Spontaneous CP violation and quark mass ambiguities

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Two entwined topics

- For what quark masses is CP spontaneously broken?
- $m_u = 0$ is not a physically meaningful concept.

M.C., PRL 92:201601 (2004) and PRL 92:162003 (2004)

Assumptions

- QCD exists and confines
- Effective chiral Lagrangians are qualitatively correct

Based on old ideas

- Dashen (1971)
- Di Vecchia and Veneziano (1980)
- Georgi and McArthur (1981)
- Kaplan and Manohar (1986)
- Banks, Nir and Seiberg (1994)
- MC (1995)

The effective meson theory $\Sigma(x) = e^{i\pi\cdot\lambda/f_\pi} \in$ SU(3)

- three flavors: up, down, strange
- λ_α: 8 generators of SU(3)
- π_α: pseudoscalar octet fields
- $\Sigma_{ij} \longleftrightarrow \overline{\psi}_{Li}\psi_{Rj}$ i, j flavor indices

Chiral symmetry $\Sigma \rightarrow g_L^\dagger \, \Sigma \, g_R$

- spontaneously broken $\langle\Sigma\rangle \sim \langle\overline{\psi}_L\psi_R\rangle \neq 0$

Chiral symmetry also explicitly broken by quark masses

$$L = \frac{f_\pi^2}{4}\mathrm{Tr}(\partial_\mu \Sigma^\dagger \partial_\mu \Sigma) - v \ \mathrm{Re}\ \mathrm{Tr}(\Sigma M)$$

$$M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

Expand to quadratic order in meson fields

- diagonalize to find meson masses $\quad m_{\pi^\pm}^2 \sim m_u + m_d$

Isospin violating $m_d - m_u$ mixes π^0 and η

$$m_{\pi^0}^2 \sim$$
$$\frac{2}{3}\left(m_u + m_d + m_s - \sqrt{m_u^2 + m_d^2 + m_s^2 - m_u m_d - m_u m_s - m_d m_s}\right)$$

$$m_\eta^2 \sim$$
$$\frac{2}{3}\left(m_u + m_d + m_s + \sqrt{m_u^2 + m_d^2 + m_s^2 - m_u m_d - m_u m_s - m_d m_s}\right)$$

$m_{\pi_0}^2$ can vanish

- requires a negative quark mass $m_u = \frac{-m_s m_d}{m_s + m_d}$

Negative quark masses do unusual things

- anomaly makes sign of mass significant

Usual case:

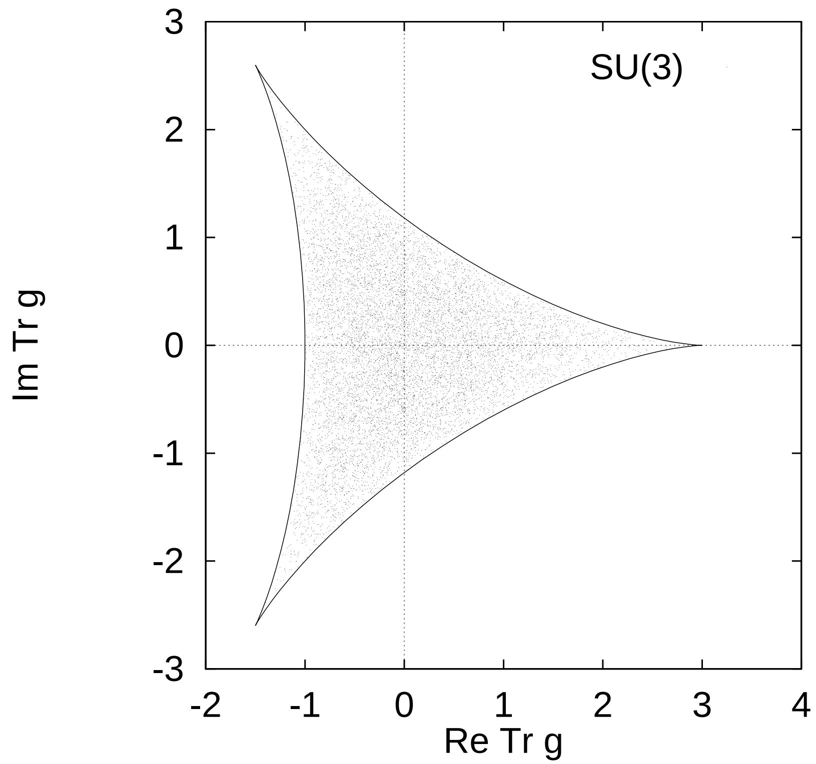
- vacuum at maximum of $\mathrm{ReTr}\Sigma$
- occurs at $\Sigma = I$

