory has given strong evidence that a non-Abelian gauge theory of quarks and gluons can exhibit an exact confinement of quarks into hadrons. Most theorists regard this as a more aesthetic situation than the possibility of almost but not quite confined constituents of the proton. Perhaps experimentalists will prove this position wrong, but persistance will be needed to persuade us.

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