



FIGURE 1. Lattices abound in the real world. Here is lattice of daisies.

So you want to be a lattice theorist?

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Abstract. For this after dinner talk I intersperse images of real lattices with a discussion of the motivations for lattice gauge theory and some current unresolved issues.

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Although lattices are frequently seen in the real world, as in Figure 1, to the particle theorist they are nothing but a mathematical trick. We constrain quarks so that rather than following arbitrary world lines, they only move in discrete hops between lattice sites. As they hop they get spun around in group space by the gauge fields, which are restricted to the lattice bonds. It is a nice framework for exploring confinement, which is related to this spinning; quarks act like kangaroos, strongly preferring to hop together in mobs.

Since the vacuum is not a crystal, this seems at first sight a rather strange thing to do. However, the lattice has several advantages, primarily in allowing calculations in situations where other methods fail. In particular, one can go far beyond the realms of perturbation theory or semi-classical methods. Furthermore, the predictions can have



FIGURE 2. Lattices can have good flavors. But beware of lurking tastes.

crucial experimental implications. These extend to many areas of particle and nuclear physics, from extracting weak matrix elements in processes involving large hadronic corrections, to understanding the behavior of matter under the extreme conditions of heavy ion collisions, and to detailed studies of hadronic structure.

And of course we get to have fun playing with big computers. Indeed, these themselves are large lattices of processors, such as the six dimensional torus that makes up the QCDOC supercomputer dedicated to lattice gauge theory. There are also more abstract reasons to study lattice gauge theory. As shown in Figure 2, lattices can have good flavors. However one should be careful of any harmful lurking tastes. Lattices are frequently seen in cities, such as the lattice of trees seen in Figure 3.

One of the fun things about lattice gauge theory is the addictive power it gives over the system. Entire lattice configurations are stored in the computer memory, and you are free to measure anything you want. In the process uncertainties can arise, and the theorist is in the unusual situation of having error bars. First of all, since we are using Monte Carlo methods, there will be statistical errors. These can be reduced by massive applications of computer time. There are also several sources of systematic error, some of which we have control over. These include finite volume and finite lattice spacing corrections, which can also be reduced by increased computer time. In practice using quarks with physical masses is quite computer intensive; so, we usually simulate with heavier than normal quarks and then do an extrapolation.

There are also some sources of error that are basically uncontrolled. One is the so-called “valence” or “quenched” approximation, wherein the feedback of internal quark loops is ignored. This is a tempting approximation since it saves a couple of orders of magnitude of computer time. But fortunately the continuing growth in computer power is now alleviating the need for this inexact approach.

Another uncontrolled source of error comes from extrapolations in the number of quark flavors. Again to save computer time, it is popular, mainly in the US, to start with a fermion formulation that has some of the naive doubling issues remaining and then do



FIGURE 3. A lattice of trees surrounded by other lattices in New York City.

an extrapolation down to the desired number of quark species. This is done by replacing the fermion determinant by a non integer power. Since the starting determinant is not a power, this procedure has not been theoretically justified. Indeed, it explicitly gives incorrect behavior in the chiral limit of small masses. I will return to this issue later.

Sometimes the lattice can reveal rather subtle issues. In particular, for many years the way chiral symmetry worked on the lattice was puzzling. We know chiral symmetry is important to the lightness of the pion, which is theoretically tied to the lightness of the up and down quarks. The lattice removes all infinities, and thus issues such as anomalies coming from divergences can be tricky. Ignoring these anomalies forces the theory to cancel them with extra species, known as doublers. But recent years have seen the development of elegant approaches that solved these problems. One tack considers our four dimensional world as an interface in five dimensions [1, 2]. An alternative extracts the essence of this interface into the slightly non-local overlap operator [3]. This satisfies an elegant modification of naive chiral symmetry. So, as indicated in Figure 4, the lattice and chiral symmetry now get along nicely.

Despite these advances, there remain some subtle unsolved problems in lattice gauge theory. One of these involves the standard model, where the weak gauge fields are coupled in a parity violating manner. Neutrinos are experimentally known to spin only to the left, but all known lattice formulations also bring in right handed partners. For example, with domain wall fermions there is naturally present an anti-wall which couples with equal strength to the gauge fields. Ad hoc Higgs fields can give the mirror particles a different mass, but they are always there. To the extent that the lattice is a technique to define a field theory, this raises worries that the usual standard model might be incomplete or even not well defined.