Negative degenerate masses:

- vacuum at minimum of $\mathrm{ReTr}\Sigma$
- $-I$ NOT in $SU(3)$
- two solutions: $\Sigma = \exp(\pm 2\pi i/3)$

CP: $\Sigma \to \Sigma^*$

- spontaneously broken



With negative quark masses $m_{\pi^0}^2$ can go negative

$$\frac{2}{3}\left(m_u + m_d + m_s - \sqrt{m_u^2 + m_d^2 + m_s^2 - m_u m_d - m_u m_s - m_d m_s}\right)$$

Vanishes at

$$m_u = \frac{-m_s m_d}{m_s + m_d}$$

- boundary for pion condensed phase $\langle \pi^0 \rangle \neq 0$
- ‘‘accidental Goldstone boson’’

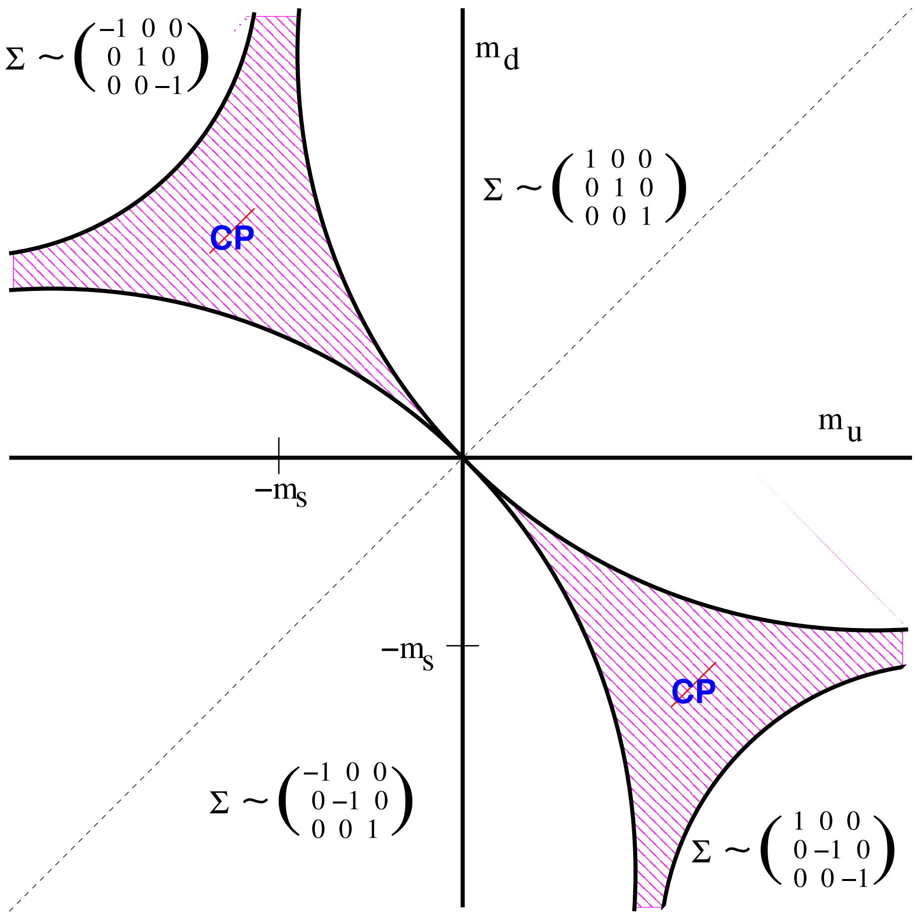
New vacuum state

$$\Sigma = \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{-i\phi_1 - i\phi_2} \end{pmatrix}$$

$$m_u \sin(\phi_1) = m_d \sin(\phi_2) = -m_s \sin(\phi_1 + \phi_2)$$

- second order transition at $m_{\pi^0} = 0$
- two degenerate vacua related by $\phi_i \leftrightarrow -\phi_i$
- CP violation appears in three-pseudoscalar couplings

(m_u, m_d) plane at fixed m_s:



Boundaries at

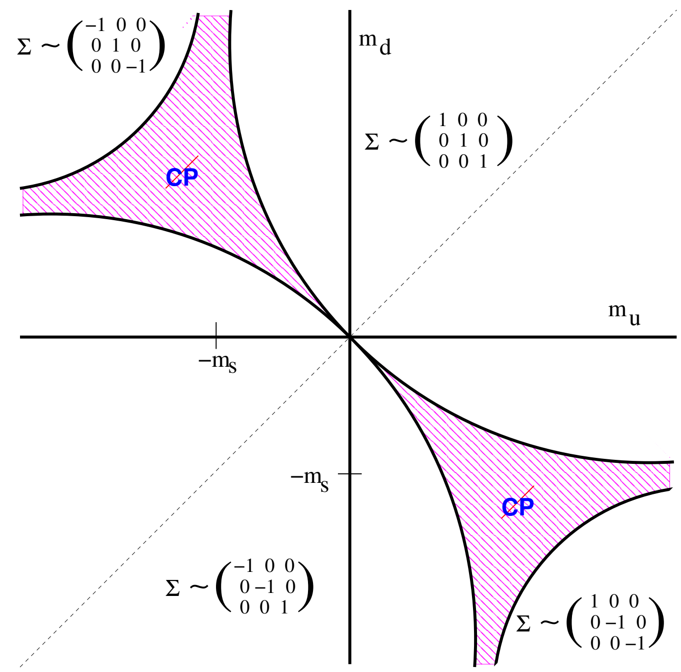
$$m_u = \frac{-m_s m_d}{\pm m_s \pm m_d}$$

Vafa and Witten: No spontaneous $\not{P}$ in the strong interactions?