The other major unsolved problem involves the properties of matter at high baryon density. Here there are no practical known algorithms for simulations. Monte Carlo methods fail because there is no positive measure for the path integral. All existing attempts to circumvent this issue require computer time growing exponentially with the



FIGURE 4. After recent advances, the lattice now embraces chiral symmetry.

system size. This is particularly frustrating in light of the rich phase diagram expected at high density, filled with exotic phenomena such as color superconductivity.

There are some lattice topics which are highly controversial. I will illustrate the issue starting from a conventional continuum discussion of how chiral symmetry works in three flavor QCD. Here a longstanding tool comes from effective chiral Lagrangians. The physics of the light pseudoscalars is nicely modeled in terms of an effective field Σ which lies in the group $SU(3)$. Incorporating quark masses into this picture involves a potential of the form $V(\Sigma) = -\text{Tr } M\Sigma$, where the mass matrix is

$$M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \quad (1)$$

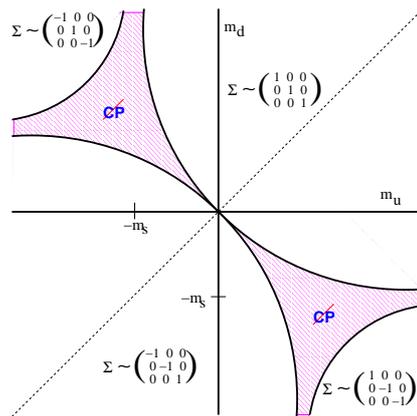


FIGURE 5. The phase structure expected for three flavor QCD as the up and down quark masses are varied at fixed strange quark mass. Spontaneous CP violation occurs in regions where the up and down quark masses differ in sign. No structure appears when just a single quark mass vanishes.



FIGURE 6. Controversial ideas came to the front at Lattice 2006.

As we vary the quark masses, minimizing this potential predicts a rich phase structure [4], sketched in Figure 5. Indeed, I discussed this structure at length during the previous meeting in this series [5]. Striking features are the regions of spontaneous CP violation where the minima of the potential are doubly degenerate at complex values of Σ .

An important feature of this diagram is the absence of any special features when only a single quark mass vanishes. The presence of the other quark masses is sufficient to stabilize the vacuum value for $\langle \Sigma \rangle$, which is real and not accompanied by any exact massless modes. This is a consequence of the anomaly at work; massless Goldstone particles require more than one quark mass to vanish at the same time.

The controversy concerns a numerical algorithm that is incapable of seeing this structure. The feelings here are rather strong, as shown in Figure 6 from Lattice 2006. The “staggered cabal” promotes using a technique known as “rooted staggered quarks.” This is the procedure mentioned above of starting with extra particles and taking a fractional power of the fermion determinant. The issue that arises is that the starting staggered formulation has an exact chiral symmetry when any single quark mass vanishes. This symmetry survives the rooting process, and demands the existence of a massless Goldstone mode where the simple effective chiral Lagrangian says there is none. Indeed, this is in direct contradiction with known anomalies [6].

The condoners of this algorithm [7] suggest, without proof, that these evils will drop away in the continuum limit as long as one avoids the zero quark mass axes in Figure 5. They argue that there is actually a plethora of extra particles, one of which is this unwanted Goldstone mode, but their total contribution cancels as the continuum limit is taken. For three flavors using independent rooted staggered quarks, there are 144 pseudoscalar bosons, out of which only the usual 9 should survive the continuum limit. This requires a loss of unitarity so that the total cross sections to produce some of these extra particles can be negative. Also the extra massless particle induces long range forces that make the algorithm non-local. And all of these unproven conjectures are being made just to save some computer time over other algorithms, such as Wilson, domain wall,



FIGURE 7. A lattice of palapas at the 2004 meeting in this series, held in Villasimius, Sardinia.

or overlap fermions, that do not so severely mutilate the qualitative chiral behavior, I conclude that rooting can be unhealthy, although the extreme contortions being tried to rescue the approach might be amusing enough to warrant a movie.

I conclude with one final reason one might want to be a lattice theorist. We often meet in very nice places to search out new lattices, such as the marble/basalt arrays here in the Azores or the environment shown in Figure 7 from the 2004 meeting in this series. And of course, as you will see tomorrow night, this meeting has a strong tradition of taking poster sessions seriously!

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