- assumes fermion determinant positive
- not true for negative quark masses

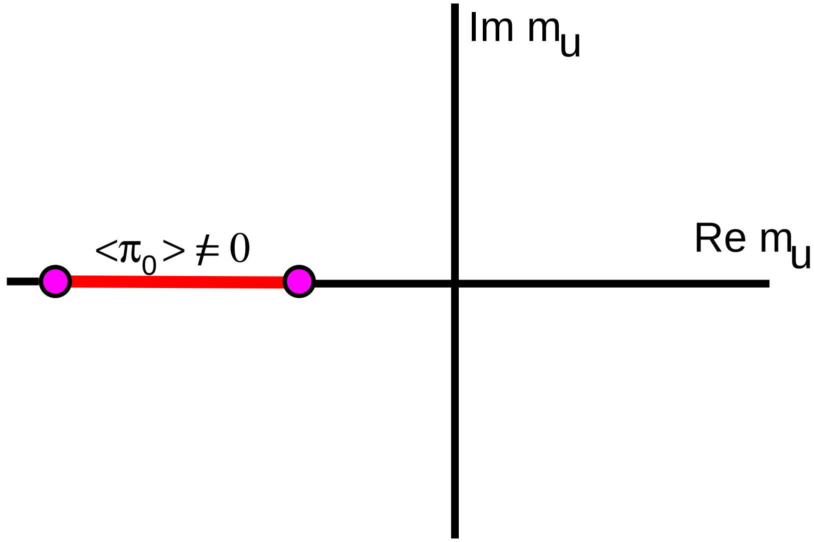
Non perturbative

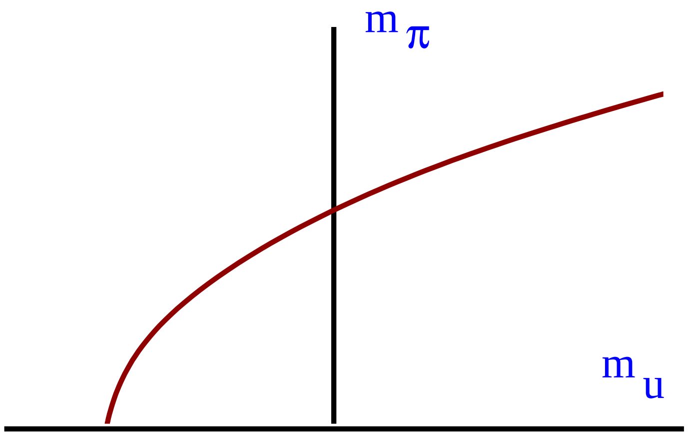
- sign of quark masses significant
- negative $|M|$ corresponds to $\theta = \pi$



Hold heavier quark masses m_s and m_d fixed

- look at complex m_u plane





Nothing significant occurs at $m_u = 0$ when $m_d \neq 0$

- First order transition along negative $\mathrm{Re}\ m$ axis
- second order critical point at non-zero $\mathrm{Re}\ m < 0$
- spontaneous breaking of CP, order parameter: $\langle \pi_0 \rangle$
- Di Vecchia and Veneziano (1980)

Does $m_u = 0$ have any physical significance?

Concept of an ‘‘underlying basic Lagrangian’’ does not exist

- must regulate divergences
- only underlying symmetries significant
- a single massless quark gives no special symmetry
    - anomaly: no exact Goldstone bosons at $m_u = 0$

Define parameters using renormalization group

- Continuum theory is a limit
    - bare parameters: coupling g and quark masses m_i
    - renormalize to zero in continuum limit

Renormalization group equations

- $a = 1/\Gamma$ cutoff $\leftrightarrow$ physical scale $1/\mu$

$$a\frac{d}{da}g = \beta(g) = \beta_0 g^3 + \beta_1 g^5 + \ldots + \text{non-perturbative}$$
$$a\frac{d}{da}m = m\gamma(g) = m(\gamma_0 g^2 + \gamma_1 g^4 + \ldots) + \text{non-perturbative}$$

$\beta_0,\ \beta_1,\ \gamma_0$ scheme independent

$$\begin{array}{llll} \beta_0 = & \frac{11-2n_f/3}{(4\pi)^2} & = .0654365977 & (n_f = 1) \\ \beta_1 = & \frac{102-12n_f}{(4\pi)^4} & = .0036091343 & (n_f = 1) \\ \gamma_0 = & \frac{8}{(4\pi)^2} & = .0506605918 & \end{array}$$

“Non-perturbative” parts

- fall faster than any power of g as $g \to 0$
- not proportional to quark mass

Solution

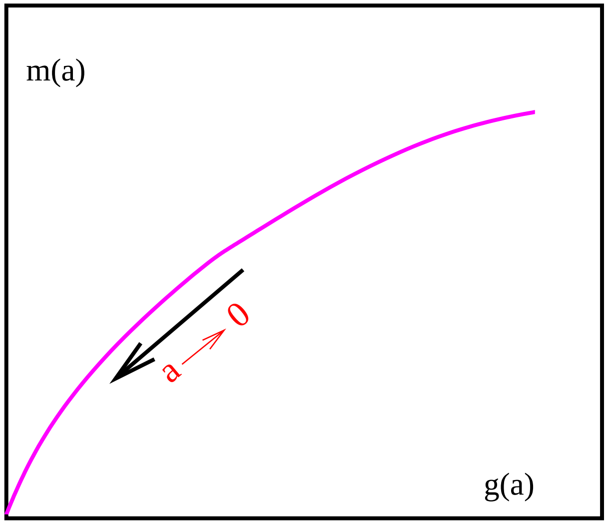
$$a = \frac{1}{\Lambda} e^{-1/2\beta_0 g^2} g^{-\beta_1/\beta_0^2} (1 + O(g^2))$$
$$m = M g^{\gamma_0/\beta_0} (1 + O(g^2))$$

Continuum limit $a \to 0$

$$g^2 \sim \frac{1}{\log(1/\Lambda a)} \to 0 \qquad \text{“asymptotic freedom”}$$
$$m \sim M \left(\frac{1}{\log(1/\Lambda a)} \right)^{\gamma_0/\beta_0} \to 0$$

$\Lambda,\ M$: "integration constants"

- Λ: "QCD scale"
- M: "renormalized quark mass"



$$\Lambda = \lim_{a\to 0} \frac{e^{-1/2\beta_0 g^2} g^{-\beta_1/\beta_0^2}}{a}$$

$$M = \lim_{a\to 0}\ m g^{-\gamma_0/\beta_0}$$

Numerical values of $\Lambda,\ M$ depend on scheme

Defining $\beta(g),\ \gamma(g)$

- fix physical quantities
- adjust bare parameters as the cutoff is removed

Use particle masses $m_i(g, m, a)$ as physical

$$a\frac{dm_i(g,m,a)}{da} = 0 = \frac{\partial m_i}{\partial g}\beta(g) + \frac{\partial m_i}{\partial m}m\gamma(g) + a\frac{\partial m_i}{\partial a}$$

Work with degenerate quarks for simplicity

- Two bare parameters $(g, m) \Rightarrow$ fix two masses
- m_p: lightest baryon
- m_π: lightest boson

$$\beta(g) = \frac{a\frac{\partial m_\pi}{\partial a}\frac{\partial m_p}{\partial m} - a\frac{\partial m_p}{\partial a}\frac{\partial m_\pi}{\partial m}}{\frac{\partial m_p}{\partial g}\frac{\partial m_\pi}{\partial m} - \frac{\partial m_\pi}{\partial g}\frac{\partial m_p}{\partial m}}$$

$$\gamma(g) = \frac{a\frac{\partial m_\pi}{\partial a}\frac{\partial m_p}{\partial g} - a\frac{\partial m_p}{\partial a}\frac{\partial m_\pi}{\partial g}}{\frac{\partial m_p}{\partial m}\frac{\partial m_\pi}{\partial g} - \frac{\partial m_\pi}{\partial m}\frac{\partial m_p}{\partial g}}$$

- includes all perturbative and non-perturbative effects
- no gauge fixing required

Physical masses map onto the integration constants

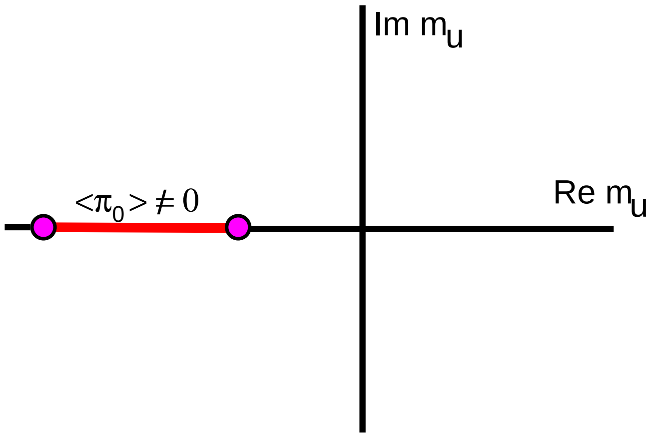
- $\Lambda = \Lambda(m_p, m_\pi) \qquad M = M(m_p, m_\pi)$
- inverting $\longrightarrow m_i = m_i(\Lambda, M)$
- dimensional analysis: $\qquad m_i = \Lambda h_i(M/\Lambda)$

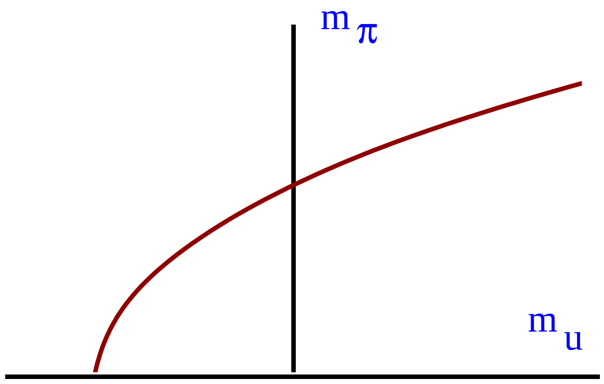
Multi-flavor theory

- expect Goldstone bosons
- $m_\pi^2 \sim m_q$
- square root singularity $\qquad h_\pi(x) \sim x^{1/2}$
- removes any additive ambiguity in defining M

The one flavor theory $m_\pi = \Lambda h_\pi(M/\Lambda)$

- no chiral symmetry
- no Goldstone bosons
- $m_\pi = 0$ occurs at negative quark mass
- $h_\pi(x)$ smooth, non-vanishing at $x = 0$

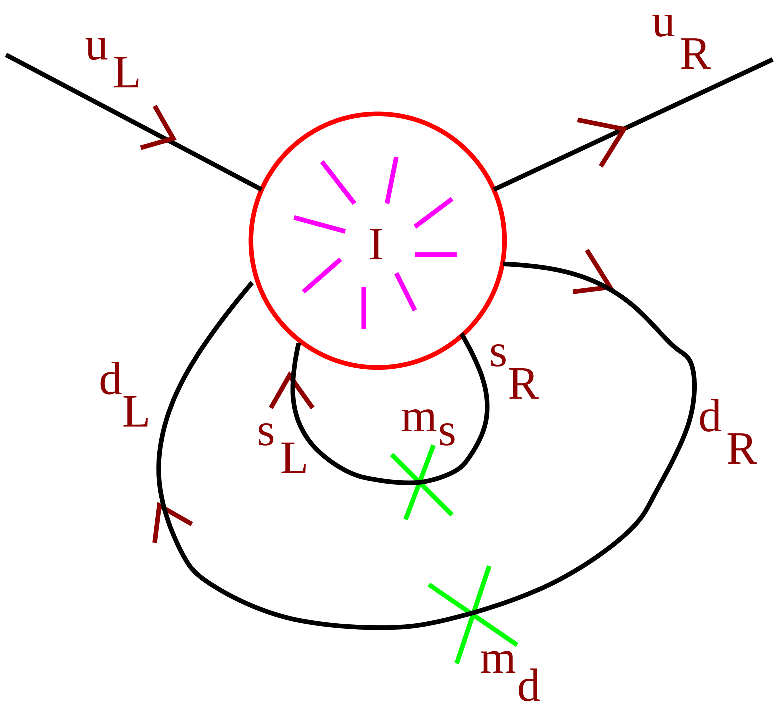




What does m=0 mean?

Non-perturbative contributions to mass flow

- not proportional to quark mass



- “instantons” flip all quark spins
- $\Delta m_u \sim \frac{m_d m_s}{\Lambda_{\rm qcd}}, \quad \Lambda_{\rm qcd}$

Eigenvalues and Topology

$$\langle\overline{\psi}\psi\rangle = \frac{1}{Z}\int(dA)\ |D|^{N_f}\ e^{-S_g(A)}\ \mathrm{Tr}D^{-1}$$

- diagonalize D in given gauge field
  - $|D| = \prod \lambda_i$
  - $\mathrm{Tr}D^{-1} = \sum \frac{1}{\lambda_i}$

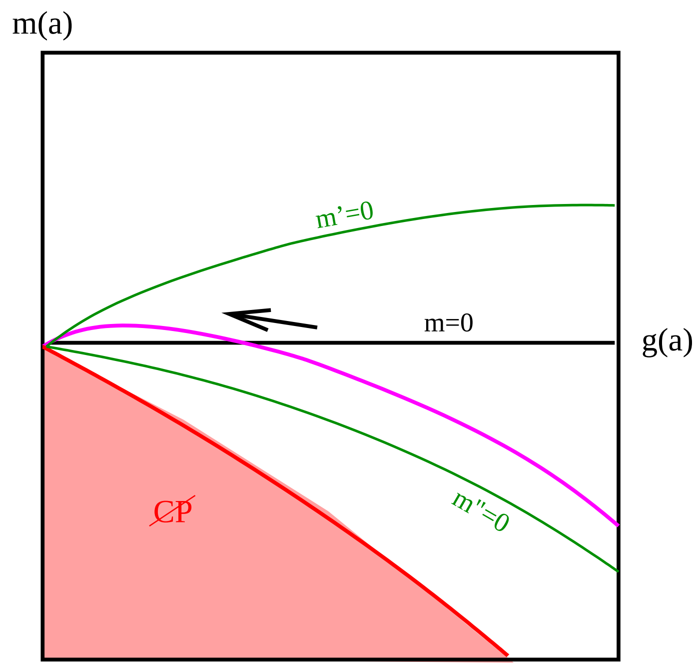
With topology, massless D has exact zero eigenvalues

- add a small mass: $\lambda_0 \sim m$
- instantons suppressed: $|D| \sim m^{N_f}$

Consider $N_f = 1$; winding number 1

- $|D| \sim m$
- $\mathrm{Tr}D^{-1} \sim \frac{1}{m} + \ldots$
  - mass factors cancel: " 't Hooft vertex"
  - constant effective mass shift

$M_u = 0$ is NOT renormalization group invariant



IF $m(a)$ passes through zero

- existence and location of zero is scheme and scale dependent
- NOT fundamental

Matching between schemes

Preserve lowest order perturbative limit as $g \to 0$ at fixed scale a

$$\tilde{g} = g + O(g^3) + \text{non-perturbative}$$

$$\tilde{m} = m(1 + O(g^2)) + \text{non-perturbative}$$

- "non-perturbative" vanishes faster than any power of g

Fixed a not the continuum limit

- $g \to 0$ at fixed a: perturbation theory on free quarks
- $a \to 0$ at fixed g: diverges
- $a, g \to 0$ on RG trajectory: confinement

Example new scheme:

- $\tilde{a} = a$
- $\tilde{g} = g$
- $\tilde{m} = m - Mg^{\gamma_0/\beta_0} \times \frac{e^{-1/2\beta_0 g^2} g^{-\beta_1/\beta_0^2}}{\Lambda a}$

Non-perturbative redefinition of parameters makes

$$\tilde{M} \equiv \lim_{a \to 0} \tilde{m}\tilde{g}^{-\gamma_0/\beta_0} = M - M = 0$$

A scheme always exists where the renormalized quark mass vanishes!

Baryon and η' masses don't vanish at $m_q = 0$

- mass contributions of form $\frac{e^{-1/2\beta_0 g^2} g^{-\beta_1/\beta_0^2}}{a}$
- must enter RG functions

Non-vanishing θ

Three bare parameters

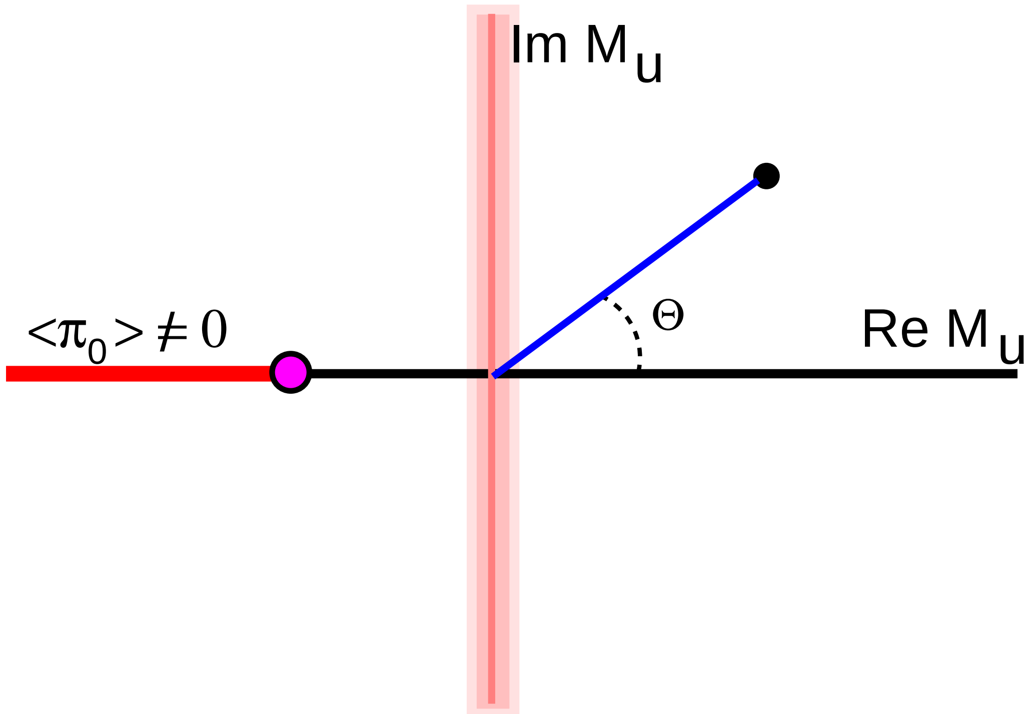
- $g \qquad \mathrm{Re}\ m_u \qquad \mathrm{Im}\ m_u$
- explicit CP violation if $\mathrm{Im}\ m_u \neq 0$

Need to fix three physical parameters

- $m_p,\ m_\pi$
- neutron electric dipole moment

Three integration constants

- $\Lambda = \lim_{a\to 0}\ \frac{e^{-1/2\beta_0 g^2} g^{-\beta_1/\beta_0^2}}{a}$
- $\mathrm{Re}\ M = \lim_{a\to 0}\ g^{\gamma_0/\beta_0}\ \mathrm{Re}\ m$
- $\mathrm{Im}\ M = \lim_{a\to 0}\ g^{\gamma_0/\beta_0}\ \mathrm{Im}\ m$



Conventional variables

- Λ
- $|M|$
- $\theta : \qquad \tan(\theta) = \frac{\mathrm{Im} M}{\mathrm{Re} M}$

With one flavor these are singular coordinates

- scheme dependent additive shift in $\mathrm{Re}\ M$ changes θ

$\mathrm{Re}\ M$ and $\mathrm{Im}\ M$ are better independent variables

- CP symmetry protects the real axis
- imaginary axis can shift

On the lattice

Renormalization flows depend on details of lattice action

- Wilson -- Staggered -- Domain wall -- Overlap

Overlap not unique

- depends on Dirac operator being projected
- starting with Wilson: input negative mass is adjustable

The one flavor theory dynamically generates a gap

- appears in the spectrum of the Dirac operator
- size of gap not protected by the overlap projection

Can $M = 0$ be preserved between schemes?

- not guaranteed by the Ginsparg-Wilson condition

Topological Susceptibility

With a GW action:

- massless quark synonymous with zero topological susceptibility

Is topological susceptibility uniquely defined for $N_f < 2$?

- Giusti, Rossi, Testa; Luscher: no perturbative infinities

Admissibility condition

- forbid plaquettes further than a finite distance δ from the origin
- removes “rough” gauge fields
- gives a unique winding number

Theorem: MC, PR D70:091501 (2004)

- admissibility incompatible with reflection positivity
- proof an extension of Grosse and Kuhnelt, 1982

CONCLUSIONS

Strong interactions can spontaneously violate CP

- large regions of parameter space
- negative quark masses

$m_u = 0$ is not a meaningful concept

- not a solution to the strong CP problem
- non-perturbative
- topological susceptibility not uniquely defined for $N_f < 2$

Available simulation algorithms cannot explore this physics

- sign problem
- the "square root trick" fails